IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received January 5, 2021, accepted January 16, 2021, date of publication January 20, 2021, date of current version January 27, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3053046

The Production Routing Problem Under Uncertain Environment

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This work was supported in part by the National Natural Science Foundation of China under Grant 71471038, in part by the Program for Huiyuan Distinguished Young Scholars, University of International Business and Economics (UIBE) under Grant 17JQ09, in part by the Fundamental Research Funds for the Central Universities in UIBE under Grant CXTD10-05, in part by the Youth Innovative Talent Projects of Guangdong's Universities Provincial Key Platforms and Significant Scientific Research Projects under Grant 2018KQNCX362, and in part by the Educational Science Research Projects for 13th Five-Year Plan in Guangdong under Grant 2020GXJK163.

ABSTRACT To reduce waste and cost as well as to promote the efficient and sustainable use of resources in supply chains, a common focus of practice and research is to integrate decisions of various operational processes such as production, inventory, distribution, and routing. The question of which is more representative of the production routing problem (PRP) then arises. In this paper, the focus is on uncertainty in the PRP, and demand and cost uncertainty are considered simultaneously. According to different decision criteria, three uncertain programming models are correspondingly formulated. Then, through researching the crisp equivalence and conversion of the proposed uncertain models, a series of crisp equivalent models are proposed under the assumptions of particular uncertainty distributions, such as linear, zigzag, and normal distributions. To verify the accuracy and usefulness of the proposed models put forward in this paper, a series of experiments are conducted. Finally, several interesting managerial aspects with respect to the relationship between the confidence level and variance of uncertain variables that are gained from the numerical experiments are highlighted. First, the overall cost of PRP grows as the confidence level increases under uncertain environment. Second, the probability that the optimal total cost of the PRP is less than or equal to a given threshold strictly increases as the threshold increases under uncertain environment. Third, in circumstances such as higher confidence level that decision makers generally pay more attention to, the growth of the variance of uncertain variables may lead to the increase of the total cost.

INDEX TERMS Integrated supply chain, production routing problem, uncertain variable, uncertain programming.

I. INTRODUCTION

In this study, the PRP is explored with respect to the integration and coordination of the production, inventory, distribution, and routing operations in supply chains under uncertain settings. It has been clear for some time that the operations integration of a supply chain generates cost savings and greater efficiency. For example, the Chandra and Fisher [1], [2] demonstrated that $3\% - 20\%$ cost savings could be achieved by solving the PRP compared to sequentially and separately solving the problem. More importantly, efficient and sustainable use of resources can only be achieved by explicitly considering the interdependencies of integrated supply-chain services [3]. Among all of the operation schedules and processes in a supply chain, production and transportation activities are regarded as the major factors having a larger impact on the supply chain's sustainability performance. The PRP, focusing on the integration of the main operation schedules in a supply chain, including production, inventory, distribution, and routing operations, initially first investigated by Lei *et al.* [4], has been well studied by both scholars and practitioners. Nananukul [5] presented the definition of PRP systematically and introduced a simpler and more efficient model with which to depict PRP soon after.

Owing to the characteristics of high degree of synergy and distinctly competitive advantage, the PRP has been a research hotspot at home and abroad, which has led to numerous research results since it was founded. The literature on the

The associate editor coordinating the review of this manuscript and approving it for publication was Zeshui Xu.

PRP is analyzed and discussed, and can be divided into four aspects: integrated optimization, algorithms research, rich PRP, and the PRP in green supply chain.

With respect to integrated optimization, several scholars carried out thorough research on the characteristics, relevant elements, and degree of advantage brought by the collaborative integration of production and transportation businesses [6]–[10]. In addition, based on the PRP, Hein *et al.* [11], Absi *et al.* [12] and Darvish *et al.* [13] made comparative studies of collaborative optimization and traditionally separate and sequential decision-making in the supply chain.

Since Lei *et al.* [4] adopted a two-phase heuristic algorithm, scholars have proposed numerous more efficient algorithms to solve the PRP, including exact solution algorithms and heuristic algorithms. Regarding the exact solution algorithms, branch-and-cut was introduced to the research of PRP by Archetti *et al.* [14] and Adulyasak *et al.* [15], focusing on vehicle scheduling with a single uncapacitated or capacitated vehicle and multiple capacitated vehicles accordingly. As the PRP can be viewed as the integration of VRP and LSP, it is a complicated combinatorial optimization problem and difficult to solve only with exact solution algorithms on a large scale. To solve this model more efficiently, some studies employing heuristic procedures appeared in recent years, such as metaheuristics [16], brand and price [17], [18], GRASP [19], the memetic algorithm [20], tabu search [21], [22], the genetic algorithm [23], and ALNS [24], [25].

The optimization model proposed by Nananukul [5] is also considered the benchmark model for solving the PRP, in which case a single commodity is made available at a single plant and then transported to multiple retailers by multiple homogeneous vehicles in multiple periods. Compared with the PRP solved by the generalized model, the rich PRPs refer to the more complicated situations, such as multiple products, multiple manufacturers, and multiple heterogeneous vehicles [26]–[29]. It is important that the vendor managed inventory policy (VMI) is usually adopted in the rich PRPs, in which the retailer's inventory is managed uniformly by the center supplier or manufacturer, and, on the basis of the inventory level, the follow-up production plans and replenishment plans are made accordingly [14]. Compared with traditional replenishment policy, the VMI policy has the advantages of quicker response, lower inventory, and more efficient use of resources.

Moreover, to promote green supply chain operations, some studies focused on PRP from environmental aspects and social impacts. Qiu *et al.* [30] introduced reverse logistics and production remanufacturing into the PRP of closed-loop supply-chain systems in 2018. Several scholars are interested in the PRP's integration with carbon emissions. Two carbon-emission-control policies, the carbon cap-and-trade policy, and carbon tax policy, are considered for the PRP in the research of [31], [32]. The costs caused by carbon emissions and energy consumption are also taken into account in the overall cost consideration in some PRPs [33], [34]. Peng *et al.* [35] focused on the PRP from the social dimension to make the integrated system more sustainable.

The above-cited work has typically focused on deterministic demands and costs. In fact, many indeterminate factors exist in the real world that cannot be ignored when making decisions. Thus, one of the most important tendencies of these studies is to explore the PRP under environments with indeterminate factors, namely randomness, fuzziness, and uncertainty, and, correspondingly, probability theory, fuzzy-set theory, and uncertainty theory. For instance, Adulyasak *et al.* [36] and Agra *et al.* [37] studied the PRP from the visual angle of randomness in which demands of retailers were described as random variables. Furthermore, fuzzy-set theory, initiated by Zadeh [38], has been introduced to the PRP by Moon *et al.* [33] with respect to fuzzy costs.

However, to the best of our knowledge, little research exists on PRP with uncertain settings. In such multi-stage business process integration in a supply chain, the costs and demands are usually unavailable to the managers and may be subject to several inherent indeterministic factors, e.g., changes of production costs, changes of market requirements, and changes of traffic states. It is necessary to identify the distributions of these parameters, which are essential for managers in making decisions, e.g., deciding whether to produce in each period, and then the amount of production, determining the distribution quantities from the plant to each retailer to satisfy the retailers' demands in each period, and choosing the most appropriate transportation routes. Theoretically, one can collect enough samples to estimate the distributions of these parameters before making decisions. However, there are many situations lacking historical data that can be referred to, such as a new product entering the market, or product promotion in a special period. Even if some data are available, an inability to obtain accurate statistics and parameter estimations may be due to some dynamic factors, such as the demand for products sold through online platforms [39]. In addition, merchants will participate in activities or adjust advertising plans from time to time, which will change the exposure rate and affect the expected sales of the product.

Practically, experience data (belief degrees) are often used to estimate the distributions in cases without historical examples by experienced experts. Some surveys have shown that, however, human beings (even the most experienced experts) usually estimate a much wider range of values than is actually required. This makes the belief degrees behave quite different from frequency, indicating that human belief degree given by managers and experts should not be treated as random or fuzzy variables [38]. Instead, uncertainty theory, initiated by Liu [40], [41] based on normality, duality, subadditivity, and product axioms, can be introduced to deal with variables estimated by human belief degrees. Uncertainty theory is mainly used to characterize human belief degree and deal with subjective uncertainty. It has been successfully used to solve many uncertain optimization decision-making problems in supply chains, such as the pricing optimization problem [42]–[44], facility location problem [45], project scheduling problem [46]–[48], and production control problem [49].

In this study, uncertainty theory is employed to depict the costs and demands estimated by expert experience data in practice to build upon previous research on the PRP. Because the situation with uncertainty existing in the PRP is not uncommon in real industries, research into it is increasingly important. How should the supply-chain managers choose the most appropriate operation schedules by using experts' estimations? What effects might the uncertain degrees of the parameters have on the supply-chain operation schedules decisions, and what, then, is the total cost? Additionally, how do the confidence levels and variance of uncertain variables influence the computational results?

To deal with these problems, three uncertain programming models based on uncertainty theory and different decision criteria under uncertain settings are built, and then the crisp equilibrium conversion is correspondingly made. Additionally, different decision criteria are considered to meet the different needs of decision makers, while in most of the existing literature only the expectations criterion is considered. Finally, numerical experiments are performed, and how the confidence levels and variance of uncertain variables affect the computational results is discussed.

The rest of this paper is organized as follows. In Section II, the problem definition is described, which includes some necessary indices and sets as well as parameters and variables of the problem involved in the objective functions and constraint conditions. Three uncertain models under different decision-making scenarios are then proposed in III. Next, the process of crisp equilibrium transformation for the above three uncertain models is performed in Section IV. In Section V, the computational results are presented and details of a comparative analysis given. The paper is concluded in Section VI.

