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Probing Homotopy Continuation Method for Solving Nonlinear Magnetic Network Equations

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ABSTRACT Nonlinear magnetic network is an effective model to solve the magnetic field of the motor. For the strong nonlinearity of magnetic network parameters, it is difficult to solve the nonlinear magnetic network equations. To overcome the divergent issue for solving the nonlinear magnetic network equations, a new method, the homotopy continuation method, is proposed. Based on the benchmark model of Testing Electromagnetic Analysis Methods Problem 20 (TEAM P20), the two-dimensional nonlinear magnetic network model is established. Next, homotopy equations of nonlinear magnetic network equations are derived, and they are solved by the homotopy continuation method. So that solving the nonlinear magnetic network equations is transformed into solving a set of homotopy equations. Then, the magnetic flux density of iron core with different saturation levels is calculated using the simple iteration method, the relaxation iteration method, Newton-Raphson method, and the homotopy continuation method, respectively. The results and convergence performance of the four methods are compared. It is proved that the proposed method can effectively expand the convergence domain and simplify the calculation process. The magnetic flux density calculated by the homotopy continuation method is compared with the finite element method in the supersaturated state, and the results are in good agreement. The relative error of the magnetic flux density of the core is less than 5%, which verifies the correctness of the proposed algorithm.

INDEX TERMS Nonlinear magnetic network, homotopy continuation method, TEAM P20, nonlinear magnetic field calculation.

I. INTRODUCTION

For electrical machines and transformers, if the current is passed through the coil, a magnetic field will be formed in the space around it. Due to the saturation characteristics of the core material, the magnetic field is nonlinear. The non-linearity of electromagnetic is a problem that scholars pay attention to. Paese *et al.* presented a method for calculating the magnetic flux density. The equivalent circuit method is a simple and straightforward numerical method that can be used to solve non-linear electromagnetic problems and can also be extended to rotary machines [1]. Zheng and Chen proposed a subspace correction method (SCM) for solving the multi-scale eddy current problem in the lamination system, which is very efficient for large-scale simulations [2]. J. Bao *et al.* proposed a hybrid model which combines Fourier modeling and magnetic equivalent circuits for the analysis of the magnetostatic field in saturated electrical machines and verified its diverse applicability to linear and rotary machines [3]. Newton-Raphson (N-R) method is a traditional method to solve nonlinear magnetic network (NMN) equations [4], which is also widely used in the finite element method (FEM) to calculate magnetic field [5], [6]. However, the key of N-R method is to solve the Jacobian matrix consisting of many partial derivatives, which have to be recalculated in each iteration. The relaxation iteration method is also a common method for solving nonlinear magnetic network equations [7], but the improper selection of relaxation factor can also lead to divergent results. Otherwise, the simple iteration method is only suitable for an unsaturated magnetic network. To solve the above problems, we took the TEAM P20 benchmark model as an example and calculated the

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nonlinear magnetic network with the homotopy continuation method for the first time.

In recent years, many scholars have studied nonlinear magnetic networks. H. Dogan *et al.* put forward a multistatic reluctance network model of permanent magnet synchronous machines and presented three different methods to calculate the air-gap reluctance [8]. Z. Q. Zhu *et al.* analysed the performance of a flux switching permanent motor (FSPM), and established a nonlinear lumped parameter magnetic circuit model for FSPM with doubly salient structure [9]. Ling Ding *et al.* presented a novel equivalent magnetic network model of permanent magnet machines, and the air-gap network is composed of rhombic elements, which can consider the rotation of permanent magnet machines [10]. Yunkai Huang *et al.* built the magnetic equivalent circuit of the stator, rotor, and permanent magnet of an axial flux permanent magnet machine, and the computational time of the proposed model is greatly reduced [11]. Wei Sun *et al.* established a parameterized dynamic magnetic equivalent circuit model, and analysed the dynamic characteristic of a novel axial-field switched reluctance motor whose rotor is segmented [12]. However, all the above mentioned did not derive the calculation method of the nonlinear magnetic network in detail, nor discussed the convergence performance of the results.

Based on the nonlinear magnetic network, Jae-Jun Lee *et al.* designed a 140 kW-class wound field synchronous motor for the electric vehicle and used the Newton-Raphson method to calculate the normalized torque values of the electric machine [13]. H. W. Derbas *et al.* established nodal-based and mesh-based magnetic equivalent circuit models of a claw-pole alternator, and derived the Newton-Raphson algorithm of both model; however, the results of the nodal-based magnetic network equations failed to converge under nonlinear operating conditions [14]. Ahmed Hemeida *et al.* given the Newton-Raphson iteration scheme of the equivalent reluctance network, and the correctness of the proposed method is verified by a 16-pole permanent magnet synchronous machine [15]. Nian Li *et al.* adopted the Newton-Raphson method to solve the nonlinear magnetic network equations of the axial field flux-switching memory machine; however, they did not describe the Jacobian matrix in detail [16]. P. Naderi *et al.* derived the Jacobian matrix in [17], and they used the Newton-Raphson method to calculate the performance under interturn short circuit fault of a salient pole synchronous motor by using magnetic equivalent circuit model, which considered the saturation effect. In fact, the scalar magnetic potential is solved by the nonlinear magnetic network equations, but the scalar potential often leads to oscillation and even divergence in Newton-Raphson iterative process [18]. The relaxation iteration method was utilized to calculate the nonlinear magnetic network in [19], which simplified the nonlinear iterative process. Gaohong Xu *et al.* deformed the iteration scheme based on the relaxation iteration method and constrained the maximum magnetic flux density to accelerate the convergence speed [20]. To improve the flexibility of modeling, Donghui Cao *et al.* proposed a magnetic network meshing for the oblique region. They used the successive over relaxation method, which is an advanced Gauss iteration method, to solve the magnetic network equations [21]. In the practical calculation, the selection of relaxation factors depends on experience, and improper relaxation factors can also lead to divergent results.

