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Fractional Low-Order Moments Based DOA Estimation With Co-Prime Array in Presence of Impulsive Noise

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ABSTRACT Direction of arrival (DOA) estimation with co-prime array is a hot issue in array signal processing. By using co-prime array structure, the degrees of freedom has been greatly improved, which can estimate much more sources than that of the conventional array structure. In the scenario of impulsive noise, the high-order (2-order or 4-order) moments of received signal do not exist, fractional low-order moments (FLOM) can be applied. In this paper, the concept of the co-prime array is extended to be applied on FLOM of received signal in presence of impulsive noise. And then MUSIC algorithm is used for DOA estimation. The proposed method is tested on the numerical data. The simulation results prove the effectiveness of the proposed method in impulsive noise scheme.

INDEX TERMS Co-prime array, impulsive noise, fractional low-order moments (FLOM), direction of arrival (DOA).

I. INTRODUCTION

Direction of arrival (DOA) estimation is a long-lasting issue for decades, which is widely used in the field of radar, sonar, source positioning, wireless communication, *etc*. [1]–[3]. The enormous amount of methods has been proposed to estimate the DOAs of signals, such as subspace-based methods MUSIC and ESPRIT [4], [5]. These methods are usually applied on uniform linear array. For N sensors, the maximum number of the estimated sources is $N-1$ due to the limited degrees of freedom (DOFs). The distance between two adjacent sensors should be less than half wavelength of received signal in order to avoid the angle ambiguity. Nevertheless, the closely space sensors can cause the mutual coupling effect, which may decrease the estimation accuracy. Therefore, it is important for the array geometry design and optimization.

To solve the above mentioned problems, the concept of co-prime array has been proposed [6], [7]. The co-prime array applies on a co-prime pairs of two uniform linear sub-arrays, the number of corresponding sensors is M and N, respectively (M and N are co-prime pairs). The two sub-arrays share the first sensor, therefore, the total number of sensors are $M+N-1$. According to [7], the co-prime array structure can achieve $O(MN)$ DOFs, which is much higher than the number of physical sensors $M+N-1$. Therefore, the co-prime array structure greatly improves the number of detectable signals [8]–[10]. Meanwhile, for co-prime array, the distance between two adjacent sensors is greater than half wavelength of received signal, thus the mutual coupling effect can also be dropped. This technique attempts to vectorize the data covariance matrix of the second-order statistics or fourth-order cumulant of the received signal and construct a virtual array involving the steering vectors with an extended aperture.

In the DOA estimation with co-prime array, the noise is assumed to obey Gaussian distribution. However, in reality, the noise often exhibits non-Gaussian properties [11]–[13]. Impulsive noise [14]–[16] is the best representative that are frequently encountered in many practical wireless radio systems, while natural phenomena such as ice cracks and

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thunderstorms are also impulsive. The probability density function (PDF) of impulsive noise has thick tails than that of the Gaussian noise. As mentioned in [17], [18], α-stable distribution ($S\alpha S$, $0 < \alpha \leqslant 2$) is suitable to describe the impulsive noise. For impulsive noise, it is of infinite variance, its second or high order moments do not exist; nevertheless, it has finite fractional lower order statistics (FLOS) [19], such as fractional lower order moments (FLOM) [17] and phased fractional lower order moments (PFLOM) [20].

In this paper, we extend the co-prime DOA estimation to the scenario of impulsive noise. Contrary to the scenario of Gaussian noise (co-prime technique applies on the data covariance matrix/high order cumulant), this work focuses on the FLOS of received signal. By studying the properties of FLOS of received signal, the general idea of co-prime technique can then be adapted for DOA estimation in presence of impulsive noise. Moreover, after co-prime technique, the power of signals replaces the snapshots (a single snapshot). As the power of signals is constant value, the problem becomes coherent DOA estimation with a single snapshot. In this situation, a Toeplitz reconstruction based methods is applied for decorrelation. The main contributions of this paper are concluded as follows:

- Apply the co-prime on fractional low-order statistics of received signal, extend this technique in the scenario of impulsive noise.
- The proposed methods extend the co-prime array technique to the scheme of fractional low-order statistics, which provide new point of view on this technique.

The paper is organized as follows: Section 2 presents DOA estimation with co-prime array in presence of additive Gaussian white noise; in section 3, the co-prime array configuration is extended to the scheme of impulsive noise, and the FLOM and phase FLOM (PFLOM) is applied. The numerical results of the proposed method are provided in Sections 4, and conclusions are drawn in section 5.