II. PROBLEM DESCRIPTION

Given a single plant and a set of retailers geographically dispersed on a grid, each retailer i has an uncertain demand $\tilde{d}_{it}$ in period t of the planning horizon that must be satisfied to a certain degree extent β_i. The plant and retailers can be regarded as a network defined on a complete directed graph $G = (N_0, E)$ with $N_0 = N \bigcup \{0\}$. N represents the set of retailers indexed by i or $j \in \{1, 2, \ldots, n\}$ and 0 represents the plant. $E = \{(i,j) : i,j \in N_0, i \neq j\}$ is the set of arcs. Over a finite set of time periods $T = \{1, 2, \ldots, \tau\}$, a single product can be produced at the plant and delivered by a set of homogeneous vehicles $K = \{1, 2, \ldots, m\}$ to the retailers. The goal is to simultaneously minimize production, inventory, and routing costs under uncertainty so that the uncertain demands of retailers are satisfied to a certain extent, and the production constraints and inventory and transportation limits cannot be violated. We summarize the indices and sets, parameters and decision variables respectively, as follows.

A. INDICES AND SETS

- i, j: Indices for retailers, where 0 corresponds to the plant.
- t: Index for periods or days, $|T| = \tau$.
- N: Set of retailers, $N_0 = N \bigcup \{0\}$.

B. PARAMETERS

- $\tilde{d}_{it}$: Uncertain demand at retailer i in period t.
- $\tilde{f}$: Uncertain fixed production setup cost.
- $\tilde{u}$: Uncertain unit production cost.
- $\tilde{h}^P$: Uncertain unit inventory holding cost at the plant.
- $\tilde{h}_i^R$: Uncertain unit inventory holding cost at retailer i.
- $\tilde{c}_{ij}$: Uncertain transportation cost from node i to node j.
- C: Production capacity of the plant.
- b_i: Initial inventory at retailer i, where 0 corresponds to the plant.
- I_{max}^P: Maximum inventory level at the plant.
- $I_{i,max}^R$: Maximum inventory level at retailer i.
- m: Number of available vehicles.
- Q: Capacity of each vehicle.
- α: Confidence level about uncertain cost.
- β_i, γ_i: Confidence level of node i (satisfaction degree of uncertain demands).
- γ: Confidence level (satisfaction degree of uncertain demands).

C. DECISION VARIABLES

- z_t: Equal to 1 if there is production at the plant in period t; 0 otherwise.
- p_t: Production quantity in period t.
- I_t^P: Inventory at the plant at end of period t.
- I_{it}^R: Inventory at retailer i at end of period t.
- x_{ijt}: Equal to 1 if a vehicle travels directly from node i to node j in period t; 0 otherwise.
- y_{it}: Load of a vehicle immediately before making a delivery to retailer i in period t.
- w_{it}: Quantity delivered to customer i in period t.

III. THREE UNCERTAIN PROGRAMMING MODELS

According to different decision criteria, three uncertain programming models are proposed correspondingly, including expected minimum cost model, (α, β)-minimum cost model, and most minimum cost model.

A. EXPECTED MINIMUM COST MODEL (EMCM)

In its most general sense, the expectations criterion is the simplest and most pervasive choice for the decision makers. The EMCM is provided as follows, and the satisfaction degree of uncertain demands is also depicted based on expectation criteria.

$$\text{Objective: } \min\ E\left[C_{Total}\right] \tag{A1}$$

$$\text{Subject to: } I_t^P = I_{t-1}^P + p_t - \sum_{i \in N} w_{it} \quad \forall t \in T \tag{A2}$$

$$E\left[\tilde{d}_{it}\right] \leq w_{it} + I^R_{it-1} - I^R_{it} \quad \forall i \in N \; \forall t \in T \tag{A3}$$

$$p_t \leq \min\left[E\left[\sum_{l=t}^{\tau}\sum_{i\in N}\tilde{d}_{il}\right], C\right] z_t \quad \forall t \in T \tag{A4}$$

$$I^P_t \leq I^P_{max}, I^R_{it} \leq I^R_{i,max}, I^P_0 = b_0, I^R_{i0} = b_i \quad \forall i \in N \; \forall t \in T \tag{A5}$$

$$\sum_{i\in N_0, i\neq j} x_{ijt} \leq 1 \quad \forall j \in N \; \forall t \in T \tag{A6}$$

$$\sum_{i\in N_0, i\neq j} x_{jit} = \sum_{i\in N_0, i\neq j} x_{ijt} \quad \forall j \in N \; \forall t \in T \tag{A7}$$

$$\sum_{j\in N} x_{0jt} \leq m \quad \forall t \in T \tag{A8}$$

$$y_{jt} \leq y_{it} - w_{it} + M(1 - x_{ijt}) \quad \forall i \in N \; \forall j \in N_0 \; \forall t \in T \tag{A9}$$

$$w_{it} \leq E\left[\sum_{l=t}^{\tau}\tilde{d}_{il}\right]\sum_{j\in N_0} x_{ijt} \quad \forall i \in N \; \forall t \in T \tag{A10}$$

$$y_{it} \leq Q \quad \forall i \in N \; \forall t \in T \tag{A11}$$

$$p_t, I^P_t, I^R_{it}, w_{it}, y_{it} \geq 0 \quad \forall i \in N \; \forall t \in T \tag{A12}$$

$$z_t, x_{ijt} \in \{0, 1\} \quad \forall i \neq j \in N_0 \; \forall t \in T \tag{A13}$$

where:

$$C_{Total} = C_P + C_I + C_T \tag{A14}$$

$$C_P = \sum_{t\in T}\left(\tilde{f} z_t + \tilde{u} p_t\right) \tag{A15}$$

$$C_I = \sum_{t\in T}\left(\tilde{h}^P I^P_t + \sum_{i\in N}\tilde{h}^R_i I^R_{it}\right) \tag{A16}$$

$$C_T = \sum_{t\in T}\sum_{(i,j)\in E}\tilde{c}_{ij} x_{ijt} \tag{A17}$$

The objective function (A1) minimizes the expectation total production, setup, inventory, and routing costs. Constraints (A2)–(A5) represent the lot-sizing part of the problem. Constraints (A2) are the inventory flow balance at the plant. The uncertain demand of retailer i at each period must be satisfied to expectation level in (A3). Constraints (A4) are the setup forcing and production capacity constraints. The constraints force the setup variable to be 1 if production takes place in a given period and limit the production quantity that must be less than the production capacity and the sum of the future expectation demands of all the retailers. Constraints (A5) limit the maximum inventory at the plant and customers, respectively, and give the initial inventory level of the plant. The inventory part of this model is controlled by the so-called maximum level (ML) policy, in which the delivery quantity can be any positive number, but the resulting inventory level cannot exceed the maximum inventory level. The remaining constraints, i.e., (A6)–(A11), are the vehicle loading and routing restrictions. Each retailer can be visited by at most one vehicle (A6). Constraints (A7) are the vehicle flow conservation. Constraints (A8) limit the number of trucks that can be used. Constraints (A9) are the vehicle loading restrictions and sub-tour elimination constraints in the form of the Miller-Tucker-Zemlin inequalities. M is a maximum value. Constraints (A10) limit the qualities delivered to retailer i, which must be less than the total expectation demand of retailer i in the future. Constraints (A11) limit the delivery qualities that must be less than the capacity of a vehicle.

B. (α, β)-MINIMUM COST MODEL ((α, β)-MCM)

However, the EMCM based on the expectations criterion can not satisfy all the needs in practical application. Decision makers may be interested in the minimum cost $\bar{W}$ with confidence level α, and the parameters β_i for retailer i can meet various satisfaction levels of its demands. Thereupon, we bring up the (α, β)-minimum cost model labeled as (α, β)-MCM.

The (α, β)-MCM is broadly similar to the EMCM, except for (B2), (B4), (B5), and (B11). The most important part of the (α, β)-MCM is to introduce the pre-determined confidence levels α in (B2), β series in (B4), and γ series in (B15) and (B16). The objective function (B1) with constraints (B2) is to minimizes the overall cost including production, setup, inventory, and routing costs at the confidence level α under uncertain environments. The uncertain demand of retailer i at time period t must be at least satisfied at the confidence level β_i in (B4). Constraints (B5) force the setup variable to be 1 if production takes place in a given period and limit the production quantity that must be less than the production capacity and U_t in (B15) which represents the sum of the future demands of all the retailers at the confidence level γ. Constraints (B11) limit the qualities delivered to retailer i at time period t, which must be less than the total future demand of retailer i at the confidence level γ_i labeled as U_{it} in (B16).

$$\text{Objective: } \min \; \bar{W} \tag{B1}$$

$$\text{Subject to: } \mathcal{M}\left\{C_{total} \leq \bar{W}\right\} \geq \alpha \tag{B2}$$

$$I^P_t = I^P_{t-1} + p_t - \sum_{i\in N} w_{it} \quad \forall t \in T \tag{B3}$$

$$\mathcal{M}\left\{w_{it} + I^R_{it-1} - I^R_{it} \geq \tilde{d}_{it}\right\} \geq \beta_i \quad \forall i \in N \; \forall t \in T \tag{B4}$$

$$p_t \leq U_t z_t \quad \forall t \in T \tag{B5}$$

$$I^P_t \leq I^P_{max}, I^R_{it} \leq I^R_{i,max}, I^P_0 = b_0, I^R_{i0} = b_i \quad \forall i \in N \; \forall t \in T \tag{B6}$$

$$\sum_{i\in N_0, i\neq j} x_{ijt} \leq 1 \quad \forall j \in N \; \forall t \in T \tag{B7}$$

$$\sum_{i\in N_0, i\neq j} x_{jit} = \sum_{i\in N_0, i\neq j} x_{ijt} \quad \forall j \in N \; \forall t \in T \tag{B8}$$

$$\sum_{j\in N} x_{0jt} \le m \quad \forall t \in T \tag{B9}$$
$$y_{jt} \le y_{it} - w_{it} + M(1-x_{ijt})$$
$$\forall i \in N\ \forall j \in N_0\ \forall t \in T \tag{B10}$$
$$w_{it} \le U_{it} \sum_{j\in N_0} x_{ijt} \quad \forall i \in N\ \forall t \in T \tag{B11}$$
$$y_{it} \le Q \quad \forall i \in N\ \forall t \in T \tag{B12}$$
$$p_t, I_t^P, I_{it}^R, w_{it}, y_{it} \ge 0 \quad \forall i \in N\ \forall t \in T \tag{B13}$$
$$z_t, x_{ijt} \in \{0,1\} \quad \forall i \ne j \in N_0\ \forall t \in T \tag{B14}$$

where:

$$U_t = \min\left\{\min\left\{W_t|\mathcal{M}\left\{\sum_{l=t}^{\tau}\sum_{i\in N}\tilde{d}_{il} \le W_t\right\} \ge \gamma\right\}, C\right\} \tag{B15}$$
$$U_t = \min\left\{W_{it}|\mathcal{M}\left\{\sum_{l=t}^{\tau}\tilde{d}_{il} \le W_{it}\right\} \ge \gamma_i\right\} \tag{B16}$$

