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# Low Complexity MIMO Channel Prediction for Fast Time-Variant Vehicular Communications Channels Based on Discrete Prolate Spheroidal Sequences

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**ABSTRACT** Channel state information (CSI) is required at the transmitter for achieving the maximum potentials of multiple-input multiple-output (MIMO) systems. In fast time-variant vehicular communications channels high data rate feedback lines are required in a frequency division duplex (FDD) transceiver for updating the transmitter with the rapidly changing CSI. Even with high data rate feedback lines, the delay caused by channel estimation and feedback may lead to outdated CSI at the transmitter. To reduce both the feedback load and CSI delay, this paper presents a reduced rank autoregressive (AR) channel predictor based on low dimensional discrete prolate spheroidal (DPS) sequences. The new subframe-wise DPS basis expansion model (DPS-BEM) channel predictor properly exploits the channel's restriction to low dimensional subspaces for reducing the prediction error and the computational complexity. The proposed channel predictor can be applied for updating the precoding matrix in time-variant MIMO systems. Simulation results demonstrate that the proposed channel predictor outperforms the DPS based minimum energy (ME) predictor at different Doppler frequencies and has better performance than the conventional Wiener predictor for slower time-variant channels and almost similar performance for very fast time-variant channels with reduced amount of computational complexity.

**INDEX TERMS** MIMO channel prediction, time varying vehicle-to-everything (V2X) channel, feedback delay, discrete prolate spheroidal sequences, precoder.

## I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) wireless communication attracted high attention during the past decades due to its capability for providing higher capacity and performance gains compared to Single-Input Single-Output (SISO) systems. High spectral efficiency can be achieved in MIMO systems through sending multiple data streams simultaneously over multiple transmit antennas without consuming extra frequency bandwidth, which is called spatial multiplexing [1]. The performance of spatial multiplexing can be further improved if the transmitted streams are matched to the propagation channel. Linear precoding is a technique that uses the available Channel State Information (CSI) at the transmitter (CSIT) for adapting the data streams to the instantaneous propagation channel [2], [3]. In frequency division duplex (FDD) systems the CSIT can be obtained through the feedback technique. However, the feedback load and feedback delay should be minimized for reliable communication with fast time-variant channels. The feedback load can be reduced by sending some quantized form of CSI through the feedback channel [2], [4], [5]. However, feedback delay causes the CSIT to become out of date and degrades the performance of the precoder especially in fast time-variant channels such as vehicle-to-everything (V2X) communications, which is characterized by a dynamic environment, high mobility, and comparatively low antenna heights on the communicating entities [6], [7]. Channel prediction has been

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proposed as a promising scheme to overcome the feedback delay problem.

Channel prediction techniques can be mostly divided into three groups, the parametric radio channel (PRC) model, the autoregressive (AR) model and basis-expansion model (BEM) [8]. The PRC approach models the time-variant channel as a sum of complex sinusoids each of which is determined with its amplitude, Doppler and phase shift. The parameters associated with each complex sinusoid is estimated using the known channel coefficients and is used for channel prediction. References [9]–[12] use the PRC method for SISO channel prediction and [13], [14] apply PRC for MIMO and Multi-user MIMO channel prediction, respectively. Recently, a data-driven deep neural network (DNN) approach is proposed to enable remote CSI inference for no explicit models of the complicated wireless channels at the expense of requiring large amounts of training data and corresponding computational power [15].

The conventional AR schemes use a linear minimum mean square error (MMSE) filter for predicting the future channel as a linear combination of the known channel coefficients [16]–[20]. This requires the knowledge of channel correlation matrix. For the case of unknown or time-variant correlation function, adaptive AR schemes have been developed which are based on adaptive filtering techniques such as least mean squares (LMS) [21], recursive least squares (RLS) [22] and Kalman filtering [23]. For AR models the computational complexity grows with the number of antennas when they are extended for MIMO channel prediction [24].

BEM is a widely used type of channel representation for time varying systems where the time varying channel taps can be approximated as a linear combination of some low dimensional orthogonal basis such as complex exponential (CE) functions, polynomials, discrete prolate spheroidal (DPS) Sequences, etc. The basis are determined based on the channel's Doppler spread and the basis coefficients are estimated using the channel information at pilot positions.

Exploiting the over-sampled complex exponential basis expansion model (CE-BEM), a subblockwise tracking scheme for the BEM coefficients using time-multiplexed (TM) periodically transmitted training symbols is proposed in [24]. Paper [24] also investigate three adaptive algorithms, including a Kalman filtering scheme based on an assumed AR model of the BEM coefficients, and two RLS schemes for BEM coefficient tracking.

Paper [25] models the user movement as the one-order unknown Markov process and proposes a spatial-temporal basis expansion model for channel tracking method in massive multiple-input multiple-output systems under both time-varying and spatial-varying circumstances. By using the polynomial basis expansion model (P-BEM), paper [26] proposes a tracking algorithm based on the fact that the changes in the channel path number and path delays are small over a few adjacent orthogonal frequency-division multiplexing (OFDM) symbols.

In [25], the transmitted symbols are segmented into overlapping blocks each containing several sub-blocks. The corresponding CE basis coefficients are updated for each block which in fact is equivalent to sub-block wise tracking of the basis coefficients since the adjacent blocks only differ by one sub-block. The sub-block wise updating of the CE-BEM coefficients for each block is performed both using a Kalman filtering scheme for an assumed first-order AR model of basis coefficients and using an RLS algorithm without considering any model for basis coefficients. A similar tracking scheme to [25] is also proposed in [27] where the extra information from the decision symbols is considered together with the information from the training sessions for improving the estimation of CE-BEM coefficients.

DPS-BEM is another commonly used BEM which is shown to outperform the polynomial-BEM and CE-BEM in approximating the Jakes' channel model for different ranges of Doppler spreads [28]. Several publications have considered the low dimensional channel estimation based on the DPS basis [29], [30]. As for the channel tracking, [31] proposes a minimum energy (ME) extrapolation-based predictor where the time-concentrated and band-limited DPS sequences are used for approximating the channel over a period of time for which the channel coefficients are known (estimated). The same estimated basis coefficients together with the same extrapolated DPS basis are used for the prediction of the future channel samples. Although ME predictor has low complexity, its performance degrades for long range prediction over fast time-variant channels [8].

