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Novel Implementation of Multi-Robot Space Exploration Utilizing Coordinated Multi-Robot Exploration and Frequency Modified Whale Optimization Algorithm

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ABSTRACT Multi-robot space exploration involves building a finite map utilizing a cluster of robots in an obstacle cluttered environment. The uncertainties are minimized by assigning tasks among robots and computing the optimum action. Such optimal trajectories are traditionally obtained utilizing deterministic or metaheuristic techniques, with each having peculiar limitations. Recently, limited work with the sub-optimal result has been done utilizing frameworks that utilize a blend of both techniques. This paper proposes a novel framework which involves the integration of deterministic Coordinated Multi-Robot Exploration (CME) and metaheuristic frequency modified Whale Optimization Algorithm (WOA) techniques, to perform search exploration that imitates the predatory behavior of whales. The frequency is dynamically adjusted utilizing a statistical objective function to tune exploitation and exploration operators. The proposed framework involves a) determination of the cost and utility functional values around individual group members utilizing deterministic CME technique, b) search space exploration to optimize and improve the overall solution utilizing frequency modified whale metaheuristic approach. The effectiveness of the proposed Frequency Modified Hybrid Whale Optimization Algorithm (FMH-WOA) is ascertained by training the multi-robotic framework in different complexity environmental conditions. The results efficacy is then demonstrated by comparing the results of the proposed methodology with those achieved from three other contemporary optimization techniques namely CME-WOA, CME-GWO, and CME-SineCosine.

INDEX TERMS Multi robotic path planning, coordinated multi-robot exploration (CME), metaheuristic, whale optimization algorithm (WOA), hybridization, CME-WOA, CME-GWO,CME-SineCosine.

I. INTRODUCTION

Mobile-robot space exploration utilizing cluster of robots have a wide spectrum utilization ranging from transportation [1], healthcare [2], industry [3], rescue [4]–[6], to all sort of Dull Dirty and Dangerous (DDD) missions [7]–[9]. Similar to Unmanned Aerial Vehicles (UAV) [10]–[14], their applications on ground is increasing at unprecedented rate. An important aspect of mobile robotics is the exploration in an environment where multiple robots build up the finite map, without any prior knowledge of map layout or position of obstacles. Exploration term in mobile robotics is defined as the search process where the robots surf the entire area to create a finite map in a minimum amount of time. Mobile robots need to keep a track of which space/area has been explored before so that they can create a global map and coordinate their positions accordingly.

This exploration can be applied to both indoor and outdoor applications for the intended purpose without the aid of human intervention or any supervision. When compared to a single robot, a group of robots increases space coverage and mitigate search and computation time [15]. However,

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the safety of robots and ensuring communication among the robots, while maneuvering in free space is an important feature in exploration. There are numerous maps available for space configuration representation that helps in the selection of algorithm. In the exploration process, space is represented by occupancy grids where the cells in grids change the value after every new position of the robot. The problem related to uncertainty of grid occupancy is resolved using onboard sensors. To determine the methodology to perform space exploration for the generation of optimal trajectory is the most critical part of such studies.

A. RELEVANT STUDIES

The optimization techniques utilized for the generation of optimal paths for the multi-robot configuration can be deterministic Coordinated Multi-Robot Exploration (CME) [16], stochastic [17] or hybrid (utilizing both deterministic and stochastic) [18] in nature. With each methodology having its own merits and limitations, different researchers have utilized either of these approaches intending to optimize the objective function.

1) DETERMINISTIC CME

Deterministic CME algorithm [19] utilize the coordination of multiple robots to explore the area employing centralized [20] technique. In centralized exploration, all robots have a common map and they all sense the search space simultaneously and interact with each other to share their progress. The cost of traveling distance is computed locally together with utility values and these values are updated in each iteration. CME based algorithms have been vastly utilized in path planning [10], [13], [21], [22] and product design [23], [24].

The search space of uncertainty is explored through sensors, during which robots maximize the knowledge of the area. It takes steps to reach the desired target by gaining knowledge from the sensor to differentiate between occupied and non-occupied spaces. The sensors have a certain range to detect the area's surroundings. This edge is called a frontier and the algorithm is known as a frontier-based methodology for exploration of autonomous robot [25]. The concept utilizes the cost of utility and frontiers for the divergence of a robot. The cost of traveling distance is computed locally together with utility values and these values are updated in each iteration.

Puig *et al.* [26] investigated deterministic optimization for CME utilizing the K-means clustering process. This allowed the robots to travel simultaneously yielding the lowest variance of average and regional waiting time during the path exploration. Benkrid and Achour [27] configured multi-robot exploration by minimizing the energy consumed by the robots. Similar work was performed by Rappaport *et al.* [28], who investigated the multi-robot exploration recharging process. Verbiest *et al.* [29] presented the implementation of frontier-based exploration for building up the map. The frontier points are defined as border points that are calculated during navigation and mapping between the occupied (known) and non-occupied (unknown) spaces. The proposed methodology was verified initially through simulations and then configured for real-world scenarios utilizing Real-Time Operating System (ROS) system.

Francisco *et al.* [30] proposed a frontier exploration algorithm for simultaneous localization and mapping, with an objective to eliminate frontier points from the map. Clara Gomez *et al.* [31] proposed integration of frontier based approach with a behavioral strategy to build a topological map. Similarly, Gomez *et al.* [32] developed a frontier-based exploration technique compatible with the ROS system for analyzing path length and execution period.

From the cited papers, it is observed that all CME based algorithms involve coordination amongst the robots and are therefore greatly dependent on the relative distance amongst the robots [33], [34]. If the distance among robots increases, the communication link breaks up and the robots cannot communicate instantly with each other. Under such cases, the centralized CME approach does not guarantee definite way-points in the map. This will affect the task assigned to individual robots and will cause coordination break down in case any of the robot(s) forgets the assignment tasks. Moreover, CME is not efficient under all map conditions, so there is no possible way to find an optimal solution except if the entire map is changed which is not possible every time.

2) STOCHASTIC ALGORITHMS

Stochastic algorithms are bio-inspired algorithms that have gained immense popularity recently, mainly due to their computational efficiency for handling complex problems [35]–[37]. Bio-inspired methodologies utilize different agents that work on nature-inspired behavior that can be exploited and applied to an optimization problem in different applications. These are aimed at finding the best optimal solution until a stopping criterion is achieved, thus making them a part of a stochastic global optimization group. They are widely utilized in control theory [38], [39] and are applied to a research field known as swarm robotics, where a known number of robots are controlled in a real-world environment in a coordinated way.

The bio-inspired computing is based on meta-heuristic methods such as Grey Wolf Optimizer (GWO) [40], Whale Optimization Algorithm (WOA) [41], Flower Pollination Algorithm (FPA) [42], Genetic Algorithm (GA) [43], Ant Colony Optimization (ACO) [44], and Particle Swarm Optimization (PSO) [45], SineCosine algorithm [46] etc. A detailed review on such algorithms can be found in [47], [48].

GWO mimics the leadership hierarchy of grey wolves. The leadership involves different wolves namely: alpha, beta, delta, and omega which depends on three steps: hunting, searching for prey, and attacking prey. FPA is a bio-inspired technique that imitates the pollination behavior of flowering plants: the process involves keeping alive the fittest species alive through reproduction. A whale optimization algorithm is a meta-heuristic optimization method, developed by Mirajalili *et al.* [41]. A complete comprehensive review on whale optimization and its applications are discussed in [49]–[51]. The algorithm is inspired by the hunting behavior of whales and many researchers utilized the WOA for the single and multi-objective purpose for mobile robot navigation and exploration.

PSO is commonly used in numerous control systems due to continuous upgrading of the velocity and position of a particle swarm. Wang *et al.* [52] presented the modified version of the PSO algorithm, such as fractional order RDPSO (FORDPSO), Darwinian PSO (DPSO), fuzzy adaptive (FORDPSO), and robotic DPSO (RDPSO). RDPSO and FORDPSO are normally used for multi-robot exploration. Dao *et al.* [53] utilized WOA to achieve multi-objective (path smoothness and path distance) function for robot navigation. The target and start locations are known as the fitness of WOA. The best global location was selected in each iteration which creates waypoints for trajectory formation. Simulation results showed the effectiveness of the proposed algorithm as it efficiently helped the robot to reach the target location in minimum time. Chhillar *et al.* [54] utilizing WOA proposed an alternative strategy by combining heuristic and classical approaches. The proposed methodology was verified through simulations. Nicola *et al.* [55] presented the integration of Ant Colony Optimization technique with Whale Optimization method for finding the optimal path for a mobile robot by achieving multi objectives of a) finding the optimal path, b) path smoothness. Kumar *et al.* [56] propose the hybridization of advanced sine cosine algorithm (ASCA) with advanced ant colony optimization algorithm (AACO) for searching the optimal path. The method was implemented in real-time with sensors, where the sensors detect the obstacles and find the global best position and in the next phase, the ACO algorithm is programmed in such a way that it evaluates and selects the next best value. By following the method robot is able to reach the target location. When the proposed method is compared with contemporary algorithms it has been found that the method performs exceptionally well, which proofs the effectiveness of the method. Qingyong Yang *et al.* [57] proposed the multigroup multistrategy SCA algorithm (MMSCA) for capacitated vehicle routing problem (CVRP). The numerical experimental results proofs the efficacy of the proposed method.

