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Finite-Time Annular Domain Bounded Control of Itô-Type Stochastic Systems With Wiener and Poisson Random Disturbance

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ABSTRACT This paper is concerned with the finite-time annular domain bounded control of Itô-type stochastic systems with Wiener and Poisson random disturbance. First, utilizing different quadratic function methods, some sufficient conditions for finite-time annular domain bounded-ness (FTADB) of the system are achieved. Second, two finite-time annular domain bounded controllers are skillfully developed to ensure the FTADB of the closed-loop system, of which one is state feedback controller and the other is dynamic output feedback controller. Furthermore, an algorithm is provided to deal with the obtained matrix inequalities. Finally, two examples are used to demonstrate the effectiveness of the theorems in this paper.

INDEX TERMS Stochastic systems, Poisson random disturbance, finite-time annular domain bounded.

I. INTRODUCTION

The control problems of stochastic systems have attracted much attention in physics, biology, engineering and other practical systems in the last decades. Among various stochastic control systems, Itô stochastic differential/difference equations play important roles. This class of systems have received considerable attention from control and mathematical communities. There are have been some results available in the literatures about stability of stochastic systems. For example, some robust state feedback controllers are proposed for linear stochastic systems with Markovian switching in [1]. It is verified [1] that the robust stabilization problem can be solved. H_2/H_∞ control for nonlinear stochastic systems based on coupled Hamilton-Jacobi equations are investigated in [2]. Then, the results are further developed in stochastic fuzzy affine systems in [3]. Some other excellent research results of stochastic systems can be found in [4]–[10] and their references.

It is known that Wiener and Poisson random process play an important role in stochastic control systems. Hence, stochastic linear systems with Wiener and Poisson random disturbance have been brought into focus [11]–[18]. For instance, stability problems for semi-Markovian switched singular stochastic systems with Wiener noise are investigated in [11]. The H_∞ control problems for systems perturbed by jump random noise, i.e., Poisson-driven stochastic systems are studied in [12]. Disturbance attenuation properties and robust H_∞ adaptive fuzzy tracking control of nonlinear systems are investigated in [13]. Based on Poisson processes, the problem of moment estimators for the parameters of Ornstein-Uhlenbeck processes is developed in [14]. Other outstanding research results can be found in [15]–[18].

When the time goes infinity, asymptotic stability is considered in most of the existing results [19]–[25]. As mentioned in [26], asymptotically stable systems may have poor transient characteristics. In practice, it is important to reach steady state in finite time, especially in communication network system [27], robot control system [28]. Fortunately, the concept of finite-time stability (FTS) is put forward [29]. Due to the advantage of the finite-time technique, there have been many nice results on it, such as the FTS of switched stochastic systems [30]–[32], FTS of stochastic delayed systems [33]–[35], FTS of stochastic Markovian jump systems [36], [37]. Besides, external disturbance is unavoidable in practice. In order to solve the problem, the concept of finite-time bounded (FTB) has been introduced [38], and many excellent research results based on finite-time bounded-ness theory have been obtained [39]–[42], [44]. For example, in [39], some sufficient conditions on finite-time stochastic bounded-ness are provided for stochastic systems with stochastic delayed interval and Markovian switching. In [40], a newly stochastic Lyapunov-Krasovskii functional and novel activation function conditions are proposed for a

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class of Markovian jump linear systems and put forward a controller to investigate the finite-time stochastic contractive bounded-ness problem. In [41], a stochastic variable satisfying the Bernoulli distribution is utilized for the problem of FTB control. In [42], the problem of FTS analysis and stochastic finite-time bounded-ness for switched linear systems are studied. However, the FTS and FTB problem in [39]–[44] only involves the upper bound of the system orbit, not its lower bound. In practice, the research of the upper bound and lower bound of system state showed the great significance. For instance, in [45], the temperature of the reactor should be within the given interval in some chemical reaction controlled systems, otherwise the reaction cannot proceed smoothly during a given time interval. In [46], to make the entire electrical power system work properly during a given time interval, transient voltage should be controlled in a finite-time interval. To deal with these phenomenon, the problem of finite-time annular domain stability and bounded control of the stochastic systems has been investigated with the lower bound of the system orbit in [47]–[49]. In [47], a quadratic function approach is developed for the FTADB of a linear stochastic system. The finite-time annular domain stability of stochastic impulsive systems was investigated in [48]. Nevertheless, the influence of Wiener and Poisson random disturbances on system performance is not considered in [47]–[49] simultaneously. Thus, how to develop a new strategy for Itô-type stochastic systems with Wiener and Poisson random disturbance such that the closed-loop systems are FTADB has not been adequately dealt with till now, which motivates the current study.

The FTADB control of Itô-type stochastic systems are investigated in this paper. The main contributions of this work can be stated as follows: i) The FTADB control of Itô-type stochastic systems with Wiener and Poisson random disturbance are studied, which generalizes some existing stochastic system. ii) By a different quadratic function method, Itô formula, Gronwall inequality, and matrix inequality technique, several sufficient criteria for FTADB of Itô-type stochastic systems are obtained under a state feedback controller (SFC) and an output feedback controller (OFC). iii) An algorithm is presented to give the relationship of the parameters under the state feedback and output feedback cases.

The paper is structured as follows. Section II introduces some basic preliminaries. Section III provides a sufficient condition for FTADB. Section IV gives some sufficient conditions for the existence of feedback controllers. Section V shows an algorithm for solving the related parameters of the two controllers. In section VI gives two simulation examples to illustrate the feasibility of the proposed theoretical results. A conclusion is shown in the last section.

Notations: M' denotes the transpose of matrix M. $M > 0(M \geq 0)$ means that M is positive-definite (positive semi-definite). $r_{max}(M)(r_{min}(M))$ means the largest (smallest) eigenvalue of matrix M. R^p means an p-dimensional Euclidean space. "$*$" represents the entries implies by the symmetry to a matrix. $diag\{\cdots\}$ represents a diagonal matrix. E represents the mathematical expectation of a random process.

II. PRELIMINARIES AND PROBLEM STATEMENT

Consider the following Itô-type stochastic linear system with Wiener and Poisson random disturbance

$$\begin{cases} dx(t)=(A_{11}x(t)+B_{11}u(t)+H_1\varpi(t))dt \\ +(A_{21}x(t)+B_{21}u(t)+H_2\varpi(t))dw(t) \\ +(A_{31}x(t)+B_{31}u(t)+H_3\varpi(t))dn(t), \\ y(t)=C_1x(t),\ x(0)=x_0 \in R^n, \end{cases} \tag{1}$$

where $A_{11}, A_{21}, A_{31}, B_{11}, B_{21}, B_{31}, H_1, H_2, H_3, C_1$ are constant matrices with appropriate dimensions. $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^p$ denote the state of the system, the control input, and the measurement output, respectively. $n(t)$ stands for the marked Poisson process with Poisson jump intensity λ. $w(t)$ is the one-dimensional standard wiener process. $\varpi(t)$ denotes the external disturbance and all $\varpi(t) \in \Re$. x_0 presents the initial state.

The class $\Re$ is defined as follows:

$$\Re=\{\varpi(t)|d\varpi(t)=F\varpi(t)dt+F_1\varpi(t)dw(t),$$
$$\varpi(0)=\varpi_0,\ \varpi_0'R_1\varpi_0 \leq f_1,\ \mathrm{E}\left[\varpi'(t)R_1\varpi(t)\right] < f\}, \tag{2}$$

where $f > 0$ and $f_1 > 0$ are given scalars, and $F > 0, F_1 > 0$, and $R_1 > 0$ are constant matrices.

Remark 1: From the above definition of f and f_1, one can see that $\Re$ actually includes a big class of signals.

Next, the concept of FTADB is introduced and more details can be found in [50] and [51].

Definition 1: Given scalars $\delta_1 > 0, \delta_2 > 0, \delta_3 > 0, \delta_4 > 0$, $T > 0$, with a matrix $R > 0$, $\delta_2 > \delta_4 > \delta_3 > \delta_1 > 0$, and a class of exogenous signals $\Re$, then the following system

$$\begin{cases} dx(t) = (A_{11}x(t) + H_1\varpi(t))dt \\ +(A_{21}x(t) + H_2\varpi(t))dw(t) \\ +(A_{31}x(t) + H_3\varpi(t))dn(t), \\ x(0) = x_0, \end{cases} \tag{3}$$

is said to be FTADB with respect to $(\delta_1, \delta_2, \delta_3, \delta_4, \Re, T, R)$, if

$$\delta_3 \leq \mathrm{E}\left[x'(0)Rx(0)\right] \leq \delta_4$$
$$\Rightarrow \delta_1 < \mathrm{E}\left[x'(t)Rx(t)\right] < \delta_2, \tag{4}$$

for all $t \in [0,\ T]$, $\varpi(t) \in \Re$.

Next, some lemmas that will be used are given in this paper.

Lemma 1 [52]: For given $\nu(x(t)) \in C^{1,2}(R^+,\ R^n)$, associated with the following stochastic system

$$dx(t) = f(x)dt + g(x)dw(t) + a(x)dn(t). \tag{5}$$

Define the infinitesimal operator $\ell\nu$ as

$$\ell\nu(x(t)) = \frac{\partial\nu(x(t))}{\partial t} + \frac{\partial\nu'(x(t))}{\partial x}f(x)$$
$$+\frac{1}{2}[g'(x)\frac{\partial^2\nu(x(t))}{\partial x^2}g(x)]$$
$$+\lambda[\nu(x(t)+a(x))-\nu(x(t))]. \tag{6}$$

Lemma 2 [51]: Let $h(t)$ be a nonnegative function, if there exist some constants $m \geq 0$ and $\eta \geq 0$, such that

$$h(t) \leq m + \eta\int_0^t h(s)ds\ \ 0 \leq t \leq T, \tag{7}$$

then

$$h(t) \leq m\ exp(\eta t)\ \ 0 \leq t \leq T. \tag{8}$$

Lemma 3 [51]: Let $h(t)$ be a nonnegative function, if there exist some constants $m \geq 0$ and $\eta \geq 0$, such that

$$h(t) \geq m + \eta \int_0^t h(s)ds\ \ 0 \leq t \leq T, \tag{9}$$

then

$$h(t) \geq m\ exp(\eta t)\ \ 0 \leq t \leq T. \tag{10}$$

III. FINITE-TIME ANNULAR DOMAIN BOUNDED-NESS

This section is to address the FTADB problem of the system (1) by selecting different quadratic functions.