However, how to ensure effective convergence and expand the convergence domain of the results of nonlinear magnetic network equations have not been discussed in the existing literature. The homotopy continuation method is an effective method for calculating nonlinear equations [22]. The principle is to introduce a continuation parameter to construct a family of homotopy equations so that the nonlinear equations start from a new system that is easy to calculate. Track towards the solution branch of each system and the solution of the original nonlinear equation is finally obtained. The homotopy continuation method can extend the convergence domain, and it is widely used in science and engineering. Abbasbandy used the homotopy analysis method to solve nonlinear equations arising in heat transfer, which provided a convenient way to control the convergence [23]. Dinesha first applied the homotopy algorithm to the dynamic simulation of large power systems [24]. A homotopy continuation method was proposed for the finite element model of indirect coupling of nonlinear electric field and temperature field in [25], and a good convergence effect was obtained. Yong and Preindl designed a novel unified position sensorless observer of permanent magnet synchronous motors based on the homotopy continuation method to identify the correct position and speed [26]. The homotopy continuation method is widely used in the power flow calculation of power systems [27], [28]. Mehta applied the homotopy continuation method to power systems and extended the method to transient stability assessment, voltage stability analysis, and other problems [29]. In this paper, we used the homotopy continuation method to calculate nonlinear magnetic network equations for the first time. By solving the homotopy equations, the convergence domain of nonlinear calculation is extended effectively, and the complicated solving process of the Newton-Raphson iteration method is avoided.

TEAM P20 is one of the benchmark model officially proposed by the International Compumag Society (ICS), and the problem is to analyse the magnetic field and electromagnetic force [30]. In this paper, the nonlinear magnetic network model of the TEAM P20 benchmark model is developed. We write a computer program of the homotopy continuation method by Matlab to solve the nonlinear magnetic network equations. The solutions are a set of data, which are the average magnetic flux density of each branch in the model. The results and iterative process of the proposed method are compared with those of the simple iteration method, relaxation iteration method, and Newton-Raphson method. It is proved that the proposed method can effectively expand the convergence domain and simplify the calculation process. Finally, Magnet 7.5, a finite element analysis software, is used to calculate the 2-D magnetic field of the TEAM P20 benchmark model. The results of the proposed algorithm are compared with those of the FEM, and the results are basically the same, which verify the effectiveness of the proposed algorithm.

II. STRUCTURE AND MATHEMATIC MODEL

A. PHYSICAL MODEL

TEAM P20 benchmark model [30] is shown in Fig.1. The model consists of two parts: the center pole and the yoke made of silicon steel. The coil is around the center pole and excited by direct current, and the gap between the silicon steel and the coil is filled with insulation.

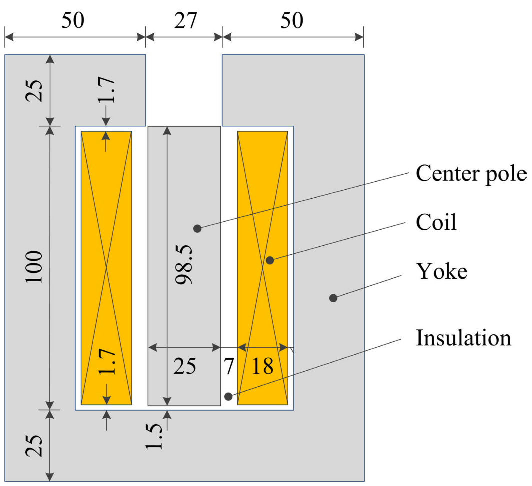

FIGURE 1. Physical model of TEAM P20.

The magnetic network model combines the advantages of the magnetic circuit theory and the finite element method to solve the magnetic field of motors. This model makes up for the shortcomings of the low accuracy of the magnetic circuit method and overcomes the disadvantages of occupying a large computer capacity and taking more time using the finite element method.

In order to establish the calculation model, the following assumptions are indicated:

1) The effect of temperature on permeability and conductivity is ignored.
2) Core eddy current loss and hysteresis effect are ignored.
3) Each magnetic permeability element is equivalent to a rectangle.
4) It is assumed that the flux directions are only horizontal and vertical.
5) The magnetic field lines are uniformly distributed in the axial direction, regardless of the axial variation.

