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Finite-Time Stability of MIMO Nonlinear Systems Based on Robust Adaptive Sliding Control: Methodology and Application to Stabilize Chaotic Motions

HA LE NHU NGOC THANH[1], MAI THE VU[2], NGOC PHI NGUYEN[3], NGUYEN XUAN MUNG[3], (Member, IEEE), AND SUNG KYUNG HONG[3]
[1]HUTECH Institute of Engineering, Ho Chi Minh City University of Technology (HUTECH), Ho Chi Minh 700000, Vietnam
[2]School of Intelligent Mechatronics Engineering, Sejong University, Seoul 05006, South Korea
[3]Faculty of Mechanical and Aerospace Engineering, Sejong University, Seoul 05006, South Korea
Corresponding authors: Sung Kyung Hong (skhong@sejong.ac.kr) and Ha Le Nhu Ngoc Thanh (hlnn.thanh@hutech.edu.vn)

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ABSTRACT This paper introduces a robust adaptive sliding mode control to solve a finite-time stability of the uncertain nonlinear systems with multiple inputs and multiple outputs (MIMO). The proposed algorithm guarantees a strict robustness and fast convergence of the system trajectories to zero in a finite time under the negative effects of uncertainties and/or external disturbances. The fundamental methodology is based on an improved modification of the super-twisting sliding technique to alleviate an undesirable influence of the chattering phenomenon. In addition, a nonlinear adaptive law is constructed to ensure a strict stability of the control system even without prior awareness of the upper bounds of uncertainties and disturbances. A general stability of the closed-loop disturbed MIMO nonlinear system is achieved by the Lyapunov theorem. Lastly, the proposed algorithm is applied to stabilize the typical chaotic behaviors of Duffing – Holmes system and Lorenz system. The advantages and effectiveness of the proposed method are clearly demonstrated through the results of numerical simulations compared with other existent methods.

INDEX TERMS Disturbed MIMO system, adaptive control, sliding mode control, finite-time stability, chaotic system.

I. INTRODUCTION

It is obviously that the undesirable influences of uncertain parameters and/or exogenous perturbations are unavoidable in many practical engineering systems, and often provide a negative performance for tracking control and stabilization control. For many decades, the robust adaptive control algorithms have been introduced by plenty of researchers for improving the tracking control performance [1]–[5]. Besides, the issue of stabilization control is also an attractive topic describing the convergence of system trajectories to a small neighborhood of the origin zero. Thus, this problem has been widely researched and applied in both practical linear and nonlinear dynamic systems [6].

The classical sliding mode control (SMC), and advanced SMC methods are renowned and advantageous robust control techniques due to its uncomplex concept, robustness, and excellent capacity to remove the negative influences of the matched/unmatched uncertainties and external disturbances on an engineering system [7]–[11]. The first step of designing SMC algorithms is to define a linear or nonlinear sliding surface which is a continuous function of system states or tracking errors, and following an effective controller is constructed in such a way that the system trajectories converge to the proposed sliding surface [12]. Because of those simple concept and great efficiency, the SMC technique has extensively applied to control a nonlinear system with highly

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guaranteeing of a robust stability and asymptotic convergence of the system state to equilibrium points in a finite time. However, it is not always possible to achieve the asymptotically stable with a finite time convergence because performance of the SMC technique strictly depends on a switching law. In order to solve this issue, various finite-time stabilization algorithms have been introduced by using a terminal sliding mode control [13], disturbance observer and second-order sliding mode control [14], [15]. However, in these researches, the singularity phenomenon may occur one the initial states of the system are inside special zones causing an amplification of the control signal [16]. In [17], [18], an extended SMC integrated with a disturbance observer was developed for *n*th order nonlinear systems with unmatched uncertainties to improve the tracking control performance. However, the control strategies cannot guarantee a robust stability of the closed-loop systems in a finite time. Furthermore, in these disturbance observers, the assumptions of an exogenous perturbation are only the harmonic signals which are not general case because an arbitrary perturbation may influence on the system at any time during the working process [19]. In addition, the studies only focus on controlling a single input and single output (SISO) nonlinear system [20]. In order to solve this problem, a terminal sliding mode control for the perturbed MIMO linear systems was introduced in [21]. However, the convergence performance of state trajectories of the closed-loop systems is quite slow when the initial states are distant from the equilibrium points. In [22], a nonlinear sliding function is proposed to improve the tracking performance for a finite time control of a disturbed MIMO nonlinear system based on the form of linear matrix inequalities (LMIs). However, the upper bound conditions of uncertainties and exogenous disturbances are strictly required for designing a control law. In [23], [24], a robust adaptive high-order SMC for class of perturbed MIMO systems was presented by using the super twisting technique. In [25], a derivative and integral terminal SMC for multiple inputs and multiple outputs system was proposed to improve the finite convergence of the system state to zero. In [26], [10], a stabilization control and robust adaptive SMC for a nonlinear system with matched and unmatched uncertainties were introduced by using a second-order sliding manifold. The technique of a second-order SMC is also applied to improve the tracking performance of a flight control based on the classical Proportional Integral Derivative (PID) sliding surface [27]. However, the overall disadvantages of these researches can be briefly described as follows: *i*) these algorithms highly concentrated on the second order SISO nonlinear systems. Therefore, it may not be suitable to apply for higher order disturbed MIMO systems in the appearance of multiple inputs; *ii)* awareness of the upper bound information must be known in advanced; *iii)* these controllers just guarantee a stability of system with a slow performance without ensuring the finite-time convergence.

Intelligent control method is also another well-known trend of study to deal with the problem of controlling the nonlinear systems in the presence of uncertainties and/or external disturbances. The conceptual methodology of this approach is to apply composite techniques between fuzzy algorithm, neural network, and SMC to stabilize the closed-loop systems [28]. In [29], a fuzzy logic algorithm is used to estimate the unknown terms, following an adaptive fuzzy siding mode control for a class of perturbed MIMO nonlinear system was developed by an input-output model to guarantee the convergence of the tracking errors to a small ball containing the origin zero. In [30], a robust control to deal with the stabilization problem and tracking control problem for a disturbed MIMO system is presented by a combination of the SMC technique and fuzzy logic algorithm. In [31], an adaptive SMC for Takagi-Sugeno fuzzy system based LMIs method is proposed to improve the control performance of the system with mismatched uncertainties and/or exogenous perturbations. However, the general drawback of these mentioned approaches is not easy to demonstrate the stability of the closed-loop system in a finite-time.

