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Decentralized Output-Feedback Controller for Uncertain Large-Scale Nonlinear Systems Using Higher-Order Switching Differentiator

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ABSTRACT A novel approximation-free differentiator-based output-feedback controller for uncertain large-scale systems (LSSs) is proposed. The considered LSS has nonautonomous and nonaffine-in-the-control subsystems which is yet to be tackled for decentralized output-feedback controller in the previous researches. The controller adopts a higher-order switching differentiator that can track the time-derivatives of a time-varying signal asymptotically. Through the differentiators, time-derivatives of output tracking errors are estimated and unstructured uncertainties in the controlled subsystems are compensated. The proposed decentralized output-feedback control formulae and the stability analysis are relatively simple in comparison to the previously proposed decentralized controllers. In this case, approximators such as fuzzy systems or neural networks are not required. The proposed controller guarantees that the tracking errors of the subsystems are asymptotically convergent to zeros and all the signals involved in the closed-loop systems are bounded.

INDEX TERMS Large-scale system, uncertain nonlinear system, decentralized controller, approximation-free, differentiator-based controller.

I. INTRODUCTION

Large-scale systems (LSSs) or interconnected systems have received much attention because many modern practical systems are described as this kind of systems [1]. For example, a multi-agent system, multi-machine power system, and modern mechanical systems are all comprised of interconnected subsystems. For the control of LSSs, decentralized control scheme which uses only locally available states without communication between remote subsystems is used, as opposed to distributed controller that requires exchange of state information for each subsystem. Thus, the decentralized controller is generally more practical because the controller usually has insufficient knowledge of the plant uncertainties and interactions between subsystems.

Influenced by the intensive research results on the controller for unknown nonlinear systems [2]–[10], typical approaches for controlling nonlinear LSSs with unstructured uncertainties are adopting universal approximators [11]–[19]. The universal approximators such as fuzzy logic systems(FLSs) and neural networks(NNs) capture and compensate for unknown functions in the controlled system dynamics. As a result, the unknown function problem is replaced by an unknown parameter problem that can apply the traditional adaptive control method. However, the control schemes that adopt the universal approximators suffer from its computational complexity and high dynamic order of the controller due to many adaptively tuning parameters. In addition, in the case of the strict- or pure-feedback systems, combining backstepping scheme with real-time tuning approximators increases considerable complexity to the resultant control law [3], [5], [12], [13], [17]. Recently, to overcome these drawbacks, prescribed performance control (PPC) [20] has been widely used to control uncertain nonlinear systems, and it has been applied to the interconnected LSSs. [21]–[23] In the case of PPC, the complexity of the controller structure is reduced considerably since universal approximators are not required. However, in the PPC schemes, backstepping design steps are still required. Moreover, most of the researches consider strict-feedback nonlinear systems. If the scope is

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narrowed down to the problem of decentralized output-feedback controller for LSS with uncertain pure-feedback nonlinear subsystems, the research results are very limited. [14], [15], [18], [19] However, their control schemes adopt FLS or NN as the approximators of the unknown functions for adaptive observers or controllers. As described above, these approximators in the closed-loop system increase dynamic order of the control laws and make stability analysis very complex. Moreover, all the previous research [14], [15], [18], [19] deals with autonomous nonlinear subsystems.

More recently, differentiator-based controllers (DBCs) for uncertain nonlinear systems have been proposed. [24]–[27] The DBC has the following advantages over the conventional controllers in regards to uncertain nonlinear systems. First, it does not require FLSs or NNs as unknown function estimators, which considerably simplifies the control law and stability proof. It is not based on the backstepping design scheme, which results in further reduction of the complexities of the overall control scheme. Second, as shown in [27], it is applicable to a general class of nonlinear systems and easy to design output-feedback controllers. According to the performance of the adopted differentiator (or time-derivative estimator), finite-time exact output tracking or asymptotic stability is guaranteed.

This paper considers interconnected LSSs whose subsystems are uncertain general nonlinear systems. Whereas the previous researches have mostly dealt with autonomous strict- or pure-feedback nonlinear subsystems, this paper considers quite general uncertain nonautonomous nonlinear subsystems including strict- or nonstrict- systems. To the best of the authors' knowledge, there is no literature that considers decentralized output-feedback controller of this general class of uncertain nonautonomous nonlinear LSSs. The differentiator-based controller proposed in [25], [27] is adopted and modified to the case of interconnected LSS control. Especially, by adopting the higher-order switching differentiator (HOSD) [28] that estimates the derivatives of a time-varying signal, the proposed controller guarantees asymptotical output tracking performance. It is not based on backstepping and it requires no universal approximators to cope with unstructured uncertainties intrinsic to the subsystems. Another advantage of the proposed controller is that there is no severe and consistent chattering or peaking in the control input. The contributions of this paper are summarized as follows.

1) Compared to the previous controllers in [14], [15], [17], [18], the LSS considered in this paper has a broader class of nonautonomous nonaffine-in-the-control nonlinear subsystems. To the best of authors' knowledge, there has been no research that has examined output-feedback decentralized controller for this class of interconnected LSS.
2) The proposed scheme combines the HOSD [28] and the differentiator-based controller [25] in order to deal with unstructured uncertainties in the controlled subsystems. No approximators such as NN or FLS are required, which considerably lowers the dynamic order and complexity of the controller.
3) The proposed output-feedback decentralized control algorithm has a relatively simplified structure. There is no lengthy or complicated control law or adaptive formula. There are only two design constants in the control formula for each subsystem, which demonstrates the compactness of the proposed controller.

The rest of this paper is organized as follows. In Section II, the dynamics of the considered LSS and its normalized system equations are described. The HOSD is also introduced in Section II. Section III describes the structure of the proposed output-feedback decentralized controller with the main theorem. Also, the Lyapunov stability analysis are presented and discussed in Section III. In Section IV, numerical simulations using two examples of LSSs are then conducted to illustrate the proposed controller's performance and compactness. Finally, Section V presents the conclusion.

