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A Disturbance-Observer-Based Sliding Mode Control for the Robust Synchronization of Uncertain Delayed Chaotic Systems: Application to Data Security

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ABSTRACT This article proposes a disturbance observer-based Sliding Mode Control (SMC) approach for the robust synchronization of uncertain delayed chaotic systems. This is done by, first, examining and analyzing the electronic behavior of the master and slave Sprott chaotic systems. Then, synthesizing a robust sliding mode control technique using a newly proposed sliding surface that encompasses the synchronization error between the master and slave. The external disturbances affecting the system were estimated using a disturbance observer. The proof of the semi-globally bounded synchronization between the master and slave was established using the Lyapunov stability theory. The efficiency of the proposed approach was first assessed using a simulation study, then, experimentally validated on a data security system. The obtained results confirmed the robust synchronization properties of the proposed approach in the presence of time-delays and external disturbances. The experimental validation also confirmed its ability to ensure the secure transfer of data.

INDEX TERMS Sliding mode control, robust synchronization control, chaotic systems, disturbance observer, data security.

I. INTRODUCTION

Due to their valuable characteristics, chaotic systems have long attracted the consideration of investigators worldwide [1]–[3]. The nonlinear, aperiodic and unstable characteristics of these systems result in their widespread applications in various fields, including secure communication [4], [5]. In chaos theory, the butterfly effect refers to the sensitive dependence on initial conditions, in which a small variation in one state can result in more bigger variations in the later state. In many applications, the chaotic behavior is undesirable because of the fact that even small disturbances may cause the states to diverge exponentially. Therefore, the chaos phenomenon should be avoided or completely suppressed in practice [6]–[9]. In the past two decades, chaos synchronization has generated important interests in applied fields such as secure communication [10], [11], electronic circuits [12], optical chaotic communication [13], chaotic CO_2 lasers [14], chaotic finance system [15], a periodically intermittent control [16], partial discharge in power cables [17], cryptosystems [18] and image encryption [19]. As a result, various control techniques have been proposed for the synchronization of chaotic systems [20]–[23]. However, most of the above mentioned approaches have neglected the effects of external disturbances and modeling inaccuracies, thus making them hard to implement in practice [24], [25]. Whereas the external disturbances always are entered to all of the practical systems in engineering, it is important to take into consideration their effect in designing of the control methods. For this reason, the disturbance-observer is adopted as an imperative technique to approximate and compensate this external influence, which can adversely affect the performance of the system [26], [27]. Therefore, some significant disturbance observers have been presented in [28]–[31].

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More recently, various control approaches such as adaptive control, sliding mode control (SMC), active control, feedback linearization control, fuzzy-logic control, output-feedback control and backstepping control have been considered in chaos control and synchronization [32]–[36]. For instance, an algorithm for the synchronization of chaotic systems using linear and nonlinear feedback control has been proposed in [37]. In that approach, a nonlinear term is considered to eliminates the nonlinear part of the syste's error and a linear term is used to stabilize the resultant linear system. In [38], dynamical behavior of chaotic flow is examined via eigenvalue, phase portraits, bifurcation figure and Lyapunov exponents. Two nonlinear control approache have been designedin [39]. The first method is a finite-time stability active control considering certain parameters and the second one is a finite-time boundless adaptive control technique which is capable to accommodate parametric uncertainties. A practical-link-based fuzzy brain emotional learning network, where its parameters have been adjusted online by adaptation laws, was proposed in [40] for the classification and synchronization control of chaotic systems. In [41], to synchronize the multi-scroll Chen chaotic systems with external disturbances in secure communication, the polynomial fuzzy-model-based procedure have been designed. A disturbance observer based feedback linearization approach has been proposed in [42] for the control of chaotic systems in the presence of external excitations. With the aim of providing the safe communication of time-delay systems, a control paradigm is suggested in [43] according to the programmable micro-controllers with digital transmission line.In [44], energy analysis of the Sprott chaotic system is done based on transformation of the Sprott system into the Kolmogorov-type system. Then, according to this analysis, a new four-dimension chaotic system with hidden equilibrium is introduced. Although, it can be observed that no control procedure is applied for synchronization of this system. In [45], a new chaotic system with interesting characteristic is derived from Sprott chaotic system, but, the synchronization problem is ignored. Additionally, by analyzing the researches [46]–[49] which are related to Sprott's chaotic system, it can be found that no comprehensive work is investigated and proposed the synchronization problem in the existence of parametric uncertainty, time-delay and external disturbance for Sprott's chaotic system.

To the best of the author's knowledge, the problem of semi-globally bounded synchronization control of Sprott chaotic systems subject to external disturbances, time-delays and parametric uncertainties, has received little consideration in the literature. It is still an open and challenging research problem. This article designs a disturbance observer-based sliding mode control scheme for the synchronizationof the Sprott chaotic system. Its mai contributions are as follows:

- A semi-globally robust sliding mode control to ensure the synchronization between the master and slave of the Sprott chaotic system in the presence of external disturbances, time-delays and parametric uncertainties.
- A sliding mode approach synthesized using a novel sliding surface that encompasses the synchronization error between the master and slave.
- Experimental validation of the proposed approach using a data security transmission system.

The remainder of the paper is organized as follows. The Sprott chaotic system is introduced in section II. The proposed disturbance observer-based sliding mode control is detailed in section III. The performance of the proposed synchronization approach is assessed using a simulation study in section IV. Experimental validation on a data security system is highlighted in section V. Some concluding remarks are finally provided in section VI.