In [44], a key approach to obtain the main results is as follows. Take the positive-definite function $\nu(S_{state}(t))$, then based on the following inequalities

$$\ell\nu(S_{state}(t)) < \alpha\nu(S_{state}(t)), \tag{11}$$

and

$$\ell\nu(S_{state}(t)) > \beta\nu(S_{state}(t)), \tag{12}$$

one can obtain the main results. Since the general quadratic functions in (11) and (12) are the same, one can find that $\nu(S_{state}(t))$ which satisfies (11) may not satisfy (12). Thus, the results obtained are conservative.

In order to deal with this problem, a different method will be introduced. Specifically, by choosing different positive quadratic functions $\nu_1(S_{state}(t))$ and $\nu_2(S_{state}(t))$, the following inequalities

$$\ell\nu_1(S_{state}(t)) < \alpha\nu_1(S_{state}(t)), \tag{13}$$

and

$$\ell\nu_2(S_{state}(t)) > \beta\nu_2(S_{state}(t)), \tag{14}$$

will be derived. Thus, the main results obtained by different quadratic function methods are better than those obtained by common approach.

Based on the different quadratic function methods, *Theorem 1* is obtained.

Theorem 1: Given positive scalars $\delta_1, \delta_2, \delta_3, \delta_4, T$, and a matrix $R > 0$, with $\delta_2 > \delta_4 > \delta_3 > \delta_1 > 0$, the system (3) is FTADB with respect to $(\delta_1, \delta_2, \delta_3, \delta_4, \Re, T, R)$, if there exist symmetric matrices $Q_1 > 0, Q_2 > 0, Q_3 > 0$, and some scalars r_i, $i = 1, 2, 3$, $\alpha \geq 0$, $\beta \geq 0$, such that the following inequalities hold

$$\begin{bmatrix} \psi & H_1 & \sqrt{\lambda}\widetilde{Q}_1(I+A_{31})' & \widetilde{Q}_1 A_{21}' \\ * & \Gamma_1 - \alpha\widetilde{Q}_2 & \sqrt{\lambda}H_3' & H_2' \\ * & * & -\widetilde{Q}_1 & 0 \\ * & * & * & -\widetilde{Q}_1 \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} \psi_1 & H_1 & \sqrt{\lambda}\widetilde{Q}_1(I+A_{31})' & \widetilde{Q}_1 A_{21}' \\ * & \beta\widetilde{Q}_2 - \Gamma_2 & \sqrt{\lambda}H_3' & H_2' \\ * & * & -\widetilde{Q}_1 & 0 \\ * & * & * & -\widetilde{Q}_1 \end{bmatrix} < 0, \tag{16}$$

$$\begin{bmatrix} r_2 f_1 - \delta_2 e^{-\alpha T} & \sqrt{\delta_4} \\ * & -r_1 \end{bmatrix} < 0, \tag{17}$$

$$\begin{bmatrix} r_3 f - \delta_3 & \sqrt{\delta_1} \\ * & -r_1 \end{bmatrix} < 0, \tag{18}$$

$$r_1 I < Q_1 < I, \tag{19}$$

$$0 < Q_2 < r_2 I, \tag{20}$$

$$0 < Q_3 < r_3 I, \tag{21}$$

where

$$\begin{aligned} \widetilde{Q}_1 &= R^{-\frac{1}{2}} Q_1 R^{-\frac{1}{2}},\quad \widetilde{Q}_2 = R_1^{\frac{1}{2}} Q_2 R_1^{\frac{1}{2}},\quad \widetilde{Q}_3 = R_1^{\frac{1}{2}} Q_3 R_1^{\frac{1}{2}}, \\ \psi &= \widetilde{Q}_1 A_{11}' + A_{11}\widetilde{Q}_1 - \lambda\widetilde{Q}_1 - \alpha\widetilde{Q}_1, \\ \Gamma_1 &= F'\widetilde{Q}_2 + \widetilde{Q}_2 F + F_1'\widetilde{Q}_2 F_1, \\ \psi_1 &= \beta\widetilde{Q}_1 + \lambda\widetilde{Q}_1 - \widetilde{Q}_1 A_{11}' - A_{11}\widetilde{Q}_1, \\ \Gamma_2 &= F'\widetilde{Q}_3 + \widetilde{Q}_3 F + F_1'\widetilde{Q}_3 F_1. \end{aligned}$$

Proof: The proof is divided into two steps.

Step 1 : $\mathrm{E}[x'(0)Rx(0)] \leq \delta_4 \Rightarrow \mathrm{E}[x'(t)Rx(t)] < \delta_2$

Choose the following quadratic function

$$\nu_1(x(t), \varpi(t)) = x'(t)\widetilde{Q}_1^{-1}x(t) + \varpi'(t)\widetilde{Q}_2\varpi(t), \tag{22}$$

where $\widetilde{Q}_1 = R^{-\frac{1}{2}} Q_1 R^{-\frac{1}{2}}$, $\widetilde{Q}_2 = R_1^{\frac{1}{2}} Q_2 R_1^{\frac{1}{2}}$, with symmetric matrices $Q_1 > 0, Q_2 > 0$.

Applying Itô formula for $\nu_1(x(t), \varpi(t))$ along the trajectory of the following system

$$\begin{cases} d\begin{bmatrix} x(t) \\ \varpi(t) \end{bmatrix} = \begin{bmatrix} A_{11} & H_1 \\ 0 & F \end{bmatrix}\begin{bmatrix} x(t) \\ \varpi(t) \end{bmatrix}dt + \begin{bmatrix} A_{21} & H_2 \\ 0 & F_1 \end{bmatrix}\begin{bmatrix} x(t) \\ \varpi(t) \end{bmatrix}dw(t) \\ \quad + \begin{bmatrix} A_{31} & H_3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ \varpi(t) \end{bmatrix}dn(t) \\ \begin{bmatrix} x(0) \\ \varpi(0) \end{bmatrix} = \begin{bmatrix} x_0 \\ \varpi_0 \end{bmatrix} \in R^{n+l}, \end{cases} \tag{23}$$

it follows

$$\begin{aligned} &\ell\nu_1(x(t), \varpi(t)) \\ &= (A_{11}x(t) + H_1\varpi(t))'\widetilde{Q}_1^{-1}x(t) \\ &\quad + x'(t)\widetilde{Q}_1^{-1}(A_{11}x(t) + H_1\varpi(t)) \\ &\quad + (A_{21}x(t) + H_2\varpi(t))'\widetilde{Q}_1^{-1}(A_{21}x(t) \\ &\quad + H_2\varpi(t)) + (F\varpi(t))'\widetilde{Q}_2\varpi(t) \\ &\quad + \varpi'(t)\widetilde{Q}_2 F\varpi(t) + (F_1\varpi(t))'\widetilde{Q}_2 F_1\varpi(t) \\ &\quad + \lambda[(x(t) + A_{31}x(t) + H_3\varpi(t))'\widetilde{Q}_1^{-1}(x(t) \\ &\quad + A_{31}x(t) + H_3\varpi(t)) - x'(t)\widetilde{Q}_1^{-1}x(t)], \end{aligned} \tag{24}$$

and one obtains

$$\begin{aligned} &\ell\nu_1(x(t), \varpi(t)) - \alpha\nu_1(x(t), \varpi(t)) \\ &= x'(t)[A_{11}'\widetilde{Q}_1^{-1} + \widetilde{Q}_1^{-1}A_{11} + A_{21}^T\widetilde{Q}_1^{-1}A_{21} \\ &\quad + \lambda(I + A_{31})'\widetilde{Q}_1^{-1}(I + A_{31}) - \lambda\widetilde{Q}_1^{-1} \\ &\quad - \alpha\widetilde{Q}_1^{-1}]x(t) + x'(t)[\widetilde{Q}_1^{-1}H_1 + A_{21}'\widetilde{Q}_1^{-1}H_2 \\ &\quad + \lambda(I + A_{31})'\widetilde{Q}_1^{-1}H_3]\varpi(t) + \varpi'(t)[H_1'\widetilde{Q}_1^{-1} \\ &\quad + H_2'\widetilde{Q}_1^{-1}A_{21} + \lambda H_3'\widetilde{Q}_1^{-1}(I + A_{31})]x(t) \\ &\quad + \varpi'(t)[F'\widetilde{Q}_2 + \widetilde{Q}_2 F + F_1'\widetilde{Q}_2 F_1 + H_2'\widetilde{Q}_1^{-1}H_2 \\ &\quad + \lambda H_3'\widetilde{Q}_1^{-1}H_3 - \alpha\widetilde{Q}_2]\varpi(t). \end{aligned} \tag{25}$$

Pre-multiplying and post-multiplying both sides of inequality (15), by $diag\{\widetilde{Q}_1^{-1}, I, \widetilde{Q}_1^{-1}, \widetilde{Q}_1^{-1}\}$, according to Schur complement, one can obtain that

$$\begin{bmatrix} \Lambda + A_{21}'\widetilde{Q}_1^{-1}A_{21} & \Lambda_1 + \widetilde{Q}_1^{-1}H_1 + A_{21}'\widetilde{Q}_1^{-1}H_2 \\ * & \Lambda_2 + H_2'\widetilde{Q}_1^{-1}H_2 \end{bmatrix} < 0, \tag{26}$$

where

$$\begin{aligned} \Lambda &= A_{11}'\widetilde{Q}_1^{-1} + \widetilde{Q}_1^{-1}A_{11} + \lambda(I + A_{31})'\widetilde{Q}_1^{-1}(I + A_{31}) \\ &\quad - \lambda\widetilde{Q}_1^{-1} - \alpha\widetilde{Q}_1^{-1}, \\ \Lambda_1 &= \lambda(I + A_{31})'\widetilde{Q}_1^{-1}H_3, \\ \Lambda_2 &= F'\widetilde{Q}_2 + \widetilde{Q}_2F + F_1'\widetilde{Q}_2F_1 + \lambda H_3'\widetilde{Q}_1^{-1}H_3 - \alpha\widetilde{Q}_2. \end{aligned}$$

