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Terminal Synergetic and State Feedback Linearization Based Controllers for Artificial Pancreas in Type 1 Diabetic Patients

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ABSTRACT Lack of insulin production by pancreas causes high blood glucose level (BGL) in the diabetic patients. For their treatment, manual insulin intake is possible only during the day timings but not feasible during the night when the patient is sleeping. Artificial pancreas (AP) is used for the automatic regulation of BGL by continuous injection of insulin. The nonlinear Bergman's Minimal Model (BMM) considers fixed meal disturbance which may actually vary continuously during medication due to meal intake or by doing exercise. This variation has been taken into account by the Extended Bergman's Minimal Model (EBMM). In this paper, two nonlinear: Terminal Synergetic and State Feedback Linearization based controllers have been proposed for AP to regulate BGL using EBMM. Asymptotic stability of the proposed controllers has been proved using Lyapunov theory. Comparison of the proposed controllers with each other and that with PID controller has been done using MATLAB/Simulink. White noise has been added as the disturbance to further analyze the output performance of the proposed controllers. The Terminal Synergetic controller which performs better than others, has also been implemented on the data of six Type 1 diabetic patients available in the literature.

INDEX TERMS Artificial pancreas, blood glucose level, Bergman minimal model, extended Bergman minimal model, terminal synergetic controller, state feedback linearization based controller.

I. INTRODUCTION

Diabetes is characterized by two types: Type 1 and Type 2. The Mellitus Type 1 Diabetes happens when internal insulin productive system fails to produce required amount of insulin resulting in hyperglycemia in which a patient suffers from high BGL. The failure of insulin production is due to the absence of β-cells in pancreas. While the Type 2 patients suffer with the ineffectiveness of insulin to their body [1]. This chronic disease is spreading worldwide. According to the World Health Organization report of the year 2012, about 1.5 million causalities were reported due to diabetes [2]. For its cure billions of USD are spent every year. In an economic survey, the recorded figure for the cure of diabetic patients was 132 billion USD in 2002 and crossed 245 billion USD by 2012 [3], [4].

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The BGL of Type 1 diabetic patients must be kept in the safe range of 80-120 mg/dL by externally infusing insulin in the patient's body because internal insulin is inadequate to maintain it in the safe range [5]. Patients having BGL below the safe range are in the state of hypoglycemia. In hyperglycemia, high BGL causes serious damage to nervous system which may result in visionary, cardiac arrest, kidneys failure etc., while in hypoglycemia, low BGL causes prolonged coma which ultimately results in death [6], [7].

It is very important to control glucose regulation in blood to get rid of such deadly outcomes. The BGL can be maintained by an automated mechanism or by manual infusion of insulin in the patient body. Manual infusion is an uncontrolled process and suitable only during day timings. There is a chance of less or over dose of insulin in the manual infusion and is not capable of catering any meal disturbance during medication. The second method is the fully autonomous AP which is "all-in-one" diabetes control system that monitors BGL using Continuous Glucose Monitor (CGM) sensor, insulin infusion pump and the controller. It works like our actual pancreas, monitoring the blood sugar level regularly and releasing insulin when the blood sugar is too high. It releases a low but continuous trickle of insulin. The CGM uses its sensor to monitor the BGL, sends this reading to the insulin infusion pump which releases calculated doses of insulin to our bloodstream if needed. As the blood sugar gets to its required level, the dose of insulin stops [8], [9]. The tests like IVGTT (Intravenous Glucose Tolerance Test), MMTT (Mixed Meal Tolerance Test) & OGTT (Oral Glucose Tolerance Test) discussed in [10], are carried out to check the performance of AP. The closed loop control method is better approach for desired BGL given that the controller has less convergence time and fewer oscillations to overcome nocturnal hypoglycemia [11].

The implementation of linear controllers require linearized model which result in local stability with unsatisfactory results [12]. The conventional PID controller has been implemented to remove steady state error in BGL [13] but the implementation results in over dose of insulin which has been rectified by using PD controller [14]. To add some robustness PID merged with fuzzy to get better results [15]. Linear Quadratic Regulator (LQR) has also been proposed for AP in [16]. Some algorithms of Model Predictive Control (MPC) have been used to get desired BGL [17], [18] but if future predictions of BGL are not good, the controller may not achieve satisfactory results. Fuzzy logic controller has also been implemented to achieve satisfactory performance by increasing fuzzy rules but computationally it becones very costly [19], [20].

