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# An Improved Cavity-Perturbation Approach for Simultaneously Measuring the Permittivity and Permeability of Magneto-Dielectric Materials in Sub-6G

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**ABSTRACT** Magneto-electric materials with low loss have prospective applications in microwave systems as they enable miniaturization and broadband impedance matching. Two example applications are antennas and filters in sub-6G communication systems. Therefore, high-accuracy and wideband testing are critical for magneto-electric materials, whose complex permittivity and permeability are usually dependent on frequency. In this paper, the mutual interference between the electric and magnetic field within magneto-electric material samples was seriously considered. It was found that the results calculated with the original perturbation formula were overestimated when the sample size was not so small or higher-order modes are used, especially when the electric or magnetic field is perpendicular to the material under test. Two methods based on perturbation, namely the iteration method and the multi-state method are proposed to reduce the impact of the mutual interference, which have been proven to be direct and effective through theoretical analysis and experiments. Finally, several rod-shaped specimens processed from several standard dielectric materials (PTFE, fused silica, Al<sub>2</sub>O<sub>3</sub>) and synthesized magneto-electric materials were measured in a fabricated cavity with a vector network analyzer. Experimental results show that the results obtained by the modified formula are more accurate than those obtained by the original formula, and are in good agreement with the data measured by other methods.

**INDEX TERMS** Magneto-electric materials, complex permittivity and permeability, iteration, multi-state, mutual interference, perturbation.

#### I. INTRODUCTION

Magneto-dielectric materials can have nontrivial or designed permittivity  $\varepsilon_r$  and permeability  $\mu_r$ , and they have been widely used for miniaturization of RF devices [1], [2] and in applications that require a material with tailored impedance. There is a strong motivation for using magnetic-dielectric materials instead of dielectric materials in microwave devices for the following reasons:

1) Both the permittivity and permeability compress the wavelength of propagating electromagnetic waves. Therefore, the physical dimensions of a transmission line or resonant structure can be made smaller.

2) In addition to miniaturization, magneto-dielectric materials can provide closer impedance matching and enhance the bandwidth.

When applied to the design of antennas [1] and filters [2], magneto-electric materials are expected to have as low loss as possible, including electrical loss and magnetic loss.

Synthesis and application of magneto-electric materials are closely related to accurate measurements of electromagnetic constitutive parameters. Compared with the non-resonant method [3]–[7], which can typically provide a broadband measurement, the resonant method is normally preferred as high precision measurement, especially for low

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FIGURE 1. Rectangular waveguide cavity and sample to be tested, and the dark red and gray rectangles correspond to the samples placed along the H and E planes respectively.

loss materials. The cavity perturbation method (CPM) is the most common resonator-based approach for characterizing magneto-dielectric materials [8]–[25]. Furthermore, a rectangular cavity (RC) was chosen as a fixture for measuring the complex permittivity and permeability of magneto-electric materials in most earlier studies [18]–[24] because the cavity structure and field distribution are very simple.

As shown in FIGURE 1, the geometric center of the cavity is usually used as the position for inserting the sample, because it is the location where the odd mode electric field and even mode magnetic field are maximum. The complex permeability determined using the rectangular cavity method requires the sample to be placed in the H-plane of the cavity [18]–[22]. However, placing a sample hole in the H-plane will disturb the surface current and cause field radiation, both of which must be avoided when designing a rectangular cavity for permeability measurements. Accordingly, in [18]-[22], the sample needs to be inserted into the cavity with the help of a customized sample support fixture, then the sample is placed parallel to the magnetic field direction by rotating the fixture. So the depolarization factor must be redefined because the sample length is smaller than the width of the cavity, and especially when the ratio of sample length to cavity width is less than 0.7 [18]. Also, the permeability and permittivity have been measured in earlier studies by placing the specimen at a non-radiating slot in the E-plane of the rectangular cavity [17], [23].