From the literature review and the cited papers, it has been observed that the conventional stochastic algorithms have relatively low accuracy and convergence rate, which makes them susceptible to fall into local optima problem easily [58]–[60]. Moreover, these bio-inspired algorithm involves the generation of non-unique solutions and therefore requires benchmark cases which unfortunately most optimization problems do not have. Reluctantly, randomness in the results is encountered causing sub-optimal solutions. Although, the existing meta-heuristic techniques are available for single and multi-robot exploration, however, they have been utilized for single robot applications [53], [61], and their impact on multi-robot configuration is yet to be ascertained. Therefore, stochastic based algorithms for multi-robot configurations, which can generate unique solutions in a near real-time environment is a potential area for research.

3) HYBRID ALGORITHMS

Hybrid algorithms for multi-robot involve utilization of both deterministic CME and stochastic Bio-inspired techniques to solve modal problems. They facilitate efficient way-point generation in the map and achieve improvements in the results that could be achieved from the utilization of individual approaches.

Wang *et al.* [62] proposed the frontier-based method with PSO and A* in two different stages. In the exploration process, the frontier points are determined around the robot and A* computes the shortest distance. The robot navigates through corners and collects exploration information and the position is updated using the PSO algorithm. The PSO algorithm is used for different task assignments in a cluttered environment, as mentioned in [26].

Nunzia Palmieri *et al.* [63] presented the two meta-heuristic approach and exploration process for mine disarming task carried out by swarm robots. The primary objective of robots is to work cooperatively to discover mines, disseminate the information among robots, and cooperatively disarm the mines. The objective also lies in the distributing regions among robots to explore the space in a minimum amount of time and collaboratively disarm the detected mines. For this purpose author integrated the Firefly algorithm (FIS-RR) and Ant-colony Optimization (ATS-RR) to achieve exploration performed by robots. The performance evaluation in terms of disarming the mines and exploring the space was checked with Particle Swarm Optimization (PSO). The obtained results proofs the proposed algorithm works better in a complex environment.

S. Sharma *et al.* [64], utilized a nature-inspired algorithm (NIA) and clustering-based distribution factor (CBDF) to explore the unknown spaces. The entire map is divided into exploration movements where robots acquire direction through the CBDF algorithm. Three nature-inspired algorithms (Bat algorithm (BA), PSO, Bacteria foraging (BFO)) were utilized and compared to determine the efficiency.

Faiza *et al.* [65] worked on the path planning for mobile robotics utilizing an integration of bio-inspired Grey Wolf Optimization algorithm with Particle Swarm Optimization (PSO), together known as HPSO-GWO. Way-points were generated for formulating a suitable and shortest trajectory for navigation of mobile robot.

4) CRITICAL OBSERVATION

Based on our review of the work done in regards to space exploration utilizing multi-robot configuration and the cited papers, it can be noted that certain areas are not fully covered in the literature and require further investigation. There has not been any conclusive study that formulates the CME algorithm for multi-robot exploration. Moreover, the hybrid algorithms formulated to date have not been fully optimized as certain limitations (area convergence, time complexity) are yet to be curtailed. Integration of algorithms although resulted in achieving a portion of intended results, but increased the computational complexity, which ultimately affected systems performance [66].

Meta-heuristic techniques such as Grey-wolf utilized in many existing hybrid algorithms although increase the convergence rate, however, the results might not be always optimal. This increases the probability to get stuck in the local/global optima problem. Furthermore, manual tuning of parameters in a hybrid algorithm makes the objective function more susceptible to slow convergence rate and low precision [67]. It, therefore, becomes extremely difficult to theoretically analyze the random decisions made by algorithms and then to implement on objective functions [60], [68]. Therefore there remains a need to explore hybrid algorithms combining CME with meta-heuristic technique to achieve optimal space convergence utilizing multi-robot configuration.

B. PAPER CONTRIBUTION

In this paper, an optimal hybrid algorithm namely *Frequency Modified Hybrid Whale Optimization Algorithm* (FMH-WOA), comprising of deterministic CME and meta-heuristic frequency modified WOA is proposed. The modified WOA has numerous merits when it comes to implementation. The algorithm has lesser parameter involvement, lesser memory requirements, a fast convergence rate due to the continuous reduction of search space, and the involvement of few judgment variables. Another advantage lies in avoiding the local optima problem as it requires only 2 parameters to be adjusted. Another added feature that makes WOA effective compared to other popular swarm intelligence algorithms is its hierarchical structure. The dominance hierarchy is formed according to the goals defined in the objective function. The objective function is classified into the cost function, evaluated cost, and the fitness function. These factors summarize how accurate the final result is compared to the given design solution. Whether the best solution is defined to be optimal among all available candidates, it should satisfy the fitness function and the cost function that is regarded as interchangeable functions of maximization and minimization. Realizing, WOA as an emerging field with immense potentials and the fact that no significant work has been done specifically in the case of multi-robot configuration, the stated algorithm was selected in this research.

Based upon the literature review and the cited papers, to the very best of our knowledge, there has not been any conclusive study that formulates the CME algorithm for multi-robot exploration. This research presents the first study to propose a methodology in which WOA and CME are utilized in an integrated manner for a multi-robot configuration. The proposed hybrid FMH-WOA algorithm is further optimized by including a statistical frequency function. This function dynamically tunes the exploration and exploitation parameters. CME being a deterministic method does not give full coverage of the map. The proposed algorithm in the manuscript (FMH-WOA) is considered as a variant of WOA because the governing equations utilized in the proposed methodology are fundamentally the same as the original WOA. The proposed algorithm in the manuscript (FMH-WOA) is considered as a variant of WOA, because the governing equations utilized in the proposed methodology are fundamentally the same as original WOA. The utility values of neighboring cells are initially evaluated and then optimized to find optimal utility values corresponding to the best fitness for each agent. The essence of maximizing the utility is that initially, each cell of the whole map had identical values. While the robots search, the utilities of their frontier cells decreases (as defined in equation 5 of the article). The robots are programmed to decline their interest to visit the cells having low utility values. This is the reason why the robots try to search for new areas, which they have not yet explored to maximize utility values. This ensures that the best fitness for each agent is selected from every iteration when all robots are coordinating to surf the area. The initially computed guess trajectories achieved through the deterministic CME technique are refined for evaluating the environment and successful coordination of robots for space exploration. Integrating the results with the frequency modified WOA algorithm, the probabilistic variation bounds on CME are determined. This provided new values for the desired task to explore the space for complete coverage. The efficacy of the results achieved through the proposed algorithm is then demonstrated by comparing the results with other hybrid techniques which involved the integration of CME with grey wolf algorithm, CME with conventional whale algorithm, and CME with SineCosine algorithm.

II. PROBLEM FORMULATION AND PROPOSED ALGORITHM

This research demonstrates the implementation of a unified framework which integrates deterministic CME and meta-heuristic frequency modified WOA, together referred to as **Stochastic Exploration.** The whole map is divided into distant grids. The precedence of surrounding cells, their utility, and cost values around the robots, is initially evaluated utilizing the CME technique. The path is then optimized utilizing a frequency modified WOA algorithm, thereby improving the overall solution. A detailed description and implementation of both techniques for the exploration of a map for the multi-robot are elaborated in this section.

A. COORDINATED MULTI-ROBOT EXPLORATION

CME utilizes the occupancy grid mapping to represent the environment [69], [70]. The sensor view of 360° with the initial position is known to the robot. The robot is located in an indoor environment and does not have any information related to the environment. Each cell of such an occupancy grid map stores certain numerical values demonstrating the previous probability that the grid cell is taken by an obstacle. In real-time, the sensor range is limited to cover the entire area for planning an optimal path for the robot. When exploring the unknown area, the most important task is to know about **Frontier cells** [71]. A frontier cell is defined as the explored cell that is an immediate neighbor of an unexplored cell. For every robot, the cost of a cell is directly proportional to the total distance from the robot to that cell. The sensor view observation of mobile robot covering cells is depicted in Figure 1. The objective function is formulated as follows:

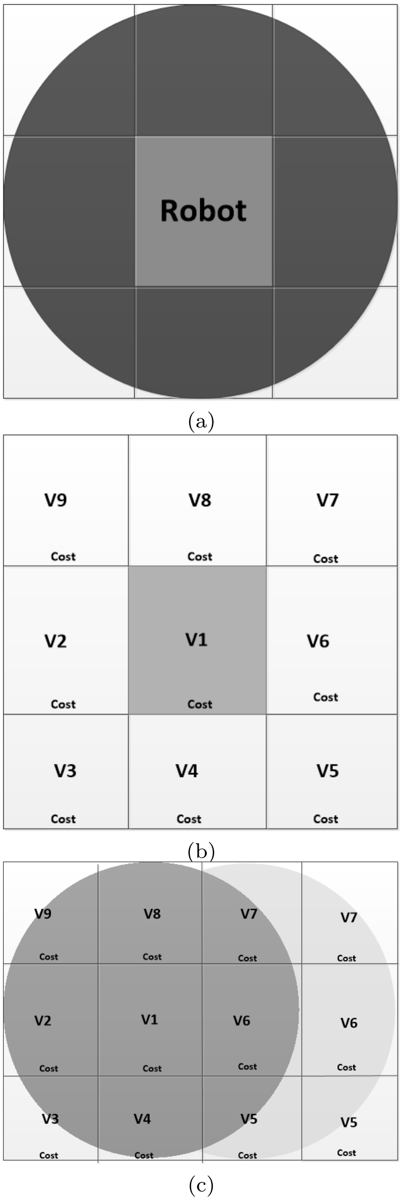


FIGURE 1. Demonstration of sensor observation in the occupancy grid map: a) the sensor shadows eight neighbor cells, b) the neighbor cells have the name V1, V2, V3, V4, V5, V6, V7, V8, V9 and cost, c) the costs V9, V2, V3 do not have the intersecting sensor observations.

