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Risk Analysis and Utility Function-Based Decision-Making Model for Spinning Reserve Allocations

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ABSTRACT Spinning reserve (SR) is an essential resource for system operators to compensate for unpredictable imbalances between load and generation caused by sudden unit outages and unforeseen fluctuations of load demand and wind power. System operators could face many alternative SR allocations determined by different methods and decide the optimal one based on the experience and consideration of risks in the power system. Considering the stochastic nature of power system components, this paper proposes a utility function-based decision-making model for SR allocations, which incorporates the whole distribution of social benefits (SBs) of the SR allocation and the risk preferences of decision-makers together. Three different utility functions are established that relate the SBs of SR allocations to risk-seeking, riskneutral, and risk-averse preferences, respectively. Besides, the risk preference degree is represented by a shape parameter. The risk of different SR allocations is analyzed and the expected utilities, which reflect the relative satisfaction, are used to determine the optimal SR allocation for the operators with various risk preferences. Incorporating both the distribution of SBs and decision-makers' risk preferences makes the final decision for optimal SR allocation more flexible than the decision strategies based only on the expected social benefit or a probabilistic reliability index (such as loss of load probability and expected load not served). The effectiveness is examined using the IEEE-RTS.

INDEX TERMS Spinning reserve, uncertainty, risk analysis, utility theory, electricity.

I. INTRODUCTION

Spinning reserve (SR), the amount of unused capacity in online units, is the most important resource to compensate for the imbalance between generation and demand without resorting to load shedding [1]. Due to wind power's randomness and unpredictability, its increasing integration is challenging for power system operations. Determining the required SR is one of the main concerns of the operators in power systems with random components. In many regional power systems, deterministic criteria are implemented by the operators to decide the required amount of SR. For example, the SR capacity is assumed to be a specific percentage of the hourly system loads or the capacity of the largest online generator [2]. Although deterministic criteria are easy to implement, they do not reflect the random behavior of

system components, which leads to inconsistent decisions and variable operating risk levels.

Probabilistic methods to address reserve assessment problems are well established in the literature [1]–[11]. These methods provide realistic evaluations of SR requirements by incorporating the probabilities of the occurrence of each contingency, unforeseen fluctuations of load or wind power in the decision-making issues. Probabilistic reliability criteria such as loss of load probability (LOLP) [3], expected load not served (ELNS) [3], [4], expected energy not supplied (EENS) [1], [5]–[6], and demand factor (the ratio of EENS to the load demand) [5]–[6] are used as reliability constraints to set the SR requirements for security-constrained unit commitment (UC) problems [1], [3]–[6]. Stochastic programming has been used to formulate and solve problems with uncertain parameters in power systems, in which the uncertainties of system components are represented by a finite set of realizations or scenarios [5]–[11]. A comprehensive evaluation

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of LOLP and EENS indices for each scenario and each hour are formulated to determine the optimal SR allocations in the stochastic–probabilistic approaches for the renewable penetrated systems [7]–[8]. Probability constrained methods are adopted to assess the reserve requirements so that a certain percentile of the total forecast error and unit outages have to be covered by the reserves [9]–[11]. In these researches, SR volumes are allocated so that these probabilistic reliability indices are within the desirable values.

However, it is complicated to set and justify a specified index, and the effect of the value of lost load (VOLL) is not considered in the involuntary load shedding of customers [12]. Consequently, the SR scheduling is not based on cost-benefit analysis and so may be suboptimal. Based on cost-benefit analysis, the authors in [13]–[15] optimize SR requirements by maximizing the expected social benefit (ESB) or minimizing the expected economic dispatch cost and EENS cost. In [16], the authors use value at risk and integrated risk management to assess the risks and make an optimal tradeoff between risks and profits in the wind power penetrated-system. Some recent researches use the conditional value at risk (CVaR) method to manage the risk and assess the reserve requirements in the wind power penetrated systems [17]–[20].

The above methods provide a more realistic evaluation and determination of SR allocations compared to the deterministic criteria. However, in the methods using predetermined risk thresholds to assess SR requirements, the remaining parameters characterizing the distribution associated with the profits or costs of the SR allocations are neglected. Rational decision-makers are concerned about both the probabilities of profits or losses of the investments. The risk attitude of the decision-maker also affects the scheduling strategies [21]. Therefore, the distributions of profits or costs of the SR allocations and decision-makers' risk attributes should be incorporated in the decision-making process. This paper is not devoted to defining the SR requirements but instead providing decision support for decision-makers with various risk preferences to determine the optimal SR allocation among alternatives.

