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Adaptive Sliding Mode Based Stabilization Control for the Class of Underactuated Mechanical Systems

FAZAL UR REHMAN[1], NADIR MEHMOOD[1], SAMI UD DIN[2], (Member, IEEE), MUHAMMAD RAFIQ MUFTI[3], AND HUMAIRA AFZAL[4]

[1]Department of Electrical Engineering, Capital University of Science and Technology, Islamabad 44000, Pakistan
[2]Department of Electrical Engineering, Namal Institute Mianwali, Mianwali 42250, Pakistan
[3]Department of Computer Science, COMSATS University Islamabad, Vehari Campus, Vehari 61100, Pakistan
[4]Department of Computer Science, Bahauddin Zakariya University, Multan 60800, Pakistan

Corresponding author: Sami Ud Din (samiuddin@ieee.org)

ABSTRACT This article presents a simple stabilizing control algorithm for a class of underactuated mechanical systems for two degrees of freedom (2DoF). In this respect, the adaptive sliding mode based strategy is proposed for the considered class. The controller, along with the adapted laws, is decided in such a way that the time derivative of a Lyapunov function grows negative. The proposed control technique has been applied to three classic benchmark 2DOF systems, the acrobot, pendoubt, and the cart-pole system. The effectiveness of the proposed strategy is proved in the light of simulation results for all said systems considered as an illustrative example.

INDEX TERMS Feedback stabilization, underactuated systems, non-holonomic systems, adaptive sliding mode control, Lyapunov function.

I. INTRODUCTION

Underactuated mechanical systems (UMS) are the systems having a fewer number of independent control actuators than the degrees of freedom (DoF) to be controlled. These systems have many practical and diverse applications in modern science and engineering. The broad application areas of underactuated systems include robotics, industry, and aerospace systems, to name a few. Apart from practical applications, these systems have been of great importance and interest in research as prototype systems for nonlinear complex systems. The reasons for underactuation include i) dynamics of the system (like spacecraft's, aerial and underwater vehicles), ii) by design (for cost and weight reduction), iii) actuator failure, iv) imposed artificially (to get the insight of high order nonlinear underactuated systems) [1]: Examples of UMS include systems designed for research like the acrobot, the pendubot, the rotating pendulum, the cart-pole system, the beam-and-ball system, the TORA system, the inertia-wheel pendulum, and many other kinds of pendulums. Examples of robotics include flexible-link joints, mobile robots, and many other kinds of manipulators, from aerospace aircraft, spacecraft, helicopters, satellites, marine vehicles, ships, and underwater and surface vehicles [2].

Control of UMS has been a challenging research problem and an active area of research for the last two decades due to highly complex and nonlinear behavior and fewer control inputs compared to DoF to be controlled. There are several control design methods for fully actuated mechanical systems because they possess some important properties like passivity and feedback linearization [3], which are helpful in the design of controllers for such systems. These methods include partial feedback linearization collocated [1], and non-collocated [4], passivity [5], adaptive and robust control [6]–[9], Sliding Mode Control (SMC) [10]–[13], fuzzy logic [14], and back-stepping [15]. The bottleneck associated with the aforementioned approaches is they are not directly applicable to the underactuated mechanical systems. Due to the non-holonomic nature, these systems are not completely feedback linearizable [1]; also, these are not locally controllable at their point of equilibrium [3]. In addition, these systems do not satisfy Brockett's necessary condition [1] (for more detail, see [1]).

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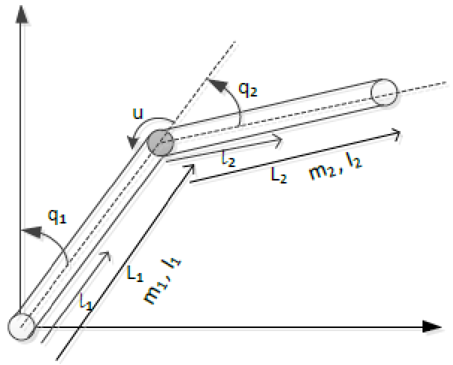

FIGURE 1. The Acrobot.

As the research in the area of UMS is not fully matured yet, therefore, challenges regarding UMS is basically categorized into two streams, i.e., theoretical challenges and practical challenges. Considering the perspective of theoretical challenge, the control research community is facing the issue of controllability and stabilization of UMS [1]. On the other hand, from the perspective of practical challenges, the control community is facing the following three aspects; i) network operated UMS, ii) DoF in complex UMS, and iii) fault detection and control [1], [2]. Among all aforementioned control approaches, sliding mode control got prime importance due to its robustness and capability to cater unknown internal and external disturbances in the presence of model uncertainties [16].

This paper focuses towards stabilizing control of 2DoF underactuated mechanical systems aiming to resolve one of the control community's theoretical challenges. In this regard, Xu [17] have proposed a global stabilization technique based on SMC. Moreover, Olfati-Saber [18] has also done remarkable work regarding the stabilization of the considered class. Integral SMC based solution is also proposed in [6], and comparative analysis with higher-order sliding modes is also posed in [10]. Increasing the order of the sliding surface makes the controller more complex, and it will further increase the computational cost. However, the proposed solution discussed in [10]–[12], [16], [17] does not cover the entire class of UMS. Different models are posed for the different classes of UMS [1], [2].

In this paper, the proposed method is based on the adaptive sliding mode technique, with a primary objective is to steer the system to any desired state from any arbitrary initial state. Moreover, computer simulation for illustrative examples shows the effectiveness of the proposed control algorithm.

