IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received January 4, 2021, accepted January 15, 2021, date of publication January 20, 2021, date of current version January 27, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3052980

Importance Measure of Equipment Task Based on Operational Dependency of SoS

TINGXUE XU[1], **YUQI CHEN**[1], **CHENG LU**[2], **HAIJUN LI**[3], **AND JIAPENG LV**[1]
[1]Navy Aeronautical University, Yantai 264001, China
[2]Naval Equipment Support Brigade, Beijing 100089, China
[3]91115 of PLA, Zhoushan 316041, China

Corresponding author: Yuqi Chen (bighammer7@163.com)

This work was supported by the National Natural Science Foundation of China under Grant 51975580.

ABSTRACT In order to evaluate the mission success of system of systems (SoS) and to develop maintenance strategies to meet mission requirements of SoS, an importance measure method of equipment task based on operational dependency of SoS is proposed. The operational dependency network of SoS is constructed based on FDNA, and the nodes and dependency relationships in the network are determined. According to the Markov process, the reliability model of a system function is constructed, and the operability level and probability of each node in the network are obtained. Meanwhile, the algorithm of Birnbaum importance measure is introduced to effectively quantify the impact of operational dependency on the task importance of each equipment system in SoS. Finally, the feasibility and effectiveness of this method is verified by a case study.

INDEX TERMS System of systems (SoS), operational dependency, importance measure, operability level.

I. INTRODUCTION

With the proposal of ‘‘network centric warfare’’, the complexity of combat system and combat mission is increasing. In the same mission, multiple equipment systems need to work independently and cooperate with each other, consequently the concepts of system of systems (SoS) [1]–[3] and system of systems engineering (SoSE) [4]–[6] emerge as the times require. Each system in SoS often shows the independence of management and operation, but it must operate continuously without faults and achieve a group of overall capabilities to complete the corresponding tasks in the mission by cooperating with each other and playing different functions, which leads to the difference of the importance measures [7] of equipment system in different combat missions of SoS. The mission importance of equipment reflects the degree to which the equipment plays a role in the specific mission of SoS, i.e. its contribution [8] to combat mission. When SoS carries out a combat mission, it is inevitable that system failures or failure events might occur. Through the reasonable analysis of task importance, we can identify key equipment systems in different combat tasks and weak links in task execution, determine the maintenance priority of component systems, and provide the basis for the maintenance decision-making of SoS [9], [10].

At present, there are few researches on this kind of problems. Researches on importance measures mainly focus on the single system [11], including structural importance [12]–[14], reliability importance [15], [16] and lifetime importance [17], [18]. However, most of the researches on importance measures or contribution degree of SoS are based on the analysis of network characteristics of SoS, and the architecture of SoS is mapped to the complex network by introducing complex network theory. The component system is transformed into the network node, thus the equipment importance analysis in the SoS is transformed into the node importance analysis in the network. It is generally divided into two categories: one is to analyze the significant characteristics of nodes, including not only single indexes such as betweenness [19], [20], node weight [21], [22], neighbor average degree [23], [24], but also the combination indexes of degree [25], [26] and coreness [27] and comprehensive indexes obtained by TOPSIS and some other methods [28]; the other is to analyze the corresponding impact of damaged node on the network [29], [30]. Although the above researches have made up for the blank of importance analysis of SoS to a certain extent and have provided certain basis for its development, there are still some deficiencies, mainly including the following aspects:

The associate editor coordinating the review of this manuscript and approving it for publication was Cristian Zambelli.

This work is licensed under a Creative Commons Attribution 4.0 License. For more information, see https://creativecommons.org/licenses/by/4.0/

(1) The SoS is driven by mission. And component systems complete the corresponding tasks by running the corresponding functions under the given combat tasks, so as to ensure the completion of the mission. The analysis of the importance of the equipment in SoS should focus on the importance of the equipment system to the combat tasks. From the perspective of operational suitability, different task capabilities are the embodiment of the combat task objectives, which are realized by different equipment systems with different functions. Therefore analysis of the importance of equipment to combat task in SoS can be converted into analysis of the importance of its functions to task capabilities.

(2) During the operation (use) of the system, whether it operates properly depends on the current operational status or operational output of other systems to a certain extent, i.e. there is operational dependency between systems and capabilities. The failure or degradation of a single system or capability in SoS will impose a certain impact on the operational level of other systems or capabilities. Therefore it is necessary to construct the operational dependency network of SoS instead of the network with general structure to analyze the task importance.

(3) At present, existing researches only consider that the equipment system is a system with ''two states''. However, the current equipment system is generally complex in structure and can demonstrate several different operational levels [31] (i.e. performance state). Only roughly dividing the equipment operational status into ''up'' and ''down'' is difficult to accurately describe the evolution law of equipment operational level during the mission. Therefore ''multi-state'' characteristics of a system should be considered when analyzing the task importance and a suitable method for analyzing degradation [32]–[34] of the equipment system is needed.

In response to these deficiencies, the operational dependency network of SoS is constructed based on the characteristics of SoS operational dependence. And based on the ''multi-state'' characteristics of the equipment, a dynamic method of analyzing the operational levels of different nodes and their corresponding probability by using Markov process is determined. According to the given task operational requirements, the task importance analysis method based on operational dependency of SoS is proposed and its specific analysis and calculation process is illustrated. Finally the method is verified through a case.

II. OPERATIONAL DEPENDENCY NETWORK OF SoS MODELING

Generally speaking, Typical representations of a SoS involve networked combinations of the constituent systems that ultimately provide SoS-level capabilities, as shown in Figure 1. At the highest level, SoS is essentially a collection of capabilities required to perform various tasks, driven by the combat mission. The execution of combat tasks requires that all systems in SoS cooperate with each other to provide corresponding task capabilities. Task capabilities required by different combat tasks are one or several elements of SoS-level capabilities. Capabilities are generated from the collection of system-level functions, which are provided by various systems. Each system generally has multiple functions. System functions are described by selecting key performance parameters, which are generally divided into six categories including c2 (command and control), battlespace awareness, maneuver, fires, communication and protection. Different system functions and task capabilities are interconnected through operational dependency and different functions of the same system are operated in different tasks. The meanings of their operational levels are also different.

At the capability level, there may be operational dependencies between different task capabilities in the same combat mission. For example, the effectiveness level of fire damage capability depends on the actual effectiveness level of target detection capability in a target damage task.

At the function level, there may be operational dependencies between different functional modules of different equipment systems performing the same combat task. For example, the ground receiving station is a bridge for data transmission between the UAV and the command system in a target detection task, so the detection function of UAV depends on the communication function of the ground receiving station to a certain extent.

In the same combat task, realizations of different task capabilities depend on normal operation of functions of equipment systems. For example, if two UAVs undertake a target detection task, the target detection capability of the task depends on depends on the target detection function of two UAVs.

According to the reliability block diagram of different functions of each system, dynamic reliability models of system functions are constructed. We can judge whether the system can meet the demands of the task by analyzing the operational levels of each function of the specific system under different tasks, so as to evaluate the overall impact of its failure or functional degradation on the whole. To achieve the judgment, whether its function is available at a given time and whether its function can continue to operate within a specified period of time both should be determined.

Functional dependency network analysis (FDNA) [35] is a dependency analysis method of SoS proposed by Garvey and Pinto in 2009. It is used to measure the ripple effects of degraded operability in one or more entities or feeder–receiver chains on SoS due to risks, so as to measure the non-operability of each entity or chain in SoS. Chen *et al.* [36], Chen *et al.* [37], and Zhang *et al.* [38] all improved the classical FDNA method, so as to conduct in-depth study on several aspects of SoS. On the basis of FDNA, Guariniello and Delaurentis [39] proposed SODA and SDDA to analyze dependency of SoS in operation and development process, and furthermore effectively analyzed the flexibility and robustness of SoS. To some extent, these studies meet the basic requirements of dependence analysis of SoS, but the explanation of nodes, dependency relations and dependency types in the network is insufficient. And the impact of dynamicity of the task and performance degradation of

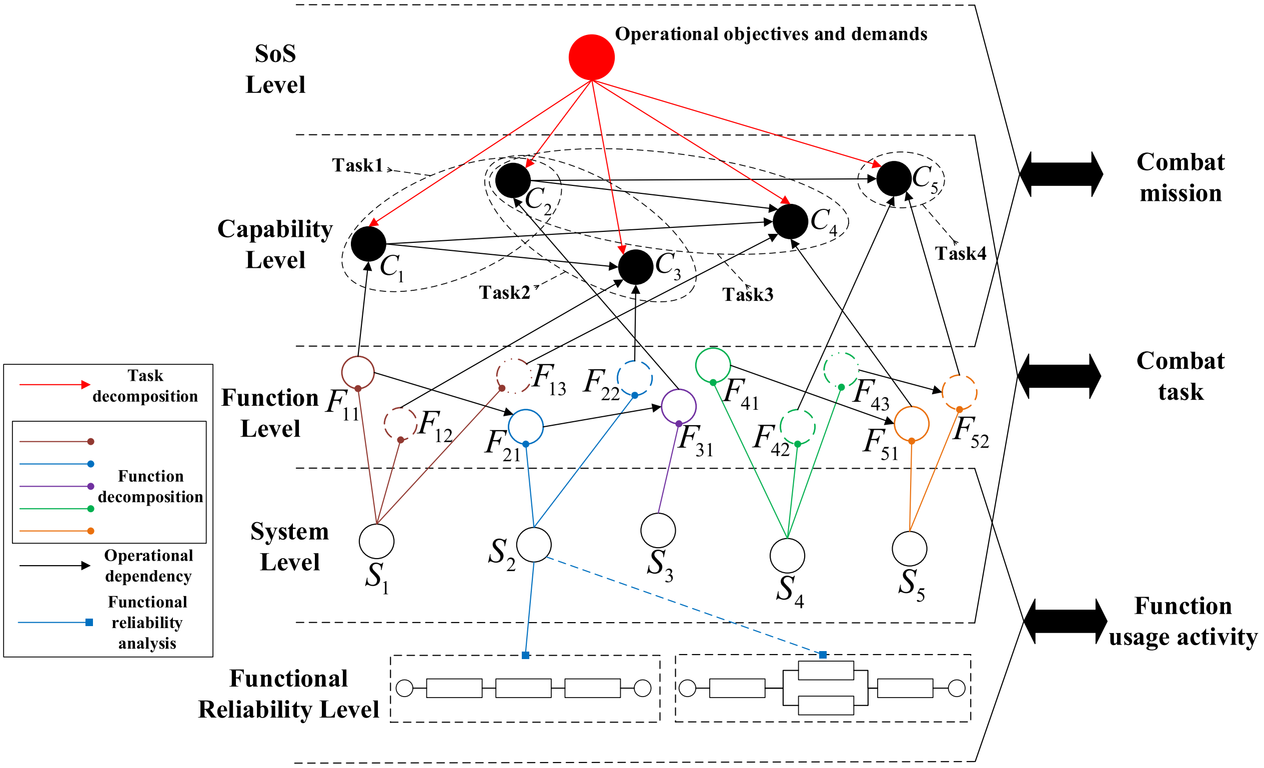


FIGURE 1. Network construction of SoS based on operational dependency.