The research's inspiration of this paper is to deal with the aforementioned drawbacks of the existent approaches. The essential methodology of this study is based on an improved modification of the super-twisting algorithm (STA) to design a robust adaptive law allowing to reimburse uncertainties and/or external perturbations changing in time or together with the system states for improvement the stability of a closed-loop perturbed MIMO nonlinear system. The main contributions of this scientific paper are briefly summarized as follows:

1) A robust adaptive control law is designed through a novel nonlinear sliding manifold which is not only to ensure that the system states strongly converge to zero in the finite-time but also to easily understand and execute to the practical system because the proposed controller is designed in the state space model of a general disturbed MIMO system.
2) Unlike the existent control strategies, the proposed algorithm is entirely possible to improve the performance of transient response and steady state response of the high-order perturbed MIMO systems.
3) The proposed algorithm always guarantees a robust stability of the system against uncertainties and external disturbances without requiring precise awareness of its upper bound information compared with the classical STA [32]–[34]. It implies that the controller allows to reimburse uncertainties/perturbations changing in time or together with the system states.
4) The singularity problem is removed by the presented controller, and the negative effect of a chattering is also eliminated due to the anti-chattering capacity of the super twisting technique.

The rest of this article is organized as follows. The problem statement and preliminaries are briefly described in Section 2. Section 3 provides the steps of designing a novel nonlinear sliding manifold and robust adaptive sliding mode controller; the stability analysis of the control system is also included in this section. The results of a numerical simulation of the chaotic systems are presented in section 4. Finally, the conclusions are given in Section 5.

II. PROBLEM STATEMENT AND PRELIMINARIES

It is proposed to consider a class of disturbed MIMO nonlinear system given by the following state space model as

$$\dot{x} = (A + \Delta A(t))\, x + (B + \Delta B(t))\, (u + f(x, t) + g(x, t)) + h(t) \tag{1}$$

where $x \in \mathbb{R}^n$ represents state vector of the system, $u \in \mathbb{R}^m$ denotes the control input; $f(x)$ and $g(x)$ denote the known and unknown smooth nonlinear functions in term of x, respectively; A and B are constant matrices of proper dimensions;$\Delta A(t)$ and $\Delta B(t)$ are the unknown time varying matrices of uncertain parameters of the system; $h(t)$ is an unknown time varying function representing an exogenous perturbation. There are some assumptions given by the following statements

Assumption 1: Pair (A, B) is completely controllable

Assumption 2: The matrix B is full rank, i.e.,rank $B = m$

Assumption 3: There exist unknown time-varying matrices with proper dimension represented by $P_A(t)$, $P_B(t)$ and $P_h(t)$ in such a way that $\Delta A(t) = BP_A(t)$, $\Delta B(t) = BP_B(t)$, $h(t) = BP_h(t)$. Thus, the disturbed MIMO nonlinear system (1) can be re-written by a simple model as follows:

$$\dot{x} = Ax + Bf\,(x, t) + B\,(u + d\,(x, u, t)) \tag{2}$$

where $d(x, u, t)$ is a lumped perturbation generated by the uncertain parameters and external disturbances as follows:

$$d(x, u, t) = P_A(t)x(t) + P_B(t)u(t) + P_B(t)f(x, t) + g(x, t) + P_B(t)g(x, t) + P_h(t)h(t) \tag{3}$$

From the *Assumption 3*, it is clear that instead of designing a controller for system (1), we can do it with the system (2).

According to the *Assumptions 1* and 2, there exists a transformation matrix T such that [35]:

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = Tx \in \begin{bmatrix} \mathbb{R}^{n-m} \\ \mathbb{R}^m \end{bmatrix} \tag{4}$$

where

$$T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}, \;\; T_2 = \left(B^T B\right)^{-1} B^T, \;\; T_1 B = 0$$

Thus, the nonlinear system (2) can be re-written by using the new transformed state variables from Eq. (4) as follows:

$$\dot{z} = A^* z + B^* f\,(z, t) + B^*\,(u + d(z, u, t)) \tag{5}$$

where

$$A^* = TAT^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{6}$$

$$B^* = TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \; B_2 \in \mathbb{R}^{m \times m} \tag{7}$$

and $A_{ij}(i, j = 1, 2)$ are matrices of proper dimensions.

The regular form of the system (5) can be re-written from Eq.(6) and Eq.(7) as follows:

$$\dot{z}_1 = A_{11} z_1 + A_{12} z_2 \tag{8a}$$

$$\dot{z}_2 = A_{21} z_1 + A_{22} z_2 + B_2 f\,(z, t) + B_2\,(u + d(z, u, t)) \tag{8b}$$

The objective/control problem of this research article is to design a robust adaptive sliding mode controller based on an improved modification of the STA. The controller must allow to reimburse uncertainties and/or external perturbations changing in time or together with the system states. An additional demand of the control algorithm is not only to guarantee the finite time stabilization of the disturbed MIMO nonlinear system, but also to remove the negative influences of a chattering phenomenon compared with the traditional SMC method.

III. MAIN RESULTS

A. DESIGN OF SLIDING SURFACE FUNCTION

The steps of designing a nonlinear sliding manifold and its stability analysis are derived in this subsection. A novel nonlinear sliding surface function is defined by

$$s = Qz + \left(\Lambda \left\|z_1\right\|^{\alpha-1} + \Gamma \left\|z_1(0)\right\|\right) z_1 \tag{9}$$

where $0 < \alpha < 1$, Λ and Γ are constant matrices with proper dimensions; $z_1(0)$ is an initial state of z_1.

Let $Q = \left[\Upsilon \;\; I\right] \in \left[\mathbb{R}^{m\times(n-m)} \;\; \mathbb{R}^{m\times m}\right]$ is a constant matrix appropriately selected; $I_{m\times m}$ is an identity matrix. The sliding manifold in Eq. (9) can be re-written by:

$$s = \Upsilon z_1 + z_2 + \left(\Lambda \left\|z_1\right\|^{\alpha-1} + \Gamma \left\|z_1(0)\right\|\right) z_1 \tag{10}$$

when the sliding surface converges to zero i.e., $s = 0$, then the Eq. (10) can be achieved as

$$z_2 = -\Upsilon z_1 - \left(\Lambda \left\|z_1\right\|^{\alpha-1} + \Gamma \left\|z_1(0)\right\|\right) z_1 \tag{11}$$

From Eq.(11), It can be seen that if the system state, z_1, is convergent to zero in a finite time when $s = 0$, then the system state, z_2, also drives to zero in a finite time. Thus, a stability of the control system can be demonstrated and discussed by Theorem 1.