II. PRELIMINARIES AND ELECTRONIC CIRCUIT OF THE SPROTT CHAOTIC SYSTEM

This section briefly describes the dynamic equations of the Sprott chaotic system and presents its electronic circuit. It also depicts the time histories of the state variables of the Sprott chaotic system obtained from both the numerical simulation and electronic circuit.

A. DESCRIPTION OF THE SPROTT CHAOTIC SYSTEMS

The Sprott chaotic system can be modeled with three-dimensional autonomous differential equations as follows [50]:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= z, \\ \dot{z} &= -ax + y^2 - bz. \end{aligned} \tag{1}$$

where $[x(t), y(t), z(t)]$ are the state variables of the chaotic system and a, b are constant parameters. In the above equation, the constant parameters are selected as $a = 1, b = 2.02$. Also, the initial conditions are considered as $x_0 = 4, y_0 = 2$ and $z_0 = 0.5$. The time responses and phase portraits of state variables of system (1) using MATLAB/Simulink software are shown in Figure 1 and Figure 2, respectively.

B. ELECTRONIC CIRCUIT OF THE SPROTT CHAOTIC SYSTEM

The Electronic circuit of the Sprott chaotic system is shown in Figure 3. The circuit is designed using the following electronic devices: resistors, capacitors, AD633 analog multiplier ICs and TL081 OPAMPs.

The circuit equations are found as

$$\begin{aligned} \dot{x} &= \frac{1}{R_8 C_1} y, \\ \dot{y} &= \frac{1}{R_9 C_2} z, \\ \dot{z} &= -\frac{1}{R_6 C_3} x + \frac{0.1}{R_5 C_3} y^2 - \frac{1}{R_7 C_3} z. \end{aligned} \tag{2}$$

The values of passive devices are chosen as $C_i = 10nF (\forall i = 1, 2, 3), R_1 = 10k, R_2 = 40k, R_3 = 4k$ and $R_4 = 19.802k$. The TL081 OPAMPs are powered by $\pm 12Vdc$

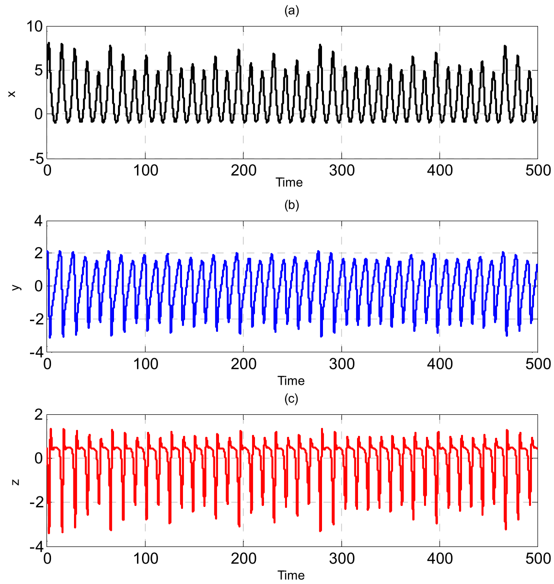

FIGURE 1. Time histories of the state variables: *(a)x*, *(b)y*, *(c)z*.

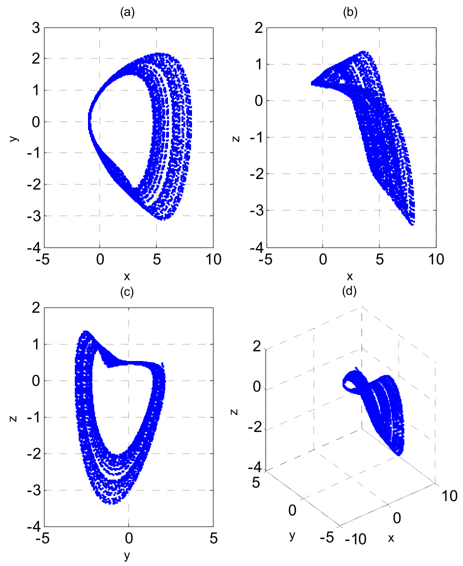

FIGURE 2. Phase portrait of the system: *(a)x* to *y*, *(b)x* to *z*, (c) *y* to *z*, *(d)x − y − z*.

power supply. Equation (2) is obtained using the following procedure:

1) Partitioning the circuit diagram into the three parts depicted in Figure 4.

2) Obtaining the outputs x, y, z as follows:

$$\begin{aligned}\frac{x}{y} &= \frac{-R_3}{R_1} \times \frac{-1}{(R_8 C_1)s} \overset{R_3=R_1}{\longrightarrow} \frac{x}{y} \\ &= \frac{1}{(R_8 C_1)s} \Rightarrow s \times x = \frac{1}{R_8 C_1} y \end{aligned} \tag{3}$$

$$\begin{aligned}\frac{y}{z} &= \frac{-R_4}{R_2} \times \frac{-1}{(R_9 C_2)s} \overset{R_4=R_2}{\longrightarrow} \frac{y}{z} \\ &= \frac{1}{(R_9 C_2)s} \Rightarrow s \times y = \frac{1}{R_9 C_2} z \end{aligned} \tag{4}$$

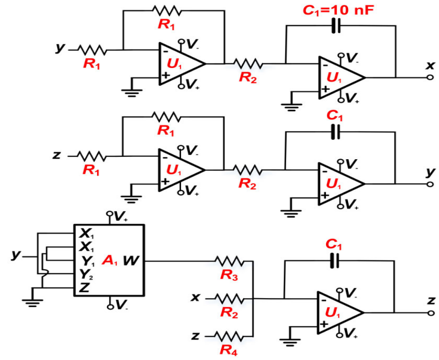

FIGURE 3. Electronic circuit of the Sprott chaotic system.