According to the hypotheses and characteristics of the material and structure of TEAM P20, the nonlinear magnetic network model, as shown in Fig. 2, is established. The black permeance is ferromagnetic permeance, and the rest is air permeance. Due to the saturation of ferromagnetic material, the ferromagnetic permeance is nonlinear. Otherwise,

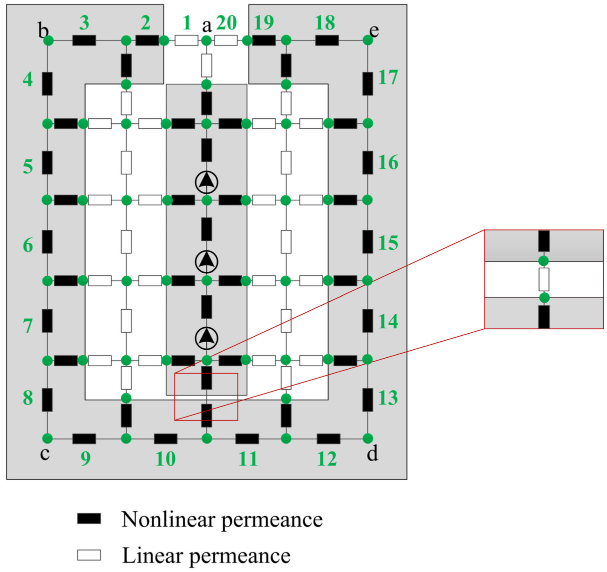

FIGURE 2. Nonlinear magnetic network of TEAM P20.

three magnetomotive force (MMF) sources are uniformly distributed on the center pole.

B. MATHEMATIC MODEL

The magnetic network is similar to the electric network in mathematical expression, and the principle and analysis methods are similar to those of circuit theory. According to Kirchhoff law, the nodal magnetic potential matrix equation can be written as

$$\boldsymbol{G}_{n\times n}\boldsymbol{\varphi}_{n\times 1} = \boldsymbol{\Phi}_{sn\times 1} \tag{1}$$

where $\boldsymbol{G}_{n\times n}$ is the nodal permeance matrix, $\boldsymbol{\Phi}_{sn\times 1}$ is the flux matrix, and $\boldsymbol{\varphi}_{n\times 1}$ is the nodal magnetic potential matrix, which is unknown.

The augmented matrix $\boldsymbol{G}_{(n+1)\times(n+1)}$ of the nodal permeance matrix $\boldsymbol{G}_{n\times n}$ can be expressed as

$$\boldsymbol{G}_{(n+1)\times(n+1)} = \begin{pmatrix} g_{1,1} & \cdots & \cdots & \cdots & g_{1,j} & \cdots & g_{1,n+1} \\ \vdots & \ddots & & & \vdots & ⋰ & \vdots \\ \vdots & & g_{i,i} & \cdots & g_{i,j} & & \vdots \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ g_{j,1} & \cdots & g_{j,i} & \cdots & g_{j,j} & & \vdots \\ \vdots & ⋰ & & & & \ddots & \vdots \\ g_{n+1,1} & \cdots & \cdots & \cdots & \cdots & \cdots & g_{n+1,n+1} \end{pmatrix} \tag{2}$$

where $g_{i,j}$ is the mutual permeance between node i and node j, which is negative. $g_{i,i}$ is the self-permeance of node i, and it is equal to the opposite of the sum of the mutual permeability between node i and the other nodes.

Since there are only n independent nodes in the equations, the last row and the last column of (2) are removed to obtain the nodal permeability matrix $\boldsymbol{G}_{n\times n}$,

$$\boldsymbol{G}_{n\times n}=\begin{pmatrix} g_{1,1} & \cdots & \cdots & \cdots & g_{1,j} & \cdots & g_{1,n} \\ \vdots & \ddots & & & \vdots & \iddots & \vdots \\ \vdots & & g_{i,i} & \cdots & g_{i,j} & & \vdots \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ g_{j,1} & \cdots & g_{j,i} & \cdots & g_{j,j} & & \vdots \\ \vdots & \iddots & & & & \ddots & \vdots \\ g_{n,1} & \cdots & \cdots & \cdots & \cdots & \cdots & g_{n,n} \end{pmatrix} \quad (3)$$

The permeance can be calculated by (4),

$$g=\mu_0\mu_r S/l \quad (4)$$

where μ_0 is the permeability of vacuum, μ_r is the relative permeability, S is the cross-sectional area of the flux through the permeance, and l is the length of the permeance along the magnetic flux direction. The calculation method of series and parallel permeance is similar to that of conductance in the circuit. The relative permeability of vacuum is 1, and the relative permeability of each ferromagnetic permeance corresponds to the B-H curve.

$\boldsymbol{\Phi}_{sn\times 1}$ is the flux matrix, which can be expressed as

$$\boldsymbol{\Phi}_{sn\times 1}=\left[\sum \Phi_{s1} \quad \ldots \quad \sum \Phi_{si} \quad \ldots \quad \sum \Phi_{sn}\right]^T \quad (5)$$

where $\sum \Phi_i$ represents the total flux of all magnetic flux sources flowing into node i. Analogous to the current source, the expression of the magnetic flux source can be written as

$$\Phi_s=F\times g \quad (6)$$

where $F = NI_f$ is the MMF. N is the turns number of excitation coil, I_f is the excitation current, g is the permeance of the same branch as the MMF.

The nodal magnetic potential φ_i can be obtained by (1), and then the magnetic flux $\phi_{i,j}$ and the average magnetic flux density $B_{i,j}$ between node i and node j can be obtained.