Theorem 1: Consider the dynamic model (8). The system state z_1, z_2 will strongly converge to zero in a finite time if the sliding manifold given in Eq.(9) is used to design a controller u, and the values of Λ, Γ, and Υ must also be chosen such that the matrices $E = A_{11} - A_{12}\Upsilon$, $A_{12}\Lambda$, and $A_{12}\Gamma$ are symmetric matrices and satisfy the following conditions:

i)

$$\lambda_{\max}\left\{E + E^T\right\} = -\chi < 0 \tag{12}$$

ii)

$$\lambda_{\min}\left\{A_{12}\Lambda\right\} = \eta > 0, \;\; \lambda_{\min}\left\{A_{12}\Gamma\right\} = \theta > 0 \tag{13}$$

where χ, η, θ are positive constants, $\lambda_{\min}\{\cdot\}$ and $\lambda_{\max}\{\cdot\}$ represent the minimum and maximum eigenvalue of a matrix.

Proof: Substituting the Eq.(11) to Eq.(8a), the dynamic system state $\dot{z}_1$ can be obtained as follows:

$$\begin{aligned}\dot{z}_1 &= A_{11}z_1 + A_{12}\left[-\Upsilon z_1 - \left(\Lambda \|z_1\|^{\alpha-1} + \Gamma \|z_1(0)\|\right) z_1\right] \\ &= Ez_1 - A_{12}\Lambda \|z_1\|^{\alpha-1} z_1 - A_{12}\Gamma \|z_1(0)\| z_1 \end{aligned} \tag{14}$$

Consider a Lyapunov function candidate as

$$V = z_1^T z_1 \tag{15}$$

The first time derivative of V can be computed by using the Eqs.(14), (15) as follows

$$\begin{aligned}\dot{V} &= \dot{z}_1^T z_1 + z_1^T \dot{z}_1 \\ &= \left(Ez_1 - A_{12}\Lambda \|z_1\|^{\alpha-1} z_1 - A_{12}\Gamma \|z_1(0)\| z_1\right)^T z_1 \\ &\quad + z_1^T \left(Ez_1 - A_{12}\Lambda \|z_1\|^{\alpha-1} z_1 - A_{12}\Gamma \|z_1(0)\| z_1\right) \\ &= z_1^T \left(E^T + E\right) z_1 - 2z_1^T \left(A_{12}\Lambda\right) z_1 \|z_1\|^{\alpha-1} \\ &\quad -2z_1^T \left(A_{12}\Gamma\right) z_1 \|z_1(0)\| \end{aligned} \tag{16}$$

It is always possible to choose the constant matrices Υ, Λ, and Γ in such a way that the matrices $E = A_{11} - A_{12}\Upsilon$, $A_{12}\Lambda$, and $A_{12}\Gamma$ are symmetric. Therefore, the following inequalities (17), (18), and (19) are always satisfied,

$$\begin{aligned}&\lambda_{\min}\left\{E^T + E\right\} \|z_1\|^2 \\ &\quad \le z_1^T \left(E^T + E\right) z_1 \le \lambda_{\max}\left\{E^T + E\right\} \|z_1\|^2 \end{aligned} \tag{17}$$

$$\begin{aligned}&\lambda_{\min}\left\{A_{12}\Lambda\right\} \|z_1\|^2 \\ &\quad \le z_1^T \left(A_{12}\Lambda\right) z_1 \le \lambda_{\max}\left\{A_{12}\Lambda\right\} \|z_1\|^2 \end{aligned} \tag{18}$$

$$\begin{aligned}&\lambda_{\min}\left\{A_{12}\Gamma\right\} \|z_1\|^2 \\ &\quad \le z_1^T \left(A_{12}\Gamma\right) z_1 \le \lambda_{\max}\left\{A_{12}\Gamma\right\} \|z_1\|^2 \end{aligned} \tag{19}$$

Substituting the Eqs.(12), (13), and Eqs.(17-19) to Eq.(16), the negative value of the function $\dot{V}$ can be proven as follows:

$$\dot{V} \le -\chi \|z_1\|^2 - 2\eta \|z_1\|^{\alpha+1} - 2\theta \|z_1\|^2 \|z_1(0)\| \tag{20}$$

From Eq.(15), expression (20) becomes

$$\dot{V} = \frac{dV}{dt} \le -\left(\chi + 2\theta \|z_1(0)\|\right) V - 2\eta V^{\gamma}, \tag{21}$$

where $\gamma = \frac{\alpha+1}{2}$, due to $0 < \alpha < 1$, thus $\frac{1}{2} < \gamma < 1$ From the inequality (21), it yields

$$\begin{aligned}\Rightarrow dt &\le -\frac{V^{-\gamma} dV}{\left(\chi + 2\theta \|z_1(0)\|\right) V^{1-\gamma} + 2\eta} \\ &= -\frac{1}{1-\gamma}\left[\frac{(1-\gamma) V^{-\gamma} dV}{\left(\chi + 2\theta \|z_1(0)\|\right) V^{1-\gamma} + 2\eta}\right] \end{aligned} \tag{22}$$

Due to $d\left(V^{1-\gamma}\right) = (1-\gamma) V^{-\gamma} dV$, thus the inequality (22) can be re-written as follows

$$dt \le -\frac{1}{1-\gamma}\left[\frac{d\left(V^{1-\gamma}\right)}{\left(\chi + 2\theta \|z_1(0)\|\right) V^{1-\gamma} + 2\eta}\right] \tag{23}$$

Let t_0 is an initial time, and $t_f > 0$ is a finite time at which the Lyapunov function, $V(t_f)$, converges to zero. It implies that $V(t) = 0$ as $t \ge t_f$. Thus, the value of t_f can be computed by integrating of the inequality (23) with the time interval $t_0 \le t \le t_f$.

$$\begin{aligned}t_f &\le t_0 - \frac{1}{1-\gamma}\int_{t_0}^{t_f}\left[\frac{d\left(V^{1-\gamma}\right)}{\left(\chi + 2\theta \|z_1(0)\|\right) V^{1-\gamma} + 2\eta}\right] \\ &= t_0 + \frac{1}{(1-\gamma)\left(\chi + 2\theta \|z_1(0)\|\right)} \\ &\quad \times \ln\left[\frac{\left(\chi + 2\theta \|z_1(0)\|\right) V^{1-\gamma}(t_0) + 2\eta}{2\eta}\right] \end{aligned} \tag{24}$$

Since $\left(\chi + 2\theta \|z_1(0)\|\right) > 0$, and $2\eta > 0$, expression (21) proves that the state trajectories z_1, z_2 of the control system are powerfully convergent to zero in finite-time t_f given in expression (24). The proof of Theorem 1 is completed.

B. ROBUST ADAPTIVE SLIDING CONTROLLER DESIGN

In this subsection, a robust adaptive sliding mode controller is designed through the nonlinear sliding manifold presented in Section 3.A. The proposed controller is constructed by using an improved modification of the super-twisting sliding mode technique to enhance a stability of the closed-loop MIMO nonlinear system and alleviating the chattering effect.