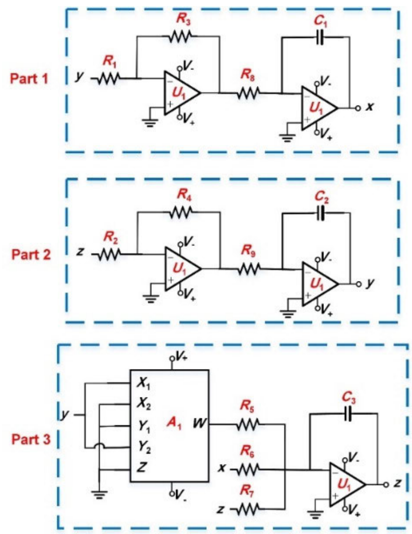

FIGURE 4. Partitions of the electronic diagram of Sprott chaotic system.

$$\begin{aligned} z &= \frac{-1}{(R_5 C_3)s} w + \frac{-1}{(R_6 C_3)s} x + \frac{-1}{(R_7 C_3)s} z \overset{w=-y^2/10}{\longrightarrow} \\ s \times z &= \frac{0.1}{R_5 C_3} y^2 + \frac{-1}{R_6 C_3} x + \frac{-1}{R_7 C_3} z \end{aligned} \tag{5}$$

3) Using the inverse Laplace transform to derive the circuit equations (2).

The dynamics of the state variables of system (1) emanating from the designed circuit are depicted in Figure 5. Note that the physical energy of capacitors $C_i (\forall i = 1, 2, 3)$ can be calculated based on equations $E_i = \frac{1}{2} C_i V_i^2 (\forall i = 1, 2, 3)$, which are displayed in III-A. It is worth noting that the op-amps are almost ideal, hence the capacitor energies can be calculated as $E_1 = \frac{1}{2} C_1 x^2$, $E_2 = \frac{1}{2} C_2 y^2$ and $E_3 = \frac{1}{2} C_3 z^2$.

III. DESIGN OF A DISTURBANCE OBSERVER-BASED SEMI-GLOBALLY ROBUST SYNCHRONIZER FOR THE SPROTT CHAOTIC SYSTEM

In what follows, a robust sliding mode synchronizer based on the disturbance observer has been proposed for the Sprott chaotic system (1) under time-delay in states, bounded external disturbances and parametric uncertainty. For this reason, the control procedure is divided into two steps. In the first

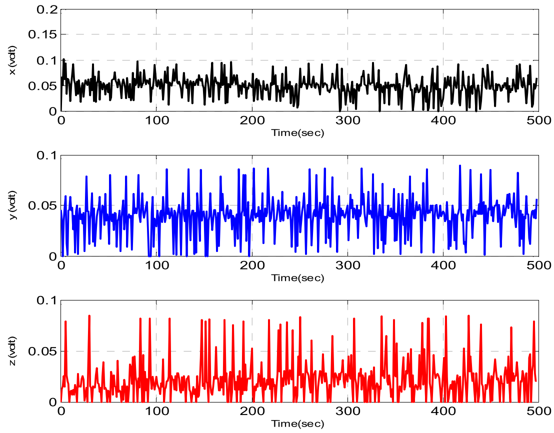

FIGURE 5. Time histories of the state variables obtained from the circuit: (*a*)*x*, (*b*)*y*, (*c*)*z*.

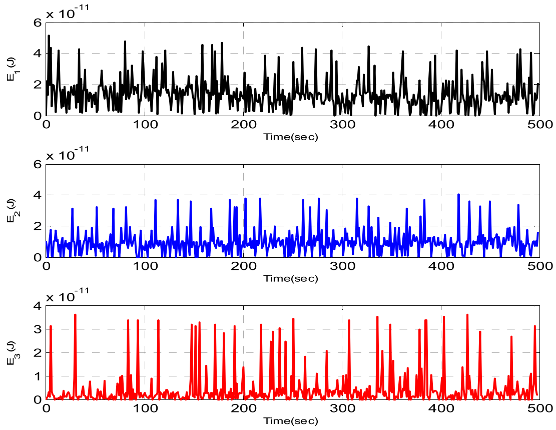

FIGURE 6. Time histories of the physical Energy of the capacitors: (a) c_1, (*b*)c_2, (c) c_3.

step, the synchronization problem is investigated. Therefore, one disturbance observer for the estimation of the exterior perturbation is designed.

A. PROBLEM DESCRIPTION AND PRELIMINARIES

In this part, the synchronization purpose between two identic systems with distinct initial conditions are examined. The first system is named master whereas the second system is named as slave. The master system of the Sprott under time-delay can be described as follows:

$$\begin{aligned}
\dot{x}_m &= y_m(t), \\
\dot{y}_m &= z_m(t), \\
\dot{z}_m &= -ax_m(t) + y_m^2(t) - bz_m(t-\tau),
\end{aligned} \tag{6}$$

and the slave system of Sprott under time-delay and parametric uncertainty is determined as

$$\begin{aligned}
\dot{x}_s &= y_s(t) + d_1 + u_1, \\
\dot{y}_s &= z_s(t) + d_2 + u_2, \\
\dot{z}_s &= -(a+\Delta a)x_s(t) + y_s^2(t) - (b+\Delta b) \\
&\quad \times z_s(t-\tau) + d_3 + u_3,
\end{aligned} \tag{7}$$

where $[x_m, y_m, z_m]^T$ and $[x_s, y_s, z_s]^T$ are the states of the master and slave systems, respectively. d_1, d_2, d_3 are the constrained disturbances, τ is time delay, the terms Δa and Δb are the parametric uncertainties and u_1, u_2, u_3 are the control inputs to be designed. Defining $D_1 = d_1$, $D_2 = d_2$ and $D_3 = d_3 - \Delta a x_s(t) - \Delta b z_s(t-\tau)$ and substituting those expressions in Eq. (7), yields:

$$\begin{aligned}
\dot{x}_s &= y_s(t) + D_1 + u_1, \\
\dot{y}_s &= z_s(t) + D_2 + u_2, \\
\dot{z}_s &= -ax_s(t) + y_s^2(t) - bz_s(t-\tau) + D_3 + u_3,
\end{aligned} \tag{8}$$