III. CALCULATION METHOD OF NONLINEAR MAGNETIC NETWORK EQUATIONS

A. HOMOTOPY CONTINUATION METHOD

1) CONSTRUCTION OF HOMOTOPY EQUATIONS

The key to the homotopy continuation method is to construct homotopy equations. For constructing homotopy equations, it is necessary to introduce the continuation parameter $t \in [0,1]$ into the equations, where homotopy equations begin with an initial equation $f_0(x)=0$ that is easy to solve with $t=0$, and gradually transitions to the original nonlinear equation $f(x)=0$ with $t=1$.

As for nonlinear equations (1), $\boldsymbol{G}$ and $\boldsymbol{\Phi}$ are both related to the relative permeability $\boldsymbol{\mu}_r$. On the other hand, $\boldsymbol{\mu}_r$ are related to the magnetic flux density $\boldsymbol{B}$. Thus, original equations (1) are equivalent to

$$\boldsymbol{F}(\boldsymbol{B})=\boldsymbol{G}(\boldsymbol{\mu}_r(\boldsymbol{B}))\boldsymbol{\varphi}-\boldsymbol{\Phi}_s(\boldsymbol{\mu}_r(\boldsymbol{B}))=\boldsymbol{0} \quad (7)$$

where $\boldsymbol{F}$ are functions defined on a certain space D of $\boldsymbol{R}^n$.

It is easy to know that the nonlinearity of (7) is caused by the nonlinearity of the relative permeability μ_r. If μ_r is a constant, (7) is a linear system. According to the continuation theory, the continuation parameter t is introduced to set up a family of functions $\boldsymbol{H}(\boldsymbol{B},t): D\times[0,1]\subset \boldsymbol{R}^{n+1}\rightarrow \boldsymbol{R}^n$ to instead of functions $\boldsymbol{F}$. Functions $\boldsymbol{H}$ meet the conditions of

$$\boldsymbol{H}(\boldsymbol{B},0)=\boldsymbol{F}_0(\boldsymbol{B}) \quad (8)$$

$$\boldsymbol{H}(\boldsymbol{B},1)=\boldsymbol{F}(\boldsymbol{B}) \quad (9)$$

where the solution $\boldsymbol{B}_0$ of $\boldsymbol{F}_0(\boldsymbol{B})=\boldsymbol{0}$ are known, and the corresponding relative permeability is constant. Therefore, $\boldsymbol{H}(\boldsymbol{B},0)=\boldsymbol{0}$ are linear equations. $\boldsymbol{H}(\boldsymbol{B},1)=\boldsymbol{0}$ are equivalent to (7). Then the original problem can be converted to solve homotopy equations (10).

$$\boldsymbol{H}(\boldsymbol{B},t)=\boldsymbol{0},\ t\in[0,1] \quad (10)$$

According to the above rules, homotopy equations of (7) can be established as (11),

$$\begin{aligned}\boldsymbol{H}(\boldsymbol{B},\boldsymbol{t})=\boldsymbol{G}(\boldsymbol{\mu}_{r0}+t(\boldsymbol{\mu}_r(\boldsymbol{B})-\boldsymbol{\mu}_{r0}))\boldsymbol{\varphi}\\ -\boldsymbol{\Phi}_s(\boldsymbol{\mu}_{r0}+t(\boldsymbol{\mu}_r(\boldsymbol{B})-\boldsymbol{\mu}_{r0})),\ t\in[0,1]\end{aligned} \quad (11)$$

where t is the continuation parameter, and $\boldsymbol{\mu}_{r0}$ are the initial relative permeability.

In this way, solving the nonlinear magnetic network equations is transformed into solving a set of homotopy equations (11).

2) SOLVING HOMOTOPY EQUATIONS

The flowchart of solving the nonlinear magnetic network based on the homotopy continuation method is shown in Fig. 3. The continuation parameter t is divided into n parts in the interval [0,1], where $0=t_0<\cdots<t_i<\cdots<t_n=1$, and n is the number of continuation. Solving homotopy equations by continuation method is a dynamic process of t from 0 to 1. When $t=0$, equations (11) are equivalent to

$$\boldsymbol{H}(\boldsymbol{B},t_0)=\boldsymbol{G}(\boldsymbol{\mu}_{r0})\boldsymbol{\varphi}-\boldsymbol{\Phi}_s(\boldsymbol{\mu}_{r0})=0, \quad (12)$$

and obviously (12) are linear equations, which can be calculated easily. Then the results are taken as the initial value $\boldsymbol{B}_1^{(0)}$ of the first continuation. In the same way, the solution $\boldsymbol{B}_i$ of $\boldsymbol{H}(\boldsymbol{B},t_i)=0$ at the i-th continuation are used as the initial value of the next continuation equations $\boldsymbol{H}(\boldsymbol{B},t_{i+1})=0$. The final solution $\boldsymbol{B}_n$ are the true solution $\boldsymbol{B}^*$ of the original equations.

In order to accelerate the convergence speed, the Steffensen iteration method is used in each continuation process, and its iteration scheme is

$$\boldsymbol{B}_i^{(k+3)}=\boldsymbol{B}_i^{(k)}-\frac{(\boldsymbol{B}_i^{(k+2)}-\boldsymbol{B}_i^{(k+1)})^2}{\boldsymbol{B}_i^{(k+2)}-2\boldsymbol{B}_i^{(k+1)}+\boldsymbol{B}_i^{(k)}} \quad (13)$$

where i is the number of continuations and k is the iteration number.