Notations: upper-case (lower-case) bold characters represent matrices (vectors). $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^*$ stand for the transpose, conjugate transpose and conjugate, respectively. diag $(\cdot)$ and vec $(\cdot)$ denote the diagonal matrices and vectorization operation, respectively. $\otimes$ and $\odot$ denote Kronecker and Khatri-Rao products, respectively. I_N represents a $N \times N$ identity matrix and $\mathbb{E}$ denotes the expectation operator. $|\cdot|$ denotes the module of a complex value.

II. DOA ESTIMATION WITH CO-PRIME ARRAY IN PRESENCE OF ADDITIVE GAUSSIAN WHITE NOISE

A. SIGNAL MODEL WITH CO-PRIME ARRAY CONFIGURATION

As shown in Fig. 1, the co-prime array configuration is presented, where M and N $M < N$ are the co-prime pairs. Two sub-arrays are considered and the sensors are arranged as follows:

$$\mathbb{L} = \{Mnd\,|0 \leqslant n \leqslant N-1\} \cup \{Nmd\,|0 \leqslant m \leqslant 2M-1\}$$

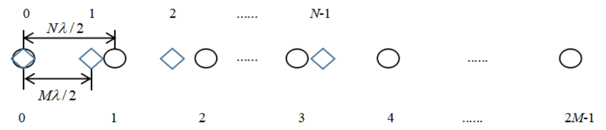

FIGURE 1. The co-prime array configuration.

where $d = \lambda/2$ with λ the wavelength of received signal. Define $\mathbf{l} = [l_1, \cdots, l_{2M+N-1}]$, $l_i \in \mathbb{L}$ as the positions of the sensors. It can be seen from Fig. 1 that the first sensor at the zeroth position is set to be the reference sensor ($l_1 = 0$) and is shared by two sub-arrays, therefore, the total number of the used sensors is $2M + N - 1$.

Assume K far field narrow band uncorrelated sources impinging on the co-prime array configuration shown in Fig. 1, the received signal vector [4] at time t ($t = 1, \cdots, T$, with t the number of snapshots) can be written as

$$\mathbf{x}(t) = \sum_{k=1}^{K} \mathbf{a}(\theta_k)s_k(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

with

- $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots s_K(t)]^T$ is the source vector.
- $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_K)]$ is the directional matrix.
- $\mathbf{a}(\theta_k) = \left[1, e^{j2\pi l_2 \sin(\theta_k)} \ldots, e^{j2\pi l_{2M+N-1} \sin(\theta_k)}\right]$ denotes direction vector corresponding to θ_k.
- $\mathbf{n}(t)$ is zero-mean Gaussian white noise with variance matrix $\sigma_n^2 \mathbf{I}_{2M+N-1}$.

Therefore, the data covariance matrix of received signal $\mathbf{x}(t)$ [4] can be expressed as

$$\mathbf{R}_x = \mathbb{E}\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\} = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma_n^2 \mathbf{I}_{2M+N-1}. \tag{2}$$

As signals are uncorrelated, the source covariance matrix $\mathbf{R}_s = \mathbb{E}\left\{\mathbf{s}(t)\mathbf{s}^H(t)\right\} = \text{diag}\left(\sigma_1^2, \cdots, \sigma_K^2\right)$ with σ_k^2 the power of the kth signal.

B. VECTORIZATION OPERATION

With the estimated data covariance matrix, the next step is to vectorize this matrix:

$$\mathbf{z} = \text{vec}(\mathbf{R}_x) = (\mathbf{A}^* \otimes \mathbf{A})\text{vec}(\mathbf{R}_s) + \sigma_n^2 \text{vec}(\mathbf{I}_{2M+N-1}). \tag{3}$$

For the uncorrelated signals, the source covariance matrix $\mathbf{R}_s$ is a diagonal matrix, therefore, (3) can be reformulated as

$$\begin{aligned}\mathbf{z} &= (\mathbf{A}^* \odot \mathbf{A})\mathbf{s} + \sigma_n^2 \text{vec}(\mathbf{I}_{2M+N-1}) \\ &= \mathbf{A}\mathbf{s} + \sigma_n^2 \mathbf{b}.\end{aligned}$$

where $\tilde{\mathbf{A}} = \left[\mathbf{a}^*(\theta_1) \otimes \mathbf{a}(\theta_1), \cdots, \mathbf{a}^*(\theta_K) \otimes \mathbf{a}(\theta_K)\right]$, $\mathbf{s} = [\sigma_1^2, \cdots, \sigma_K^2]^T$ is the single snapshot signal vector and $\mathbf{b} = \text{vec}(\mathbf{I}_{2M+N-1})$. The vector $\mathbf{z}$ can be regarded as the received signal from a virtual array, whose directional matrix is $\tilde{\mathbf{A}}$. According to [21], $\mathbf{z}$ contains a lot of redundant information. By removing these redundant items, we get new data information $\mathbf{z}_1$, which can be expressed as

$$\mathbf{z}_1 = \mathbf{A}_1\mathbf{s} + \sigma_n^2 b_1 \tag{4}$$

Therefore, by using the co-prime technique, the array aperture is greatly improved, and more signals can be estimated. However, after vectorization, the virtual signal $\mathbf{s}$ is a single snapshot. The rank of noise free data covariance matrix drops to 1, like coherent DOA estimation. In the situation, subspace-based methods, like MUSIC and ESPRIT fail to estimate the DOA of signals.