1) OBJECTIVE FUNCTION

The popular deterministic approach is utilized for reaching the next frontier point by calculating the optimal path from the current robot position to the next cell is utilized. In 2-dimensional occupancy grid map, the tuple (x,y) corresponds to the x_{th} and y_{th} cell in the direction of x and y axis. The cost of these grid cells *(x,y)* (containing robot position) is calculated utilizing occupancy probability, Euclidean distance, and the sensor observation. They can be found out in two steps, as depicted in equations 1, 2 and 4.

1) **Initialization**: Initialize 0 grid cell containing robot location, otherwise ∞.

$$V_{x,y} = \begin{cases} 0, if\ (x, y) is robot current position \\ \infty, otherwise \end{cases} \tag{1}$$

2) **Loop**: For the grid cells (x,y), perform

$$V_{x,y} = min\{V_{x+\Delta x, y+\Delta y} + \sqrt{\Delta x^2 + \Delta y^2}. \tag{2}$$

$$P(occ_{x+\Delta x}, y + \Delta y)\} \tag{3}$$

$$V_{x,y} = min\{\sqrt{\Delta x^2 + \Delta y^2}.P(occ_{x+\Delta x}, y + \Delta y)\} \tag{4}$$

where, $\Delta x,\ \Delta y \epsilon [-1, 0, 1] \wedge P(occ_{x+\Delta x}, y+\Delta y)\epsilon[0, occ_{max}]$ and occ_{max} is the maximum probability value of the grid cell.

According to the above equations, if the grid cell has been explored before for checking the availability of obstacles, then the cost of the grid cell in the last step is appended to the cost in the new position. If the beam is opened to the grid cell primarily, then the grid cell is designated as a frontier cell eliminating the backward costs in the previous steps.

The occupancy probability of grid cells are as follows:

- The occupancy probability 1 represents that the grid cell is occupied by the obstacle.
- The occupancy probability 0.5, shows the uncertainty of an unknown/unexplored grid cell.
- The occupancy probability 0, shows the certainty of a grid cell that it is not occupied by an obstacle.

The occupancy values of the grid cells decline when the sensor beam touches the grid cell at a certain distance (refer to equation 2 of [34]). For a single robot, searching the minimum cost for determining the next position is easy, however, in multi-robot, a coordinated integration is required for determining the next position because every robot move is linked with the other robots. For this, CME (Coordinated Multi-Robot Exploration) with the notion of utility values is utilized for distributing tasks among robots. The purpose is to mitigate the cost among neighboring cells to find the optimal next best position of the robot.

2) MAXIMIZING THE UTILITY

Maximizing the utility values is defined as: in the beginning, every grid cell has the same utility values in the map. When the robots surf the map, the utility values of their frontier cell decreases, refer to equation 5. The robots only visit those grid cells having higher utility values to explore new positions in a map to maximize utility values.

$$U_j^{gc} = U_{j-1}^{gc} - \Sigma P(\| occ_{x,y}^{u} - occ_{x,y}^{u'} \|) \tag{5}$$

The grid cell utility value U_j^{gc} is the state of previous modification U_{j-1}^{gc}. This value is modified during the

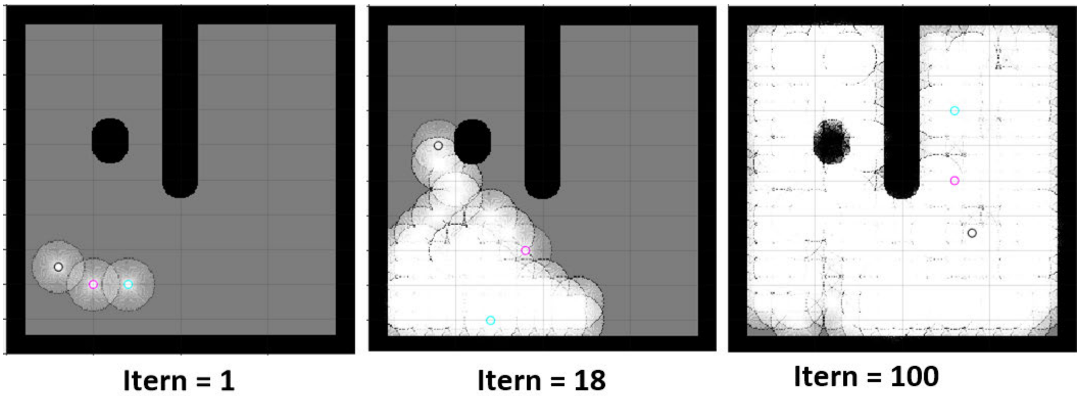


FIGURE 2. Coordinated Multi-Robot Exploration.

exploration phase done by the robots and has a value equal to the probability occupancy of the chosen selected grid cell minus the robot's current position. This maximum value is selected using equation 6.

$$(j, gc) = max(U_j^{gc} - V_{x,y}) \tag{6}$$

Towards the collaborative cooperation, all the robots start their journey earliest for their sensors scan reach the first iteration Figure 2. This allows the divergence in search space by decreasing the utilities of chosen targets. In Figure 2, the search space is $20m \times 20m$ in size with ray length equals to $1.5m$. The primarily presented research on CME demonstrates fast exploration for a multi-robot system. Nonetheless, some room for improvement was found, firstly, the cost parameters are mitigated if there is more than one identical value, then the last value is selected, secondly, if the utility and cost values are the same, then the robot gets stuck at one position. To avoid this problem, a solution is needed for the robot to search unexplored areas.

B. FREQUENCY MODIFIED WHALE OPTIMIZATION ALGORITHM (FMH-WOA)

The conceptual framework of WOA is as follows:-

$$\vec{D} = |\vec{C}.\vec{X}^{*}(t) - \vec{X}(t)| \tag{7}$$

$$\vec{X}(t+1) = |\vec{X(t)} - \vec{A}.\vec{D}| \tag{8}$$

$$A = C = \vec{2} * rand() \tag{9}$$

where t is number of current iterations, $C = [2 \vec{*} rand()]$ and $A = [2a * \vec{rand}() - a]$ are coefficient vectors, $\vec{X}^{*}$ is the position vector of best solution of the prey, $\vec{X}$ is the position vector, the values of a linearly decreased from 2 to 0 during iteration, and $rand()$ are random numbers in the range [0,1]. By adjusting the values of A and C vectors, suitable places around the best agent can be found out using the current position.

The humpback whales swim around the prey within the shrinking circle in a spiral shape. For modeling this simultaneous behavior, there is a probability of 50% to choose between either spiral shape model or either shrinking encircling model for updating the position of whales during the optimization problem. The mathematical model can be written as:

$$\vec{X}_{t+1} = \begin{cases} \vec{X}^{*}(t) - \vec{A}.\vec{D}, & \text{if } p < 0.5 \\ \vec{D}' e^{bl} cos(2\pi l) + \vec{X}^{*}(t), & \text{if } p \geqslant 0.5 \end{cases} \tag{10}$$

where p is a random number defining the shape of a logarithmic circle. $\vec{A}$ are the coefficient vectors with random values between 1 and -1, forcing search agents to move away from prey and a linearly decrease from 2 to 0 to enforce exploitation and exploration. If $| \vec{A} > 1 |$ enforces the exploitation and perform global search, mathematically can be found using equations 11 & 12, and if $| \vec{A} < 1 |$ than half of the iterations are dedicated to exploitation. So $\vec{A}$ is utilized for a transition between exploration and exploitation.

$$\vec{D} = | \vec{C}\vec{rand} - \vec{X} | \tag{11}$$

$$\vec{X}(t+1) = \vec{rand} - \vec{A}\vec{D} \tag{12}$$

The objective of attacking and killing prey for whales in nature has high-efficient optimization. The WOA has numerous merits when it comes to its implementation. The only disadvantage this algorithm has, that in unimodal problems, in the beginning, it hasten towards finding the optimal solution but eventually slows down mainly due to the extensive diversity issue. For speedy convenience of the audience, the conventional WOA technique is depicted in Algorithm 1.

WOA's performance is compared to different population-based algorithms utilizing benchmark functions on Table 2, Table 3, Table 4 in the original paper [41]. The fundamental aim lies in killing the prey, which is related to its life reduction *fmin* by whales attacking the prey. Table 2 depicts a scenario where 30 whales individually attack the prey one after the other. The *fmin* will be zero when the last whale kills the prey. In Tables 3 and 4, whales attack the prey collectively, and any injury dealt by the whales on the prey varies. The negative values depict the energy/power is wasted for killing the prey and a value greater than zero indicates that the prey is still alive.