Assumption 4: [32]–[34], The unknown uncertain parameters and external perturbations, $d(z, u, t)$, effecting on the MIMO nonlinear system (5) are the smooth function and bounded by

$$\|d(z, u, t)\| \le \kappa \|s\|^{1/2} \tag{25}$$

where $\kappa \in \mathbb{R}^+$ is a unknown constant.

Remark 1: The *Assumption 4* was also given in the previous researches [32]–[34]. However, in those studies, the controller gains strictly depend on the value of κ. Thus, in order to design an efficient controller ensuring a robust stability of the closed-loop system, either the coefficient κ must be known in advance or the controller gains must be selected by trial-and-error method regardless of parameter κ. However, it is almost impossible way to precisely know κ due to the diversity and complexity of the working environment [36]. Furthermore, although it is obvious that the controller gains must be appropriately chosen according to the bound of perturbations through the trial-and-error method, it is not clear how to pick the appropriate gains. The problem is solved by a typical robust adaptive controller using the super-twisting sliding technique given in [37]. However, it is also difficult to find out the proper controller gains since there are many parameters that need to be fine-tuned and furthermore the algorithm is only appropriate for controlling the SISO system. Hence, in this subsection a robust controller based an adaptive law of κ is proposed to overcome these existent drawbacks for controlling the MIMO system.

Remark 2: In this approach, the *Assumption 4* is also used to analyze a stability of the control system. However, the real upper bound of $d(z, u, t)$ is not precisely known in advance. It implies that a prior knowledge of the coefficient, κ, is not required. Instead of that, the value, κ, will be approximated through an adaptive law using the novel sliding manifold presented in Eq.(9) and a robust controller given in Theorem 2. Furthermore, the approximate value is also used as an adaptive gain to guarantee a rapid adaptation and robust stability of the control system with perturbations raising in time or together with the system states. The detail method is described and discussed through Theorem 2 and its proof.

Theorem 2: Consider the disturbed MIMO nonlinear system (8), the motion on sliding manifold function given in Eq.(9) is strongly convergent to zero if the controller, u, and adaptive law, $\hat{\kappa}$, are chosen as the following equations (26), (27), and equation (28).

$$\begin{aligned} u &= -\left(QB^*\right)^{-1} \\ &\quad\times\Big[QA^*z+QB^*f(z,t)+\left(\Lambda\alpha\left\|z_1\right\|^{\alpha-1}+\Gamma\left\|z_1(0)\right\|\right)A_1^*z \\ &\quad+\left(k_1+\left\|QB^*\right\|\hat{\kappa}\right)\|s\|^{-1/2}s+k_3s-\sigma\Big] \quad (26)\\ \dot{\sigma} &= -k_2\|s\|^{-1}s \quad (27)\\ \dot{\hat{\kappa}} &= \frac{\|QB^*\|\,\|s\|^{1/2}}{2\delta},\ \text{where } \delta>0 \quad (28)\end{aligned}$$

where $A_1^* = \left[A_{11}\ A_{12}\right]$; $k_1, k_2, k_3 > 0$ are constants; $\hat{\kappa}$ is an adaptive law estimating value of κ.

Proof:

Let $\xi_1, \xi_2 \in \mathbb{R}$ are new variables defined by functions

$$\begin{cases}\xi_1 = \|s\|^{-1/2}s \rightarrow \|\xi_1\| = \|s\|^{1/2}\\ \xi_2 = \sigma\end{cases} \quad (29)$$

According to Eq.(29), if the new state variables, (ξ_1, ξ_2), are convergent to zero, it implies that the sliding function, s, also converges to zero. The first-time derivative of (ξ_1, ξ_2) can be achieved from Eqs.(27), and (29) as follows,

$$\begin{cases}\dot{\xi}_1 = \dfrac{\dot{s}}{2\|s\|^{1/2}} = \dfrac{\dot{s}}{2\|\xi_1\|}\\ \dot{\xi}_2 = -k_2\|s\|^{-1}s = -k_2\dfrac{\|s\|^{-1/2}s}{\|s\|^{1/2}} = -k_2\dfrac{\xi_1}{\|\xi_1\|}\end{cases} \quad (30)$$

The first derivative of sliding function s can be computed from Eqs.(9), (5) and Eq.(8a) as follows

$$\begin{aligned}\dot{s} &= Q\dot{z}+\left(\Lambda\alpha\|z_1\|^{\alpha-1}+\Gamma\|z_1(0)\|\right)\dot{z}_1\\ &= QA^*z+QB^*f(z,t)+QB^*u+QB^*d(z,u,t)\\ &\quad+\left(\Lambda\alpha\|z_1\|^{\alpha-1}+\Gamma\|z_1(0)\|\right)A_1^*z \quad (31)\end{aligned}$$

Substituting the controller u given in Eq.(26) and Eq.(29) to Eq.(31), $\dot{s}$ becomes

$$\dot{s} = -\left(k_1+\left\|QB^*\right\|\hat{\kappa}\right)\xi_1 - k_3s+\xi_2+QB^*d(z,u,t) \quad (32)$$

Consider a positive Lyapunov function as

$$V_2 = \frac{1}{2}\xi_1^T\xi_1+\frac{1}{4k_2}\xi_2^T\xi_2+\frac{1}{2}\delta\tilde{\kappa}^2 \quad (33)$$

where $\tilde{\kappa}$ is an adaptive error and its value is $\tilde{\kappa} = \kappa-\hat{\kappa}$.

The first time derivative of the Lyapunov function, $\dot{V}_2$, can be obtained by Eq(33) and Eq.(30) as

$$\begin{aligned}\dot{V}_2 &= \frac{1}{2}\left(\dot{\xi}_1^T\xi_1+\xi_1^T\dot{\xi}_1\right)+\frac{1}{4k_2}\left(\dot{\xi}_2^T\xi_2+\xi_2^T\dot{\xi}_2\right)+\delta\tilde{\kappa}\dot{\tilde{\kappa}}\\ &= \frac{1}{2}\left(\frac{\dot{s}^T}{2\|\xi_1\|}\xi_1+\xi_1^T\frac{\dot{s}}{2\|\xi_1\|}\right)\\ &\quad+\frac{1}{4k_2}\left(-k_2\frac{\xi_1^T}{\|\xi_1\|}\xi_2-\xi_2^Tk_2\frac{\xi_1}{\|\xi_1\|}\right)-\delta\tilde{\kappa}\dot{\hat{\kappa}}\\ &= \frac{\xi_1^T}{2\|\xi_1\|}\dot{s}-\frac{\xi_1^T}{2\|\xi_1\|}\xi_2-\delta\tilde{\kappa}\dot{\hat{\kappa}} \quad (34)\end{aligned}$$