Following are the main contributions regarding this paper:

- The proposed algorithm is kept generic and can be applicable to the class of underactuated mechanical systems having 2DoF.
- The proposed control algorithm has been applied to three systems (the acrobot, pendoubt, and the cart-pole system) considered as illustrative examples for the considered class.
- The simulation results prove the effectiveness and efficiency of the proposed control strategy.

The rest of the article is organized as follows. Section 2 presents the dynamical model of underactuated mechanical systems with n-DoF and 2DoF, respectively. Section 3 presents a description of the control problem, along with some insight of the proposed control algorithm. Illustrative examples and simulation results are listed in section 4, whereas section 6 concludes the paper.

II. DYNAMICAL MODEL OF UNDERACTUATED MECHANICAL SYSTEMS

A. GENERAL N DEGREE OF FREEDOM (N DoF) UNDERACTUATED MECHANICAL SYSTEMS

Euler-Lagrange equation of motion for the underactuated mechanical system of order n is given as follows [1]:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = F(q)\,u \tag{1}$$

where $\mathcal{L}$ is the Lagrangian of the system, wheres (2) represents the difference in kinetic K and potential energy V, as follows:

$$\mathcal{L}(q, \dot{q}) = K - V = \frac{1}{2}\dot{q}^T M(q)\,\dot{q} - V(q) \tag{2}$$

In (1), $q \in \Re^n$ displays the configuration vector, $u \in \Re^m$ represents control input and $F(q) = [f_1(q), \ldots f_m(q)]^T$ is the matrix of external forces. The dynamics of equation (1) conceivably modified as:

$$M(q)\,\ddot{q} + C(q, \dot{q})\,\dot{q} + G(q) = F(q)\,u \tag{3}$$

where $M(q)$ represents the inertia matrix, and the gravitational term is displayed as $G(q)$. Coriolis and centrifugal terms are posed as $C(q, \dot{q})\,\dot{q}$. The system is called as fully actuated if $m = rank\,(F) = n$ however, if $m < n$, the system is considered as underactuated. For the general case $F(q) = [F_1(q), F_2(q)]^T$ and partitioning $q = [q_1, q_2]^T$ according to $F(q)$, where $q_1 \in \Re^{n-m}$ and $q_2 \in \Re^m$, dynamics (3) can be written as [3], [5]:

$$\begin{cases} m_{11}\ddot{q}_1 + m_{12}\ddot{q}_2 + c_1 + g_1 = F_1(q)\,u \\ m_{21}\ddot{q}_1 + m_{22}\ddot{q}_2 + c_2 + g_2 = F_2(q)\,u \end{cases} \tag{4}$$

where $M(q) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$ is inertia matrix, $c_1(q, \dot{q}) \in \Re^{n-m}$ and $c_2(q, \dot{q}) \in \Re^m$ are the centrifugal and coriolis terms, $g_1(q) \in \Re^{n-m}$ and $g_2(q) \in \Re^m$ represents gravitational terms, and control inputs are presented as $u \in \Re^m$, produced by m actuators.

B. TWO DEGREES OF FREEDOM (2DOF) UNDERACTUATED MECHANICAL SYSTEMS

For 2DOF underactuated mechanical system, $n = 2$ and $m = 1$, so in the system (4), $q \in \Re^2, u \in \Re^1$, $q_1 \in \Re^1$, and