We tested the WOA on unimodal function f1 from Table 2 [41] (sphere function). The convergence rate is fast during initial iterations, depicted in simulation run, (refer Figure 3(a),(b),(c)). The 2-d space is considered with 5×5 bounds, 20 iterations are required for 10 population size to reach the optimal zero solution. The leader whale trend is shown in Figure 3(d).

Algorithm 1 Pseudo Code for WOA

Initialize whale population X_i (i=1,2,3…n)
Calculate fitness value of each search agent and find best search agent X^*
while t < maximum number of iterations **do**
  for each search agent **do**
    if p<0.5 **then**
      if $|\vec{A} < 1|$ **then**
        $\vec{X}(t+1) = C * \vec{X^*}_t - \vec{A}.\vec{D}$
      else if $|\vec{A} \geqslant 1|$ **then**
        $\vec{X}(t+1) = X_{\vec{rand}} - \vec{A}.\vec{D}$
      end if
    else if p>0.5 **then**
      $\vec{X}(t+1) = \vec{D} * exp^{b.l} * cos(2\pi l) + X^*(t)$
    end if
  end for
  Evaluate fitness of $X(t+1)$ and update X^*
end while

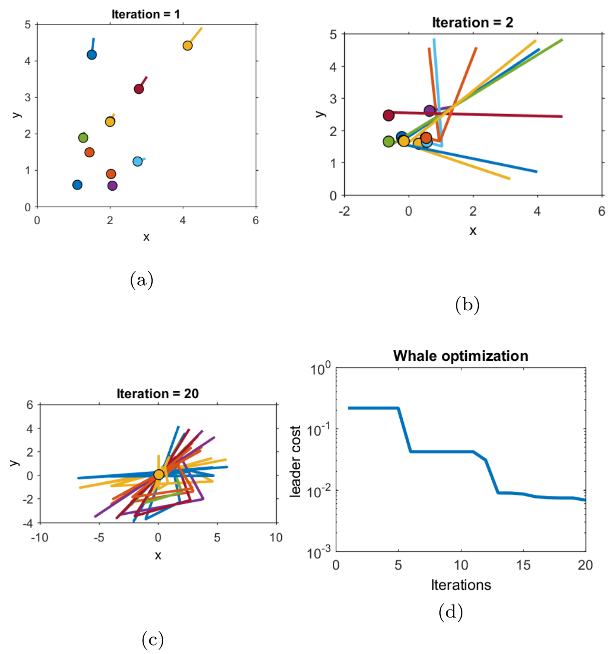


FIGURE 3. WOA performance that calculates the cost using sphere function in iteration 1 (a), in iteration 1 (b), and in iteration 2 (c). and in iteration 20, (d) The leader whale trajectory in the search of the optimal solution. Population = 10. The leader whale in iteration 1 is 2.6349, iteration 2 is 1.7958, and iteration 20 is 0.093259. The runtime of 20 iterations is 2369.618595 sec.

C. INTEGRATION OF CME WITH FREQUENCY MODIFIED WOA

In this subsection, the hierarchy of Frequency Modified WOA integrated with CME is presented. The conventional whale algorithm is modified to include a frequency-based parameter for efficient map exploration. Instead of using constant values for the parameters A & C, they are configured as a statistical function. Frequency as a parameter is added for optimum tuning of the function values for the exploration and exploitation phase. Depending on the control parameters involved, the best position of all robots is defined. The algorithm generates certain random parameters for determining the maximum positions of the robot that alters the order. This stochastic approach works without a priori information about space that needs to be surfed to find the solutions.

Algorithm 2 explains the proposed hybrid exploration. The utility value is kept as 1. The robot surrounding sensory window is divided into eight vector cells, V_c, where V_c is $V1_{x,y}, \ldots\ldots, V8_{x,y}$. These cells are the new candidate's positions for the robot. The stochastic technique computes the cost and minuses the utilities from the cost for the eight cells using equation 6. The meta-heuristic optimizer then calculates the utility value for leader whale after vetting the candidates with priorities. These priorities changes due to f1 and f2, refer to step 8 and the occupancy probability values of the dominated grid cells. f_{max} is a dynamic parametric value assigned to the whales when they have to attack the prey while whales will get f_{min} value when they kill all the preys. The numeric ranges for this parameter is $fmin = 0$ &$fmax = 2$. In the proposed algorithm, when the entire space has been surfed the frequency function tries to converge due to the presence of exponential function.

The hunting operators for the surrounding environment are defined by equation 13 and 14.

$$D_{lead,i} = |f1(i).P_{lead,i}(occ_{x+\Delta x,y+\Delta y}) - P_i(occ_{x+\Delta x,y+\Delta y})| \tag{13}$$

$$X_{1,i} = P_{lead,i}(occ_{x+\Delta x,y+\Delta y}) - f2(i).D_{lead,i} \tag{14}$$

The next best robot position X(t+1) is automatically fed to the best whale, designated as leader whale and the maximum value is assigned to that whale. So the utility values of neighbor cells are reduced by equation 5. For the next iteration, new random values through modified frequency parameters are generated.

The value of P decides the model shape. Humpback whales encircle their prey based on two strategies: (i) they make a spiral shape, (ii) they make shrinking encircle. There is a 50% probability for a whale to choose either a spiral shape model or a shrinking encircling model for updating its position during the optimization problem. The parameter A in the original WOA is changed with statistical frequency function (refer to algorithm 2, step 8, the description can be found in section III, subsection C, para 2). The FMH-WOA continuously updates the next best whale value during the iteration. The costs and utilities of cells around a robot provides knowledge over each

Algorithm 2 Coordinated Multi-Robot Exploration With WOA

1: Initialize the total number of robots nRbt, initial position (Sp) and iterations (iter), sensor range (SR), fmin=0, fmax=2
2: Utility value = 1
3: **while** t < iter **do**
4: **for** all nRbt **do**
5: Set coordinates of Vc
6: Calculate cost of Vc
7: Perform subtraction, U_j^{gc} and Vc
8: Adjust the frequency function $\Big(f1 = fmin + (fmax - fmin) * exp(\tau * rand())\Big)$, $\Big(f2 = (fmax - fmin) * exp(\tau * rand()) - fmax\Big)$, $\tau = 0.01$
9: Find best whale using step 7 (refer to, algo 1)
10: Find leader whale from step 9(refer to, algo 1)
11: Find X(t+1) i.e. distance to leader whale
12: Change robot position X(t+1)
13: Reduce the utility value U_j^{gc}
14: **end for**
Evaluate Frequency function 'f'
15: **end while**

robot step. The statistical frequency function of FMH-WOA selects the best whales. Then, this function obliges the robot to move according to the best cell value that is formulated from the occupancy probability. For the next best solution and to determine the next move of the robot, FMH-WOA is used.

It remains to mention that the control parameter utilized in CME-WOA and CME-GWO algorithms are A & C and they drive the algorithm. Based on these values the algorithms search for their prey. These parameters linearly decrease from 2 to 0, which demonstrates when to stop the hunting. In the proposed algorithm FMH-WOA both these control parameters are modified and a statistical frequency-based function is introduced to dynamically cater to the changing requirements. The tuned value of these statistical function giving the most optimal values are mentioned in Algorithm 2 (step 8).

The unexplored areas in the map have greater utility value, those areas than those of explored cells. When the cost of explored cells is subtracted from the utility values of unexplored cells, the maximum value becomes more attractive for the next robot position. In the proposed hybrid stochastic method the hierarchical order of whales is set by the statistical frequency parameter.

III. RESULTS AND DISCUSSION

In this section, we present the results for the proposed hybrid multi-coordinated exploration based on frequency modified WOA. Different complexity area maps are utilized to assess the algorithm performance. The map complexity is varied by adding obstacles and their relative orientation.

For comparison purposes, the dimensions of all the maps are kept constant at $20 \times 20m$. In the maps, the black occupied area shows the presence of an obstacle and the white area shows the explored area. Furthermore, the maps are divided into sections namely 0-40%, 40-75% and 75-100%, according to the area explored by the robots. The efficacy of Frequency modified hybrid WOA **(FMH-WOA)** is subsequently validated on different complexity areas ranging from normal to highly dense & cluttered conditions. To demonstrate the improvements achieved, three other hybrid algorithms namely a) deterministic CME combined with meta-heuristic conventional WOA **(CME-WOA)** and b) deterministic CME combined with meta-heuristic Grey Wolf Optimization **(CME-GWO)** and c) deterministic CME combined with meta-heuristic conventional SineCosine **(CME-SineCosine)** is then implemented under similar conditions. The results of FMH-WOA are then compared to those of CME-WOA, CME-GWO, and CME-SineCosine to analyze the potential benefits.