Substituting the Eq.(32) to Eq.(34), $\dot{V}_2$ becomes

$$\begin{aligned}\dot{V}_2 &= \frac{1}{2\|\xi_1\|}\begin{pmatrix}-\left(k_1+\|QB^*\|\hat{\kappa}\right)\|\xi_1\|^2-k_3\xi_1^Ts\\ +\xi_1^T\xi_2+\xi_1^TQB^*d(z,u,t)\end{pmatrix}\\ &\quad-\frac{\xi_1^T}{2\|\xi_1\|}\xi_2-\delta\tilde{\kappa}\dot{\hat{\kappa}} \quad (35)\end{aligned}$$

From Eq.(29), it can be seen that,

$$\xi_1\|s\|^{1/2} = s \Leftrightarrow \|\xi_1\|^2\|s\|^{1/2} = \xi_1^Ts \Leftrightarrow \|\xi_1\|^3 = \xi_1^Ts \quad (36)$$

Therefore, from inequality (25), Eq.(29) and Eq.(36), the following expressions always satisfy

$$\begin{cases}\xi_1^Ts = \|\xi_1\|^3\\ \|d(z,u,t)\| \le \kappa\|\xi_1\|\end{cases} \quad (37)$$

Substituting the Eq.(28) and Eq.(37) to Eq.(35), the negative value of $\dot{V}_2$ can be expressed by

$$\begin{aligned}\dot{V}_2 &\le \frac{1}{2\|\xi_1\|}\begin{pmatrix}-k_1\|\xi_1\|^2-\hat{\kappa}\|QB^*\|\,\|\xi_1\|^2-k_3\|\xi_1\|^3\\ +\kappa\|QB^*\|\,\|\xi_1\|^2\end{pmatrix}\\ &\quad-\frac{\tilde{\kappa}}{2}\|QB^*\|\,\|\xi_1\|\\ &\le -\frac{1}{2}\left(k_1\|\xi_1\|+k_3\|\xi_1\|^2\right)\\ &\Rightarrow \dot{V}_2 \le 0,\ \forall k_1,k_3>0 \quad (38)\end{aligned}$$

From the result of expression (38), we can confirm that the motion on sliding manifold function given in Eq.(9) is powerfully convergent to zero by using the presented controller in Eqs.(26), (27) and Eq.(28). From Theorem 1, it exhibited that the system states, z_1, z_2, of a disturbed MIMO nonlinear system (5) are strongly convergent to zero in finite time as the sliding manifold $s = 0$.

Remark 3: From Eq.(26), we can see that the coefficient $\hat{\kappa}$, which is an estimate of κ in *Assumption 4*, is used as an adaptive gain to stabilize the control system. However, the value of $\hat{\kappa}$ can be easily obtained from Eq.(28) by fine-tuning δ regardless of the original parameter κ. Thus, it is obvious that the prior knowledge of κ is not necessary. In other words, the real bound of uncertainties and/or external perturbations does not actually need to know in the proposed controller.

IV. SIMULATION RESULTS AND DISCUSSIONS

In this Section, the presented control algorithm is verified through two typical examples of stabilizing the chaotic systems. The first example is to control the disturbed Duffing – Holmes dynamic model representing an uncertain SISO nonlinear system. Next, the proposed algorithm is also applied to stabilize a perturbed MIMO nonlinear system with the Lorenz model as given in example 2. In addition, the performance of system trajectories is compared with the previous algorithms such as terminal SMC based linear matrix inequality (TSMC-LMI) in [22], and SMC with a nonlinear disturbance observer (SMC-NDO) in [28], to emphasize the contribution of the proposed method.

A. EXAMPLE 1 (DISTURBED SISO NONLINEAR MODEL)

Consider a Duffing – Holmes system [38], which is a typical example of the second-order nonlinear dynamical model describing a complex and chaotic motion,

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -c_1 z_1 - c_2 z_2 - z_1^3 + l\cos(\omega t) \end{cases} \quad (39)$$

where c_1, c_2 are constants, l is an excitation magnitude, and ω is an excitation frequency. The dynamic model (39) can be formulated by a matrix form as follows:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -c_1 & -c_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left(-z_1^3 + l\cos\omega t \right) \quad (40)$$

The constant parameters of the system (40) are chosen as $c_1 = -1, c_2 = 0.073,\ l = 3.97, \omega = 0.68$, the initial state of the system is $z_0 = \left[z_1(0)\ z_2(0) \right]^T = \left[1\ -5 \right]^T$. The complex and chaotic behavior of the uncontrolled Duffing-Holmes system is described by a phase portrait and state trajectories exhibited in Figures 1 and 2. In order to verify efficiency of the presented algorithm for a disturbed SISO system, a control input, u, is added to the system (40) according to the regular form of a closed-loop system given in Eq.(5). The system matrices and the effect of an uncertainty on the dynamic model can be described as follows: $A^* = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -0.073 \end{bmatrix}, B^* = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, A_1^* = \left[0\ 1 \right], f(z,t) = -z_1^3 + l\cos\omega t$; the uncertainty, $d(z,u,t)$, is given by $d(z,u,t) = \left(1/2\right)\cos(5\pi t) + \left(1/5\right)\sin z_1 + 2z_1 z_2 + 0.1u$.

The control objective of the first illustrative example is to design a controller in such a way that the state trajectories (z_1, z_2) of the chaotic system (40) are convergent to zero in a finite time. The coefficients of the proposed sliding manifold are selected as $Q = \left[0.52\ 1 \right]$, $\Lambda = 5.2$, $\alpha = 0.85$, and $\Gamma = 8.0$. The parameters of controller and adaptive law are chosen as $k_1 = 15,\ k_2 = 0.1,\ k_3 = 0.1, \delta = 0.85$, $\hat{\kappa}(0) = 1.0$. The simulation results of the controlled Duffing – Holmes system are exhibited in Figures 3-7.