C. TOEPLITZ RECONSTRUCTION METHOD

Spatial smoothing techniques are the common way to decorrelate the correlation between signals, but with aperture loss [22]. In the section, a Toeplitz reconstruction method is applied [23] for decorrelation. The first step is to construct the uniform linear array. As mentioned in [10], the $[-M(N+1)+1, M(N+1)-1]$th elements of the virtual array (4) make up a uniform linear array with $2M(N+1)-1$ elements. The received signal on this uniform linear array can be written as

$$\tilde{\mathbf{z}}_1 = \tilde{\mathbf{A}}_1\mathbf{s} + \sigma_n^2\tilde{b}_1 \tag{5}$$

where $\tilde{\mathbf{A}}_1$ is the direction matrix of $2M(N+1)-1$ elements and $\tilde{b}_1$ is a $(2M(N+1)-1)\times 1$ vector.

Based on the received signal model from the constructed uniform linear array (5), the following step is to reconstruct a rank restored data covariance matrix by using Toeplitz reconstruction method [23]:

$$\mathbf{R}_T = \begin{bmatrix} \tilde{\mathbf{z}}(0) & \tilde{\mathbf{z}}(-1) & \cdots & \tilde{\mathbf{z}}(1-S) \\ \tilde{\mathbf{z}}(1) & \tilde{\mathbf{z}}(0) & \cdots & \tilde{\mathbf{z}}(2-S) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{z}}(S-1) & \tilde{\mathbf{z}}(S-2) & \cdots & \tilde{\mathbf{z}}(0) \end{bmatrix} \tag{6}$$

where $S = M(N+1)-1$. After applying Toeplitz reconstruction method, the rank restored data covariance matrix $\mathbf{R}_T$ is obtained. Subspace based method can then be applied for DOA estimation.

III. DOA ESTIMATION WITH CO-PRIME ARRAY IN PRESENCE OF IMPULSIVE NOISE

A. α STABLE DISTRIBUTION

When noise in (1) is impulsive noise, the data covariance matrix or high order cumulant of received signal does not exist. The performance of Gaussian white noise based DOA estimation methods can be significantly decreased. According to [17], [18], α-stable distribution ($S\alpha S$, $0 < \alpha \leqslant 2$) is suitable to describe the impulsive noise. The parameter α stands for the characteristic exponent of the $S\alpha S$ distribution, which is used to determine the sharpness of impulsive noise. The smaller the value of α, the thicker the tails. When $\alpha = 2$, $S\alpha S$ distribution becomes a Gaussian distribution, and Cauchy distribution for $\alpha = 1$. Therefore, with the change of α, $S\alpha S$ distribution with different probability density function (PDF) can be found.

As mentioned in [17], [18], impulsive noise with α-stable distribution has finite FLOM. Therefore, instead of using high order moments, in this paper, we extend the co-prime array technique to FLOM and PFLOM, and propose Toeplitz- FLOM, Toeplitz-PFLOM combined with MUSIC algorithm for DOA estimation.

B. TOEPLITZ-FLOM METHOD

According to the basic idea of FLOM presented in [17], the $(i,\ j)$th element of FLOM $\mathbf{R}_{\mathrm{FLOM}}$ can be defined as

$$R_{\mathrm{FLOM}}(i,\ j) = \mathbb{E}\left\{x_i(t)\left|x_j(t)\right|^{p-2}x_j^*(t)\right\}, \tag{7}$$

where $i, j = 1, \cdots, 2M+N-1$, $x_i(t)$ is the ith element of $\mathbf{x}$ and $1 < p < \alpha \leq 2$ is the order of the moments. The FLOM of received signal $\mathbf{R}_{\mathrm{FLOM}}$ can then be expressed as [11]

$$\mathbf{R}_{\mathrm{FLOM}} = \mathbf{A}_{\mathrm{FLOM}}\Lambda\mathbf{A}_{\mathrm{FLOM}}^H + \gamma\mathbf{I}_{2M+N-1} \tag{8}$$

where Λ is a fractional lower-order correlation matrix of signals; for the uncorrelated signals, Λ is a diagonal matrix. γ is a scalar, which based on statistics of the signal and noise components of the received signal vector and Λ.