C. MOST MINIMUM COST MODEL (MMCM)

Sometimes, decision makers may be interested in maximizing the confidence level of the event such that the total cost does not exceed a pre-determined value W_0, which can be described as the MMCM.

$$\text{Objective: } \max\ \mathcal{M}\{C_{Total} \le W_0\} \tag{C1}$$
$$[8pt]\ \text{Subject to: } I_t^P = I_{t-1}^P + p_t - \sum_{i\in N} w_{it} \quad \forall t \in T \tag{C2}$$
$$\mathcal{M}\left\{w_{it} + I_{it-1}^R - I_{it}^R \ge \tilde{d}_{it}\right\} \ge \beta_i$$
$$\forall i \in N\ \forall t \in T \tag{C3}$$
$$p_t \le U_t z_t \quad \forall t \in T \tag{C4}$$
$$I_t^P \le I_{max}^P, I_{it}^R \le I_{i,max}^R, I_0^P = b_0, I_{i0}^R = b_i$$
$$\forall i \in N\ \forall t \in T \tag{C5}$$
$$\sum_{i\in N_0, i\ne j} x_{ijt} \le 1 \quad \forall j \in N\ \forall t \in T \tag{C6}$$
$$\sum_{i\in N_0, i\ne j} x_{jit} = \sum_{i\in N_0, i\ne j} x_{ijt} \quad \forall j \in N\ \forall t \in T \tag{C7}$$
$$\sum_{j\in N} x_{0jt} \le m \quad \forall t \in T \tag{C8}$$
$$y_{jt} \le y_{it} - w_{it} + M(1-x_{ijt})$$
$$\forall i \in N\ \forall j \in N_0\ \forall t \in T \tag{C9}$$
$$w_{it} \le U_{it} \sum_{j\in N_0} x_{ijt} \quad \forall i \in N\ \forall t \in T \tag{C10}$$
$$y_{it} \le Q \quad \forall i \in N\ \forall t \in T \tag{C11}$$
$$p_t, I_t^P, I_{it}^R, w_{it}, y_{it} \ge 0 \quad \forall i \in N\ \forall t \in T \tag{C12}$$
$$z_t, x_{ijt} \in \{0,1\} \quad \forall i \ne j \in N_0\ \forall t \in T \tag{C13}$$

where:

$$U_t = \min\left\{\min\left\{W_t|\mathcal{M}\left\{\sum_{l=t}^{\tau}\sum_{i\in N}\tilde{d}_{il} \le W_t\right\} \ge \gamma\right\}, C\right\} \tag{C14}$$
$$U_t = \min\left\{W_{it}|\mathcal{M}\left\{\sum_{l=t}^{\tau}\tilde{d}_{il} \le W_{it}\right\} \ge \gamma_i\right\} \tag{C15}$$

The MMCM is much the same as (α, β)-MCM, but only in its decision objective. The objective function (C1) is to maximize the the possibility of the overall cost being less than or equal to W_0, which is a pre-determined value reflecting the safety margin of the total cost.

IV. CRISP EQUIVALENT TRANSFORMATION

Owing to the complexity of uncertainty settings, one should transform the above uncertain models to an equivalent crisp form based on uncertain theory.

A. CRISP EQUIVALENT TRANSFORMATION FOR EMCM

Let $\tilde{f}, \tilde{u}, \tilde{h}^P, \tilde{h}_i^R, \tilde{c}_{ij}$, and $\tilde{d}_{it}$ be independent uncertain variables, with the uncertainty inverse distributions Φ_1^{-1}, Φ_2^{-1}, Φ^{P-1}, Φ_i^{R-1}, Φ_{ij}^{-1}, and Υ_{it}^{-1}, respectively. Then, referring to uncertainty theory by Liu *et al.* [50] and Yang [51],

$$E[C_{Total}] = E[C_P] + E[C_I] + E[C_T], \tag{1}$$
$$E[C_P] = \sum_{t\in T} z_t \int_0^1 \Phi_1^{-1}(\alpha)d\alpha + \sum_{t\in T} p_t \int_0^1 \Phi_2^{-1}(\alpha)d\alpha, \tag{2}$$
$$E[C_I] = \sum_{t\in T} I_t^P \int_0^1 \Phi^{P-1}(\alpha)d\alpha + \sum_{t\in T}\sum_{i\in N} I_{it}^R \int_0^1 \Phi_i^{R-1} d\alpha, \tag{3}$$
$$E[C_T] = \sum_{t\in T}\sum_{(i,j)\in E} x_{ijt} \int_0^1 \Phi_{ij}^{-1} d\alpha. \tag{4}$$

According to Liu [41], the constraint (A3) can be converted to the crisp form as follows:

$$\int_0^1 \Upsilon_{it}^{-1} d\alpha \le w_{it} + I_{it-1}^R - I_{it}^R. \tag{5}$$

In the same way, the constraint (A4) can be converted to the crisp form as follows:

$$p_t \le \min\left\{\sum_{l=t}^{\tau}\int_0^1 \Upsilon_{il}^{-1} d\alpha, C\right\} z_t. \tag{6}$$

Similarly, the constraint (A10) can be converted to the crisp form as follows:

$$w_{it} \le \left\{\sum_{l=t}^{\tau}\int_0^1 \Upsilon_{il}^{-1} d\alpha\right\} \sum_{j\in N_0} x_{ijt}. \tag{7}$$

1) LINEAR UNCERTAIN SITUATION

According to Liu [40], who put forward the definitions and properties of commonly used linear uncertain variables, we can transform the above uncertain EMCM into a crisp equivalence class. Let uncertain variables satisfy the following distributions:

$$\tilde{u} \sim \mathcal{L}(a_1, b_1);\ \tilde{f} \sim \mathcal{L}(a_2, b_2);\ \tilde{h}^P \sim \mathcal{L}(a^P, b^P);$$
$$\tilde{h}_i^R \sim \mathcal{L}(a_i^R, b_i^R);\ \tilde{c}_{ij} \sim \mathcal{L}(a_{ij}, b_{ij});\ \tilde{d}_{it} \sim \mathcal{L}(a_{it}^d, b_{it}^d).$$

With the above assumptions and uncertainty theory by Liu [41], the cost function in the EMCM is simplified as follows:

$$E\left[C_P\right] = 0.5\sum_{t\in T}\left\{(a_1+b_1)z_t+(a_2+b_2)p_t\right\}, \tag{8}$$

$$E\left[C_I\right] = 0.5\sum_{t\in T}\left\{(a^P+b^P)I_t^P+\sum_{i\in N}(a_i^R+b_i^R)I_{it}^R\right\}, \tag{9}$$

$$E\left[C_T\right] = 0.5\sum_{t\in T}\sum_{(i,j)\in E}\left(a_{ij}+b_{ij}\right)x_{ijt}. \tag{10}$$

Then, the constraint (A3) can be converted to the linear crisp form as follows:

$$\left(a_{it}^d+b_{it}^d\right)\le w_{it}+I_{it-1}^R-I_{it}^R. \tag{11}$$

The constraint (A4) can be converted to the linear crisp form as follows:

$$p_t\le \min\left\{0.5\sum_{l=t}^{\tau}\sum_{i\in N}\left(a_{il}^d+b_{il}^d\right), C\right\}z_t. \tag{12}$$

In a similar way, the constraint (A10) can be converted to the linear crisp form as follows:

$$w_{it}\le 0.5\sum_{l=t}^{\tau}\left(a_{il}^d+b_{il}^d\right)\sum_{j\in N_0}x_{ijt}. \tag{13}$$

2) ZIGZAG UNCERTAIN SITUATION

In a similar way, based on the zigzag uncertain variable that Liu [40] put forward, the zigzag form of crisp equivalence class can be obtained. Let uncertain variables satisfy the following distributions:

$$\tilde{u}\sim \mathcal{Z}(a_1,b_1,c_1);\tilde{f}\sim \mathcal{Z}(a_2,b_2,c_2);$$
$$\tilde{h}^P\sim \mathcal{Z}(a^P,b^P,c^P);\ \tilde{h}_i^R\sim \mathcal{Z}(a_i^R,b_i^R,c_i^R);$$
$$\tilde{c}_{ij}\sim \mathcal{Z}(a_{ij},b_{ij},c_{ij});\ \tilde{d}_{it}\sim \mathcal{Z}(a_{it}^d,b_{it}^d,c_{it}^d).$$