By (25) and (26), one obtains

$$\ell\nu_1(x(t), \varpi(t)) < \alpha\nu_1(x(t), \varpi(t)). \tag{27}$$

Then, integrate from 0 to t with $t \in [0, T]$ and take the mathematical expectation on both sides of (27), one has

$$\begin{aligned} &\mathrm{E}\nu_1(x(t), \varpi(t)) \\ &\quad < \mathrm{E}\nu_1(x(0), \varpi(0)) + \alpha \int_0^t E\nu_1(x(s), \varpi(s))ds. \end{aligned} \tag{28}$$

By Lemma 2, it can be concluded that

$$\mathrm{E}\nu_1(x(t), \varpi(t)) < \mathrm{E}\nu_1(x(0), \varpi(0))e^{\alpha t}. \tag{29}$$

Applying the given conditions, we obtain

$$\begin{aligned} &\mathrm{E}\nu_1(x(0), \varpi(0))e^{\alpha t} \\ &\quad = \mathrm{E}[x'(0)\widetilde{Q}_1^{-1}x(0) + \varpi'(0)\widetilde{Q}_2\varpi(0)]e^{\alpha t} \\ &\quad = \mathrm{E}[x'(0)R^{\frac{1}{2}}Q_1^{-1}R^{\frac{1}{2}}x(0) + \varpi'(0)R_1^{\frac{1}{2}}Q_2R_1^{\frac{1}{2}}\varpi(0)]e^{\alpha t} \\ &\quad \le \mathrm{E}[r_{\max}(Q_1^{-1})\delta_4 + r_{\max}(Q_2)f_1]e^{\alpha T} \\ &\quad = [\frac{1}{r_{\min}(Q_1)}\delta_4 + r_{\max}(Q_2)f_1]e^{\alpha T} \\ &\quad < [\frac{1}{r_1}\delta_4 + r_2f_1]e^{\alpha T}, \end{aligned} \tag{30}$$

$$\begin{aligned} &\mathrm{E}\nu_1(x(t), \varpi(t)) \\ &\quad = \mathrm{E}[x'(t)\widetilde{Q}_1^{-1}x(t) + \varpi'(t)\widetilde{Q}_2\varpi(t)] \\ &\quad = \mathrm{E}[x'(t)R^{\frac{1}{2}}Q_1^{-1}R^{\frac{1}{2}}x(t) \\ &\qquad + \varpi'(t)R_1^{\frac{1}{2}}Q_2R_1^{\frac{1}{2}}\varpi(t)] \\ &\quad \ge \mathrm{E}[r_{\min}(Q_1^{-1})x'(t)Rx(t) \\ &\qquad + r_{\min}(Q_2)\varpi'(t)R_1\varpi(t)] \\ &\quad = \mathrm{E}[\frac{1}{r_{\max}(Q_1)}x'(t)Rx(t) + r_{\min}(Q_2)\varpi'(t)R_1\varpi(t)] \\ &\quad > \mathrm{E}[x'(t)Rx(t)]. \end{aligned} \tag{31}$$

According to (29), (30) and (31), it yields

$$\mathrm{E}[x'(t)Rx(t)] < (\frac{1}{r_1}\delta_4 + r_2f_1)e^{\alpha T}. \tag{32}$$

From (17), one finds

$$(\frac{1}{r_1}\delta_4 + r_2f_1)e^{\alpha T} < \delta_2. \tag{33}$$

According to (32) and (33), one can obtain

$$\mathrm{E}[x'(t)Rx(t)] < \delta_2. \tag{34}$$

Step 2 : $\delta_3 \le \mathrm{E}[x'(0)Rx(0)] \Rightarrow \delta_1 < \mathrm{E}[x'(t)Rx(t)]$

Consider a function as follows

$$\nu_2(x(t), \varpi(t)) = x'(t)\widetilde{Q}_1^{-1}x(t) + \varpi'(t)\widetilde{Q}_3\varpi(t), \tag{35}$$

where $\widetilde{Q}_1 = R^{-\frac{1}{2}}Q_1R^{-\frac{1}{2}}$, $\widetilde{Q}_3 = R_1^{\frac{1}{2}}Q_3R_1^{\frac{1}{2}}$, with symmetric positive definite matrices Q_1, Q_3 being solutions (15)-(21).

Applying Itô formula for $\nu_2(x(t), \varpi(t))$ along the trajectory of the system of (23), one has

$$\begin{aligned} &\ell\nu_2(x(t), \varpi(t)) \\ &\quad = (A_{11}x(t) + H_1\varpi(t))'\widetilde{Q}_1^{-1}x(t) \\ &\qquad + x'(t)\widetilde{Q}_1^{-1}(A_{11}x(t) + H_1\varpi(t)) \\ &\qquad + (A_{21}x(t) + H_2\varpi(t))'\widetilde{Q}_1^{-1}(A_{21}x(t) + H_2\varpi(t)) \\ &\qquad + (F\varpi(t))'Q_3\varpi(t) + \varpi'(t)Q_3F\varpi(t) \\ &\qquad + (F_1\varpi(t))'Q_3F_1\varpi(t) \\ &\qquad + \lambda[(x(t) + A_{31}x(t) + H_3\varpi(t))'\widetilde{Q}_1^{-1}(x(t) \\ &\qquad + A_{31}x(t) + H_3\varpi(t)) - x'(t)\widetilde{Q}_1^{-1}x(t)], \end{aligned} \tag{36}$$

and it follows

$$\begin{aligned} &\beta\nu_2(x(t), \varpi(t)) - \ell\nu_2(x(t), \varpi(t)) \\ &\quad = x'(t)[\beta\widetilde{Q}_1^{-1} + \lambda\widetilde{Q}_1^{-1} - A_{11}'\widetilde{Q}_1^{-1} - \widetilde{Q}_1^{-1}A_{11} \\ &\qquad - A_{21}'\widetilde{Q}_1^{-1}A_{21} - \lambda(I + A_{31})'\widetilde{Q}_1^{-1}(I + A_{31})]x(t) \\ &\qquad - x'(t)[\widetilde{Q}_1^{-1}H_1 + A_{21}'\widetilde{Q}_1^{-1}H_2 \\ &\qquad + \lambda(I + A_{31})'\widetilde{Q}_1^{-1}H_3]\varpi(t) \\ &\qquad - \varpi'(t)[H_1'\widetilde{Q}_1^{-1} + H_2'\widetilde{Q}_1^{-1}A_{21} \\ &\qquad + \lambda H_3'\widetilde{Q}_1(I + A_{31})]x(t) \\ &\qquad + \varpi'(t)[\beta\widetilde{Q}_3 - F'\widetilde{Q}_3 - \widetilde{Q}_3F - F_1'\widetilde{Q}_3F_1 \\ &\qquad - H_2'\widetilde{Q}_1^{-1}H_2 - \lambda H_3'\widetilde{Q}_1^{-1}H_3]\varpi(t) \end{aligned} \tag{37}$$

Pre-multiplying and post-multiplying both sides of inequality (15), by $diag\{\widetilde{Q}_1^{-1}, I, \widetilde{Q}_1^{-1}, \widetilde{Q}_1^{-1}\}$, according to Schur complement, one can obtain that

$$\begin{bmatrix} \Sigma + A_{21}'\widetilde{Q}_1^{-1}A_{21} & \Sigma_1 + \widetilde{Q}_1^{-1}H_1 + A_{21}'\widetilde{Q}_1^{-1}H_2 \\ * & \Sigma_2 + H_2'\widetilde{Q}_1^{-1}H_2 \end{bmatrix} < 0, \tag{38}$$

where

$$\begin{aligned} \Sigma &= \beta\widetilde{Q}_1^{-1} + \lambda\widetilde{Q}_1^{-1} - A_{11}'\widetilde{Q}_1^{-1} - \widetilde{Q}_1^{-1}A_{11} \\ &\quad + \lambda(I + A_{31})'\widetilde{Q}_1^{-1}(I + A_{31}), \\ \Sigma_1 &= \lambda(I + A_{31})'\widetilde{Q}_1^{-1}H_3, \\ \Sigma_2 &= \beta\widetilde{Q}_3 - F'\widetilde{Q}_3 - \widetilde{Q}_3F - F_1'\widetilde{Q}_3F_1 + \lambda H_3'\widetilde{Q}_1^{-1}H_3. \end{aligned}$$

Considering (37) and (38), one obtains

$$\ell\nu_2(x(t), \varpi(t)) > \beta\nu_2(x(t), \varpi(t)). \tag{39}$$

Then, integrating from 0 to t with $t \in [0, T]$ and taking the mathematical expectation on both sides of (39), one has

$$\begin{aligned} &\mathrm{E}\nu_2(x(t), \varpi(t)) \\ &\quad > \mathrm{E}\nu_2(x(0), \varpi(0)) + \beta \int_0^t E\nu_2(x(s), \varpi(s))ds. \end{aligned} \tag{40}$$

Using Lemma 3, it yields

$$\mathrm{E}v_2(x(t),\varpi(t)) > \mathrm{E}v_2(x(0),\varpi(0))e^{\beta t}. \tag{41}$$