By applying vectorization operation on $\mathbf{R}_{\mathrm{FLOM}}$, it can be deduced that

$$\begin{aligned}\mathbf{z}_{\mathrm{FLOM}} &= \mathrm{vec}(\mathbf{R}_{\mathrm{FLOM}}) \\ &= (\mathbf{A}_{\mathrm{FLOM}}^* \odot \mathbf{A}_{\mathrm{FLOM}})\mathbf{s}_{\mathrm{FLOM}} + \gamma\,\mathrm{vec}(\mathbf{I}_{2M+N-1}) \\ &= \tilde{\mathbf{A}}_{\mathrm{FLOM}}\mathbf{s}_{\mathrm{FLOM}} + \gamma\mathbf{b}\end{aligned} \tag{9}$$

where $\mathbf{s}_{\mathrm{FLOM}}$ is a $K \times 1$ vector, whose elements are the diagonal elements of matrix Λ. $\mathbf{z}_{\mathrm{FLOM}}$ can be considered as the received signal from a virtual array by co-prime array technique in the scenario of FLOM.

Similar to the case of additive Gaussian white noise, a uniform linear array ($2M(N+1)-1$ elements) is built by using the $[-M(N+1)+1, M(N+1)-1]$th elements of this virtual array. Therefore, the received signal can then be written as

$$\tilde{\mathbf{z}}_{\mathrm{FLOM}} = \tilde{\mathbf{A}}_{\mathrm{FLOM1}}\mathbf{s}_{\mathrm{FLOM}} + \gamma\tilde{\mathbf{b}} \tag{10}$$

where $\tilde{\mathbf{A}}_{\mathrm{FLOM1}}$ is the the $[-M(N+1)+1, M(N+1)-1]$th sub-matrix of $\tilde{\mathbf{A}}_{\mathrm{FLOM}}$. Then, by applying Toeplitz reconstruction method on $\tilde{\mathbf{z}}_{\mathrm{FLOM}}$, we construct the new data covariance matrix with restored rank. Afterwards, subspace-based method MUSIC can be used for DOA estimation.

C. TOEPLITZ-PFLOM METHOD

It can be seen from the definition of FLOM that the selection of exponent parameter α is restricted within interval $[1, 2]$. The proposed Toeplitz-FLOM fails when $\alpha \in [0, 1]$. Therefore, in this subsection, the PFLOM is applied with $\alpha \in [0, 2]$. According to [20], the (i, j)th element of PFLOM matrix $\mathbf{R}_{\mathrm{PFLOM}}$ can be expressed as

$$R_{\mathrm{PFLOM}}(i,\ j) = \mathbb{E}\left\{x_i^{\langle b\rangle}(t)\cdot x_j^{\langle -b\rangle}(t)\right\}. \tag{11}$$

For a complex value z, the operation $\langle b\rangle$ is defined as

$$z^{\langle b\rangle} = \begin{cases} \dfrac{|z|^{b+1}}{z^*}, & z \neq 0 \\ 0, & z = 0 \end{cases} \tag{12}$$

where $0 < b < \frac{\alpha}{2}$ is the order of the moments. Therefore, the PFLOM of received signal $\mathbf{R}_{\text{PFLOM}}$ can be expressed as

$$\mathbf{R}_{\text{PFLOM}} = \mathbf{A}_{\text{PFLOM}} \Phi \mathbf{A}_{\text{PFLOM}}^H + \gamma_p \mathbf{I}_{2M+N-1} \tag{13}$$

where Φ is the phase fractional lower-order correlation matrix of the signals, which is a diagonal matrix for uncorrelated signals. γ_p is a scalar, which based on statistics of the signal and noise components.

Similar to the Toeplitz-FLOM method, we apply vectorization operation on $\mathbf{R}_{\text{PFLOM}}$ and deduce that

$$\begin{aligned} \mathbf{z}_{\text{PFLOM}} &= \text{vec}(\mathbf{R}_{\text{PFLOM}}) \\ &= (\mathbf{A}_{\text{PFLOM}}^* \odot \mathbf{A}_{\text{PFLOM}}) \mathbf{s}_{\text{PFLOM}} \\ &\quad + \gamma_p \text{vec}(\mathbf{I}_{2M+N-1}) \\ &= \tilde{\mathbf{A}}_{\text{PFLOM}} \mathbf{s}_{\text{PFLOM}} + \gamma_p \mathbf{b} \end{aligned} \tag{14}$$

and

$$\tilde{\mathbf{z}}_{\text{PFLOM}} = \tilde{\mathbf{A}}_{\text{PFLOM1}} \mathbf{s}_{\text{PFLOM}} + \gamma_p \tilde{\mathbf{b}} \tag{15}$$