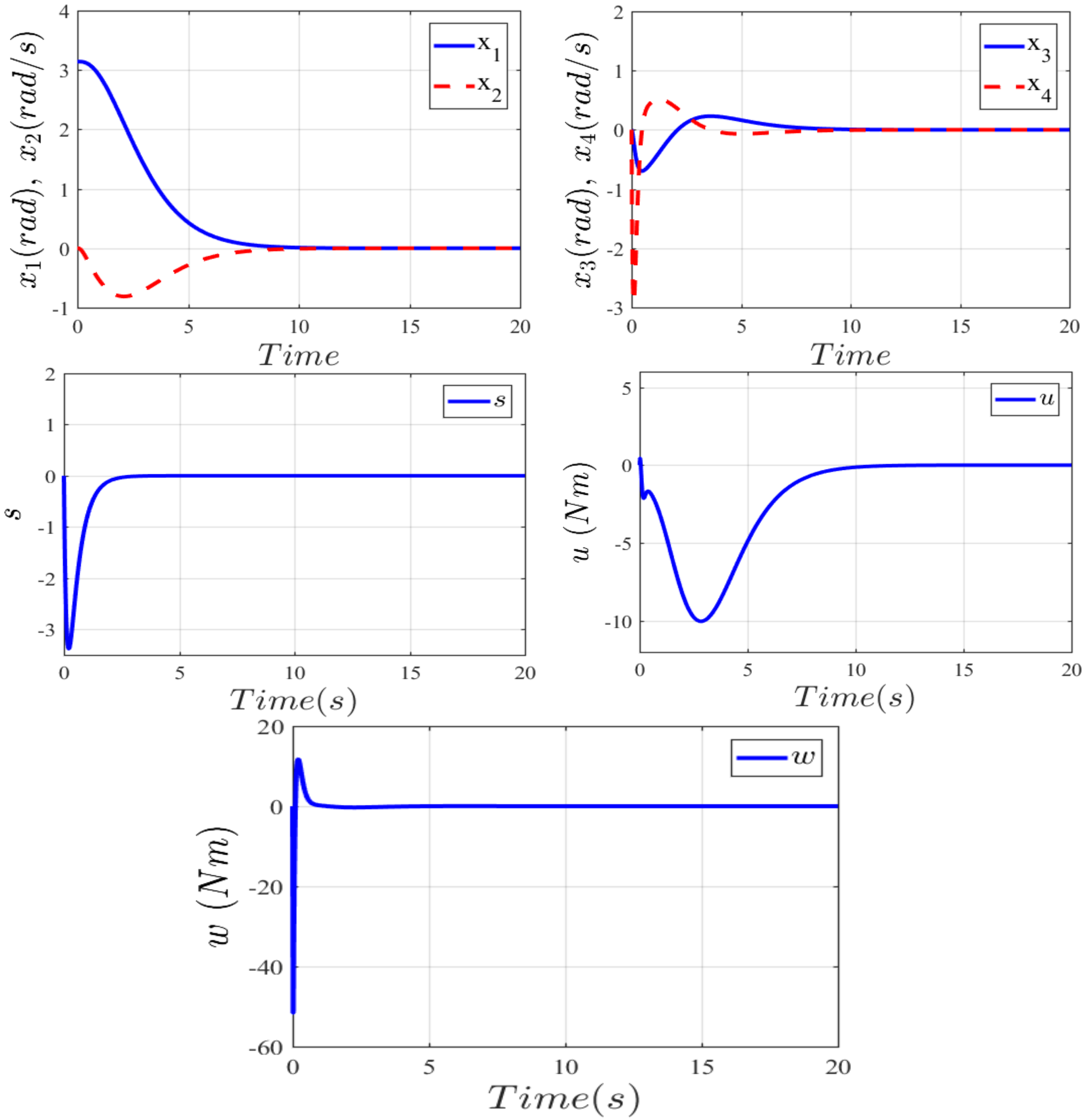

FIGURE 2. Convergence of system's states considering initial condition $(x_1(0), \ldots, x_4(0)) = (\pi, 0, 0, 0)$. Time response of sliding surface s and control effort.

$q_2 \in \mathfrak{R}^1$. Depending upon whether $F_1(q) = 0$ or $F_2(q) = 0$, system (4) takes one of the following two forms:

$$\begin{cases} m_{11}\ddot{q}_1 + m_{12}\ddot{q}_2 + c_1 + g_1 = 0 \\ m_{21}\ddot{q}_1 + m_{22}\ddot{q}_2 + c_2 + g_2 = u \end{cases} \tag{5}$$

for $F_1(q) = 0$ and $F_2(q) = I_1$. In (5), Coriolis and gravity are represented by "c" and "g" respectively. Input is displayed as u, where $q, \dot{q}$ points toward states of the system.

In this article, the considered case is when

$$\begin{cases} m_{11}\ddot{q}_1 + m_{12}\ddot{q}_2 + c_1 + g_1 = u \\ m_{21}\ddot{q}_1 + m_{22}\ddot{q}_2 + c_2 + g_2 = 0 \end{cases} \tag{6}$$

for $F_1(q) = I_1$ or $F_2(q) = 0$.

Solving the first equation in (5) for $\ddot{q}_1$ and $\ddot{q}_2$, and substituting the result in the second equation, (5) can be written as:

$$\begin{cases} \bar{m}_{11}\ddot{q}_1 + \bar{c}_1 + \bar{g}_1 = u \\ \bar{m}_{22}\ddot{q}_2 + \bar{c}_2 + \bar{g}_2 = u \end{cases} \tag{7}$$

where

$$\begin{cases} \bar{m}_{11}(q) = m_{21} - m_{22}m_{12}^{-1}m_{11} \\ \bar{c}_1(q,\dot{q}) = c_2 - m_{22}m_{12}^{-1}c_1 \\ \bar{g}_1(q) = g_2 - m_{22}m_{12}^{-1}g_1 \\ \bar{m}_{22}(q) = m_{22} - m_{21}m_{11}^{-1}m_{12} \\ \bar{c}_2(q,\dot{q}) = c_2 - m_{21}m_{11}^{-1}c_1 \\ \bar{g}_2(q) = g_2 - m_{21}m_{11}^{-1}g_1 \end{cases} \tag{8}$$

Since $q_1 \in \Re^1$, and $q_2 \in \Re^1$. system (7) is a set of two second-order systems in state variables $q_1 \in \Re^1$, and $q_2 \in \Re^1$. Writing $x = [x_1, x_2, x_3, x_4]^T = q = [q_1, \dot{q}_1, q_2, \dot{q}_2]^T$, the system shown in (7) can be displayed as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(x) + b_1(x)u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(x) + b_2(x)u \end{cases} \tag{9}$$

where

$$\begin{cases} f_1(x) = -\bar{m}_{11}^{-1}(\bar{c}_1 + \bar{g}_1) \\ b_1(x) = \bar{m}_{11}^{-1} \\ f_2(x) = -\bar{m}_{22}^{-1}(\bar{c}_2 + \bar{g}_2) \\ b_2(x) = \bar{m}_{22}^{-1} \end{cases} \tag{10}$$

are the nonlinear nominal functions. A similar treatment can be applied to convert system (6) to form (9) with the following transformation:

$$\begin{cases} f_1(x) = -\bar{m}_{11}^{-1}(\bar{c}_1 + \bar{g}_1) \\ b_1(x) = \bar{m}_{11}^{-1} \\ f_2(x) = -\bar{m}_{22}^{-1}(\bar{c}_2 + \bar{g}_2) \\ b_2(x) = \bar{m}_{22}^{-1} \end{cases} \tag{11}$$

where

$$\begin{cases} \bar{m}_{11}(q) = m_{11} - m_{12}m_{22}^{-1}m_{21} \\ \bar{c}_1(q,\dot{q}) = c_1 - m_{12}m_{22}^{-1}c_2 \\ \bar{g}_1(q) = g_1 - m_{12}m_{22}^{-1}g_2 \\ \bar{m}_{22}(q) = m_{12} - m_{11}m_{21}^{-1}m_{22} \\ \bar{c}_2(q,\dot{q}) = c_1 - m_{11}m_{21}^{-1}c_2 \\ \bar{g}_2(q) = g_1 - m_{11}m_{21}^{-1}g_2 \end{cases} \tag{12}$$