In this paper we develop a new DPS-BEM channel predicting scheme which can be applied for reduced feedback load and feedback delay precoder design in fast time-variant V2X communications. The proposed channel predictor assumes non-overlapping transmitted frames and applies a sub-frame wise tracking approach for updating the DPS-basis coefficients based on a Q-order AR modelling of the basis coefficients. The proposed scheme differs from the sub-block wise CE-BEM tracking approaches of [25], [27] in that they exploit the CE basis for the overall channel variation in each frame which implies the basis duration being the same as the frame duration and increases the required number of CE basis for channel modelling. Since the frame length is large, overlapping frames are considered for better tracking of the basis coefficients. However, for the proposed DPS-BEM tracking scheme a few number of DPS basis which are time-limited to the sub-frame length are used for exploiting the channel variation inside each sub-frame and an AR model is applied for tracking the channel variation between subframes without any requirement for considering overlapping frames. Moreover, the CE-BEM tracking schemes of [25], [27] consider time-multiplexed training sessions inside each subframe for updating the basis coefficients, while the proposed frame structure only assumes known channel coefficients at the beginning of each frame. The detailed contributions are as follows:

1) We formulate the channel prediction problem considering a DPS-BEM of the time-variant channel coefficients over each sub-frame and an AR model of the time-variant DPS basis coefficients over the whole transmitted frame.

2) The error contribution from the AR prediction of the DPS basis coefficients and the DPS modelling of the channel coefficients are investigated and an algorithm for obtaining the optimal sub-frame length (correspondingly the optimal AR modelling order) and the optimal number of basis per sub-frame is proposed through minimizing the total prediction error of the transmitted frame.

The paper is organized as follows. The MIMO system model is presented in Section II. Section III reviews the DPS sequences, the ME and AR channel predictors. In Section IV, the new DPS-AR channel predictor and its application to precoder design are presented. Section V presents simulation results and the conclusion is provided in Section VI.

*Notation:* We denote a scalar by $a$, a column vector by $a$ with its $i$th element $[a]_i$ and a matrix by $\mathbf{A}$ with its $(i,j)$th element $[\mathbf{A}]_{i,j}$. The inverse, transpose and Hermitian transpose of a matrix $\mathbf{A}$ is denoted by $\mathbf{A}^{-1}$, $\mathbf{A}^T$ and $\mathbf{A}^H$, respectively. $a^*$ denotes the complex conjugate of $a$. We denote the set of all real numbers by $\mathbb{R}$, the set of all integers by $\mathbb{Z}$ and the set of non-negative integers $\{0, 1, \cdots, n-1\}$ by $\mathbb{I}_n$. The smallest integer that is greater than or equal to $a \in \mathbb{R}$ is denoted by $\lceil a \rceil$.

## II. SYSTEM MODEL

A precoded MIMO system with $M_{T_x}$ transmit antennas and $M_{R_x}$ receive antennas is shown in Fig. 1. The input bit stream is first QPSK modulated and then it is demultiplexed into $K$-spatial data streams represented by $\{d_i[m]\}_{i=1}^{K}$ at time $m$. Note that $K \le M_{R_x} \le M_{T_x}$ in general and for a full-rate system $K = M_{R_x} = M_{T_x}$ which is assumed in the rest of this paper without loss of generality. Considering $d[m] = [d_1[m], d_2[m], \cdots, d_K[m]]^T$ as a vector which contains the $m$th symbol of all data streams, this vector is then multiplied by the precoder matrix $\mathbf{F}[m]$ to obtain the transmitted symbols as

$$x[m] = \mathbf{F}[m]d[m] \tag{1}$$

where $x[m] = \left[x_1[m], x_2[m], \cdots, x_{M_{T_x}}[m]\right]^T$ contains the symbols transmitted from $M_{T_x}$ transmit antennas at time $m$. Similarly, the corresponding received vector is represented by $y[m] = \left[y_1[m], y_2[m], \cdots, y_{M_{R_x}}[m]\right]^T$ where for a single tap channel,

$$y[m] = \mathbf{H}[m]x[m] + z[m] \tag{2}$$

$\mathbf{H}[m]$ is the $M_{R_x} \times M_{T_x}$ channel matrix and $z[m]$ is the corresponding additive white Gaussian noise vector at time $m$. The received vector, $y[m]$, is first multiplied by the combiner matrix $\mathbf{G}[m]$ and then is processed for data detection.

Precoder and combiner matrices are designed to make decoupling of data steams possible at the receiver. To avoid performance degradation, the precoder and combiner should be matched to the channel matrix. The optimal precoder and combiner can be obtained from the eigenvectors of the channel matrix [2]. For the slow time-varying channels, the precoder matrix $\mathbf{F}[m]$ is considered to be constant over each transmitted frame of length $M_f$, i.e., for $m \in \mathbb{I}_{M_f}$. So, it can be calculated once based on the estimated channel matrix at the beginning of each frame. The precoder matrix is then fed back to the transmitter directly in infinite feedback rate systems or it is first quantized based on a predefined codebook and the quantization index is fed back in finite feedback rate systems. The same precoder will be used through the whole frame. However, in fast time-variant channels, $\mathbf{F}[m]$ becomes out of date very quickly and it is required to be updated even inside one frame which is impossible due to channel estimation and feedback delays. To overcome the feedback delay problem, it is proposed to predict the channel coefficients and design the precoder in advance [2], [5]. In the next section we will investigate two channel prediction schemes which can be used for precoder design in fast time-variant channels.

Note that, here after, $h_{k,l}[m]$ represents the channel impulse response for $(k, l)$-antenna pair at time $m$. Moreover, we consider independent and identically distributed (i.i.d.) Rayleigh fading channel based on the Jakes' model for each antenna pair with no spatial correlation between different antennas as in [32].

## III. ME AND AR CHANNEL PREDICTORS

In this section a brief introduction to DPS sequences as well as the ME and AR channel predictors is presented which then will be used for introducing the proposed DPS-AR channel predictor in the next section.