As the mathematical model of Type 1 diabetic patients is nonlinear in nature so, it is better to design a nonlinear controller. The nonlinear based controllers perform better while dealing with complex and nonlinear models as compared to the linear ones. Backstepping Control (BSC) is a nonlinear recursive scheme to stabilize nonlinear systems with strict feedback form. In [21], BSC algorithm has been proposed for automatic regulation of BGL but the convergence time is large. To get better convergence time, the BSC is updated by adding adaptation of glucose parameter but even with the good convergence speed some large overshoots/undershoots have been observed in BGL [22]. In [23], the Integral Backstepping (IBS) has been proposed using EBMM [24] but IBS is a computationally complex, costly and have steady state error when subjected to additional white noise. Sliding Mode Control (SMC) has also been proposed for BGL stabilization [25] but the controller's performance is compromised due to chattering effect and is again computationally costly.

The insulin disturbance due to meal intake or by doing exercise during medication, is dynamic and can't be considered to be constant [26]. AP being a small device, has a slow micro-processor which can afford to simulate an algorithm which is simple and easy to implement and computationally less expensive.

In this paper, EBMM which considers time varying meal disturbance represented by an additional dynamical state of the system, has been used for designing Lyapunov based nonlinear Terminal Synergetic Controller (TSC) and State Feedback Linearization based Controller (SFC) which are computationally less costly, much simpler to design and easy to implement than the recursive computationally costly BSC, IBS and SMC (which requires switching about the sliding surface with theoretically infinite frequency), have been proposed for AP. Furthermore the EBMM has been perturbed by adding white noise in it and then the performance of each proposed controller has been evaluated to validate their ability to handle perturbations. Finally the efficiency of the proposed TSC has also been evaluated in simulations using the varied data provided by six different diabetic patients in the literature [27].

Salient features of this research paper are as follows:

- Proposed two different nonlinear controllers to cater for all the non-linearties present in the system without linearization.
- Proposed TSC which is advanced and improved Lyapunov based synergetic controller which ensures convergence of macro-variable to zero.
- For the comparison purposes, SFC has also been proposed whose output performance has been compared with TSC.
- Perturbation has been added in the system as white noise to analyze the performance of each proposed controller under such conditions.
- Output performance of the proposed TSC controller has also been checked using data of six different diabetic patients available in literature.

The remaining part of the paper has been arranged as follows; Section II describes the nonlinear mathematical models for Type 1 diabetic patients. Section III describe the problem statement and mathematical analysis of the proposed nonlinear controllers for AP using EBMM. Section IV explains the simulations results and comparisons. Finally section V contains the concluding remarks.

II. NONLINEAR MATHEMATICAL MODELS FOR TYPE 1 DIABETIC PATIENTS

The internal insulin and glucose regulatory process of a human body has been shown in Fig. 1. Pancreas is responsible for maintaining BGL in the safe range. In case of a high BGL pancreas releases insulin which is infused in the blood through liver. On the other hand glucagon is infused in the blood by liver in case of low BGL. This is how the BGL is kept at safe level.

A. BERGMAN'S MINIMAL MODEL (BMM)

R. N. Bergman proposed a basic three sates mathematical representation for the Type 1 diabetic patients in [28] with a fixed value of meal disturbance which is given by the eqs (1)-(3):

$$\dot{x}_1 = -p_1 x_1 - x_2(x_1 + G_b) + d(t) \tag{1}$$

$$\dot{x}_2 = -p_2 x_2 + p_3 x_3 \tag{2}$$

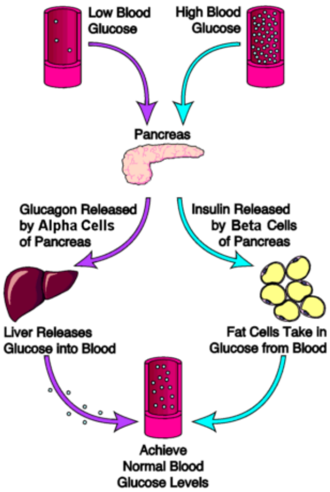

FIGURE 1. BGL mechanism in Human Beings [29].

$$\dot{x}_3 = -n(x_3 + I_b) + u(t) \tag{3}$$

where x_1 represents BGL which must satisfy $20 \leq x_1 \leq 600$ (mg/dl), otherwise the patient may die if it is out of this range and x_2 & x_3 are remote insulin concentration and plasma insulin concentration respectively and $u(t)$ is external insulin infusion rate. The system parameters along with their values used for simulation results have been detailed in Table-1.

