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A Novel Delay-Product-Type Functional Method to Extended Dissipativity Analysis for Markovian Jump Neural Networks

XIAOPING HUANG[1], CAIYUN WU[2], YUZHONG WANG[3], AND WENDONG LI[2]

[1]School of Computer Science, Northwestern Polytechnical University, Xi'an 710072, China
[2]School of Equipment Engineering, Shenyang Ligong University, Shenyang 110159, China
[3]School of Computer Science and Engineering, Northeastern University, Shenyang 110819, China

Corresponding author: Caiyun Wu (wu_cai_yun@126.com)

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ABSTRACT This paper studies the problem of extended dissipativity analysis for Markovian jump neural networks (MJNNs) with time-varying delay. Combining Wirtinger-based double integral inequality and S-procedure lemma, a novel double integral-based delay-product-type (DIDPT) Lyapunov functional is constructed in this paper, which avoids the incomplete components in the existing works. Then, based on parameter-dependent reciprocally convex inequality (PDRCI) and the novel DIDPT, a new extended dissipativity condition is obtained for MJNNs. A numerical example is employed to illustrate the advantages of the proposed method.

INDEX TERMS Markovian jump neural networks, extended dissipativity, time-varying delay, delay-product-type functional, S-procedure lemma.

I. INTRODUCTION

From the modeling of the biological brain, the concept of neural networks (NNs) is proposed, which have been successfully used in various areas, such as signal processing, associative memories, and pattern recognition [1]–[8]. As the prerequisite in many applications, stability is of great importance [9]–[16]. In the implementation of (large scale) neural networks, since finite switching speed of amplifiers and communication speed between neurons, the presence of time delays is unavoidable, which may cause a stable neural network oscillated [17]–[20]. Thus, it is a meaningful topic for the dynamic performance analysis for NNs with time delay [21]–[23].

On the other hand, Markov jump systems (MJSs), experience abrupt changes in their parameters and structure resulting from abrupt environmental disturbances, component failures or repairs, modifications of the operating point, which have been an attractive research topic where a large amount of theoretical results on MJSs have been reported to deal with a variety of problems [24]–[27]. Meanwhile, a large number of NNs may experience abrupt parameter changes in their structures caused by sudden environment changes, component or interconnection failures and so on. NNs under this situation usually have finite modes which may switch (or jump) from one to another in a random way. Such jumping between different modes can be determined by a Markov chain, and these kinds of NNs are known as Markovian jump neural networks (MJNNs) [28]. Very recently, by multiplying time-varying delay-dependent matrices with some nonintegral terms in [29], a new method named as delay-product-type (DPT) functional method is proposed to reduce the conservatism, which has been used for NNs or MJNNs in [30]–[32]. However, the introduced matrices are positive definite since the positive definiteness of LKF should be ensured. In this case, the negative definiteness of the final conditions is constrained. In order to avoid the issue, by introducing integral terms connected with the nonintegral quadratic terms, two novel DPT functionals are constructed for the extended dissipativity analysis for MJNNs in [33], respectively. However, in [33], only the single integral-based DPT Lyapunov functional is constructed. As a result, a double integral-based DPT (DIDPT) functional is proposed in [34], in which the double integrals are introduced in augmented forms. It should be pointed out that the lower and upper bounds of the DIDPT are time-varying so as to more coupling

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system information can be utilized, which is an efficient way to capture the time-varying information. However, the time-varying delay-dependent matrices connected with non-integral terms are incomplete with some zero components such that the relationships on system information can not be fully utilized. Thus, solving the insufficiency motivates current study.

In addition, to estimate $-\int_{t-h}^{t}\dot{x}^T(s)R\dot{x}(s)ds$, the time-varying delay in the denominators is commonly treated through the traditional reciprocally convex inequality (TRCI), which is proposed in [35]. Then, an improved version of TRCI is proposed in [36], [37], which is named improved reciprocally convex inequality (IRCI). Further, by introducing two independent parameters, a parameter-dependent RCI (PDRCI) is proposed in [38], which covers those [36], [37] as special cases, which leads to less conservative results without additional decision variables.

With above analysis, this paper is concerned with the extended dissipativity analysis for MJNNs with time delay. By using Wirtinger-based double integral inequality and S-procedure lemma, a novel double integral-based DPT (DIDPT) functional is constructed, which avoids the zero components in [34]. In this case, the coupling information among states and time varying can be fully utilized. Based on PDRCI and the novel DIDPT functional, a new condition is proposed to ensure MJNNs to be stochastically stable and extended dissipative. A numerical example is illustrated to demonstrate the proposed method.

Notation: Throughout this paper, $\mathbb{R}^n$ represents the n-dimensional Euclidean space; T and $\perp$ denote the superscripts of transpose and the right orthogonal complement of a matrix; '*' in LMIs represents the symmetric term of the matrix, respectively; $col[X, Y]$ denotes $[X^T, Y^T]^T$; $diag\{\ldots\}$ represents a block diagonal matrix; $He[X]$ means $X + X^T$; $Co\{a_1, a_2, a_3, a_4\}$ stands for a polytope generated by these four vertices; The superscripts T and -1 stand for transpose and inverse of a matrix, respectively; $P > 0(\geq 0)$ means that P is a positive-definite (semi-positive-definite) matrix; I and 0 denote the identity matrix and zero matrix with compatible dimensions, respectively. $\mathcal{L}_2[0, \infty)$ refers to the space of square-integrable vector functions over $[0, \infty)$.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the following MJNNs with time-varying delay:

$$\begin{cases}\dot{x}(t) = -A(r_t)x(t) + W_0(r_t)g(W_2x(t)) \\ \qquad +W_1(r_t)g(W_2x(t-d(t))) + B_1w(t) \\ y(t) = Cx(t) + D_1x(t-d(t)) + D_2g(W_2x(t)) \\ \qquad +B_2w(t)\end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $w(t) \in \mathbb{R}^w$ is the disturbance input, which belongs to $\mathcal{L}_2[0, +\infty)$; $y(t) \in \mathbb{R}^y$ is the output; $A(r_t)$, $W_0(r_t)$, $W_1(r_t)$, B_1, B_2, C, D_1, D_2, and $W_2 = col[W_{21}, W_{22}, \cdots, W_{2n}]$ are the known interconnection weight matrices. $r_t(t \geq 0)$ is a right continuous Markov chain having values in a finite space $S = \{1, 2, \ldots, m\}$ with TRM $\Pi \triangleq [\pi_{ij}]$ given by

$$Pr\{r_{t+\delta} = j|r_t = i\} = \begin{cases}\pi_{ij}\delta + o(\delta), & j \neq i \\ 1 + \pi_{ii}\delta + o(\delta), & j = i\end{cases} \tag{2}$$

where $\delta > 0$, $\lim_{\delta\to 0} o(\delta)/\delta = 0$, $\pi_{ij} \geq 0$, and $\pi_{ii} = -\sum_{j=1,j\neq i}^{m}\pi_{ij}(j \neq i)$.