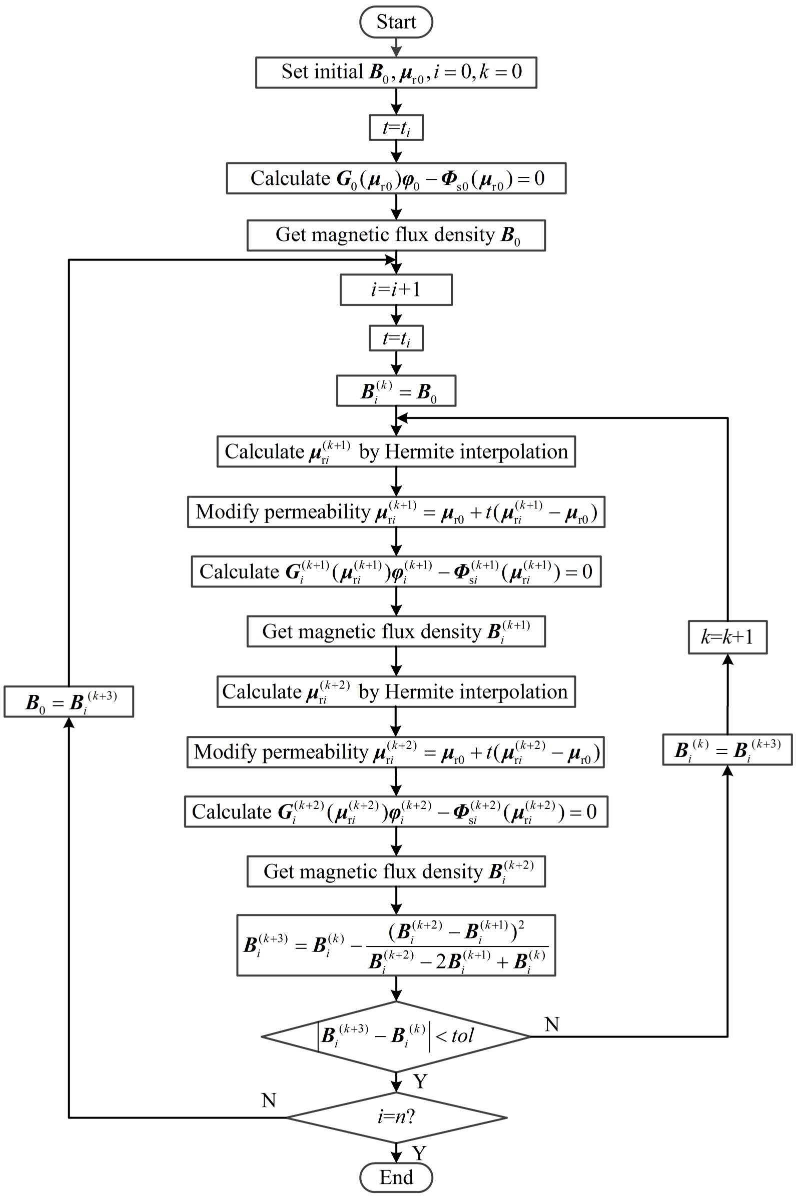

FIGURE 3. **Flowchart of homotopy continuation method.**

When the Steffensen convergence condition is satisfied, one continuation process ends, and the next continuation process begins. Until $t = 1$, the calculation stops when meeting the convergence condition. The Steffensen convergence condition is shown as

$$| \boldsymbol{B}_i^{(k+3)} - \boldsymbol{B}_i^{(k)} | < tol \tag{14}$$

where tol is the tolerance. We chose 1×10^{-6} as the tolerance in this paper.

It can be known from the B-H curve that the relative permeability μ_r changes with the change of magnetic flux density. The nonlinearity of the magnetic network equations is caused by the nonlinearity of μ_r of the ferromagnetic material. The higher degree of nonlinearity, the less likely it is that the calculation results will converge. The homotopy continuation method starts with a simple solution of the physical field; that is, the calculation starts from a linear magnetic field that does not consider saturation and gradually transitions to calculate a non-linear magnetic field, and finally, the distribution of the magnetic field is obtained.

B. OTHER METHODS TO CALCULATE NONLINEAR MAGNETIC NETWORK

Simple iteration, also called fixed-point iteration, is a conventional method to calculate nonlinear equations [31]. The expression of simple iteration is

$$\boldsymbol{B}^{(k+1)} = f(\boldsymbol{B}^{(k)}), \quad k = 0, 1, 2, \ldots, N \tag{15}$$

$$\boldsymbol{B}^{*} = \boldsymbol{B}^{N} \tag{16}$$

where $\boldsymbol{B}^{(k)}$ is the k-th iteration, and $f(\boldsymbol{B}^{(k)})$ is the iterated function. When $\boldsymbol{B}^{(N+1)}$ and $\boldsymbol{B}^{(N)}$ are infinite approximation, $\boldsymbol{B}^{*} = \boldsymbol{B}^{(N)}$ can be regarded as the true solution.

Relaxation iteration is an accelerated iteration method, and the iteration scheme is

$$\boldsymbol{B}^{(k+1)} = \boldsymbol{B}^{(k)} + \omega(\boldsymbol{B}^{(k+1)} - \boldsymbol{B}^{(k)}) \tag{17}$$

where ω is the relaxation factor. In order to compare the convergence performance of different methods under different saturation levels more accurately, the same relaxation factor should be selected for calculation. We chose 0.5 as the relaxation factor in this paper.