Utility-based methods have been applied for the decisions in power systems [22]–[23]. In the expected utility framework, the whole distribution of returns (risk and return) is simultaneously considered, and there is no need to define risk [24]. This paper proposes a utility function-based decision-making model for optimal SR allocation in the wind power penetrated-system. The variable representing the social benefit (SB) of the SR allocation is established by considering every realization of the uncertainties from unit outages and net load forecast errors. Three different utility functions are established, which relate the SBs of the SR allocation to decision-makers' risk-seeking (RS), risk-neutral (RN), and risk-averse (RA) preferences, respectively. In these utility functions, the subjective risk preference degrees, particularly the RA, RN, and RS preferences, are represented by different values of the shape parameter. The expected

FIGURE 1. Discretization of the probability distribution of the net load forecast error.

utilities that reflect the relative satisfaction are then used to determine the optimal SR allocation. The proposed model is tested on IEEE-RTS and compared with the decision strategies that order the SR allocations according to the expected social benefit and the reliability indexes, such as LOLP and EENS.

The rest of the paper is organized as follows: Section 2 models the uncertainties of unit outages and forecast errors of wind power and load. Section 3 introduces the SBs of the SR allocations and the risk measure that characterizes the risk associated with the SR allocations. Section 4 formulates the utility function-based decision-making model for SR allocations and the shape parameter that represents the risk preference degrees of decision-makers. Section 5 includes simulation and data analysis from the IEEE-RTS. And Section 6 is devoted to the conclusions.

II. UNCERTAINTY MODEL OF POWER SYSTEM

A. UNCERTAINTY MODEL OF LOAD AND WIND POWER

Both the forecast errors of load and wind power can be modeled as Gaussian random variables [15], [17]. The probability density function of load and wind power forecast errors are given by

$$
f_{\varepsilon_l}(\varepsilon_l) = \frac{1}{\sqrt{2\pi}\sigma_l} e^{-(\varepsilon_l)^2/(2\sigma_l^2)}
$$
 (1)

$$
f_{\varepsilon_{w}}\left(\varepsilon_{w}\right) = \frac{1}{\sqrt{2\pi}\,\sigma_{w}}e^{-(\varepsilon_{w})^{2}/\left(2\sigma_{w}^{2}\right)}\tag{2}
$$

where ε_l and ε_w are the load and wind power forecast errors, respectively. σ_l and σ_w are the standard deviations of load and wind power forecast errors, respectively. The net load forecast error ε*^d* equals the difference between load and wind power forecast errors [17].

$$
\varepsilon_d = \varepsilon_l - \varepsilon_w \tag{3}
$$

As these forecast errors are uncorrelated and assumed to follow the same distribution, the standard deviation of net load forecast error σ_d is given as follows [15], [25]

$$
\sigma_d = \sqrt{(\sigma_l)^2 + (\sigma_w)^2}
$$
 (4)

The continuously-valued net load forecast error model just developed is not computationally practical to set the SR requirements, so the continuous variable is approximated by a set of discrete variables [26]. The probability distribution of forecast error is equally divided into several intervals, as shown in Fig. 1. The net load forecast error of interval

FIGURE 2. Two-state model of generator.

k takes the value of the midpoint of the interval, and its probability is given by

$$
\pi_k = \frac{1}{\sigma_d \sqrt{2\pi}} \int\limits_{l_k}^{u_k} e^{\frac{-(x)^2}{2\sigma_d^2}} dx \tag{5}
$$

where u_k and l_k are the upper/lower bounds of the interval k respectively.