The time-varying delay $d(t)$ satisfies

$$0 \leq d(t) \leq h, \quad -\mu \leq \dot{d}(t) \leq \mu \tag{3}$$

where h and μ are constants. $g(W_2(x(t)))$ is the neuron activation function and satisfies

$$k_l^- \leq \frac{g_l(a_1) - g_l(a_2)}{a_1 - a_2} \leq k_l^+, a_1 \neq a_2 \tag{4}$$

$$k_l^- \leq \frac{g_l(a)}{a} \leq k_l^+, \quad l = 1, 2, \cdots, n \tag{5}$$

where k_l^- and k_l^+ are constants.

In this paper, we will propose a sufficient condition with less conservativeness to ensure the MJNNs (1) to be stochastically stable and extended dissipative. To this end, the preconditions are given as follows.

Assumption 1: [24], [25] Matrices Ψ_1, Ψ_2, Ψ_3, and Ψ_4 satisfy the following conditions

$$\Psi_1 = \Psi_1^T \leq 0, \quad \Psi_3 = \Psi_3^T, \quad \Psi_4 = \Psi_4^T \tag{6}$$

$$B_2^T\Psi_1B_2 + He[B_2^T\Psi_2] + \Psi_3 > 0 \tag{7}$$

$$(||\Psi_1|| + ||\Psi_2||) \cdot ||\Psi_4|| = 0. \tag{8}$$

Definition 1: [24], [25] For prescribed matrices Ψ_1, Ψ_2, Ψ_3, and Ψ_4 satisfying Assumption 1, MJNNs (1) are said to be extended dissipative, if there exists a scalar ϱ such that the following inequality holds for all nonzero $w(t) \in \mathcal{L}_2[0, +\infty)$ and any $T \geq t$

$$\int_0^T (y^T(t)\Psi_1y(t) + 2y^T(t)\Psi_2w(t) + w^T(t)\Psi_3w(t))dt \geq y^T(t)\Psi_4y(t) + \varrho, \tag{9}$$

Lemma 1: (S-Procedure Lemma) [39] Denote the set $Z = \{z\}$ and let $F(z)$, $Y_1(z)$, $Y_2(z)$,..., $Y_k(z)$ be some functionals or functions. Definite domain D as

$$D = \{z \in Z : Y_1(z) \geq 0, Y_2(z) \geq 0, \ldots, Y_k(z) \geq 0\}$$

and the two following conditions

(I) $F(z) \geq 0, \forall z \in D$,

(II) $\exists\sigma_1 \geq 0, \sigma_2 \geq 0, \ldots, \sigma_k \geq 0$ such that

$$S(\sigma, z) = F(z) - \sum_{j=1}^{k}\sigma_jY_j(z) \geq 0, \forall z \in Z.$$

Then (II) implies (I).

Lemma 2: (PDRIC) [38] For real scalars $\beta \in (0, 1)$, $\kappa_1 \geq 1$, $\kappa_2 \geq 1$, real symmetric positive definite matrices

$R_1, R_2 \in \mathbb{R}^{m\times m}$, if there exist real matrices $S \in \mathbb{R}^{m\times m}$, then the following matrix inequality holds

$$\begin{bmatrix} \frac{1}{\beta}R_1 & 0 \\ 0 & \frac{1}{1-\beta}R_2 \end{bmatrix} \geq \begin{bmatrix} R_1 + (1-\beta)\kappa_1 T_1 & S \\ * & R_2 + \beta\kappa_2 T_2 \end{bmatrix} \quad (10)$$

where $T_1 = R_1 - SR_2^{-1}S^T$, $T_2 = R_2 - S^T R_1^{-1} S$.

For simplicity, the following notations are used:

$$\begin{aligned}
h_d(t) &= h - d(t) \\
v_1(t) &= \frac{1}{d(t)} \int_{t-d(t)}^{t} x(s)ds \\
v_2(t) &= \frac{1}{h_d(t)} \int_{t-h}^{t-d(t)} x(s)ds \\
v_3(t) &= \frac{1}{d^2(t)} \int_{-d(t)}^{0} \int_{t+\theta}^{t} x(s)dsd\theta \\
v_4(t) &= \frac{1}{h_d^2(t)} \int_{-h}^{-d(t)} \int_{t+\theta}^{t-d(t)} x(s)dsd\theta \\
\xi_1(t) &= col[x(t), g(W_2 x(t)), \dot{x}(s)] \\
\xi_2(t) &= col[x(t), x(t-d(t)), v_1(t)] \\
\xi_3(t) &= col[x(t-d(t)), x(t-h), v_2(t)] \\
\xi_4(t) &= col[x(t), v_1(t), v_3(t)] \\
\xi_5(t) &= col[x(t-d(t)), v_2(t), v_4(t)] \\
\eta_1(t) &= [x(t), x(t-d(t)), x(t-h)] \\
\eta_2(t) &= [g(W_2x(t)), g(W_2x(t-d(t))), g(W_2x(t-h))] \\
\eta_3(t) &= [v_1(t), v_2(t), v_3(t), v_4(t)] \\
\eta_4(t) &= [\dot{x}(t), \dot{x}(t-d(t)), \dot{x}(t-h)] \\
\eta(t) &= col[\eta_1(t), \eta_2(t), \eta_3(t), \eta_4(t)].
\end{aligned}$$

III. MAIN RESULTS

This section focuses on a sufficient criterion such that the system (1) is stochastically stable and extended dissipative.

Firstly, recall the double integral-based delay-product-type (DIDPT) functional $V^*(x_t)$ in [34], that is

$$V^*(x_t) = V_G(x_t) + V_F(x_t) \quad (11)$$

where

$$\begin{aligned}
V_G(x_t) &= \frac{d(t)}{2} \int_{-d(t)}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) G_1 \dot{x}(s) dsd\theta \\
&\quad + \frac{h_d(t)}{2} \int_{-h}^{-d(t)} \int_{t+\theta}^{t-d(t)} \dot{x}^T(s) G_2 \dot{x}(s) dsd\theta \\
V_F(x_t) &= -\frac{d(t)}{h^2} \xi_4^T(t) F_1 \xi_4(t) - \frac{h_d(t)}{h^2} \xi_5^T(t) F_2 \xi_5(t) \\
F_k &= \begin{bmatrix} \frac{3}{2}G_k & 0 & -3G_k \\ * & 3G_k & -6G_k \\ * & * & 18G_k \end{bmatrix}, \quad k = 1, 2
\end{aligned}$$

and G_1, G_2 are the positive definite matrices with appropriate dimensions. Although the DIDPT functional constructed in [34] can increase the information of time-varying delay, the coupling information has not been fully considered. Note that there are some zero components in F_k. In this case, the coupling information among $x(t)$, $x(t-d(t))$, $v_1(t)$, and $v_2(t)$ lacks. Because of $F_k \geq 0$, in the following, for semi-positive definite matrices $Y_k = \begin{bmatrix} Y_{11k} & Y_{12k} \\ * & Y_{22k} \end{bmatrix}$, based on S-procedure lemma, if there exist scalars $\sigma_k \geq 0$, F_k can be deduced by

$$\begin{aligned}
Z_k &= F_k - \sigma_k \begin{bmatrix} Y_k & 0 \\ 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} \frac{3}{2}G_k - \sigma_k Y_{11k} & -\sigma_k Y_{12k} & -3G_k \\ * & 3G_k - \sigma_k Y_{22k} & -6G_k \\ * & * & 18G_k \end{bmatrix} \geq 0. \quad (12)
\end{aligned}$$