II. PROBLEM FORMULATION

A. CONSIDERED NONLINEAR SYSTEM

This paper considers an interconnected general nonlinear system whose dynamics are uncertain, nonautonomous, and nonaffine-in-the-control. The dynamics of N interconnected subsystems of the large-scale nonlinear system are as follows.

$$\left.\begin{aligned}\dot{\mathbf{x}}_j &= \mathbf{f}_j(\mathbf{x}_j, u_j, t, \tilde{\mathbf{x}}_j)\\ y_j &= h_j(\mathbf{x}_j, t)\end{aligned}\right\}, \quad j = 1, 2, \cdots, N \tag{1}$$

where $\mathbf{f}_j$ and h_j are unknown smooth functions, $\mathbf{x}_j = [x_{j,1}, x_{j,2}, \cdots, x_{j,n_j}]^T$ is a state vector of jth subsystem, n_j is the dynamic order of the jth subsystem, y_j and u_j denote the output and input of jth subsystem, respectively. $\tilde{\mathbf{x}}_j$ denotes the total state vectors of remote subsystems defined as follows

$$\tilde{\mathbf{x}}_j = [\mathbf{x}_1^T, \cdots, \mathbf{x}_{j-1}^T, \mathbf{x}_{j+1}^T, \cdots, \mathbf{x}_N^T]^T. \tag{2}$$

That is, it is a collection of state vectors except for the jth one. Throughout this paper, the index $j \in \{1, 2, \cdots, N\}$ denotes a unique index of a subsystem. Note that the system in consideration is of a relatively broader class of nonlinear systems including strict- and pure-feedback systems. It is also nonautonomous. That is, $\mathbf{f}_j$ and h_j are the functions of time explicitly. This class of systems may contain time-varying parameters, additive or multiplicative disturbances, etc. Only the output y_j of the subsystem is assumed to be measurable. The control objective is driving y_j to track the desired output $y_j^d(t)$ that is a smooth function of t while also maintaining all the signals in the closed-loop system to be bounded.

In practical engineering systems, all the states tend to be maintained in prescribed bounded operation regions, and the control inputs are also bounded due to physical limitations.

Assumption 1: The following open set includes the whole operation region of the jth subsystem (1)

$$\Omega_j = \left\{\mathbf{x}_j, u_j \middle| |\mathbf{x}_j| < b_j^x, u_j < b_j^u\right\} \tag{3}$$

where b_j^x and b_j^u are positive bounding constants.

The total set is defined as the union of all the Ω_js as follows.

$$\Omega = \cup_{j=1}^{N} \Omega_j \tag{4}$$

Assumption 2: The relative degree of the system (1) is n_j for all $j = 1, \cdots, N$. That is, all the subsystems have full relative degrees, and the control inputs appear first in the $y^{(n_j)}$ equation.

B. NORMALIZING ORIGINAL SYSTEM

Let the tracking error of the jth subsystem be $e_j \triangleq y_j - y_j^d$. Then, the original subsystem (1) can be normalized with respect to the tracking error into the following Brunovsky form.

$$\begin{aligned} \dot{e}_{j,i} &= g_{j,i}(\mathbf{x}_j, t, \tilde{\mathbf{x}}_j) \\ &\triangleq e_{j,i+1}, \ i = 1, 2, \cdots, n_j - 1 \\ \dot{e}_{j,n_j} &= g_{j,n_j}(\mathbf{x}_j, u_j, t, \tilde{\mathbf{x}}_j) \end{aligned} \tag{5}$$

with $e_{j,1} = e_j$ where

$$\begin{aligned} g_{j,1}(\mathbf{x}_j, t, \tilde{\mathbf{x}}_j) &= \frac{\partial h_j}{\partial \mathbf{x}_j}\mathbf{f}_j + \frac{\partial h_j}{\partial t} - \dot{y}_j^d(t) \\ g_{j,i}(\mathbf{x}_j, t, \tilde{\mathbf{x}}_j) &= \frac{\partial g_{j,i-1}}{\partial \mathbf{x}_j}\mathbf{f}_j + \frac{\partial g_{j,i-1}}{\partial \tilde{\mathbf{x}}_j}\tilde{\mathbf{f}}_j \\ &\quad + \frac{\partial g_{j,i-1}}{\partial t}, \ i = 2, \cdots, n_j - 1 \\ g_{j,n_j}(\mathbf{x}_j, u_j, t, \tilde{\mathbf{x}}_j) &= \frac{\partial g_{j,n_j-1}}{\partial \mathbf{x}_j}\mathbf{f}_j + \frac{\partial g_{j,n_j-1}}{\partial \tilde{\mathbf{x}}_j}\tilde{\mathbf{f}}_j \\ &\quad + \frac{\partial g_{j,n_j-1}}{\partial t} \end{aligned} \tag{6}$$

with

$$\tilde{\mathbf{f}}_j \triangleq [\mathbf{f}_1^T, \cdots, \mathbf{f}_{j-1}^T, \mathbf{f}_{j+1}^T, \cdots, \mathbf{f}_N^T]^T. \tag{7}$$

Note that, from Assumption 2, control input appears only in $g_{j,n_n}(\cdot)$. Also note that $g_{j,i}$s are all smooth functions since $\mathbf{f}_j$s and h_js are all assumed to be smooth.

For the controllability of the considered system, it is required the following assumption.

Assumption 3: The following inequality holds for all $j = 1, \cdots, N$

$$\frac{\partial g_{j,n_j}(\mathbf{x}_j, u_j, t, \tilde{\mathbf{x}}_j)}{\partial u_j} > 0 \tag{8}$$

on the compact set $\Omega \times [0, \infty)$.

This assumption for the controllability of the system (1) is widely adopted in the literature. (e.g., assumption 1 in [4], assumption 4 in [3], assumption 1 in [5], etc.)

C. INTRODUCTION OF HOSD

In the proposed controller, the HOSD [28], [29] is adopted in its design. The switching differentiator is first proposed in [29] to estimate the time-derivative of a time-varying signal. In [28], the differentiator is extended to observe higher order time-derivatives. The HOSD has the property of asymptotic convergence, which results in no peaking or chattering in the estimated signals. In [27], a more compact form of HOSD, which contains only one design constant while maintaining tracking performance is proposed, and subsequently adopted in this paper.

Before introducing HOSD dynamics, some definitions are required. Let Φ be a set of all strictly increasing infinite time sequences such that

$$\Phi \triangleq \{(t_i)_{i=0}^{\infty} | t_0 = 0, t_i < t_{i+1} \forall i \in \mathbb{N}_0\} \tag{9}$$

where $\mathbb{N}_0 = \{0, 1, 2, \cdots\}$. For a sequence $T = (t_i) \in \Phi$, Ω_T denotes a set of functions that are discontinuous at some or all t_i.