Remark: Among two degrees of freedom, the Acrobot, TORA, and the Inertial-Wheel Pendulum can be written in the form shown in (5). On the other hand, systems like an overhead crane, the Pendoubt, ball on a beam system, and the Cart-pole systems may reflect as the system shown in (6). Interestingly, all aforementioned systems can be represented as (9), but with f_1, f_2, b_1, and b_2 needs to be defined accordingly as given by (8, 10, 11, and 12) respectively.

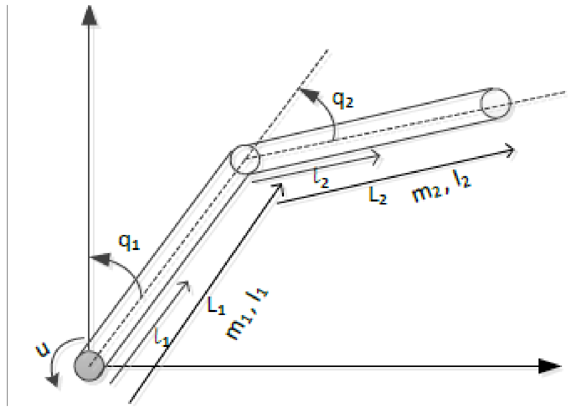

FIGURE 3. The Pendubot.

III. THE CONTROL PROBLEM AND THE PROPOSED CONTROL ALGORITHM

A. THE CONTROL PROBLEM

Given the desired set point $x_{des} \in \Re^4$ establish a feedback approach in times of controls $u : \Re^4 \rightarrow \Re$ such that the desired set point x_{des} is an attractive set for the system (9), so that there exists an $\epsilon > 0$, such that $x(t; t_0, x_0) \rightarrow x_{des}$, as $t \rightarrow \infty$ for any initial condition $(t_0, x_0) \in \Re^+ \times B(x_{des}, \in)$.

For set-point regulation of 2DOF underactuated mechanical systems, we take $x_{des} = [x_{1des}, 0, x_{3des}, 0]^T$ which can be accomplished by an appropriate interpretation of the coordinate system.

B. THE PROPOSED ALGORITHM

Step 1: Choose $u = \frac{1}{b_2(x)}\{-f_2(x) + x_1\}$. Then system (9) can be written as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \beta(x) + \alpha(x)x_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = x_1 \end{cases}$$

where,

$$\beta(x) = f_1(x) - \alpha(x)f_2(x) \;\&\; \alpha(x) = \frac{b_1(x)}{b_2(x)} \tag{13}$$

Step 2: Adding and subtracting w in the last equation $\dot{x}_2 = \beta(x) + \alpha(x)x_1$, we have $\dot{x}_2 = \beta(x) + \alpha(x)x_1 + w - w$. Let the second w is unknown (and it may be computed adaptively) later. Assume $\hat{w}$ be the estimated value of w and $\tilde{w} = w - \hat{w}$ be the estimation error of w. Therefore, the system shown in (13) can be written as:

$$\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = x_1 \\ \dot{x}_1 = x_2 \\ \dot{x}_2 = \beta(x) + \alpha(x)x_1 + w - \hat{w} - \tilde{w} \end{cases} \tag{14}$$

Step 3: Construct the switching/sliding surface for the system defined in (14) as:

$$s = x_3 + 3x_4 + 3x_1 + x_2$$

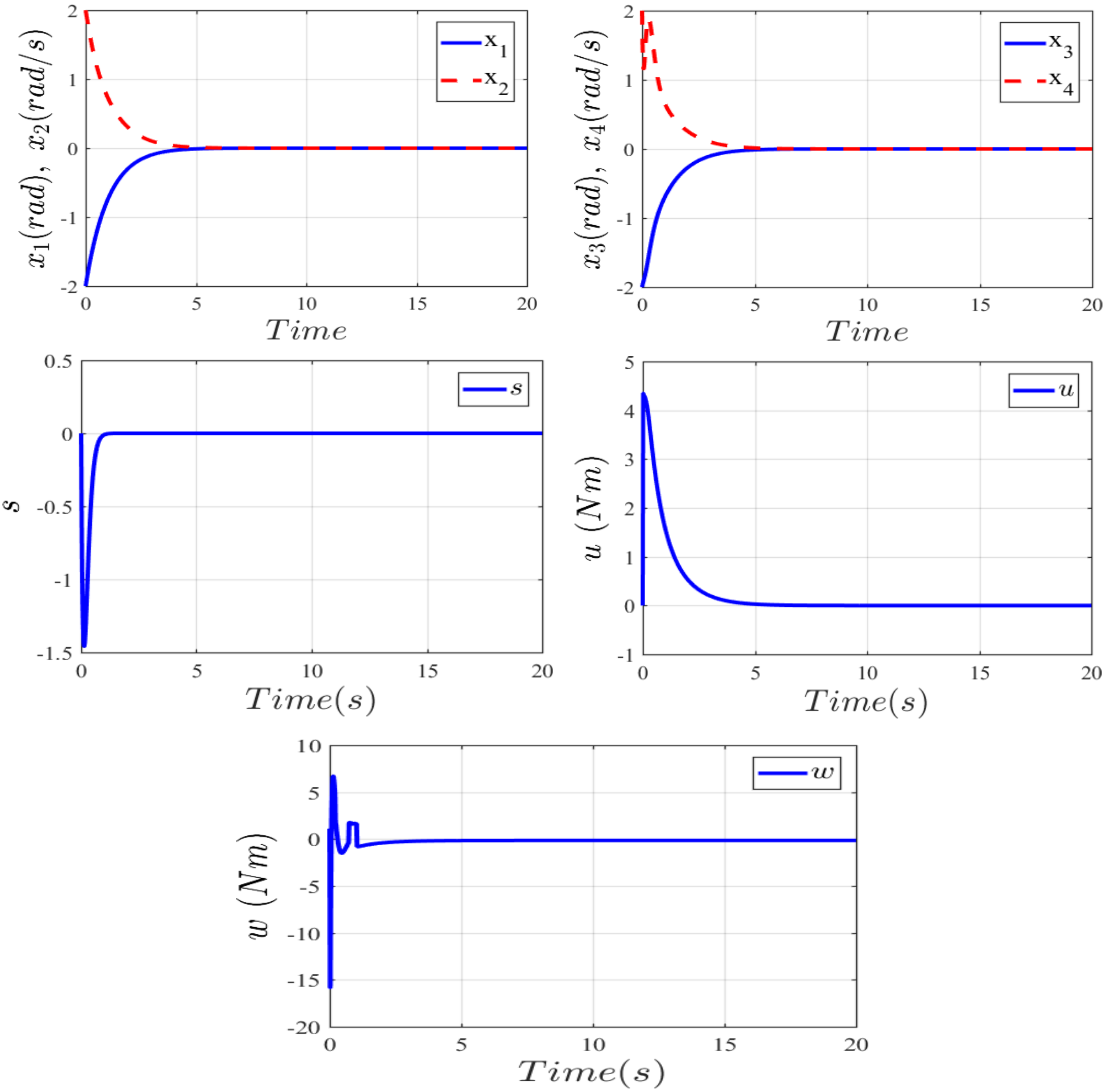

FIGURE 4. Convergence of system's states considering initial condition $(x_1(0), \ldots, x_4(0)) = (-2, 2, -2, 2)$. Time response of sliding surface s and control effort u and w.

then

$$\begin{aligned}\dot{s} &= \dot{x}_3 + 3\dot{x}_4 + 3\dot{x}_1 + \dot{x}_2 \\ &= x_4 + 3x_1 + 3x_2 + f_1(x) + w - \hat{w} - \tilde{w}\end{aligned}$$

By considering the Lyapunov function like;

$$V = \frac{1}{2}s^2 + \frac{1}{2}\tilde{w}^2,$$

One may able to devise the adaptive laws for $\hat{w}$, $\tilde{w}$ and compute w such that $\dot{V} < 0$ (for detail see [19]). Since

$$\begin{aligned}\dot{V} &= s\dot{s} + \tilde{w}\dot{\tilde{w}} \\ &= s\left\{x_4 + 3x_1 + 3x_2 + f_1(x) + w - \hat{w} - \tilde{w}\right\} + \tilde{w}\dot{\tilde{w}} \\ &= sx_4 + 3sx_1 + 3sx_2 + sf_1(x) + ws - s\hat{w} - s\tilde{w} + \tilde{w}\dot{\tilde{w}} \\ &= s\left\{x_4 + 3x_1 + 3x_2 + f_1(x) + w - \hat{w}\right\} + \tilde{w}\{\dot{\tilde{w}} - s\}\end{aligned} \tag{14b}$$

To make $\dot{V} < 0$, substitute following values in (14b),

$$w = -x_4 - 3x_1 - 3x_2 - f_1(x) + \hat{w} - ksign(s) - ks$$
$$\dot{\tilde{w}} = s - k_1\tilde{w}, \quad k_1 > 0 \text{ and } \dot{\hat{w}} = -s + k_1\tilde{w}, \quad k_1 > 0,$$

We have

$$\dot{V} = -ks^2 - k|s| - k_1\tilde{w}^2 < 0. \tag{15}$$

From the aforementioned equation, one concludes that $s, \tilde{w} \to 0$. Since $s \to 0$, therefore $x \to 0$ and $\hat{w} \to w$.

In the upcoming section, the aforementioned strategy is applied to the three benchmarks UMS belongs to the considered class.

IV. ILLUSTRATIVE EXAMPLES

The examples of 2DoF underactuated mechanical systems are presented in this section. The Acrobot is of the first form (5), and the Pendubot and the cart- pole system are from the second form (6).