Then, we obtain

$$\begin{aligned}
&\frac{d(t)}{2} \int_{-d(t)}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) G_1 \dot{x}(s) dsd\theta \\
&\quad \geq \frac{1}{d(t)} \xi_4^T(t) Z_1 \xi_4(t) \geq \frac{d(t)}{h^2} \xi_4^T(t) Z_1 \xi_4(t) \\
&\quad \frac{h_d(t)}{2} \int_{-h}^{-d(t)} \int_{t+\theta}^{t-d(t)} \dot{x}^T(s) G_2 \dot{x}(s) dsd\theta \\
&\quad \geq \frac{1}{h_d(t)} \xi_5^T(t) Z_2 \xi_5(t) \geq \frac{h_d(t)}{h^2} \xi_5^T(t) Z_1 \xi_5(t).
\end{aligned}$$

Further, the following double integral-based DPT (DIDPT) functional can be constructed.

Proposition 1: For MJNNs (1) subject to (3), given symmetric positive matrices G_1, G_2, symmetric semi-positive matrices Y_1, Y_2 and Z_1, Z_2 satisfying (12), the following functional is positive definite

$$V_0(x_t) = V_G(x_t) + V_Z(x_t) \quad (13)$$

where

$$\begin{aligned}
V_G(x_t) &= \frac{d(t)}{2} \int_{-d(t)}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) G_1 \dot{x}(s) dsd\theta \\
&\quad + \frac{h_d(t)}{2} \int_{-h}^{-d(t)} \int_{t+\theta}^{t-d(t)} \dot{x}^T(s) G_2 \dot{x}(s) dsd\theta \\
V_Z(x_t) &= -\frac{d(t)}{h^2} \xi_4^T(t) Z_1 \xi_4(t) - \frac{h_d(t)}{h^2} \xi_5^T(t) Z_2 \xi_5(t)
\end{aligned}$$

Remark 1: In Proposition 1, a new DIDPT Lyapunov functional is constructed, which has the following advantages. (i) Compared with the SIDPT Lyapunov functionals in [28], [30], [33], the novel DIDPT functional (13) further strengthens the links among the vectors $\dot{d}(t)$, $v_3(t)$, and $v_4(t)$. (ii) By using the Wirtinger-based double integral inequality (WDII) [18] and S-procedure lemma, $V_Z(x_t)$ is connected to $V_G(x_t)$, which can avoid additional matrices (see (13) for details). Moreover, the nonintegral terms are negative positiveness, which relax the positive definite requirements of the matrices to be solved in the functional. Thus, the functional (13) is not only effective for conservatism reducing but also effective for calculation complexity reducing. (iii) Compared with the DIDPT functional in [34], the zero components have been avoided in case that extra information among $x(t)$, $x(t-d(t))$, $v_1(t)$ and $v_2(t)$ can be fully coupled. As a result, the novel DIDPT (13) can lead to less conservative results.

Based on the above-mentioned proposition and Lemma 2, the sufficient condition is provided in the following theorem, under which MJNN (1) is stochastically stable and extended dissipative.

Theorem 1: For given scalars $\alpha \in (0, 1)$, h, and μ, $\sigma_k \geq 0$, $\kappa_k \geq 1$, matrices Ψ_1, Ψ_2, Ψ_3, Ψ_4 satisfying Assumption 1, MJNNs (1) are stochastically stable and extended dissipative if there exist symmetric positive-definite matrices $P_i \in \mathbb{R}^{3n\times 3n}$, $Q_k \in \mathbb{R}^{3n\times 3n}$, R_k, T_k, M_k, G_k, $L_i \in \mathbb{R}^{n\times n}$, $N_i \in \mathbb{R}^{n_w\times n_w}$, symmetric semi-positive definite matrices Y_{11k}, Y_{22k}, positive definite diagonal matrices Λ_c, Δ_c, $V_k = diag\{v_{k1}, v_{k2}, \cdots, v_{kn}\} \in \mathbb{R}^{3n\times 3n}$, and any matrices Y_{12k} such that the following inequalities hold for all $(d(t), \dot{d}(t)) \in Co\{(0, 0), (0, \mu), (h, 0), (h, -\mu)\}$, $i \in S$, $c = 1, 2, 3$, and $k = 1, 2$:

$$(1-\mu)R_1 + hT_1 - \mu M_1 > 0 \quad (14)$$

$$R_2 - \mu M_2 > 0, T - \frac{\mu}{2}G_1 > 0, T - \frac{\mu}{2}G_2 > 0 \quad (15)$$

$$\begin{bmatrix} (\wp_i^{\perp})^T\Omega(0,0)\wp_i^{\perp} & \sqrt{\kappa_1}(\wp_i^{\perp})^T\Pi_{14}^T S \\ * & -h\Re_2(0,0) \end{bmatrix} < 0 \quad (16)$$

$$\begin{bmatrix} (\wp_i^{\perp})^T\Omega(0,\mu)\wp_i^{\perp} & \sqrt{\kappa_1}(\wp_i^{\perp})^T\Pi_{14}^T S \\ * & -h\Re_2(0,\mu) \end{bmatrix} < 0 \quad (17)$$

$$\begin{bmatrix} (\wp_i^{\perp})^T\Omega(h,0)\wp_i^{\perp} & \sqrt{\kappa_2}(\wp_i^{\perp})^T\Pi_{15}^T S^T \\ * & -h\Re_2(h,0) \end{bmatrix} < 0 \quad (18)$$

$$\begin{bmatrix} (\wp_i^{\perp})^T\Omega(h,-\mu)\wp_i^{\perp} & \sqrt{\kappa_2}(\wp_i^{\perp})^T\Pi_{15}^T S^T \\ * & -h\Re_2(h,-\mu) \end{bmatrix} < 0 \quad (19)$$