According to Liu [41], three parts of the cost function $E\left[C_{Total}\right]$ are simplified as follows:

$$E\left[C_P\right] = 0.25\sum_{t\in T}\left\{(a_1+2b_1+c_1)z_t+(a_2+2b_2+c_2)p_t\right\}, \tag{14}$$

$$E\left[C_I\right] = 0.25\sum_{t\in T}(a^P+2b^P+c^P)I_t^P+0.25\sum_{t\in T}\sum_{i\in N}(a_i^R +2b_i^R+c_i^R)I_{it}^R, \tag{15}$$

$$E\left[C_T\right] = 0.25\sum_{t\in T}\sum_{(i,j)\in E}\left(a_{ij}+2b_{ij}+c_{ij}\right)x_{ijt}. \tag{16}$$

Then, the constraint (A3) can be converted to the zigzag crisp form as follows:

$$0.25\left(a_{it}^d+2b_{it}^d+c_{it}^d\right)\le w_{it}+I_{it-1}^R-I_{it}^R. \tag{17}$$

The constraint (A4) can be converted to the zigzag crisp form as follows:

$$p_t\le \min\left\{0.25\sum_{l=t}^{\tau}\sum_{i\in N}(a_{il}^d+2b_{il}^d+c_{il}^d), C\right\}z_t. \tag{18}$$

In a similar way, (A10) can be converted to the zigzag crisp form as follows:

$$w_{it}\le 0.25\sum_{l=t}^{\tau}\left(a_{il}^d+2b_{il}^d+c_{il}^d\right)\sum_{j\in N_0}x_{ijt}. \tag{19}$$

3) NORMAL UNCERTAIN SITUATION

In a similar way, according to the normal uncertain variable in Liu [40], the normal form of the crisp equivalence class can be obtained. Let uncertain variables satisfy the following distributions:

$$\tilde{u}\sim \mathcal{N}(e_1,\sigma_1);\tilde{f}\sim \mathcal{N}(e_2,\sigma_2);\tilde{h}^P\sim \mathcal{N}(e^P,\sigma^P);$$
$$\tilde{h}_i^R\sim \mathcal{N}(e_i^R,\sigma_i^R);\tilde{c}_{ij}\sim \mathcal{N}(e_{ij},\sigma_{ij});\tilde{d}_{it}\sim \mathcal{N}(e_{it}^d,\sigma_{it}^d).$$

According to Liu [41], three parts of the cost function $E\left[C_{Total}\right]$ are expressed as follows:

$$E\left[C_P\right] = \sum_{t\in T}\left(e_1z_t+e_2p_t\right), \tag{20}$$

$$E\left[C_I\right] = \sum_{t\in T}\left(e^PI_t^P+\sum_{i\in N}e_i^RI_{it}^R\right), \tag{21}$$

$$E\left[C_T\right] = \sum_{t\in T}\sum_{(i,j)\in E}e_{ij}x_{ijt}. \tag{22}$$

Then, the constraint (A3) can be converted to the normal crisp form as follows:

$$e_{it}^d\le w_{it}+I_{it-1}^R-I_{it}^R. \tag{23}$$

The constraint (A4) can be converted to the normal crisp form as follows:

$$p_t\le \min\left\{\sum_{l=t}^{\tau}\sum_{i\in N}e_{il}^d, C\right\}z_t. \tag{24}$$

In a similar way, the constraint (A10) can be converted to the normal crisp form as follows:

$$w_{it}\le \sum_{l=t}^{\tau}e_{il}^d\sum_{j\in N_0}x_{ijt}. \tag{25}$$

B. CRISP EQUIVALENT TRANSFORMATION FOR (α,β)-MCM

In the same way, one can transform the additional uncertain constraints (B2) to its crisp equivalent for the (α,β)-MCM, and specify it in linear, zigzag, and normal uncertain environments separately. With the above assumptions and uncertainty theory by Liu [41] and Liu [52], (B2) in model (α,β)-MCM is simplified as

$$\sum_{t\in T}\Phi_1^{-1}(\alpha)z_t+\sum_{t\in T}\Phi_2^{-1}(\alpha)p_t+\sum_{t\in T}\Phi^{P^{-1}}(\alpha)I_t^P +\sum_{t\in T}\sum_{i\in N}\Phi_i^{R^{-1}}(\alpha)I_{it}^R+\sum_{t\in T}\sum_{(i,j)\in E}\Phi_{ij}^{-1}(\alpha)x_{ijt}\le \bar{W}. \tag{26}$$

Then, the crisp form of constraint (B4) can be obtained as follows:

$$\Upsilon_{it}^{-1}(\beta_i)\le w_{it}+I_{it-1}^R-I_{it}^R. \tag{27}$$

In a similar way, the crisp forms of (B15) and (B16) can be correspondingly obtained as follows:

$$U_t = \sum_{l=t}^{\tau}\sum_{i\in N}\Upsilon_{il}^{-1}(\gamma), \quad (28)$$

$$U_{it} = \sum_{l=t}^{\tau}\Upsilon_{il}^{-1}(\gamma_i). \quad (29)$$

1) LINEAR UNCERTAIN SITUATION

Let uncertain variables satisfy the following linear distributions:

$$\tilde{u} \sim \mathcal{L}(a_1, b_1); \tilde{f} \sim \mathcal{L}(a_2, b_2); \tilde{h}^P \sim \mathcal{L}(a^P, b^P);$$
$$\tilde{h}_i^R \sim \mathcal{L}(a_i^R, b_i^R); \tilde{c}_{ij} \sim \mathcal{L}(a_{ij}, b_{ij}); \tilde{d}_{it} \sim \mathcal{L}(a_{it}^d, b_{it}^d).$$

For convenience, the mathematical formulas are defined as follows:

$$\begin{aligned} a(X) &= a\left(z_t, p_t, I_t^P, I_{it}^R, x_{ijt}\right) \\ &= \left(a_1 z_t + a_2 p_t + a^P I_t^P + \sum_{i\in N} a_i^R I_{it}^R + \sum_{(i,j)\in E} a_{ij}x_{ijt}\right), \end{aligned} \quad (30)$$

$$\begin{aligned} b(X) &= b\left(z_t, p_t, I_t^P, I_{it}^R, x_{ijt}\right) \\ &= \left(b_1 z_t + b_2 p_t + b^P I_t^P + \sum_{i\in N} b_i^R I_{it}^R + \sum_{(i,j)\in E} b_{ij}x_{ijt}\right). \end{aligned} \quad (31)$$

With the above assumptions and uncertainty theory by Liu [52], (B2) in the (α, β)-MCM is simplified as follows:

$$\sum_{t\in T}\{(1-\alpha)a(X) + \alpha b(X)\} \le \bar{W}. \quad (32)$$

Then, the linear crisp form of constraint (B4) can be obtained as follows:

$$(1-\beta_i)\, a_{il}^d + \beta_i b_{il}^d \le w_{it} + I_{it-1}^R - I_{it}^R. \quad (33)$$

In a similar way, the linear crisp forms of (B15) and (B16) can be correspondingly obtained as follows:

$$U_t = \sum_{l=t}^{\tau}\sum_{i\in N}(1-\gamma)a_{il}^d + \gamma b_{il}^d, \quad (34)$$

$$U_{it} = \sum_{l=t}^{\tau}((1-\gamma_i)a_{il}^d + \gamma_i b_{il}^d. \quad (35)$$

2) ZIGZAG UNCERTAIN SITUATION

Let uncertain variables satisfy the following zigzag distributions:

$$\tilde{u} \sim \mathcal{L}(a_1, b_1); \tilde{f} \sim \mathcal{L}(a_2, b_2); \tilde{h}^P \sim \mathcal{L}(a^P, b^P);$$
$$\tilde{h}_i^R \sim \mathcal{L}(a_i^R, b_i^R); \tilde{c}_{ij} \sim \mathcal{L}(a_{ij}, b_{ij}); \tilde{d}_{it} \sim \mathcal{L}(a_{it}^d, b_{it}^d).$$

Then, the mathematical formula is defined as follows:

$$\begin{aligned} c(X) &= c\left(z_t, p_t, I_t^P, I_{it}^R, x_{ijt}\right) \\ &= \left(c_1 z_t + c_2 p_t + c^P I_t^P + \sum_{i\in N} c_i^R I_{it}^R + \sum_{(i,j)\in E} c_{ij}x_{ijt}\right). \end{aligned} \quad (36)$$

The chance constraint (B2) in the (α, β)-MCM is simplified as follows with the above assumptions and uncertainty theory by Liu [52]:

$$\sum_{t\in T}\{(1-2\alpha)a(X) + 2\alpha b(X)\} \le \bar{W}, \text{ if } \alpha < 0.5,$$
$$\sum_{t\in T}\{(2-2\alpha)b(X) + (2\alpha-1)c(X)\} \le \bar{W}, \text{ if } \alpha \ge 0.5. \quad (37)$$

Then, the zigzag crisp form of constraint (B4) can be obtained as follows:

$$\begin{aligned} &(1-2\beta_i)\, f a_{il}^d + 2f\beta_i b_{il}^d + (2-2\beta_i)\, g b_{il}^d + (2\beta_i - 1)\, g c_{il}^d \\ &\quad \le w_{it} + I_{it-1}^R - I_{it}^R, \end{aligned} \quad (38)$$

where

$$f = \begin{cases} 1, & \text{if } \beta_i < 0.5, \\ 0, & \text{if } \beta_i \ge 0.5, \end{cases}$$

$$g = \begin{cases} 0, & \text{if } \beta_i < 0.5, \\ 1, & \text{if } \beta_i \ge 0.5. \end{cases}$$