Newton-Raphson iteration method is a traditional method to solve nonlinear magnetic network equations. The iteration scheme of the Newton-Raphson method is

$$\varphi_{n+1} = \varphi_n - [J(\varphi_n)]^{-1}\boldsymbol{f}(\varphi_n) \tag{18}$$

$$\boldsymbol{f}(\varphi_n) = \boldsymbol{G\varphi} - \boldsymbol{\Phi}_s \tag{19}$$

where J is the Jacobian matrix.

IV. CALCULATION RESULTS AND VERIFICATION

A. DIFFERENT METHODS TO CALCULATE NONLINEAR MAGNETIC NETWORK

The saturation level of the core is affected by the excitation current flowing in the coil. Simple iteration, relaxation method, Newton-Raphson method, and homotopy continuation method are compared to solve the nonlinear magnetic network at different saturation levels. By changing the excitation current I_f, the MMF can be changed to simulate different iron core saturation levels.

1) UNSATURATED STATE

When the total MMF is 600A, the iron core is unsaturated. The simple iteration, the relaxation method, Newton-Raphson method, and the homotopy continuation method are compared to solve the nonlinear magnetic network equations. The iterative processes of the maximum magnetic flux density calculated by different methods are shown in Fig. 4. All four methods obtain convergent solutions. The simple iteration method converges first, the relaxation method and the hotomopy continuation method require more iterations, while the Newton-Raphson method needs the most time to converge. For easy to analyse, we selected the path a→b→c→d→e→a as the analysis object, which is shown in Fig. 2, and the branches are numbered from 1 to 20 along the path. The magnetic flux density of each branch along the path is shown in Fig.5. The symmetry of the magnetic circuit results in a symmetrical magnetic flux density. The magnetic flux density of branch 1 and branch 20 is the smallest because there is an air gap. As can be seen from Fig.5, the results obtained by the four methods are consistent.

2) SATURATED STATE

When the total MMF is 6000A, the center pole is saturated. The iterative processes of the maximum magnetic flux density calculated by different methods are shown in Fig. 6. In the saturation state, the results of the simple iteration method always oscillate up and down, and the calculation results diverge. Both the homotopy continuation method and the relaxation iteration method obtain a convergent solution quickly; however, the Newton Raphson method obtains a convergent solution after a long time. The magnetic flux density of each branch is shown in Fig. 7. The results obtained

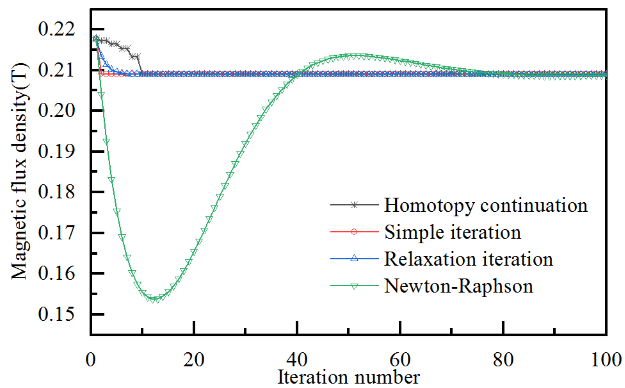

FIGURE 4. The iterative process under unsaturated state.

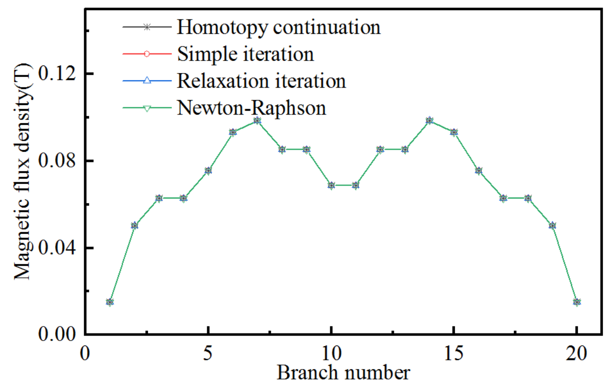

FIGURE 5. Magnetic flux density under unsaturated state.

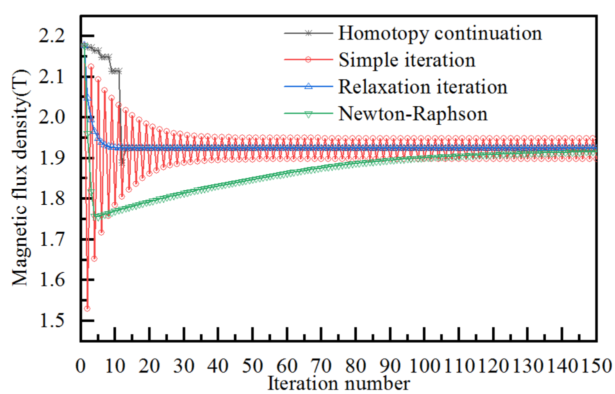

FIGURE 6. The iterative process under saturated state.