Definition 1: [28] For $T = (t_i) \in \Phi$, define the set of functions as follows:

$$\overline{\Omega}_T^L \triangleq \left\{ f(t) \middle| f(t) \in \Omega_T, \sup_{\substack{t_i \le t < t_{i+1} \\ \forall i \in \mathbb{N}_0}} |f(t)| \le L < \infty \right\} \tag{10}$$

where $L > 0$ is a constant. The functions in $\overline{\Omega}_T^L$ are bounded in the piecewise sense (BPWS) below L.

Lemma 1: [27] Suppose the time-derivatives of a time-varying signal $a(t)$ are BPWS such that $a^{(i+1)} \in \overline{\Omega}_T^{L_i^*}$ for $i = 1, 2, \cdots, n$ where L_i^*s are positive constants and $T \in \Phi$. $a^{(n+2)}$ is also assumed to be BPWS. Consider the following HOSD dynamics

$$\left.\begin{aligned} \dot{\alpha}_i &= \beta_i L e_{\alpha_i} + \sigma_i \\ \dot{\sigma}_i &= L\, sgn(e_{\alpha_i}) \end{aligned}\right\}, \quad i = 1, 2, \cdots, n \tag{11}$$

where $e_{\alpha_i} = \sigma_{i-1} - \alpha_i$ with $\sigma_0 = a$. If the design constants are selected such that $\beta_i > 0$ and $L > \max_{i=1,\cdots,n} L_i^*$, then:

$$\sigma_i(t) \to a^{(i)}, \quad i = 1, 2, \cdots, n \tag{12}$$

holds.

Detailed proof of Lemma 1 is shown in [28]. In [27], appropriately selected constants β_js up to $j = 6$ have been presented as

$$\beta_1 = 10, \beta_2 = 7, \beta_3 = 5.5, \beta_4 = 4.8, \beta_5 = 4.4, \beta_6 = 4.2. \tag{13}$$

As described in [27], the only design constant L must be increased to improve the estimation performance of the HOSD.

III. CONTROLLER DESIGN

As described earlier, the objectives of the controllers are that the y_js track desired output y_j^ds and that all the time-varying signals involved in the closed-loop system remain bounded. The HOSD (11) is adopted in every subsystem, and they are described as follows

$$\left.\begin{aligned} \dot{\alpha}_{j,i} &= \beta_i L_j e_{\alpha_{j,i}} + \sigma_{j,i} \\ \dot{\sigma}_{j,i} &= L_j\, sgn(e_{\alpha_{j,i}}) \end{aligned}\right\}, \quad i = 1, 2, \cdots, n_j \tag{14}$$

for $j = 1, \cdots, N$ where $e_{\alpha_{j,i}} \triangleq \sigma_{j,i-1} - \alpha_{j,i}$ with $\sigma_{j,0} = a_j$. The constants β_is in (13) are commonly used in all subsystems. The feeding signal a_j into the jth HOSD is generated as described in the following subsection.

A. CONTROL INPUT FILTERING

The following simple linear time-invariant(LTI) filter is adopted to generate the signal $a_j(t)$ that is injected into the HOSD (14).

$$\begin{aligned} \dot{w}_{j,i} &= -c_j w_{j,i} + w_{j,i+1}, \ i = 1, \cdots, n_j - 1 \\ \dot{w}_{j,n_j} &= -c_j w_{j,n_j} + u_j \end{aligned} \tag{15}$$

where $c_j > 0$ is a design constant. Note that, due to the stabilizing terms of $-c_j w_{j,i}$ for $i = 1, \cdots, n_j$ in (15) with positive c_j, it is guaranteed that the states $w_{j,i}$ of the LTI filter (15) are bounded since u_j is bounded based on Assumption 1.

Lemma 2: With Assumption 1, the following inequalities hold

$$|w_{j,i}| < \frac{b_j^u}{c_j^{n_j - i + 1}} \tag{16}$$

for $i = 1, \cdots, n_j$.

Detailed proof of Lemma 2 is described as in [30].

The input signal to the HOSD is generated as

$$a_j = e_j - w_{j,1}. \tag{17}$$

From Lemma 1, the following holds

$$\begin{aligned} \sigma_{j,1} &= \dot{a}_j + d_{j,1}(t) \\ &= \dot{e}_j - p_1(\mathbf{w}_j) - w_{j,2} + d_{j,1}(t) \end{aligned} \tag{18}$$

$$\begin{aligned} \sigma_{j,2} &= \ddot{a}_j + d_{j,2}(t) \\ &= \ddot{e}_j - p_2(\mathbf{w}_j) - w_{j,3} + d_{j,2}(t) \end{aligned} \tag{19}$$

$$\vdots$$

$$\begin{aligned} \sigma_{j,n_j-1} &= a_j^{(n_j-1)} + d_{j,n_j-1}(t) \\ &= e_j^{(n_j-1)} - p_{n_j-1}(\mathbf{w}_j) - w_{j,n_j} + d_{j,n_j-1}(t) \end{aligned} \tag{20}$$

$$\begin{aligned} \sigma_{j,n_j} &= a_j^{(n_j)} + d_{j,n_j}(t) \\ &= e_j^{(n_j)} - p_{n_j}(\mathbf{w}_j) - u_j + d_{j,n_j}(t) \end{aligned} \tag{21}$$

where $\mathbf{w}_j = [w_{j,1}, w_{j,2}, \cdots, w_{j,n_j}]^T$, and $d_{j,i}(t)$s denote estimation errors that disappear asymptotically, i.e., $d_{j,i}(t) \to 0$. The terms of $p_i(\mathbf{w}_j)$s are the polynomials of the elements of $\mathbf{w}_j$ that are easily calculated for $i = 1, \cdots, 6$ as follows:

$$p_1(\mathbf{w}_j) = -c_j w_{j,1} \tag{22}$$

$$p_2(\mathbf{w}_j) = c_j^2 w_{j,1} - 2c_j w_{j,2} \tag{23}$$

$$p_3(\mathbf{w}_j) = -c_j^3 w_{j,1} + 3c_j^2 w_{j,2} - 3c_j w_{j,3} \tag{24}$$

$$\begin{aligned} p_4(\mathbf{w}_j) = {} & c_j^4 w_{j,1} - 4c_j^3 w_{j,2} + 6c_j^2 w_{j,3} \\ & - 4c_j w_{j,4} \end{aligned} \tag{25}$$

$$\begin{aligned} p_5(\mathbf{w}_j) = {} & -c_j^5 w_{j,1} + 5c_j^4 w_{j,2} - 10c_j^3 w_{j,3} \\ & + 10c_j^2 w_{j,4} - 5c_j w_{j,5} \end{aligned} \tag{26}$$

$$\begin{aligned} p_6(\mathbf{w}_j) = {} & c_j^6 w_{j,1} - 6c_j^5 w_{j,2} + 15c_j^4 w_{j,3} \\ & - 20c_j^3 w_{j,4} + 15c_j^2 w_{j,5} - 6c_j w_{j,6} \end{aligned} \tag{27}$$

As described in [27], the value of c_j does not have a crucial effect on the performance of the controller. Therefore, the c_js are typically chosen as 1 in order to simplify the calculations of $p_i(\mathbf{w}_j)$s.

B. CONTROL LAW AND STABILITY ANALYSIS

The tracking error vector for the jth subsystem is defined as $\mathbf{e}_j = [e_j, \dot{e}_j, \cdots, e_j^{(n_j-1)}]^T \in \mathbb{R}^{n_j}$ for $j = 1, \cdots N$ and its estimate is available using (18)-(20) as follows

$$\hat{\mathbf{e}}_j = \begin{bmatrix} e_j \\ \sigma_{j,1} + p_1(\mathbf{w}_j) + w_{j,2} \\ \vdots \\ \sigma_{j,n_j-1} + p_{n_j-1}(\mathbf{w}_j) + w_{j,n_j} \end{bmatrix} \in \mathbb{R}^{n_j}. \tag{28}$$

which tracks $\mathbf{e}_j$ asymptotically by Lemma 1. Therefore, the following equality holds

$$\mathbf{e}_j = \hat{\mathbf{e}}_j - \mathbf{d}_j(t) \tag{29}$$

where $\mathbf{d}_j(t) \triangleq [0, d_{j,1}(t), \cdots, d_{j,n_j-1}(t)]^T$. Considering (21), the decentralized control law of jth subsystem is determined as

$$u_j = -\sigma_{j,n_j} - p_{n_j}(\mathbf{w}_j) - \mathbf{k}_j^T \hat{\mathbf{e}}_j \tag{30}$$

where $\mathbf{k}_j = [k_{j,1}, k_{j,2}, \cdots, k_{j,n_j}]^T$ is selected such that the polynomial

$$s^{n_j} + k_{j,n_j} s^{n_j-1} + \cdots + k_{j,2} s + k_{j,1} \tag{31}$$

is Hurwitz. To reduce the number of design constants, the elements of the vector $\mathbf{k}_j$ can be chosen such that the following equality holds

$$(s + \kappa_j)^{n_j} = s^{n_j} + k_{j,n_j} s^{n_j-1} + \cdots + k_{j,2} s + k_{j,1} \tag{32}$$

with $\kappa_j > 0$. Thus, if κ_j is once selected, the vector $\mathbf{k}_j$ is directly calculated. Therefore, the proposed controller has only two design constants $\kappa_j > 0$ in (32) and $L_j > 0$ in (14) since the design constant c_j in (15) is chosen typically as 1.

C. MAIN THEOREM AND SOME REMARKS

The following theorem describes the main result of the proposed controller.

Theorem 1: Consider the system (1) under Assumption 1 and Assumption 2. The control input (30) using the HOSD (11) and input filter (15) makes the tracking error vector $\mathbf{e}_j$ to be asymptotically stable.

Proof: From (21) and (29), it is evident that the control input (30) becomes

$$\begin{aligned} u_j &= -\sigma_{j,n_j} - p_{n_j}(\mathbf{w}_j) - \mathbf{k}_j^T \hat{\mathbf{e}}_j \\ &= -\{e_j^{(n_j)} - p_{n_j}(\mathbf{w}_j) - u_j + d_{j,n_j}(t)\} \\ &\quad - p_{n_j}(\mathbf{w}_j) - \mathbf{k}_j^T \mathbf{e}_j - \mathbf{k}_j^T \mathbf{d}_j(t) \\ &= -e_j^{(n_j)} + u_j - \mathbf{k}_j^T \mathbf{e} - d_{j,n_j}(t) - \mathbf{k}_j^T \mathbf{d}_j(t) \end{aligned} \tag{33}$$

from which the following equality is induced

$$e_j^{(n_j)} = -\mathbf{k}_j^T \mathbf{e}_j + \delta_j(t) \tag{34}$$

where $\delta_j(t) \triangleq -d_{j,n_j}(t) - \mathbf{k}_j^T \mathbf{d}_j(t)$. In vector form

$$\dot{\mathbf{e}}_j = \mathbf{A}_j \mathbf{e}_j + \mathbf{b}_j \delta_j(t) \tag{35}$$

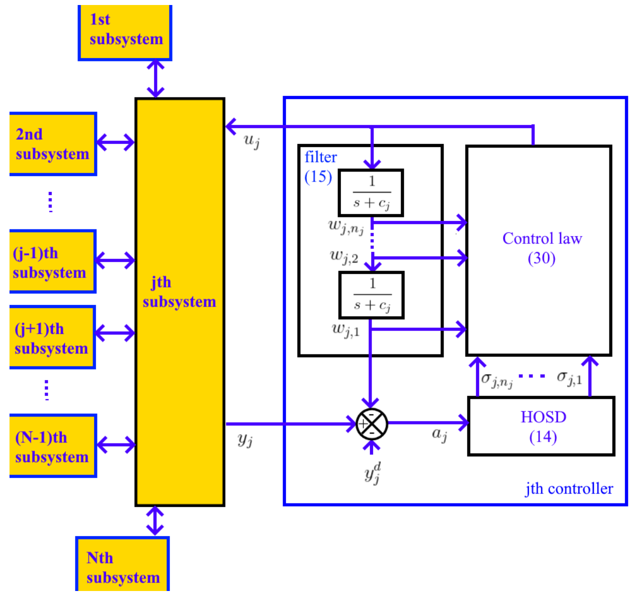

FIGURE 1. **Overall block diagram of the proposed decentralized controller.**

where

$$\mathbf{A}_j = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ -k_{j,1} & -k_{j,2} & -k_{j,3} & \cdots & -k_{j,n_j} \end{bmatrix}, \quad \mathbf{b}_j = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \tag{36}$$