the system on operational dependency is not considered. The related parameters of the analysis of dependency need to be further explored.

A. NETWORK CONSTRUCTION

Each equipment system in SoS is the undertaker and executor of a task, and the task capability is also the result of the cooperation of different functions of each equipment system. Therefore, before the network construction, the following definitions are given:

$$SoS = \{S_1, S_2, \ldots, S_n\} \tag{1}$$

$$S_i = \left\{F_{i1}, F_{i2}, \ldots, F_{im_i}\right\}, \quad i = 1, 2, \ldots, n \tag{2}$$

$$C = \left\{C_1, C_2, \ldots, C_p\right\} \tag{3}$$

where: SoS represents a certain system of systems, which is composed of n equipment systems; S_i represents the ith equipment system, which includes m_i functions; C represents the set of task capabilities in one task, and each task capability corresponds to several functions of some equipment systems that perform the task.

Operational dependency network is represented by $G = \{N, E\}$, which is mainly composed of nodes and edges. The node represents the equipment system or task capability, which is represented by $N = \{N_S, N_C\}$, where N_S is the set of system nodes and N_C is the set of capability nodes. These nodes can be a feeder node or a receiver node, or both. It should be emphasized that since the equipment system has multiple functions, for the system node, the jth function node of S_i is represented by $N_{F_{ij}}$, that is:

$$N_{S_i} = \left\{N_{F_{i1}}, N_{F_{i2}}, \ldots, N_{F_{im_i}}\right\}, \quad i = 1, 2, \ldots, n \tag{4}$$

The edge represents the dependency between nodes and $E = \{E_1, E_2, \ldots, E_n\} \subseteq N \times N$ represents all edges. From the above description, it can be included that the operational dependency network is actually a description of the dependency between system functions and task capabilities. Therefore, system nodes can be decomposed into corresponding function nodes to accurately construct the operational dependence network, as shown in Figure 2.

The related concepts in the network can be defined as follows:

1) NODE

According to the description of operational dependency network in Figure 1 and Figure 2, nodes in the network are divided into two categories: function nodes and capability nodes.

The function node represents the specific function of the equipment system. It is formed by the cooperation of various components of the system after a certain series and parallel connection. It is the external expression of the system structure and belongs to the entity node. It can be roughly divided into c2 nodes, battlespace awareness nodes, maneuver nodes, fire nodes, communication nodes and protection nodes.

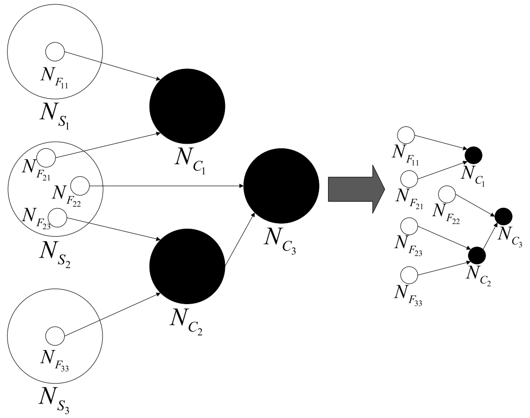


FIGURE 2. **Dependency network of function node and capability node.**

The capability node mainly refers to the basic capabilities needed to complete the task and belongs to virtual node. It is formed by the cooperation of the relevant equipment systems operating their respective functions according to objectives and demands of the task. Therefore it depends on functions of equipment systems in SoS.

In addition, from the perspective of dependency, it can be divided into feeder nodes and receiver nodes, which is similar to the ‘‘parent-child’’ relationship. The operational level of the receiver node depends on the operational level of the feeder node. The feeder node supplies its output to one or more receiver nodes.

2) OPERABILITY LEVEL *O*

In operational dependency networks, operability refers to the ability of nodes to operate reliably according to pre-defined operational demands. The operability level is its corresponding measurement, ranging from 0 to 100.

The operability level of capability node is defined as the level of combat effectiveness (O_C), which represents the size of task capability.

The operability level of function node is defined as the operational performance level of function (O_F), which represents the performance level of system function (i.e. system function state). It transforms the performance level of system function into value or utility. For different function nodes, the size and dimension of performance parameters corresponding to different physical quantities are different, and they are uniformly expressed as dimensionless metrics through processing in the network.

3) BASELINE OPERABILITY LEVEL O_B

O_B refers to the operability level of the receiver node operating alone in SoS when all dependencies are unavailable. The basic operational level of capability node is defined as the basic level of combat effectiveness (O_B^C). O_B^C generally takes the value of 0 because capability node depends on function node. The basic operability level of function node is defined as the basic operational performance level of function, and its value is calculated according to the operational dependency network.

4) INDEPENDENT OPERATIONAL PERFORMANCE LEVEL O_F^I

The classical FDNA method assumes that the original operability level of nodes is 100, which is obviously unreasonable in practice. In the mission executing process of SoS, function nodes themselves will have faults or performance degradation, resulting in O_F failing to reach the optimal value, that is, the function nodes themselves have multiple degraded states. This situation does not exist when calculating O_C in the network because O_C completely depends on O_F and the capability node has no performance degradation phenomenon itself. Therefore, O_F^I represents the independent operational performance level of the function node and ranges from 0 to 100, which is basically consistent with O_F. However, the difference between them is that O_F^I does not consider any dependency relationship in SoS and only considers the impact of performance degradation of the function node on operability level. And O_F is the final output of the operability level of the function node during the execution of the task by considering the operational dependency and O_F^I.

5) DEPENDENCY RELATION

Dependency relation is a condition or state existing between two nodes. In the operational dependency network, it means that the operation (use) of one node depends on the operation (use) of another node to a certain extent. It is usually divided into single dependency relation (one-to-one) and multi dependency relation (one to many).

B. CALCULATION AND ANALYSIS OF OPERATIONAL DEPENDENCY

FDNA usually uses a set of parameters $\{SOD_{i,j}, COD_{i,j}\}$ to construct the dependency calculation function of SoS. $SOD_{i,j}$ represents the strength of dependency between nodes N_i and N_j, which is used to describe the additional contribution of the dependency relationship to the basic operability level of the receiver node (i.e. characterizing the auxiliary characteristics of the dependency relationship). It contains a parameter called strength of dependency fraction α_{ij} for the corresponding calculation and $0 \leq \alpha_{ij} \leq 1.COD_{i,j}$ represents the criticality of dependency between two nodes, which is used to describe the restriction of the dependency relationship on the basic operability level of the receiving node (i.e. characterizing the constraint characteristics of the dependency relationship). It contains a parameter called criticality of dependency parameter β_{ij} for the corresponding calculation and $0 \leq \beta_{ij} \leq 100$.

Suppose that there is a dependency relationship between node N_j and feeder nodes $N_1, N_2, N_3, \ldots, N_h$, to design the basic algorithm of operational dependency analysis, some other variables and parameters are also defined as follows:

O_j The operability level of N_j

$f(O_1, O_2, O_3, \ldots, O_h)$ The function of operability level of N_j based on strength of dependency

$g(O_1, O_2, O_3 \ldots, O_h)$ The function of operability level of N_j based on criticality of dependency

$O_{SOD_{ij}} (i = 1, 2, \ldots, h)$ The value of operability level between nodes N_j and N_i based on $SOD_{i,j}$

$O_{B_{ij}}$ The value of basic operability level of node N_j when the operability level of feeder node N_i is 0

O_j^i The operability level of node N_j when the operability level of N_i in the feeder node set is 100 and other nodes are 0

$O_{COD_{ij}} (i = 1, 2, \ldots, h)$ The value of operability level between nodes N_j and N_i based on $COD_{i,j}$

According to the FDNA weakest link rule, the operability level of $N_j$is measured as follows:

$$\begin{aligned} O_j = \mathrm{Min}(&f(O_1, O_2, O_3, \ldots, O_h), \\ &g(O_1, O_2, O_3 \ldots, O_h)) \end{aligned} \tag{5}$$

$$f(O_1, O_2, O_3, \ldots, O_h) = O_{SOD_j} \tag{6}$$

$$\begin{aligned} O_{SOD_j} = \mathrm{Average}(&O_{SOD_{1j}}, \\ &O_{SOD_{2j}}, \ldots, O_{SOD_{hj}}) \end{aligned} \tag{7}$$

$$\begin{aligned} O_{SOD_{ij}} &= \alpha_{ij} O_i + 100(1 - \alpha_{ij}), \\ &\quad i = 1, 2, \ldots, h \end{aligned} \tag{8}$$

In equation (8), the larger the value of α_{ij}, the greater the contribution of the feeder node, and the stronger the dependency of the receiver node on the feeder node.

$$O_{B_{ij}} = 100(1 - \alpha_{ij}), \quad i = 1, 2, \ldots, h \tag{9}$$

$$O_{B_j} = \mathrm{Average}(O_{B_{1j}}, O_{B_{2j}}, \ldots, O_{B_{hj}}) \tag{10}$$

When N_j has multiple dependency relationships, the final value of its basic operability level is the average value of N_j operability levels in each dependency relationship, as shown in equation (10).