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & -C^T\Psi_4 D_2 & -C^T\Psi_4 B_2 \\ * & \Gamma_{22} & -D_1^T\Psi_4 D_2 & -D_1^T\Psi_4 B_2 \\ * & * & \Gamma_{33} & -D_2^T\Psi_4 B_2 \\ * & * & * & \Gamma_{44} \end{bmatrix} > 0 \quad (20)$$

where

$$\Omega(d,\dot{d}) = \sum_{l=1}^{7}\Omega_l, d = d(t), h_d = h - d(t)$$

$$\Omega_1 = \Pi_1^T\sum_{j\in S}\pi_{ij}P_j\Pi_1 + He[\Pi_1^T P_i\Pi_2]$$

$$\Omega_2 = \Pi_3^T Q_1\Pi_3 - \Pi_5^T Q_2\Pi_5 - (1-\dot{d})\Pi_4^T(Q_1 - Q_2)\Pi_4$$

$$\Omega_3 = He[e_4^T(V_1 - V_2)W_2 e_{11} + e_1^T W_2^T(K_2V_2 - K_1V_1)W_2 e_{11}]$$

$$\begin{aligned}\Omega_4 = {} & e_{11}^T\Big(d(R_1+M_1) + \frac{h^2}{2}T + \frac{dh}{2}G_1\Big)e_{11} \\ & + e_{12}^T\Big((1-\dot{d})(h_d M_2 - dM_1) \\ & + h_d(1-\dot{d})R_2 + h(1-\dot{d})\frac{h_d}{2}G_2)\Big)e_{12} \\ & - h_d e_{13}^T M_2 e_{13} - \frac{\dot{d}}{h}\Big(\Pi_6^T U_1\Pi_6 - \Pi_7^T U_2\Pi_7\Big) \\ & - \frac{1}{h}He[\Pi_6^T U_1\Pi_8 + \Pi_7^T U_2\Pi_9] \\ & - \frac{\dot{d}}{h^2}(\Pi_{10}^T Z_1\Pi_{10} - \Pi_{11}^T Z_2\Pi_{11}) \\ & - \frac{1}{h^2}He[\Pi_{10}^T Z_1\Pi_{12} + \Pi_{11}^T Z_2\Pi_{13}]\end{aligned}$$

$$\Omega_5 = -\frac{1}{h}\Pi_{16}^T\Re\Pi_{16} - \Pi_{17}^T\mathfrak{T}_1(d,\dot{d})\Pi_{17} - \Pi_{18}^T\mathfrak{T}_2(d,\dot{d})\Pi_{18}$$

$$\begin{aligned}\Omega_6 = {} & \sum_{l=1}^{3}He[(e_{l+3} - K_1W_2e_l)^T\Lambda_l(K_2W_2e_l - e_{l+3})] \\ & + \sum_{l=1}^{2}He\Big[\Big((e_{l+3} - e_{l+4}) - K_1W_2(e_l - e_{l+1})\Big)^T \\ & \times\Delta_l\Big(K_2W_2(e_l - e_{l+1}) - (e_{l+3} - e_{l+4})\Big)\Big] \\ & + He\Big[\Big((e_4 - e_6) - K_1W_2(e_1 - e_3)\Big)^T \\ & \times\Delta_3\Big(K_1W_2(e_1 - e_3) - (e_4 - e_6)\Big)\Big]\end{aligned}$$

$$\Omega_7 = -e_0^T\Psi_1 e_0 - He[e_0^T\Psi_2 e_{14}] - e_{14}^T\Psi_3 e_{14}$$

$$\Re = \begin{bmatrix} (1 + \frac{(h-d)\kappa_1}{h})\Re_1(d,\dot{d}) & S \\ * & (1 + \frac{d\kappa_2}{h})\Re_2(d,\dot{d}) \end{bmatrix}$$

$$\Re_1(d,\dot{d}) = diag\{\Re_1^0(t), 3\Re_1^0(t)\}$$

$$\Re_2(d,\dot{d}) = diag\{\Re_2^0(t), 3\Re_2^0(t)\}$$

$$\mathfrak{T}_1(d,\dot{d}) = diag\{2\mathfrak{T}_1^0, 4\mathfrak{T}_1^0\}, \mathfrak{T}_2(d,\dot{d}) = diag\{2\mathfrak{T}_2^0, 4\mathfrak{T}_2^0\}$$

$$\Re_1^0(t) = (1-\dot{d})R_1 + h_d T + \frac{(1-\dot{d})d}{2}G_1 - \dot{d}M_1$$

$$\Re_2^0(t) = R_2 + \dot{d}M_2 + \frac{h_d}{2}G_2$$

$$\mathfrak{T}_1^0(t) = T - \frac{\dot{d}}{2}G_1, \mathfrak{T}_2^0(t) = T + \frac{\dot{d}}{2}G_2$$

$$K_1 = diag\{k_1^-, k_2^-, \cdots, k_n^-\}$$

$$K_2 = diag\{k_1^+, k_2^+, \cdots, k_n^+\}$$

$$P_{1i} = [I, 0, 0]P_i[I, 0, 0]^T$$

$$\Gamma_{11} = \alpha P_{1i} - C^T\Psi_4 C, \Gamma_{12} = -C^T\Psi_4 D_1$$

$$\Gamma_{22} = (1-\alpha)P_{1i} - D_1^T\Psi_4 D_1$$

$$\Gamma_{33} = L_i - D_2^T\Psi_4 D_2$$

$$\Gamma_{44} = N_i - B_2^T\Psi_4 B_2$$

$$\Pi_1 = col[e_1, e_2, e_3]$$

$$\Pi_2 = col[e_{11}, (1-\dot{d})e_{12}, e_{13}]$$

$$\Pi_{l+3} = col[e_{l+1}, e_{l+4}, e_{l+11}], l = 0, 1, 2$$

$$\Pi_{6+l} = col[e_{l+1}, e_{l+2}, e_{l+7}], l = 0, 1$$

$$\Pi_8 = col[de_{11}, d(1-\dot{d})e_{12}, e_1 - (1-\dot{d})e_2 - \dot{d}e_7]$$

$$\Pi_9 = col[h_d(1-\dot{d})e_{12}, h_d e_{13}, (1-\dot{d})e_2 - e_3 + \dot{d}e_8]$$

$$\Pi_{l+10} = col[e_{l+1}, e_{l+7}, e_{l+9}], l = 0, 1$$

$$\Pi_{12} = col[de_{11}, e_1 - (1-\dot{d})e_2 - \dot{d}e_7, e_1 - (1-\dot{d})e_7 - 2\dot{d}e_9]$$

$$\Pi_{13} = col[h_d e_{12}, (1-\dot{d})e_2 - e_3 + \dot{d}e_8, (1-\dot{d}) \times e_2 - e_8 + 2\dot{d}e_{10}]$$

$$\Pi_{14} = col[e_1 - e_2, e_1 + e_2 - 2e_7]$$

$$\begin{aligned}
\Pi_{15} &= col[e_2 - e_3, e_2 + e_3 - 2e_8] \\
\Pi_{16} &= col[\Pi_{14}, \Pi_{15}] \\
\Pi_{17} &= col[e_1 - e_7, -\frac{1}{2}e_1 - e_7 + 3e_9] \\
\Pi_{18} &= col[e_2 - e_8, -\frac{1}{2}e_2 - e_8 + 3e_{10}] \\
\wp_i &= -A_i e_1 + W_{0i}e_4 + W_{1i}e_5 + B_1 e_{14} - e_{11} \\
e_0 &= Ce_1 + D_1 e_2 + D_2 e_4 + B_2 e_{14} \\
e_l &= [0_{n\times(l-1)n}, I_n, 0_{n\times(14-l)n}], l = 1, 2, \cdots, 14.
\end{aligned}$$