There exist positive definite matrixes $\mathbf{P}_j$ and $\mathbf{Q}_j$ such that $\mathbf{A}_j^T\mathbf{P}_j + \mathbf{P}_j\mathbf{A}_j = -\mathbf{Q}_j$ holds. Defining Lyapunov function $V_j = \mathbf{e}_j^T\mathbf{P}_j\mathbf{e}_j$, its time derivative is derived as

$$\begin{aligned} \dot{V}_j &= -\mathbf{e}_j^T\mathbf{Q}_j\mathbf{e}_j + 2\mathbf{e}_j^T\mathbf{P}_j\mathbf{b}_j\delta_j(t) \\ &\le -\lambda_{min}(\mathbf{Q}_j)|\mathbf{e}_j|^2 + 2|\mathbf{e}_j|\lambda_{max}(\mathbf{P}_j)|\delta_j(t)| \end{aligned} \tag{37}$$

From the last inequality, it is determined that if $|\mathbf{e}_j| > \lambda_j|\delta(t)|$ where $\lambda_j = \frac{2\lambda_{max}(\mathbf{P}_j)}{\lambda_{min}(\mathbf{Q}_j)}$, then $\dot{V}_j < 0$. This means that, since $\delta_j(t)$ converges at zero asymptotically, the $|\mathbf{e}_j|$ is also asymptotically stable. It is trivially concluded that the total Lyapunov function that is defined as $V = \sum_{j=1}^N V_j$ is also stable. □

Remark 1: Although the jth subsystem itself is affected by the other subsystems, the jth controller (30) is a fully decentralized one since no real-time signal of kth subsystem ($k \neq j$) is required. Actually, the jth control law is derived by using the jth output signal only, which is illustrated in Fig. 1 representing the overall block diagram of the closed-loop system.

Remark 2: It is worth noting that the proposed controller uses no universal approximators such as NNs or FLSs despite the unstructured uncertainties in the subsystem.

Remark 3: The time-derivatives of the desired output $y_j^d(t)$ are not required to be available. In practical situations, this is may be desirable since it may be hard to obtain time-derivatives of the desired output signal. The only condition for the $y_j^d(t)$ is that it is differentiable up to the n_jth order.

Remark 4: The proposed decentralized controller assumes that all the subsystems have a full relative degree. However, from [27], it is expected that the controller is also applicable to systems whose relative degree is less than the system dynamic order as long as the internal zero dynamics are stable.

IV. NUMERICAL SIMULATIONS

A. TWO-INVERTED PENDULUM EXAMPLE

In this section, the numerical simulation of two connected inverted pendulums is performed to illustrate the design procedure and performance of the proposed controller. The state-space equations of the system are as follows.

$$\Sigma_1 : \begin{cases} \dot{x}_{1,1} = x_{1,2} \\ \dot{x}_{1,2} = \left(\dfrac{m_1\zeta H}{J_1} - \dfrac{\eta H}{2J_1}\right)\sin(x_{1,1}) \\ + \dfrac{\eta H}{2J_1}(l - v) \\ + \dfrac{sat(u_1)}{J_1} + \dfrac{\eta H^2}{4J_1}\sin(x_{2,1}) + \Delta_1(t) \\ y_1 = x_{1,1} \end{cases} \tag{38}$$

$$\Sigma_2 : \begin{cases} \dot{x}_{2,1} = x_{2,2} \\ \dot{x}_{2,2} = \left(\dfrac{m_2\zeta H}{J_2} - \dfrac{\eta H}{2J_2}\right)\sin(x_{2,1}) \\ + \dfrac{\eta H}{2J_2}(l - v) \\ + \dfrac{sat(u_2)}{J_2} + \dfrac{\eta H^2}{4J_2}\sin(x_{1,1}) + \Delta_2(t) \\ y_2 = x_{2,1} \end{cases} \tag{39}$$

where the system outputs $x_{j,1}(j = 1, 2)$ are the vertical angular displacements that are available for measurement. The states of $x_{j,2}(j = 1, 2)$ are the angular velocities that are assumed to be unavailable. The inputs u_js ($j = 1, 2$) are torques that are generated by servomotors and Δ_js ($j = 1, 2$) are external unknown disturbances that are assumed to be $\Delta_1(t) = 0.1\sin(t)$ and $\Delta_2(t) = 0.2 + 0.1\cos(2t)$. The parameter $\zeta = 9.8\ m/s^2$ is the gravitational acceleration, $\eta = 100\ N/m$ is the spring constant, $H = 0.5\ m$ is the pendulum height, $l = 0.5m$ is the length of spring, $J_1 = 0.5\ kg \cdot m^2$ and $J_2 = 0.625\ kg \cdot m^2$ represent the moments of inertia, and $v = 0.4m < l$ is the distance between the hinges of the pendulums. The masses of the pendulums are $m_1 = 2$ kg and $m_2 = 2.5$ kg respectively. The control inputs are assumed to be saturated as $sat(u_j) = sgn(u_j)\min(|u_j|, b_j^u)$ with $b_1^u = b_2^u = 25$ where b_j^us are the maximum torques of the servomotors.