A. EXAMPLE1: THE ACROBOT

As displayed in Figure 1, the acrobot is a two-link manipulator with a single actuator at the elbow. For the dynamical model of the Acrobot, $F(q) = [F_1(q), F_2(q)]^T = [0, 1]^T$ in (4), and the equations of motion are given by (5) as:

$$\begin{aligned} m_{11}\ddot{q}_1 + m_{12}\ddot{q}_2 + c_1 + g_1 &= 0 \\ m_{21}\ddot{q}_1 + m_{22}\ddot{q}_2 + c_2 + g_2 &= u \end{aligned} \tag{16}$$

with $m_{11}, m_{12}, m_{21}, m_{22}, c_1, g_1, c_2,$ *and* g_2 as follows:

$$\begin{aligned} m_{11}(q_2) &= I_1 + I_2 + m_1 l_1^2 + m_2\left(L_1^2 + l_2^2\right) \\ &\quad + 2m_2L_1l_2\cos(q_2) \\ m_{12}(q_2) &= I_2 + m_2l_2^2 + m_2L_1l_2\cos(q_2) \\ m_{21}(q_2) &= m_{12}(q_2) \\ m_{22}(q_2) &= I_2 + m_2l_2^2 \\ c_1(q,\dot{q}) &= -m_2L_1l_2\sin(q_2)(2\dot{q}_1\dot{q}_2 + \dot{q}_2^2) \\ c_2(q,\dot{q}) &= m_2L_1l_2\sin(q_2)(\dot{q}_1^2) \\ g_1(q_1,q_2) &= -(m_1l_1 + m_2L_1)g\sin(q_1) \\ &\quad - m_2l_2g\sin(q_1+q_2) \\ g_2(q_1,q_2) &= -m_2l_2g\sin(q_1+q_2) \end{aligned} \tag{17}$$

Using the definition of (8), equation (9) can be rewritten as (10) in terms of f_1, f_2, b_1 *and* b_2. Physical parameters for Acrobot are adopted from [20], as:
$m_1 = 1\,(kg)$, $m_2 = 1\,(kg)$, $L_1 = 1\,(m)$, $L_2 = 2\,(m)$, $l_1 = 0.5\,(m)$, $l_2 = 1\,(m)$, $I_1 = 0.0833\,(kg.m^2)$, $I_2 = 0.33\,(kg.m^2)$, *and* $g = 9.8(m.sec^{-2})$.

The control assignment is to swing the Acrobat from the stable equilibrium (downward) state ($q_1 = \pi, q_2 = 0$) to the upright unstable equilibrium state ($q_1 = 0, q_2 = 0$). Figure 2 shows the simulation results for the Acrobat with the proposed control algorithm.

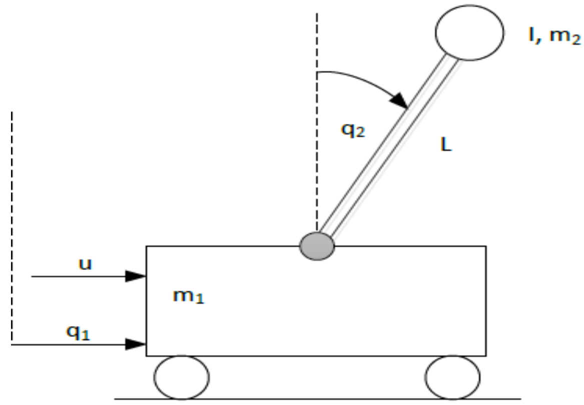

FIGURE 5. The cart-pole system.

B. EXAMPLE2: THE PENDUBOT

The pendubot is also a two-link manipulator with a sole actuator at the base, as displayed in Figure 3. For the dynamical model of the Pendubot, $F(q) = [F_1(q), F_2(q)]^T = [1, 0]^T$ in (4), and the equations of motion are given by (6) as:

$$\begin{aligned} m_{11}\ddot{q}_1 + m_{12}\ddot{q}_2 + c_1 + g_1 &= u \\ m_{21}\ddot{q}_1 + m_{22}\ddot{q}_2 + c_2 + g_2 &= 0 \end{aligned}$$

with $m_{11}, m_{12}, m_{21}, m_{22}, c_1, g_1, c_2,$ *and* g_2 the same as for the Acrobot, given by (17). The f_1, f_2, b_1 *and* b_2 required in the general form (9) are now given by (11) combined with the definition (12). We chose the physical parameters of the Pendubot, adopted from [20], as:
$m_1 = 0.39\,(kg)$, $m_2 = 1.59\,(kg)$, $L_1 = 0.2\,(m)$, $L_2 = 0.08\,(m)$, $l_1 = 0.1\,(m)$, $l_2 = 0.04\,(m)$, $I_1 = 0.0423\,(kg.m^2)$, $I_2 = 0.0711\,(kg.m^2)$, *and* $g = 9.8(m.sec^{-2})$.

Here the control objective is to swing the Pendubot from the stable (downward) equilibrium state ($q_1 = \pi, q_2 = 0$) to the upright second unstable state of equilibrium ($q_1 = 0, q_2 = 0$). Fig. 4 shows the simulation results for the Pendubot with the proposed control algorithm. Moreover, Fig. 4 shows the simulation for any non-zero initial state.