The presented controller is applied to stabilize the chaotic behavior right after $t \geq 0$. For easy understanding, we only exhibit the simulation results in the short time interval $t \geq 0$ and $t \leq 7s$. As shown in Figures 3 and 4, the responses of $z_1,\ z_2$ powerfully converge to zero compared with the other methods such as TSMC-LMI and SMC-NDO even though the upper bound of uncertainty, $d(z,u,t)$, is not known in advance. The chattering effect is efficiently eliminated as shown in Figure 5. The trajectory of sliding surface of the proposed method rapidly converges to zero without any significant oscillations in comparison with the controllers

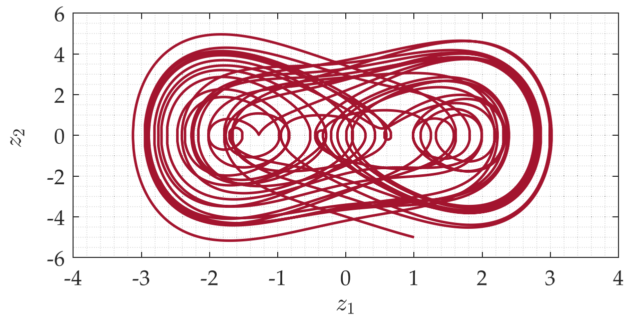

FIGURE 1. Phase portrait of the Duffing – Holmes system.

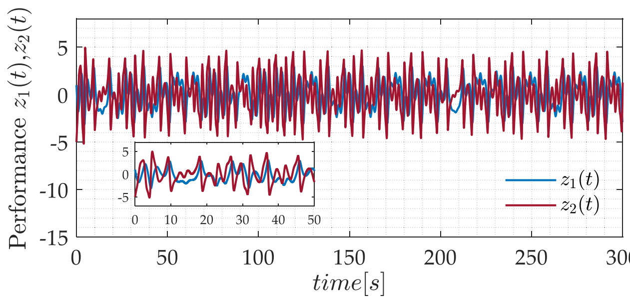

FIGURE 2. State trajectories $z_1(t)$, $z_2(t)$ of the uncontrolled Duffing – Holmes system.

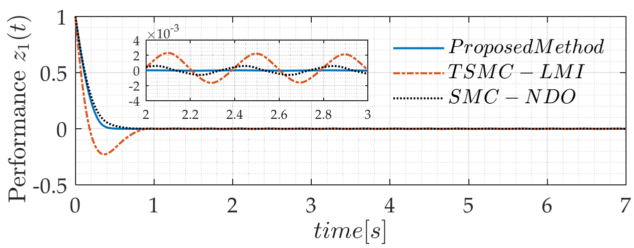

FIGURE 3. Performance $z_1(t)$ of the controlled Duffing – Holmes system compared with *TSMC-LMI* and *SMC-NDO*.

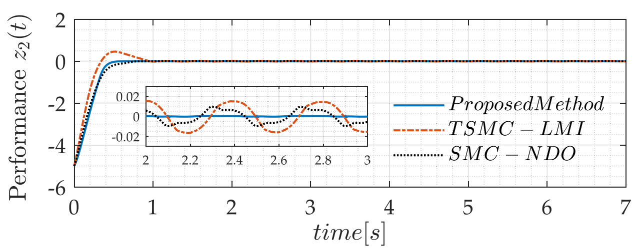

FIGURE 4. Performance $z_2(t)$ of the controlled Duffing – Holmes system compared with *TSMC-LMI* and *SMC-NDO*.

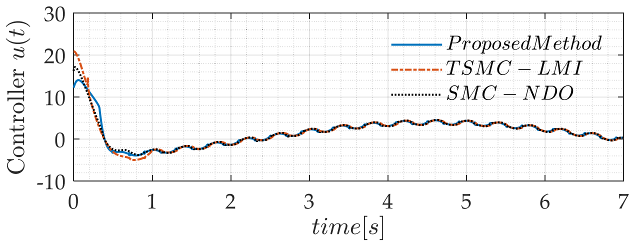

FIGURE 5. Controller $u(t)$ for the controlled Duffing – Holmes system.

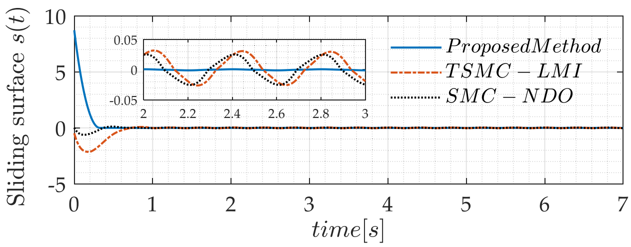

FIGURE 6. Sliding surface $s(t)$ to control the Duffing – Holmes system.

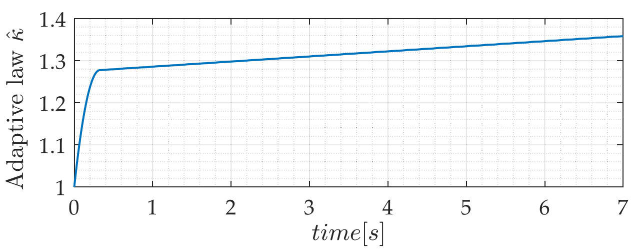

FIGURE 7. Adaptive law $\hat{\kappa}$ to control the Duffing – Holmes system.

of TSMC-LMI and SMC-NDO as exhibited in Figure 6. The adaptive gain $\hat{\kappa}$ is automatically adjusted to guarantee a strong convergence of the system states to zero as shown in Figure 7. This example demonstrated that the proposed algorithm is more effective control performance than the methods of TSMC-LMI and SMC-NDO in controlling the SISO system.

B. EXAMPLE 2 (DISTURBED MIMO NONLINEAR SYSTEM)

Next, the proposed controller is also applied to stabilize a disturbed MIMO nonlinear system. A third order of Lorenz model [39] is considered. The system model is given as follows:

$$\begin{aligned} \dot{z}_1 &= -az_1 + az_2 \\ \dot{z}_2 &= bz_1 - z_2 - z_1 z_3 \\ \dot{z}_3 &= -cz_3 + z_1 z_2 \end{aligned} \tag{41}$$

where a and b denote the Prandtl and Rayleigh numbers, respectively, and c is physical dimension factor. The system (41) can be re-written under the matrix form

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} -a & a & 0 \\ b & -1 & 0 \\ 0 & 0 & -c \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -z_1 z_3 \\ z_1 z_2 \end{bmatrix} \tag{42}$$

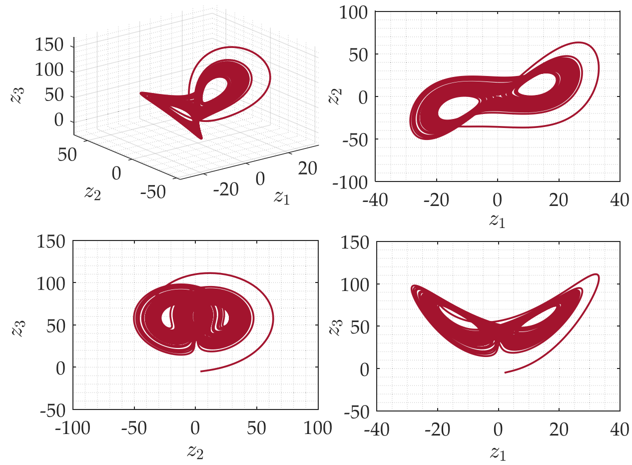

FIGURE 8. Phase portrait of the uncontrolled Lorenz system.