The design procedure of the controllers is as follows. It is worth noting that the actual dynamic equations and contained disturbances are unknown to the controller. For illustrative purposes, the output is regulated to the origins. Thus, the desired outputs $y_1^d(t)$ and $y_2^d(t)$ are all zeros for all $t \ge 0$. For the controller for Σ_1, the HOSD, input filer and control input formula is determined as follows.

$$\mathcal{D}_1 : \begin{cases} \dot{\alpha}_{1,1} = 10L_1 e_{\alpha 1,1} + \sigma_{1,1} \\ \dot{\sigma}_{1,1} = L_1\, sgn(e_{\alpha 1,1}) \\ \dot{\alpha}_{1,2} = 7L_1 e_{\alpha 1,2} + \sigma_{1,2} \\ \dot{\sigma}_{1,2} = L_1\, sgn(e_{\alpha 1,2}) \end{cases}$$

$$\mathcal{F}_1 : \begin{cases} \dot{w}_{1,1} = -c_1 w_{1,1} + w_{1,2} \\ \dot{w}_{1,2} = -c_1 w_{1,2} + u_1 \end{cases}$$

$$u_1 = -\sigma_{1,2} - p_2(\mathbf{w}_1) - \mathbf{k}_1^T \hat{\mathbf{e}}_1 \tag{40}$$

where

$$\begin{aligned} e_{\alpha 1,1} &= a_1(t) - \alpha_{1,1} \\ e_{\alpha 1,2} &= \sigma_{1,1} - \alpha_{1,2} \\ \mathbf{w}_1 &= \begin{bmatrix} w_{1,1} \\ w_{1,2} \end{bmatrix} \\ \hat{\mathbf{e}}_1 &= \begin{bmatrix} e_1 \\ \sigma_{1,1} + p_1(\mathbf{w}_1) + w_{1,2} \end{bmatrix} \end{aligned} \tag{41}$$

and $p_1(\mathbf{w}_1)$, $p_2(\mathbf{w}_1)$ are defined as in (22), (23) respectively. The design constants are chosen as $L_1 = 12$, $\kappa_1 = 10$ (that is, $\mathbf{k}_1 = [100, 20]^T$), and $c_1 = 1$. Note that, as depicted in Fig. 1, the equations in (40) uses y_1 only to generate control input of the first system (38).

The following HOSD, filter, and control law constitutes the controller for Σ_2.

$$\mathcal{D}_2 : \begin{cases} \dot{\alpha}_{2,1} = 10L_2 e_{\alpha 2,1} + \sigma_{2,1} \\ \dot{\sigma}_{2,1} = L_2\, sgn(e_{\alpha 2,1}) \\ \dot{\alpha}_{2,2} = 7L_2 e_{\alpha 2,2} + \sigma_{2,2} \\ \dot{\sigma}_{2,2} = L_2\, sgn(e_{\alpha 2,2}) \end{cases}$$

$$\mathcal{F}_2 : \begin{cases} \dot{w}_{2,1} = -c_2 w_{2,1} + w_{2,2} \\ \dot{w}_{2,2} = -c_2 w_{2,2} + u_2 \end{cases}$$

$$u_2 = -\sigma_{2,2} - p_2(\mathbf{w}_2) - \mathbf{k}_2^T \hat{\mathbf{e}}_2 \tag{42}$$

where

$$\begin{aligned} e_{\alpha 2,1} &= a_2(t) - \alpha_{2,1} \\ e_{\alpha 2,2} &= \sigma_{2,1} - \alpha_{2,2} \\ \mathbf{w}_2 &= \begin{bmatrix} w_{2,1} \\ w_{2,2} \end{bmatrix} \\ \hat{\mathbf{e}}_2 &= \begin{bmatrix} e_2 \\ \sigma_{2,1} + p_1(\mathbf{w}_2) + w_{2,2} \end{bmatrix} \end{aligned} \tag{43}$$

and $p_1(\mathbf{w}_2)$, $p_2(\mathbf{w}_2)$ is defined as in (22), (23) respectively. The design constants are selected as $L_2 = 12$, $\kappa_2 = 10$ (that is, $\mathbf{k}_2 = [100, 20]^T$), and $c_2 = 1$. It is also noted that, as depicted in Fig. 1, the equations in (42) uses y_2 only to generate control input of the second system (45). The initial states of HOSDs and input filters are all zeros. The initial conditions of the systems states are $\mathbf{x}_1(0) = [-3, 0.5]^T$ and $\mathbf{x}_2(0) = [3, -0.7]^T$. The simulations have been performed using python libraries such as NumPy, SciPy and matplotlib [31].

The simulation results are expressed as in Figs. 2-6. As in Fig. 2 and Fig. 3, it is illustrated that the system outputs $x_{j,1}$ and $x_{j,2}$ show that the outputs of the subsystems are regulated to the origins after short transient periods. It is also shown in Figs. 4 - 6 that the control inputs as well as all the state variables of the HOSDs and input filters are bounded.

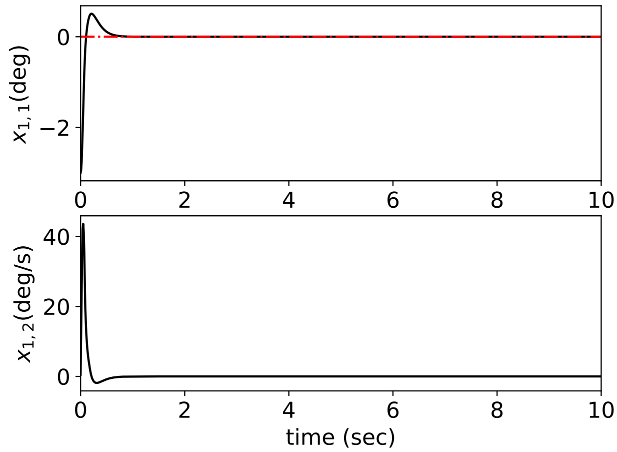

FIGURE 2. Trajectories of $x_{1,1}$ and $x_{1,2}$ of system Σ_1.

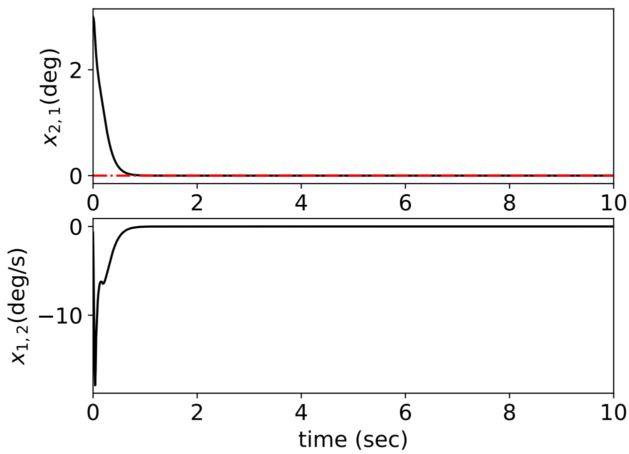

FIGURE 3. Trajectories of $x_{2,1}$ and $x_{2,2}$ of system Σ_2.