By (19) and (21), one has

$$\begin{aligned}
&\mathrm{E}v_2(x(0),\varpi(0))e^{\beta t}\\
&= \mathrm{E}[x'(0)\widetilde{Q}_1^{-1}x(0)+\varpi'(0)\widetilde{Q}_3\varpi(0)]e^{\beta t}\\
&= \mathrm{E}[x'(0)R^{\frac{1}{2}}Q_1^{-1}R^{\frac{1}{2}}x(0)+\varpi'(0)R_1^{\frac{1}{2}}Q_3R_1^{\frac{1}{2}}\varpi(0)]e^{\beta t}\\
&\geq r_{\min}(Q_1^{-1})\delta_3=\frac{1}{r_{\max}(Q_1)}\delta_3>\delta_3. \qquad (42)\\
&\mathrm{E}v_2(x(t),\varpi(t))\\
&= \mathrm{E}[x'(t)\widetilde{Q}_1^{-1}x(t)+\varpi'(t)\widetilde{Q}_3\varpi(t)]\\
&= \mathrm{E}[x'(t)R^{\frac{1}{2}}Q_1^{-1}R^{\frac{1}{2}}x(t)\\
&\quad +\varpi'(t)R_1^{\frac{1}{2}}Q_3R_1^{\frac{1}{2}}\varpi(t)]\\
&\leq \mathrm{E}[r_{\max}(Q_1^{-1})x'(t)Rx(t)\\
&\quad +r_{\max}(Q_3)\varpi'(t)R_1\varpi(t)]\\
&= \mathrm{E}[\frac{1}{r_{\min}(Q_1)}x'(t)Rx(t)+r_{\max}(Q_3)\varpi'(t)R_1\varpi(t)]\\
&< \frac{1}{r_1}\mathrm{E}[x'(t)Rx(t)]+r_3f. \qquad (43)
\end{aligned}$$

By (41), (42) and (43),

$$\delta_3 < \frac{1}{r_1}\mathrm{E}\left[x'(t)Rx(t)\right]+r_3f. \tag{44}$$

From (18), one has

$$\delta_1-\delta_3r_1+r_3fr_1<0, \tag{45}$$

and (45) is equivalent to

$$\delta_1<(\delta_3-r_3f)r_1. \tag{46}$$

From (44) and (46), it is easy to obtain

$$\delta_1<\mathrm{E}[x'(t)Rx(t)]. \tag{47}$$

This completes the proof. □

Remark 2: For system (1) with $u(t) = 0$ (i.e. Itô-type stochastic linear with Wiener and Poisson random disturbance), the FTADB can be tested by *Theorem 1*.

IV. FINITE-TIME ANNULAR DOMAIN BOUNDED CONTROLLER DESIGN

In this section, in order to deal with the system (1) is FTADB, different quadratic functions are constructed to design the SFC and OFC.

A. STATE FEEDBACK FINITE-TIME ANNULAR DOMAIN BOUNDED CONTROLLER DESIGN

For system (1), consider the following SFC

$$u(t)=Kx(t), \tag{48}$$

where K is the feedback controller gain to be designed.

Substituting (48) into system (1), one gets

$$\begin{cases} dx(t)=[(A_{11}+B_{11}K)x(t)+H_1\varpi(t)]dt\\ +[(A_{21}+B_{21}K)x(t)+H_2\varpi(t)]dw(t)\\ +[(A_{31}+B_{31}K)x(t)+H_3\varpi(t)]dn(t)\\ \;y(t)=C_1x(t),\quad x(0)=x_0\in R^n. \end{cases} \tag{49}$$

Next, the following theorem provides sufficient conditions to guarantee the system (49) is FTADB.

Theorem 2: Given a matrix $R>0$, and positive scalars δ_1, δ_2, δ_3, δ_4, T, with $\delta_2>\delta_4>\delta_3>\delta_1>0$, the system (49) is FTADB with respect to $(\delta_1,\delta_2,\delta_3,\delta_4,\Re,T,R)$, if there exist symmetric matrices $Q_1>0$, $Q_2>0$, $Q_3>0$, some scalars $r_i, i=1,2,3$, $\alpha\geq 0$, $\beta\geq 0$, and a suitable dimensions matrix U, such that the following inequalities hold

$$\begin{bmatrix} \Delta & H_1 & \Delta_1 & \widetilde{Q}_1A_{21}'+U'B_{21}'\\ * & \Gamma_1-\alpha\widetilde{Q}_2 & \sqrt{\lambda}H_3' & H_2'\\ * & * & -\widetilde{Q}_1 & 0\\ * & * & * & -\widetilde{Q}_1 \end{bmatrix}<0, \tag{50}$$

$$\begin{bmatrix} \Delta_2 & H_1 & \Delta_1 & \widetilde{Q}_1A_{21}'+U'B_{21}'\\ * & \beta\widetilde{Q}_2-\Gamma_2 & \sqrt{\lambda}H_3' & H_2'\\ * & * & -\widetilde{Q}_1 & 0\\ * & * & * & -\widetilde{Q}_1 \end{bmatrix}<0, \tag{51}$$

where

$$\begin{aligned}
\widetilde{Q}_1 &= R^{-\frac{1}{2}}Q_1R^{-\frac{1}{2}},\quad \widetilde{Q}_2=R_1^{\frac{1}{2}}Q_2R_1^{\frac{1}{2}},\quad \widetilde{Q}_3=R_1^{\frac{1}{2}}Q_3R_1^{\frac{1}{2}},\\
\Delta &= \widetilde{Q}_1A_{11}'+A_{11}\widetilde{Q}_1+U'B_{11}'+B_{11}U-\lambda\widetilde{Q}_1-\alpha\widetilde{Q}_1,\\
\Delta_1 &= \sqrt{\lambda}\widetilde{Q}_1(I+A_{31})'+\sqrt{\lambda}U'B_{31}',\\
\Delta_2 &= \beta\widetilde{Q}_1+\lambda\widetilde{Q}_1-\widetilde{Q}_1A_{11}'-A_{11}\widetilde{Q}_1-U'B_{11}'-B_{11}U.
\end{aligned}$$

In this case, a desired controller gain is given by $K = U\widetilde{Q}_1^{-1}$.

Proof: By replacing A_{11} by $A_{11}+B_{11}K$, A_{21} by $A_{21}+B_{21}K$ and A_{31} by $A_{31}+B_{31}K$ in *Theorem 2*. One can obtain that conditions (15), (16) and

$$\begin{bmatrix} \Delta^* & H_1 & \Delta_1^* & \widetilde{Q}_1(A_{21}+B_{21}K)'\\ * & \Gamma_1-\alpha\widetilde{Q}_2 & \sqrt{\lambda}H_3' & H_2'\\ * & * & -\widetilde{Q}_1 & 0\\ * & * & * & -\widetilde{Q}_1 \end{bmatrix}<0, \tag{52}$$

$$\begin{bmatrix} \Delta_2^* & H_1 & \Delta_1^* & \widetilde{Q}_1(A_{21}+B_{21}K)'\\ * & \beta\widetilde{Q}_2-\Gamma_2 & \sqrt{\lambda}H_3' & H_2'\\ * & * & -\widetilde{Q}_1 & 0\\ * & * & * & -\widetilde{Q}_1 \end{bmatrix}<0, \tag{53}$$

hold. Let $U = K\widetilde{Q}_1$, it can be seen that (52) and (53) are derived from (50) and (51), where

$$\begin{aligned}
\Delta^* &= (A_{11}+B_{11}K)\widetilde{Q}_1+\widetilde{Q}_1(A_{11}+B_{11}K)'-\lambda\widetilde{Q}_1-\alpha\widetilde{Q}_1,\\
\Delta_1^* &= \sqrt{\lambda}\widetilde{Q}_1(I+A_{31}+B_{31}K)',\\
\Delta_2^* &= \beta\widetilde{Q}_1+\lambda\widetilde{Q}_1-\widetilde{Q}_1(A_{11}+B_{11}K)'-(A_{11}+B_{11}K)\widetilde{Q}_1.
\end{aligned}$$

This completes the proof. □

B. DYNAMIC OUTPUT FEEDBACK FINITE-TIME ANNULAR DOMAIN BOUNDED CONTROLLER DESIGN

It is well known that the SFC may fail, when the system states are not fully accessible. Therefore, we propose an OFC.

Assumption 1: There exists a SFC $v(t)=Kx(t)$ which has been designed using the results of *Theorem 2*.

An observer-based controller with appropriate dimensions is selected as follows

$$\begin{cases} d\hat{x}(t)=[A_{11}\hat{x}(t)+B_{11}v(t)+L(y(t)-C_1\hat{x}(t))]dt \\ +[A_{21}\hat{x}(t)+B_{21}v(t)+L(y(t)-C_1\hat{x}(t))]dw(t) \\ +[A_{31}\hat{x}(t)+B_{31}v(t)+L(y(t)-C_1\hat{x}(t))]dn(t) \\ v(t)=K\hat{x}(t),\ \hat{x}(0)=\hat{x}_0 \in R^n, \end{cases} \tag{54}$$

where $\hat{x}(t)$ is the estimate of the state of $x(t)$ and L is an estimator gain matrix with appropriate dimensions.

Define $e(t) = x(t) - \hat{x}(t)$, then one gets the error system as follows

$$\begin{aligned} de(t) = {} & [(A_{11}-LC_1)e(t)+H_1\varpi(t)]dt \\ & +[(A_{21}-LC_1)e(t)+H_2\varpi(t)]dw(t) \\ & +[(A_{31}-LC_1)e(t)+H_3\varpi(t)]dn(t). \end{aligned} \tag{55}$$

In general, it is required to satisfy $\mathrm{E}[e'(t)Re(t)] < 1$, $t \in [0, T]$.