B. UNCERTAINTY MODEL OF CONVENTIONAL UNIT

In Fig. 2, a two-state model [1] is used, assuming that failure and repair times are exponentially distributed with parameters and respectively. Applying the parameters failure rate λ_i and repair time μ_i of generation units, the unavailability UTi , and availability *AT i* of unit *i* in period *T* can be approximated by the two-state Markov model as follows [7], [8].

$$
U_i(T) = 1 - e^{-\lambda_i T} \approx \lambda_i T = ORR_i \tag{6}
$$

$$
A_i(T) = 1 - U_i(T) = 1 - ORR_i \tag{7}
$$

Suppose *S* is the set of states representing the realizations of all the unit outages and forecast errors of net load. Then, the probability of a realization of net load forecast error and generation outages in state s ($s \in S$) p_s , is obtained by

$$
p_s = \pi_{(s)} \cdot \prod_{j \in G_s^U} U_j \prod_{i \in G_s^A} (1 - U_i)
$$
 (8)

where $\pi(s)$ is the probability of net load forecast error $\varepsilon_{d,s}$ in state *s*. G_s^U and G_s^A are the sets of unavailable units and available units in state *s*, respectively.

III. SOCIAL BENEFIT AND RISK OF SPINNING RESERVE ALLOCATIONS

It is necessary to schedule SR capacities to maintain the system's reliability due to unit outages and unforeseen fluctuations of wind power and load demands. In this section, the variable representing the SB of the SR allocation is established. The standard deviation of this variable is used as the risk measure of the SR allocation.

A. SOCIAL BENEFIT OF THE SPINNING RESERVE **ALLOCATION**

After the SR volumes are determined in the day-ahead dispatch schedule, the deployment of these volumes is affected by the power imbalance caused by the unit outages and the power fluctuations of net load in real-time. For the SR allocation $A = (R_1, R_2, \dots, R_{|G|})^T$, its SB in state *s* is obtained as the difference between its benefits and costs (capacity cost and deployment cost), which is given by

$$
w_s = \eta_{B,s} - c_{R,s} - c_{E,s} \tag{9}
$$

where w_s , $\eta_{B,s}$, $c_{R,s}$, and $c_{E,s}$ are the SB, benefit, capacity cost, and reserve deployment cost of the SR allocation in state *s*, respectively. *G* is the set of all generating units. The positive value of w_s means that the SR allocation is profitable in state *s*, while a negative value indicates a deficit.

$$
\eta_{B,s} = Voll \cdot \left(P^0_{loss,s} - P_{loss,s} \right) \tag{10}
$$

The benefit of SR is measured by the reduction in the cost of load interruptions [14]. *Voll* is the VOLL. P_{loss}^0 and $P_{loss,s}$ are the load interruptions before and after purchasing the SR volumes in state *s*, respectively.

$$
c_{R,s} = C_R = \sum_{i \in G} \rho_i^R \cdot R_i \tag{11}
$$

where ρ_i^R and R_i are the SR cost and allocated SR volume of unit *i*, respectively. Since the SR volumes are determined before the realization of every state, the capacity costs in every state are the same and equal to the SR purchase cost *CR*.

The reserve deployment cost varies with the power imbalance caused by an increase in net load or unit outages. The power imbalance in state *s Pimb*,*^s* , is given by

$$
P_{imb,s} = \max(\varepsilon_{d,s} + \sum_{j \in G_s^U} P_j, 0)
$$
 (12)

where P_i is the power production of unit *j* in the day-ahead dispatch schedule. The SR volumes will be deployed to compensate for the power imbalance in two cases: [\(1\)](#page-1-0) The power imbalance is within the total SR volumes. Thus, parts of these SR volumes are needed, which will be deployed economically to minimize the deployment costs, and the remaining SRs are unused. [\(2\)](#page-1-0) The power imbalance exceeds the total SR volume. In this case, all the SR volumes will be deployed, and part of the load demand has to be curtailed to restore the power balance. According to the above cases, the reserve deployment cost is formulated as follows.

$$
c_{E,s} = \min_{r_{i,s}} \sum_{i \in G_s^A} \rho_i^E \cdot r_{i,s} \tag{13}
$$

$$
\text{s.t.} \sum_{i \in G_s^A} r_{i,s} = \min(P_{imb,s}, \sum_{i \in G_s^A} R_i) \tag{14}
$$

$$
0 \le r_{i,s} \le R_i, \quad \forall i \in G_s^A \tag{15}
$$

where ρ_i^E is the energy cost of unit *i*, and $r_{i,s}$ is the deployed SR volume of unit *i* in state *s*. The SR volumes accommodating the power imbalance are limited by the allocated SR capacities (14)–[\(15\)](#page-2-0). The lost load $P_{loss,s}$ corresponding to the reserve deployment, is the part of the load that could not be supplied:

$$
P_{loss,s} = P_{imb,s} - \sum_{i \in G_s^A} r_{i,s} \tag{16}
$$

The distribution of SBs is then obtained by calculating every w_s in [\(9\)](#page-2-1) and the corresponding probability p_s in [\(8\)](#page-2-2), respectively. The ESB of the SR allocation, *Eb*, is given by

$$
E_b = \sum_{s \in S} p_s \cdot w_s \tag{17}
$$

B. RISK MEASURE

The risk occurs when the SBs vary from the ESB. The SR allocations obtained by different methods show different risks. Thus, risk measures are needed to characterize the risk associated with the given decisions and enable us to compare different decisions in terms of the risk involved [27]. The variance, or standard deviation, first proposed by Harry Markowitz, has been the most common measure of risk among academics and practitioners alike [24]. The standard deviation of SBs σ_{SB} is used as the risk measure in this paper, which is given by

$$
\sigma_{SB} = \sqrt{\sum_{s \in S} p_s \cdot (E_b - w_s)^2}
$$
 (18)

This standard deviation indicates possible deviations of the realized SBs from the ESB. Hence a high standard deviation indicates that there exists a high risk of experiencing a benefit or loss that is different from *Eb*.

IV. UTILITY FUNCTION-BASED DECISION-MAKING FOR SPINNING RESERVE ALLOCATIONS

This section presents the utility function-based decisionmaking process for decision-makers with different risk preferences to decide between different SR allocations. The utility is a measure of the relative satisfaction from the consumption of goods or services in economics. In the expected utility framework, one does not analyze risk and return separately but instead considers the whole distribution of returns simultaneously [24]. Also, the risk preferences of decision-makers can be expressed in the model.

A. RISK PREFERENCE

Let $u(w)$ denote the utility function of SB w , u' and u'' denote the first and second derivatives of $u(w)$. The relationship between utility functions and risk attitudes are given as follows [22], [28].

1) $u(w)$ is a non-decreasing function $u' \ge 0$, which means that the decision-makers prefer more wealth to less wealth.

2) The decision-maker is risk-averse (RA) with $u'' \leq$ 0, which means that other things being equal, this decision-maker dislikes uncertainty or risk.

3) The decision-maker is risk-neutral (RN) with $u'' = 0$.

4) The decision-maker is risk-seeking (RS) with $u'' \ge 0$, which means this decision-maker will be sensitive to possible opportunities.

The typical utility curves of these three risk preferences are illustrated in Fig. 3.

B. MEASURE OF RISK PREFERENCE

The Arrow-Pratt measure of absolute risk aversion (ARA) is a widely used measure of risk aversion in current day economic analysis [29]–[30]. The ARA coefficient $R_a(w)$ of the utility function $u(w)$ at benefit *w* is given by [30]:

$$
R_a(w) = -\frac{u^{''}(w)}{u'(w)}\tag{19}
$$

FIGURE 3. Diagrams of utility curves of different risk attitudes.

Based on the definition in [\(19\)](#page-3-0), the relationship between $R_a(w)$ and risk preference can be summarized as follows. For a risk-averse decision-maker, $R_a(w) \geq 0$, it reflects the degree of concavity of $u(w)$, and hence the strength or intensity of risk aversion [29]. A larger $R_a(w)$ value indicates a greater degree of risk aversion. For a risk-neutral decision-maker, $R_a(w) = 0$, it means $u(w)$ is linear. When $R_a(w) \leq 0$, it reflects the degree of convexity of $u(w)$ for a risk-seeking decision-maker. A lower $R_a(w)$ value indicates a greater degree of risk-seeking preference.

C. UTILITY FUNCTION

Different utility functions can be used to study decisionmakers' behavior, such as linear utility, quadratic utility, and exponential utility [30]. The utility functions constructed in the paper are based on linear utility and exponential utility because of their mathematical tractability. This paper assumes that the range of utility functions is [0, 1], where 1 means the decision-maker is most satisfied, and 0 means extremely uncomfortable. Combined with the fuzzy logic, the utility functions constructed in this paper are given as follows:

1) The utility function of the RA decision-maker

*u*¹ (*w*)

$$
= \begin{cases} 0, & w \le w_{\min} \\ \gamma_1 \left(1 - e^{-\alpha_1 \beta (w - w_{\min})} \right), & w_{\min} < w < w_{\max} \\ 1, & w \ge w_{\max} \end{cases} \tag{20}
$$

where $\alpha_1 > 0$, $\gamma_1 = \frac{1}{1 - e^{-\alpha_1}}$.