Proof: Choose the following LKF candidate:

$$V(x_t) = \sum_{l=0}^{6} V_l(x_t) \tag{21}$$

where

$$\begin{aligned}
V_1(x_t) &= \eta_1^T(t)P_i\eta_1(t) \\
V_2(x_t) &= \int_{t-d(t)}^{t} \xi_1^T(s)Q_1\xi_1(s)ds + \int_{t-h}^{t-d(t)} \xi_1^T(s)Q_2\xi_1(s)ds \\
V_3(x_t) &= 2\sum_{l=1}^{n}\int_0^{W_{2l}x_l(t)} [v_{1l}(g_l(s) - k_l^- s) \\
&\quad +v_{2l}(k_l^+ s - g_l(s))]ds \\
V_4(x_t) &= \int_{-d(t)}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta \\
&\quad + \int_{-h}^{-d(t)}\int_{t+\theta}^{t-d(t)} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta \\
V_5(x_t) &= \int_{-h}^{0}\int_{\theta}^{0}\int_{t+u}^{t} \dot{x}^T(s)T\dot{x}(s)dsdud\theta \\
V_6(x_t) &= d(t)\int_{t-d(t)}^{t} \dot{x}^T(s)M_1\dot{x}(s)ds \\
&\quad +h_d(t)\int_{t-h}^{t-d(t)} \dot{x}^T(s)M_2\dot{x}(s)ds \\
&\quad -\frac{d(t)}{h}\xi_2^T(t)U_1\xi_2(t) - \frac{h_d(t)}{h}\xi_3^T(t)U_2\xi_3(t).
\end{aligned}$$

Three steps will be given as follows.

Step 1: Positive definiteness of $V(x_t)$.

From [33], it follows that $V_6(x_t)$ is positive definite. From Proposition 1, $V_0(x_t)$ is positive definite. Thus, the positive definiteness of (21) can be ensured.

Step 2: Stability analysis. Let $\mathfrak{L}$ be the infinitesimal operator along system (1). It yields

$$\sum_{c=1}^{3}\mathfrak{L}V_c(x_t) = \eta^T(t)(\Omega_1 + \Omega_2 + \Omega_3)\eta(t) \tag{22}$$

$$\begin{aligned}
\sum_{l=4}^{5}\mathfrak{L}V_l(x_t) &= \dot{x}^T(t)(d(t)R_1 + \frac{h^2}{2}T)\dot{x}(t) \\
&\quad +h_d(t)(1-\dot{d}(t))\dot{x}^T(t-d(t)) \\
&\quad \times R_2\dot{x}(t-d(t)) \\
&\quad -(1-\dot{d}(t))\int_{t-d(t)}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds \\
&\quad -\int_{t-h}^{t-d(t)} \dot{x}^T(s)R_2\dot{x}(s)ds \\
&\quad -\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)T\dot{x}(s)dsd\theta.
\end{aligned} \tag{23}$$

$$\begin{aligned}
\mathfrak{L}V_6(x_t) &= \dot{d}(t)\int_{t-d(t)}^{t} \dot{x}^T(s)M_1\dot{x}(s)ds \\
&\quad -\dot{d}(t)\int_{t-h}^{t-d(t)} \dot{x}^T(s)M_2\dot{x}(s)ds \\
&\quad +d(t)\dot{x}^T(t)M_1\dot{x}(t) - h_d(t)\dot{x}^T(t-h) \\
&\quad \times M_2\dot{x}(t-h) - (1-\dot{d}(t))\dot{x}^T(t-d(t)) \\
&\quad \times(d(t)M_1 - h_d(t)M_2)\dot{x}(t-d(t)) \\
&\quad -\frac{\dot{d}(t)}{h}\xi_2^T(t)U_1\xi_2(t) - \frac{2d(t)}{h}\xi_2^T(t)U_1\dot{\xi}_2(t) \\
&\quad +\frac{\dot{d}(t)}{h}\xi_3^T(t)U_2\xi_3(t) - \frac{2h_d(t)}{h}\xi_3^T(t)U_2\dot{\xi}_3(t).
\end{aligned} \tag{24}$$

$$\begin{aligned}
\mathfrak{L}V_0(x_t) &\le \dot{x}^T(t)\frac{d(t)h}{2}G_1\dot{x}(t) \\
&\quad +(1-\dot{d}(t))\dot{x}^T(t-d(t))\frac{h_d(t)h}{2}G_2\dot{x}(t-d(t)) \\
&\quad -\frac{\dot{d}(t)}{h^2}\xi_4^T(t)Z_1\xi_4(t) - \frac{2d(t)}{h^2}\xi_4^T(t)Z_1\dot{\xi}_4(t) \\
&\quad +\frac{\dot{d}(t)}{h^2}\xi_5^T(t)Z_2\xi_5(t) - \frac{2h_d(t)}{h^2}\xi_5^T(t)Z_2\dot{\xi}_5(t) \\
&\quad -\frac{(1-\dot{d}(t))d(t)}{2}\int_{t-d(t)}^{t} \dot{x}^T(s)G_1\dot{x}(s)ds \\
&\quad -\frac{h_d(t)}{2}\int_{t-h}^{t-d(t)} \dot{x}^T(s)G_2\dot{x}(s)ds \\
&\quad +\frac{\dot{d}(t)}{2}\int_{-d(t)}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)G_1\dot{x}(s)dsd\theta \\
&\quad -\frac{\dot{d}(t)}{2}\int_{-h}^{-d(t)}\int_{t+\theta}^{t-d(t)} \dot{x}^T(s)G_2\dot{x}(s)dsd\theta.
\end{aligned} \tag{25}$$

Recalling (23), in order to capture more information of time delay, dividing $[0, h]$ into $[0, d(t)] \cup [d(t), h]$, it yields

$$\begin{aligned}
&-\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)T\dot{x}(s)dsd\theta \\
&= -\int_{-d(t)}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)T\dot{x}(s)dsd\theta \\
&\quad -h_d(t)\int_{t-d(t)}^{t} \dot{x}^T(s)T\dot{x}(s)ds \\
&\quad -\int_{-h}^{-d(t)}\int_{t+\theta}^{t-d(t)} \dot{x}^T(s)T\dot{x}(s)dsd\theta.
\end{aligned} \tag{26}$$

Summing up (23)-(26), it follows that

$$\begin{aligned}
\sum_{c=4}^{6}\mathfrak{L}V_c(x_t) + \mathfrak{L}V^*(x_t) &\le \eta^T(t)\Omega_4\eta(t) \\
&\quad -\int_{t-d(t)}^{t} \dot{x}^T(s)\mathfrak{R}_1^0(t)\dot{x}(s)ds \\
&\quad -\int_{t-h}^{t-d(t)} \dot{x}^T(s)\mathfrak{R}_2^0(t)\dot{x}(s)ds
\end{aligned}$$

$$-\int_{-d(t)}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\mathfrak{T}_1^0(t)\dot{x}(s)dsd\theta$$
$$-\int_{-h}^{-d(t)}\int_{t+\theta}^{t-d(t)}\dot{x}^T(s)\mathfrak{T}_2^0(t)\dot{x}(s)dsd\theta. \quad (27)$$