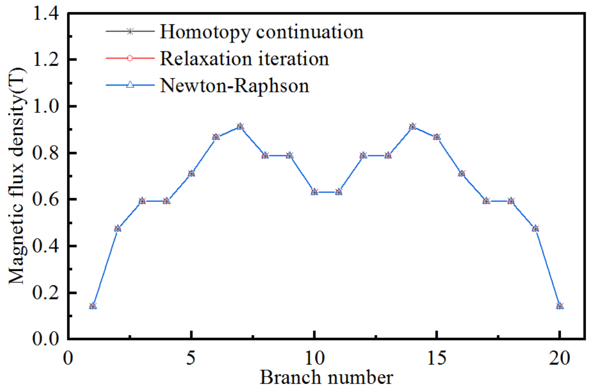

FIGURE 7. Magnetic flux density under saturated state.

by the relaxation iteration method, Newton-Raphson method, and homotopy continuation method are consistent.

3) SUPERSATURATED STATE

When the MMF is 10000A, the center pole is supersaturated. As shown in Fig. 8, the highly nonlinear magnetic network equation causes the calculation results of the simple iteration method to not converge. Although the relaxation iteration method can change the convergence performance, the inappropriate selection of the relaxation factor still fails to converge. In the supersaturated state, only the results calculated by the Newton-Raphson method and the homotopy continuous method converge. The magnetic flux density at each branch on the path is shown in Fig. 9.

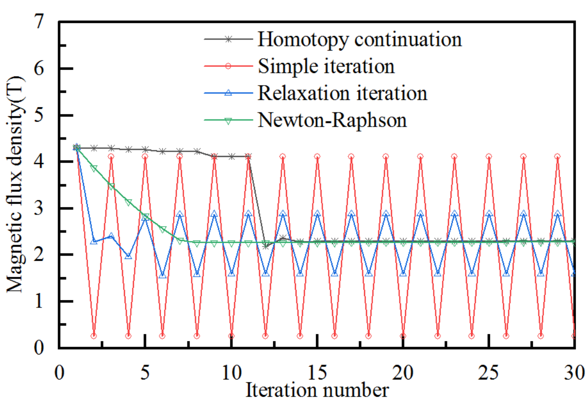

FIGURE 8. The iterative process under supersaturated state.

Fig. 10 shows the iteration number of the three methods under different saturation levels, where the continuation step is 0.2. Table 1 gives the calculation time of different methods. It can be seen from the above calculations that the results of the simple iteration converge when the iron core is at an unsaturated state. As the level of saturation increases, the convergence effect of the simple iteration and the relaxation iteration method becomes worse. When the relaxation factor is not selected properly, the results will also diverge. Newton-Raphson method converges to a reasonable solution,

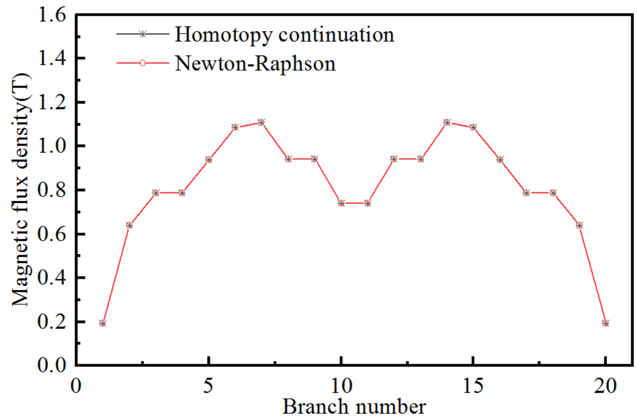

FIGURE 9. Magnetic flux density under supersaturated state.

TABLE 1. The calculation time of different methods.

MMF	600A	6000A	10000A
Homotopy continuation	0.20s	0.22s	0.62s
Newton-Raphson	1.73s	5.43s	5.68s
Relaxation iteration	0.20s	0.44s	
Simple iteration	0.07s		

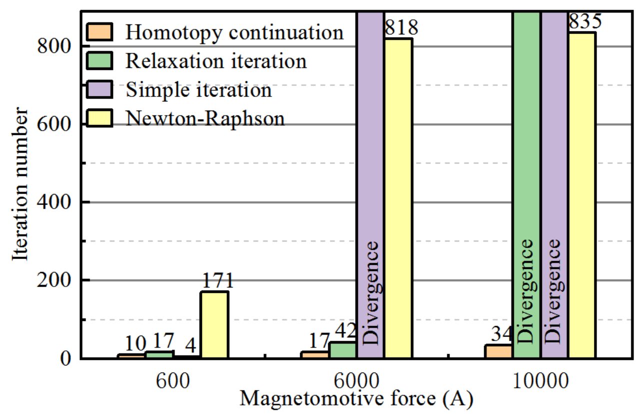

FIGURE 10. Iteration number of different methods.

but the construction of Jacobian matrix is extremely complex, and it takes a much longer time to calculate. It is important to select an appropriate initial value of the Newton-Raphson method to get a reasonable result. However, the homotopy continuation method does not have high requirements for the selection of initial values. When the continuation parameter t is introduced to construct the homotopy equation, the solving process of the original nonlinear equations is divided into several continuation processes. The solution after each continuation can enter the local convergence domain to ensure subsequent calculation convergence.

B. INFLUENCE OF DIFFERENT CONTINUATION STEP SIZES AND DIFFERENT INITIAL VALUE

In order to analyse the influence of the selected initial value and continuation step size on the results, we set the initial relative permeability as 0, 200, 500, 1000, and 2000, respectively, and compared the iteration number of different continuation step sizes at several initial values under saturation state.