2) The utility function of the RN decision-maker

$$
u_2(w) = \begin{cases} 0, & w \le w_{\min} \\ \beta(w - w_{\min}), & w_{\min} < w < w_{\max} \\ 1, & w \ge w_{\max} \end{cases}
$$
 (21)

3) The utility function of the RS decision-maker

$$
u_3(w)
$$

=
$$
\begin{cases} 0, & w \le w_{\min} \\ \gamma_3 \left(e^{\alpha_3 \beta (w - w_{\min})} - 1 \right), & w_{\min} < w < w_{\max} \\ 1, & w \ge w_{\max} \end{cases}
$$
 (22)

where $\alpha_3 > 0, \gamma_3 = \frac{1}{e^{\alpha_3} - 1}.$

In [\(20\)](#page-3-1)-[\(22\)](#page-3-2), *wmax* and *wmin* are the SBs that the decision-makers are most and least satisfied, respectively. β is a scaling factor for the normalization, it equals the ratio of the utility range to the SB range and $\beta = 1/(w_{max} - w_{min}) \alpha_1, 0$, and $-\alpha_3$ are the shape parameters that characterize the concavity or convexity of utility functions. The shape parameters reflect the subjective risk preference degrees. The scaling factor β depends on the objective distribution of SB and the selected utility range, while the shape parameter depends on the subjective risk preference. γ_1 and γ_3 are the calibration coefficients that satisfy $u(w_{max}) = 1$.

The feature of the above utility functions is that they exhibit constant ARA coefficients. The constant ARA coefficients of the RA, RN, and RS utility functions are $\alpha_1\beta$, 0, $-\alpha_3\beta$, which are the productions of their shape parameters $(\alpha_1, 0, \alpha_2)$ and $-\alpha_3$) and the scaling factor β , respectively. Besides, both the RA and RS preferences approach RN preference as the respective shape parameters (α_1 and $-\alpha_3$) approach 0. Take $u'_1(w)$ as an example.

$$
u'_{1}(w) = \frac{\alpha_{1}\beta e^{-\alpha_{1}\beta(w-w_{\min})}}{1 - e^{-\alpha_{1}}}, \forall w \in (w_{\min}, w_{\max})
$$
 (23)

Let $f(\alpha_1)$ denote $\alpha_1 \beta e^{-\alpha_1 \beta (w - w_{\text{min}})}$, and $g(\alpha_1)$ denote $1 - e^{-\alpha_1}$. The limits of *f* (α_1) and *g* (α_1) as α_1 approaches 0 are 0.

$$
\lim_{\alpha_1 \to 0} \alpha_1 \beta e^{-\alpha_1 \beta (w - w_{\min})} = \lim_{\alpha_1 \to 0} \left[1 - e^{-\alpha_1} \right] = 0 \quad (24)
$$

Then $g'(\alpha_1) \neq 0$, $\forall \alpha_1 > 0$ and

$$
\frac{\lim_{\alpha_1 \to 0} f'(\alpha_1)}{\lim_{\alpha_1 \to 0} g'(\alpha_1)} = \frac{\lim_{\alpha_1 \to 0} \beta e^{-\alpha_1 \beta (w - w_{\min})} [1 - \alpha_1 (w - w_{\min})]}{\lim_{\alpha_1 \to 0} e^{-\alpha_1}}
$$

$$
= \beta, \forall w \in (w_{\min}, w_{\max})
$$
(25)

Replace α_1 by $-\alpha_3$, the limits of $u'_1(w)$ and $u'_3(w)$ as α_1 and $-\alpha_3$ approach 0 are β [\(26\)](#page-4-0). The limits of $u''_1(w)$ and $u''_3(w)$ as α_1 and $-\alpha_3$ approach 0 are 0 [\(27\)](#page-4-0).