Let $z(t) = [x'(t)\ e'(t)\ \varpi'(t)]'$; then one gets the following augmented system

$$\begin{cases} dz(t) = \tilde{A}_1 z(t)dt + \tilde{A}_2 z(t)dw(t) + \tilde{A}_3 z(t)dn(t), \\ z(0) = [x_0'\ e_0'\ \varpi_0']' \in R^{2n+l}, \end{cases} \tag{56}$$

where

$$\tilde{A}_1 = \begin{bmatrix} A_{11}+B_{11}K & -B_{11}K & H_1 \\ 0 & A_{11}-LC_1 & H_1 \\ 0 & 0 & F \end{bmatrix},\quad \tilde{A}_2 = \begin{bmatrix} A_{21}+B_{21}K & -B_{21}K & H_2 \\ 0 & A_{21}-LC_1 & H_2 \\ 0 & 0 & F_1 \end{bmatrix},\quad \tilde{A}_3 = \begin{bmatrix} A_{31}+B_{31}K & -B_{31}K & H_3 \\ 0 & A_{31}-LC_1 & H_3 \\ 0 & 0 & 0 \end{bmatrix}. \tag{57}$$

According to *Assumption 1*, the following theorem is given to solve the sufficient conditions of the existence of L

Theorem 3: Given a matrix $R > 0$, and positive scalars δ_1, δ_2, δ_3, δ_4, T, with $\delta_2 > \delta_4 > \delta_3 > \delta_1 > 0$, the system (56) is FTADB with respect to $(\delta_1, \delta_2, \delta_3, \delta_4, \Re, T, R)$, if there exist symmetric matrices $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, some scalars $\tau_i > 0$, $i = 1, 2, \cdots, 7$, $\alpha \geq 0$, $\beta \geq 0$, a suitable dimensions matrix V, and $L = \widetilde{Q}_2^{-1}V$, such that the following inequalities hold

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & 0 & 0 \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} \\ * & * & \Omega_{33} & 0 & 0 \\ * & * & * & -\widetilde{Q}_2 & 0 \\ * & * & * & 0 & -\widetilde{Q}_2 \end{bmatrix} < 0, \tag{58}$$

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & 0 & 0 \\ * & \Upsilon_{22} & \Upsilon_{23} & \Upsilon_{24} & \Upsilon_{25} \\ * & * & \Upsilon_{33} & 0 & 0 \\ * & * & * & -\widetilde{Q}_2 & 0 \\ * & * & * & 0 & -\widetilde{Q}_2 \end{bmatrix} < 0, \tag{59}$$

$$\tau_4 I < Q_1 < \tau_1 I,\quad 0 < Q_2 < \tau_2 I,\quad 0 < Q_3 < \tau_3 I, \tag{60}$$

$$\tau_6 I < Q_4 < \tau_5 I, \tag{61}$$

$$0 < Q_5 < \tau_7 I, \tag{62}$$

$$\tau_1\delta_4 + \tau_2 + \tau_3 f_1 \leq \delta_2 \exp(-\alpha T)\tau_4, \tag{63}$$

$$\delta_1\tau_5 - \delta_3\tau_6 + \tau_2 + \tau_7 f < 0, \tag{64}$$

where

$$\begin{aligned}
\widetilde{Q}_1 &= R^{\frac{1}{2}} Q_1 R^{\frac{1}{2}},\quad \widetilde{Q}_2 = R^{\frac{1}{2}} Q_2 R^{\frac{1}{2}},\quad \widetilde{Q}_3 = R_1^{\frac{1}{2}} Q_3 R_1^{\frac{1}{2}}, \\
\widetilde{Q}_4 &= R^{\frac{1}{2}} Q_4 R^{\frac{1}{2}},\quad \widetilde{Q}_5 = R_1^{\frac{1}{2}} Q_5 R_1^{\frac{1}{2}}, \\
\Omega_{11} &= (A_{11}+B_{11}K)'\widetilde{Q}_1 + \widetilde{Q}_1(A_{11}+B_{11}K) \\
&\quad + (A_{21}+B_{21}K)'\widetilde{Q}_1(A_{21}+B_{21}K) \\
&\quad + \lambda[(A_{31}+B_{31}K)'\widetilde{Q}_1 + \widetilde{Q}_1(A_{31}+B_{31}K) \\
&\quad + (A_{31}+B_{31}K)'\widetilde{Q}_1(A_{31}+B_{31}K)] - \alpha\widetilde{Q}_1, \\
\Omega_{12} &= -\widetilde{Q}_1 B_{11}K - (A_{21}+B_{21}K)'\widetilde{Q}_1 B_{21}K - \lambda[\widetilde{Q}_1 B_{31}K \\
&\quad + (A_{31}+B_{31}K)'\widetilde{Q}_1 B_{31}K], \\
\Omega_{13} &= \widetilde{Q}_1 H_1 + (A_{21}+B_{21}K)'\widetilde{Q}_1 H_2 \\
&\quad + \lambda[\widetilde{Q}_1 H_3 + (A_{31}+B_{31}K)'\widetilde{Q}_1 H_3], \\
\Omega_{22} &= A_{11}'\widetilde{Q}_2 + \widetilde{Q}_2 A_{11} - C_1'V' - VC_1 + K'B_{21}'\widetilde{Q}_1 B_{21}K \\
&\quad + \lambda[A_{31}\widetilde{Q}_2 + \widetilde{Q}_2 A_{31} - C_1'V' - VC_1 + K'B_{31}'\widetilde{Q}_1 B_{31}K] \\
&\quad - \alpha\widetilde{Q}_2, \\
\Omega_{23} &= \widetilde{Q}_2 H_1 + A_{21}'\widetilde{Q}_2 H_{21} - C_1'V'H_2 - K'B_{21}'\widetilde{Q}_1 H_2 \\
&\quad + \lambda[\widetilde{Q}_1 H_3 + A_{31}'\widetilde{Q}_2 H_3 - C_1'V'H_3 - K'B_{31}'\widetilde{Q}_1 H_3], \\
\Omega_{24} &= A_{21}'\widetilde{Q}_2 - C_1'V', \\
\Omega_{25} &= \sqrt{\lambda}A_{31}'\widetilde{Q}_2 - \sqrt{\lambda}C_1'V', \\
\Omega_{33} &= F'\widetilde{Q}_3 + \widetilde{Q}_3 F + H_2'\widetilde{Q}_1 H_2 + H_2'\widetilde{Q}_2 H_2 + F_1'\widetilde{Q}_3 F_1 \\
&\quad + \lambda[H_3'\widetilde{Q}_1 H_3 + H_3'\widetilde{Q}_2 H_3] - \alpha\widetilde{Q}_3, \\
\Upsilon_{11} &= \beta\widetilde{Q}_4 - (A_{11}+B_{11}K)'\widetilde{Q}_4 - \widetilde{Q}_4(A_{11}+B_{11}K) \\
&\quad - (A_{21}+B_{21}K)'\widetilde{Q}_4(A_{21}+B_{21}K) \\
&\quad - \lambda[(A_{31}+B_{31}K)'\widetilde{Q}_4 + \widetilde{Q}_4(A_{31}+B_{31}K) \\
&\quad + (A_{31}+B_{31}K)'\widetilde{Q}_4(A_{31}+B_{31}K)], \\
\Upsilon_{12} &= \widetilde{Q}_4 B_{11}K + (A_{21}+B_{21}K)'\widetilde{Q}_4 B_{21}K \\
&\quad + \lambda[\widetilde{Q}_4 B_{31}K + (A_{31}+B_{31}K)'\widetilde{Q}_4 B_{31}K], \\
\Upsilon_{13} &= -\widetilde{Q}_4 H_1 - (A_{21}+B_{21}K)'\widetilde{Q}_4 H_2 \\
&\quad - \lambda[\widetilde{Q}_4 H_3 + (A_{31}+B_{31}K)'\widetilde{Q}_4 H_3], \\
\Upsilon_{22} &= -A_{11}'\widetilde{Q}_2 - \widetilde{Q}_2 A_{11} + C_1'V' + VC_1 - K'B_{21}'\widetilde{Q}_2 B_{21}K \\
&\quad - \lambda[A_{31}\widetilde{Q}_2 + \widetilde{Q}_2 A_{31} - C_1'V' - VC_1 \\
&\quad + K'B_{31}'\widetilde{Q}_2 B_{31}K] + \beta\widetilde{Q}_2, \\
\Upsilon_{23} &= -\widetilde{Q}_2 H_1 - A_{21}'\widetilde{Q}_2 H_2 + C_1'V'H_2 + K'B_{21}'\widetilde{Q}_4 H_2 \\
&\quad - \lambda[\widetilde{Q}_2 H_3 + A_{31}'\widetilde{Q}_2 H_3 - C_1'V'H_3 - K'B_{31}'\widetilde{Q}_4 H_3], \\
\Upsilon_{24} &= A_{21}'\widetilde{Q}_2 - C_1'V', \\
\Upsilon_{25} &= \sqrt{\lambda}A_{31}'\widetilde{Q}_2 - \sqrt{\lambda}C_1'V', \\
\Upsilon_{33} &= -F'\widetilde{Q}_5 - \widetilde{Q}_5 F - H_2'\widetilde{Q}_4 H_2 - H_2'\widetilde{Q}_2 H_2 - F_1'\widetilde{Q}_5 F_1 \\
&\quad - \lambda[H_3'\widetilde{Q}_4 H_3 + H_3'\widetilde{Q}_2 H_3] + \beta\widetilde{Q}_5.
\end{aligned}$$

Proof: The proof is divided into two steps.

$$\textit{Step}1:\ \mathrm{E}[x'(t_0)Rx(t_0)] \leq \delta_4 \Rightarrow \mathrm{E}[x'(t)Rx(t)] < \delta_2$$

Let $\Theta = diag\{\widetilde{Q}_1, \widetilde{Q}_2, \widetilde{Q}_3\}$, $\widetilde{Q}_1 > 0$, $\widetilde{Q}_2 > 0$ and $\widetilde{Q}_3 > 0$ being solutions to (58)-(64).

Define $z(t) = [x'(t)\ e'(t)\ \varpi'(t)]'$, one has

$$\begin{aligned} v_3(z(t)) &= z'(t)\Theta z(t) \\ &= x'(t)\widetilde{Q}_1 x(t) + e'(t)\widetilde{Q}_2 e(t) \\ &\quad + \varpi'(t)\widetilde{Q}_3\varpi(t), \end{aligned} \tag{65}$$

where $\widetilde{Q}_1 = R^{\frac{1}{2}}Q_1R^{\frac{1}{2}}$, $\widetilde{Q}_2 = R^{\frac{1}{2}}Q_2R^{\frac{1}{2}}$, $\widetilde{Q}_3 = R_1^{\frac{1}{2}}Q_3R_1^{\frac{1}{2}}$.