The odd mode magnetic field and even mode electric field within the sample at the center of the cavity were assumed to be zero when gathering permittivity and permeability measurements in the aforementioned studies. If this condition cannot be satisfied when measuring one of the parameters, the electric and magnetic fields will act on the magnetoelectric material samples at the same time, thereby reducing the accuracy of the test results. When higher-order modes and finite samples are used in the test, the aforementioned restrictions cannot be easily satisfied, especially when the magnetic or electric field is perpendicular to the sample.

The iteration method and the multi-state method based on the CPM are naturally thought of and proposed to solve the above problem. The first method is to use test results from adjacent odd and even modes to perform cross-iteration calculations. The second one is by setting up two full perturbation equations based on two different positions where the sample placed, and then both the complex permittivity and permeability can be extracted by solving simultaneous equations.

The paper is organized as follows. The conventional cavity perturbation equations used to determine the complex permittivity and permeability of materials along the E and H planes is discussed in Section II. The influence of mutual interference between the electric field and magnetic field within samples during the measurement is analyzed, and two improved cavity perturbation techniques that can reduce interference are presented in Section III. To show the validity of the proposed methods, the resonance parameters obtained from simulation, calculation, and measurements from a double-ridged waveguide cavity are compared in Section IV. Experimental results and measured data from various samples under test are presented in Section V. Conclusions are offered in Section VI.

## II. BASIC APPROACH

The basic equation for the change in resonant frequency due to material perturbations is as follow [8]

$$\frac{\omega_0 - \omega}{\omega_0} = \frac{\int_{V_S} \left[ (\varepsilon - \varepsilon_0) \vec{E}_0^* \cdot \vec{E} + (\mu - \mu_0) \vec{H}_0^* \cdot \vec{H} \right] dV}{\int_{V_C} \left( \varepsilon_0 \vec{E}_0^* \cdot \vec{E} + \mu_0 \vec{H}_0^* \cdot \vec{H} \right) dV}.$$
(1)

where  $\omega$  and  $\omega_0$  are the complex resonant frequency of the cavity with and without a sample;  $E_0$ ,  $H_0$ , and E, H are the electric and magnetic fields before and after the perturbation, respectively. The complex permittivity and permeability of the sample to be tested are  $\varepsilon = \varepsilon_0 \varepsilon_r = \varepsilon_0 (\varepsilon'_r - j\varepsilon''_r)$  and  $\mu = \mu_0 \mu_r = \mu_0 \times (\mu'_r - j\mu''_r)$ , respectively, while  $\varepsilon_0$  and  $\mu_0$  represent the dielectric constant and permeability of a vacuum. The integral regions  $V_S$  and  $V_C$  in the formula are the volume of the perturbation medium and the cavity, respectively.

Equation (1) is derived from Maxwell's equations and holds when

1: The walls of the cavity can be regarded as perfectly conducting.

2: 
$$V_C - V_S \approx V_C$$
.

3:  $(\omega \varepsilon - \omega_0 \varepsilon_0) \approx \omega_0(\varepsilon - \varepsilon_0)$  and  $(\omega \mu - \omega_0 \mu_0) \approx \omega_0(\mu - \mu_0)$ .

Since the above conditions are usually guaranteed to be met, Equation (1) is an relatively rigorous. When the perturbation is caused by the medium, the following condition is approximately satisfied.

4: The distribution of the electromagnetic field outside the sample area does not change.

$$\frac{\omega_0 - \omega}{\omega_0} = \frac{\int_{V_S} \left[ (\varepsilon_r - 1) \vec{E}_0^* \cdot \vec{E} + (\mu_r - 1) \vec{H}_0^* \cdot \vec{H} \right] dV}{\int_{V_C} \left( \left| \vec{E}_0 \right|^2 + \left| \vec{H}_0 \right|^2 \right) dV}.$$
(2)

Considering the influence of the polarization field in the medium, the field in the sample can be written as

$$\vec{E} = \frac{1}{1 + N_e(\varepsilon_r - 1)}\vec{E}_0.$$
(3)

$$\vec{H} = \frac{1}{1 + N_m(\mu_r - 1)} \vec{H}_0.$$
(4)

where  $N_e$  and  $N_m$  are depolarization factors of the electric and magnetic fields, respectively, which mainly depend on the sample geometry, electromagnetic parameters of the sample, and applied field [26]. For several common types, such as dielectric spheres or long thin cylindrical dielectric rods placed in a uniform external field, it can be solved under the quasi-static field.