### A. DPS SEQUENCES

DPS sequences were first introduced by Slepian in 1978 [33], for approximation, prediction and estimation of band limited signals [28]. The DPS sequences $\{u_i[m, W, M]\}_{i=0}^{M-1}$ with time concentration to $\mathbb{I}_M$ and band limitation to $W = [-f_d T_s, f_d T_s]$ are the solutions to

$$\sum_{\ell=0}^{M-1} C[\ell-m, W]u_i[\ell, W, M] = \lambda_i(W, M)u_i[m, W, M] \tag{3}$$

where $m \in \mathbb{Z}$, $f_d$ is the maximum Doppler frequency and $T_s$ is the sampling period. It is shown in [31] that $C[n, W]$ is proportional to the covariance function of a process with constant spectrum over $W$ and can be evaluated as

$$C[n, W] = \int_W e^{j2\pi n\nu} d\nu \qquad \text{for } n \in \mathbb{Z} \tag{4}$$

and

$$\lambda_i(W, M) = \frac{\sum_{m=0}^{M-1} |u_i[m, W, M]|^2}{\sum_{m=-\infty}^{\infty} |u_i[m, W, M]|^2} \tag{5}$$

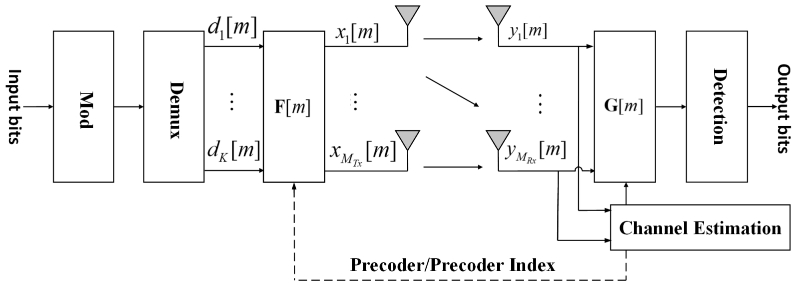



**FIGURE 1.** Precoded MIMO system at time *m*.

represents the energy concentration of $u_i[m, W, M]$ within $\mathbb{I}_M$. While being band limited to $W = [-f_d T_s, f_d T_s]$, hence

$$u_i[m, W, M] = \int_{-f_d T_s}^{f_d T_s} U_i(v) e^{j2\pi m v} dv, \tag{6}$$

where $U_i(v)$ is the spectrum of $u_i[m, W, M]$, given by

$$U_i(v) = \sum_{m=-\infty}^{\infty} u_i[m, W, M] e^{-j2\pi m v}. \tag{7}$$

The DPS sequence is the unique sequence $u_0[m, W, M]$ that is band-limited and most time-concentrated in a given interval with length $M$, $u_1[m, W, M]$ is the next sequence having maximum energy concentration among the DPS sequences orthogonal to $u_0[m, W, M]$, and so on. Thus, the DPS sequences show that a set of orthogonal sequences exists that is exactly band limited and simultaneously posses a high (but not complete) time concentration in a certain interval with length $M$. The eigenvalues $\lambda_i(W, M)$ are a measure for this energy concentration expressed by (5).

DPS sequences have a double orthogonality property: They are orthogonal over the finite set $\{0, 1, \cdots, M-1\}$ and the infinite set $\mathbb{Z} = \{-\infty, \cdots, \infty\}$ simultaneously. This means that the DPS sequences $\{u_i[m, W, M]\}_{i=0}^{M-1}$ are orthogonal on the set $m \in \{0, 1, \cdots, M-1\}$ and orthogonal on the set $m \in \{-\infty, \cdots, \infty\} = \mathbb{Z}$. More specifically

$$\begin{aligned}\sum_{m=0}^{M-1} & u_i[m, W, M] u_j[m, W, M] \\ &= \lambda_i(W, M) \sum_{m=-\infty}^{\infty} u_i[m, W, M] u_j[m, W, M] \\ &= \delta_{i,j}, \ i, j = 0, 1, \cdots, M-1. \end{aligned} \tag{8}$$

This remarkable property enables parameter estimation without the drawbacks of windowing in the case of the CE sequences.

### B. ME CHANNEL PREDICTOR

A low complexity ME channel predictor is proposed in [31] which is based on a subspace spanned by time-concentrated and band limited DPS sequences and it is shown that for a fading process with constant spectrum over its support the ME predictor is identical to a reduced-rank maximum-likelihood predictor.

For $(k, l)$-antenna pair, the ME predictor uses the band-limitation of the fading channel and models the channel over the first $M$ known coefficients of each frame as a linear combination of $D_t$ dominant DPS basis

$$h_{k,l}[m] = \sum_{i=0}^{D_t-1} u_i[m, W, M] \psi_i^{k,l} \quad \mathit{for} \quad m \in \mathbb{I}_M \tag{9}$$

where $D_t$ is the essential dimension of the subspace spanned by DPS sequences and is given by [33],

$$D_t(W, M) = \lceil |W| M \rceil + 1. \tag{10}$$

with $|W| = 2 f_d T_s$ indicating the maximum normalized support of the signal in frequency domain. For a highly oversampled fading process $|W| \ll 1$ and the process can be approximated by $D_t(W, M) \ll M$ DPS basis which reduces the amount of computational complexity. The basis coefficients $\{\psi_i^{k,l}\}_{i=0}^{D_t-1}$ are estimated as

$$\Psi_{k,l} = \mathbf{U}^H h_{k,l} \tag{11}$$

where $h_{k,l} = [h_{k,l}[0], \cdots, h_{k,l}[M-1]]^T$ contains the known channel coefficients, $\Psi_{k,l} = [\psi_0^{k,l}, \cdots, \psi_{D_t-1}^{k,l}]^T$ and $[\mathbf{U}]_{m,i} = u_i[m, W, M]$ for $m \in \mathbb{I}_M$ and $i \in \mathbb{I}_{D_t}$.

In the ME predictor, the estimated basis coefficients from (11) are used for predicting the future channel coefficients by extrapolating the DPS basis $\{u_i[m, W, M]\}_{i=0}^{D_t-1}$ over $m = M, \cdots, M_f - 1$ based on (3) and using the same linear combination of the extrapolated basis presented in (9). As the DPS sequences are most energy concentrated in $\mathbb{I}_M$, among the infinitely many ways for extending the band-limited channel samples over $m \in \mathbb{Z} \setminus \mathbb{I}_M$, the ME predictor is the only one that extends the channel in an ME continuation sense [31].