Therefore, the value of α_{ij} can be determined from the above, as shown in equation (11):

$$\alpha_{ij} = \frac{O_j^i - O_{B_j}}{O_j^i} \times 100\% \tag{11}$$

$$g(O_1, O_2, O_3 \ldots, O_h) = O_{COD_j} \tag{12}$$

$$\begin{aligned} O_{COD_j} = \mathrm{Min}(&O_{COD_{1j}}, O_{COD_{2j}}, \ldots, \\ &O_{COD_{hj}}) \end{aligned} \tag{13}$$

$$O_{COD_{ij}} = O_i + \beta_{ij}, i = 1, 2, \ldots, h \tag{14}$$

When the operability level of the feeder node is 0, the operability level of the receiver node will be degraded from its basic operability level, which leads to a certain loss of $(100 - \beta_{ij})$. The smaller the value of β_{ij}, the greater the constraints of the feeder node on the receiver node and the higher its criticality.

The basic algorithm of operational dependency analysis is aforementioned, but it is found that the algorithm has some defects in practical use. Firstly, the influence of O_F^I on the dependency relationship is not considered; secondly, the basic parameters used in FDNA are not enough to accurately describe the operational dependency network; thirdly, it cannot be applied to the analysis and calculation of various kinds of dependency relationships. Therefore, the algorithm of FDNA needs to be improved to ensure its accuracy and effectiveness.

For the single dependency relationship, the feeder node is generally the function node, and the receiving node is the function node or capability node. Therefore, the calculation of the operability level of the receiver node will be different mainly due to different types of receiver nodes.

In equation (8), the function of operability level based on strength of dependency is only controlled by a single parameter α. However, the performance level of equipment system will degrade over time in the process of task execution, which will lead to the decrease of operability level of system function. As a feeder node, it will affect the operability level of receiver node, and as a receiver node, it will also affect its own operability level. Therefore, in this paper, the function degradation coefficient δ is introduced to improve the function of operability level based on strength of dependency combined with reference [40] and its calculation formula is given as follows:

$$\delta = \frac{O_F^I}{100} \times 100\% \tag{15}$$

As a capability node, it is completely affected by the function node and has no fault and performance degradation. Assuming that the feeder node is $N_i$and the receiver node is N_j, the improved function of operability level based on strength of dependency is as follows:

$$\text{Function node}: \begin{cases} O_{SOD_j} = \delta_j \alpha_{ij} O_i + O_{F_j}^I (1 - \alpha_{ij}) \\ \delta_j = \dfrac{O_{F_j}^I}{100} \times 100\% \end{cases} \tag{16}$$

$$\text{Capability node}: O_{SOD_j} = \alpha_{ij} O_i + 100(1 - \alpha_{ij}) \tag{17}$$

In equation (14), the function of operability level based on criticality of dependency is only controlled by a single parameter β. Assuming that the feeder node is N_i and the receiver node is N_j, the piecewise linear model of O_j with O_i can be obtained according to the classical FDNA method, as shown in Figure 3. In the COD zone, the slope of the function of operability level of N_j is always 1 and O_j only depends solely on O_i without any contributions by the independent operational performance level of N_j.

In practice [41], it can be found that O_j can increase faster than O_i in COD zone, the influence of criticality of dependency should be limited to a smaller area, and β_{ij} should gradually decrease with the decrease of $O_{F_j}^I$. Therefore in this paper the impact of dependency parameter $\gamma (1 \leq \gamma \leq 100)$ is introduced to improve the function of operability level based on criticality of dependency combined with δ as follows:

$$\text{Function node}: O_{COD_j} = \frac{100}{\gamma_{ij}} O_i + \delta_j \beta_{ij} \tag{18}$$

$$\text{Capability node}: O_{COD_j} = \frac{100}{\gamma_{ij}} O_i + \beta_{ij} \tag{19}$$

The smaller the value of γ_{ij}, the smaller the influence of criticality of dependency.

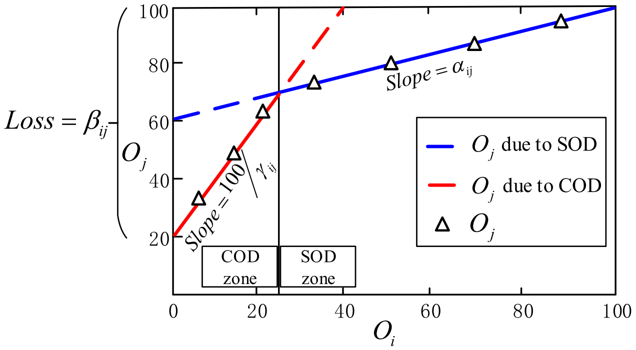


FIGURE 3. **The piecewise linear model of operability level of single dependency in FDNA.**

For multiple dependency relationships, the influence of types of node and dependency relationship on the result should be considered when calculating the operability level of receiver node.

In the multiple dependency relationship, it can be divided into "or" dependency and "and" dependency. Assume that the receiver node is N_j, and the set of feeder nodes is $\{N_1, N_2, N_3, \ldots, N_h\}$:

(1) If there is "and" dependency between feeder nodes, that is, all the feeder nodes make a certain contribution to the receiver node, and the unavailability (failure) of any feeder node will lead to the operability level of the receiver node being 0, then the calculation formula of O_{COD_j} is as follows:

$$O_{COD_j} = \begin{cases} \text{Min}(O_{COD_{1j}}, O_{COD_{2j}}, \ldots, O_{COD_{hj}}), & \prod_{i=1}^{h} O^I_{F_i} \neq 0 \\ 0, & \prod_{i=1}^{h} O^I_{F_i} = 0 \end{cases} \tag{20}$$

(2) If there is "or" dependency between feeder nodes, that is, all the feeder nodes can contribute to the receiver node, when and only if all feeder nodes are unavailable, the operability level of receiver node is 0, then the calculation formula of O_{COD_j} is as follows:

$$O_{COD_j} = \begin{cases} \text{Min}(O_{COD_{1j}}, O_{COD_{2j}}, \ldots, O_{COD_{hj}}), & \sum_{i=1}^{h} O^I_{F_i} \neq 0 \\ 0, & \sum_{i=1}^{h} O^I_{F_i} = 0 \end{cases} \tag{21}$$

If the dependency relationship between the set of feeder nodes and the receiver node is of mixed type, we need to modify equations (20) and (21) to obtain the final calculation formula, which will not be repeated here.

The formation of capability depends on the operation and exertion of system functions. In SOD zone, each function node has auxiliary dependency on the capability node and has a certain contribution to the level of combat effectiveness of capability node. Its calculation formula is roughly the same as that in single dependency relationship. See equation (17) for details.

In COD zone, if all the function nodes in the feeder node set are failed, the level of combat effectiveness of capability node will be reduced to 0. While the capability node in the feeder node set generally plays an auxiliary role to the receiver node, and if it is unavailable, it will not have a devastating impact on the receiver node. In addition, β_{ij} is defined as the constraint on N_j caused by the failure of N_i when other feeder nodes operate normally, while β_{ij}should cause certain loss when operability levels of other function nodes decrease. Therefore the weight should be used to judge the loss caused by the complete failure of any function node on the final level of combat effectiveness of the capability node. Based on the reference [41], in this paper the function weight coefficient ω and the criticality weight coefficient μ are introduced to improve the function of operability level based on criticality of dependency as follows:

$$\begin{cases} O_{COD_{ij}} = \dfrac{100}{\gamma_{ij}} O_i + \omega_{ij}\beta_{ij} \\ \omega_{ij} = \dfrac{\sum\limits_{k=1 k\neq i}^{h} \mu_{kj} O^I_{F_k}}{100 \sum\limits_{k=1 k\neq i}^{h} \mu_{kj}} \times 100\% \\ \mu_{kj} = \dfrac{\beta_{kj}}{\sum\limits_{i=1}^{h} \beta_{ij}} \times 100\% \\ i = 1, 2, \ldots, h \end{cases} \tag{22}$$

Equation (22) is generally applicable to the case that the receiver node is a capability node. If the receiver node is a function node, $\omega_{ij}\beta_{ij}$ in equation (22) should be replaced by $\omega_{ij}\delta_j\beta_{ij}$, so as to consider the corresponding impact of degradation of independent operational performance level of nodes.

In addition, the methods of obtaining values of parameters of operational dependency are generally divided into two categories: data-based approaches and expert judgement-based approaches. Data-based approaches generally use available data to perform full factorial design or fractional factorial design of experiments and then use the obtained results to perform data fitting or parametric regression analysis to obtain various parameters in operational dependency network. Expert judgement-based approaches are suitable for the situation where the data is difficult to obtain. Based on the method and data given in reference [41], in this paper expert judgement-based approaches are mainly used to tune the parameters in operational dependency network of SoS.

C. INDEPENDENT OPERATIONAL PERFORMANCE LEVEL MODELING OF FUNCTION NODE

According to the previous section, performance of functional modules will degrade during operation. So only function nodes in the network have independent operational performance level (state) that can be described dynamically and continuously.