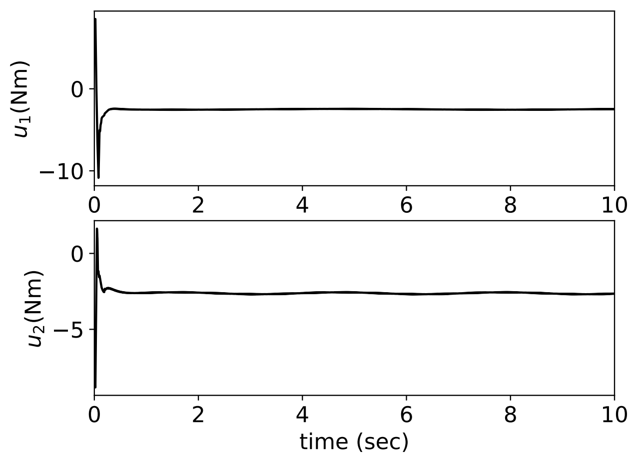

FIGURE 4. Trajectories of control inputs u_1 of Σ_1 and u_2 of Σ_2.

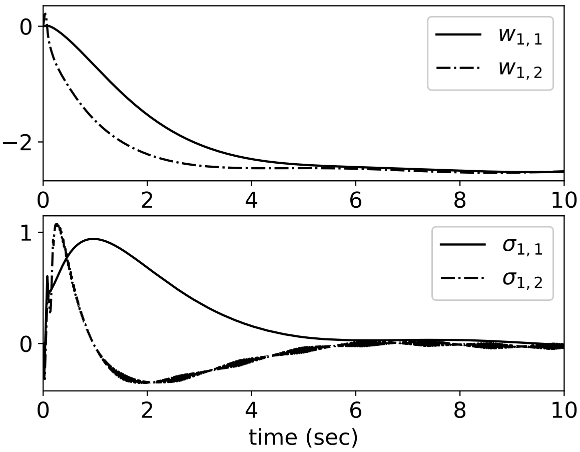

FIGURE 5. Trajectories of $w_{1,1}, w_{1,2}$ in filter $\mathcal{F}_1$ and $\sigma_{1,1}, \sigma_{1,2}$ in HOSD $\mathcal{D}_1$ of Σ_1.

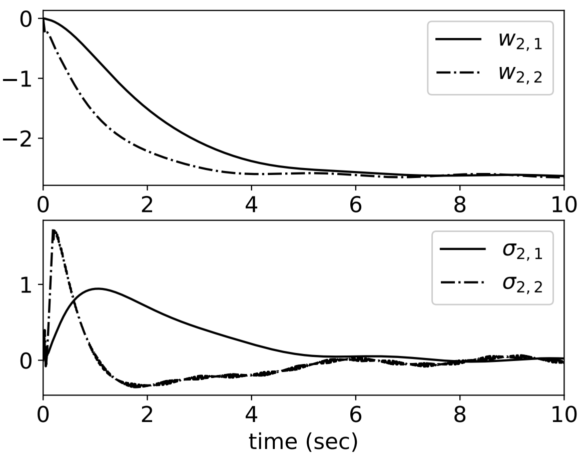

FIGURE 6. Trajectories of $w_{2,1}, w_{2,2}$ in filter $\mathcal{F}_2$ and $\sigma_{2,1}, \sigma_{2,2}$ in HOSD $\mathcal{D}_2$ of Σ_2.

Remark 5: There are two design constants (i.e., $\kappa_j > 0$ and $L_j > 0$ for $j = 1, 2$) in each decentralized controller. The performance of HOSD tends to be better as the design constant L_j becomes larger. However, if L_j is too large, chattering occurs in the differential estimates. Therefore, after performing the simulation severally, the L_j value was properly selected such that chattering did not occur and sufficient estimation performance was achieved.

B. SECOND EXAMPLE

The second example is the following LSS with pure-feedback nonlinear subsystems that have unmatched disturbances and interconnections.

$$\Sigma_3 : \begin{cases} \dot{x}_{1,1} = x_{1,1} + (1 + 0.2x_{1,2}^2)x_{1,2} + \sin(\frac{t}{10}) \\ \quad + x_{2,1}x_{2,2} \\ \dot{x}_{1,2} = x_{1,1}x_{1,2} + u_1 + \frac{u_1^3}{7} + \cos(\frac{t}{15}) + x_{2,2}^2 \\ y_1 = x_{1,1} \end{cases} \quad (44)$$

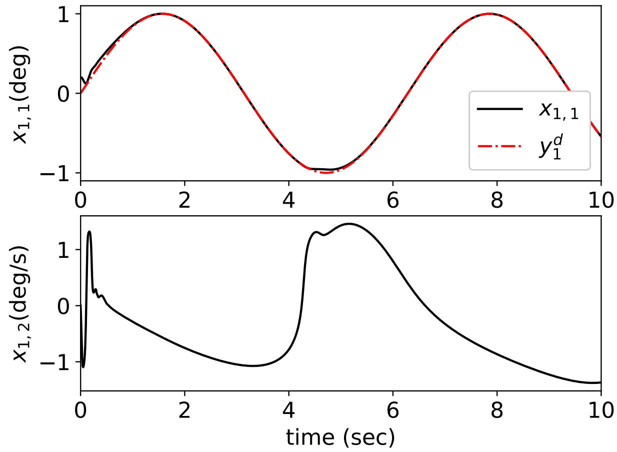

FIGURE 7. Trajectories of $y_1 (= x_{1,1})$ and $x_{1,2}$ of system Σ_3.