The robot's position on the map is considered arbitrary. So the primary principle is to take care of the full map to be fully explored. Equation 15 is used for computing the total explored grid cells (T_{gc}) in percentage form:

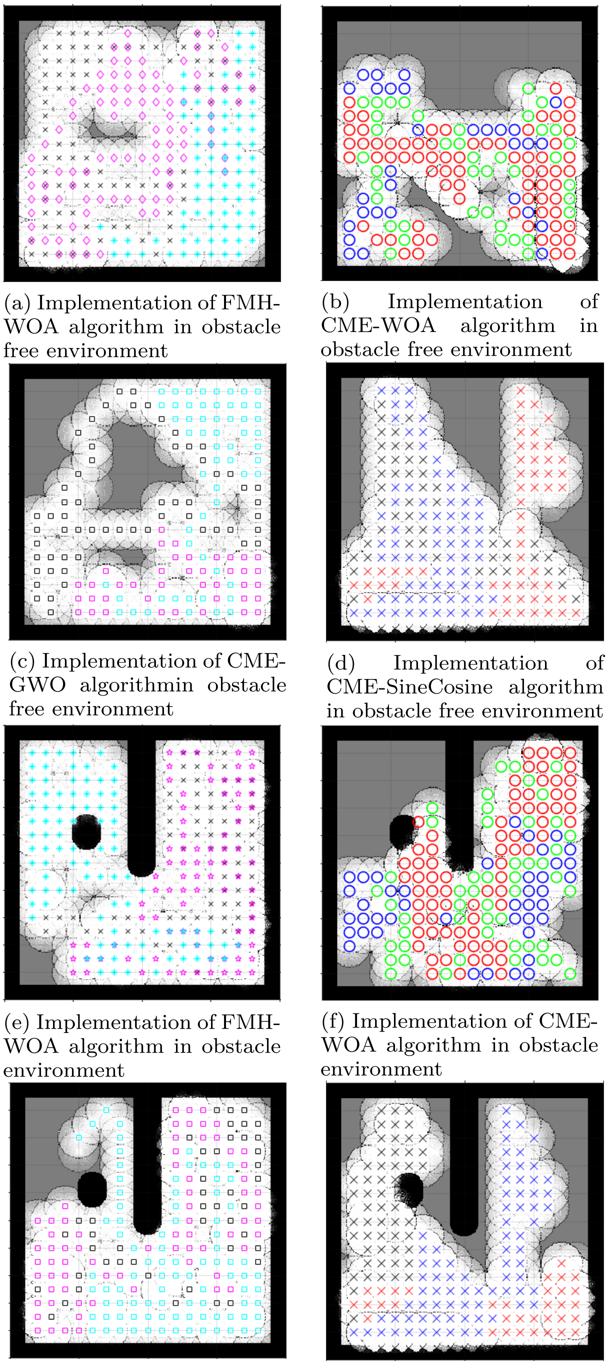
$$T_{gc} = \frac{Total_{unexplored} - Total_{explored}}{Total_{unexplored}} * 100 \qquad (15)$$

Based on this parameter, an assessment of the area being explored by the multi-robot configuration is being made. It can have the best value of 1 which represents 100% area being explored and the minimum value of zero which corresponds to no area being explored. Also, $Total_{unexplored}$ is the total unexplored area having some utility value that depicts the area free from obstacles, while $Total_{explored}$ is the total explored area. Comparison between different methods is done by computing the T_{gc} value at the end of the simulation. The parameters set include; the number of iterations, map size, obstacles, sensor range, number of robots, and the start position of the robot.

A. NOMINAL COMPLEXITY MAP

Figure 4 depicts implementation of FMH-WOA in an area with relatively less number of obstacles. Firstly, the algorithm is implemented with no obstacle (as referenced in Figure 4(a), 4(b), 4(c)), 4(d)) and then obstacles are introduced (refer Figure 4(e), 4(f), 4(g)), 4(h)).

Use of a hybrid strategy in which the addition of random weights in utility value forces the robots to predict the next position often leads to obstacle collision. This problem arises when the robot is occupied by the same utility values or/and occupied by the obstacle from the neighboring cell. Figure 4 demonstrate a similar situation, however, the random parameters A and C of the FMH-WOA algorithm helps in finding the new position for the robot due to producing continuously random values. In our proposed methodology, the introduced statistical function for tuning modal parameters fastened the search process for identifying the new position for the robot.



(a) Implementation of FMH-WOA algorithm in obstacle free environment
(b) Implementation of CME-WOA algorithm in obstacle free environment
(c) Implementation of CME-GWO algorithmin obstacle free environment
(d) Implementation of CME-SineCosine algorithm in obstacle free environment
(e) Implementation of FMH-WOA algorithm in obstacle environment
(f) Implementation of CME-WOA algorithm in obstacle environment
(g) Implementation of CME-GWO algorithm in obstacle environment
(h) Implementation of CME-SineCosine in obstacle environment

FIGURE 4. **Environment exploration by FMH-WOA and its comparison with CME-WOA, CME-GWO and CME-SineCosine algorithms: Nominal complexity map.**

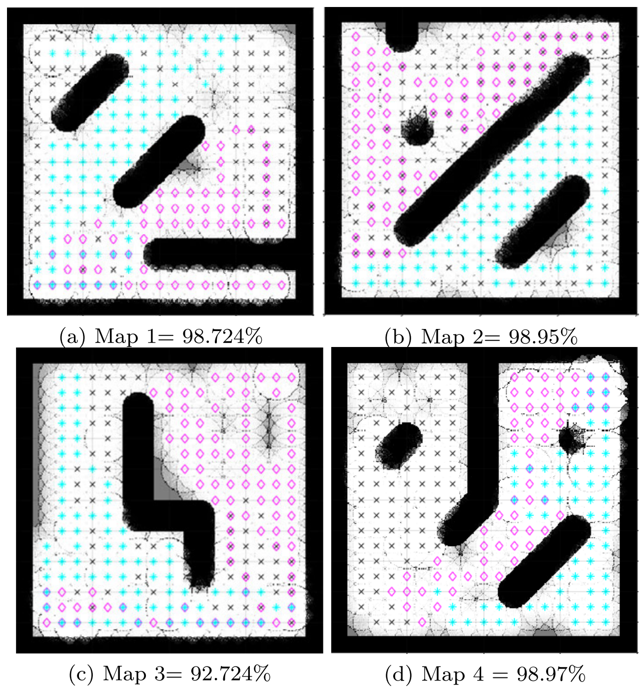
It is evident that a total of 98.3% and 97.1% area has been surfed 4 (a) & (d) for the two cases respectively.

The obtained results from FMH-WOA depicts the model effectivity and viability. It is also observed that if the number of iterations is increased to 120 or 150, then the proposed method produces even better results. The efficacy of the results are then compared with those achieved from **CME-WOA** (refer Figure 4)(b) & 4(f) and **CME-GWO** (refer Figure 4)(c) & 4(g) and **CME-SineCosine** (refer Figure 4)(d) & 4(h). It is evident from the referred figures that the proposed methodology Figure (4)(a) & (e) produces better results than all contemporary methodologies. The algorithm is more efficient in exploring the area, consumes less energy, and requires the least amount of time for space exploration.

The trade-off is often observed in such scenarios where the computational time of the algorithm gets affected by one or more parameters when we balance two objectives together but this trade-off can be neglected when certain/desired objectives are fulfilled.

B. MID-RANGE COMPLEXITY MAP

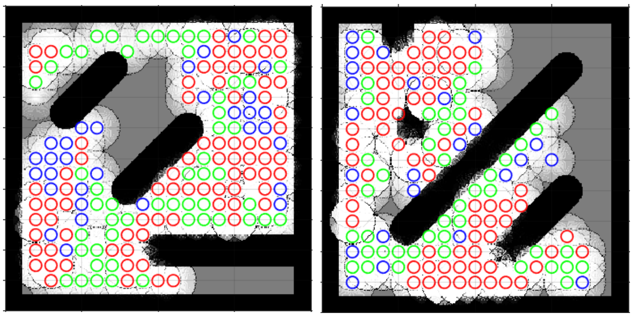
Figure 5 depicts moderately cluttered obstacle environment in which the performance of the proposed FMH-WOA algorithm is evaluated. The efficacy of these results obtained from proposed algorithm are then compared with **CME-WOA** (refer Figure 6) and **CME-GWO** (refer Figure 7) and **CME-SineCosie** (refer Figure 8).



(a) Map 1= 98.724%
(b) Map 2= 98.95%
(c) Map 3= 92.724%
(d) Map 4 = 98.97%

FIGURE 5. **Implementation of FMH-WOA algorithm: Mid range complexity map.**

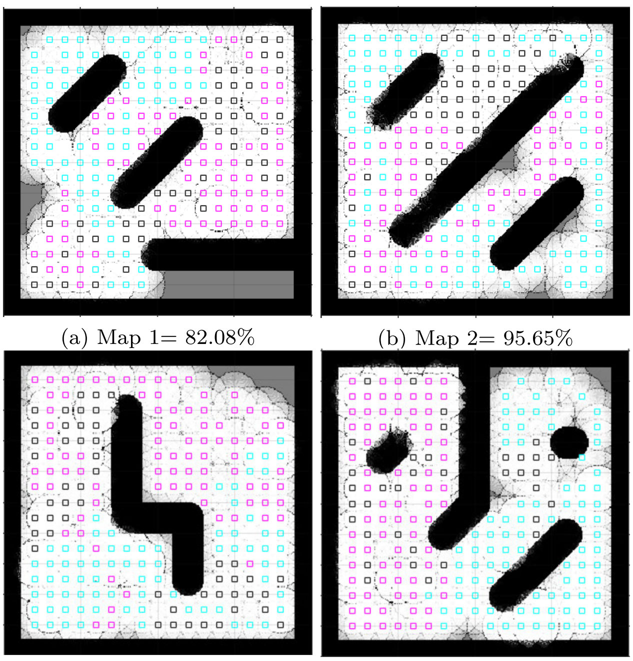
It is evident from the referred figures that the FMH-WOA produces results that are better than all the contemporary techniques. The algorithm is more efficient in exploring the area, consumes less energy, and requires the least amount of time for space exploration. It can be seen that the proposed methodology fasten the search process for identifying the new position for the robot. FMH-WOA surfs Map 1 having 2 barriers and a tunnel obstacle (at lower left corner) with the