### C. AR CHANNEL PREDICTOR

Comparing different channel prediction algorithms, it is concluded in [34] that the AR model based predictors, known as Wiener predictors [35], outperform the PRC model based schemes both for synthesized and measured radio channels at least for the narrowband case. Assuming the channel is known over the first $M$ samples of each frame, the Wiener predictor predicts the channel at $m \in \{M, \cdots, M_f - 1\}$ using the weighted linear combination of the $M$ most recent channel coefficients as follows,

$$\hat{h}_{k,l}[m] = w_{k,l}[m]^H \tilde{h}_{k,l}[m] \tag{12}$$

where $w_{k,l}[m]$ is the $M \times 1$ weighting vector and $\tilde{h}_{k,l}[m]$ is defined as

$$\tilde{h}_{k,l}[m] = \left[\hat{h}_{k,l}[m-1], \hat{h}_{k,l}[m-2], \cdots, \hat{h}_{k,l}[m-M]\right]^T \tag{13}$$

which we call it the history vector with $\hat{h}_{k,l}[m']$ being the known channel coefficient for $m' \in \{0, \cdots, M-1\}$ and the previously predicted channel coefficient for $m' \in \{M, \cdots, M_f - 1\}$. The weighting vector $w_{k,l}[m]$ is obtained through minimizing the mean square error (MSE) between the predicted channel and real channel at time $m$,

$$w[m] = \mathrm{argmin}_w \mathbb{E}\left\{\| h[m] - w[m]^H \tilde{h}[m] \|^2\right\} \tag{14}$$

which leads to

$$w[m] = \mathbf{R}_{hh}[m]^{-1} r_{hh}[m] \tag{15}$$

and the $M \times M$ matrix $\mathbf{R}_{hh}$ and the $M \times 1$ vector $r_{hh}$ are defined as

$$\begin{aligned} [\mathbf{R}_{hh}[m]]_{n,\ell} &= \mathbb{E}\left\{h[m-n]h^*[m-\ell]\right\} \\ [r_{hh}[m]]_n &= \mathbb{E}\left\{h[m]h^*[m-n]\right\} \quad \text{for } n, \ell = 1, \cdots, M \end{aligned} \tag{16}$$

Note that for notation simplicity, the antenna pair index $(k, l)$ is omitted in (14)-(16) and in the rest of the paper.

## IV. PROPOSED DPS BASED AR PREDICTOR

In this section a new DPS based AR channel predictor is presented. The new predictor takes advantage of the low computational complexity induced by the DPS sequences and the performance gain of AR model, simultaneously. We will also investigate the application of the proposed scheme for the precoder design in time-variant channels.

### A. DPS-AR CHANNEL PREDICTOR

Consider the transmitted frame over each antenna is divided into $Q_f$ subframes of $M_s = M_f/Q_f$ length shown in Fig. 2. The first $Q = M/M_s$ subframes contain the $M$ perfectly known channel samples, and the channel coefficients of the rest of $Q_f - Q$ subframes are to be predicted. We use the DPS sequences to model the channel's time-variation inside each subframe. It is clear from (3) that for each subframe the DPS basis depend on the length of the subframe and the normalized Doppler spread of the channel. Considering the same maximum Doppler frequency over the whole frame and the same length for all subframes, the DPS basis would be the same for all subframes and only the basis coefficients would change from one subframe to another one. Using $D_s$ dominant DPS basis, the channel impulse response inside the $q$th subframe can be approximated as follows for each antenna pair,

$$h[(q-1)M_s+m] \approx \sum_{i=0}^{D_s-1} u_i[m, W, M_s]\psi_i[q], \quad m \in \mathbb{I}_{M_s} \tag{17}$$

where $m$ indicates the $m$th sampling time inside each subframe and $\psi_i[q]$ is the $i$th basis coefficient for subframe $q$. As mentioned previously, the basis $\{u_i[m, W, M_s], i \in \mathbb{I}_{D_s}, m \in \mathbb{I}_{M_s}\}$ are independent of $q$ and they are the same for all subframes. So for tracking the time-variation of the channel, we only require to track the changes of the basis coefficients $\{\psi_i[q]\}_{i=0}^{D_s-1}$ over adjacent subframes. In order to predict the basis coefficients for future subframes we will use the AR model. For the $q$th subframe, the $i$th basis coefficient, $\psi_i[q]$, is predicted from,

$$\hat{\psi}_i[q] = w_i[q]^H \tilde{\Psi}_i[q] \tag{18}$$

where $\tilde{\Psi}_i[q]$ is a $Q \times 1$ history vector which contains the $i$th coefficient of the $Q$ recent predicted/known subframes and is defined as

$$\tilde{\Psi}_i[q] = \left[\hat{\psi}_i[q-1], \cdots, \hat{\psi}_i[q-Q]\right]^T. \tag{19}$$

Using the same idea as (14), the MMSE weighting vector $w_i[q]$ is obtained from

$$w_i[q] = \mathbf{R}_i[q]^{-1} r_i[q] \tag{20}$$

where

$$\begin{aligned} [\mathbf{R}_i[q]]_{n,\ell} &= \mathbb{E}\left\{\psi_i[q-n]\psi_i^*[q-\ell]\right\} \\ [r_i[q]]_n &= \mathbb{E}\left\{\psi_i[q]\psi_i^*[q-n]\right\} \quad \text{for } n, \ell = 1, \ldots, Q \end{aligned} \tag{21}$$

and $\mathbb{E}\left\{\psi_i[q]\psi_i^*[q']\right\}$ indicates the amount of the correlation between the $i$th basis coefficient of the $q$th and $q'$th subframes. For estimating the correlation between the basis coefficients of different subframes we can write the following vector-matrix relationship for the $q$th subframe based on (17) and by considering $D_s = M_s$,

$$h[q] = U_t \Psi_t[q] \tag{22}$$

where $U_t$ is an $M_s \times M_s$ matrix with $[U_t]_{m,i} = u_i[m, W, M_s]$ for $m, i \in \mathbb{I}_{M_s}$, $h[q]$ contains the channel coefficients of the $q$th subframe with $[h[q]]_m = h[(q-1)M_s+m]$ and $[\Psi_t[q]]_i = \psi_i[q]$. Multiplying both sides of (22) by their Hermitian at $q'$ and taking the expectation of them the following relationship is obtained between the correlation function of the channel and the correlation function of the basis coefficient of the $q$th and $q'$th subframes,

$$R_{\Psi_t}[q, q'] = U_t^H R_h[q, q'] U_t \tag{23}$$

where $R_{\Psi_t}[q, q'] = \mathbb{E}\{\Psi_t[q]\Psi_t[q']^H\}$ and $R_h[q, q'] = \mathbb{E}\{h[q]h[q']^H\}$. Note that for a stationary channel $R_h[q, q'] = R_h[q - q']$ and consequently $R_{\Psi_t}[q, q'] = R_{\Psi_t}[q - q']$ which results in the same $w_i[q]$ for different value of $q$.