$$
\lim_{\alpha_3 \to 0} u'_3(w) = \lim_{\alpha_1 \to 0} u'_1(w) = \beta \n= u'_2(w), \quad \forall w \in (w_{\min}, w_{\max})
$$
\n(26)\n
$$
\lim_{u''_3}(u'_3(w)) = \lim_{u \to 0} u''_1(w)
$$

$$
u_3(w) = \frac{\ln u_1(w)}{\alpha_1 \to 0}
$$

= $u_3'(w) = 0$, $\forall w \in (w_{\min}, w_{\max})$ (27)

Equations [\(26\)](#page-4-0) and [\(27\)](#page-4-0) indicate that both the RA and RS utility functions approximate the linear RN utility function as α_1 and $-\alpha_3$ approach 0. Therefore, differential risk preferences, including the RA, RN, and RS preferences, can be represented by merely the shape parameter. Table 1 illustrates the relationship between the risk preference degree and the shape parameter. A positive value of the shape parameter means the decision-maker is RA. And a larger value of the shape parameter indicates a higher degree of riskaverse. Meanwhile, a negative value of the shape parameter means the decision-maker is RS. And a lower value of the shape parameter indicates a higher degree of risk-seeking preference.

D. DECISION-MAKING FOR SR ALLOCATIONS

Suppose the decision-maker has to decide between *N* different SR allocations $\Omega = (\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_N)^T$, which are obtained by available methods. The decision-making process for these SR allocations is given as follows.

First, the SBs of each SR allocation in each state are obtained by [\(9\)](#page-2-1). Second, the utilities of these SBs are obtained by [\(20\)](#page-3-1)–[\(22\)](#page-3-2) for decision-makers with particular risk preferences. Then, the summation of $w_{n,s}$ weighted by probabilities of the corresponding states is the expected utility (EU) of the *n*-th SR allocation *Eu*·*n*:

$$
E_{u \cdot n} = \sum_{s \in S} p_s \cdot u(w_{n,s}), \quad \forall n = 1, 2, ..., N \qquad (28)
$$

Based on the utility maximization principle, the best choice is the one that provides the highest utility (satisfaction) to the decision-maker. Suppose the EU of the *q*-th SR allocation is the largest in the EU set, then the final decision-making process is given by

$$
q = \arg\max_{n} \{E_{u \cdot n}\}\tag{29}
$$

Consequently, the *q*-th SR allocation is the best choice for the decision-maker with a particular risk preference.

V. CASE STUDY

In this section, the proposed utility function-based decision-making model for SR allocations is tested on the IEEE-RTS [31]. There are 26 thermal generators in this base system, and the hydro units have been removed [1], [12]. Generators' data, including size, type, forced outage rate (FOR), and production cost, are given in [31]. VOLL is assumed to be \$200/MWh [4]. Generation units offer SR costs at the rates of 50% of their energy production costs [13]. For a given hour, the system load and wind power are 2280 MW and 150 MW, respectively. The standard deviations of load and wind power forecast errors are set at 3% and 10%, respectively [17]. In this paper, the net load forecast error is divided into seven intervals [15], and 26 first-order generation contingencies are studied [12]. The simulation has been achieved in the Matlab 2014a environment [32].

Table 2 gives each unit's energy production schedule, energy and SR costs, and three different SR allocations during the study period. A_1 is determined by the deterministic method, and a minimum SR requirement of 10% of the load

 $\alpha_3 \rightarrow$

demand (228 MW) is satisfied. **A**² is determined by maximizing the ESB. A_3 is determined by a probabilistic method, in which the risk threshold of LOLP is 0.01.

A. DISTRIBUTION OF SOCIAL BENEFITS

Based on the data given above, the discrete SBs and the corresponding probabilities of the above three SR allocations are obtained by [\(8\)](#page-2-2) and [\(9\)](#page-2-1), respectively. Then both the probability distributions and cumulative distributions of the SBs are obtained, which are shown in Fig. 4. Some essential characteristics of the SB allocations are shown in Table 2, where SB^{\min} and SB^{\max} denote the minimum and maximum SBs of the SR allocations, respectively.

The SBmins in **A**1, **A**2, and **A**³ are −\$724, −\$1819, and −\$2650 with high probabilities of 0.5223, 0.5127, and 0.5127, respectively (the sixth column in Table 3). These SBmins are equal to the SR capacity costs, resulting from the situation where none of the purchased SR volumes is deployed in the real world. Thus, there is no benefit of the SR.