where $\mathbf{s}_{\text{PFLOM}}$ is a $K \times 1$ vector, whose elements are the diagonal elements of matrix Φ. $\tilde{\mathbf{A}}_{\text{PFLOM1}}$ is the $[-M(N+1)+1, M(N+1)-1]$th sub-matrix of $\tilde{\mathbf{A}}_{\text{PFLOM}}$. $\mathbf{z}_{\text{PFLOM}}$ is considered as the received signal from a virtual array by co-prime array technique in the scenario of PFLOM, and $\tilde{\mathbf{z}}_{\text{PFLOM}}$ is the received signal model from the $[-M(N+1)+1, M(N+1)-1]$th elements of this virtual array (a uniform linear array with $2M(N+1)-1$ elements). Then, we apply Toeplitz reconstruction method to restore the rank of the data covariance matrix of $\tilde{\mathbf{z}}_{\text{PFLOM}}$ and use MUSIC algorithm to esstimate the DOAs of incoming signals.

D. GENERAL STEPS OF THE PROPOSED METHOD

To conclude, the general steps of the Toeplitz-FLOM and Toeplitz-PFLOM are shown as following:

- Estimate the FLOM $\mathbf{R}_{\text{FLOM}}$ and PFLOM $\mathbf{R}_{\text{PFLOM}}$ of received signal by (7) and (11), respectively.
- Apply vectorization operation on FLOM and PFLOM of received signal, and obtain the signal models $\tilde{\mathbf{z}}_{\text{FLOM}}$ and $\tilde{\mathbf{z}}_{\text{PFLOM}}$ from virtual arrays with the improved aperture.
- Apply Toeplitz reconstruction method on $\tilde{\mathbf{z}}_{\text{FLOM}}$ and $\tilde{\mathbf{z}}_{\text{PFLOM}}$, and estimate the rank restore data covariance matrices.
- Apply MUSIC algorithm on rank restore data covariance matrices for DOA estimation.

IV. NUMERICAL RESULT

In this section, the performance of the proposed methods is tested on the simulated data with four different simulations. A co-prime array with two sub-arrays is considered. One is with 8 isotropic sensors ($M = 4$) and the distance between two adjacent sensors is equal to $5\lambda/2$ ($N = 5$); another is with 5 isotropic sensors and the distance between two adjacent sensors is equal to 2λ. Therefore, the total number of sensors of co-prime array configure is $2M + N - 1 = 12$. In the simulation, the sample sign covariance matrix (SCM) or normalized covariance matrix [11] combined with Toeplitz reconstruction method is taken as comparison (Toeplitz-SCM). The above mentioned methods share similar computational complexity in DOA estimation. Instead of signal to noise ratio (SNR), the generalized SNR (GSNR) [15] is applied in the scheme of impulsive noise as follows:

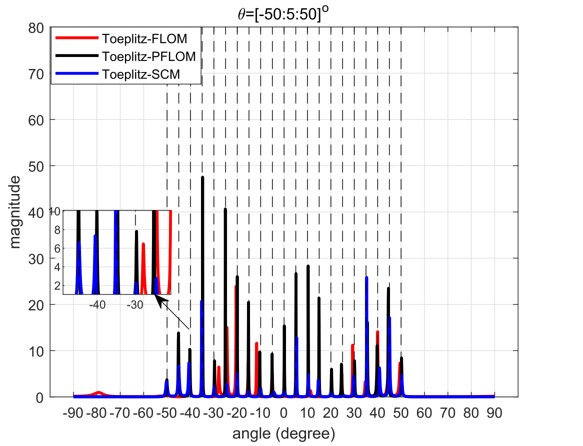

FIGURE 2. Case I pseudo-spectrum of Toeplitz-FLOM, Toeplitz-PFLOM and Toeplitz-SCM, GSNR = −5 dB, $\alpha = 1.2$.

$$\text{GSNR} = 10 \log \left(\frac{\mathbb{E}\left\{ |s(t)|^2 \right\}}{\gamma} \right).$$

A. FIRST SIMULATION

In the first simulation, the pseudo-spectrums of the proposed methods are estimated for multiple sources with different exponent parameter α. Two cases (I and II) are studied with $\alpha = 1.2$ and 0.6. The number of snapshots is 600.

Case I: 21 sources are studied, their DOAs range from -50° to 50° with an interval of 5°. The characteristic exponent $\alpha = 1.2$ and GSNR is fixed at -5 dB.

Fig. 2 plots the pseudo-spectrum of Toeplitz-FLOM, Toeplitz-PFLOM, Toeplitz-SCM. The peaks correspond to the estimated DOA of signals by three methods. It can be seen for Fig. 2 that Toeplitz-PFLOM and Toeplitz-SCM have better accuracy than that of Toeplitz-FLOM when $\alpha = 1.2$, since the Toeplitz-FLOM estimates the DOAs of signals with bias. In addition, the Toeplitz-PFLOM and Toeplitz-SCM shares similar performance in this situation.