TABLE 1. Model parameters with numerical values.

Parameters	Description	Values
p_1	Glucose Effectiveness	0 $min^{-}1$
p_2	Delay in Insulin Action	0.015 $min^{-}1$
p_3	Patient Parameter	0.000013 $mUL^{-}1min^{-}2$
p_4	Insulin Degradation Rate	0.021 $min^{-}1$
p_5	Meal Disturbance	0.05 $min^{-}1$
I_b	Basal Plasma Insulin	7 $mUL^{-}1$
G_b	Basal Plasma Glucose	80 $mgdL^{-}1$

B. EXTENDED BERGMAN'S MINIMAL MODEL (EBMM)

To deal with the dynamic meal disturbances an EBMM has been proposed [24] which is given by the eqs (4)-(7):

$$\dot{x}_1 = -p_1(x_1 - G_b) - x_1x_2 + x_4 \tag{4}$$
$$\dot{x}_2 = -p_2x_2 + p_3(x_3 - I_b) \tag{5}$$
$$\dot{x}_3 = -p_4(x_3 - I_b) + u(t) \tag{6}$$
$$\dot{x}_4 = -p_5x_4 \tag{7}$$

where p_5 is the meal disturbance factor and x_4 represents the variable meal disturbance effect on BGL. Note that in both the models eqs (3) and (6) are same.

III. PROBLEM STATEMENT AND CONTROLLERS DESIGNING

For the blood glucose regulation, an automated system of AP with a suitable controller is required which can provide essential amount of insulin when needed. AP provides controlled and automated insulin infusion to patient's body by monitoring BGL using its sensor. Conventional manual infusion is feasible only during the day timings but there is a chance of over/under dosage to the patient. AP not only provides facility of autonomous mechanism for insulin infusion while patient is sleeping but also provides a calculated required amount of insulin to avoid over dosage. As the system given by eqs (4)-(7) is nonlinear in nature due to cross product term's x_1x_2 so, for the global asymptotic stability of the system, it would be better to design a nonlinear controller. The proposed closed loop control scheme for AP has been shown in Fig. 2.

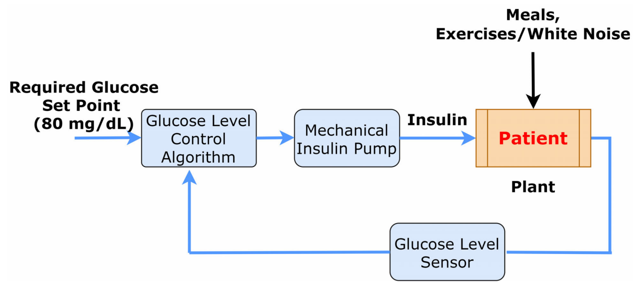

FIGURE 2. Close loop Control Scheme.

A. TERMINAL SYNERGETIC CONTROLLER DESIGN

Synergetic controller ensures convergence of the system states to their reference points when time approaches infinity while the TSC has the additional benefit of the convergence in finite time. The macro-variable for TSC is taken as function of errors. The error for the desired BGL is defined as:

$$e = x_1 - x_{1d} \tag{8}$$

where x_{1d} is the reference value for the desired BGL. As EBMM contains only one input, so we can define a single macro-variable $\xi = \xi(x, t)$ to get the desired design specifications which is given by:

$$\xi = \ddot{e} + S_1\dot{e} + S_0e \tag{9}$$

where S_1 and S_0 are positive gain parameters. The ASC forces the system to the designed manifold $\xi = 0$. The manifold constraint for the states of the system given in eqs (4)-(7) would be driven to specified manifold in finite time and is defined as:

$$T\dot{\xi}^{\frac{p}{q}} + \xi = 0 \tag{10}$$

where $T > 0$ while p and q are positive odd integers that satisfy the $1 < \frac{p}{q} < 2$ condition. The value of $\dot{\xi}$ from eq (10) is given as:

$$\dot{\xi} = -(\frac{\xi}{T})^{\frac{q}{p}} \tag{11}$$

Now by taking the time derivative of ξ from eq (9), we have

$$\dot{\xi} = \dddot{e} + S_1\ddot{e} + S_0\dot{e} \tag{12}$$

By third derivative of the eq (8) w.r.t.time, we get

$$\dddot{e} = \dddot{x}_1 - \dddot{G}_b = \dddot{x}_1 \tag{13}$$

Taking third derivative of the eq (4), we have

$$\begin{aligned}\dddot{x}_1 &= p_1^2\dot{x}_1 + 2p_1(\dot{x}_1x_2 + x_1\dot{x}_2) - p_1\dot{x}_4 + p_1G_b\dot{x}_2 + \dot{x}_1x_2^2 \\ &\quad + 2x_1x_2\dot{x}_2 - \dot{x}_2x_4 - x_2\dot{x}_4 + p_2(\dot{x}_1x_2 + x_1\dot{x}_2) \\ &\quad - p_3(\dot{x}_1x_3) - p_3x_1(-p_4(x_3 - I_b)) - p_3x_1u(t) \\ &\quad + p_3I_b\dot{x}_1 - p_5\dot{x}_4\end{aligned} \tag{14}$$

Now let us represent

$$\begin{aligned}\lambda(t) &= p_1^2\dot{x}_1 + 2p_1(\dot{x}_1x_2 + x_1\dot{x}_2) - p_1\dot{x}_4 + p_1G_b\dot{x}_2 + \dot{x}_1x_2^2 \\ &\quad + 2x_1x_2\dot{x}_2 - \dot{x}_2x_4 - x_2\dot{x}_4 + p_2(\dot{x}_1x_2 + x_1\dot{x}_2) \\ &\quad - p_3(\dot{x}_1x_3) - p_3x_1(-p_4(x_3 - I_b)) + p_3I_b\dot{x}_1 - p_5\dot{x}_4\end{aligned} \tag{15}$$

Then eq (14) takes the form

$$\dddot{x}_1 = \lambda(t) - p_3x_1u(t) \tag{16}$$

Using eq (16) in eq (12), we have

$$\dot{\xi} = \lambda(t) - p_3x_1u(t) + S_1\ddot{e} + S_0\dot{e} \tag{17}$$

Now considering the same macro-variable as in eq (9) for the design of TSC as well. By comparing eq (11) and eq (17) we get the control input $u(t)$ for the TSC as follows:

$$u(t) = \frac{1}{p_3x_1}[(\frac{\xi}{T})^{\frac{q}{p}} + \lambda(t) + S_1\ddot{e} + S_0\dot{e}] \tag{18}$$

The control law $u(t)$ derived in eq (18) ensures the system states to converge on their respective equilibrium points with the convergence rate depending on the p and q parameters. Since the state variable x_1 represents BGL, it is supposed to be at higher value and the proposed controller brings it down to the safe range of $80-120\ mg/dl$. It remains always a positive and never reaches at zero because BGL at zero means the death of a patient which restricts control input to get infinite.

The stability of the system has been proved by considering a positive definite Lyapunov candidate function and the following Lemma 1 ensures the finite time convergence of macro-variable ξ to zero [30].

Lemma 1: Let us consider positive definite function of Lyapunov, which satisfies the following inequality:

$$\dot{V}(t) \leq -\beta V^{\gamma}(t), \forall\ t \geq t_0, V(t_0) \geq 0, \tag{19}$$

where $\beta \geq 0$ and $0 \leq \gamma \leq 1$ are the constants. For any initial time t_0, $V(t)$ satisfies the following inequality as:

$$V^{1-\gamma} \leq V^{1-\gamma}(t_0) - \beta(1-\gamma)(t - t_0), \quad t_0 < t < t_1 \tag{20}$$

and $V(t) \equiv 0, \forall\ t \geq t_1$ with the value of t_1 given as:

$$t_1 = t_0 + \frac{V^{1-\gamma}(t_0)}{\beta(1-\gamma)} \tag{21}$$

From Lemma 1, t_1 is the time at which the TSC manifold converges to zero in finite time, can be obtained as:

$$t_1 = \frac{V^{(\frac{p-q}{2p})}(t_0)}{T_1(\frac{p-q}{2p})} \tag{22}$$

B. STATE FEEDBACK LINEARIZATION BASED CONTROLLER DESIGN

The SFC is input output linearization algorithm which utilizes the knowledge of state vector for computing the control action of the given dynamical system. The states of the system can be stabilized at the origin using SFC. Our ultimate goal is to get desired BGL. So, the output can be taken as:

$$y = x_1 \tag{23}$$

By taking time derivative of eq (23), we get

$$\dot{y} = \dot{x}_1 \tag{24}$$

Using eq (4) in eq (24), we have

$$\dot{y} = -p_1(x_1 - G_b) - x_1x_2 + x_4 \tag{25}$$

Since eq (25) does not have any control input $u(t)$ so, we compute another time derivative of eq (25) given by:

$$\ddot{y} = -p_1\dot{x}_1 - \dot{x}_1x_2 - x_1\dot{x}_2 + \dot{x}_4 \tag{26}$$

Now by substituting $\dot{x_1}$, $\dot{x_2}$ and $\dot{x_4}$ from the eq (4), eq (5) and eq (7) respectively in eq (26), we get

$$\begin{aligned}\ddot{y} &= p_1^2(x_1 - G_b) + p_1x_1x_2 - p_1x_4 + p_1x_2(x_1 - G_b) \\ &\quad + x_1x_2^2 - x_2x_4 + p_2x_1x_2 - p_3x_1(x_3 - I_b) - p_5x_4\end{aligned} \tag{27}$$

As we can observe in eq (27), there is still no control input $u(t)$ present so, we again compute its time derivative, we have

$$\begin{aligned}\dddot{y} &= p_1^2\dot{x}_1 + 2p_1(\dot{x}_1x_2 + x_1\dot{x}_2) - p_1\dot{x}_4 + p_1G_b\dot{x}_2 + \dot{x}_1x_2^2 \\ &\quad + 2x_1x_2\dot{x}_2 - \dot{x}_2x_4 - x_2\dot{x}_4 + p_2(\dot{x}_1x_2 + x_1\dot{x}_2) \\ &\quad - p_3(\dot{x}_1x_3 + x_1\dot{x}_3) + p_3I_b\dot{x}_1 - p_5\dot{x}_4\end{aligned} \tag{28}$$

By substituting value of $\dot{x_3}$ from the eq (6) in eq (28), we get

$$\begin{aligned}\dddot{y} &= p_1^2\dot{x}_1 + 2p_1(\dot{x}_1x_2 + x_1\dot{x}_2) - p_1\dot{x}_4 + p_1G_b\dot{x}_2 + \dot{x}_1x_2^2 \\ &\quad + 2x_1x_2\dot{x}_2 - \dot{x}_2x_4 - x_2\dot{x}_4 + p_2(\dot{x}_1x_2 + x_1\dot{x}_2) - p_3\dot{x}_1x_3 \\ &\quad - p_3x_1(-p_4(x_3 - I_b) + u(t)) + p_3I_b\dot{x}_1 - p_5\dot{x}_4\end{aligned} \tag{29}$$

As the control input $u(t)$ appears in the eq (29) at the third derivative of output y. Hence, the relative degree of the system is 3. The reference matrix R is defined as:

$$R = \begin{bmatrix} x_{1ref} \\ \dot{x}_{1ref} \\ \ddot{x}_{1ref} \end{bmatrix}$$

where x_{1ref} is desired BGL. Hence, the errors in matrix form are defined as:

$$e = \begin{bmatrix} x_1 - x_{1ref} \\ \dot{x}_1 - \dot{x}_{1ref} \\ \ddot{x}_1 - \ddot{x}_{1ref} \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Using error matrix we can write the error eq as:

$$e_1 = x_1 - x_{1ref} \tag{30}$$

From the eq (30), we can write

$$\dot{e}_1 = e_2 \tag{31}$$

and

$$\dot{e}_2 = e_3 \tag{32}$$

By computing time derivative of e_3 from the eq (32) and comparing it with eq (28), we have

$$\dot{e}_3 = \dddot{x}_1 - \dddot{x}_{1ref} = \dddot{y} \tag{33}$$

Using eq (29) in eq (33), we get

$$\begin{aligned}\dot{e}_3 &= p_1^2\dot{x}_1 + 2p_1(\dot{x}_1x_2 + x_1\dot{x}_2) - p_1\dot{x}_4 + p_1G_b\dot{x}_2 + \dot{x}_1x_2^2 \\ &\quad + 2x_1x_2\dot{x}_2 - \dot{x}_2x_4 - x_2\dot{x}_4 + p_2(\dot{x}_1x_2 + x_1\dot{x}_2) - p_3(\dot{x}_1x_3) \\ &\quad - p_3x_1(-p_4(x_3 - I_b) + u(t)) + p_3I_b\dot{x}_1 - p_5\dot{x}_4 \end{aligned} \tag{34}$$