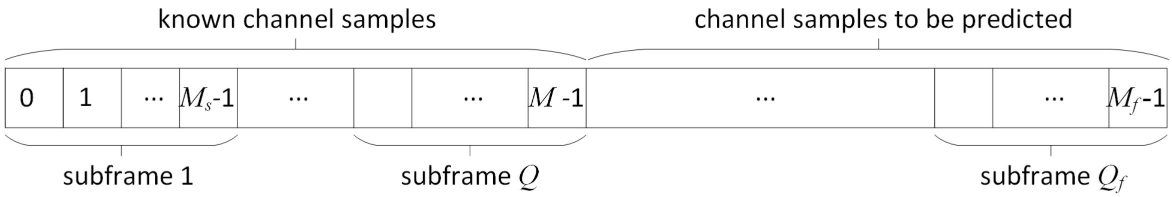



**FIGURE 2.** Transmitted frame structure for each antenna.

### B. MEAN SQUARE ERROR OF THE PREDICTOR

As discussed in the previous section, the channel inside each subframe is modelled by $D_s$ DPS basis and each basis coefficient is predicted using a Q-order AR model. So the predicted channel impulse response of the $q$th subframe is modelled as,

$$\hat{h}[q] = U_{D_s}\hat{\Psi}_{D_s}[q] \tag{24}$$

where $U_{D_s}$ is an $M_s \times D_s$ matrix containing the first $D_s$ dominant DPS basis as its columns and $\hat{\Psi}_{D_s}[q]$ is a $D_s \times 1$ vector containing the corresponding predicted basis coefficients at the $q$th subframe which differs from the original basis coefficient vector $\Psi_{D_s}[q]$ by the AR prediction error vector $e_{AR}[q]$ as follows

$$\hat{\Psi}_{D_s}[q]=\Psi_{D_s}[q]+e_{AR}[q] \quad \text{for} \quad q=Q+1,\cdots,Q_f \tag{25}$$

where for $i = 1, 2, \cdots, D_s$,

$$\begin{aligned}[e_{AR}[q]]_i &= \hat{\psi}_i[q] - \psi_i[q] \\ &= \underbrace{w_i[q]^H\Psi_i[q] - \psi_i[q]}_{e_M[q,i]} + \underbrace{w_i[q]^H e_i[q]}_{e_P[q,i]}. \end{aligned} \tag{26}$$

In (26) $\hat{\psi}_i[q]$ is replaced by $w_i[q]^H\tilde{\Psi}_i[q]$ according to (18) and the error contribution from the previously predicted coefficients, resulting in the so-called AR propagation error $e_P[q, i]$, is reflected in the $Q \times 1$ vector $e_i[q] = \tilde{\Psi}_i[q] - \Psi_i[q]$ with $[e_i[q]]_j = [e_{AR}[q-j]]_i$ being its $j$th element. $e_M[q, i]$ is called the AR modelling error which is the result of the prediction in an MMSE sense using the MMSE weighting vector $w_i[q]$.

Using (22), (24) and (25), the mean square error of the $q$th block is defined as,

$$\begin{aligned}\text{MSE}[q] &= \mathbb{E}\left\{\| h[q] - \hat{h}[q] \|^2\right\} \\ &= \underbrace{\mathbb{E}\left\{\| V^H h[q] \|^2\right\}}_{\text{MSE}_{\text{DPS}}[q]} + \underbrace{\mathbb{E}\left\{\| e_{AR}[q] \|^2\right\}}_{\text{MSE}_{\text{AR}}[q]}\end{aligned} \tag{27}$$

where in deriving (27) we used the fact that the DPS basis are orthonormal and independent of the AR prediction error and $V = [u_{D_s}, \ldots, u_{M_s-1}]$ contains the $M_s - D_s$ less dominant DPS basis spanning the subspace orthogonal to the signal subspace spanned by columns of $U_{D_s}$. $\text{MSE}_{\text{DPS}}[q]$ is the mean square of DPS reconstruction error caused by using the limited number of basis for modelling the channel inside each subframe which according to (23) can be simplified to,

$$\text{MSE}_{\text{DPS}}[q]=\sum_{i=D_s}^{M_s-1} u_i^H R_h[q,q]u_i=\sum_{i=D_s}^{M_s-1}\mathbb{E}\{|\psi_i[q]|^2\} \tag{28}$$

where for a stationary channel $\text{MSE}_{\text{DPS}}[q]$ is independent of the subframe index $q$. Using (26), $\text{MSE}_{\text{AR}}[q]$ can be also obtained from,

$$\text{MSE}_{\text{AR}}[q] = \sum_{i=0}^{D_s-1}\mathbb{E}\left\{|e_M[q,i] + e_P[q,i]|^2\right\} \tag{29}$$

It is clear that for a given normalized channel bandwidth $W$, $\text{MSE}_{\text{DPS}}[q]$ and $\text{MSE}_{\text{AR}}[q]$ can be controlled by the subframe length $M_s$ and the number of DPS basis per subframe $D_s$. While increasing the number of basis would decrease $\text{MSE}_{\text{DPS}}[q]$, it would increase the AR prediction error since more coefficients are required to be predicted. As for the subframe length, choosing $M_s$ to be too small would increase the AR modelling error as the history vector is more correlated specifically for low Doppler frequencies which can result in a rank deficient matrix $\mathbf{R}_i[q]$ in (20) and less accurate weighting vector $\mathbf{w}_i[q]$ estimation. Smaller subframe length also increases the number of subframes to be predicted which can lead to a bigger AR propagation error. On the other hand, very large subframe length may also increase the AR modelling error due to the increased distance between the history samples which may not be correlated enough with the basis coefficient to be predicted especially in fast time variant channels.

So for the normalized bandwidth $W$ and $M$ known channel coefficients at the beginning of each frame, the subspace dimension and the subframe length which minimize the total MSE over the frame of length $M_f$ with $Q_f$ subframes can be obtained from the following optimization problem,

$$\begin{aligned}&[M_s^{opt}(W), D_s^{opt}(W)] \\ &= \underset{\substack{M_s \text{is a factor of } M \\ D_s\in\{1,2,\ldots,M_s\}}}{\text{argmin}} \\ &\sum_{q=Q+1}^{Q_f}\left(\sum_{i=D_s}^{M_s-1}\mathbb{E}\{|\psi_i[q]|^2\}+\sum_{i=0}^{D_s-1}\mathbb{E}\left\{|e_M[q,i]+e_P[q,i]|^2\right\}\right)\end{aligned} \tag{30}$$

In (27) $M_s$ should be a factor of $M$ such that the $M$ known samples at the beginning of each frame can be divided into an integer number of subframes represented by $Q = M/M_s$. Moreover, for any chosen $M_s$, $D_s$ should be a member of $1, 2, \ldots, M_s$. So, for solving (27), it is only required to iterate between limited possible choices of $M_s$ and $D_s$ and choose the optimal pair which minimizes the MSE over the whole frame. Algorithm 1 presents the search procedure for optimal values of $M_s$ and $D_s$ for any given values of $M$, $M_f$ and $W$.