C. EXAMPLE3: THE CART-POLE SYSTEM

The cart-pole system is displayed in Figure 5, where F is considered as a control input (applied along q_1) and the joint q_2 is unactuated. The dynamical equations for the cart-pole systems can be written (see [1]) as:

$$\begin{cases} (m_1+m_2)\ddot{q}_1 + m_2l_2\cos(q_2)\ddot{q}_2 - m_2l_2\sin(q_2)\dot{q}_2^2 = F \\ m_2l_2\cos(q_2)\ddot{q}_1 + \left(m_2l_2^2 + I_2\right)\ddot{q}_2 - m_2l_2g\sin(q_2) = 0 \end{cases}$$

For the dynamical model of the cart-pole system, $F(q) = [F_1(q), F_2(q)]^T = [1, 0]^T$ in (4), and the equations of motion can be written as:

$$\begin{cases} m_{11}\ddot{q}_1 + m_{12}\ddot{q}_2 + c_1 + g_1 = u \\ m_{21}\ddot{q}_1 + m_{22}\ddot{q}_2 + c_2 + g_2 = 0 \end{cases}$$

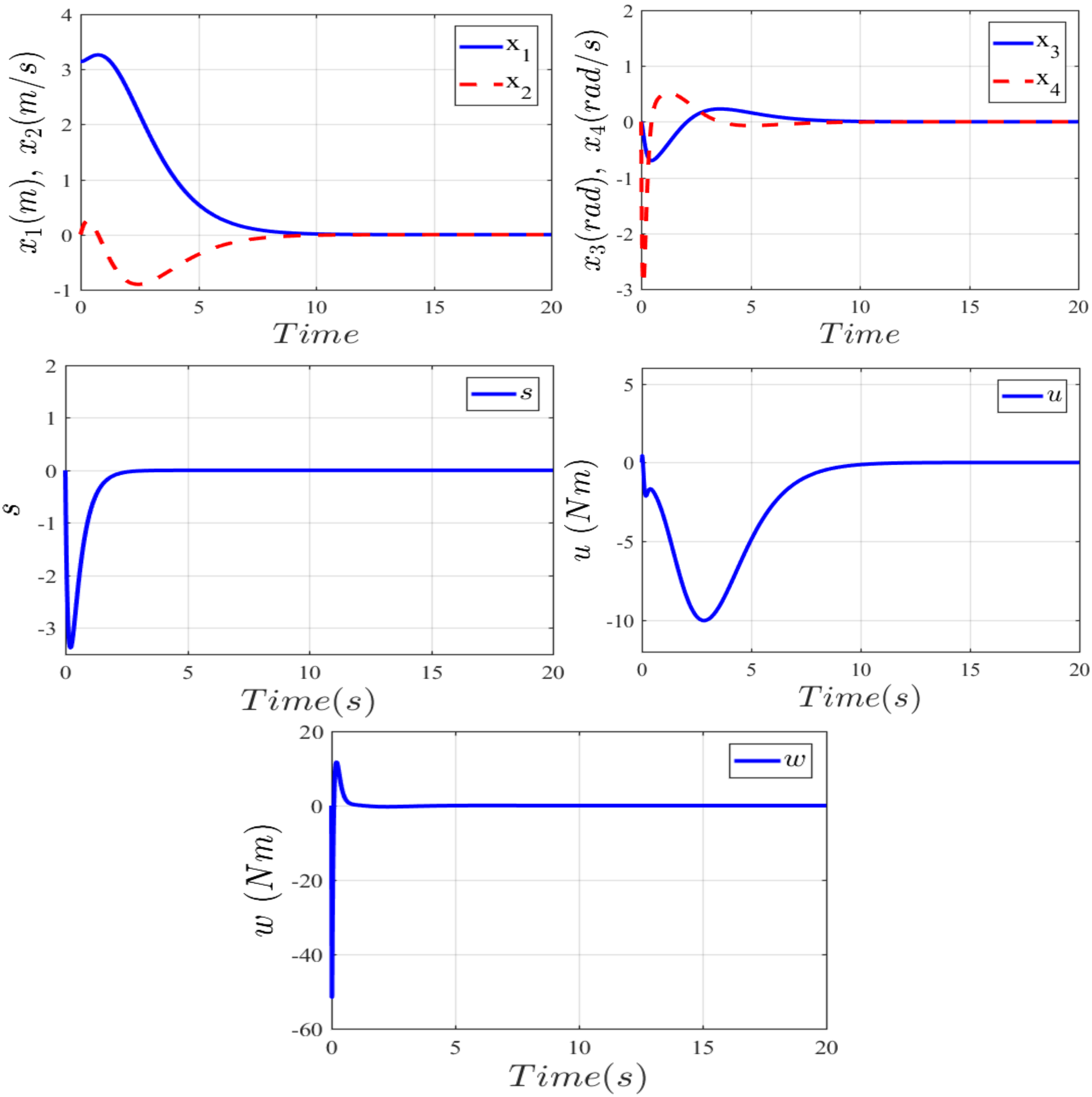

FIGURE 6. **Convergence of system's states considering initial condition $(x_1(0), \ldots, x_4(0)) = (\pi, 0, \pi/2, 0)$. Time response of sliding surface s and control effort *u*.**

with $m_{11}, m_{12}, m_{21}, m_{22}, c_1, g_1, c_2,$ *and* g_2 are given as:

$$\begin{cases} m_{11} = m_1 + m_2, & m_{12} = m_2 l_2 cos q_2, \\ m_{21} = m_{12}, & m_{22} = m_2 l_2^2 + I_2, \\ c_1 = -m_2 l_2 \sin(q_2)\,\dot{q}_2^2, & \\ g_1 = 0, & c_2 = 0, \text{ and} \\ & g_2 = -m_2 g l_2 \sin(q_2) \end{cases}$$

$$\begin{cases} c_1 = -m_2 l_2 \sin(q_2)\,\dot{q}_2^2 \\ \bar{m}_{11} = m_{11} - m_{12} m_{21}/m_{22}, \\ \bar{c}_1 = c_1 - m_{12} c_2/m_{22}, \quad \bar{g}_1 = g_1 - m_{11} g_2/m_{22} \\ \bar{m}_{22} = m_{12} - m_{11} m_{22}/m_{21}, \quad \bar{c}_2 = c_1 - m_{11} c_2/m_{21} \\ \bar{g}_2 = g_1 - m_{11} g_2/m_{21}, \end{cases}$$

$$f_1 = -\frac{\bar{c}_1 + \bar{g}_1}{\bar{m}_{11}} \text{ and } b_1 = 1/\bar{m}_{11}$$

$$f_2 = -(\bar{c}_2 + \bar{g}_2)/\bar{m}_{22} \text{ and } b_1 = 1/\bar{m}_{22}$$