Now, take

$$\dot{e}_3 = -v \tag{35}$$

where v is the control law which can stabilize errors e_1, e_2 and e_3 defined as:

$$v = F_1e_1 + F_2e_2 + F_3e_3 \tag{36}$$

where F_1, F_2 and F_3 are positive constants. Now by using eqs (31), (32) and (35) in error matrix, we get

$$\dot{e} = \begin{bmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\end{bmatrix} = \begin{bmatrix} e_2 \\ e_3 \\ -F_1e_1 - F_2e_2 - F_3e_3\end{bmatrix}$$

or

$$\dot{e} = \begin{bmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -F_1 & -F_2 & -F_3\end{bmatrix}\begin{bmatrix}e_1\\ e_2\\ e_3\end{bmatrix}$$

Hence, error matrix can be written as:

$$\dot{e} = Ae \tag{37}$$

Using eq (35) and eq (36), we have

$$\dot{e}_3 = -F_1e_1 - F_2e_2 - F_3e_3 \tag{38}$$

The control law $u(t)$ can be derived by comparing eq (34) and eq (38) which is given by:

$$\begin{aligned}u(t) &= \frac{1}{p_3x_1}[F_1e_1 + F_2e_2 + F_3e_3 + p_1^2\dot{x}_1 + 2p_1(\dot{x}_1x_2 \\ &\quad + x_1\dot{x}_2) - p_1\dot{x}_4 + p_1G_b\dot{x}_2 + \dot{x}_1x_2^2 + 2x_1x_2\dot{x}_2 - \dot{x}_2x_4 \\ &\quad - x_2\dot{x}_4 + p_2(\dot{x}_1x_2 + x_1\dot{x}_2) - p_3(\dot{x}_1x_3) - p_3x_1(-p_4(x_3 \\ &\quad - I_b)) + p_3I_b\dot{x}_1 - p_5\dot{x}_4]\end{aligned} \tag{39}$$

The control law $u(t)$ given by eq (39) ensures stability of the system. To check the asymptotic stability of the system, matrix A given by eq (37) must be a Hurwitz. As we know the design parameters F_1, F_2 and F_3 are positive constants which ensure matrix A is Hurwitz. Hence, all the errors e_1, e_2 and e_3 approach to zero therefore x_1 tracks x_{1ref}.

IV. SIMULATION RESULTS AND ANALYSIS

In these results the proposed controllers given by eqs (18) and (39) have been simulated in MATLAB/Simulink software. The horizontal axis represents time (*seconds*) while the vertical axis represents BGL (mg/dl). Initial value for the BGL is supposed to be 230 mg/dL for each proposed controller.

The numerical values of model parameters used in simulations for all the controllers have been taken form Table-1. A reference signal of 80 mg/dl has been defined for the tracking of BGL, as its optimum safe range is between $80 - 120\, mg/dl$ to avoid state of hyper-glycemia/hypoglycemia.

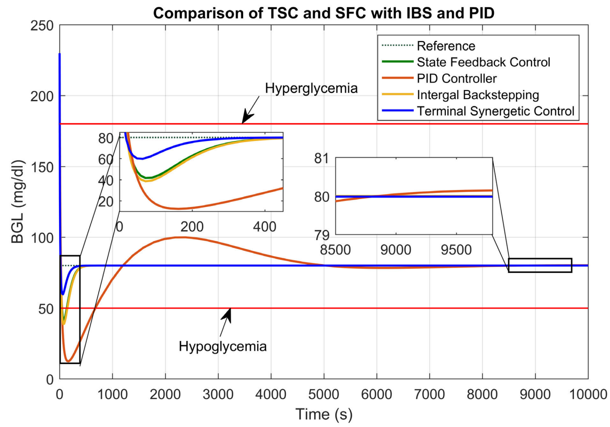

FIGURE 3. TSC and SFC comparison with IBS and PID Controller.