In addition, for most complex equipment systems, the number of failures in arbitrary time intervals in many practical cases can be described as a Poisson process and the time up to the failure and repair time are often exponentially distributed. The Markov process has unique advantages in describing the state transition of such systems. Therefore, based on the reliability analysis of system function and considering its characteristics of multistate, this paper constructs a state model of function node by using Markov process in order to simplify the calculation. Firstly, the following assumptions are made:

(1) The sojourn time of performance state of each system function in SoS obeys exponential distribution.

(2) Each system in SoS is non-repairable.

(3) The function F_{mk} of equipment system S_m has n_{mk} states (i.e. n_{mk} discrete independent operational performance levels) in the degradation process, and the transition intensity strength $\lambda_{i,j}^{mk}$ from state i to state j is a constant, where $i, j \in 1, 2, \ldots, n_{mk}$. 1 is the failure state of F_{mk}, n_{mk} is the perfect state of F_{mk}, and states $2 \sim n_{mk} - 1$ are intermediate states between state 1 and state n_{mk}.

(4) It is assumed that the degenerate stochastic process of F_{mk} starts from the perfect state n_{mk}, that is, the initial conditions are as follows:

$$\begin{cases} p_{n_{mk}}^{mk}(0) = 1 \\ p_1^{mk}(0) = p_2^{mk}(0) = \ldots = p_{n_{mk}-1}^{mk}(0) = 0 \end{cases} \tag{23}$$

According to the above assumptions, the Markov [42] process of independent operational performance state (level) of function F_{mk} of equipment system S_m under the condition of gradual and abrupt failure is constructed as shown in Figure 4.

Given the state transition intensity matrix of F_{mk}, transformation theorem of Laplace-Stieltjes and equation (23), probability of F_{mk} in each performance state at any time t can be obtained by solving the following differential equations of Kolmogorov [43]:

$$\begin{cases} \dfrac{\mathrm{d}p_{n_{mk}}^{mk}(t)}{\mathrm{d}t} = -p_{n_{mk}}^{mk}(t) \displaystyle\sum_{e=1}^{n_{mk}-1} \lambda_{n_{mk},e}^{mk} \\ \dfrac{\mathrm{d}p_i^{mk}(t)}{\mathrm{d}t} = \displaystyle\sum_{e=i+1}^{n_{mk}} \lambda_{e,i}^{mk} p_e^{mk}(t) - p_i^{mk}(t) \sum_{e=1}^{i-1} \lambda_{i,e}^{mk}, \\ \qquad i = 2, 3, \ldots, n_{mk} - 1 \\ \dfrac{\mathrm{d}p_1^{mk}(t)}{\mathrm{d}t} = \displaystyle\sum_{e=2}^{n_{mk}} \lambda_{e,1}^{mk} p_e^{mk}(t) \end{cases} \tag{24}$$

By plugging the above calculation results into the network, the final probability of node N_j in different operability levels at any time t under the influence of operational dependency can be obtained.

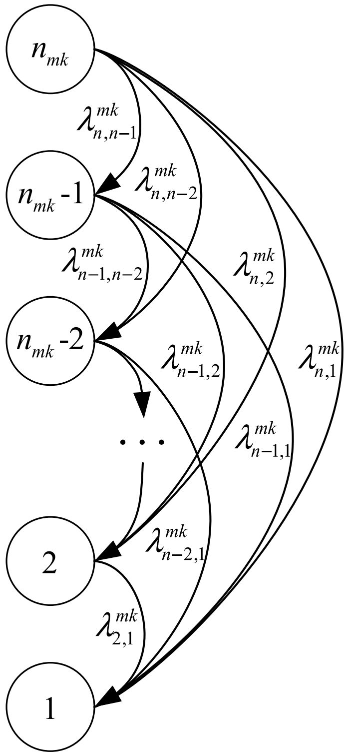


FIGURE 4. Markov process of independent operational performance state(level) of function of equipment system under the condition of gradual and abrupt failure.

III. IMPORTANCE MEASURE OF EQUIPMENT TASK IN THE NETWORK

The specific process of SoS performing combat mission can be transformed into the process of normal operation of equipment system functions to form the required task capabilities. Therefore, the equipment task importance in SoS can be transformed into importance of function nodes relative to capability nodes in the network. Combat task execution is a dynamic process, and the task importance of equipment must change with the task time This paper improves the Birnbaum importance measure and proposes the importance measure of equipment task in the operational dependency network from the perspective of functional reliability analysis.

In 1969, Birnbaum first proposed the Birnbaum importance measure of reliability based on two-state system, which is used to describe the influence of component reliability changes on system reliability changes:

$$\begin{aligned} I_B(i; p) &= \frac{\partial R(p)}{\partial p_i} \\ &= \Pr\{\varphi(X) = 1 \,|X_i = 1\} - \Pr\{\varphi(X) = 1 \,|X_i = 0\} \end{aligned} \tag{25}$$

where: p_i is the reliability of component i. p is a vector composed of reliability of all components in the system. $R(p)$ is the reliability of the system.

The Birnbaum importance measure is only a static analysis method. Adding time factor can extend Birnbaum importance measure to B-TDL importance measure:

$$I_{B-TDL}(i;\bar{F}(t)) = R(1_i,\bar{F}(t)) - R(0_i,\bar{F}(t)) \tag{26}$$

According to the above section, most of equipment systems in SoS have multiple operability levels during the task execution, and the success of the task depends on whether the current operability level of each system meets demands of the task. While the traditional importance measure only supports the "two-state" characteristics of the system, based on the importance measure of multi-state system proposed in reference [44], the calculation formula of B-TDL importance measure of function node N_i relative to capability node N_j is given as follows:

$$I^{j}_{B-TDL}(i;t) = \frac{\sum_{k=1}^{n_i} \left| \Pr\left\{O_j(t) \geq w_j \,\middle|\, \phi(O^{I}_{F_i}(t)) = b_{ik}\right\} - \Pr\left\{O_j(t) \geq w_j\right\}\right|}{n_i - 1} \tag{27}$$

where: w_j is the task demand of capability node N_j, which is considered as a constant in this paper. n_i is the total number of independent operational performance states of N_i. b_{ik} represents that the independent operational performance level of N_i is in the kth state and $O^{I}_{F_i}(t)$ is the independent operational performance level of N_i at time t, function $\phi(\cdot)$ is used to establish the mapping relationship between them. $(n_i - 1)$ is the normalization of importance measure according to the number of states of the node.

The process of the importance measure of equipment task based on operational dependency in SoS can be given from the above, as shown in Figure 5.

IV. CASE STUDY

In this paper, the case and corresponding data in reference [41] are adopted to explain and verify the proposed method.

In a small Naval Warfare scenario, a MH-60 helicopter and a ship equipped with detection and combat system constitute a SoS to perform a task. The enemy target is an enemy ship heading for the coast. Both helicopters and ships can detect the target, and the combat system on the ship will carry out fire strike on it. In this task, the commander is concerned about the degree of fire attack suffered by the enemy ship. The problem will be analyzed according to the corresponding process proposed in this paper.

A. CONSTRUCTION OF OPERATIONAL DEPENDENCY NETWORK

According to the scenario, five nodes in the network can be determined, which are function node of helicopter detection N_1, function node of ship detection N_2, capability node of target detection N_3, function node of ship combat N_4 and capability node of fire damage N_5. N_1 and N_2 work together to form the target detection capability in the task, that is, both N_1 and N_2 contribute to N_3 and form "or" dependency. In order to improve the degree of fire damage to enemy ships, on the one hand, combat function of the ship is needed to be effective; on the other hand, it also needs the capability of target detection to improve the damage accuracy. Therefore N_3 and N_4 make a certain contribution to N_5 and form the corresponding dependency relationship. The operational dependency network of this scenario is shown in Figure 6.

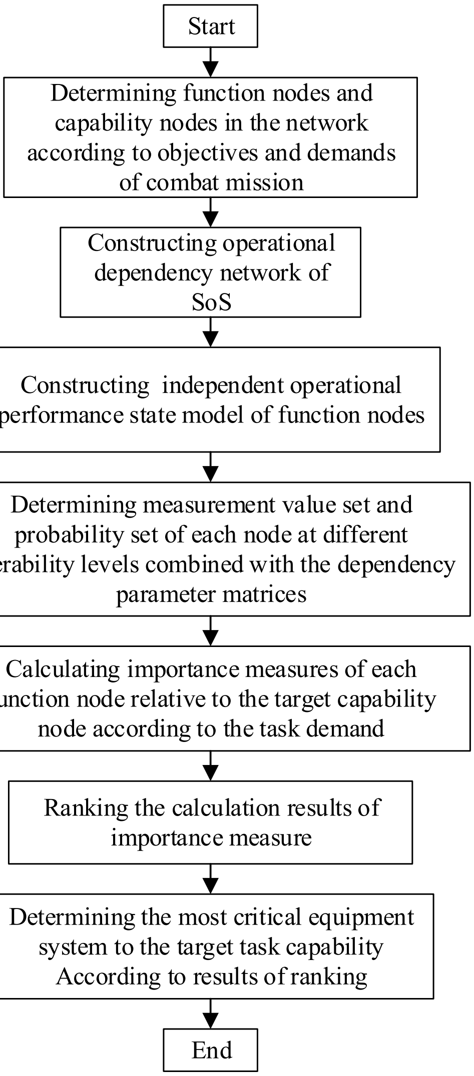


FIGURE 5. Analysis process of importance measure of equipment task based on operational dependency.