FIGURE 4. Distributions of social benefits. (a) for R_1 , (b) for R_2 , and (c) for A₃.

Meanwhile, the probabilities of SBs less than the ESBs are 0.627, 0.5959, and 0.5959 for **A**1, **A**2, and **A**3, respectively.

The SBmaxs in **A**1, **A**2, and **A**³ are \$43 427, \$74 540, and \$88 729, with low probabilities of 0.0631, 0.0232, and 0.0103 (the eighth column in Table 3), respectively. The more the SR volumes are purchased, the better the system is capable of reducing the interruption costs from large power imbalances, so the larger the SBmax will be. The above results show that both SB^{min} and SB^{max} vary between these three SR allocations, even though the ESBs of these three SR allocations are close to each other.

Fig. 5 illustrates the probability distributions of SBs of **A**³ when VOLLs are set at 100, 200, and 300 \$/MWh, respectively. VOLL is the parameter representing the cost of unserved power, so only the benefit component of the SB increases with the increase of VOLL, while the cost component and the probability of each state are independent

SR allocation	Total SR /MW	$C_R/\$	E_b /\$	SB^{min}/S	P_{SB} ^{min}	$SB^{max}/\$$	P_{SB} max
A	228	724	9314	-724	0.5223	43427	0.0631
A ₂	400	1819	10305	-1819	0.5127	74540	0.0232
A,	483.4	2650	9809	-2650	0.5127	88729	0.0103

TABLE 3. Statistical results of social benefits of the SR allocations.

FIGURE 5. Probability distributions of SBs of A₃ for different VOLLs.

of VOLL. Three observations are obtained. First, the maximum discrete losses are the same −\$2650 with the same probability of 0.5127. Second, the discrete SBs increase with the increase of VOLL except for the minimum one. The ESBs corresponding to these three VOLLs are \$3336, \$9809, and \$16 282, respectively. Finally, the LOLP and EENS of **A**³ do not change with different VOLLs. The above results confirm that only a reliability index could not cover the above characteristics of SBs of different SR allocations.

B. UTILITY-BASED DECISION-MAKING FOR SR ALLOCATIONS

In this section, *wmax* and *wmin* are set at −\$10 000 and \$100 000 respectively, according to the ranges of SBs in Fig. 4. Then, the shape parameter is the only variable that influences the utility-based decisions for these SR allocations. The shape parameters are supposed to be 4 (highly RA), 0, and −4 (highly RS) for the RA, RN, and RS decisionmakers, respectively. The decision results based on reliability indices (LOLP and EENS), ESB, and EU are compared and presented in Table 4.

As seen in Table 4, **A**¹ requires the least capacity cost and the ESB of A_1 is also the least. A_2 is the most cost-effective choice among the three because of the maximized expected return. A_3 is the most reliable choice because of the least LOLP and EENS, but it requires the largest capacity cost of \$2650, and the probability of A_3 resulting in no benefit is as high as 0.5127.

FIGURE 6. Discrete utilities and the utility curves for different shape parameters of A₃.

When considering the risk attributes, the order of the SR allocations determined by EUs is the same as that determined by ESBs for the RN decision-maker. Thus A_2 is most favored for the RN decision-maker. However, the orders are quite different for the RA and RS decision-makers. The RA decision-maker is unwilling to spend too much to purchase SR capacities and prefers A_1 because A_1 shows the least risk (minimum σ_{SB}). In contrast, the RS decision-maker is willing to seek potential benefits by reducing the interruption costs from generation outages and forecast errors, and thus prefers A_3 , which has the maximum σ_{SB} . The difference in the decisions for SR allocations confirms the flexibility of the proposed model, as compared to the decision strategies based only on ESBs or the reliability indices such as LOLP or EENS.

C. EFFECT OF THE SHAPE PARAMETER

The risk attributes of decision-makers play an essential role in the utility-based decision for SR allocations. Fig. 6 illustrates the utility distributions for different shape parameters $(\pm 10, \pm 2, \pm 1, 0)$ of A₃. As seen in Fig. 6, the larger the absolute value of the shape parameter, the greater is the curvature of the utility curve. In comparison, the utility curves become straight as the shape parameters approach 0. The decision results with these different shape parameters are listed in Table 5.