The f_1, f_2, b_1 and b_2 required in the general form (9) are now given by (11) combined with the definition (12). The physical parameters of the considered system are adopted from [20], as:

$$m_1 = m_2 = 1,\ l_2 = 0.75,\ I_2 = \left(\frac{4}{3}\right) m_2 l_2^2,\ g = 9.8$$

The control objective here is to swing the cart-pole system from a stable equilibrium (downward) state ($q_3 = \pi, q_4 = 0$) (to upright unstable equilibrium state ($q_3 = 0, q_4 = 0$). Figure 6 shows the simulation results for the cart-pole system with the proposed control algorithm.

V. CONCLUSION

A stabilization algorithm based on adaptive sliding mode is accomplished in this work. The aforesaid algorithm is made generic to become applicable to all underactuated mechanical systems belongs to the considered class of two degrees of freedom system. The proposed strategy is applied to the benchmark systems of the acrobot, the pendoubt, and the cart-pole system. The authenticity and effectiveness of the proposed technique is established via simulation result.

In light of [1] and [10], considering the future direction, adaptive terminal-based sliding mode control may be considered for the class of underactuated mechanical systems with 2DoF, along with hardware implementation to compare the results from the existing literature like [1]. Moreover, this work can be expanded in terms of the time-delay system, in addition to comparative analysis with modern techniques.

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FAZAL UR REHMAN received the M.Sc. and M.Phil. degrees in mathematics from Bahauddin Zakariya University, Multan, Pakistan, in 1986 and 1990, respectively, and the M.Eng. and Ph.D. degrees in control systems from the Department of Electrical Engineering, McGill University, Montreal, Canada, in 1993 and 1997, respectively.

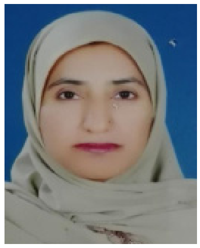
He joined the Faculty of Electronic Engineering, Ghulam Ishaq Khan Institute of Engineering Sciences and Technology, Pakistan, as an Assistant Professor, in January 1998, where he worked as an Assistant Professor, from 1998 to 2001, and as an Associate Professor, from 2002 to 2006. He joined Mohammad Ali Jinnah University, Islamabad, Pakistan, as a Professor, and the HOD of the Department of Electronic Engineering, in January 2007. His research interests include nonlinear control systems, nonholonomic control systems, multi-rate digital signal processing, and optimal control.

NADIR MEHMOOD received the B.Sc. degree in electronics engineering from UET, Peshawar, in 2015, and the M.S. degree in electrical engineering from the Capital University of Science and Technology (CUST), Islamabad, Pakistan, in 2019. He is currently working as a Lab Engineer with the Electrical Engineering Department, CUST. His research interests include nonlinear control and sliding mode control.

SAMI UD DIN (Member, IEEE) received the B.S. degree in electrical engineering from the Federal Urdu University of Arts, Science and Technology, Islamabad, Pakistan, in 2009, the M.S. degree in electronics engineering (Control Systems) from Muhammad Ali Jinnah University, Islamabad, in 2012, and the Ph.D. degree in electrical engineering with a specialization in control systems from the Capital University of Science and Technology, Islamabad, in 2019.

He was with the Department of Electrical Engineering, The University of Lahore, Islamabad, till October 2019, as an Assistant Professor. He is currently with the Department of Electrical Engineering, Namal Institute Mianwali, Mianwali, Pakistan. His research interests include non-linear control, sliding mode control, under-actuated systems, chaotic systems, robotics, antenna design, and power systems.

MUHAMMAD RAFIQ MUFTI received the M.Sc. degree in computer science from Bahauddin Zakariya University, Multan, Pakistan, in 1994, the M.Sc. degree in computer engineering from CASE, University of Engineering and Technology, Taxila, Pakistan, in 2007, and the Ph.D. degree in electronic engineering from Mohammad Ali Jinnah University (MAJU), Islamabad, in 2012. He is currently a Faculty Member with COMSATS University Islamabad, Vehari Campus, Pakistan. His research interests include sliding mode control, fractional control, neural networks, cognitive radio networks, and network security.



HUMAIRA AFZAL received the M.Sc. degree (Hons.) in computer science from Bahauddin Zakariya University, Multan, Pakistan, in 1997, the M.Sc. degree in computer engineering from CASE, University of Engineering and Technology, Taxila, Pakistan, in 2010, and the Ph.D. degree in computer science from the School of Electrical Engineering and Computer Science, University of Bradford, U.K., in August 2014. She is currently working as an Assistant Professor with the Department of Computer Science, Bahauddin Zakariya University. Her research interests include MAC protocol design for the cognitive radio network, performance modeling, queuing theory, network security, software engineering, and sliding mode control.