Assumption 1: For the external disturbances, without less of generality, the following inequality exists [51]:

$$\left\| \dot{D}_i(x,t) \right\| \leq \delta_i \quad \forall i = 1, 2, 3 \tag{9}$$

where $\delta_i{}'$s are known and positive constants.

Lemma1: For the bounded initial conditions, if there exists a continuous positive-definite Lyapunov function $V(x)$ satisfying $\pi_1(\|x\|) \leq V(x) \leq \pi_2(\|x\|)$ such that $\dot{V}(x) \leq -\kappa V(x) + c$, where $\pi_1, \pi_2 \colon \mathbb{R}^n \to \mathbb{R}$ are class Kfunctions and c is a positive constant, then the solution $x(t)$ is uniformly bounded [52].

B. DESIGN OF A DISTURBANCE OBSERVER-BASED SEMI-GLOGALLY ROBUST SYNCHRONIZER CONTROL FOR THE SPROTT CHAOTIC SYSTEM

In this section, a disturbance observer is developed to estimate the exterior perturbations affecting the chaotic systems (6) and (7). To this end, define the synchronization error between the master and slave systems as follows:

$$\begin{aligned}
e_1 &= x_s - x_m, \\
e_2 &= y_s - y_m, \\
e_3 &= z_s - z_m,
\end{aligned} \tag{10}$$

Taking the time-derivative of the equation (10) yields

$$\begin{aligned}
\dot{e}_1 &= \dot{x}_s - \dot{x}_m, \\
\dot{e}_2 &= \dot{y}_s - \dot{y}_m, \\
\dot{e}_3 &= \dot{z}_s - \dot{z}_m,
\end{aligned} \tag{11}$$

where substituting (6) and (7) into (11), one obtains

$$\begin{aligned}
\dot{e}_1 &= y_s(t) + D_1 + u_1 - y_m(t), \\
\dot{e}_2 &= z_s(t) + D_2 + u_2 - z_m(t), \\
\dot{e}_3 &= -ax_s(t) + y_s^2(t) - bz_s(t-\tau) + D_3 + u_3 \\
&\quad + ax_m(t) - y_m^2(t) + bz_m(t-\tau),
\end{aligned} \tag{12}$$

After simplifications and applying (10) in (12), one achieves

$$\begin{aligned}
\dot{e}_1 &= e_2 + D_1 + u_1, \\
\dot{e}_2 &= e_3 + D_2 + u_2, \\
\dot{e}_3 &= -ae_1 - be_{3\tau} + y_s^2(t) - y_m^2(t) + D_3 + u_3.
\end{aligned} \tag{13}$$

where $e_{3\tau} = z_s(t-\tau) - z_m(t-\tau)$. In order to design the disturbance observer, a supporting variable has been introduced as

$$\eta_i = D_i - k_i e_i. \quad \forall i = 1, 2, 3 \tag{14}$$

where $k_i > 1, \forall i = 1, 2, 3$. Taking the time-derivative of (14) gives

$$\begin{aligned} \dot{\eta}_1 &= \dot{D}_1 - k_1 \dot{e}_1, \\ \dot{\eta}_2 &= \dot{D}_2 - k_2 \dot{e}_2, \\ \dot{\eta}_3 &= \dot{D}_3 - k_3 \dot{e}_3, \end{aligned} \tag{15}$$

Now, by substituting (13) into the above equation yields

$$\begin{aligned} \dot{\eta}_1 &= \dot{D}_1 - k_1 \dot{e}_1 = \dot{D}_1 - k_1(e_2 + D_1 + u_1), \\ \dot{\eta}_2 &= \dot{D}_2 - k_2 \dot{e}_2 = \dot{D}_2 - k_2(e_3 + D_2 + u_2), \\ \dot{\eta}_3 &= \dot{D}_3 - k_3 \dot{e}_3 = \dot{D}_3 - k_3(-ae_1 - be_{3\tau} + y_s^2(t) \\ &\quad -y_m^2(t) + D_3 + u_3), \end{aligned} \tag{16}$$

By considering the equation (14), we have

$$D_i = \eta_i + k_i e_i \quad \forall i = 1, 2, 3 \tag{17}$$

where by substituting (17) into (16), it results

$$\begin{aligned} \dot{\eta}_1 &= \dot{D}_1 - k_1 \dot{e}_1 = \dot{D}_1 - k_1(e_2 + \eta_1 + k_1 e_1 + u_1), \\ \dot{\eta}_2 &= \dot{D}_2 - k_2 \dot{e}_2 = \dot{D}_2 - k_2(e_3 + \eta_2 + k_2 e_2 + u_2), \\ \dot{\eta}_3 &= \dot{D}_3 - k_3 \dot{e}_3 = \dot{D}_3 - k_3(-ae_1 - be_{3\tau} + y_s^2(t) \\ &\quad -y_m^2(t) + \eta_3 + k_3 e_3 + u_3). \end{aligned} \tag{18}$$