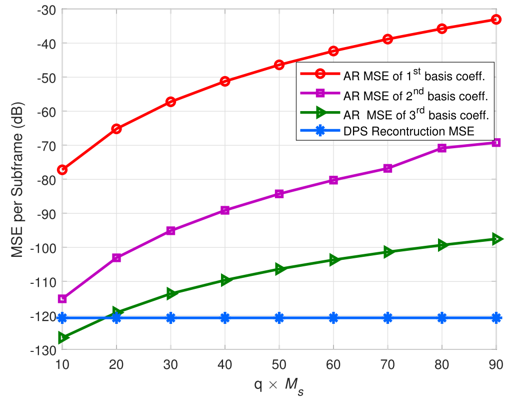
**Algorithm 1** Algorithm for Calculating $M_s^{opt}(W)$ and $D_s^{opt}(W)$

**Input:** $M, M_f, W$
**Output:** $M_s^{opt}(W), D_s^{opt}(W)$

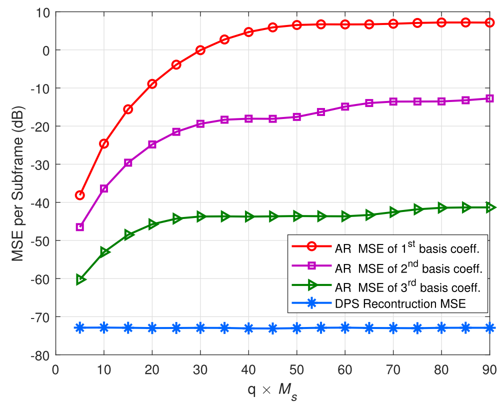
1: **Initialisation**: $k = 1$, $M_s(k) = M$
2: $Q = \frac{M}{M_s(k)}$, $Q_f = \frac{M_f}{M_s(k)}$
3: Find $D_s(k) \in \{1, \cdots, M_s(k)\}$ which minimizes (30)
4: $k \leftarrow k + 1$
5: $D_s(k) \leftarrow D_s(k-1)$
6: Find $M_s(k) \in \{m | m \text{ is a factor of } M \text{ and } m \geq D_s(k)\}$ which minimizes (30)
7: **while** $M_s(k) \neq M_s(k-1)$ **do**
8: $\quad k \leftarrow k + 1$
9: $\quad M_s(k) \leftarrow M_s(k-1)$
10: $\quad$ Repeat 1 to 5
11: **end while**
12: **return** $M_s^{opt}(W) = M_s(k)$ and $D_s^{opt}(W) = D_s(k)$

Assuming $M = 30$ and $M_f = 120$, Table 1 shows the optimum values of $M_s$, $D_s$ and the corresponding value of $Q = M/M_s$, for different normalized Doppler frequencies. We consider a traditional MIMO communication system which operates at carrier frequency of 5.9 GHz, data rate of 100 kBd (kilo-Bauds), therefore $T_s = 10\ \mu$s, and a varying Doppler spread $f_d$ in the range of 0 to 2000 Hz, or the normalized Doppler spread $f_d T_s$ from 0 to 0.02 (Corresponding to the maximum mobile velocity of 366 km/h). It is clear that by increasing the maximum normalized Doppler frequency the optimal subframe length decreases and the ratio of $D_s/M_s$ increases. In fact when the channel varies more rapidly, the correlation between the basis coefficients of different subframes will decrease which requires smaller subframe length for reducing the AR prediction error. Therefore, for controlling the AR prediction error, the basis coefficients are required to be sampled with a higher sampling rate which is equivalent to reducing the subframe length. Moreover, to reduce the DPS reconstruction error at high Doppler frequencies more DPS basis per subframe are required for better tracking of the time-variation of the channel in each subframe.

For two different normalized Doppler frequencies and the optimum values of Table 1, Fig. 3 shows the AR prediction mean square error for each of the basis coefficients as well as the DPS reconstruction mean square error for each subframe $q \in \{Q+1, \cdots, Q_f\}$ over the prediction length of $M_f - M = 90$ channel samples. It can be seen from Fig. 3, the DPS reconstruction error is constant over different subframes since the




(a)




(b)

**FIGURE 3. Basis coefficients' AR prediction error and DPS modelling error per each subframe $q \in \{Q+1, \cdots, Q_f\}$ over the prediction length of $M_f - M = 120 - 30 = 90$ channel samples for (a) $f_d T_s = 0.001$, $M_s = 10$, $Q = 3$, $Q_f = 12$, (b) $f_d T_s = 0.02$, $M_s = 5$, $Q = 6$, $Q_f = 24$.**

**TABLE 1. Optimal subframe length and subspace dimension for $M = 30$ and $M_f = 120$.**

| $f_d T_s$ | 0.001 | 0.002 | 0.005 | 0.01 | 0.015 | 0.02 |
|---|---|---|---|---|---|---|
| $M_s$ | 10 | 10 | 6 | 6 | 6 | 5 |
| $D_s$ | 3 | 3 | 3 | 3 | 3 | 3 |
| $Q$ | 3 | 3 | 5 | 5 | 5 | 6 |

simulated channel is wide sense stationary. The small amount of $\mathrm{MSE_{DPS}}[q]$ indicates the proper performance of the time variation modelling of the channel inside each subframe using adequate number of DPS basis. As for the AR prediction error, it is clear the less dominant basis have smaller prediction error as they have less contribution in channel modelling (have smaller basis coefficients).

For evaluating the prediction quality of the proposed scheme over the samples of each subframe, we can also calculate the amount of error at the $m$th sample of the $q$th subframe as follows,

$$
\begin{aligned}
e[q, m] &= h[(q-1)M_s + m] - \hat{h}[(q-1)M_s + m] \\
&= \underbrace{h[(q-1)M_s + m] - \sum_{i=0}^{D_s - 1} u_i[m]\psi_i[q]}_{e_{DPS}[m]}
\end{aligned}
$$