Considering (3), (14), (15), positive definite matrices R_k, M_k, G_k, and $T_k (k = 1, 2)$, it yields

$$\mathfrak{R}_1^0(t) > 0, \mathfrak{R}_2^0(t) > 0, \mathfrak{T}_1^0(t) > 0, \mathfrak{T}_2^0(t) > 0.$$

By using Wirtinger-based single integral inequalities (WSIIs) [17], the R-, M-, G-, and T-dependent single integral terms in (27) can be respectively estimated as

$$-\int_{t-d(t)}^{t}\dot{x}^T(s)\mathfrak{R}_1^0(t)\dot{x}(s)ds \le -\frac{1}{d(t)}\eta^T(t)\Pi_{14}^T\mathfrak{R}_1(d(t),\dot{d}(t))\Pi_{14}\eta(t) \quad (28)$$

and

$$-\int_{t-h}^{t-d(t)}\dot{x}^T(s)\mathfrak{R}_2^0(t)\dot{x}(s)ds \le -\frac{1}{h_d(t)}\eta^T(t)\Pi_{15}^T\mathfrak{R}_2(d(t),\dot{d}(t))\Pi_{15}\eta(t). \quad (29)$$

Then, by using Lemma 2, we can obtain

$$-\frac{1}{d(t)}\Pi_{14}^T\mathfrak{R}_1(d(t),\dot{d}(t))\Pi_{14}-\frac{1}{h_d(t)}\Pi_{15}^T \times\mathfrak{R}_2(d(t),\dot{d}(t))\Pi_{15}\le-\frac{1}{h}\Pi_{16}^T(\mathfrak{R}-\mathfrak{I})\Pi_{16} \quad (30)$$

where

$$\mathfrak{I}=diag\{\frac{v_1h_d(t)}{h}S\mathfrak{R}_2^{-1}(d(t),\dot{d}(t))S^T, \frac{v_2d(t)}{h}S^T\mathfrak{R}_1^{-1}(d(t),\dot{d}(t))S\}.$$

By using WDII [18], the T- and G-dependent double integral terms in (27) can be respectively estimated as

$$-\int_{-d(t)}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\mathfrak{T}_1^0(t)\dot{x}(s)dsd\theta \le -\eta^T(t)\Pi_{17}\mathfrak{T}_1(d(t),\dot{d}(t))\Pi_{17}\eta(t) \quad (31)$$

$$-\int_{-h}^{-d(t)}\int_{t+\theta}^{t-d(t)}\dot{x}^T(s)\mathfrak{T}_2^0(t)\dot{x}(s)dsd\theta \le -\eta^T(t)\Pi_{18}\mathfrak{T}_2(d(t),\dot{d}(t))\Pi_{18}\eta(t). \quad (32)$$

Recalling (27) and summing up (30)-(32), we have

$$\sum_{c=4}^{6}\mathfrak{L}V_c(x_t)+\mathfrak{L}V_0(x_t) \le \eta^T(t)(\Omega_4+\Omega_5+\frac{1}{h}\Pi_{16}^T\mathfrak{I}\Pi_{16})\eta(t). \quad (33)$$

Taking consideration of (4) and (5), the following inequalities can be obtained for $c = 1, 2, 3$

$$\lambda_c(s) = 2[g(W_2x(s)) - K_1W_2x(s)]^T \times\Lambda_c[K_2W_2x(s) - g(W_2x(s))] \ge 0$$
$$\delta_c(s_1, s_2) = 2[g(W_2x(s_1)) - g(W_2x(s_2)) - K_1W_2(x(s_1) - x(s_2))]^T \times\Delta_c[K_2W_2(x(s_1) - x(s_2)) - g(W_2x(s_1)) + g(W_2x(s_2))] \ge 0.$$

Thus, the following inequalities hold

$$\lambda_1(t) + \lambda_2(t - d(t)) + \lambda_3(t - h) \ge 0\delta(t, t - d(t)) + \delta(t - d(t), t - h) + \delta(t, t - h) \ge 0. \quad (34)$$

In addition, we introduce the cost function as follows

$$J_T=\int_0^T(y^T(t)\Psi_1y(t)+2y^T(t)\Psi_2w(t)+w^T(t)\Psi_3w(t))dt. \quad (35)$$

Recalling (22), (33), (34), and (35), we can obtain

$$\int_0^T\mathfrak{L}V(x_t)dt-J_T\le\int_0^T\eta^T(t)\Upsilon(d(t),\dot{d}(t))\eta(t)dt \quad (36)$$

where

$$\Upsilon(d(t),\dot{d}(t))=\sum_{l=1}^{7}\Omega_l+\frac{1}{h}\Pi_{16}^T\mathfrak{I}\Pi_{16}.$$

Note that $\Upsilon(d, \dot{d})$ is a linear matrix-value based on $d(t) \in [0, h]$ and $\dot{d}(t) \in [-\mu, \mu]$, respectively. It can be written as

$$\Upsilon(d(t),\dot{d}(t))=\dot{d}(t)(d(t)\ell_1+\ell_2)+d(t)\ell_3+\ell_4 \quad (37)$$

where $\ell_l(l = 1, 2, \ldots, 4)$ are some real matrix combinations irrespective of $d(t)$ and $\dot{d}(t)$. Thus, two allowable delay sets $\mathbb{H}_1 = Co\{(0, -\mu), (0, \mu), (h, -\mu), (h, \mu)\}$ and $\mathbb{H}_2 = Co\{(0, 0), (0, \mu), (h, 0), (h, -\mu)\}$ can be used to solve the condition (37). It is pointed out [41] that $(0, -\mu)$ and (h, μ) in $\mathbb{H}_1$ are inappropriate due to the fact that it is impossible for the time delay $d(t)$ to achieve the maximum h at the time when $\dot{d}(t) = \mu > 0$ and the minimum 0 at time when $\dot{d}(t) = -\mu < 0$. Thus, for any $(d(t), \dot{d}(t)) \in \mathbb{H}_2$, for any $(d(t), \dot{d}(t))$, if (16)-(19) are satisfied, we have

$$\begin{cases}\Upsilon(0,0)<0\\ \Upsilon(0,\mu)<0\\ \Upsilon(h,0)<0\\ \Upsilon(h,-\mu)<0\end{cases}\Rightarrow\Upsilon(d(t),\quad\dot{d}(t))<0. \quad (38)$$

Considering $\wp_i\eta(t) = 0$, by using Finsler's Lemma [12], it follows

$$(\wp_i^{\perp})^T\Upsilon(d(t),\dot{d}(t))\wp_i^{\perp}<0\Rightarrow\eta^T(t)\Upsilon(d(t),\dot{d}(t))\eta(t)<0.$$

For $\Psi_1 \le 0$, there exists a scalar $\nu \ge 0$ such that the following inequality holds under $w(t) = 0$

$$\mathfrak{L}V(x_t)<-\nu|\eta(t)|^2$$

which means that MJNNs (1) are stochastically stable.