Substituting (3) and (4) into (2), the full perturbation formula can be obtained:

$$\frac{w_0 - w}{w_0} = \frac{\varepsilon_r - 1}{1 + N_e(\varepsilon_r - 1)} C_e + \frac{\mu_r - 1}{1 + N_m(\mu_r - 1)} C_h.$$
 (5)

where  $C_e$  and  $C_h$  are shape factors of the material under test (MUT) corresponding to the electric and magnetic fields, respectively. And these two parameters are given by (6) and (7).

$$C_{e} = \frac{\int_{V_{s}} \left| \vec{E}_{0} \right|^{2} dV}{2 \left| \int_{V_{s}} \left| \vec{E}_{0} \right|^{2} dV}.$$
 (6)

$$C_{h} = \frac{\int_{V_{S}} \left| \vec{H}_{0} \right|^{2} dV}{2 \int_{V_{C}} \left| \vec{H}_{0} \right|^{2} dV}.$$
(7)

In general, when measuring the permittivity, the sample is placed at the location where the electric field is maximized and the magnetic field interacting with the sample is considered to be zero. Similarly, when measuring the permeability, the sample is placed at the location where the magnetic field is maximized, and ignore the interaction between the electric field and the sample. Based on the above condition, the following CPM equations can be used to calculate the permittivity and permeability independently.

$$\frac{w_0 - w}{w_0} = (\varepsilon_r - 1) \frac{1}{1 + N_e(\varepsilon_r - 1)} C_e.$$
 (8)

$$\frac{w_0 - w}{w_0} = (\mu_r - 1) \frac{1}{1 + N_m(\mu_r - 1)} C_h.$$
 (9)

For a lossy sample, the complex resonant frequency can be separated into real and imaginary parts as follow:

$$\frac{w_0 - w}{w_0} = \frac{f_0 - f}{f_0} + \frac{j}{2} \left( \frac{1}{Q_0} - \frac{1}{Q} \right).$$
(10)

where  $f_0$ , f, and  $Q_0$ , Q are the resonance frequency and quality factor of the unperturbed and perturbed cavities, respectively. Then (8) and (9) can be expressed as four implicit expressions by substituting the complex forms of frequency, permittivity, and permeability:

$$\frac{f_0 - f}{f_0} = \frac{\left(\varepsilon_r' - 1\right) \left\{ N_e \left(\varepsilon_r' - 1\right) + 1 \right\} + N_e \varepsilon_r''^2}{\left[ 1 + N_e (\varepsilon_r' - 1) \right]^2 + \left(N_e \varepsilon_r''\right)^2} \times C_e.$$
(11)

$$\frac{f_0}{f_0} = \frac{(r + r) (r + r)}{\left[1 + N_m \left(\mu'_r - 1\right)\right]^2 + \left(N_m \mu''_r\right)^2} \times C_h.$$
(13)

$$\frac{1}{Q} - \frac{1}{Q_0} = \frac{\mu_r''}{\left[1 + N_m \left(\mu_r' - 1\right)\right]^2 + \left(N_m \mu_r''\right)^2} \times 2C_h.$$
(14)

Strictly speaking, it is impossible to ensure complete independence between electricity and magnetism in the sample. Therefore, the mutual interference between the electric and magnetic fields within magneto-electric material samples and the impact on the measurement accuracy must be discussed. And the results of the theoretical analysis will be presented in the next section, and two improved methods based on CPM are derived.