The parameter matrices of α_{ij}, β_{ij} and γ_{ij} in the network are given as follows:

$$\alpha_{ij} = \begin{bmatrix} 0 & 0 & 0.28 & 0 & 0 \\ 0 & 0 & 0.88 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.89 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{28}$$

$$\beta_{ij} = \begin{bmatrix} 0 & 0 & 9.03 & 0 & 0 \\ 0 & 0 & 85.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 60 \\ 0 & 0 & 0 & 0 & 39.7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{29}$$

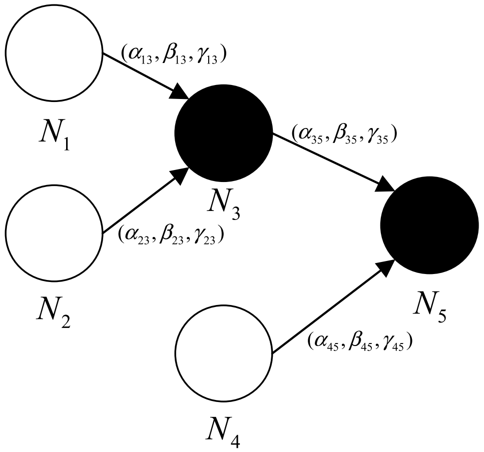


FIGURE 6. Example of operational dependency network in SoS.

$$\gamma_{ij}=\begin{bmatrix}0&0&100&0&0\\0&0&57.6&0&0\\0&0&0&0&100\\0&0&0&0&52.1\\0&0&0&0&0\end{bmatrix}\tag{30}$$

According to the above conditions, we can get functions of combat effectiveness level of capability node as follows:

$$N_3\quad\begin{cases}O_{C_3}=\mathrm{Min}(f(O_{F_1},O_{F_2}),g(O_{F_1},O_{F_2}))\\ f(O_{F_1},O_{F_2})=O_{SOD_3}=\mathrm{Average}(O_{SOD_{13}},O_{SOD_{23}})\\ g(O_{F_1},O_{F_2})=O_{COD_3}=\begin{cases}\mathrm{Min}(O_{COD_{13}},\\ \quad O_{COD_{23}}), & \sum\limits_{i=1}^{2}O_{F_i}^{I}\neq 0\\ 0, & \sum\limits_{i=1}^{2}O_{F_i}^{I}=0\end{cases}\\ O_{SOD_{i3}}=\alpha_{i3}O_{F_i}+100(1-\alpha_{i3})\\ O_{COD_{i3}}=\frac{100}{\gamma_{i3}}O_{F_i}+\omega_{i3}\beta_{i3}\\ \omega_{13}=\frac{O_{F_2}^{I}}{100}\times 100\%\\ \omega_{23}=\frac{O_{F_1}^{I}}{100}\times 100\%\\ i=1,2\end{cases}\tag{31}$$

$$N_5\quad\begin{cases}O_{C_5}=\mathrm{Min}(f(O_{C_3},O_{F_4}),g(O_{C_3},O_{F_4}))\\ f(O_{C_3},O_{F_4})=O_{SOD_5}=\mathrm{Average}(O_{SOD_{35}},O_{SOD_{45}})\\ g(O_{C_3},O_{F_4})=O_{COD_5}=\begin{cases}\mathrm{Min}(O_{COD_{35}},\\ \quad O_{COD_{45}}), & O_{F_4}^{I}\neq 0\\ 0, & O_{F_4}^{I}=0\end{cases}\\ O_{SOD_{35}}=\alpha_{35}O_{C_3}+100(1-\alpha_{35})\\ O_{SOD_{45}}=\alpha_{45}O_{F_4}+100(1-\alpha_{45})\\ O_{COD_{35}}=\frac{100}{\gamma_{35}}O_{C_3}+\omega_{35}\beta_{35}\\ O_{COD_{45}}=\frac{100}{\gamma_{45}}O_{F_4}+\omega_{45}\beta_{45}\\ \omega_{35}=\frac{O_{F_4}^{I}}{100}\times 100\%,\ \omega_{45}=\frac{O_{C_3}}{100}\times 100\%\end{cases}\tag{32}$$

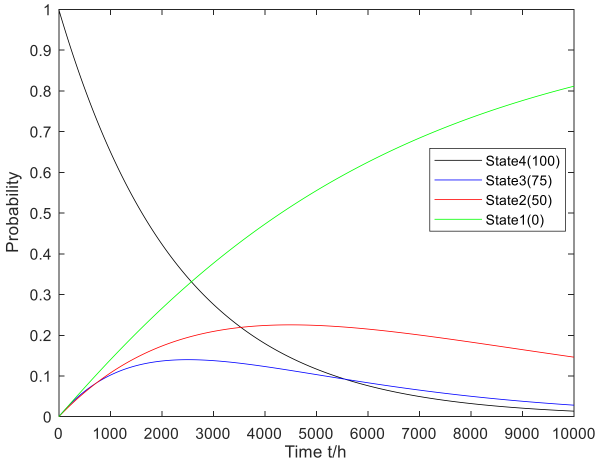


FIGURE 7. Probability curves of N_1 at different independent operational performance levels.

N_1, N_2 and N_4 are different functions subordinate to the same or different equipment systems and have multiple independent operational performance levels. N_1 contains four states with values of 0, 50, 75, and 100. N_2 contains three states with values of 0, 60 and 100. N_4 contains two states with values of 0 and 100. Combined with functions of combat effectiveness level of N_3 and N_5, vectors of combat effectiveness levels can be obtained respectively as follows:

$$\begin{aligned}O_{C_3}=[&0\quad 5.418\quad 9.03\quad 42.85\\ &52.5\quad 55.418\quad 56\quad 59.03\\ &78.9\quad 84.03\quad 82.4\quad 100]\end{aligned}\tag{33}$$

$$\begin{aligned}O_{C_5}=[&0\quad 55.5\quad 57.911\quad 59.5184\quad 74.5683\\ &78.8625\quad 80.161\quad 80.42\quad 81.7684\\ &90.6105\quad 92.168\quad 92.8934\quad 100]\end{aligned}\tag{34}$$

B. ANALYSIS OF OPERATIONAL LEVEL OF NODE BASED ON OPERATIONAL DEPENDENCY

In this paper, the degradation parameters without repairing of each function node are given as shown in Table 1. By plugging these parameters into equation (24) and using the transformation and inverse transformation theorem of Laplace-Stieltjes, the probability function of each function node at each independent operational performance level can be obtained. Taking N_4 as an example, the set of state probability functions is as follows:

$$\begin{cases}\Pr\left\{\phi(O_{F_4}^{I}(t))=1\right\}=1-e^{-1.96\times 10^{-4}t}\\ \Pr\left\{\phi(O_{F_4}^{I}(t))=2\right\}=e^{-1.96\times 10^{-4}t}\end{cases}\tag{35}$$

Through the above calculation, probability curves of N_1, N_2 and N_4 at different independent operational performance levels over time are shown in Figure 7, 8 and 9.

It can be seen from Figure 7, Figure 8 and Figure 9 that probabilities of independent operational performance levels of N_1, N_2 and N_4 in each state will change continuously

TABLE 1. Values of degradation parameters of function nodes.

Node number	State number	Operability level	Instantaneous transition intensity （$\times 10^{-5}/h$）					
			λ_{43}	λ_{42}	λ_{41}	λ_{32}	λ_{31}	λ_{21}
N_1	1	0	15.2	13.2	14.5	16.5	20.5	16.8
	2	50	15.2	13.2	14.5	16.5	20.5	16.8
	3	75	15.2	13.2	14.5	16.5	20.5	16.8
	4	100	15.2	13.2	14.5	16.5	20.5	16.8
N_2	1	0	-	-		19.6	14.3	20.2
	2	60	-	-	-	19.6	14.3	20.2
	3	100	-	-	-	19.6	14.3	20.2
N_4	1	0	-	-	-	-	-	18
	2	100	-	-	-	-	-	18

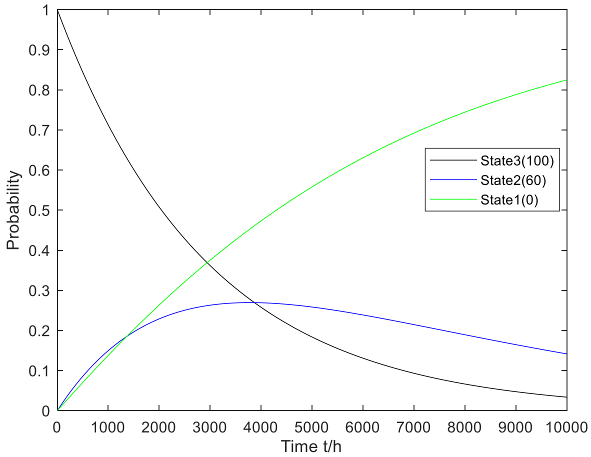


FIGURE 8. Probability curves of N_2 at different independent operational performance levels.

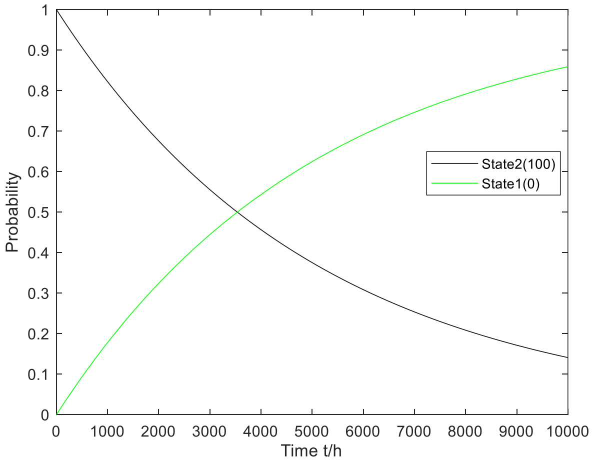


FIGURE 9. Probability curves of N_4 at different independent operational performance levels.

according to the change of time. And independent operational performance level of the function node will decrease with the increase of time. Therefore in the operational dependency

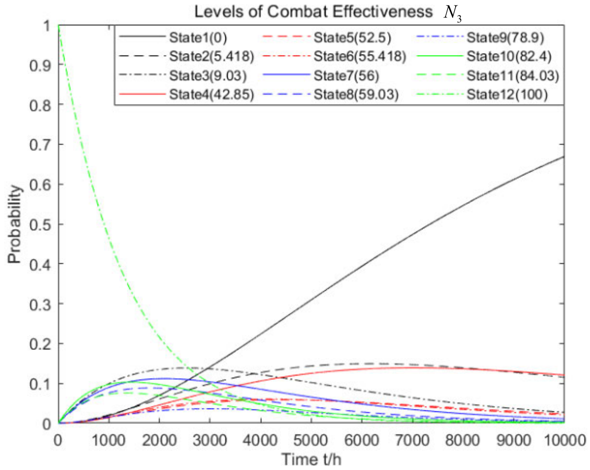


FIGURE 10. Probability curves of N_3 at different levels of combat effectiveness.

network, this kind of node is bound to affect the actual level of combat effectiveness of the capability node.