In a similar way, the zigzag crisp forms of (B15) and (B16) can be correspondingly obtained as follows:

$$U_t = \sum_{l=t}^{\tau}\sum_{i\in N}(1-2\gamma)\, a_{il}^d + 2\gamma b_{il}^d, \text{ if } \gamma < 0.5,$$
$$U_t = \sum_{l=t}^{\tau}\sum_{i\in N}(2-2\gamma)\sum_{i\in N} b_{il}^d + (2\gamma-1)c_{il}^d, \text{ if } \gamma \ge 0.5, \quad (39)$$

$$\begin{aligned} U_{it} = \sum_{l=t}^{\tau}&((1-2\gamma_i)fa_{il}^d + 2\gamma_i f b_{il}^d + (2-2\gamma_i)bg_{il}^d + (2\gamma_i \\ &-1)gc_{il}^d), \end{aligned} \quad (40)$$

where

$$f = \begin{cases} 1, & \text{if } \gamma_i < 0.5, \\ 0, & \text{if } \gamma_i \ge 0.5, \end{cases}$$

$$g = \begin{cases} 0, & \text{if } \gamma_i < 0.5, \\ 1, & \text{if } \gamma_i \ge 0.5. \end{cases}$$

3) NORMAL UNCERTAIN SITUATION

Let uncertain variables satisfy the following normal distributions:

$$\tilde{u} \sim \mathcal{N}(e_1, \sigma_1); \tilde{f} \sim \mathcal{N}(e_2, \sigma_2); \tilde{h}^P \sim \mathcal{N}(e^P, \sigma^P);$$
$$\tilde{h}_i^R \sim \mathcal{N}(e_i^R, \sigma_i^R); \tilde{c}_{ij} \sim \mathcal{N}(e_{ij}, \sigma_{ij}); \tilde{d}_{it} \sim \mathcal{N}(e_{it}^d, \sigma_{it}^d).$$

Then, the mathematical formulas are defined as follows:

$$\begin{aligned} e(X) &= e\left(z_t, p_t, I_t^P, I_{it}^R, x_{ijt}\right) \\ &= \left(e_1 z_t + e_2 p_t + e^P I_t^P + \sum_{i\in N} e_i^R I_{it}^R + \sum_{(i,j)\in E} e_{ij} x_{ijt}\right), \end{aligned} \quad (41)$$

$$\begin{aligned} d(X) &= d\left(z_t, p_t, I_t^P, I_{it}^R, x_{ijt}\right) \\ &= \left(\sigma_1 z_t + \sigma_2 p_t + \sigma^P I_t^P + \sum_{i\in N} \sigma_i^R I_{it}^R + \sum_{(i,j)\in E} \sigma_{ij} x_{ijt}\right). \end{aligned} \quad (42)$$

The chance constraint (B2) in the (α, β)-MCM is simplified as follows with the above assumptions and uncertainty theory by Liu [52]:

$$\sum_{t\in T}\left\{e(X) + \frac{\sqrt{3}}{\pi} ln \frac{\alpha}{1-\alpha} d(X)\right\}. \quad (43)$$

Then, the normal crisp form of constraint (B4) can be obtained as follows:

$$e_{it}^d + \frac{\sqrt{3}\sigma_{it}^d}{\pi} ln \frac{\beta_i}{1-\beta_i} \leq w_{it} + I_{it-1}^R - I_{it}^R. \quad (44)$$

In a similar way, the normal crisp forms of (B15) and (B16) can be correspondingly obtained as follows:

$$U_t = \sum_{l=t}^{\tau}\sum_{i\in N}\left(e_{il}^d + \frac{\sqrt{3}\sigma_{il}^d}{\pi} ln \frac{\gamma}{1-\gamma}\right), \quad (45)$$

$$U_{it} = \sum_{l=t}^{l}\left(e_{il}^d + \frac{\sqrt{3}\sigma_{il}^d}{\pi} ln \frac{\gamma_i}{1-\gamma_i}\right). \quad (46)$$

C. CRISP EQUIVALENT TRANSFORMATION FOR MMCM

In the same way, one can transform the additional uncertain constraints (C1) to their crisp equivalent for the MMCM, and specify them in linear, zigzag, and normal uncertain environments separately. Let ξ represent the left-hand side of the mathematical formula in the inequality (C1), which is also an uncertain variable based on uncertainty theory. $\Psi(x)$ represents the uncertain distribution of the uncertain variable ξ. Then, the mathematical formula (C1) can be simplified as follows:

$$\begin{aligned} \mathcal{M}\Bigg\{&\sum_{t\in T}\tilde{f} z_t + \sum_{t\in T}\tilde{u} p_t + \sum_{t\in T}\tilde{h}^P I_t^P + \sum_{t\in T}\sum_{i\in N}\tilde{h}_i^R I_{it}^R \\ &+ \sum_{t\in T}\sum_{(i,j)\in E}\tilde{c}_{ij} x_{ijt} \leq W_0\Bigg\} = \Psi(W_0). \end{aligned} \quad (47)$$

1) LINEAR UNCERTAIN SITUATION

Let uncertain variables satisfy the following linear distributions:

$$\tilde{u} \sim \mathcal{L}(a_1, b_1); \tilde{f} \sim \mathcal{L}(a_2, b_2); \tilde{h}^P \sim \mathcal{L}(a^P, b^P);$$
$$\tilde{h}_i^R \sim \mathcal{L}(a_i^R, b_i^R); \tilde{c}_{ij} \sim \mathcal{L}(a_{ij}, b_{ij}); \tilde{d}_{it} \sim \mathcal{L}(a_{it}^d, b_{it}^d).$$

With the above assumptions and uncertainty theory by Liu [40], we can draw the following conclusion:

$$\xi \sim \mathcal{L}\left(\sum_{t\in T} a(X), \sum_{t\in T} b(X)\right). \quad (48)$$

Then, the following specific uncertainty distribution of $\Psi(W_0)$ in linear circumstances is provided as follows:

$$\begin{cases} 0, & \text{if } W_0 \leq a(X), \\ \dfrac{W_0 - \sum_{t\in T} a(X)}{\sum_{t\in T}(b(X) - a(X))}, & \text{if } \sum_{t\in T} a(X) \leq W_0 \leq \sum_{t\in T} b(X), \\ 1, & \text{if } W_0 \geq \sum_{t\in T} b(X). \end{cases} \quad (49)$$

2) ZIGZAG UNCERTAIN SITUATION

Let uncertain variables satisfy the following zigzag distributions:

$$\tilde{u} \sim \mathcal{Z}(a_1, b_1, c_1); \tilde{f} \sim \mathcal{Z}(a_2, b_2, c_2);$$
$$\tilde{h}^P \sim \mathcal{Z}(a^P, b^P, c^P); \tilde{h}_i^R \sim \mathcal{Z}(a_i^R, b_i^R, c_i^R);$$
$$\tilde{c}_{ij} \sim \mathcal{Z}(a_{ij}, b_{ij}, c_{ij}); \tilde{d}_{it} \sim \mathcal{Z}(a_{it}^d, b_{it}^d, c_{it}^d).$$

With the above assumptions and uncertainty theory by Liu [40], the following conclusion can be obtained:

$$\xi \sim \mathcal{Z}\left(\sum_{t\in T} a(X), \sum_{t\in T} b(X), \sum_{t\in T} c(X)\right). \quad (50)$$

Then, the following specific uncertainty distribution of $\Psi(W_0)$ is provided as follows:

$$\begin{cases} 0, & \text{if } W_0 \leq a(X), \\ \dfrac{W_0 - \sum_{t\in T} a(X)}{2\sum_{t\in T} g(X)}, & \text{if } \sum_{t\in T} a(X) \leq W_0 \leq \sum_{t\in T} b(X), \\ \dfrac{W_0 + \sum_{t\in T} n(X)}{2\sum_{t\in T} h(X)}, & \text{if } \sum_{t\in T} b(X) \leq W_0 \leq \sum_{t\in T} c(X), \\ 1, & \text{if } W_0 \geq \sum_{t\in T} c(X). \end{cases} \quad (51)$$

For convenience, the mathematical formulas are expressed as follows:

$$g(X) = b(X) - a(X), \quad (52)$$
$$h(X) = c(X) - b(X), \quad (53)$$
$$n(X) = c(X) - 2b(X). \quad (54)$$

3) NORMAL UNCERTAIN SITUATION

Let uncertain variables satisfy the following normal distributions:

$$\tilde{u} \sim \mathcal{N}(e_1, \sigma_1); \tilde{f} \sim \mathcal{N}(e_2, \sigma_2); \tilde{h}^P \sim \mathcal{N}(e^P, \sigma^P);$$
$$\tilde{h}_i^R \sim \mathcal{N}(e_i^R, \sigma_i^R); \tilde{c}_{ij} \sim \mathcal{N}(e_{ij}, \sigma_{ij}); \tilde{d}_{it} \sim \mathcal{N}(e_{it}^d, \sigma_{it}^d).$$

Similarly, with the above assumptions and uncertainty theory by Liu [40], it can be concluded that

$$\xi \sim \mathcal{N}\left(\sum_{t\in T} e(X), \sum_{t\in T} d(X)\right). \tag{55}$$

The specific uncertainty distribution of $\Psi(W_0)$ can be described as follows:

$$\Psi(W_0) = \left(1 + exp\left(\frac{\sum_{t\in T} \pi e(X) - W_0}{\sum_{t\in T} \sqrt{3}d(X)}\right)\right)^{-1}. \tag{56}$$

V. NUMERICAL RESULTS

The crisp deterministic models have been implemented in C++ using ILOG Concert and CPLEX 12.8. The goals of these experiments are to solve the nine crisp equivalent models after crisp equivalent conversion from the three uncertain models proposed in Section three, and to analyze how the confidence levels and variance of uncertain variables influence the computational results.

Since much of the previous research on the PRP was done only under deterministic circumstances, the uncertainties may have gone undetected, being unsuitable for researching the uncertain PRP. To solve these uncertain decision models, the uncertain instance is proposed based on the classic instance from Archetti *et al.* [53], which has been considered one classical instance used to solve the PRP with exact solution algorithms and heuristics.

TABLE 1. Distributions of uncertain linear variables.