#### **III. IMPROVED CAVITY PERTURBATION METHOD**

In this work, the double-ridge waveguide cavity is selected as the test sensor to realize a wide-band test in sub 6GHz (0.7GHz-6GHz) frequency range, because it can be used in a smaller size and achieve a wider operating frequency band compared with the RC.

In this paper, the sample is still placed along the E plane instead of the H plane when measuring permeability. There are two reasons. First and foremost, the large size of the waveguide cavity in the 0.7GHz makes the preparation of a full-length (about 200mm) and thin rod sample extremely difficult even if a double-ridged waveguide is used. And radiation holes should be avoided to be introduced. Secondly, when the sample length is much smaller than cavity width, it is quite difficult to accurately calculate the depolarization factor [26], [27], especially for the double-ridged waveguide cavity.

 $TE_{10p}$  modes in the double-ridge cavity have sinusoidal electric and magnetic field distribution along the waveguide axis. Therefore, the magnetic field can be ignored in a very small area where the electric field is maximum and vice versa, thus the permittivity and permeability can be measured separately using odd and even modes. However, when a magneto-electric sample is placed in the cavity, the measured results are composed of the sum of the perturbations caused by the electric and magnetic fields acting on the sample, respectively. This will cause the results calculated with (11) - (14) to be overestimated when the sample size is not so small or higher-order modes are used during measurement, especially when the electric or magnetic field is perpendicular to the MUT.

### A. MUTUAL INTERFERENCE EVALUATION

First, mutual interference needs to be analyzed quantitatively. We set the dielectric constant and permeability to be equal, and the sample volume is set to be sufficiently small to guarantee a fractional change in the resonant frequency perturbation is about 0.001 [25]. Therefore,  $C_e$  and  $C_h$  can



**FIGURE 2.** Perturbation analysis and evaluation of electrical and magnetic interference for a square rod sample.

be assumed as constant. Then we can directly calculate the perturbation of permeability and permittivity in odd or even modes, respectively.

FIGURE 2(a) describes the ratio of the magnetic field perturbation to the total perturbation in the sample region for odd modes and different permeability values. As shown in FIGURE 2(c), the error is introduced because this part of the magnetic field perturbation is not deducted in the permittivity calculation.

FIGURE 2(b) shows the ratio of the electric field perturbation to the total perturbation in the sample region for even modes and different permittivity values. Error shown in FIGURE 2(d) is introduced because this part of the electric field perturbation is not deducted in the permeability calculation.

From FIGURE 2(c) and FIGURE 2(d), one can see that, if the mutual interference in odd modes or even modes is not considered, the influence on the result cannot be negligible. And the influence of the electric field perturbation on the calculation of permeability in even mode in FIGURE 2(d) is particularly significant, the reason is that the sample is placed perpendicular to the magnetic field, which makes the effective perturbation caused by the magnetic field small.

Meanwhile, a similar analysis is made for the rectangular plate sample with the ratio of transverse dimension of 1:2, and the results are shown in FIGURE 3. It can be found that the larger the sample size along the Z direction, the greater the mutual interference between electricity and magnetism, which conforms to the characteristics of electromagnetic field distribution.

## **B. INTERFERENCE CORRECTION**

After the impact of mutual interference on the measurement results is analyzed, it needs to be corrected. Two methods namely the iteration method and the multi-state method are proposed here.

## 1) THE ITERATION METHOD

A simple correction method for mutual interference involves iterative calculations using test results of adjacent odd and even modes.



FIGURE 3. Perturbation analysis and evaluation of electrical and magnetic interference for a rectangular sheet sample.