For $v_3(z(t))$, according to Itô formula along with the state trajectory of (56), one obtains

$$\begin{aligned} &\ell v_3(z(t)) \\ &= (\tilde{A}_1 z(t))'\Theta z(t) + z'(t)\Theta(\tilde{A}_1 z(t)) \\ &\quad + (\tilde{A}_2 z(t))'\Theta(\tilde{A}_2 z(t)) \\ &\quad + \lambda[(z(t) + \tilde{A}_3 z(t))'\Theta(z(t) + \tilde{A}_3 z(t)) \\ &\quad - z(t)'\Theta z(t)] \\ &= z(t)'[\tilde{A}_1'\Theta + \Theta\tilde{A}_1 + \tilde{A}_2'\Theta\tilde{A}_2 + \lambda(\tilde{A}_3'\Theta + \Theta\tilde{A}_3 \\ &\quad + \tilde{A}_3'\Theta\tilde{A}_3)]z(t) \\ &= [x(t)'\ e(t)'\ \varpi(t)']\bar{Z}[x(t)'\ e(t)'\ \varpi(t)']', \end{aligned} \tag{66}$$

where

$$\bar{Z} = \begin{bmatrix} \Pi_{11} & \Omega_{12} & \Omega_{13} \\ * & \Pi_{22} & \Pi_{23} \\ * & * & \Pi_{33} \end{bmatrix}, \tag{67}$$

$$\begin{aligned} \Pi_{11} &= (A_{11} + B_{11}K)'\widetilde{Q}_1 + \widetilde{Q}_1(A_{11} + B_{11}K) \\ &\quad + (A_{21} + B_{21}K)'\,\widetilde{Q}_1\,(A_{21} + B_{21}K) \\ &\quad + \lambda[(A_{31} + B_{31}K)'\widetilde{Q}_1 + \widetilde{Q}_1(A_{31} + B_{31}K) \\ &\quad + (A_{31} + B_{31}K)'\,\widetilde{Q}_1\,(A_{31} + B_{31}K)], \\ \Pi_{22} &= (A_{11} - LC_1)'\widetilde{Q}_2 + \widetilde{Q}_2(A_{11} - LC_1) \\ &\quad + (A_{21} - LC_1)'\widetilde{Q}_2(A_{21} - LC_1) + K'B_{21}'\widetilde{Q}_1B_{21}K \\ &\quad + \lambda[(A_{31} - LC_1)'\widetilde{Q}_2 + \widetilde{Q}_2(A_{31} - LC_1) \\ &\quad + (A_{31} - LC_1)'\widetilde{Q}_2(A_{31} - LC_1) + K'B_{31}'\widetilde{Q}_1B_{31}K], \\ \Pi_{23} &= \widetilde{Q}_2H_1 + (A_{21} - LC_1)'\widetilde{Q}_2H_2 - K'B_{21}'\widetilde{Q}_1H_2 \\ &\quad + \lambda[\widetilde{Q}_1H_3 + (A_{31} - LC_1)'\widetilde{Q}_2H_3 - KB_{31}'\widetilde{Q}_1H_3], \\ \Pi_{33} &= F'\widetilde{Q}_3 + \widetilde{Q}_3F + H_2'\widetilde{Q}_1H_2 + H_2'\widetilde{Q}_2H_2 + F_1'\widetilde{Q}_3F_1 \\ &\quad + \lambda[H_3'\widetilde{Q}_1H_3 + H_3'\widetilde{Q}_2H_3], \end{aligned}$$

which leads to

$$\ell v_3(z(t)) - \alpha v_3(z(t)) = z'(t)\bar{Z}^* z(t), \tag{68}$$

where

$$\bar{Z}^* = \begin{bmatrix} \Pi_{11} - \alpha\widetilde{Q}_1 & \Omega_{12} & \Omega_{13} \\ * & \Pi_{22} - \alpha\widetilde{Q}_2 & \Pi_{23} \\ * & * & \Pi_{33} - \alpha\widetilde{Q}_3 \end{bmatrix}.$$

According to Schur complement, and let $V = \widetilde{Q}_2L$, condition (58) can be rewritten as

$$\bar{Z}^* < 0. \tag{69}$$

It is obvious that (68) and (69) give

$$\ell v_3(z(t)) < \alpha v_3(z(t)). \tag{70}$$

Then, integrate from 0 to t with $t \in [0, T]$ and take the mathematical expectation on both sides of (70), one has

$$\mathrm{E}v_3(z(t)) < \mathrm{E}v_3(z(0)) + \alpha\int_0^t \mathrm{E}v_3(z(s))ds. \tag{71}$$

According to Lemma 2, one has

$$\mathrm{E}v_3(z(t)) < \mathrm{E}v_3(z(0))e^{\alpha t}. \tag{72}$$

Considering (60), one obtains

$$\begin{aligned} &\mathrm{E}v_3(z(0))e^{\alpha t} \\ &= \mathrm{E}[z'(0)\Theta z(0)]e^{\alpha t} \\ &= \mathrm{E}[x'(0)\widetilde{Q}_1x(0) + e'(0)\widetilde{Q}_2e(0) \\ &\quad + \varpi'(0)\widetilde{Q}_3\varpi(0)]e^{\alpha t} \\ &= \mathrm{E}[x'(0)R^{\frac{1}{2}}Q_1R^{\frac{1}{2}}x(0) + e'(0)R^{\frac{1}{2}}Q_2R^{\frac{1}{2}}e(0) \\ &\quad + \varpi'(0)R_1^{\frac{1}{2}}Q_3R_1^{\frac{1}{2}}\varpi(0)]e^{\alpha t} \\ &\leq [r_{\max}(Q_1)\delta_4 + r_{\max}(Q_2) + r_{\max}(Q_3)f_1]e^{\alpha T} \\ &< [\tau_1\delta_4 + \tau_2 + \tau_3f_1]e^{\alpha T}, \end{aligned} \tag{73}$$

$$\begin{aligned} &\mathrm{E}v_3(z(t)) \\ &= \mathrm{E}[z'(t)\Theta z(t)] \\ &= \mathrm{E}[x'(t)\widetilde{Q}_1x(t) + e'(t)\widetilde{Q}_2e(t) + \varpi'(t)\widetilde{Q}_3\varpi(t)] \\ &= \mathrm{E}[x'(t)R^{\frac{1}{2}}Q_1R^{\frac{1}{2}}x(t) + e'(t)R^{\frac{1}{2}}Q_2R^{\frac{1}{2}}e(t) \\ &\quad + \varpi'(t)R_1^{\frac{1}{2}}Q_3R_1^{\frac{1}{2}}\varpi(t)] \\ &\geq \mathrm{E}[r_{\min}(Q_1)\,x'(t)Rx(t) + r_{\min}(Q_2)\,e'(t)Re(t) \\ &\quad + r_{\min}(Q_3)\,\varpi'(t)R_1\varpi(t)] \\ &\geq \mathrm{E}[r_{\min}(Q_1)\,x'(t)Rx(t)] \\ &> \tau_4\mathrm{E}[x'(t)Rx(t)]. \end{aligned} \tag{74}$$

According to (72), (73) and (74), it follows

$$\mathrm{E}[x'(t)Rx(t)] < \frac{1}{\tau_4}[\tau_1\delta_4 + \tau_2 + \tau_3f_1]\,e^{\alpha T}. \tag{75}$$

According to (75) and (63), one obtains $\mathrm{E}[x'(t)Rx(t)] < \delta_2$ for all $t \in [0, T]$.

Step 2 : $\delta_3 \leq \mathrm{E}[x'(t_0)Rx(t_0)] \Rightarrow \delta_1 < \mathrm{E}[x'(t)Rx(t)]$

Let $\bar{\Theta} = diag\{\widetilde{Q}_4, \widetilde{Q}_2, \widetilde{Q}_5\}$, $\widetilde{Q}_4 > 0$, $\widetilde{Q}_2 > 0$ and $\widetilde{Q}_5 > 0$ being solutions to (58)-(64), and $z(t) = [x'(t)\ e'(t)\ \varpi'(t)]'$; one gets

$$\begin{aligned} v_4(z(t)) &= z'(t)\bar{\Theta}z(t) \\ &= x'(t)\widetilde{Q}_4x(t) + e'(t)\widetilde{Q}_2e(t) \\ &\quad + \varpi'(t)\widetilde{Q}_5\varpi(t), \end{aligned} \tag{76}$$

where $\widetilde{Q}_4 = R^{\frac{1}{2}}Q_4R^{\frac{1}{2}}$, $\widetilde{Q}_2 = R^{\frac{1}{2}}Q_2R^{\frac{1}{2}}$, $\widetilde{Q}_5 = R_1^{\frac{1}{2}}Q_5R_1^{\frac{1}{2}}$.

For $v_4(z(t))$, according to Lemma 1, one can obtain

$$\begin{aligned} &\ell v_4(z(t)) \\ &= (\tilde{A}_1 z(t))'\bar{\Theta}z(t) + z'(t)\bar{\Theta}(\tilde{A}_1 z(t)) \\ &\quad + (\tilde{A}_2 z(t))'\bar{\Theta}(\tilde{A}_2 z(t)) \\ &\quad + \lambda[(z(t) + \tilde{A}_3 z(t))'\bar{\Theta}(z(t) + \tilde{A}_3 z(t)) \\ &\quad - z(t)'\bar{\Theta}z(t)] \\ &= z(t)'[\tilde{A}_1'\bar{\Theta} + \bar{\Theta}\tilde{A}_1 + \tilde{A}_2'\bar{\Theta}\tilde{A}_2 + \lambda(\tilde{A}_3'\bar{\Theta} + \bar{\Theta}\tilde{A}_3 \\ &\quad + \tilde{A}_3'\bar{\Theta}\tilde{A}_3)]z(t) \\ &= [x'(t)\ e'(t)\ \varpi'(t)]\tilde{Z}[x'(t)\ e'(t)\ \varpi'(t)]', \end{aligned} \tag{77}$$

where

$$\tilde{Z} = \begin{bmatrix} \Xi_{11} & -\Upsilon_{12} & -\Upsilon_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33} \end{bmatrix}, \tag{78}$$