(a) Map 1= 72.56% (b) Map 2= 85.79%

(c) Map 3= 53.48% (d) Map 4 = 67.89%

FIGURE 6. **Implementation of CME-WOA algorithm: Mid range complexity map.**



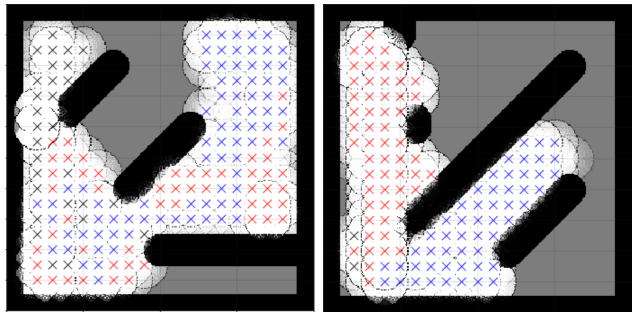
(a) Map 1= 82.08% (b) Map 2= 95.65%

(c) Map 3= 91.45% (d) Map 4 = 90.06%

FIGURE 7. **Implementation of CME-GWO algorithm: Mid range complexity map.**

efficacy of 98.72%. Similarly, for Maps 2,3, and 4, the area coverage is 98.95%, 92.72%, and 98.97% respectively.

The proposed algorithm demonstrated an inherent capability to move around corners without getting stuck. When the same scenario was applied on CME-WOA, CME-GWO, and CME-SineCosine, all the algorithms did not surf the corner.



(a) Map 1= 65.31% (b) Map 2= 57.73%

(c) Map 3= 69.41% (d) Map 4= 58.23%

FIGURE 8. **Implementation of CME-SineCosine algorithm: Mid range complexity map.**

Moreover, they also got stuck for which multiple simulation runs have to be performed (refer map 1 of Figure 6 & 7). The time complexity for the proposed algorithm and contemporary algorithms are also considered (refer to Table 1). The longer period does not affect the efficiency of the algorithm because the primary objective of the proposed algorithm involves map coverage utilizing the coordination of robots.

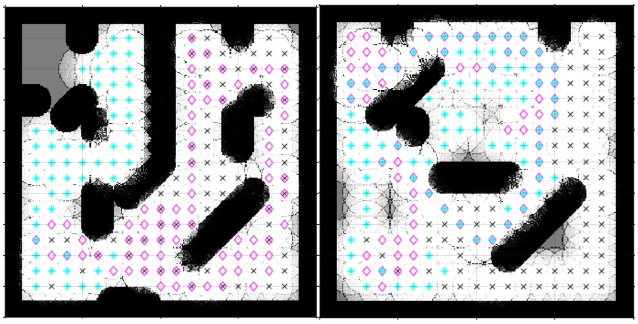
C. DENSE AND CLUTTERED ENVIRONMENT

The attractive feature of a robot is its maneuverability in a highly crowded environment. For single robot configuration, it is relatively easy as compared to multiple robot configurations especially when their motion is coordinated with each other. The chances to get stuck at any point and being hit by the obstacle or inability to avoid obstacle makes this whole task much challenging. When the proposed algorithm is tested in a highly dense and crowded environment, the obtained results are astonishing. This can be easily visualized from Figure 9. The area coverage of Maps 5 and 6 are 94.56% and 98.79% respectively, which elucidates the effectiveness of the proposed algorithm.

To compare the results, the same map was explored employing the other three techniques **CME-WOA** (refer Figure 10) and **CME-GWO** (refer Figure 11) and **CME-SineCosine** (refer Figure 12). The efficacy of the map in all these cases greatly depended on the simulation runs. It is also observed that for the proposed FMH-WOA, the simulation runs were minimum as compared to CME-WOA, CME-GWO, and CME-SineCosine. e.g to fully explore Map No 5, FMH-WOA needed a maximum of 2 runs, whereas

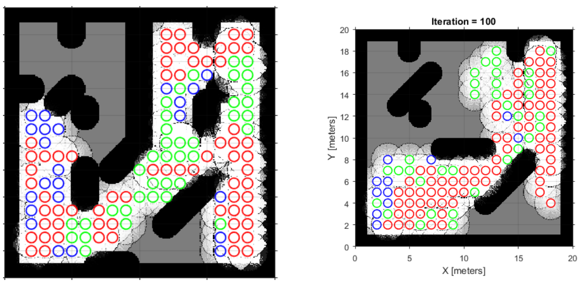
TABLE 1. Summary of Simulation runs required for successful implementation of all algorithms.

Map No	FMH-WOA			CME-WOA			CME-GWO			CME-SineCosine		
	Explored Area%	Failed Run	Time Consumed (m)	Explored Area%	Failed Run	Time Consumed (m)	Explored Area%	Failed Run	Time Consumed (m)	Explored Area%	Failed Run	Time Consumed (m)
Map 1	98.72%	0	2.023	72.56%	6	3.539	82.08%	7	2.804	65.31%	6	4.589
Map 2	98.95%	0	2.876	85.79%	2	3.945	95.65%	9	2.765	57.73%	5	3.689
Map 3	92.72%	0	2.839	53.48%	2	2.924	91.45%	3	2.486	69.41%	8	4.214
Map 4	98.97%	1	2.392	67.89%	5	2.758	90.06%	6	2.559	58.23%	8	3.011
Map 5	94.56%	2	2.536	59.65%	8	2.755	88.43%	11	4.002	45.75%	9	4.586
Map 6	98.79%	1	2.878	66.5%	4	2.527	92.89%	6	3.838	55.36%	7	4.998



(a) Map 5= 94.56% (b) Map 6= 98.79%

FIGURE 9. Implementation of FMH-WOA algorithm: Dense & cluttered environment.



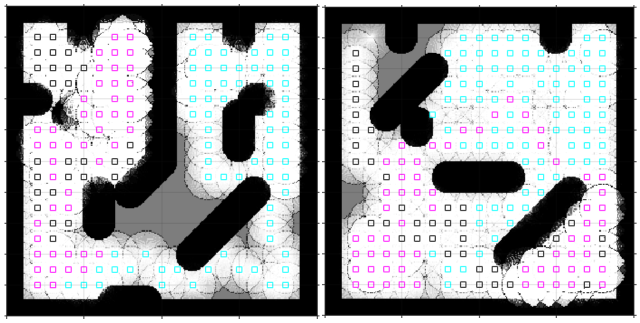
(a) Map 5: 59.65% (b) Map 6: 66.5%

FIGURE 10. Implementation of CME-WOA algorithm: Dense & cluttered environment.

CME-WOA, CME-GWO, and CME-SineCosine needed 8, 11, and 9 runs respectively.

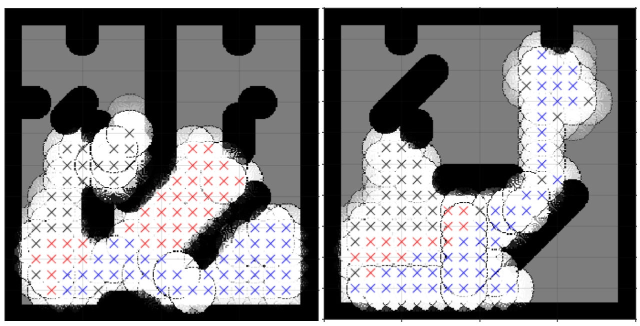
D. SUMMARY OF RESULTS

The entire results of section III are summarized and collected in Table 1 for the speedy reference of the audience. In Table 1, all conducted comparisons among the proposed **FMH-WOA** algorithm and the referenced algorithms **CME-WOA**, **CME-GWO** and **CME-SineCosine** are presented. Also, the table references the figures, where the simulations have been depicted. It is noticeable, that **FMH-WOA** results for space exploration show greater space coverage as compared to the other algorithms. Moreover proposed algorithm requires a lesser number of trial runs for simulation



(a) Map 5= 88.43% (b) Map 6= 92.89%

FIGURE 11. Implementation of CME-GWO algorithm: Dense & cluttered environment.



(a) Map 5= 45.75% (b) Map 6= 55.36%

FIGURE 12. Implementation of CME-Sine-Cosine algorithm: Dense & cluttered environment.

whereas the CME-WOA, CME-GWO and CME-SineCosine require numerous attempts for space exploration. Remarkably, the proposed **FMH-WOA** is superior, when compared to all the other algorithms in every single result.