For odd modes, the full perturbation equation (5) can be written as:

$$\frac{\left(\varepsilon_{ri}^{o}-1\right)C_{e}^{o}}{1+N_{e}(\varepsilon_{ri}^{o}-1)} = \frac{w_{0}^{o}-w_{c}^{o}}{w_{0}^{o}} - \frac{\left(\mu_{r(i-1)}^{e}-1\right)C_{h}^{o}}{1+N_{m}(\mu_{r(i-1)}^{e}-1)}.$$
 (15)

Similarly, for adjacent even modes, the full perturbation equation is written as:

$$\frac{\left(\mu_{ri}^{e}-1\right)C_{h}^{e}}{1+N_{m}(\mu_{ri}^{e}-1)} = \frac{w_{0}^{e}-w_{c}^{e}}{w_{0}^{e}} - \frac{\left(\varepsilon_{ri-1}^{o}-1\right)C_{e}^{e}}{1+N_{e}(\varepsilon_{ri-1}^{o}-1)},$$
 (16)

where the superscript letters e and o represent even and odd modes, respectively, and i is the number of iterations. When i = 0, the above two equations simplify to (8) and (9), which means the results of odd mode and even mode that calculated by (8) and (9) will be used as the initial iteration data. Table 1 shows the iteration results when the samples are rod-shaped and sheet-shaped. And the relative complex permittivity and permeability are set to 5-j0.05and 5-j0.5, respectively. It can be seen from the calculated results that the iterative method can effectively take the influence of interference into consideration. Besides, when the order of iteration is changed, the result is completely the same.

# 2) THE MULTI-STATE METHOD

Another way to reduce interference is called the multi-state method, which is to establish full perturbation equations at different positions and solve them simultaneously. The first position is the middle of the cavity. For measuring the permeability at 0.7 GHz and keeping enough distance between the sample and the coupling ring, the second position is selected as 2.0 mm from the short-circuit end. The perturbation at the center is

$$\frac{w_0 - w_c}{w_0} = \left[\frac{(\varepsilon_r - 1) C_{ec}}{1 + N_{ec}(\varepsilon_r - 1)} + \frac{(\mu_r - 1) C_{hc}}{1 + N_{mc}(\mu_r - 1)}\right].$$
 (17)

The perturbation at the side is

$$\frac{w_0 - w_s}{w_0} = \left[\frac{(\varepsilon_r - 1) C_{es}}{1 + N_{es}(\varepsilon_r - 1)} + \frac{(\mu_r - 1) C_{hs}}{1 + N_{ms}(\mu_r - 1)}\right].$$
 (18)

Sample dimension: 3.0mm×3.0mm								
Modes	s $TE_{1,0,10}$ $TE_{1,0,11}$ $TE_{1,0,11}$		$TE_{1,0,12}$					
Iterations	$\mu_r$	$\varepsilon_r$ $\varepsilon_r$		$\mu_r$				
0	5.222	5.012	5.012	5.325				
	<i>-j</i> 0.542	-j0.051 -j0.051		- <i>j</i> 0.561				
1	4.999	5.000	5.000	4.999				
1	- <i>j</i> 0.500	- <i>j</i> 0.050	- <i>j</i> 0.050	- <i>j</i> 0.500				
2	5.000	5.000	5.000	5.000				
2	- <i>j</i> 0.500	- <i>j</i> 0.050	- <i>j</i> 0.050	- <i>j</i> 0.500				
	Sample dir	mension: 2.0m	m×4.0mm					
Modes	$TE_{1,0,10}$	$TE_{1,0,11}$	$TE_{1,0,11}$	$TE_{1,0,12}$				
Iterations	$\mu_r$	$\varepsilon_r$	$\varepsilon_r$	$\mu_r$				
0	5.636	5.017	5.017	5.961				
0	<i>-j</i> 0.633	- <i>j</i> 0.051	- <i>j</i> 0.051	- <i>j</i> 0.707				
1	4.998	4.999	4.999	4.997				
	- <i>j</i> 0.499	- <i>j</i> 0.050	- <i>j</i> 0.050	-j0.499				
2	5.000	5.000	5.000	5.000				
	- <i>j</i> 0.500	- <i>j</i> 0.050	- <i>j</i> 0.050	- <i>j</i> 0.500				