To design the nonlinear disturbance observer, the estimation of the variable $\eta_i, \forall i = 1, 2, 3$ is recommended as

$$\begin{aligned} \dot{\hat{\eta}}_1 &= -k_1(e_2 + \hat{\eta}_1 + k_1 e_1 + u_1), \\ \dot{\hat{\eta}}_2 &= -k_2(e_3 + \hat{\eta}_2 + k_2 e_2 + u_2), \\ \dot{\hat{\eta}}_3 &= -k_3(-ae_1 - be_{3\tau} + y_s^2(t) \\ &\quad -y_m^2(t) + \hat{\eta}_3 + k_3 e_3 + u_3). \end{aligned} \tag{19}$$

where $\hat{\eta}_i, \forall i = 1, 2, 3$ are the estimations of $\eta_i, \forall i = 1, 2, 3$. Then, the approximation of $d_i(t)(\forall i = 1, 2, 3)$ can be found using equation (17). Define:

$$\tilde{\eta}_i = \eta_i - \hat{\eta}_i = D_i - \hat{D}_i = \tilde{d}_i . \forall i = 1, 2, 3 \tag{20}$$

Differentiating (20) and considering (18) and (19) yields

$$\begin{aligned} \dot{\tilde{\eta}}_1 &= \dot{D}_1 - k_1(e_2 + \eta_1 + k_1 e_1 + u_1) \\ &\quad + k_1(e_2 + \hat{\eta}_1 + k_1 e_1 + u_1), \\ \dot{\tilde{\eta}}_2 &= \dot{D}_2 - k_2(e_3 + \eta_2 + k_2 e_2 + u_2) + k_2(e_3 + \hat{\eta}_2 \\ &\quad + k_2 e_2 + u_2), \\ \dot{\tilde{\eta}}_3 &= \dot{D}_3 - k_3(-ae_1 - be_{3\tau} + y_s^2(t) - y_m^2(t) + \eta_3 \\ &\quad + k_3 e_3 + u_3) + k_3(-ae_1 - be_{3\tau} + y_s^2(t) \\ &\quad - y_m^2(t) + \hat{\eta}_3 + k_3 e_3 + u_3), \end{aligned} \tag{21}$$

After some simplifications and removing the same expressions form Eq. (21), one obtains

$$\dot{\tilde{\eta}}_i = \dot{D}_i - k_i \tilde{\eta}_i. \quad \forall i = 1, 2, 3 \tag{22}$$

Now, the PI sliding surfaces are defined as follow:

$$s_i = e_i + k_I \int_0^t e_i. \tag{23}$$

where k_I is the a positive constant. Taking time-derivative of (23), we have

$$\dot{s}_i = \dot{e}_i + k_I e_i. \tag{24}$$

where substituting (13) in (24), it yields:

$$\begin{aligned} \dot{s}_1 &= e_2 + D_1 + u_1 + k_I e_1, \\ \dot{s}_2 &= e_3 + D_2 + u_2 + k_I e_2, \\ \dot{s}_3 &= -ae_1 - be_{3\tau} + y_s^2(t) \\ &\quad -y_m^2(t) + D_3 + u_3 + k_I e_3. \end{aligned} \tag{25}$$

Finally, the robust sliding mode synchronizer based on the disturbance observer with semi-globally bounded of errors (13) is designed as follow:

$$\begin{aligned} u_1 &= -e_2 - \hat{D}_1 - A_{01} s_1 - k_I e_1, \\ u_2 &= -e_3 - \hat{D}_2 - A_{02} s_2 - k_I e_2, \\ u_3 &= ae_1 + be_{3\tau} - y_s^2(t) \\ &\quad + y_m^2(t) - \hat{D}_3 - A_{03} s_3 - k_I e_3. \end{aligned} \tag{26}$$

where $A_{0i} > 0.5 \forall i = 1, 2, 3$ are the design parameters. With substituting the control inputs (26) in Eq.(25), one obtains:

$$\dot{s}_i = D_i - \widehat{D}_i - A_{0i} s_i. \tag{27}$$

Theorem 1: Let the master and slave chaotic systems be defined as (6) and III-B. As well, the external disturbances are estimated by (19) and the PI sliding surfaces are defined as (23). Then, the sliding surface as well as synchronization-error between master and slave systems arebounded using the robust controller (26) based on the nonlinear disturbance observer.

Proof: The Lyapunov function is defined as

$$V_i = \frac{1}{2} s_i^2 + \frac{1}{2} \tilde{\eta}_i^2. \quad \forall i = 1, 2, 3 \tag{28}$$

where the time-derivative of V_i can be found as

$$\dot{V}_i = s_i \dot{s}_i + \tilde{\eta}_i \dot{\tilde{\eta}}_i, \quad \forall i = 1, 2, 3 \tag{29}$$

Substituting (22) and (27) in the above equation yields

$$\dot{V}_i = s_i \left(D_i - \widehat{D}_i - A_{0i} s_i \right) + \tilde{\eta}_i (\dot{D}_i - k_i \tilde{\eta}_i) \tag{30}$$

or

$$\dot{V}_i = -A_{0i} s_i^2 + s_i (D_i - \widehat{D}_i) + \tilde{\eta}_i \dot{D}_i - k_i \tilde{\eta}_i^2 \tag{31}$$