Case II: 17 sources are studied, their DOAs range from -40° to 40° with an interval of 5°. The characteristic exponent $\alpha = 0.6$. Because Toeplitz-FLOM only works when $1 < \alpha < 2$. In this case, only Toeplitz-PFLOM and Toeplitz-SCM are considered. GSNR is set to be 0 dB and 5 dB.

Figs. 3-4 shows the pseudo-spectrum of Toeplitz-PFLOM and Toeplitz-SCM when GSNR = 0 dB and = 5 dB. When GSNR = 0 dB, Toeplitz-SCM is not able to detect all the DOAs of signals correctly, some peaks do not correspond to the true DOAs of signals. While Toeplitz-PFLOM remains robust in this situation. When GSNR= 5 dB, similar performance can be found.

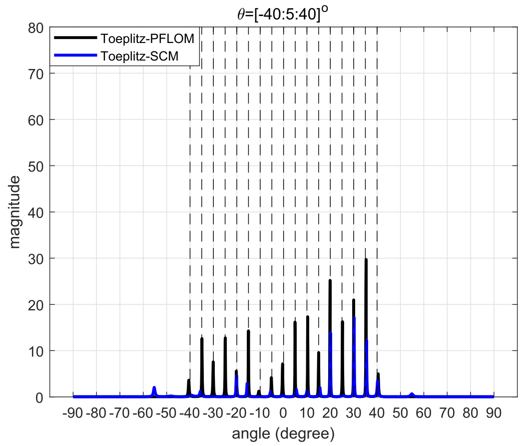

FIGURE 3. Case II pseudo-spectrum of Toeplitz-PFLOM and Toeplitz-SCM, GSNR = 0 dB, $\alpha = 0.6$.

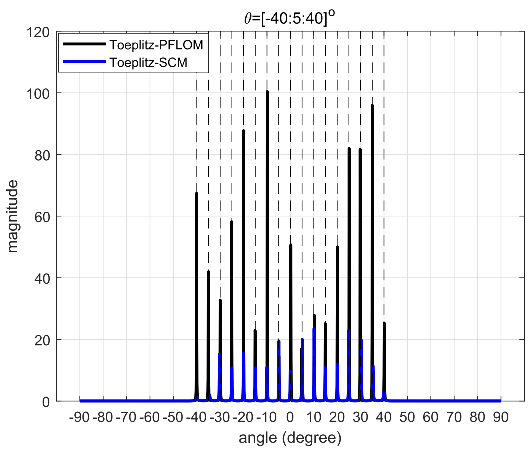

FIGURE 4. Case II pseudo-spectrum of Toeplitz-PFLOM and Toeplitz-SCM, GSNR = 5 dB, $\alpha = 0.6$.

B. SECOND SIMULATIONS

In the second simulation, the statistics performance of the proposed methods versus characteristic exponent α is assessed with a Monte-Carlo process of 500 independent runs. The Root Mean Square Error (RMSE) of the estimated DOA is defined as follows:

$$\mathrm{RMSE} = \sqrt{\frac{1}{KJ}\sum_{k=1}^{K}\sum_{j=1}^{J}\left(\hat{\theta}_{kj} - \theta_k\right)^2},$$

where $\hat{\theta}_{kj}$ denotes the estimated DOA of the kth incoming signal for the jth run of the algorithm, and J represents the total number of Monte-Carlo trials. 3 sources are considered, the corresponding DOAs are $10^\circ,\ 20^\circ,\ 30^\circ$, respectively. Two cases are studied: *a*. $1 < \alpha \leq 2$ and *b*. $0.1 < \alpha \leq 1$. GSNR is fixed at -5 dB, the number of snapshot is 600.

Case a: Fig. 5 indicates the RMSEs of estimated DOA versus α, $1 < \alpha \leq 2$ by Toeplitz-FLOM, Toeplitz-PFLOM and Toeplitz-SCM. It can be seen that RMSEs are continuously decreasing with α increases. The proposed Toeplitz-PFLOM provides the best performance with the lowest RMSE at each α among three methods, especially when α is small.

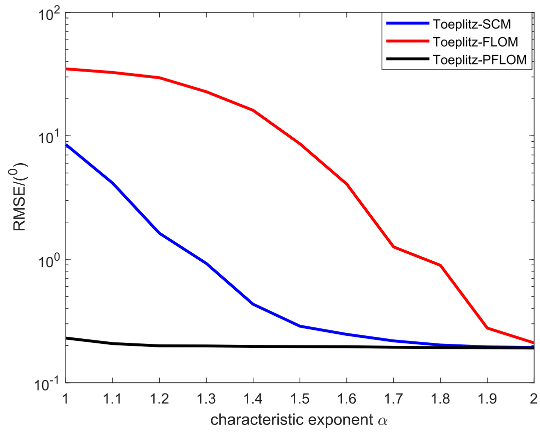

FIGURE 5. Case *a* RMSE of DOA estimation versus characteristic exponent α.