A brief comparison of all the proposed controllers with IBS (already proposed in the literature [23]) and PID controller has been made in Fig. 3 to analyze their output performance. It can be observed that TSC is tracking its reference level smoothly with almost zero steady state error. Initially it undergoes very short-timed transient with an undershoot of 21 mg/dl but then it converges to the reference level quickly with the settling time of almost 6.5 *mins*. In case of SFC, the undershoot of 38 mg/dl has been observed with the settling time of approximately 8.3 *mins*. In case of IBS, the steady state position has been attained in time 11.5 *mins* and an undershoot of 40 mg/dL. While it can be clearly observed that PID has an oscillatory behavior having larger settling time of 172 *mins* before going to steady state.

In this comparison, it has been observed that the output result in case of TSC is much improved with a faster settling time without any steady state error. The performance of SFC is also satisfying with good convergence speed and almost zero steady state error. So, the performance of TSC and SFC validates the purpose of this paper which is to provide a simple, computationally less costly and better solution for the problem as compared to recursive IBS. It has been deduced that the performance of PID is not satisfactory as compared to all the proposed controllers in terms of settling time, steady state error and overshoots/undershoots.

To check the proposed controllers how they behave when there is a noise in data, the BGL of the system has been perturbed by adding white noise given by Fig. 4 having noise

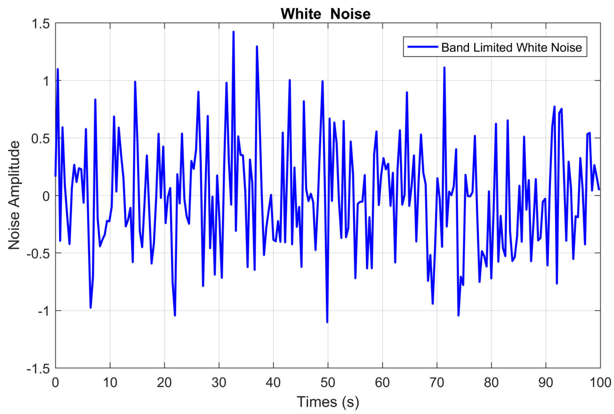

FIGURE 4. White Noise.

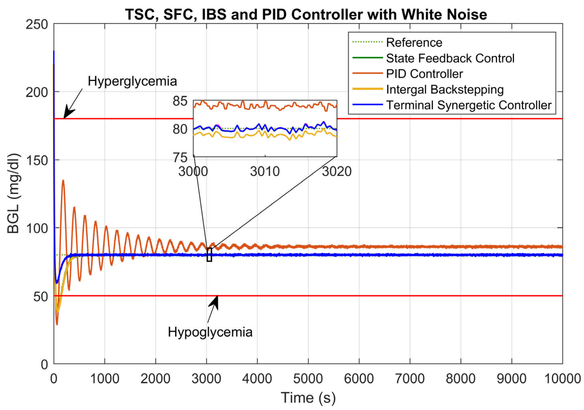

FIGURE 5. TSC, SFC, IBS and PID Controller performance with White Noise.

power of 1.6136 *dB* and sample time of 0.43 *seconds* with seed of [23341].

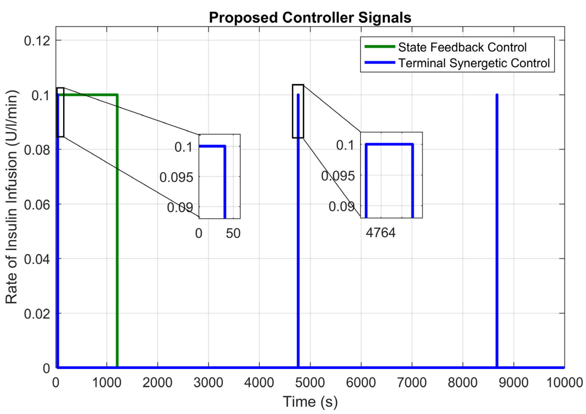

FIGURE 6. Proposed TSC and SFC Controller Signals.

In the presence of such perturbations TSC and SFC perform nicely not much affected by noise as shown in the Fig. 6. IBS exhibits some steady state error in its response with the presence of white noise. On the other hand PID controller performed unsatisfactory in the presence of white noise it exhibits oscillations of larger amplitude and steady state error.