From assumption 1 and equation (20), we can get:

$$\dot{V}_i \leq -A_{0i} s_i^2 - k_i \tilde{\eta}_i^2 + s_i \tilde{\eta}_i + \tilde{\eta}_i \delta_i \tag{32}$$

where using Young's inequality ($c_1 c_2 \leq \frac{c_1^\alpha}{\alpha} + \frac{c_2^\beta}{\beta}$), we have

$$\begin{aligned} s_i \tilde{\eta}_i &\leq 0.5 s_i^2 + 0.5 \tilde{\eta}_i^2 \\ \tilde{\eta}_i \delta_i &\leq 0.5 \tilde{\eta}_i^2 + 0.5 \delta_i^2 \end{aligned} \tag{33}$$

while substituting (33) into (32), we have

$$\dot{V}_i \leq -A_{0i}s_i^2 - k_i\tilde{\eta}_i^2 + 0.5s_i^2 + 0.5\tilde{\eta}_i^2 + 0.5\delta_i^2 + 0.5\tilde{\eta}_i^2 \quad (34)$$

Now, by taking some simplifications, one can obtain

$$\dot{V}_i \leq -(A_{0i} - 0.5)s_i^2 - (k_i - 1)\tilde{\eta}_i^2 + 0.5\delta_i^2 \quad (35)$$

By considering the Lyapunov function, hence, it can be concluded that

$$\dot{V}_i \leq -\sigma_i V_i + c_i, \quad (36)$$

where $\sigma_i = \min(\lambda_{min}(A_{0i} - 0.5), \lambda_{min}(k_i - 1))$ and $c_i = 0.5\delta_i^2$. Now, from Lemma 1, it is obvious that sliding surface s_i and synchronization error e_i are bounded. This completes the proof. □

IV. SIMULATION RESULTS

To highlight the effectiveness of the proposed approach, we carry out a set of simulations and comparison analysis with the controller and observer proposed in [1]. Based on the Sprott's chaotic system, the constant parameters are given as $a = 1, b = 2.02$ and the preliminary conditions for master system are considered as $[x_m(0), y_m(0), z_m(0)]^T = [4, 2, 0.5]^T$. Moreover, the preliminary conditions for the slave system are given as $[x_s(0), y_s(0), z_s(0)]^T = [-3.9, -1.9, -0.4]^T$. The design parameters are specified as $k_1 = 20$, $k_2 = 25$, $k_3 = 30$ and $a_{0i} = 50, \forall i = 1, 2, 3$. It should be mentioned that the parameters of the control strategy have been obtained by trial and error approach. The external disturbances are considered as $D_1(x_s, t) = 1.5x_s \sin(t), D_1(y_s, t) = 1.5y_s \sin(t)\, and D_1(z_s, t) = 1.5z_s \sin(t) - \Delta a x_s(t) - \Delta b z_s(t - \tau)$, where, $\Delta a = 0.01sin(a)$ and $\Delta b = 0.01sin(b)$, also, the time delay is $\tau = 0.005$. The tracking performance of the master and slave systems is highlighted in Figure 7. Note the perfect tracking performance of the proposed approach. The time histories of the control inputs are depicted in Figure 8. Note that these signals have suitable amplitudes without chattering. The time responses of the errors between the master and slave systems are displayed in Figure 9. These results confirm the bounded of the synchronization errors around zero. The estimations of exterior disturbances using the disturbance observer are displayed in Figure 10, which shows that the disturbance estimation can be accomplished properly. The time responses of the sliding surfaces are shown in V-A. Note the boundedness of switching surface around zero.

It can be concluded from the simulation results that the proposed technique provides more accurate transient responses than the method presented in [1]. These results prove the efficiency and feasibility of the proposed technique.

V. IMPLEMENTATION TO A DATA SECURITY SYSTEM

In what follows, we implement the concept to ensure the secure transfer of data from the origin to the destination. For this aim, an encrypted signal is generated by adding the Sprott chaotic signal to the data to be transferred. The encrypted

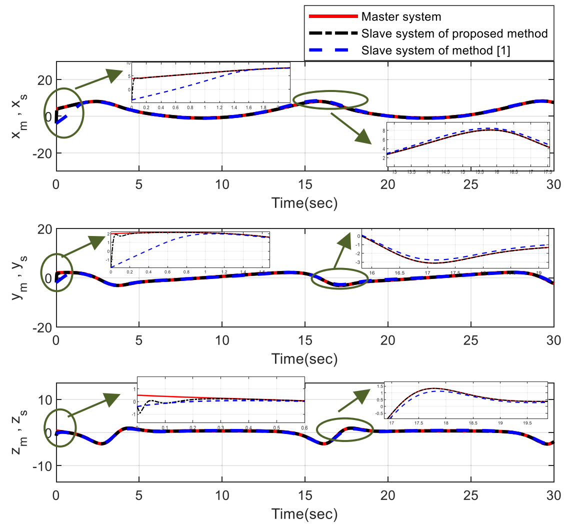

FIGURE 7. Tracking of master and slave systems using the proposed controller.