From above, probability functions of N_3 and N_5 at each level of combat effectiveness can be obtained. Probability curves of N_3 and N_5 at different levels of combat effectiveness over time are shown in Fig. 10 and 11.

It can be seen from Figure 10 and Figure 11 that probabilities of levels of combat effectiveness of N_3 and N_5 in each state will change with changes of independent operational performance levels of N_1, N_2 and N_4. If the actual level of combat effectiveness of the capability node cannot meet the current task requirement, the task may fail. Therefore in the importance analysis, we must consider impacts of the current independent operational performance level of the function node and operational dependency in SoS.

C. IMPORTANCE MEASURE BASED ON OPERATIONAL DEPENDENCY

We select N_3 and N_5 as target nodes and use equation (27) to analyze the importance of each function node relative to N_3

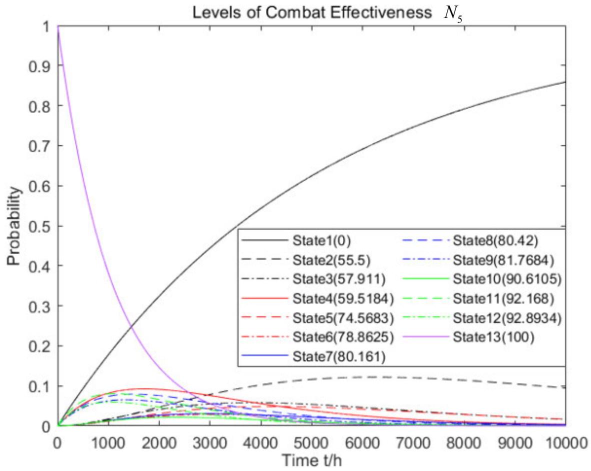


FIGURE 11. Probability curves of N_5 at different levels of combat effectiveness.

TABLE 2. Relationships of operability level of nodes (the capability of target detection).

Independent operational performance level of function node		Combat effectiveness level
N_1	N_2	N_3
0	0	0
0	60	5.418
0	100	9.03
50	0	42.85
50	60	55.418
50	100	59.03
75	0	52.5
75	60	78.9
75	100	84.03
100	0	56
100	60	82.4
100	100	100

and N_5. Thus, the importance of helicopter and ship relative to different task capabilities can be ranked.

In this case, N_3 only depends on N_1 and N_2. Therefore, we need to measure the importance of N_1 and N_2 relative to N_3. Firstly, according to the operational dependency network, the relationship between independent operational performance level of the two and the level of combat effectiveness of N_3 is given, as shown in Table 2.

It is assumed that the task demand of N_3 is 60, that is, when O_{C_3} is not less than 60, the SoS can successfully execute the task of target detection. The B-TDL importance measure $I^3_{B-TDL}(1;t)$ and $I^3_{B-TDL}(2;t)$ of N_1 and N_2 can be obtained from the data in Table 2. Importance measure-time curves of N_1 and N_2 are shown in Figure 12.

Assuming that the task demand of N_5 is 75, the B-TDL importance measure of N_1, N_2 and N_4 relative to N_5 can be obtained according to the above method. Importance measure-time curves of each function node are shown in Figure 13.

It can be seen from Figure 12 and Figure 13 that importance measures of all function nodes relative to capability nodes decrease with the extension of time. Each task capability in SoS is generated by the cooperation of several different systems playing their respective functions. A function of a single system from down state to up state cannot make SoS produce the required task capability, that is, SoS cannot guarantee to complete the task in the mission. From the perspective of functional reliability, the function of a single system from down state to up state cannot improve the mission success of SoS. Therefore, the importance measure of each system function tends to zero as time approaches infinity, which also conforms to the physical meaning of Birnbaum importance measure.

For the capability of target detection in SoS, the importance measure of detection function of helicopter system is always higher than that of detection function of ship system. On the one hand, the influence of helicopter and ship detection function degradation on capability of target detection is slightly different; on the other hand, the operational dependency network has a certain impact on the task capability. Therefore at any time t, the order of importance measure of the two

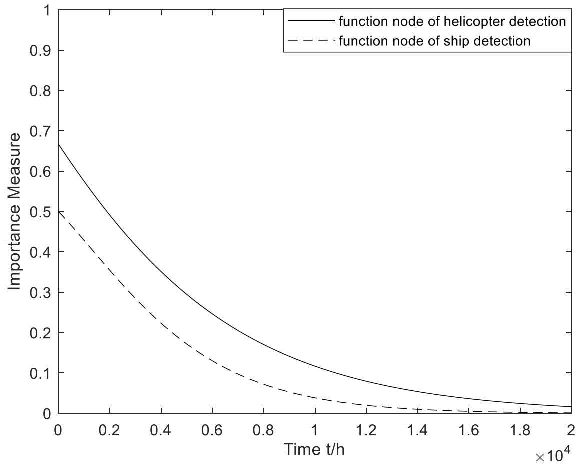


FIGURE 12. Curves of importance measure of function nodes to target detection capability.

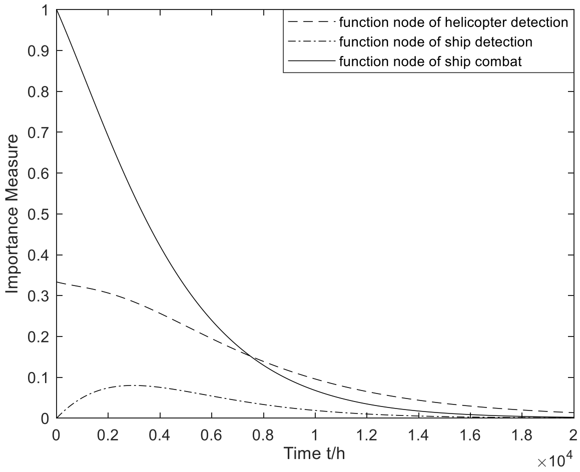


FIGURE 13. Curves of importance measure of function nodes to fire damage capability.

function nodes relative to the capability of target detection is as follows:

$$I^3_{B-TDL}(1; t) > I^3_{B-TDL}(2; t) \tag{36}$$

Therefore, in order to ensure the success of the task of target detection in SoS, helicopter system should be paid more attention in maintenance decision.

For the capability of fire damage in SoS, the order of importance measures of the three function nodes will change with time. Take $t = 1000h$ and $t = 10000h$ as an example, when $t = 1000h$, the order of importance measures of the three function nodes relative to the capability of fire damage is as follows:

$$\begin{cases} I^5_{B-TDL}(1; 1000) = 0.32 \\ I^5_{B-TDL}(2; 1000) = 0.05 \\ I^5_{B-TDL}(4; 1000) = 0.85 \end{cases} \tag{37}$$

$$I^5_{B-TDL}(4; 1000) > I^5_{B-TDL}(1; 1000) > I^5_{B-TDL}(2; 1000) \tag{38}$$

When $t = 10000h$, the order of importance measures of the three function nodes relative to the capability of fire damage is as follows:

$$\begin{cases} I^5_{B-TDL}(1; 10000) = 0.096 \\ I^5_{B-TDL}(2; 10000) = 0.019 \\ I^5_{B-TDL}(4; 10000) = 0.068 \end{cases} \tag{39}$$

$$I^5_{B-TDL}(1; 10000) > I^5_{B-TDL}(4; 10000) > I^5_{B-TDL}(2; 10000) \tag{40}$$

At the beginning of the task of fire damage in SoS (i.e. t tends to 0), the independent operational performance level of each equipment system is in a good state. The level of combat effectiveness of the capability of fire damage mainly depends on the normal operation of the function of ship combat. However, with the increase of time, independent operational performance levels of N_1 and N_2 decrease rapidly, which greatly affect the level of combat effectiveness of N_5. Therefore, it is necessary to formulate maintenance strategies consistent with importance measure ranking in order to improve the mission success of SoS.