$$\begin{aligned}
\Xi_{11} &= (A_{11} + B_{11}K)'\widetilde{Q}_4 + \widetilde{Q}_4(A_{11} + B_{11}K) \\
&\quad + (A_{21} + B_{21}K)'\,\widetilde{Q}_4\,(A_{21} + B_{21}K) \\
&\quad + \lambda[(A_{31} + B_{31}K)'\widetilde{Q}_4 + \widetilde{Q}_4(A_{31} + B_{31}K) \\
&\quad + (A_{31} + B_{31}K)'\,\widetilde{Q}_4\,(A_{31} + B_{31}K)], \\
\Xi_{22} &= (A_{11} - LC_1)'\widetilde{Q}_2 + \widetilde{Q}_2(A_{11} - LC_1) \\
&\quad + (A_{21} - LC_1)'\widetilde{Q}_2(A_{21} - LC_1) + K'B_{21}'\widetilde{Q}_1 B_{21}K \\
&\quad + \lambda[(A_{31} - LC_1)'\widetilde{Q}_2 + \widetilde{Q}_2(A_{31} - LC_1) \\
&\quad + (A_{31} - LC_1)'\widetilde{Q}_2(A_{31} - LC_1) + K'B_{31}'\widetilde{Q}_4 B_{31}K], \\
\Xi_{23} &= \widetilde{Q}_2 H_1 + (A_{21} - LC_1)'\widetilde{Q}_2 H_2 - K'B_{21}'\widetilde{Q}_4 H_2 \\
&\quad + \lambda[\widetilde{Q}_2 H_3 + (A_{31} - LC_1)'\widetilde{Q}_2 H_3 - K'B_{31}'\widetilde{Q}_4 H_3], \\
\Xi_{33} &= F'\widetilde{Q}_5 + \widetilde{Q}_5 F + H_2'\widetilde{Q}_4 H_2 + H_2'\widetilde{Q}_2 H_2 + F_1'\widetilde{Q}_5 F_1 \\
&\quad + \lambda[H_3'\widetilde{Q}_4 H_3 + H_3'\widetilde{Q}_2 H_3],
\end{aligned}$$

which leads to

$$\beta v_4(z(t)) - \ell v_4(z(t)) = z'(t)\tilde{Z}^* z(t), \tag{79}$$

where

$$\tilde{Z}^* = \begin{bmatrix} \beta\widetilde{Q}_4 - \Xi_{11} & \Upsilon_{12} & \Upsilon_{13} \\ * & \beta\widetilde{Q}_2 - \Xi_{22} & -\Xi_{23} \\ * & * & \beta\widetilde{Q}_5 - \Xi_{33} \end{bmatrix}.$$

Let $V = \widetilde{Q}_2 L$ and by using Schur complement, condition (59) can be rewritten as

$$\tilde{Z}^* < 0. \tag{80}$$

It is obvious that (79) and (80) give

$$\beta v_4(z(t)) < \ell v_4(z(t)). \tag{81}$$

Then, integrate from 0 to t with $t \in [0, T]$ and take the mathematical expectation on both sides of (81), one has

$$\mathrm{E}v_4(z(0)) + \beta \int_0^t \mathrm{E}v_4(z(s))ds < \mathrm{E}v_4(z(t)). \tag{82}$$

By Lemma 3, one has

$$\mathrm{E}v_4(z(0))e^{\beta t} < \mathrm{E}v_4(z(t)). \tag{83}$$

Considering (61) and (62), one obtains

$$\begin{aligned}
&\mathrm{E}v_4(z(0))e^{\beta t} \\
&\quad = \mathrm{E}[z'(0)\bar{\Theta}z(0)]e^{\beta t} \\
&\quad = \mathrm{E}[x'(0)\widetilde{Q}_4 x(0) + e'(0)\widetilde{Q}_2 e(0) \\
&\qquad + \varpi'(0)\widetilde{Q}_5\varpi(0)]e^{\beta t} \\
&\quad = \mathrm{E}[x'(0)R^{\frac{1}{2}}Q_4R^{\frac{1}{2}}x(0) + e'(0)R^{\frac{1}{2}}Q_2R^{\frac{1}{2}}e(0) \\
&\qquad + \varpi'(0)R_1^{\frac{1}{2}}Q_5R_1^{\frac{1}{2}}\varpi(0)]e^{\beta t} \\
&\quad \geq \mathrm{E}[r_{\min}(Q_4)x'(0)Rx(0) + r_{\min}(Q_2)e'(0)Re(0) \\
&\qquad + r_{\min}(Q_5)\varpi'(0)R_1\varpi(0)] \\
&\quad > \tau_6\delta_3,
\end{aligned} \tag{84}$$

$$\begin{aligned}
&\mathrm{E}v_4(z(t)) \\
&\quad = \mathrm{E}[z'(t)\bar{\Theta}z(t)] \\
&\quad = \mathrm{E}[x'(t)\widetilde{Q}_4 x(t) + e'(t)\widetilde{Q}_2 e(t) + \varpi'(t)\widetilde{Q}_5\varpi(t)] \\
&\quad = \mathrm{E}[x'(t)R^{\frac{1}{2}}Q_4R^{\frac{1}{2}}x(t) + e'(t)R^{\frac{1}{2}}Q_2R^{\frac{1}{2}}e(t) \\
&\qquad + \varpi'(t)R_1^{\frac{1}{2}}Q_5R_1^{\frac{1}{2}}\varpi(t)] \\
&\quad \leq \mathrm{E}[r_{\max}(Q_4)\,x'(t)Rx(t) + r_{\max}(Q_2)\,e'(t)Re(t) \\
&\qquad + r_{\max}(Q_5)\,\varpi'(t)R_1\varpi(t)] \\
&\quad < \tau_5\mathrm{E}[x'(t)Rx(t)] + \tau_2 + \tau_7 f.
\end{aligned} \tag{85}$$

From (83), (84) and (85), one finds

$$\frac{\delta_3\tau_6 - \tau_2 - \tau_7 f}{\tau_5} < \mathrm{E}[x'(t)Rx(t)]. \tag{86}$$

Equation (64) gives

$$\delta_1 < \frac{\delta_3\tau_6 - \tau_2 - \tau_7 f}{\tau_5}. \tag{87}$$

So it is easy obtained that

$$\delta_1 < \mathrm{E}[x'(t)Rx(t)], \tag{88}$$

for all with $t \in [0, T]$.

This completes the proof. □

Remark 3: It can be seen from the above that the values of α and β determine the feasibility of Theorem 3. The selection process of α and β is provided in below section.

V. NUMERICAL ALGORITHMS

An algorithm is presented for the results of the paper in this section. The specifics of the stragegy are as follows.

Algorithm 1

Step 1. Set the values of δ_1, δ_2, δ_3, δ_4, $\Re$, T, and R.

Step 2. Take a series of $\alpha_p(p = 1, \cdots, n)$ and a series of $\beta_q(q = 1, \cdots, m)$.

Step 3. Let $p = 1$ and set $\alpha_1 = 0$.

Step 4. Let $q = 1$ and set $\beta_1 = 0$.

Step 5. If (α_p, β_q) such that the conditions (17)-(23) are feasible, then store (α_p, β_q) into $(X(p), Y(q))$ and $\beta_q = \beta_{q+1}$; go to *Step 5*; otherwise, go to the next step.

Step 6. If $p + 1 < n$, then $\alpha_p = \alpha_{p+1}$ and take β_q; return to *Step 5*; otherwise, skip to *Step 7*.

Step 7. Break.

Remark 4: From Algorithm 1, one can get the feasible solution area surrounded by α and β.

VI. NUMERICAL EXAMPLES

In this part, an numerical example with their Matlab simulations are provided to show the effectiveness of the obtained results. The parameters of the system (1) are defined as:

$$A_{11} = \begin{bmatrix} 1.21 & -2.27 \\ 2.57 & 0.82 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0.16 & -0.45 \\ 0.12 & -0.37 \end{bmatrix},$$

$$A_{31} = \begin{bmatrix} 0.5 & 0.2 \\ -0.25 & 0.3 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 2 & -0.8 \\ 1.5 & 0.1 \end{bmatrix},$$

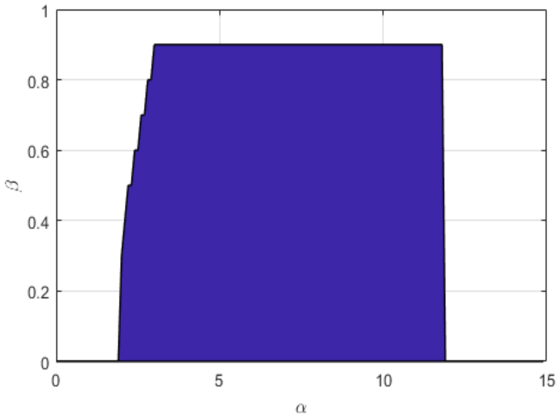

FIGURE 1. A region by α and β in A.

$$B_{21}=\begin{bmatrix}0.9 & 0.5\\ -1 & 1\end{bmatrix},\quad B_{31}=\begin{bmatrix}0.35 & 0.21\\ -0.4 & 0.75\end{bmatrix},$$
$$H_1=\begin{bmatrix}1.1 & 0.05\\ 0.06 & 0.2\end{bmatrix},\quad H_2=\begin{bmatrix}-0.01 & 0.03\\ 0.02 & -0.12\end{bmatrix},$$
$$H_3=\begin{bmatrix}-0.04 & 0.02\\ -0.01 & 0.08\end{bmatrix},\quad F=\begin{bmatrix}0.5 & -0.3\\ -0.6 & 0.5\end{bmatrix},$$
$$F_1=\begin{bmatrix}0.13 & 0.5\\ 0.15 & 0.18\end{bmatrix}, C_1=\begin{bmatrix}1 & 2\end{bmatrix},\ x(0)=\begin{bmatrix}1.5 & -1.5\end{bmatrix}'.$$

and $\delta_1=1$, $\delta_2=35$, $\delta_3=4$, $\delta_4=5$, $f=0.5$, $f_1=0.1$, $T=0.3$, $\lambda=0.25$, $R=R_1=I$.

A. STATE FEEDBACK FINITE-TIME ANNULAR DOMAIN BOUNDED CONTROLLER DESIGN

Applying Algorithm 1 to *Theorem 2*, one can get the feasible solution area surrounded by α and β (See Figure 1 for details).