The signals of the proposed TFC, ASC and SFC have been given in the Fig. (6). To avoid the control signal from getting infinite and to restrict the high input dose from controller, they have been limited by using saturation block in the simulations. The insulin dose input in AP must be restricted by practically using saturation within the controller such that BGL remains in the range $20-600\ (mg/dl)$, otherwise the death of the patient may occur if it is out of this range. Each controller calculates the required amount of insulin to be injected in the patient body and generate corresponding control signal using its mathematical algorithm. In the beginning, the input from controllers is high because the BGL is initially at higher level. After that as the BGL starts reducing and tracking the reference value, the control input is decreases and ultimately becomes zero as BGL is kept at desired level.

TABLE 2. Performance comparison of controllers.

Controller	Settling Time (min)	Steady State Error	Overshoots/ Undershoots (mg/dl)
TSC	6.5 or less	No	21
SFC	8.3 or less	No	38
IBS	11.5 or less	No	40
PID	172 or greater	Yes	68

A brief comparison for the performance of all controllers has been made in Table-2 to justify simulation results.

TABLE 3. Integral absolute error and control energy of controllers.

Controller	Integral Absolute Error (IAE)	Control Energy
TSC	1.8	0.00018
SFC	10.1	0.00101
IBS	1.9	0.00019
PID	$2.3\text{x}10^3$	0.23

TABLE 4. Patient parameter values.

System Parameters	Patient I	Patient II	Patient III
p_1	0	0	0
p_2	0.0107	0.0072	0.0142
p_3	$5.3\text{x}10^{-6}$	$2.16\text{x}10^{-6}$	$9.94\text{x}10^{-6}$
p_4	0.264	0.2465	0.2814
p_5	0.4	0.45	0.56
I_b	7	7	7
G_b	80	80	80
I_0	50	55	60
G_0	220	200	180
D_0	11.3	10	9.7

The Integral Absolute Error (IAE) and the control energy of the proposed controllers have been given in Table-3 for analysis about the energy consumption of each controller to achieve desired BGL. It is clear from the data of Table-3 that TSC has lowest IAE and its control energy exertion is also low. While on the other hand PID has higher IAE and energy

TABLE 5. Patient parameter values.

System Parameters	Patient IV	Patient V	Patient VI
p_1	0	0	0
p_2	0.083	0.0095	0.0230
p_3	3.3x10^{-6}	4.16x10^{-6}	10.2x10^{-6}
p_4	0.273	0.310	0.3140
p_5	0.49	0.50	0.59
I_b	7	7	7
G_b	80	80	80
I_0	57	63	58
G_0	200	210	190
D_0	9.9	9.2	10.8

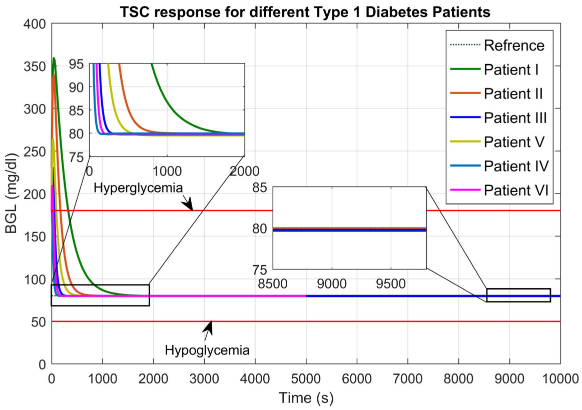

FIGURE 7. Tracking Response of TSC for different Type 1 patients.

consumption. SFC in terms of both the parameters is also better than PID but overall TSC is much better. Furthermore to observe the tracking response of the proposed TSC under different parametric conditions, we have considered data of six Type 1 diabetic patients available in the literature [27] given by the Table-4 and Table-5. The Fig. 7 shows that the TSC efficiently monitors and tracks the reference level of BGL for the data of these six Type 1 diabetic patients very nicely.

V. CONCLUSION

In this paper, two Lyapunov based nonlinear controllers: Terminal Synergetic and State Feedback Linearization based controller, have been proposed for the automatic tracking of BGL using Extended Bergman's Minimal Model. Their asymptotic stability has been proved by using Lyapunov theory. The performance of the proposed controllers has been analyzed by the simulations results using MATLAB/Simulink environment. From these results, it has been observed that all the proposed nonlinear controllers track the reference value quite nicely. The Terminal Synergetic Controller outperforms all the rest by having almost zero steady state error, lesser settling and convergence time with satisfactory undershoots/overshoots as compared to other controllers. In future, the proposed controllers can be implemented with unknown model parametric adaptation technique to enhance the controller's performance.

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