In addition, the orders of importance measure will change with the change of the values of parameters of operational dependency. Here the importance measures of the three function nodes relative to N_5 are taken as an example for analysis and calculation. The changed parameter matrices of α^{new}_{ij}, β^{new}_{ij} and γ^{new}_{ij} in the network are given as follows:

$$\alpha^{new}_{ij} = \begin{bmatrix} 0 & 0 & 0.48 & 0 & 0 \\ 0 & 0 & 0.52 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.56 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{41}$$

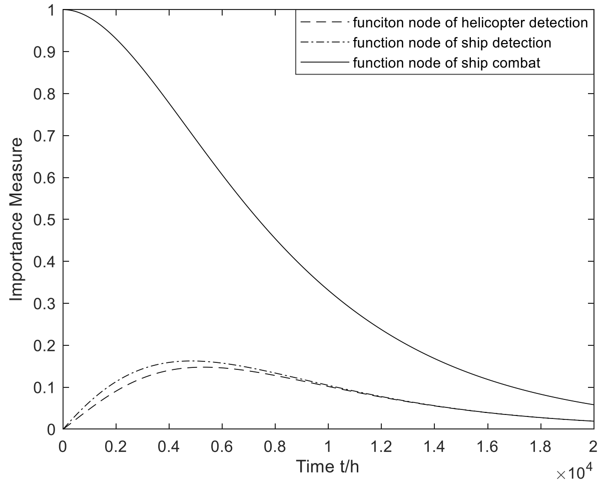


FIGURE 14. **Curves of importance measure of function nodes to fire damage capability with changed values of parameters of operational dependency.**

$$\beta^{new}_{ij} = \begin{bmatrix} 0 & 0 & 65.8 & 0 & 0 \\ 0 & 0 & 64.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 55 \\ 0 & 0 & 0 & 0 & 42.6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{42}$$

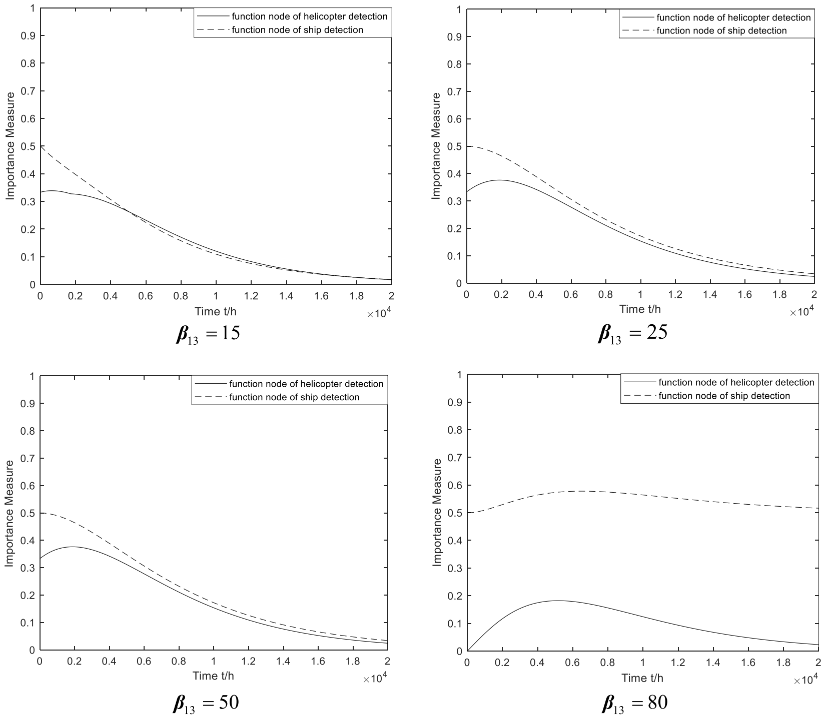
$$\gamma^{new}_{ij} = \begin{bmatrix} 0 & 0 & 64.8 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 74.6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{43}$$

Assuming that other conditions remain unchanged, importance measure-time curves of each function node can be obtained, as shown in Figure 14.

It can be seen from Figure 14 that the importance measure of N_4 relative to the capability of fire damage is the largest at any time $t$in this case because of the change of the values of parameters of operational dependency. Therefore, the ship has higher priority of maintenance in the SoS and its functional module of combat especially needs more attention.

Then the importance measures of the two function nodes relative to N_3 are taken as an example by using variable-controlling approach to analyze impacts of changes of operational dependency parameters on importance measures of equipment task. Here only operational dependency parameter β_{13} related to N_1 is changed.

In the simulation, we find that changes of β_{13} will make the results change, as shown in Figure 15. When $\beta_{13} = 15$, the orders of importance measure will continue to change over time and importance measures of N_1 and $N_2$are equal when $t = 4871h$. When $\beta_{13} = 25$ or $\beta_{13} = 50$, the importance measure of N_2 will be slightly higher than that of N_1. When $\beta_{13} = 80$, the importance measure of N_2 will be obviously higher than that of N_1. As space is limited, we do not do in-depth research on sensitivity analysis of operational dependency parameters in this paper.



$\beta_{13} = 15$ $\beta_{13} = 25$

$\beta_{13} = 50$ $\beta_{13} = 80$

FIGURE 15. Curves of importance measure of function nodes to fire damage capability with changed values of β_{13} of operational dependency.

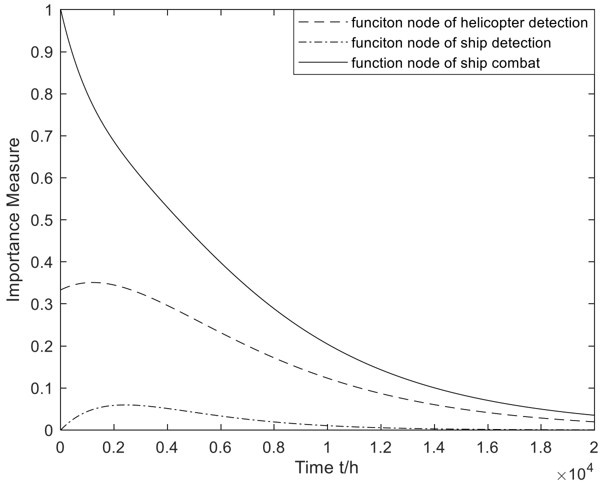


FIGURE 16. Curves of importance measure of function nodes to fire damage capability by using method proposed in reference [38].

D. COMPARATIVE ANALYSIS OF TWO METHODS

Here the importance measures of the three function nodes relative to N_5 are taken as an example for comparative analysis of the method proposed in this paper and the method proposed in reference [41].

Assuming that all conditions remain unchanged, according to the method of calculation of operational dependency proposed in reference [41], importance measure-time curves of each function node can be obtained, as shown in Figure 16.

It can be seen from Figure 16 that the results obtained by using the method proposed in reference [41] are different from those obtained by the method proposed in this paper. This is because the method proposed in reference [41] did not consider the corresponding effects of the changes of independent operational performance level of function nodes on SOD zone and COD zones in operational dependency analysis. This will affect the ranking results of importance measures of function nodes relative to the target capability node, so that it cannot meet the requirements of accurate maintenance of SoS.

V. CONCLUSION

Aiming at the common phenomenon that SoS has operational dependency in the process of mission execution, this paper improves FDNA to construct the operational dependency network of SoS, and puts forward an importance measure of equipment task based on operational dependency network;

This paper determines a dynamic method of analyzing the operational levels of different nodes and their corresponding probability by using Markov process, and proposes analysis process of importance measure of equipment task in SoS by improving the Birnbaum importance measure.

The effectiveness of the proposed method is verified by a case study. It is found that the importance measure of each system is affected by operational dependency, performance degradation and task time, and the ranking results may change with time. The method proposed in this paper provides some guidance for the formulation of maintenance strategies in SoS.

In addition, the proposed method performs well in scalability and is suitable for the analysis and research of operational dependency of SoS containing a large number of equipment systems. And the proposed method can be used to analyze mission success, maintenance decision-making and importance measure of SoS considering operational dependency.

We will analyze the sensitivity of operational dependency parameters in the following research, so as to effectively measure impacts of operational dependency on importance analysis in SoS.

REFERENCES

[1] P. C. Luo, J. L. Zhou, G. Jin, *Evaluation and Analysis Method of Combat Effectiveness and Combat Capability of Weapons System of Systems*. Beijing, China: National Defense Industry Press, 2014.

[2] United States Government US Army, "Manual for the operation of the joint capabilities integration and development system—JCIDS," CreateSpace Independ. Publishing Platform, Charleston, SC, USA, Tech. Rep., 2014.

[3] N. Kilicay and C. H. Dagli, "Methodologies for understanding behavior of system of systems," presented at the 2nd Annu. Syst. Syst. Eng. Conf., 2006.

[4] W. M. Zhang, *Theory and Method of System of Systems Engineering*. Beijing, China: Science Press, 2010.

[5] M. Butterfield and J. Pearlman, "Creation of a system of systems on global scale: The evolution of GEOSS," presented at the 2nd Annu. Syst. Syst. Eng. Conf., 2006.

[6] C. Keating, R. Rogers, R. Unal, D. Dryer, A. Sousa-Poza, R. Safford, W. Peterson, and G. Rabadi, "System of systems engineering," *Eng. Manage. J.*, vol. 15, no. 3, pp. 36–45, 2003.

[7] Z. Xu, J. E. Ramirez-Marquez, Y. Liu, and T. Xiahou, "A new resilience-based component importance measure for multi-state networks," *Rel. Eng. Syst. Saf.*, vol. 193, Jan. 2020, Art. no. 106591, doi: 10.1016/j.ress.2019.106591.

[8] X. Zan, C. L. Chen, S. X. Zhang, W. L. Chen, and L. J. Zhang, "Dynamic evaluation method for equipment important degree considering weight-evolving," *Syst. Eng. Electron.*, vol. 39, no. 9, pp. 2022–2030, 2017.

[9] H. Dui, S. Si, S. Wu, and R. C. M. Yam, "An importance measure for multistate systems with external factors," *Rel. Eng. Syst. Saf.*, vol. 167, pp. 49–57, Nov. 2017.

[10] C. L. Chen, X. Zan, S. X. Zhang, Y. H. Cao, and W. L. Chen, "Evaluation method for equipment importance degree based on multi-dimensional relationship complex networks," *Acta Armamentarii*, vol. 38, no. 6, pp. 1168–1177, 2017.

[11] T. Kim and J. Song, "Generalized reliability importance measure (GRIM) using Gaussian mixture," *Rel. Eng. Syst. Saf.*, vol. 173, pp. 105–115, May 2018.

[12] L. He, Z. Lu, and X. Li, "Failure-mode importance measures in structural system with multiple failure modes and its estimation using copula," *Rel. Eng. Syst. Saf.*, vol. 174, pp. 53–59, Jun. 2018.

[13] W. Kuo and X. Zhu, "Relations and generalizations of importance measures in reliability," *IEEE Trans. Rel.*, vol. 61, no. 3, pp. 659–674, Sep. 2012.