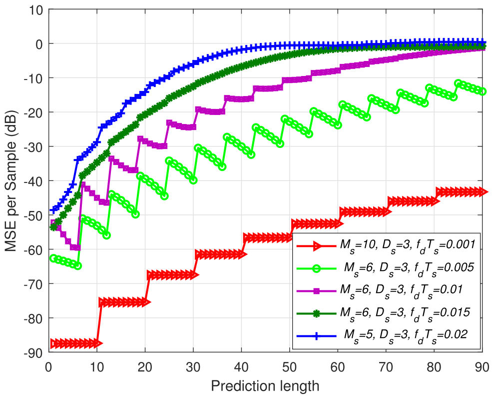



**FIGURE 4.** MSE of the proposed DPS-AR prediction scheme over 90 channel samples.

$$\underbrace{-\sum_{i=0}^{D_s-1} u_i[m]\,[e_{AR}[q]]_i}_{e_{AR}[m]} \tag{31}$$

where $e_{DPS}[m]$ and $e_{AR}[m]$ are associated with the DPS modelling error and the AR prediction error of different basis coefficients, respectively. As discussed, the error contribution from the DPS modelling is fairly small and it can be concluded that the error at each sample is mainly controlled by the second term which is a linear combination of the AR prediction error of different basis coefficients weighted by the corresponding DPS basis values at time $m$. Since the DPS basis may not be all strictly ascending or descending functions over the subframe length and can take both positive and negative values, the error at each sample inside each subframe can follow an increasing, decreasing or even constant trend due to constructive or destructive contribution from the weighted prediction error of different basis coefficients. Fig. 4 indicates the error per sample for different normalized Doppler frequencies and the optimal values of Table 1. It can be seen from Fig. 4, the amount of $\mathrm{MSE}_{\mathrm{DPS}}[q]$ will increase along with the increasing of the normalized Doppler frequency.

### C. APPLICATION TO PRECODER DESIGN

The DPS based AR channel predictor which is proposed in the previous section can be used to reduce the feedback rate and the feedback delay of sending the CSI to the transmitter for precoder design in time-variant channels. Based on the proposed scheme shown in Fig. 5, for a MIMO system with $M_{T_x}$ transmit antennas and $M_{R_x}$ receive antennas, it is only required to feedback the $D_s$ basis coefficients of the first $Q$ known/estimated subframes for each antenna pair which results in total feedback load of $M_{T_x} \times M_{R_x} \times Q \times D_s$ basis coefficients. These coefficients are used for precoder design over the subsequent subframes of each frame. The relative amount of feedback load reduction compared to the case of sending back the channel impulse response of the first $M$ samples of each frame is in the order of $F_R = O(M_s/D_s)$ per each antenna pair and the total amount of feedback load reduction over all antennas in terms of the number of coefficients required to be fed back is $M_{T_x} \times M_{R_x} \times (M - Q \times D_s)$. Based on the optimum values of $M_s$ and $D_s$ presented in Table 1, Fig. 6 evaluates the relative feedback load reduction with MIMO sizes. We can observe from the figure that the proposed DPS-AR predictor offers significant amount of feedback load reduction specially for slower time-variant channels and the relative feedback load reduction of the proposed DPS-AR predictor will increase along with the increasing of MIMO sizes.

It should be also noted that, the DPS sequences are the same for different subframes and can be calculated once and stored at the transmitter based on the known maximum normalized Doppler frequency and the chosen subframe length. Once the channel is predicted for each subframe the precoder can be obtained using the SVD decomposition of the channel matrix. BEM can also be used to reduce the feedback load for heterogeneous multiuser transmissions by capturing the time-variation of the multiuser downlink channels and reducing the number of the channel parameters [36].

## V. SIMULATION RESULTS

In this section the performance of the proposed DPS-AR channel predictor is compared with that of the AR and the ME channel predictors. We consider the same MIMO system with parameters described in Section IV.

Fig. 7 shows the normalized MSE (NMSE) for different prediction schemes at the prediction length $p$ which is defined as,

$$\mathrm{NMSE}[p] = \frac{\sum_{m=M+1}^{M+p} \mathbb{E}\{|h[m] - h_{pre}[m]|^2\}}{p \sum_{m=M+1}^{M+p} \mathbb{E}\{|h[m]|^2\}} \tag{32}$$

where $h_{pre}[m]$ is the predicted channel sample at time $m$.

It can be seen from Fig. 7 that the ME predictor has lower accuracy both for low and high maximum normalized Doppler frequencies compared to the proposed DPS-AR predictor. This is due to the fact that ME predictor assumes that most of the energy of the channel is contained in the known samples at the beginning of each frame and extends the channel over the future time instants in an ME continuation sense. Therefore, ME predictor has low complexity but would not guarantee the best prediction accuracy for the signals that do not have low energy over their prediction interval. Furthermore, the proposed predictor also outperforms the AR approach for low to medium normalized Doppler frequencies and it has almost the same performance as the AR method for high normalized Doppler frequencies. The reason for that is the fact that in lower Doppler frequencies, the history vector in the AR predictor contains more correlated samples resulting in inaccurate weighting vector estimation. However, for the proposed DPS-AR scheme the basis coefficients are sampled far enough from each other based on the

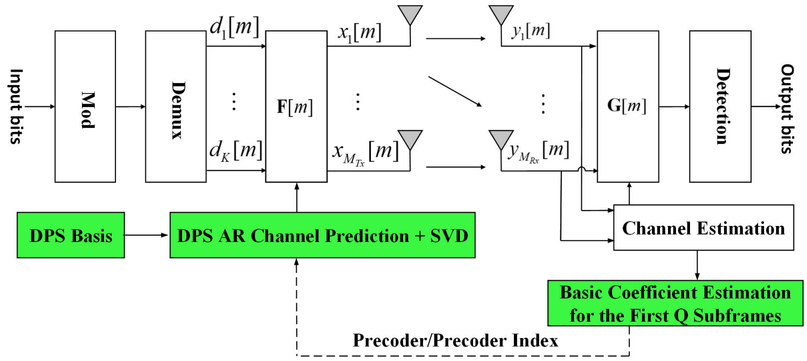



**FIGURE 5.** Proposed Precoded MIMO system.

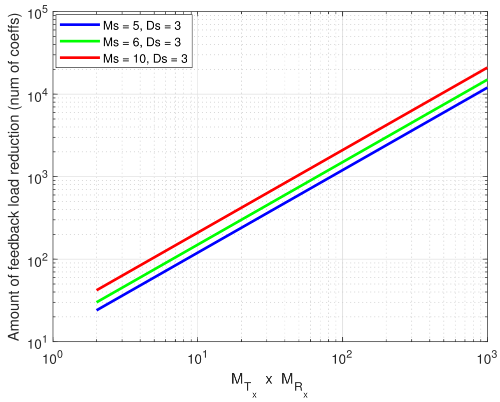



**FIGURE 6.** Relative feedback load reduction for different MIMO sizes.

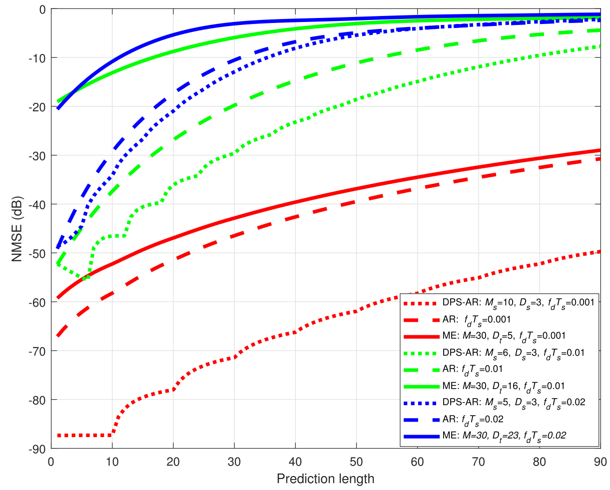



**FIGURE 7.** NMSE comparison of different predictors at different prediction length and for different Doppler frequencies.