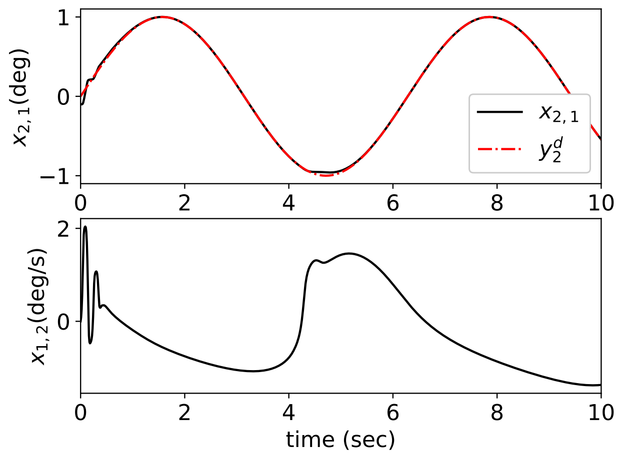

FIGURE 8. Trajectories of $y_2 (= x_{2,1})$ and $x_{2,2}$ of system Σ_4.

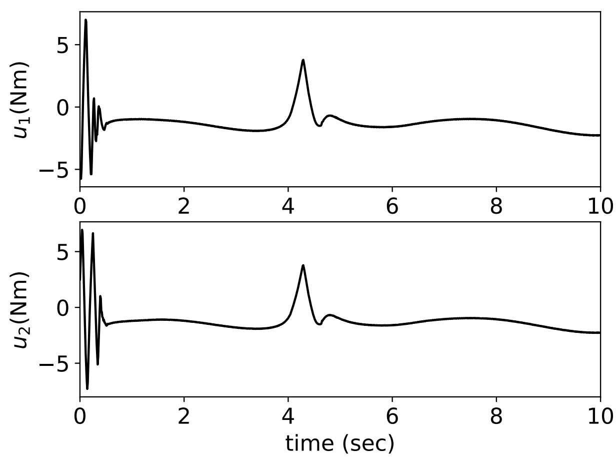

FIGURE 9. Control inputs u_1 of Σ_3 and u_2 of Σ_4.

$$\Sigma_4 : \begin{cases} \dot{x}_{2,1} = x_{2,1} + (1 + 0.2x_{2,2}^2)x_{2,2} + \sin(\frac{t}{10}) \\ \quad + x_{1,1}x_{1,2} \\ \dot{x}_{2,2} = x_{2,1}x_{2,2} + u_2 + \frac{u_2^3}{7} + \cos(\frac{t}{15}) + x_{1,2}^2 \\ y_2 = x_{2,1} \end{cases} \quad (45)$$

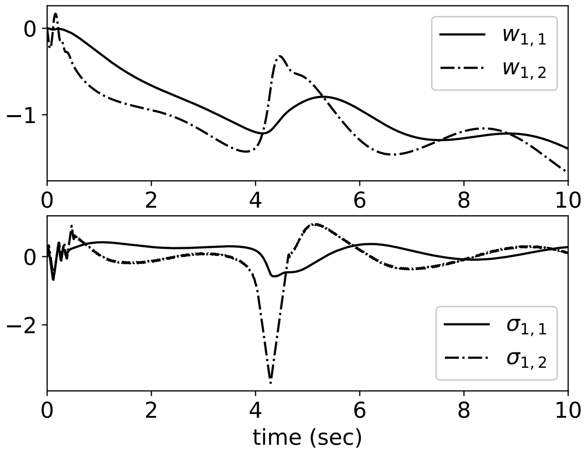

FIGURE 10. Trajectories of $w_{1,1}$, $w_{1,2}$ in filter $\mathcal{F}_1$ and $\sigma_{1,1}$, $\sigma_{1,2}$ in HOSD $\mathcal{D}_1$.

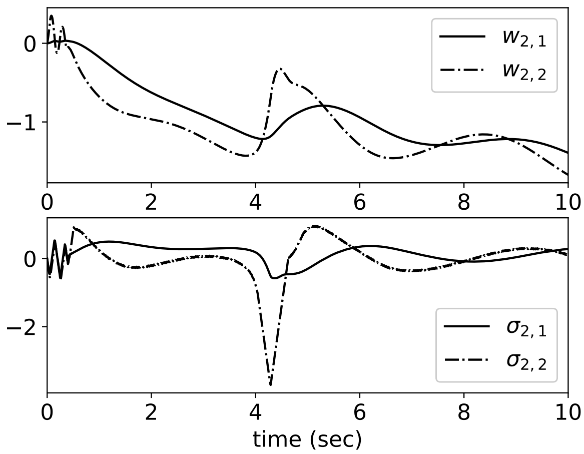

FIGURE 11. Trajectories of $w_{2,1}$, $w_{2,2}$ in filter $\mathcal{F}_2$ and $\sigma_{2,1}$, $\sigma_{2,2}$ in HOSD $\mathcal{D}_2$.

where $x_{j,1}$,$x_{j,2}$ are state variables and u_j, y_j are control input and output of jth subsystem ($j = 1, 2$), respectively. In this example, the desired outputs are $y_1^d(t) = y_2^d(t) = \sin(t)$. The controllers have the same structures as the ones used in the former subsection IV-A since the dynamic order of the subsystems are identical. Control formulas (40) and (42) are adopted again for Σ_3 and Σ_4 respectively with $\kappa_1 = \kappa_2 = 5$ and $L_1 = L_2 = 11$. The initial state vectors are $\mathbf{x}_1(0) = [0.2, 0]^T$ and $\mathbf{x}_2(0) = [-0.1, 0]^T$. The simulation results are presented as in Figs. 7 - 11. It is illustrated that the subsystem outputs track the desired signal well in Fig. 7 and Fig. 8. Apparently, the remaining time-varying signals in the closed-loop systems are bounded as shown in Figs. 9-11.

V. CONCLUSION

A novel differentiator-based decentralized controller for interconnected LSS with uncertain nonautonomous and non-affine nonlinear subsystems is proposed. The nonlinear subsystems in consideration are quite general and contain broad classes of modern controlled systems. To the best of the authors' knowledge, there are no research results of decentralized output-feedback controller design studies for such classes of LSSs. The proposed controller adopts HOSD that can estimate the time-derivatives of a time-varying signal asymptotically for the purpose of compensating uncertainties in the controlled subsystems. No universal approximators such as FLS or NN are required, and the control scheme is not based on backstepping. As a result, the proposed output-feedback decentralized controller has a relatively simple formulae, and the resulting stability analysis is straightforward. The proposed controller guarantees that the tracking errors of the subsystems are asymptotically convergent at zero, and that all the signals involved are bounded. Herein, the numerical simulations performed illustrate the performance and compactness of the proposed control scheme.

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