[14] F. C. Meng, "Relationships of Fussell–Vesely and Birnbaum importance to structural importance in coherent systems," *Rel. Eng. Syst. Saf.*, vol. 67, no. 1, pp. 55–60, Jan. 2000.

[15] X. Huang, F. P. A. Coolen, T. Coolen-Maturi, and Y. Zhang, "A new study on reliability importance analysis of phased mission systems," *IEEE Trans. Rel.*, vol. 69, no. 2, pp. 522–532, Jun. 2020.

[16] P. Wei, F. Liu, and C. Tang, "Reliability and reliability-based importance analysis of structural systems using multiple response Gaussian process model," *Rel. Eng. Syst. Saf.*, vol. 175, pp. 183–195, Jul. 2018.

[17] S. Si, M. Liu, Z. Jiang, T. Jin, and Z. Cai, "System reliability allocation and optimization based on generalized Birnbaum importance measure," *IEEE Trans. Rel.*, vol. 68, no. 3, pp. 831–843, Sep. 2019.

[18] Y. Fu, T. Yuan, and X. Zhu, "Importance-measure based methods for component reassignment problem of degrading components," *Rel. Eng. Syst. Saf.*, vol. 190, Oct. 2019, Art. no. 106501, doi: 10.1016/j.ress.2019.106501.

[19] D. Zhu, D. Wang, S.-U. Hassan, and P. Haddawy, "Small-world phenomenon of keywords network based on complex network," *Scientometrics*, vol. 97, no. 2, pp. 435–442, Nov. 2013.

[20] C. Shao, P. Cui, P. Xun, Y. Peng, and X. Jiang, "Rank correlation between centrality metrics in complex networks: An empirical study," *Open Phys.*, vol. 16, no. 1, pp. 1009–1023, Dec. 2018.

[21] M. Mayo, A. Abdelzaher, and P. Ghosh, "Long-range degree correlations in complex networks," *Comput. Social Netw.*, vol. 2, no. 1, pp. 1–13, Dec. 2015.

[22] K. Jiang-Tao, H. Jian, G. Jian-Xing, and L. Er-Yu, "Evaluation methods of node importance in undirected weighted networks based on complex network dynamics models," *Acta Phys. Sinica*, vol. 67, no. 9, 2018, Art. no. 098901.

[23] J. Ai, L. Li, Z. Su, L. Jiang, and N. Xiong, "Node-importance identification in complex networks via neighbors average degree," in *Proc. Chin. Control Decis. Conf. (CCDC)*, May 2016, pp. 1298–1303.

[24] J. Bae and S. Kim, "Identifying and ranking influential spreaders in complex networks by neighborhood coreness," *Phys. A, Stat. Mech. Appl.*, vol. 395, pp. 549–559, Feb. 2014.

[25] J. Liu, Q. Xiong, W. Shi, X. Shi, and K. Wang, "Evaluating the importance of nodes in complex networks," *Phys. A, Stat. Mech. Appl.*, vol. 452, pp. 209–219, Jun. 2016.

[26] L. Zhu, S. L. Fang, Q. Hu, S. Luo, and F. H. Fan, "Evaluation method for time-varying satellite topology network node importance," *Syst. Eng. Electron.*, vol. 39, no. 6, pp. 1274–1279, 2017.

[27] L. Lü, T. Zhou, Q.-M. Zhang, and H. E. Stanley, "The H-index of a network node and its relation to degree and coreness," *Nature Commun.*, vol. 7, no. 1, pp. 1–7, Apr. 2016.

[28] J. Hu, Y. Du, H. Mo, D. Wei, and Y. Deng, "A modified weighted TOPSIS to identify influential nodes in complex networks," *Phys. A, Stat. Mech. Appl.*, vol. 444, pp. 73–85, Feb. 2016.

[29] G. Q. Cheng, Y. Z. Lu, M. X. Zhang, and J. C. Huang, "Node importance evaluation and network vulnerability analysis on complex network," *J. Nat. Univ. Defense Technol.*, vol. 39, no. 1, pp. 120–127, 2017.

[30] J. S. Wang, X. P. Wu, B. Yan, and J. W. Guo, "Improved method of node importance evaluation based on node contraction in complex networks," *Procedia Eng.*, vol. 15, pp. 1600–1604, Dec. 2011.

[31] A. Lisnianski, I. Frenkel, Y. Ding, *Multi-state System Reliability Analysis and Optimization for Engineers and Industrial Managers*. London, U.K.: Springer, 2010.

[32] B. Cai, H. Fan, X. Shao, Y. Liu, G. Liu, Z. Liu, and R. Ji, "Remaining useful life re-prediction methodology based on Wiener process: Subsea christmas tree system as a case study," *Comput. Ind. Eng.*, Nov. 2020, Art. no. 106983, doi: 10.1016/j.cie.2020.106983.

[33] B. Cai, X. Kong, Y. Liu, J. Lin, X. Yuan, H. Xu, and R. Ji, "Application of Bayesian networks in reliability evaluation," *IEEE Trans. Ind. Informat.*, vol. 15, no. 4, pp. 2146–2157, Apr. 2019, doi: 10.1109/TII.2018.2858281.

[34] B. Cai, M. Xie, Y. Liu, Y. Liu, and Q. Feng, "Availability-based engineering resilience metric and its corresponding evaluation methodology," *Rel. Eng. Syst. Saf.*, vol. 172, pp. 216–224, Apr. 2018.

[35] P. R. Garvey and C. A. Pinto, "Introduction to functional dependency network analysis," presented at the 2nd Int. Symp. Eng. Syst., 2009.

[36] Y. Chen, Y. Y. Li, X. Z. Qin, and M. Z. Wang, "Ripple effect analysis for system of systems using FDNA," *Fire Control Command Control*, vol. 42, no. 10, pp. 14–18, 2017.

[37] K. Chen, Y. Y. Li, Y. F. Luo, and M. Z. Wang, "Evaluating system of systems resilience based on functional dependency network analysis," *J. Command Control*, vol. 2, no. 3, pp. 256–260, 2016.

[38] W. X. Zhang, Q. Li, H. Hou, and W. Wang, "System of systems safety analysis method for GNSS," *J. Nat. Univ. Defense Technol.*, vol. 37, no. 2, pp. 92–98, 2015.

[39] C. Guarinello and D. Delaurentis, "Dependency analysis of system-of-systems operational and development networks," *Precedia Comput. Sci.*, vol. 16, no. 2, pp. 264–274, 2013.

[40] W. X. Zhang, "A weapon system of systems safety analysis method based on complex interaction networks," Ph.D. dissertation, Nat. Univ. Defense Technol., Changsha, Hunan, China, Oct. 2015.

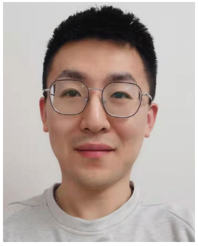
[41] C. Guariniello, "Supporting space systems design via systems dependency analysis methodology," Ph.D. dissertation, Purdue Univ., West Lafayette, IN, USA, 2016.

[42] Z. Qu, Q. Xie, Y. Liu, Y. Li, L. Wang, P. Xu, Y. Zhou, J. Sun, K. Xue, and M. Cui, "Power cyber-physical system risk area prediction using dependent Markov chain and improved grey wolf optimization," *IEEE Access*, vol. 8, pp. 82844–82854, 2020.

[43] Z. Q. Li, T. X. Xu, Q. Dong, and Y. D. Liu, "Markov model based reliability assessment of non-repairable element with multiple states," *Electron. Opt. Control*, vol. 24, no. 9, pp. 58–63, 2017.

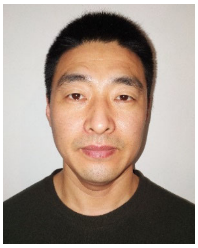
[44] T. F. Xiahou, Y. Liu, H. D. Zhang, and C. L. Zhang, "Birnbaum importance measure of multi-state systems under epistemic uncertainty," *J. Mech. Eng.*, vol. 54, no. 8, pp. 223–232, 2018.

TINGXUE XU was born in 1962. He received the M.S. degree in control, guidance and simulation of aircraft from the Nanjing University of Aeronautics and Astronautics, Nanjing, China, and the Ph.D. degree in theory and engineering of aerospace propulsion from Navy Aeronautical University, Yantai, China. He is currently a Professor with Navy Aeronautical University. He has established a research group and studied theory of equipment general quality characteristics and maintenance decision-making of equipment for years. His research interest includes theory and application of equipment support.

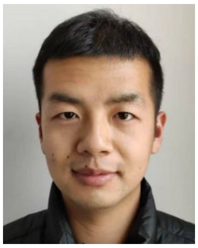


YUQI CHEN was born in 1992. He received the B.S. degree in support and command of equipment and the M.S. degree in armament science and technology from the Ordnance Engineering College, Shijiazhuang, China, in 2014 and 2017, respectively. He is currently pursuing the Ph.D. degree in application and engineering of weapon system with Navy Aeronautical University. His current research interests include theory and application of equipment support, reliability engineering, and so on.

CHENG LU was born in 1990. He received the M.S. and Ph.D. degrees from Navy Aeronautical University, in 2016 and 2019, respectively. He is currently an Engineer with Naval Equipment Support Brigade. His research interests include theory and application of equipment support, maintenance decision-making of equipment, and so on.



HAIJUN LI was born in 1978. He received the M.S. and Ph.D. degrees from Navy Aeronautical University, in 2009 and 2014, respectively. He is currently an Engineer with Navy Aeronautical University. His research interests include pattern recognition, reliability engineering, and so on.



JIAPENG LV was born in 1994. He received the M.S. degree in control engineering from Navy Aeronautical University, in 2019, where he is currently pursuing the Ph.D. degree in control science and engineering. His current research interests include fault detection and isolation.

• • •