Time complexity is another measuring parameter of such algorithms that defines their efficiency. The primary objective of any algorithm is to complete the desired task fully in minimum time. An algorithm, therefore, taking lesser time is considered energy efficient. To evaluate the same aspect, the time complexity for the proposed algorithm and three contemporary algorithms is computed and the results are summarized in Table 1. As evident from the results, the proposed algorithm is computationally effective as it takes lesser time to maximally surf the entire area. All three contemporary algorithms take a much longer time for area exploration and additionally do not fully explore the space. Another prominent characteristic of the proposed FMH-WOA is that it takes lesser simulation runs for execution in comparison with the other three algorithms which need much more simulation runs for the complete execution.

IV. CONCLUSION

The study formulates the exploration of unknown spaces in an environment by robot motion with sensor coverage. An integrated approach of **FMH-WOA** for multi-robot exploration was presented which combines the deterministic CME and bio-inspired WOA techniques. To further optimize the results, the random parameters involved in WOA were modified by adding the Frequency modified function. This statistical parameter facilitated in speedy convergence rate. Through simulations, it was demonstrated that the proposed algorithm works efficiently in all environmental conditions.

The results achieved were then compared with three contemporary techniques **CME-WOA**, **CME-GWO** and **CME-SineCosine**. As demonstrated in Table 1, the proposed **FMH-WOA** algorithm depicted promising results, which were superior to the contemporary techniques in every single aspect. Space exploration showed enhanced area coverage with a lesser number of trial runs for simulation with the least time required for execution. From application perspective, the proposed algorithm will have widespread utility in operations where human presence is considered dangerous or undesirable.

It is imperative that the investigation performed in this study for multi-robot utilization will serve as a baseline that supports the idea of integration of deterministic CME based approach with powerful bio-inspired WOA technique. This shall open a new era for research, as the distinct advantages of both deterministic and bio-inspired techniques can now be integrated into a single framework. Studies can then be performed to integrate other bio-inspired techniques with deterministic CME based algorithm.

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REFERENCES

[1] S. Siuhi and J. Mwakalonge, "Opportunities and challenges of smart mobile applications in transportation," *J. Traffic Transp. Eng. (English Ed.)*, vol. 3, no. 6, pp. 582–592, Dec. 2016.
[2] H. Egami and T. Matsumoto, "Mobile money use and healthcare utilization: Evidence from rural uganda," *Sustainability*, vol. 12, no. 9, p. 3741, May 2020.
[3] G. Fragapane, D. Ivanov, M. Peron, F. Sgarbossa, and J. O. Strandhagen, "Increasing flexibility and productivity in industry 4.0 production networks with autonomous mobile robots and smart intralogistics," *Ann. Oper. Res.*, pp. 1–19, Feb. 2020.
[4] S. G. Tzafestas, "Mobile robot path, motion, and task planning," in *Introduction to Mobile Robot Control*, 2014, pp. 429–478.
[5] F. Gul and W. Rahiman, "An integrated approach for path planning for mobile robot using bi-RRT," *IOP Conf. Ser., Mater. Sci. Eng.*, vol. 697, Dec. 2019, Art. no. 012022.
[6] F. Gul, S. S. N. Alhady, and W. Rahiman, "A review of controller approach for autonomous guided vehicle system," *Indonesian J. Electr. Eng. Comput. Sci.*, vol. 20, no. 1, pp. 552–562, 2020.
[7] A. Masłowski, "Training in military robotics and EOD unmanned systems," Tech. Rep., 2014.
[8] E. Vassev and M. Hinchey, *Autonomy Requirements Engineering for Space Missions*. New York, NY, USA: Springer, 2014.
[9] W. Gao, W. Wang, H. Zhu, S. Zhao, G. Huang, and Z. Du, "Irradiation test and hardness design for mobile rescue robot in nuclear environment," *Ind. Robot, Int. J. Robot. Res. Appl.*, vol. 46, no. 6, pp. 851–862, Oct. 2019.
[10] I. Mir, S. A. Eisa, and A. Maqsood, "Review of dynamic soaring: Technical aspects, nonlinear modeling perspectives and future directions," *Nonlinear Dyn.*, vol. 94, no. 4, pp. 3117–3144, 2018.
[11] I. Mir, A. Maqsood, and S. Akhtar, "Dynamic modeling & stability analysis of a generic UAV in glide phase," in *Proc. MATEC Web Conf.*, vol. 114. Les Ulis, France: EDP Sciences, 2017, Art. no. 01007.
[12] I. Mir, H. Taha, S. A. Eisa, and A. Maqsood, "A controllability perspective of dynamic soaring," *Nonlinear Dyn.*, vol. 93, pp. 1–16, Dec. 2018.
[13] I. Mir, A. Maqsood, S. A. Eisa, H. Taha, and S. Akhtar, "Optimal morphing—Augmented dynamic soaring maneuvers for unmanned air vehicle capable of span and sweep morphologies," *Aerosp. Sci. Technol.*, vol. 79, pp. 17–36, Aug. 2018.
[14] I. Mir, A. Maqsood, and S. Akhtar, "Optimization of dynamic soaring maneuvers for a morphing capable UAV," in *Proc. AIAA Inf. Syst.-AIAA Infotech @ Aerosp.*, Jan. 2017, p. 0678.
[15] R. K. Ramachandran and S. Berman, "Automated construction of metric maps using a stochastic robotic swarm leveraging received signal strength," 2019, *arXiv:1903.05392*. [Online]. Available: http://arxiv.org/abs/1903.05392
[16] R. Gayathri and V. Uma, "Performance analysis of robotic path planning algorithms in a deterministic environment," *Environ., Int.*, vol. 19, no. 4, pp. 83–108, 2019.
[17] Q. M. Ta and C. C. Cheah, "Stochastic control for orientation and transportation of microscopic objects using multiple optically driven robotic fingertips," *IEEE Trans. Robot.*, vol. 35, no. 4, pp. 861–872, Aug. 2019.
[18] K. Albina and S. G. Lee, "Hybrid stochastic exploration using grey wolf optimizer and coordinated multi-robot exploration algorithms," *IEEE Access*, vol. 7, pp. 14246–14255, 2019.
[19] L. Palmieri, L. Bruns, M. Meurer, and K. O. Arras, "Dispertio: Optimal sampling for safe deterministic motion planning," *IEEE Robot. Autom. Lett.*, vol. 5, no. 2, pp. 362–368, Apr. 2020.
[20] Y. Xiao, J. Hoffman, T. Xia, and C. Amato, "Learning multi-robot decentralized Macro-Action-Based policies via a centralized Q-Net," 2019, *arXiv:1909.08776*. [Online]. Available: http://arxiv.org/abs/1909.08776
[21] X. Li, H. Qiu, S. Jia, and Y. Gong, "Dynamic algorithm for safe and reachable frontier point generation for robot exploration," in *Proc. IEEE Int. Conf. Mechatronics Autom.*, Aug. 2016, pp. 2088–2093.
[22] L. Freda and G. Oriolo, "Frontier-based probabilistic strategies for sensor-based exploration," in *Proc. IEEE Int. Conf. Robot. Autom.*, Apr. 2005, pp. 3881–3887.
[23] H. Stone and K. Wong, "Preliminary design of a tandem-wing tail-sitter UAV using multi-disciplinary design optimization," in *Proc. AUVSI*, Jul. 1996, pp. 163–178.
[24] K.-Y. Kim and S.-J. Seo, "Shape optimization of forward-curved-blade centrifugal fan with Navier-Stokes analysis," *J. Fluids Eng.*, vol. 126, no. 5, pp. 735–742, Sep. 2004.
[25] B. Yamauchi, "A frontier-based approach for autonomous exploration," in *Proc. IEEE Int. Symp. Comput. Intell. Robot. Autom. CIRA Towards New Comput. Princ. Robot. Autom.*, Jul. 1997, pp. 146–151.
[26] D. Puig, M. A. Garcia, and L. Wu, "A new global optimization strategy for coordinated multi-robot exploration: Development and comparative evaluation," *Robot. Auto. Syst.*, vol. 59, no. 9, pp. 635–653, Sep. 2011.
[27] A. Benkrid and N. Achour, "A novel approach for coordinated multi-robot exploration," in *Proc. 6th Int. Conf. Syst. Control (ICSC)*, May 2017, pp. 509–513.
[28] M. Rappaport and C. Bettstetter, "Coordinated recharging of mobile robots during exploration," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst. (IROS)*, Sep. 2017, pp. 6809–6816.
[29] K. Verbiest, S. A. Berrabah, and E. Colon, "Autonomous frontier based exploration for mobile robots," in *Intelligent Robotics and Applications*. New York, NY, USA: Springer, 2015, pp. 3–13.