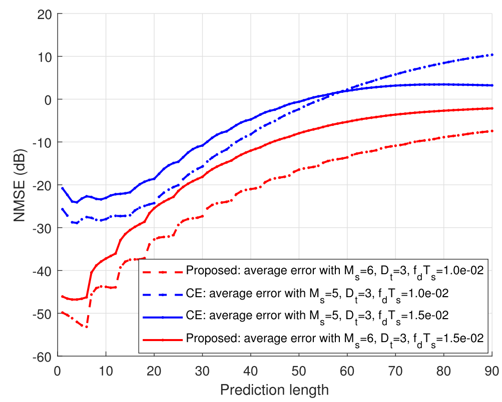



**FIGURE 8.** NMSE comparison of CE-BEM for different doppler frequencies.

selected subframe length which increases the accuracy of the weighting vector estimation and reduces the AR prediction error of the basis coefficients. On the other hand, by proper selection of the number of basis the DPS reconstruction error is controlled to be small enough and consequently the total prediction error of the proposed DPS-AR scheme would be smaller than that of the AR model. Increasing the Doppler frequency, however reduces the correlation in the history vector of the AR predictor leading AR predictor to reach its best performance and reduces the performance gap between the AR and DPS-AR schemes.

Fig. 8 and Fig. 9 shows the NMSE comparison between proposed DPS-AR scheme and CE-AR scheme [25], [27] at different prediction lengths and for different Doppler frequencies in Rayleigh fading channel and COST2100 channel [37]. It should be noted that in [25] and [27], the transmitted symbols are segmented into over-lapping blocks with each containing several sub-blocks. The corresponding CE basis coefficients are updated for each block which in fact is equivalent to sub-block wise tracking of the basis coefficients since the adjacent blocks only differ by one sub-block. For the sake of fairness, the performance of the proposed DPS-AR scheme and CE-AR scheme are compared under the same frame structure proposed in Section III. It can be seen from Fig. 8 and Fig. 9, the proposed DPS-AR scheme works well under both channel models.

In terms of the computational complexity it is clear from (15) and (20) that the conventional AR method requires to calculate an $M$-order MMSE filter, $w[m]$, for each $m \in \{M, \cdots, M_f - 1\}$, while the proposed scheme needs to calculate a $Q$-order MMSE filter, $w_i[q]$, for each $i \in \{0, \cdots, D_s - 1\}$ and each $q \in \{Q+1, \cdots, Q_f\}$. The computational complexity of the MMSE filter design for the conventional AR model is in the order of $C_{AR} = O(M^3(M_f - M))$

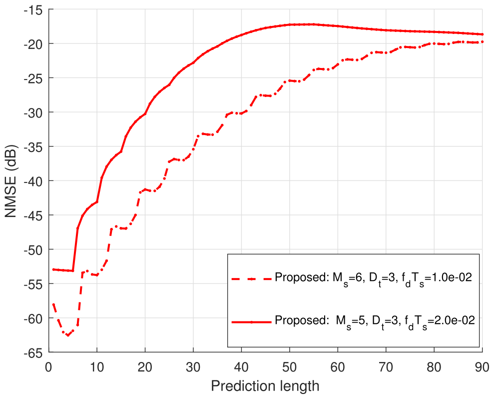



**FIGURE 9.** **NMSE comparison of CE-BEM for different doppler frequencies (COST2100 channel).**

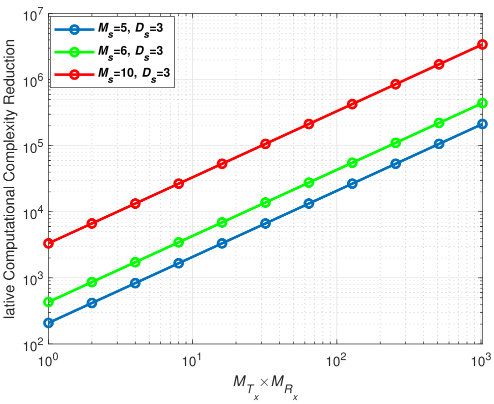



**FIGURE 10.** **Relative computational complexity reduction for different MIMO sizes.**

and for the proposed reduced rank DPS-AR scheme is in the order of $C_{DPS-AR} = O(D_s Q^3 \frac{M_f - M}{M_s})$. Fig. 10 shows the relative amount of computational complexity reduction $C_R = C_{AR}/C_{DPS-AR} = M_s^4/D_s$ for different MIMO sizes. So, although for more fast time variant channels the DPS-AR method has almost the same performance as the AR predictor, it still outperforms the AR method in terms of the computational complexity.

## VI. CONCLUSION

In this paper we have proposed a low complexity channel predictor which uses the DPS basis in combination with an AR model to track the V2X channel's time variation. The same DPS basis are used to model the time-variant channel inside different subframes and the AR model is used for tracking the time variation of the basis coefficients in each transmitted frame. The subframe length and the number of DPS basis used for channel modelling in each subframe are obtained by minimizing the MSE of the proposed predictor and it is shown that the bigger the normalized Doppler frequency is, the smaller should be the sub-frame length and the bigger should be the number of basis per subframe. The simulation results demonstrates the better prediction accuracy of the DPS-AR scheme in comparison to the ME predictor for different prediction length and different ranges of maximum normalized Doppler frequency. Moreover, comparing to AR predictor the DPS-AR approach has better performance both in terms of computational complexity and MSE for low to moderate time variant channels and has almost the same performance for fast time variant channels with lower complexity. The proposed channel predictor can be applied for predicting the precoder in time varying MIMO channels with reduced amount of the required feedback load. For future work, we will discuss the effect of the noise on the proposed DPS-AR channel predictor.

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