Step 3: Extended dissipativity analysis. From (36), one has

$$J_T\ge\int_0^T\mathfrak{L}V(x_t)dt=V(x_T)-V(x_0) \ge x^T(T)P_{1i}x(T)-V(x_0). \quad (39)$$

In view of (20), we have $\Gamma_{33} = L_i - D_2^T\Psi_4D_2 > 0$ and $\Gamma_{44} = N_i - B_2^T\Psi_4B_2 > 0$. Since L_i and N_i are symmetric positive definite matrices, $\Psi_4 \geq 0$ is guaranteed. Obviously, $\Psi_4 < 0$ holds for any positive definite matrices L_i and N_i. For unconstrained parameter Ψ_4, two cases will be considered as follows.

a: $\Psi_4 = 0$, setting $\varrho \leq -V(x_0)$, from (9) and (39), we have

$$J_T \geq \varrho. \tag{40}$$

b: $\Psi_4 \neq 0$, from Assumption 1, we have $\Psi_1 = \Psi_2 = 0$, $\Psi_3 > 0$. Together with (39), we obtain

$$J_T = \int_0^T w^T(t)\Psi_3 w(t)dt \geq 0. \tag{41}$$

Considering $0 \leq t \leq T$ and $0 \leq t - d(t) \leq T$, for symmetric positive definite matrices L_i, N_i, and Ψ_3, it yields for all $i \in S$

$$\begin{aligned} J_T &\geq J_t \geq x^T(t)P_{1i}x(t) - V(x_0) \\ &\geq x^T(t)P_{1i}x(t) - V(x_0) \\ &\quad -g^T(W_2x(t))L_i g(W_2x(t)) - w^T(t)N_i w(t) \end{aligned} \tag{42}$$

$$\begin{aligned} J_T &\geq J_{t-d(t)} \\ &\geq x^T(t-d(t))P_{1i}x(t-d(t)) - V(x_0) \\ &\quad -g^T(W_2x(t))L_i g(W_2x(t)) - w^T(t)N_i w(t). \end{aligned} \tag{43}$$

Thus, for a positive scalar $\alpha \in (0, 1)$, we can get

$$\begin{aligned} J_T &\geq (1-\alpha)x^T(t-d(t))P_{1i}x(t-d(t)) \\ &\quad +\alpha x^T(t)P_{1i}x(t) - V(x_0) \\ &\quad -g^T(W_2x(t))L_i g(W_2x(t)) - w^T(t)N_i w(t). \end{aligned} \tag{44}$$

Then, it yields

$$\begin{aligned} y^T(t)\Psi_4 y(t) &= (1-\alpha)x^T(t-d(t))P_{1i}x(t-d(t)) \\ &\quad +\alpha x^T(t)P_{1i}x(t) + w^T(t)N_i w(t) \\ &\quad +g^T(W_2x(t))L_i g(W_2x(t)) - \chi^T(t)\Theta\chi(t) \end{aligned} \tag{45}$$

where

$$\chi(t) = col[x(t), x(t-d(t)), g(W_2x(t)), w(t)]$$

$$\Theta = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & -C^T\Psi_4 D_2 & -C^T\Psi_4 B_2 \\ * & \Gamma_{22} & -D_1^T\Psi_4 D_2 & -D_1^T\Psi_4 B_2 \\ * & * & \Gamma_{33} & -D_2^T\Psi_4 B_2 \\ * & * & * & \Gamma_{44} \end{bmatrix}.$$

Combining $\Theta > 0$, (44) and (45), we have

$$\begin{aligned} J_T \geq y^T(t)\Psi_4 y(t) &- 2g^T(W_2x(t))L_i g(W_2x(t)) \\ &-2w^T(t)N_i w(t) - V(x_0). \end{aligned} \tag{46}$$

Recalling (5), we have $|g(W_2x(t))| \leq ||K_2W_2|| \cdot |x(t)|$. Then, two cases for $\Psi_4 \neq 0$ will be given as follows.

b(1): For $\Psi_4 > 0$, recalling $w(t) \in \mathcal{L}_2[0, \infty)$, there exists a scalar $\varrho \leq 0$ such that the following relationship holds for $\kappa \leq \varrho - 2\max_i ||K_2W_2||^2 \cdot ||L_i|| \cdot |x(t)|^2 - 2\max_i ||N_i|| \cdot |w(t)|^2 - V(x_0)$

$$y^T(t)\Psi_4 y(t) \geq \sup_{0 \leq t \leq T} y^T(t)\Psi_4 y(t) + \kappa. \tag{47}$$

Together with (46), we have

$$J_T \geq \sup_{0 \leq t \leq T} y^T(t)\Psi_4 y(t) + \varrho. \tag{48}$$

b(2): For $\Psi_4 < 0$, we obtain

$$\sup_{0 \leq t \leq T} y^T(t)\Psi_4 y(t) \leq 0. \tag{49}$$

Combining (41) and (49), we have

$$J_T = \int_0^T w^T(t)\Psi_3 w(t)dt \geq \sup_{0 \leq t \leq T} y^T(t)\Psi_4 y(t) + \varrho. \tag{50}$$

Recalling (48) and (50), it yields

$$J_T = \int_0^T w^T(t)\Psi_3 w(t)dt \geq \sup_{0 \leq t \leq T} y^T(t)\Psi_4 y(t) + \varrho. \tag{51}$$

It follows from Definition 1 that system (1) is extended dissipative. Summarizing the above three steps, this completes the proof.

Remark 2. Based on Proposition 1 and PDRCI, a novel condition is proposed in Theorem 1 to ensure MJNNs (1) to be stochastically stable and extended dissipative. Several merits of Theorem 1 can be concluded as follows. (i) More information of time delay can be captured since its derivative is associated with $d(t)$, h, $\dot{d}(t)$, $v_1(t)$, $v_2(t)$, $v_3(t)$, and $v_4(t)$. Moreover, the double integrals are considered and zero components are avoided in this paper, which can lead to more less conservative results than those in [33], [34]. (ii) The PDRCI instead of those in [33], [34] is presented to handle the integral term $-\int_{t-h}^{t} \dot{x}^T(s)R\dot{x}(s)ds$. In this case, two parameters κ_1 and κ_2 can be chosen freely and independently in Theorem 1. Then, a better solution can be gotten by adjusting the two parameters.

Remark 3. Note that in [34], by introducing symmetric positive definite matrices L_i and N_i, the constraints $D_2 = 0$, $||B_2|| \cdot ||\Psi_4|| = 0$ on the structure of MJNNs (1) are overcome and $\Psi_4 \geq 0$ is ensured. As seen in (20) of present paper. If $\Psi_4 \geq 0$, $L_i \leq 0$, and $N_i \leq 0$, we can obtain $\Gamma_{33} \leq 0$ and $\Gamma_{44} \leq 0$. In this case, condition (20) is invalid. In order to overcome the issue, setting $D_2 = B_2 = 0$ is the common way in previous works such as in [33], which obviously leads to some limitations in practical engineering. As a result, with two positive definite matrices L_i and N_i, the obtained result of present paper is more general and practical than that in [33].