According to Figure 1, let $\alpha=5$, $\beta=0.4$, and solving (17)-(21) and (50)-(51), one obtains

$$r_1=0.2959,\qquad r_2=53.4000,\qquad r_3=6.6104,$$
$$Q_1=\begin{bmatrix}0.8278 & -0.0034\\ -0.0034 & 0.8624\end{bmatrix},\ Q_2=\begin{bmatrix}25.1521 & -2.1231\\ -2.1231 & 25.9419\end{bmatrix},$$
$$Q_3=\begin{bmatrix}5.0297 & -0.6646\\ -0.6646 & 2.0367\end{bmatrix},\ U=\begin{bmatrix}0.0241 & 0.2392\\ -0.2957 & 0.4024\end{bmatrix}.$$

Thus, one can obtain the feedback gain matrix as follows

$$K=UQ_1^{-1}=\begin{bmatrix}0.0302 & 0.2774\\ -0.3554 & 0.4651\end{bmatrix}.$$

Figure 2 depicts the influence of Poisson jump intensity λ on the system (1). From Figure 2, when $\delta_2=35$ and $\lambda=0.7322$, one can obtain *tmin* is 0, that is there is no solution to the system of matrix inequalities when $\lambda>0.7322$. Furthermore, considering the external disturbance $\varpi(t)=sin(t)$, one can obtain, $4=\delta_3\le \mathrm{E}\left[x'(t_0)Rx(t_0)\right]=4.5\le\delta_4$, and we can get $1=\delta_1<\mathrm{E}\left[x'(t)Rx(t)\right]<\delta_2=35$. Then, the simulation results are shown in Figure 3. From Figure 3, one can get the system of (1) is FTADB with respect to (1, 35, 4, 5, $\Re$, 0.3, I).

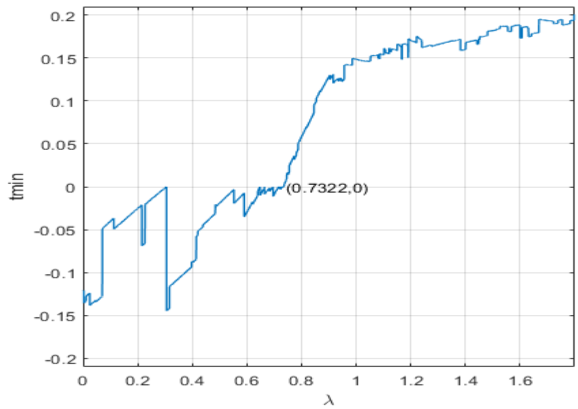

FIGURE 2. When $\lambda\in[0, 1.8]$, the value of *tmin* in *A*.

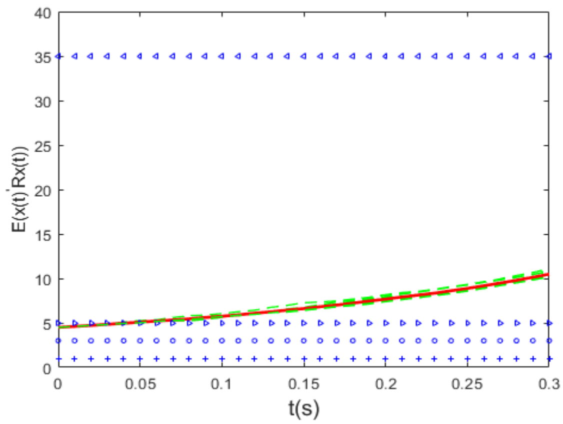

FIGURE 3. The evolution of $\mathrm{E}[x(t)'Rx(t)]$ of the closed-loop system of (1) in *A*.

B. DYNAMIC OUTPUT FEEDBACK FINITE-TIME ANNULAR DOMAIN BOUNDED CONTROLLER DESIGN

Based on SFC design, an observer-based dynamic controller $v(t)=K\hat{x}(t)$ is chosen. Applying Algorithm 1 to *Theorem 3*, one can get the feasible solution area surrounded by α and β (See Figure 4 for details).

According to Figure 4, let $\alpha=6$, $\beta=0.4$, and solving (58)-(64), we obtain

$$\tau_1=142.6099,\qquad \tau_2=48.3473,\qquad \tau_3=88.4166,$$
$$\tau_4=134.7766,\qquad \tau_5=114.9056,\qquad \tau_6=70.6907,$$
$$\tau_7=199.6924,$$
$$Q_1=\begin{bmatrix}138.5253 & -0.0135\\ -0.0135 & 138.4162\end{bmatrix},\ Q_2=\begin{bmatrix}31.2164 & 1.7243\\ 1.7243 & 37.1193\end{bmatrix},$$
$$Q_3=\begin{bmatrix}34.3755 & -2.8616\\ -2.8616 & 27.6025\end{bmatrix},\ Q_4=\begin{bmatrix}79.4562 & 2.8600\\ 2.8600 & 88.6498\end{bmatrix},$$
$$Q_5=\begin{bmatrix}177.7823 & -8.0007\\ -8.0007 & 16.5713\end{bmatrix},\ V=\begin{bmatrix}-2.4203 & -4.5547\end{bmatrix}'.$$

Hence, the observe gain matrix is given by

$$L=\widetilde{Q}_2^{-1}V=\begin{bmatrix}-0.0709 & -0.1194\end{bmatrix}'.$$

Figure 5 depicts the influence of Poisson jump intensity λ on the system (1). From figure 5, when $\delta_2=35$ and $\lambda=1.66$,

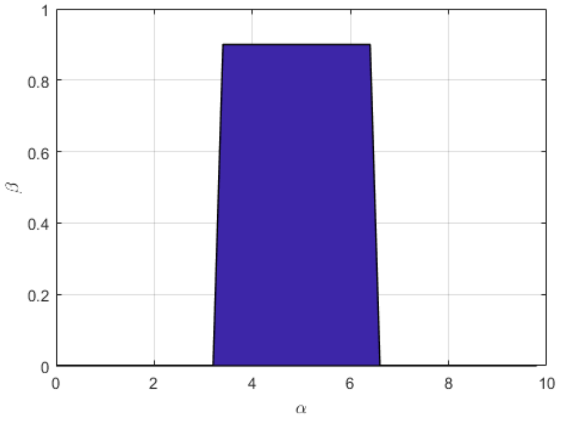

FIGURE 4. A region by α and β in B.

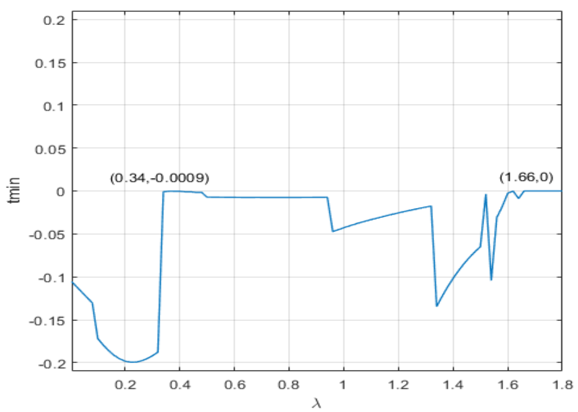

FIGURE 5. When $\lambda \in [0, 1.8]$, the value of $tmin$ in B.

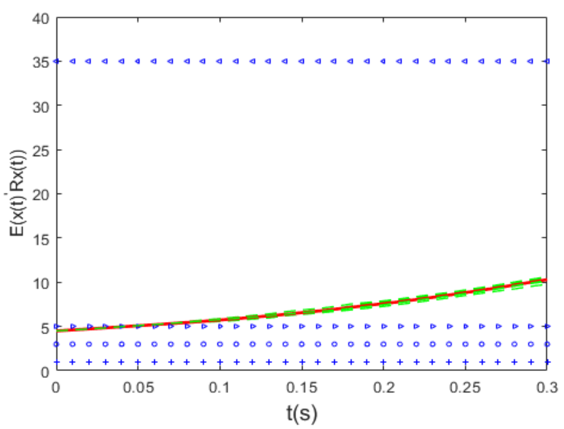

FIGURE 6. The evolution of $E[x(t)'Rx(t)]$ of the system (1) in B.

one can obtain $tmin$ is 0, that is, there is no solution to the system of matrix inequalities when λ>1.66.

Moreover, considering the external disturbance $\varpi(t) = sin(t)$, one can obtain Figure 6 and Figure 7. Specifically, Figure 6 shows that the system (1) is FTADB with respect to $(1, 35, 4, 5, \Re, 0.3, I)$. The evolution of $E[e'(t)Re(t)]$ of the error system of (55), and $E[e'(t)Re(t)] < 1$ are shown in Figure 7.

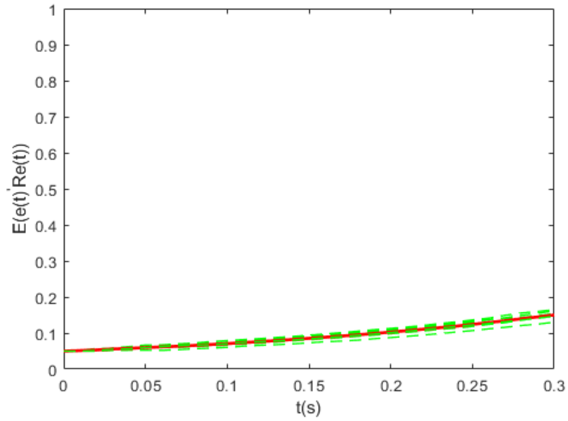

FIGURE 7. The evolution of $E[e'(t)Re(t)]$ in B.

VII. CONCLUSION

Finite-time annular domain bounded control problems of Itô-type stochastic systems with Wiener and Poisson random disturbance are investigated in this paper. Then, using different quadratic function methods, a SFC and an OFC are obtained, respectively. Several sufficient conditions are derived under different controllers. And one numerical example and their Matlab simulations are given to illustrate the feasibility of the proposed theoretical results. In the future, we will study the finite-time control problem subject to some other more complex systems such as Takagi-Sugeno fuzzy system, network system, linear variable parameter system, and so on.

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