The system parameters are chosen as $a = 8$, $b = 60$, and $c = 10/3$. The initial states of the chaotic system are given by $z_1(0) = 2$, $z_2(0) = 4$ and $z_3(0) = -5$. The complex and chaotic motion of an uncontrolled Lorenz system can be easily described by the phase portraits (Figure 8), and state trajectories of z_1, z_2, z_3 (Figure 9). In order to demonstrate the effectiveness of the proposed method with a disturbed MIMO nonlinear system, the control inputs are added to the system (42) according to the presented regular form in Eq.(5). The system matrices and uncertainties are given as follows

$$\begin{aligned} A^* &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} -a & a & 0 \\ b & -1 & 0 \\ 0 & 0 & -c \end{bmatrix} \\ &= \begin{bmatrix} -8 & 8 & 0 \\ 60 & -1 & 0 \\ 0 & 0 & -10/3 \end{bmatrix}, \\ A_{11} &= -8, A_{12} = \begin{bmatrix} 8 & 0 \end{bmatrix}, \textit{and } B^* = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}; \\ f(z,t) &= \begin{bmatrix} -z_1 z_3 \\ z_1 z_2 \end{bmatrix} \end{aligned}$$

and the influences of uncertainties and external perturbations, $d(z, u, t)$, on the Lorenz system are given by

$$\begin{aligned} &d(z,u,t) \\ &= \|s\|^{1/2} \begin{bmatrix} \left(\frac{1}{2}z_1^2 + \begin{bmatrix} \sin(2t) & 0.2 & 1 \end{bmatrix} z + \frac{t}{6} + \sin^2 2t\right) \\ \frac{1}{2}z_1^2 + \frac{3}{10}\cos z_1 + \frac{1}{2}z_2 z_3 \end{bmatrix} \\ &\quad + \begin{bmatrix} 0.1 & 0.5 \\ 0 & 0.7 \end{bmatrix} u \end{aligned}$$

The coefficients of a sliding manifold are selected as follows: $Q = \begin{bmatrix} 1.05 & 1 & 0 \\ -0.25 & 0 & 1 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}$, $\Gamma = \begin{bmatrix} 0.02 \\ 0.2 \end{bmatrix}$, $\alpha = 0.75$.

The other parameters are selected as $k_1 = 2.1$, $k_2 = 0.2$, $k_3 = 10$, $\delta = 0.15$, and $\hat{\kappa}(0) = 1.0$. The presented controller is applied to stabilize the chaotic system right after $t \geq 0$.

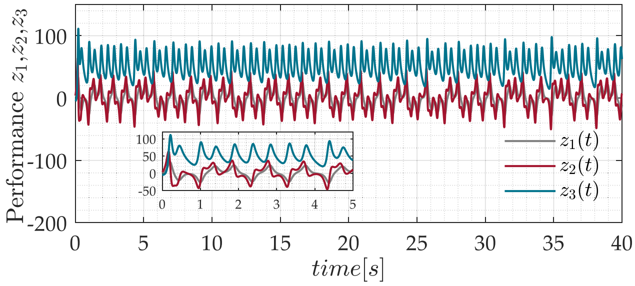

FIGURE 9. State trajectories $z_1(t)$, $z_2(t)$, and $z_3(t)$ of the uncontrolled Lorenz system.

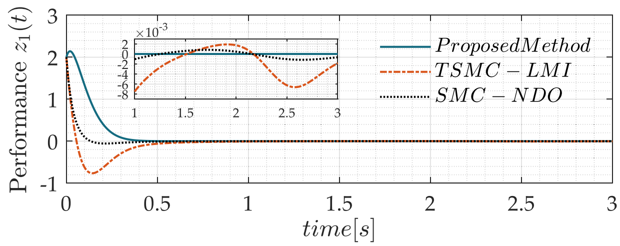

FIGURE 10. Performance $z_1(t)$ of the controlled Lorenz system compared with *TSMC-LMI* and *SMC-NDO*.

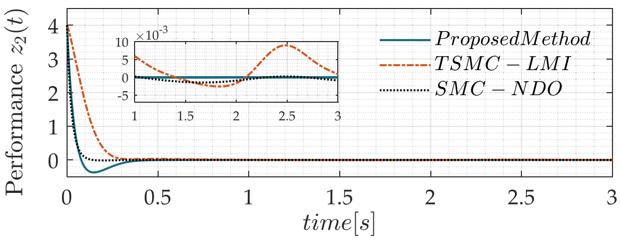

FIGURE 11. Performance $z_2(t)$ of the controlled Lorenz system compared with *TSMC-LMI* and *SMC-NDO*.

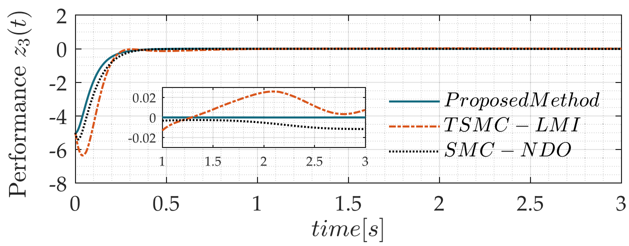

FIGURE 12. Performance $z_3(t)$ of the controlled Lorenz system compared with *TSMC-LMI* and *SMC-NDO*.

For easy understanding, we exhibit the results of simulation in the short time interval $t \in [0, 3]$sec. The simulation results of the controlled Lorenz system are exhibited in Figures 10-17.

As shown in Figures 10-12, the state trajectories of z_1, z_2, and z_3 are excellently convergent to zero compared with the other methods of TSMC-LMI and SMC-NDO even though the upper bound of uncertainty and exogeneous disturbance, $d(z, u, t)$, is not known in advance. The proposed controllers u_1 and u_2 strongly remove the chattering effect as shown

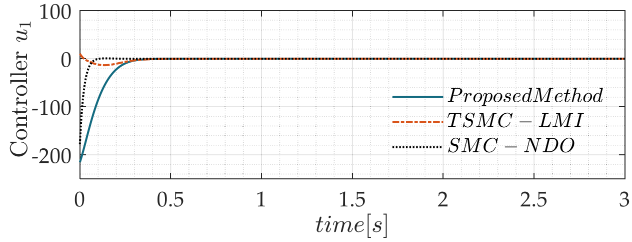

FIGURE 13. Controller $u_1(t)$ for Lorenz system.