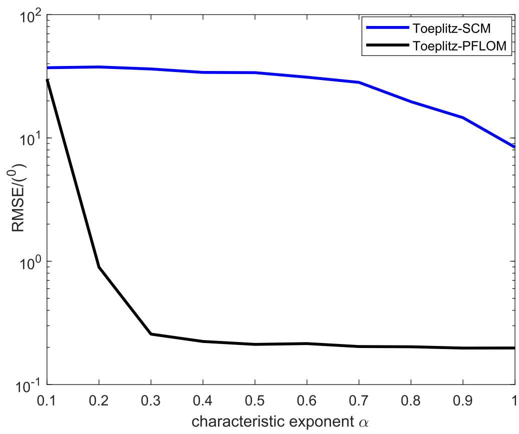

FIGURE 6. Case *b* RMSE of DOA estimation versus characteristic exponent α.

Case b: Only Toeplitz-PFLOM and Toeplitz-SCM are taken into consideration. Fig. 6 indicates the RMSEs of estimated DOA versus α, $0.1 < \alpha \leq 1$ by Toeplitz-PFLOM and Toeplitz-SCM. As shown in Fig. 6, similar to Case *a*, the RMSEs are also decreasing with α. The proposed Toeplitz-PFLOM performs better than that of Toeplitz-SCM within $0.1 < \alpha \leq 1$.

C. THIRD SIMULATION

In the third simulation, decorrelation method spatial smoothing is applied. Then the methods can be called SS-FLOM and SS-PFLOM, respectively. Their performance is compared with that of Toeplitz-FLOM and Toeplitz-PFLOM. The simulation parameter is set as follows: 3 sources are considered with DOAs $[10^\circ,\ 20^\circ,\ 30^\circ]$, characteristic exponent $\alpha = 1.2$ and snapshots $T = 600$. GSNR varies from -10 to 0 dB with a Monte-Carlo process of 500 independent runs.

Fig. 7 plots the performance of Toeplitz-FLOM, Toeplitz-PFLOM, SS-FLOM and SS-PFLOM. Similar performance can be found because spatial smoothing and Toeplitz reconstruction have similar decorrelation power.

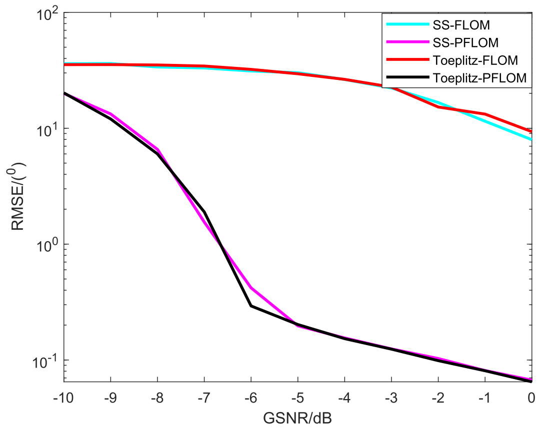

FIGURE 7. RMSE of Toeplitz-FLOM, Toeplitz-PFLOM, SS-FLOM and SS-PFLOM versus GSNRs.

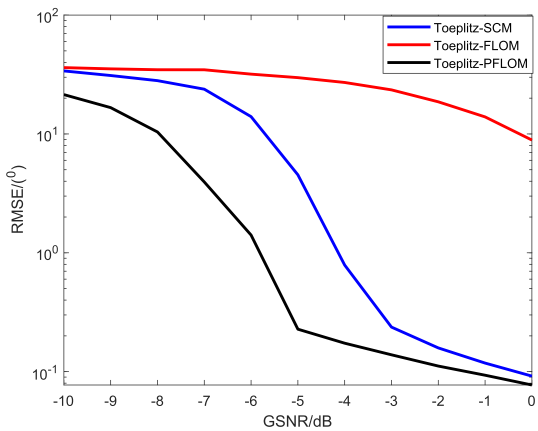

FIGURE 8. RMSE of the estimated DOAs versus GSNRs, $\alpha = 1.2$.

D. FOUR SIMULATION

In the fourth simulation, the statistics performance of the proposed methods versus GSNRs with a Monte-Carlo process of 500 independent runs. 3 incoherent sources are considered with DOAs [10°, 20°, 30°]. The characteristic exponent $\alpha = 1.2$ and 0.6, respectively.