Parameters	Variables
$\tilde{d_{it}}$	$\mathcal{L}(d_{it}(1-\epsilon^{ld}), d_{it}(1+\epsilon^{ld}))$
$\tilde{f}$	$\mathcal{L}(f(1-\epsilon^{lf}), f(1+\epsilon^{lf}))$
$\tilde{p}$	$\mathcal{L}(p(1-\epsilon^{lp}), p(1+\epsilon^{lp}))$
$\tilde{h}_0$	$\mathcal{L}(h_0(1-\epsilon^{lh}), h_0(1+\epsilon^{lh}))$
$\tilde{h}_i$	$\mathcal{L}(h_i(1-\epsilon^{lh}), h_i(1+\epsilon^{lh}))$
$\tilde{c}_{ij}$	$\mathcal{L}(c_{ij}(1-\epsilon^{lc}), c_{ij}(1+\epsilon^{lc}))$

TABLE 2. Distributions of uncertain zigzag variables.

Parameters	Variables
$\tilde{d_{it}}$	$\mathcal{Z}(d_{it}(1-\epsilon_1^{ld}), d_{it}(1-\epsilon_2^{ld}), d_{it}(1+\epsilon_1^{ld}+2\epsilon_2^{ld}))$
$\tilde{f}$	$\mathcal{Z}(f(1-\epsilon_1^{lf}), f(1-\epsilon_2^{lf}), f(1+\epsilon_1^{lf}+2\epsilon_2^{lf}))$
$\tilde{p}$	$\mathcal{Z}(p(1-\epsilon_1^{lp}), p(1-\epsilon_2^{lp}), p(1+\epsilon_1^{lp}+2\epsilon_2^{lp}))$
$\tilde{h}_0$	$\mathcal{Z}(h_0(1-\epsilon_1^{lh}), h_0(1-\epsilon_2^{lh}), h_0(1+\epsilon_1^{lh}+2\epsilon_2^{lh}))$
$\tilde{h}_i$	$\mathcal{Z}(h_i(1-\epsilon_1^{lh}), h_i(1-\epsilon_2^{lh}), h_i(1+\epsilon_1^{lh}+2\epsilon_2^{lh}))$
$\tilde{c}_{ij}$	$\mathcal{Z}(c_{ij}(1-\epsilon_1^{lc}), c_{ij}(1-\epsilon_2^{lc}), c_{ij}(1+\epsilon_1^{lc}+2\epsilon_2^{lc}))$

This uncertain variables have been generated and are presented in Tables 1, 2, and 3, including uncertain demands $\tilde{d_{it}}$, uncertain production setup cost $\tilde{f}$, uncertain unit production cost $\tilde{p}$, uncertain inventory costs $\tilde{h_0}$ for the plant and $\tilde{h_i}$ for the retailer i, and uncertain translation cost $\tilde{c_{ij}}$ between nodes i and j, with the series parameters values of ϵ being between 0 and 1, ϵ_1 greater than ϵ_2, and σ greater than 0, which represent the value of the variances of uncertain variables. Furthermore, the pre-defined series confidence levels of α and β always vary between 0 and 1. Because the series parameter values of γ are always equivalent to β, its parameter values are no longer listed in this section for simplicity. Compared to the deterministic instance from Archetti *et al.* [53], the major differences are that the variables (the costs of production, inventory, and transportation, as well as the demands of retailers in each period) are uncertain, and, correspondingly, confidence levels and correlation parameters are introduced.

TABLE 3. Distributions of uncertain normal variables.

Parameters	Variables
$\tilde{d_{it}}$	$\mathcal{N}(d_{it}, \sigma^d)$
$\tilde{f}$	$\mathcal{N}(f, \sigma^f)$
$\tilde{p}$	$\mathcal{N}(p, \sigma^p)$
$\tilde{h}_0$	$\mathcal{N}(h_0, \sigma^h)$
$\tilde{h}_i$	$\mathcal{N}(h_i, \sigma^h)$
$\tilde{c}_{ij}$	$\mathcal{N}(c_{ij}, \sigma^c)$

TABLE 4. Results of MMCM under linear circumstance.

W_0	20000	25000	30000	35000	40000
Results	0.175	0.343	0.512	0.680	0.849

TABLE 5. Results of MMCM under zigzag circumstance.

W_0	10000	15000	20000	25000	30000
Results	0.047	0.257	0.468	0.545	0.597
W_0	35000	40000	45000	50000	
Results	0.650	0.703	0.756	0.808	

A. RESULTS OF BASIC EXPERIMENTS

For solving the EMCM, three series examples that separately describe linear, zigzag, and normal distributions are presented, and calculation results are correspondingly obtained by solving the deterministic forms of the EMCM on a computer. The series parameter values of ϵ are set to 0.5 in the linear circumstance. In the zigzag circumstance, ϵ_1 and ϵ_2 series parameter values are equivalent to 0.7 and 0.3, respectively. All variances in the normal circumstance are 0.5. One can learn by experiments that the EMCM under three situations (linear, zigzag, and normal) has the same expectation total cost being 29652, which is ascribed to their identical expectations. Similarly, the experimental computing results of the MMCM under linear, zigzag, and normal distributions are shown in Tables 4, 5 and 6, respectively. Results show that the probability that the optimal total cost of MMCM is less than or equal to W_0, strictly increases as the value of W_0 increases in three uncertain distributions.

TABLE 6. Results of MMCM under normal circumstance.

W_0	27000	27500	28000	28500	29000	29500
Results	0.020	0.041	0.082	0.156	0.278	0.445
W_0	30000	30500	31000	31500	32000	
Results	0.625	0.776	0.888	0.937	0.969	

TABLE 7. (α, β)-MCM total cost under linear circumstance with $\epsilon = 0.5$ and $\beta \leq 0.5$.

	β=0.1	β=0.2	β=0.3	β=0.4	β=0.5
$\alpha = 0.1$	11551.1	12986.9	14381.8	16308.5	17791.2
$\alpha = 0.2$	13476.3	15151.4	16778.7	19026.6	20756.4
$\alpha = 0.3$	15401.4	17315.9	19175.7	21744.7	23721.6
$\alpha = 0.4$	17326.6	19480.4	21572.6	24462.8	26686.8
$\alpha = 0.5$	19251.8	21644.9	23969.6	27180.9	29652.0
$\alpha = 0.6$	21177.0	23809.4	26366.6	29899.0	32617.2
$\alpha = 0.7$	23102.2	25973.9	28763.5	32617.1	35582.4
$\alpha = 0.8$	25027.3	28138.4	31160.5	35335.2	38547.6
$\alpha = 0.9$	26952.5	30302.9	33557.4	38053.3	41512.8

TABLE 8. (α, β)-MCM total cost under linear circumstance with $\epsilon = 0.5$ and $\beta > 0.5$.

	β=0.6	β=0.7	β=0.8	β=0.9
$\alpha = 0.1$	20458.3	21917.6	23732.7	25241.5
$\alpha = 0.2$	23868.0	25570.6	27688.2	29448.4
$\alpha = 0.3$	27277.8	29223.5	31643.6	33655.4
$\alpha = 0.4$	30687.5	32876.5	35602.5	37862.3
$\alpha = 0.5$	34097.2	36529.4	39558.3	42069.2
$\alpha = 0.6$	37506.9	40182.3	43509.9	46276.1
$\alpha = 0.7$	40916.6	43835.3	47465.4	50483.0
$\alpha = 0.8$	44326.4	47488.2	51420.8	54690.0
$\alpha = 0.9$	47736.1	51141.2	55376.3	58896.9

B. CONFIDENCE-LEVEL ANALYSIS

The (α, β)-MCM focuses attention on the different confidence levels compared with the EMCM. Through changing test parameters, the effect of different level parameters on the (α, β)-MCM was studied. The results of experiments on linear, zigzag, and normal distributions can be seen in Tables 7-12, respectively. It is not difficult to find that the (α, β)-MCM total costs increase with the series parameter values of α and β. Specifically, the total cost of (α, β)-MCM grows with the increase of the confidence level α in uncertain cost environments, and also increases with the increase of the confidence level β in uncertain demand environments. The experimental result is mainly in accordance with the prospective desire.

C. SENSITIVITY ANALYSIS

In this section, details of the comparison analysis of variance are presented. The series parameters of ϵ used to solve the proposed model under three common distributions, i.e., linear, zigzag, and normal, are in given in Tables 13-18, respectively, reflect variations to some extent. And it is found that the different variations give different results in the experiments. By tweaking the series parameters of ϵ separately about uncertain demand (UD), uncertain setup cost (US),

TABLE 9. (α, β)-MCM total cost under zigzag circumstance with $\epsilon_1 = 0.7$, $\epsilon_2 = 0.3$, and $\beta \leq 0.5$.

	β=0.1	β=0.2	β=0.3	β=0.4	β=0.5
$\alpha = 0.1$	5331.5	5959.3	6802.7	7486.7	8225.1
$\alpha = 0.2$	6453.9	7213.9	8234.9	9062.8	9956.7
$\alpha = 0.3$	7576.3	8468.5	9667.0	10639.0	11688.2
$\alpha = 0.4$	8698.7	9723.1	11099.2	12215.1	13419.8
$\alpha = 0.5$	9821.2	10977.7	12531.3	13791.2	15151.4
$\alpha = 0.6$	14310.8	15996.0	18260.0	20095.8	22077.8
$\alpha = 0.7$	18800.5	21014.4	23988.6	26400.4	29004.2
$\alpha = 0.8$	23290.2	26032.8	29717.2	32704.9	35930.5
$\alpha = 0.9$	27779.8	31051.2	35445.8	39009.5	42856.9

TABLE 10. (α, β)-MCM total cost under zigzag circumstance with $\epsilon_1 = 0.7$, $\epsilon_2 = 0.3$, and $\beta > 0.5$.