Both Fig. 6 and Table 5 show that the utilities and EUs decrease as the shape parameter decreases. As expected, the RA decision-maker tends to be risk-neutral with a decrease in α_1 (approaching 0), and the optimal SR allocation changes from A_1 to A_2 when α_1 is less than 3.

SR allocation	C_R	LOLP	EENS	E_b	σ_{SB}	E_u		
		$\%$	MWh			u _I	u ₂	u_3
A ₁	724	10.02	13.20	9314	13843	0.4596	0.1756	0.0252
A ₂	1819	2.25	2.21	10305	17758	0.4543	0.1846	0.0344
A_3	2650	0.46	0.41	9809	18704	0.4382	0.1801	0.0367
Order	123	321	321	231	$\overline{}$	123	231	321

TABLE 4. Result comparison of the optimal SR allocation determined by different decision strategies.

TABLE 5. Utility-based decision results for different shape parameters.

Risk preference / Shape parameter			E_u	Order	Optimal SR.	
		${\bf A}_1$ A,		A_3		allocation
	10	0.7226	0.7033	0.6802	123	
RA	2	0.319	0.3215	0.3109	2 1 3	2
	1	0.2447	0.251	0.2434	2 1 3	$\overline{2}$
RN	θ	0.1756	0.1846	0.1801	2 3 1	2
	-1	0.1175	0.1281	0.1262	2 3 1	$\overline{2}$
RS	-2	0.0738	0.0847	0.0848	321	3
	-10	0.0007	0.0034	0.0061	3 2 1	3

TABLE 6. Utility-based decision results for different FORs.

Meanwhile, the RS decision-maker tends to be risk-neutral with an increase in $-\alpha_3$ (approaching 0), and the optimal SR allocation changes from A_3 to A_2 when $-\alpha_3$ is larger than −2. The difference in optimal decisions for the RA and RS decision-makers becomes apparent when the absolute values of the shape parameters become large. Thus, the model can reveal the changing trend of the satisfaction degree of the SR allocations with the change of shape parameter and provide decision support for the operators with various risk preferences.

D. EFFECT OF THE FOR AND FORECAST ERROR

Both the unit outages, forecast errors of wind power and load demand affect the optimal SR allocation. In this sub-section, the decision results between two different decision-makers are compared, whose shape parameters are −2 (slightly RS) and 0 (RN), respectively. Table 6 shows the utility-based decision results when the FOR is scaled by factors of 0.5, 1.0, and 1.5 times. Table 7 shows the utility-based decision results when both load and wind power forecast errors are

TABLE 7. Utility-based decision results for forecast errors.

scaled by factors of 0.5, 1.0, and 1.5 times. As expected, both decision-makers prefer more SR capacities when the FOR or forecast errors increase. This is because the increased uncertainty from either the unit outages or net load forecast errors leads to higher interruption costs. Conversely, when the units are more reliable or the forecast errors are lower, the SR requirements are lower. Besides, compared to the RN decision-makers, the RS ones seek possible opportunities and prefer more reserve capacities when dealing with uncertainty.

VI. CONCLUSION

Various methods have been proposed to determine the optimal SR allocation while achieving a balance between reliability and economy in recent years. Before making the final decision between alternative SR allocations, the decision-maker considers both the ESB of the SR allocations and the experience and risk attributes in the system with random components.

In this paper, decision-makers' risk attributes are addressed, and a utility function-based decision-making model for SR allocations is proposed. The distribution of SBs of the SR allocations is simultaneously considered in the model, and the risk preference degree is represented by the shape parameter. Simulation results show that the proposed model can reveal the changing trend of the satisfaction degree of the SR allocations with the change of shape parameter and provide decision support for decision-makers with particular risk preferences. The SR allocation with the largest ESB is also the one with the largest EU for the RN decisionmaker. Meanwhile, the optimal SR allocations for the RA and RS decision-makers vary with their shape parameters. The model provides operators with another choice for optimal SR allocation besides decision strategies based on maximum

expected return or a specified reliability index. It should be noted that although the standard deviation of SBs is used to analyze the risk of the SR allocations, there is no need to define risk or set a risk threshold in the proposed model. Further research will focus on studying a multi-objective optimization problem for the optimal SR allocation considering both the expected operational cost and expected utility in the market environment.

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