Remark 4. It is worth pointing out that the conservativeness of a stability criterion of system (1) is dependent on the choice of LKF and the bound on its derivative. In this paper, the main contribution focuses on the construction of DIDPT functional. Actually, Lyapunov method is a fruitful field in the stability analysis of time-delay systems [42], [43]. Note that the authors in [23] show that the conservatism of a stability criterion can be reduced by increasing the ply of integral terms in LKF. Thus, a suitable multiple integral delay-product-type LKF can further reduce the conservatism of the stability criterion.

IV. EXAMPLES

Consider MJNNs (1) with parameters from [33]

$$A_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},$$

TABLE 1. Extended dissipative performance.

Performance	Ψ_4	Ψ_1	Ψ_2	Ψ_3
H_∞	0	-I	0	γ^2I
L_2-L_∞	I	0	0	γ^2I

$$A_2=\begin{bmatrix}2.2 & 0\\ 0 & 1.8\end{bmatrix}, K_1=D_2=0$$
$$W_{01}=\begin{bmatrix}1 & 1\\ -1 & -1\end{bmatrix}, W_{02}=\begin{bmatrix}-1 & -1\\ 0.5 & -1\end{bmatrix}$$
$$W_{11}=\begin{bmatrix}0.88 & 1\\ 1 & 1\end{bmatrix}, W_{12}=\begin{bmatrix}-0.5 & 0.6\\ 0.7 & 0.8\end{bmatrix}$$
$$W_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, B_1=\begin{bmatrix}0.0403\\ 0.6771\end{bmatrix}, K_2=diag\{0.4,0.8\}$$
$$C=\begin{bmatrix}-0.3775 & -0.2959\end{bmatrix}, D_1=\begin{bmatrix}0.2532 & -0.1684\end{bmatrix}.$$

Choose $\Pi=\begin{bmatrix}-4 & 4\\ 5 & -5\end{bmatrix}$ and $\Psi_l(l=1,2,3,4)$ in Table 1. For simplicity, setting $\sigma=\sigma_1=\sigma_2$ and $\kappa=\kappa_1=\kappa_2$, the following two cases will be considered in this example as special cases of extended dissipativity.

1) H_∞ performance: Choosing $B_2=0.1184$, the minimum H_∞ performance of γ and NDVs calculated by different methods are illustrated in Table 2.

TABLE 2. Minimum H_∞ performance γ for $h=3$.

Criteria	$\mu=0.8$	$\mu=0.9$	NDVs
[33],Theorem 3	0.8429	2.7968	125
[34],Theorem 1	0.6113	1.7642	252
Theorem 1($\sigma=0,\kappa=1$)	0.6113	1.7642	252
Theorem 1($\sigma=1,\kappa=1$)	0.5902	1.7449	272
Theorem 1($\sigma=0.5,\kappa=1$)	0.6451	1.8004	272
Theorem 1($\sigma=0.1,\kappa=1$)	0.6747	1.8291	272
Theorem 1($\sigma=0.1,\kappa=3$)	0.6971	1.8512	272

2) L_2-L_∞ performance: (1) Choosing $B_2=0$, the minimum L_2-L_∞ performance of γ and NDVs calculated by different criteria are illustrated in Table 3. From Cases 1)-2), summaries are given as follows. (i) From Tables 2-3, it can be seen that the utilization of coupling information of the DIDPT (13) can be adjusted by choosing different σ. (ii) The PDRCI plays a key role in reducing conservatism. (iii) The proposed result of present paper for MJNNs (1) is less conservative than that in [33], [34]. In addition, setting $\mu=0.8$, $\kappa_1=3$, and $\kappa_2=5$, the minimum H_∞ and L_2-L_∞ performance indices are 0.6934 and 0.4872, which show the effectiveness of PDRCI in [38].

TABLE 3. Minimum L_2-L_∞ performance γ for $h=3$.

Criteria	$\mu=0.8$	$\mu=0.9$	NDVs
[33],Theorem 3	0.6684	1.3591	125
[34],Theorem 1	0.4241	0.8744	252
Theorem 1($\sigma=0,\kappa=1$)	0.4241	0.8744	252
Theorem 1($\sigma=1,\kappa=1$)	0.4002	0.8454	272
Theorem 1($\sigma=0.5,\kappa=1$)	0.4417	0.8503	272
Theorem 1($\sigma=0.1,\kappa=1$)	0.4710	0.8795	272
Theorem 1($\sigma=0.1,\kappa=3$)	0.4952	0.8894	272

V. CONCLUSION

In this paper, the problem of the extended dissipative analysis of Markovian jump neural networks (MJNNs) has been investigated by using delay-product-type (DPT) functional method. By using Wirtinger-based double integral inequality and S-procedure lemma, a novel double integral-based DPT (DIDPT) functional is constructed. The incomplete components in [34] have been avoided in case that extra information of system information can be coupled. Together with the parameter-dependent reciprocally convex inequality and the Wirtinger-based integral inequality to estimate the derivative of the constructed LKF, a delay-dependent extended dissipativity condition is derived for the delayed MJNNs. A numerical example is employed to illustrate the advantages of the proposed method. Extending DIDPT functional to multiple integral delay-product-type functional deserves further investigation.

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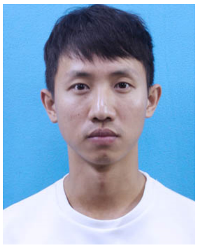
XIAOPING HUANG received the Ph.D. degree in computer architecture from Northwestern Polytechnical University, Xi'an, in 2011.

He is currently an Associate Professor with Northwestern Polytechnical University. His current research interests include the concurrent learning algorithm and real-time systems.

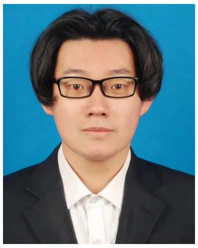
CAIYUN WU received the Ph.D. degree in control theory and applications from Northeastern University, Shenyang, China, in 2014.

From 2002, she has been a Teacher with Shenyang Ligong University, where she is currently a Professor. Her research interests include switched systems, adaptive control, and the concurrent learning algorithm.



YUZHONG WANG received the B.Sc. degree in mathematics and the M.Sc. degree in operations research and cybernetics from Northeastern University, Shenyang, China, in 2016 and 2018, respectively, where he is currently pursuing the Ph.D. degree with the School of Computer Science and Engineering.

His research interests include switched systems, descriptor systems, sliding mode control, and event-triggered adaptive control.



WENDONG LI received the B.S. degree in vehicle engineering from Hainan University, Hainan, China, in 2018. He is currently pursuing the master's degree with the Shenyang Ligong University.

His research interests include switched systems and adaptive control.