	β=0.6	β=0.7	β=0.8	β=0.9
$\alpha = 0.1$	12125.3	15411.3	18555.2	22224.1
$\alpha = 0.2$	14678.0	18655.7	22461.5	26902.9
$\alpha = 0.3$	17230.7	21900.2	26367.9	31581.6
$\alpha = 0.4$	19783.4	25144.7	30274.2	36260.4
$\alpha = 0.5$	22336.1	28389.2	34180.6	40939.1
$\alpha = 0.6$	32547.0	41367.1	49806.0	59654.2
$\alpha = 0.7$	42757.8	54345.0	65431.4	78369.2
$\alpha = 0.8$	52968.6	67322.9	81056.8	97084.2
$\alpha = 0.9$	63179.4	80300.8	96682.2	115799.0

TABLE 11. (α, β)-MCM total cost under normal circumstance with $\sigma = 0.5$ and $\beta \leq 0.5$.

	β=0.1	β=0.2	β=0.3	β=0.4	β=0.5
$\alpha = 0.1$	28602.6	28731.5	28817.3	28887.5	28952.0
$\alpha = 0.2$	28857.9	28988.0	29074.4	29145.3	29210.4
$\alpha = 0.3$	29027.5	29158.4	29245.4	29316.6	29382.1
$\alpha = 0.4$	29166.6	29298.1	29385.5	29457.1	29522.8
$\alpha = 0.5$	29294.3	29426.3	29514.0	29586.0	29652.0
$\alpha = 0.6$	29421.9	29554.5	29642.6	29714.9	29781.2
$\alpha = 0.7$	29561.0	29694.2	29782.7	29855.3	29921.9
$\alpha = 0.8$	29730.6	29864.6	29953.7	30026.7	30093.6
$\alpha = 0.9$	29985.9	30121.0	30210.8	30284.4	30352.0

TABLE 12. (α, β)-MCM total cost under normal circumstance with $\sigma = 0.5$ and $\beta > 0.5$.

	β=0.6	β=0.7	β=0.8	β=0.9
$\alpha = 0.1$	30598.0	30682.0	30784.5	30938.6
$\alpha = 0.2$	30894.8	30979.7	31083.2	31238.9
$\alpha = 0.3$	31092.1	31177.5	31281.7	31438.5
$\alpha = 0.4$	31253.8	31339.7	31444.5	31602.1
$\alpha = 0.5$	31402.2	31488.5	31593.9	31752.3
$\alpha = 0.6$	31550.6	31637.4	31743.2	31902.4
$\alpha = 0.7$	31712.3	31799.6	31906.0	32066.1
$\alpha = 0.8$	31909.6	31997.4	32104.5	32265.7
$\alpha = 0.9$	32206.4	32295.1	32403.2	32551.7

TABLE 13. (α, β)-MCM under linear circumstance with $\alpha = 0.6$, $\beta = 0.8$, and $\epsilon \leq 0.5$.

	ϵ=0.1	ϵ=0.2	ϵ=0.3	ϵ=0.4	ϵ=0.5
UD	36393.1	38044.3	39639.8	41221.5	43509.9
US	43034.1	43149.9	43274.1	43390.0	43509.9
UP	41975.3	42355.8	42740.5	43129.4	43509.9
UI	42574.3	42808.2	43042.1	43276.0	43509.9
UT	43300.1	43352.6	43405.0	43457.5	43509.9

TABLE 14. (α, β)-MCM under linear circumstance with $\alpha = 0.6$, $\beta = 0.8$, and $\epsilon > 0.5$.

	ϵ=0.6	ϵ=0.7	ϵ=0.8	ϵ=0.9
UD	45160.2	46870.1	48547.9	50309.7
US	43629.9	43750.0	43869.9	43989.9
UP	43898.8	44279.4	44668.3	45048.8
UI	43743.9	43977.8	44211.7	44445.6
UT	43562.4	43614.9	43667.3	43719.8

TABLE 15. (α, β)-MCM total cost under zigzag circumstance with $\alpha = 0.6$, $\beta = 0.8$, $\epsilon_2 = 0.01$, and $\epsilon_1 \leq 0.5$.

	ϵ_1=0.1	ϵ_1=0.2	ϵ_1=0.3	ϵ_1=0.4	ϵ_1=0.5
UD	33843.6	35335.6	36822.9	38258.3	40394.0
US	33843.6	33915.3	33975.3	34035.3	34095.3
UP	33843.6	34119.8	34384.2	34648.7	34913.2
UI	33843.6	34061.7	34279.8	34488.1	34696.7
UT	33843.6	33891.1	33938.6	33986.1	34033.6

TABLE 16. (α, β)-MCM total cost under zigzag circumstance with $\alpha = 0.6$, $\beta = 0.8$, $\epsilon_2 = 0.01$, and $\epsilon_1 > 0.5$.

	ϵ_1=0.6	ϵ_1=0.7	ϵ_1=0.8	ϵ_1=0.9
UD	41914.4	43512.6	45041.4	46668.6
US	34155.3	34215.3	34275.3	34335.3
UP	35177.6	35442.1	35706.5	35971.0
UI	34905.4	35114.1	35322.8	35531.4
UT	34081.1	34128.6	34176.1	34223.6

TABLE 17. (α, β)-MCM total cost under normal circumstance with $\alpha = 0.6$, $\beta = 0.8$, and $\sigma \leq 5$.

	σ=1	σ=2	σ=3	σ=4	σ=5
UD	33158.9	34231.7	35278.4	36394.0	37608.0
US	33158.9	33159.3	33159.7	33160.2	33160.6
UP	33158.9	33259.4	33359.9	33460.5	33561.0
UI	33158.9	33531.5	33904.1	34266.0	34626.3
UT	33158.9	33164.2	33169.6	33174.9	33180.3

uncertain unit production cost (UP), uncertain inventory cost (UI), and uncertain transportation cost (UT), the experimental results presented are obtained. In the experiment, we observe the influence of the variances of uncertain variables on the total costs of (α, β)-MCM by adjusting the size of series parameters, such as ϵ in linear distribution, ϵ_1 and ϵ_2 in zigzag distribution, and σ in normal distribution. Because decision-makers are usually most interested in higher confidence levels in realistic applications, the confidence level of α is set to 0.6 and β is set to 0.8. Results show that the growth of the variances of uncertain demands or costs both lead to the increase of the total cost.

TABLE 18. (α, β)-MCM total cost under normal circumstance with $\alpha = 0.6$, $\beta = 0.8$, and $\sigma > 5$.

	σ=6	σ=7	σ=8	σ=9	σ=10
UD	38803.6	39922.3	41321.6	42425.3	43497.6
US	33161.1	33161.5	33162.0	33162.4	33162.9
UP	33661.5	33762.1	33862.6	33963.2	34063.7
UI	34986.6	35347.0	35707.3	36067.6	36427.9
UT	33185.7	33191.0	33196.4	33201.8	33207.1

VI. CONCLUSION

In this paper, the PRP under uncertain circumstances with lacking historical data is studied, to the best of our knowledge, for the first time. Based on the uncertainty theory and three uncertain decision criteria, three original uncertain models are proposed, i.e., the EMCM, (α, β)-MCM, and MMCM. In addition, the deterministic equivalents of the uncertain decisions model are presented under the assumption that all of the uncertain variables, which include cost series and demand series, are considered as independent uncertain variables, which provides a general means for understanding the PRP under uncertain settings. So these three common uncertain distribution conditions (linear, zigzag, and normal) can be understood more deeply and used more reasonably, the models applied are designed. Moreover, an instance with uncertain settings based on a classic instance is used to solve the PRP to, in turn, solve these aforementioned models more effectively. Finally, numerical experiments are shown to reveal the application of the models and the effects of confidence levels and variance. The conclusion shows that the overall cost of the optimal solution of PRP grows with the increase of the confidence levels in both uncertain demand and cost environments and is simultaneously affected by the variances of uncertain variables. Meanwhile, the probability that the optimal total cost of the PRP is less than or equal to a given threshold strictly increases as the threshold increases in uncertain environments. When the confidence level is high, as the variances of uncertain demands or costs increase, it then causes the total cost of the PRP to increase.

REFERENCES

[1] P. Chandra, "A dynamic distribution model with warehouse and customer replenishment requirements," *J. Oper. Res. Soc.*, vol. 44, no. 7, pp. 681–692, Jul. 1993.

[2] P. Chandra and M. L. Fisher, "Coordination of production and distribution planning," *Eur. J. Oper. Res.*, vol. 72, no. 3, pp. 503–517, Feb. 1994.

[3] V. Schmid, K. F. Doerner, and G. Laporte, "Rich routing problems arising in supply chain management," *Eur. J. Oper. Res.*, vol. 224, no. 3, pp. 435–448, Feb. 2013.

[4] L. Lei, S. Liu, A. Ruszczynski, and S. Park, "On the integrated production, inventory, and distribution routing problem," *IIE Trans.*, vol. 38, no. 11, pp. 955–970, Nov. 2006.