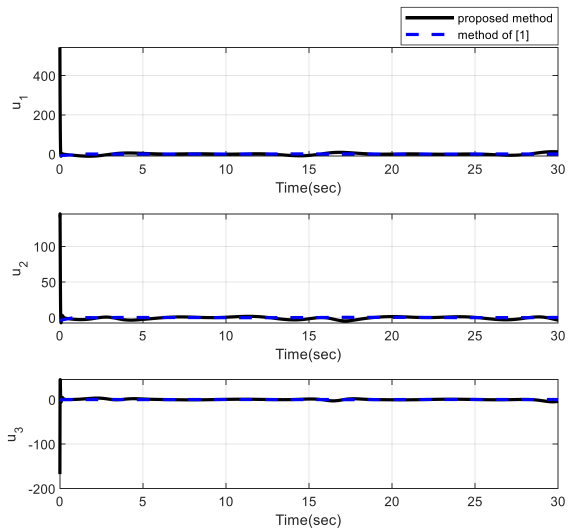

FIGURE 8. Control inputs.

signal is the sent to the destination using a transmitter. At the receiver end, the main signal is recovered by subtracting the chaotic signal from the encrypted signal. In this work, the pulse generator is considered as the main signal which is supposed to be sent and $y$axis of Sprott chaotic system which has been introduced in the previous section is considered as the chaotic signal to be added to the main signal to produce the encrypted signal. The main steps of the proposed technique are illustrated as follows:

A. ADDING CHAOS TO THE MAIN SIGNAL

As mentioned above, for the creation of the encrypted signal, the chaotic signal related to $y$axis of Sprott chaotic system which is shown in Figure 12 is added to the main signal which is displayed in Figure 13. The block diagram of this system is presented in Figure 14 and its schematic circuit is displayed in V-B. The main part of this circuit is related to

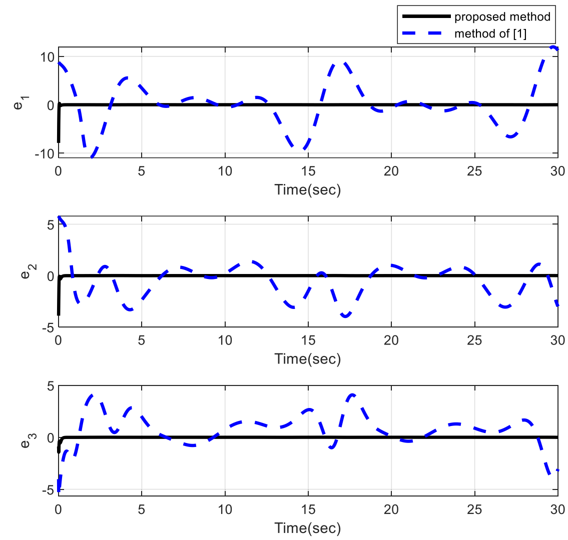

FIGURE 9. Time history of synchronization errors via recommended controller.

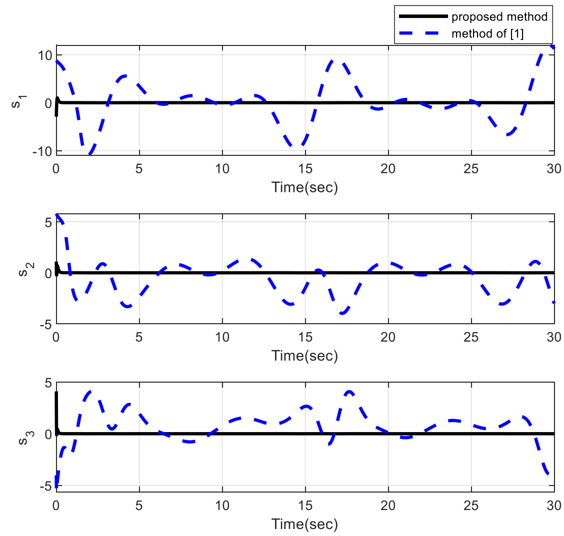

FIGURE 11. Sliding surfaces.

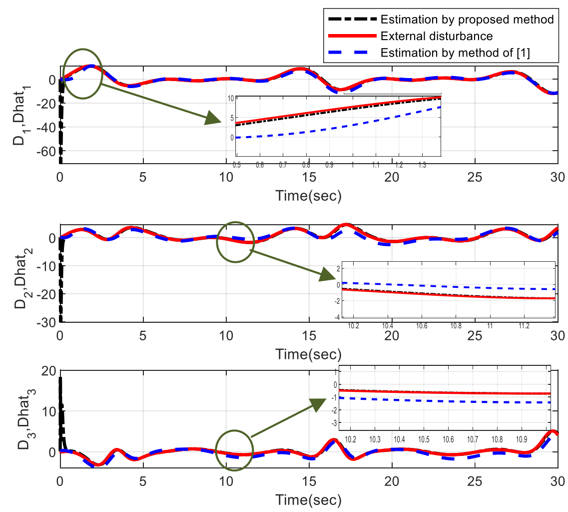

FIGURE 10. Estimation of the external disturbances.

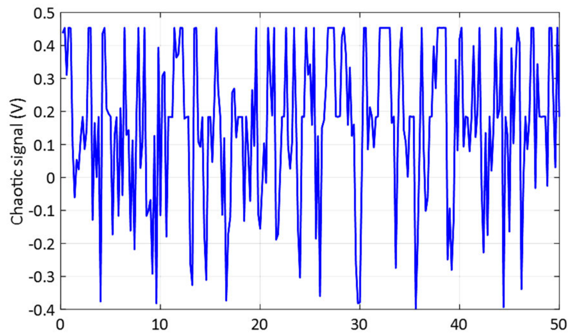

FIGURE 12. Chaotic signal related to *y* axis of Sprott chaotic system.

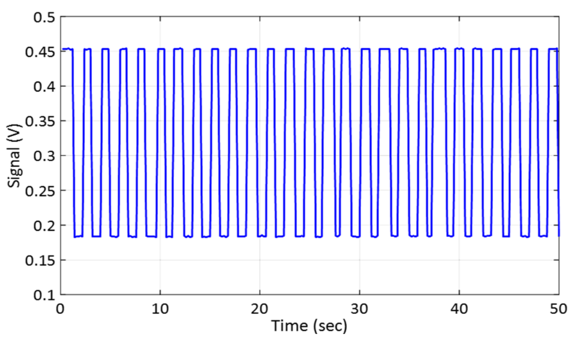

FIGURE 13. Main signal.