1) RMSE VERSUS GSNR

In this case, the number of snapshots is set to be 600. GSNR varies from -10 to 0 dB. Figs. 8-9 show the RMSEs of the estimated DOAs of signals versus GSNR with $\alpha = 1.2$ and 0.6, respectively. As expected, when $\alpha = 1.2$ and 0.6, the RMSEs is decreasing with GSNR increases for all methods. The proposed Toeplitz-PFLOM is much more robust to the impulsive noise (with lower RMSE) than that of Toeplitz-FLOM and Toeplitz-SCM, especially at low GSNR level. With high GSNR, the performance of the above mentioned methods tends towards the same.

2) RMSE VERSUS NUMBER OF SNAPSHOTS

In the simulation, the performance of the proposed methods versus the number of snapshots is evaluated with $\alpha = 1.2$ and 0.6. GSNR is set to be -5 dB. Figs. 10-11 provide the RMSEs of the estimated DOAs of signals as function of

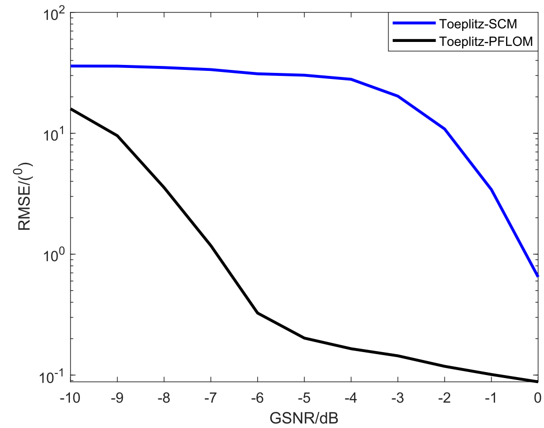

FIGURE 9. RMSE of the estimated DOAs versus GSNRs, $\alpha = 0.6$.

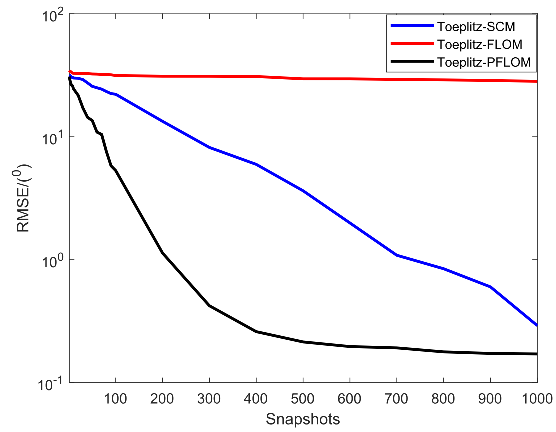

FIGURE 10. RMSE of the estimated DOAs versus snapshots, $\alpha = 1.2$.

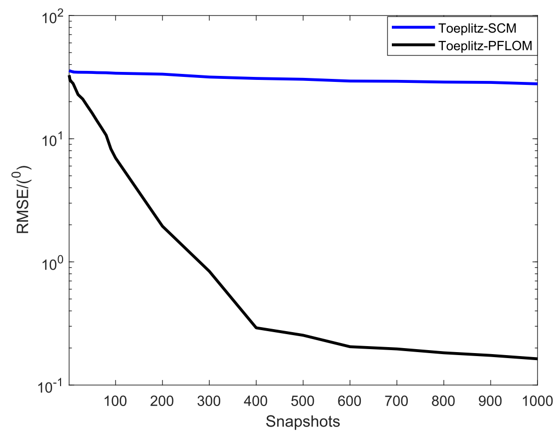

FIGURE 11. RMSE of the estimated DOAs versus snapshots, $\alpha = 0.6$.

number of snapshots. when the number of snapshots is small, all the methods fail to estimate the DOAs of signals. With an increasing number of snapshots, the RMSEs decrease. The proposed Toeplitz-PFLOM has a more significant decrease of the RMSE than that of other methods.

It can be concluded that the proposed Toeplitz-PFLOM has good accuracy for DOA estimation of co-prime array configuration in the scenario of $S\alpha S$ distribution noise (impulsive noise) with various of α, GSNR and number of snapshots.

V. CONCLUSION

In this paper, two methods: Toeplitz-FLOM and Toeplitz-PFLOM are proposed for DOA estimation with co-prime array configuration in presence of impulsive noise. The proposed methods extend the co-prime array technique to the scheme of fractional low-order statistics, which provide new point of view on this technique. The high order statistics based co-prime array techniques may also be adapted to the scenario of fractional low-order statistics. Simulation results demonstrate the effectiveness of the proposed methods in a wide range of environment of impulsive noise.

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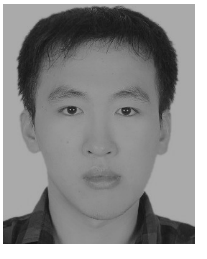

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