[5] A. Nananukul, "Lot-sizing and inventory routing for a production-distribution supply chain," Ph.D. dissertation, Dept. Ind. Eng., UT-Austin., Austin, TX, USA, 2008.
[6] Q. H. Zhao, S. Chen, S. C. H. Leung, and K. K. Lai, "Integration of inventory and transportation decisions in a logistics system," *Transp. Res. E, Logistics Transp. Rev.*, vol. 46, no. 6, pp. 913–925, Nov. 2010.
[7] M. Stålhane, H. Andersson, M. Christiansen, and K. Fagerholt, "Vendor managed inventory in tramp shipping," *Omega*, vol. 47, pp. 60–72, Sep. 2014.
[8] T. Bektaş, G. Laporte, and D. Vigo, "Integrated vehicle routing problems," *Comput. Oper. Res.*, vol. 55, p. 126, Mar. 2015.
[9] C. Archetti and M. G. Speranza, "The inventory routing problem: The value of integration," *Int. Trans. Oper. Res.*, vol. 23, no. 3, pp. 393–407, May 2016.
[10] J. F. Côté, G. Guastaroba, and M. G. Speranza, "The value of integrating loading and routing," *Eur. J. Oper. Res.*, vol. 257, no. 1, pp. 89–105, Feb. 2017.
[11] F. Hein and C. Almeder, "Quantitative insights into the integrated supply vehicle routing and production planning problem," *Int. J. Prod. Econ.*, vol. 177, pp. 66–76, Jul. 2016.
[12] N. Absi, C. Archetti, S. Dauzère-Pérès, D. Feillet, and M. G. Speranza, "Comparing sequential and integrated approaches for the production routing problem," *Eur. J. Oper. Res.*, vol. 269, no. 2, pp. 633–646, Sep. 2018.
[13] M. Darvish and L. C. Coelho, "Sequential versus integrated optimization: Production, location, inventory control, and distribution," *Eur. J. Oper. Res.*, vol. 268, no. 1, pp. 203–214, Jul. 2018.
[14] C. Archetti, L. Bertazzi, G. Laporte, and M. G. Speranza, "A branch-and-cut algorithm for a vendor-managed inventory-routing problem," *Transp. Sci.*, vol. 41, no. 3, pp. 382–391, Aug. 2007.
[15] Y. Adulyasak, J.-F. Cordeau, and R. Jans, "Formulations and branch-and-cut algorithms for multivehicle production and inventory routing problems," *Informs J. Comput.*, vol. 26, no. 1, pp. 103–120, Feb. 2014.
[16] D. L. T. Kyee and N. H. A. Moin, "Matheuristic approach for production-inventory-distribution routing problem," *Chiang Mai J. Sci.*, vol. 45, no. 2, pp. 1145–1160, Jun. 2018.
[17] J. F. Bard and N. Nananukul, "A branch-and-price algorithm for an integrated production and inventory routing problem," *Comput. Oper. Res.*, vol. 37, no. 12, pp. 2202–2217, Dec. 2010.
[18] J. F. Bard and N. Nananukul, "Heuristics for a multiperiod inventory routing problem with production decisions," *Comput. Ind. Eng.*, vol. 57, no. 3, pp. 713–723, Oct. 2009.
[19] M. Boudia, M. A. O. Louly, and C. Prins, "A reactive GRASP and path relinking for a combined production–distribution problem," *Comput. Oper. Res.*, vol. 34, no. 11, pp. 3402–3419, Nov. 2007.
[20] M. Boudia and C. Prins, "A memetic algorithm with dynamic population management for an integrated production–distribution problem," *Eur. J. Oper. Res.*, vol. 195, no. 3, pp. 703–715, Jun. 2009.
[21] J. F. Bard and N. Nananukul, "The integrated production–inventory–distribution–routing problem," *J. Scheduling*, vol. 12, no. 3, pp. 257–280, Jun. 2009.
[22] V. A. Armentano, A. L. Shiguemoto, and A. Løkketangen, "Tabu search with path relinking for an integrated production–distribution problem," *Comput. Oper. Res.*, vol. 38, no. 8, pp. 1199–1209, Aug. 2011.
[23] A. Senoussi, S. Dauzère-Pérès, N. Brahimi, B. Penz, and N. K. Mouss, "Heuristics based on genetic algorithms for the capacitated multi vehicle production distribution problem," *Comput. Oper. Res.*, vol. 96, pp. 108–119, Aug. 2018.
[24] Y. Adulyasak, "Models and solution algorithms for production routing problems," Ph.D. dissertation, HEC Montréal, Univ. Montréal, Montreal, QC, Canada, 2012.
[25] Y. Adulyasak, J.-F. Cordeau, and R. Jans, "Optimization-based adaptive large neighborhood search for the production routing problem," *Transp. Sci.*, vol. 48, no. 1, pp. 20–45, Feb. 2014.
[26] N. Brahimi and T. Aouam, "Multi-item production routing problem with backordering: A MILP approach," *Int. J. Prod. Res.*, vol. 54, no. 4, pp. 1076–1093, Feb. 2016.
[27] Q. Zhang, A. Sundaramoorthy, I. E. Grossmann, and J. M. Pinto, "Multiscale production routing in multicommodity supply chains with complex production facilities," *Comput. Oper. Res.*, vol. 79, pp. 207–222, Mar. 2017.
[28] T. Bayley, H. Süral, and J. H. Bookbinder, "A hybrid benders approach for coordinated capacitated lot-sizing of multiple product families with set-up times," *Int. J. Prod. Res.*, vol. 56, no. 3, pp. 1326–1344, Feb. 2018.
[29] Y. Qiu, L. Wang, X. Xu, X. Fang, and P. M. Pardalos, "Formulations and branch-and-cut algorithms for multi-product multi-vehicle production routing problems with startup cost," *Expert Syst. Appl.*, vol. 98, no. 15, pp. 1–10, May 2018.
[30] Y. Qiu, M. Ni, L. Wang, Q. Li, X. Fang, and P. M. Pardalos, "Production routing problems with reverse logistics and remanufacturing," *Transp. Res. E, Logistics Transp. Rev.*, vol. 111, pp. 87–100, Mar. 2018.
[31] Y. Shuang, A. Diabat, and Y. Liao, "A stochastic reverse logistics production routing model with emissions control policy selection," *Int. J. Prod. Econ.*, vol. 213, pp. 201–216, Jul. 2019.
[32] Y. Zhang, H. Alshraideh, and A. Diabat, "A stochastic reverse logistics production routing model with environmental considerations," *Ann. Oper. Res.*, vol. 271, no. 2, pp. 1023–1044, Sep. 2018.
[33] I. Moon, Y. Jeong, and S. Saha, "Fuzzy bi-objective production-distribution planning problem under the carbon emission constraint," *Sustainability*, vol. 8, no. 8, p. 798, Aug. 2016.
[34] X. Fang, Y. Du, and Y. Qiu, "Reducing carbon emissions in a closed-loop production routing problem with simultaneous pickups and deliveries under carbon cap-and-trade," *Sustainability*, vol. 9, no. 12, p. 2198, Nov. 2017.
[35] X. Peng, S. Ji, and T. Ji, "Promoting sustainability of the integrated production-inventory-distribution system through the Physical Internet," *Int. J. Prod. Econ.*, vol. 58, no. 22, pp. 1–20, Nov. 2019.
[36] Y. Adulyasak, J.-F. Cordeau, and R. Jans, "Benders decomposition for production routing under demand uncertainty," *Oper. Res.*, vol. 63, no. 4, pp. 851–867, Aug. 2015.
[37] A. Agra, C. Requejo, and F. Rodrigues, "A hybrid heuristic for a stochastic production-inventory-routing problem," *Electron. Notes Discrete Math.*, vol. 64, pp. 345–354, Feb. 2018.
[38] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965.
[39] R. Wang, G. Nan, L. Chen, and M. Li, "Channel integration choices and pricing strategies for competing dual-channel retailers," *IEEE Trans. Eng. Manag.*, early access, Jul. 27, 2020, doi: 10.1109/TEM.2020.3007347.
[40] B. Liu, *Uncertainty Theory*, 2nd ed. Berlin, Germany: Springer, 2007.
[41] B. Liu, *Uncertainty Theory: A Branch of Mathematics for Modeling Human Uncertainty*. Berlin, Germany: Springer, 2010.
[42] J. Zhou, F. Yang, and K. Wang, "Multi-objective optimization in uncertain random environments," *Fuzzy Optim. Decis. Making*, vol. 13, no. 4, pp. 397–413, Mar. 2014.
[43] H. Ke, Y. Wu, H. Huang, and Z. Chen, "Pricing decision in a two-echelon supply chain with competing retailers under uncertain environment," *J. Uncertainty Anal. Appl.*, vol. 5, no. 1, p. 5, Mar. 2017.
[44] J. Shen and K. Zhu, "Uncertain supply chain problem with price and effort," *Int. J. Fuzzy Syst.*, vol. 20, no. 4, pp. 1145–1158, Apr. 2018.
[45] Y. Gao, "Uncertain models for single facility location problems on networks," *Appl. Math. Model.*, vol. 36, no. 6, pp. 2592–2599, Jun. 2012.
[46] H. Ke, "A genetic algorithm-based optimizing approach for project time-cost trade-off with uncertain measure," *J. Uncertainty Anal. Appl.*, vol. 2, no. 1, p. 8, Apr. 2014.
[47] L. Wang, H. Huang, and H. Ke, "Chance-constrained model for RCPSP with uncertain durations," *J. Uncertainty Anal. Appl.*, vol. 3, no. 1, p. 12, Sep. 2015.
[48] W. Ma, Y. Che, H. Huang, and H. Ke, "Resource-constrained project scheduling problem with uncertain durations and renewable resources," *Int. J. Mach. Learn. Cybern.*, vol. 7, no. 4, pp. 613–621, Aug. 2016.
[49] H. Ke, T. Su, and Y. Ni, "Uncertain random multilevel programming with application to production control problem," *Soft Comput.*, vol. 19, no. 6, pp. 1739–1746, Jun. 2015.
[50] Y. H. Liu and M. Ha, "Expected value of function of uncertain variables," *J. Uncertain Syst.*, vol. 4, no. 3, pp. 181–186, Sep. 2010.
[51] X. Yang, "On comonotonic functions of uncertain variables," *Fuzzy Optim. Decis. Making*, vol. 12, no. 1, pp. 89–98, Mar. 2013.
[52] B. Liu, "Some research problems in uncertainty theory," *J. Uncertain Syst.*, vol. 3, no. 1, pp. 3–10, Jan. 2009.
[53] C. Archetti, L. Bertazzi, G. Paletta, and M. G. Speranza, "Analysis of the maximum level policy in a production-distribution system," *Comput. Oper. Res.*, vol. 38, no. 12, pp. 1731–1746, Dec. 2011.

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