[30] F. M. Campos, M. Marques, F. Carreira, and J. M. F. Calado, "A complete frontier-based exploration method for pose-SLAM," in *Proc. IEEE Int. Conf. Auto. Robot Syst. Competitions (ICARSC)*, Apr. 2017, pp. 79–84.
[31] C. Gomez, A. C. Hernandez, and R. Barber, "Topological frontier-based exploration and map-building using semantic information," *Sensors*, vol. 19, no. 20, p. 4595, Oct. 2019.
[32] C. Gómez, A. C. Hernández, J. Crespo, and R. Barber, "A ROS-based middle-cost robotic platform with high-performance," *Proc. Int. Acad. Technol., Edu. Develop. (IATED)*, Barcelona, Spain, 2015, pp. 6–8.
[33] W. Burgard, M. Moors, D. Fox, R. Simmons, and S. Thrun, "Collaborative multi-robot exploration," in *Proc. IEEE Int. Conf. Robot. Automat. Symposia (ICRA)*, vol. 1, Apr. 2000, pp. 476–481.
[34] W. Burgard, M. Moors, C. Stachniss, and F. E. Schneider, "Coordinated multi-robot exploration," *IEEE Trans. Robot.*, vol. 21, no. 3, pp. 376–386, Jun. 2005.
[35] V. Magnago, P. Corbalan, G. P. Picco, L. Palopoli, and D. Fontanelli, "Robot localization via odometry-assisted ultra-wideband ranging with stochastic guarantees," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst. (IROS)*, Nov. 2019, pp. 1607–1613.
[36] T. Alam, M. M. Rahman, P. Carrillo, L. Bobadilla, and B. Rapp, "Stochastic multi-robot patrolling with limited visibility," *J. Intell. Robotic Syst.*, vol. 97, no. 2, pp. 411–429, Feb. 2020.
[37] L. Xie, Y. Miao, S. Wang, P. Blunsom, Z. Wang, C. Chen, A. Markham, and N. Trigoni, "Learning with stochastic guidance for robot navigation," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 32, no. 1, pp. 166–176, Jan. 2021.
[38] R. Oikawa, M. Takimoto, and Y. Kambayashi, "Distributed formation control for swarm robots using mobile agents," in *Proc. IEEE 10th Jubilee Int. Symp. Appl. Comput. Intell. Informat.*, May 2015, pp. 111–116.
[39] D. Xu, X. Zhang, Z. Zhu, C. Chen, and P. Yang, "Behavior-based formation control of swarm robots," *Math. Problems Eng.*, vol. 2014, pp. 1–13, Jun. 2014.
[40] S. Mirjalili, S. M. Mirjalili, and A. Lewis, "Grey wolf optimizer," *Adv. Eng. Softw.*, vol. 69, pp. 46–61, Mar. 2014.
[41] S. Mirjalili and A. Lewis, "The whale optimization algorithm," *Adv. Eng. Softw.*, vol. 95, pp. 51–67, May 2016.
[42] X.-S. Yang, M. Karamanoglu, and X. He, "Multi-objective flower algorithm for optimization," *Procedia Comput. Sci.*, vol. 18, pp. 861–868, Jan. 2013.
[43] E. Bonabeau, D. D. R. D. F. Marco, M. Dorigo, G. Théraulaz, and G. Theraulaz, *Swarm Intelligence: From Natural to Artificial Systems*, no. 1. London, U.K.: Oxford Univ. Press, 1999.
[44] M. Dorigo, M. Birattari, and T. Stutzle, "Ant colony optimization," *IEEE Comput. Intell. Mag.*, vol. 1, no. 4, pp. 28–39, Nov. 2006.
[45] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Proc. IEEE ICNN*, vol. 4, Nov./Dec. 1995, pp. 1942–1948.
[46] S. Mirjalili, "SCA: A sine cosine algorithm for solving optimization problems," *Knowl.-Based Syst.*, vol. 96, pp. 120–133, Mar. 2016.
[47] F. Gul, W. Rahiman, S. S. Nazli Alhady, and K. Chen, "A comprehensive study for robot navigation techniques," *Cogent Eng.*, vol. 6, no. 1, Jan. 2019, Art. no. 1632046.
[48] C. Qu, Z. Zeng, J. Dai, Z. Yi, and W. He, "A modified sine-cosine algorithm based on neighborhood search and greedy levy mutation," *Comput. Intell. Neurosci.*, vol. 2018, pp. 1–19, Jul. 2018.
[49] F. S. Gharehchopogh and H. Gholizadeh, "A comprehensive survey: Whale optimization algorithm and its applications," *Swarm Evol. Comput.*, vol. 48, pp. 1–24, Aug. 2019.
[50] H. M. Mohammed, S. U. Umar, and T. A. Rashid, "A systematic and meta-analysis survey of whale optimization algorithm," *Comput. Intell. Neurosci.*, vol. 2019, pp. 1–25, Apr. 2019.
[51] S. Mostafa Bozorgi and S. Yazdani, "IWOA: An improved whale optimization algorithm for optimization problems," *J. Comput. Design Eng.*, vol. 6, no. 3, pp. 243–259, Jul. 2019.
[52] D. Wang, H. Wang, and L. Liu, "Unknown environment exploration of multi-robot system with the FORDPSO," *Swarm Evol. Comput.*, vol. 26, pp. 157–174, Feb. 2016.
[53] T.-K. Dao, T.-S. Pan, and J.-S. Pan, "A multi-objective optimal mobile robot path planning based on whale optimization algorithm," in *Proc. IEEE 13th Int. Conf. Signal Process. (ICSP)*, Nov. 2016, pp. 337–342.
[54] A. Chhillar and A. Choudhary, "Mobile robot path planning based upon updated whale optimization algorithm," in *Proc. 10th Int. Conf. Cloud Comput., Data Sci. Eng. (Confluence)*, Jan. 2020, pp. 684–691.
[55] G. Nicola, N. Pedrocchi, S. Mutti, P. Magnoni, and M. Beschi, "Optimal task positioning in multi-robot cells, using nested meta-heuristic swarm algorithms," *Procedia CIRP*, vol. 72, pp. 386–391, Jan. 2018.
[56] S. Kumar, D. R. Parhi, M. K. Muni, and K. K. Pandey, "Optimal path search and control of mobile robot using hybridized sine-cosine algorithm and ant colony optimization technique," *Ind. Robot, Int. J. Robot. Res. Appl.*, vol. 47, no. 4, pp. 535–545, Apr. 2020.
[57] Q. Yang, S.-C. Chu, J.-S. Pan, and C.-M. Chen, "Sine cosine algorithm with multigroup and multistrategy for solving CVRP," *Math. Problems Eng.*, vol. 2020, pp. 1–10, Mar. 2020.
[58] C. Qu and W. He, "A double mutation cuckoo search algorithm for solving systems of nonlinear equations," *Int. J. Hybrid Inf. Technol.*, vol. 8, no. 12, pp. 433–448, Dec. 2015.
[59] Y.-C. Wu, W.-P. Lee, and C.-W. Chien, "Modified the performance of differential evolution algorithm with dual evolution strategy," in *Proc. Int. Conf. Mach. Learn. Comput.*, vol. 3, 2011, pp. 57–63.
[60] Y. Wang, D. Li, Y. Lu, Z. Cheng, and Y. Gao, "Improved flower pollination algorithm based on mutation strategy," in *Proc. 9th Int. Conf. Intell. Hum.-Mach. Syst. Cybern. (IHMSC)*, Aug. 2017, pp. 337–342.
[61] M. Petrović, Z. Miljković, and A. Jokić, "A novel methodology for optimal single mobile robot scheduling using whale optimization algorithm," *Appl. Soft Comput.*, vol. 81, Aug. 2019, Art. no. 105520.
[62] Y. Wang, A. Liang, and H. Guan, "Frontier-based multi-robot map exploration using particle swarm optimization," in *Proc. IEEE Symp. Swarm Intell.*, Apr. 2011, pp. 1–6.
[63] N. Palmieri, F. de Rango, X. S. Yang, and S. Marano, "Multi-robot cooperative tasks using combined nature-inspired techniques," in *Proc. 7th Int. Joint Conf. Comput. Intell.*, 2015, pp. 74–82.
[64] S. Sharma, A. Shukla, and R. Tiwari, "Multi robot area exploration using nature inspired algorithm," *Biologically Inspired Cognit. Archit.*, vol. 18, pp. 80–94, Oct. 2016.
[65] F. Gul, W. Rahiman, S. S. N. Alhady, A. Ali, I. Mir, and A. Jalil, "Meta-heuristic approach for solving multi-objective path planning for autonomous guided robot using PSO–GWO optimization algorithm with evolutionary programming," *J. Ambient Intell. Humanized Comput.*, pp. 1–18, Sep. 2020.
[66] A. E. Ezugwu, O. J. Adeleke, A. A. Akinyelu, and S. Viriri, "A conceptual comparison of several Metaheuristic algorithms on continuous optimisation problems," *Neural Comput. Appl.*, vol. 32, no. 10, pp. 6207–6251, May 2020.
[67] S. Xiao-Hua and Y. Chun-Ming, "Application of bat algorithm to permutation flow-shop scheduling problem," *Ind. Eng. J.*, vol. 16, no. 1, p. 119, 2013.
[68] V. Selvi and D. R. Umarani, "Comparative analysis of ant colony and particle swarm optimization techniques," *Int. J. Comput. Appl.*, vol. 5, no. 4, pp. 1–6, Aug. 2010.
[69] H. P. Moravec, "Sensor fusion in certainty grids for mobile robots," in *Sensor Devices and Systems for Robotics*. New York, NY, USA: Springer, 1989, pp. 253–276.
[70] A. Zelinsky, R. A. Jarvis, J. C. Byrne, and S. Yuta, "Planning paths of complete coverage of an unstructured environment by a mobile robot," in *Proc. Int. Conf. Adv. Robot.*, vol. 13, 1993, pp. 533–538.
[71] B. Yamauchi, "Frontier-based exploration using multiple robots," in *Proc. 2nd Int. Conf. Auto. Agents AGENTS*, 1998, pp. 47–53.

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