As can be seen from table 2, at the same continuation step, iteration numbers are different with different initial values. On the other hand, the iteration number increases with the decrease of the continuation step size at the same initial relative permeability. This is because the results must be converged in each continuation, and the smaller the continuation step size, the more times of continuation, so the iteration number increases.

TABLE 2. Iteration number of different continuation step sizes and different initial relative permeability.

Continuation step size	Initial relative permeability μ_{r0} 0	200	500	1000	2000
0.5	22	10	11	12	9
0.2	73	24	23	19	31
0.1	118	55	45	43	53
0.01	1480	340	438	331	261
0.001	5387	2797	2496	2796	2230

In some cases, in fact, the excessive continuation step size will lead to non-convergence, because the solution obtained in the previous continuation does not fall into the local region of convergence of the next continuation. By reducing the continuation step size, the solution is easier to track the solution branch, and gradually approaches the final nonlinear solution.

C. METHOD VALIDATION

The correctness of a new algorithm can be verified through existing algorithms or experiments. We verify the correctness of the proposed method by the finite element method (FEM), which is a well-recognized method for analyzing electromagnetic fields with high accuracy. The magnetic flux density along the path when the center pole is supersaturated is compared with the FEM. The 2-D magnetostatic field distribution obtained by the finite element software MagNet 7.5 is shown in Fig. 11, where the air volume boundary is flux tangential boundary. In 2-D FEM analysis, the elements are shaped like triangles defined by three vertices, with a maximum element size of 5 millimeters, 4478 elements, and 770 knots. In order to compare with the calculation results of the nonlinear magnetic network method, according to Figure 2, there are 20 branches along the path a→b→c→d→e→a in Figure 11, as shown by the red solid line. The sampler in the finite element software is used to sample the magnetic flux density of each branch, and 50 points are uniformly sampled along the path for each branch. The average magnetic flux density of each branch is taken as the reference value. The comparison results of the two methods are shown in Fig. 12. The comparison results of the two methods are shown in Fig. 12. The black curve shows the calculation results of FEM, and the red curve shows those of nonlinear magnetic network equations. The green vertical lines show the relative error of the magnetic

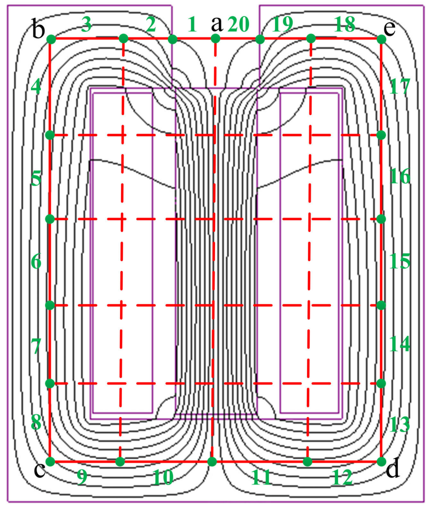

FIGURE 11. Magnetic flux density by FEM.

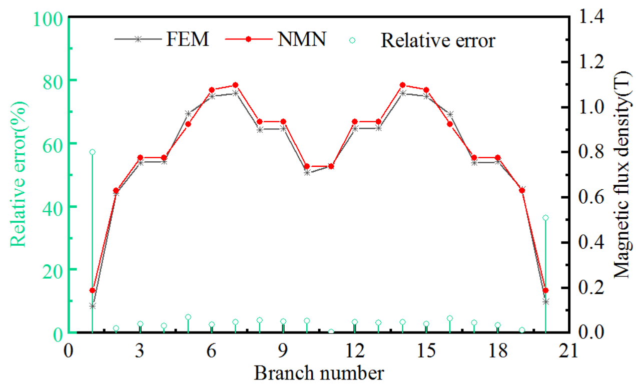

FIGURE 12. Comparison with finite element method.

flux density of each branch of the two calculation methods. The relative error of the flux density from branch 2 to branch 19 obtained by the two calculation methods is less than 5%, while the relative error of branch 1 and branch 20 is large. As branch 1 and branch 20 are located in the air gap position, a large flux leakage will be generated. Therefore, the equivalent cross-sectional area at the air gap cannot be accurately determined, which leads to large errors. In fact, the flux density of each branch calculated by the proposed method is basically identical to the results of FEM which verifies the correctness of the algorithm.

V. CONCLUSION

This paper established a nonlinear magnetic network model of the TEAM P20 benchmark model, and the homotopy continuation method is proposed to solve nonlinear magnetic network equations. The following conclusions are obtained.

1) Homotopy continuation method can effectively solve the nonlinear magnetic network equations, improve the convergence of nonlinear magnetic network calculations, and expand the convergence range.

2) Homotopy continuation method can avoid the complicated calculation of the Jacobian matrix of the Newton-Raphson iteration method, and can reduce the calculation time by nearly 10 times or more.
3) Initial value and continuation step size affect the iteration number of the solution. In general, the larger the continuation step size, the fewer the number of iterations. The smaller the continuation step size, the easier it is to follow the solution branch, but the number of iterations will increase accordingly.
4) Homotopy continuation method has clear physical meaning to solve the nonlinear magnetic network equations. The calculation starts from a linear magnetic field and gradually transitions to a nonlinear magnetic field considering saturation.

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