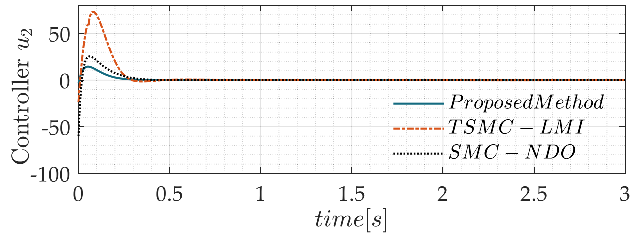

FIGURE 14. Controller $u_2(t)$ for Lorenz system.

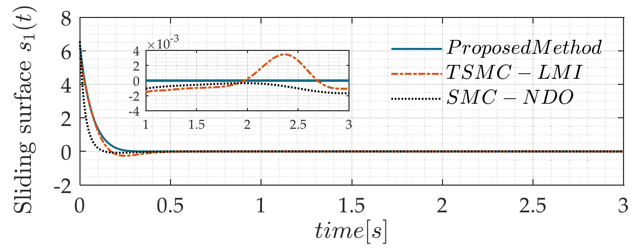

FIGURE 15. Sliding surface $s_1(t)$ for the Lorenz system.

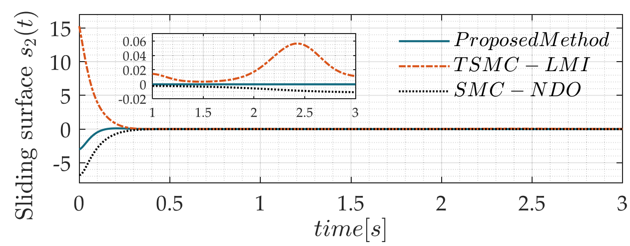

FIGURE 16. Sliding surface $s_2(t)$ for the Lorenz system.

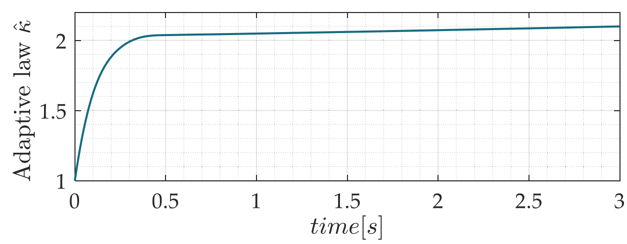

FIGURE 17. Adaptive law $\hat{\kappa}$ for the Lorenz system.

in Figures 13-14. The trajectories of sliding surface s_1, s_2 of the proposed method rapidly and accurately converge to zero without any significant oscillations compared with the algorithms of TSMC-LMI and SMC-NDO as exhibited in Figures 15 and 16. The self-adjustment of an adaptive gain $\hat{\kappa}$ (Figure 17) guaranteed an excellent convergence of the state trajectories to zero. This example confirmed that the control performance of the disturbed MIMO nonlinear systems is significantly improved by using the proposed method.

V. CONCLUSION

In this study, we introduced a robust adaptive sliding control method for stabilization problem of the multiple inputs and multiple outputs nonlinear systems influenced by uncertainties and external disturbances. A novel sliding manifold and an adaptive law are proposed to guarantee a strict stability and fast convergence of the state trajectories to zero in a finite-time without any awareness of the upper bound condition of uncertainties and external perturbations. A general stability of the closed-loop disturbed MINO nonlinear system is demonstrated through the Lyapunov theorem. The effectiveness and advantages of the introduced method are verified and confirmed through the numerical simulations compared with the existent methods. From the above results, the stabilization problem of a disturbed MIMO systems is excellently resolved by the proposed algorithm. However, the research did not consider the time delays of the control system. Thus, this topic will be conducted in the future study.

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HA LE NHU NGOC THANH received the B.S. and M.S. degrees from the HCMC University of Technology and Education, Vietnam, in 2011 and 2015, respectively, both in mechanical engineering, and the Ph.D. degree in mechanical and aerospace engineering from Sejong University, Seoul, South Korea, in 2019. From 2019 to 2020, he was an Assistant Professor with the Faculty of Intelligent Mechatronics Engineering, Sejong University. He is currently a Lecturer with the Vietnam–Korea Institute of Technology, and the HUTECH Institute of Engineering, Ho Chi Minh City University of Technology (HUTECH), Vietnam. His research interests include nonlinear control, mechatronics, and robotics. He received an Outstanding Research Award from Sejong University, in 2019. He also serves as a Guest Editor for *Sensors* journal (MDPI) and *Applied Sciences* journal (MDPI).

MAI THE VU received the B.S. degree in naval architecture and marine engineering from the Ho Chi Minh City University of Technology, Vietnam, in 2013, and the Ph.D. degree from the Department of Convergence Study on the Ocean Science and Technology, Korea Maritime and Ocean University, South Korea, in 2019. He is currently an Assistant Professor with the Department of Unmanned Vehicle Engineering, Sejong University, South Korea. His research interests include underwater vehicles and robotics, multi-body dynamic modeling, linear and nonlinear control, trajectory tracking, path planning, obstacle avoidance, and intelligent navigation.

NGOC PHI NGUYEN received the B.S. degree in mechatronics engineering from the HCMC University of Technology and Education, in 2012, the M.S. degree in mechatronics engineering from Vietnamese-German University, Vietnam, in 2015, and the Ph.D. degree in aerospace engineering from Sejong University, Seoul, South Korea, in 2020. He is currently an Assistant Professor with the Department of Aerospace Engineering, Sejong University. His research interests include fault-tolerant control, nonlinear control, intelligent control, and formation control.

NGUYEN XUAN MUNG (Member, IEEE) received the B.S. degree in mechatronics from the Hanoi University of Science and Technology, Hanoi, Vietnam, in 2014, and the M.S. and Ph.D. degrees from Sejong University, Seoul, South Korea, in 2017 and 2021, respectively, both in aerospace engineering. From 2014 to 2015, he was with Samsung Electronics Vietnam Company Ltd., Vietnam. He is currently an Assistant Professor with the Department of Aerospace Engineering, Sejong University. His research interests include unmanned aerial vehicles, control systems, automation, and robotics.

SUNG KYUNG HONG received the B.S. and M.S. degrees from Yonsei University, Seoul, South Korea, in 1987 and 1989, respectively, both in mechanical engineering, and the Ph.D. degree from Texas A&M University, College Station, in 1998. From 1989 to 2000, he was with the Unmanned Aerial Vehicle System Division and the Flight Dynamics and Control Laboratory of the Agency for Defense Development (ADD), South Korea. He is currently a Full Professor with the Department of Aerospace Engineering, Sejong University, South Korea. His research interests include fuzzy logic controls, inertial sensor applications, and flight control systems.