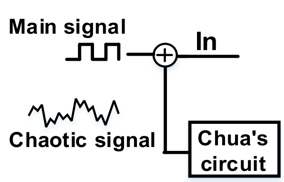

FIGURE 14. Block diagram of adder section.

the adder including two resistors which have responsibility for adding chaos to the main signal. The buffer circuit is used for isolating the circuit of chaotic signal from adder circuit and the amplifier is applied for amplification of adder output signal with appropriate coefficient. The output signal voltage of transmitter which has been shown in V-B can be written as

$$V_{out,TX} = (\frac{R_c + R_d}{R_c(R_a + R_b)})\,[R_a V_{main} + R_b V_{ch}] \qquad (37)$$

where V_{main} and V_{ch} are the main and chaotic signals, respectively. Assuming all resistors are equal, the transmitter output is exactly equal to sum of these two signals.

B. SENDING THE ENCRYPTED SIGNAL WITH TRANSMITTER

The encrypted signal is sent to the destination using the transmitter system depicted in Figure 16.

Wireless transfer of the encrypted signal is performed using the ESP8266 Module. Arduino software and

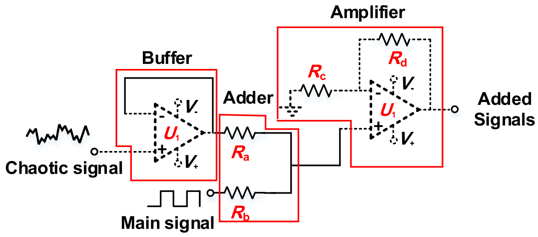

FIGURE 15. Schematic circuit of adder section.

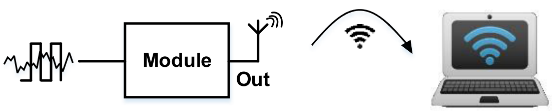

FIGURE 16. Schematic diagram of the transmitter.

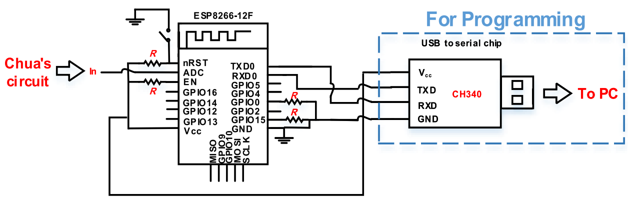

FIGURE 17. Transmitter circuits.

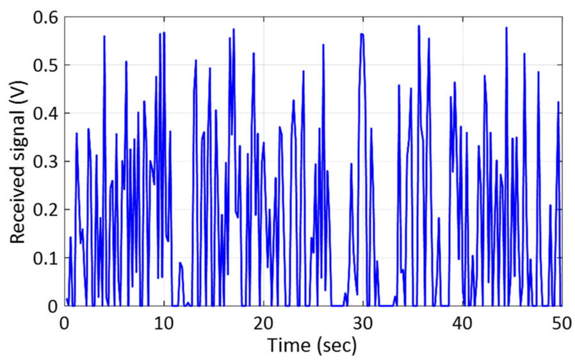

FIGURE 18. Signal received at the destination.

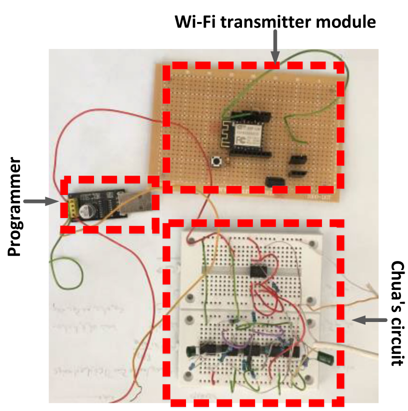

FIGURE 19. Circuit boards for chaotic signal generator, transmitter and programmer.

Module CH340 were considered in programming ESP8266 (Figure 17). Visual Studio, a programming software for windows, was used to extract and display the data received by the module on a computer.

The signal received at the destination is depicted in V-C.

C. RECOVERING THE MAIN SIGNAL FROM THE ENCRYPTED SIGNAL

The final step consists on extracting the main signal from the encrypted one. This is done by subtracting the chaotic signal from the encrypted signal. Figure 19 depicts the circuit board of the transmitter modules.

VI. CONCLUSION

This article proposes a disturbance observer-SMC approach for the robust synchronization of uncertain delayed chaotic systems. To this end, it first examined and analyzed the electronic behavior of the Sprott chaotic system. Then, proposed a robust sliding mode control technique for the synchronization of the master and slave Sprott chaotic systems subject to external disturbances, time-delays and parametric uncertainties. It also proposed a disturbance observer to accurately estimate the external disturbances affecting the system. The proof of the semi-globally bounded synchronization between the master and slave systems was established using the Lyapunov stability theory. The efficiency of the proposed approach was first assessed using a simulation study, then, experimentally validated on a data security system. The obtained results confirmed the robust synchronization properties of the proposed approach in the presence of time-delays and external disturbances. The experimental validation also confirmed its ability to ensure the secure transfer of data. Establishing finite-time synchronization between the master and slave chaotic systems and design of an adaptive-tuning control scheme for the unknown upper bounds of external disturbances are among the topics that will be considered in our future research.

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