#### TABLE 1. Results of iterative calculation.

Simultaneously solving (16) and (17) gives the following implicit expressions:

$$(\varepsilon_r - 1) \frac{1}{1 + N_{ec}(\varepsilon_r - 1)} = \frac{\frac{w_0 - w_c}{w_0} C_{hs} - \frac{w_0 - w_s}{w_0} C_{hc}}{C_{ec} C_{hs} - C_{es} C_{hc}}.$$
 (19)  
$$(\mu_r - 1) \frac{1}{1 + N_{mc}(\mu_r - 1)} = \frac{\frac{w_0 - w_s}{w_0} C_{ec} - \frac{w_0 - w_c}{w_0} C_{es}}{C_{hs} C_{ec} - C_{hc} C_{es}}.$$
 (20)

Among them,  $C_{ec}$ ,  $C_{hc}$ ,  $C_{es}$ , and  $C_{hs}$  are shape factors for the electric and magnetic fields corresponding to the middle and side positions, respectively. The equations (19) and (20) can theoretically eliminate the influence of mutual interference.

The even-odd mode iteration method and the multi-state method have proven to be practical and very effective ways for correcting the influence of mutual interference between electricity and magnetism. The main differences between the two methods are, the iteration method requires the permittivity and permeability corresponding to adjacent modes (frequencies) do not change significantly, while the multi-state method does not rely on this assumption. But in the multi-state method, the MUT needs to be placed in two different positions for testing.

## **IV. NUMERICAL SIMULATION**

ANSYS HFSS was used to calculate the eigenmodes of the double-ridged cavity shown in Figure 4(a). When the field of the mode under consideration is symmetric, as is the case in  $TE_{10p}$  modes, the symmetrical boundary condition can be used, thus only one-quarter of the cavity needs to be used in the calculation, as shown in Figure 4(b). This will significantly reduce the computation time and suppresses asymmetric modes, which is helpful for pattern recognition.



**FIGURE 4.** Simulation models of (a) a full double-ridged waveguide cavity and (b) a quarter of a double-ridged waveguide cavity and its dimensions.

TABLE 2. Resonant frequency and quality factor.

	c	c	c	0	0
Modes	Jcal	Jsim	Jmea	$Q_{sim}$	$Q_{mea}$
Middes	[GHz]	[GHz]	[GHz]		
$TE_{1,0,1}$	0.7058	0.7054	0.7054	15923	15949
$TE_{1,0,2}$	1.0751	1.0746	1.0747	16764	16731
$TE_{1,0,3}$	1.5006	1.5001	1.5004	18704	18358
$TE_{1,0,4}$	1.9458	1.9453	1.9456	20468	19737
$TE_{1,0,5}$	2.3998	2.3992	2.3996	21540	20925
$TE_{1,0,6}$	2.8584	2.8577	2.8582	23530	22381
$TE_{1,0,7}$	3.3197	3.3190	3.3195	25286	22824
$TE_{1,0,8}$	3.7826	3.7819	3.7824	26772	24046
$TE_{1,0,9}$	4.2467	4.2460	4.2466	28275	24999
$TE_{1,0,10}$	4.7117	4.7106	4.7115	29613	25638
$TE_{1,0,11}$	5.1772	5.1762	5.1771	31055	26309
$TE_{1,0,12}$	5.6431	5.6420	5.6431	31975	28127
$TE_{1,0,13}$	6.1094	6.1083	6.1094	33010	29150
$TE_{1,0,14}$	6.5760	6.5746	6.5760	34336	30723

The simulated characteristic frequency, calculated results by mode-matching technique (MMT) [24], and measured resonance frequency are compared in Table 2.

The differences between the resonance frequency measured and that determined from simulations are less than 0.02%, and the differences between the measured values and

TABLE 3.	Comparison	of measured	results with	different	methods for	or dielectric materi	ials.
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Dielectric materials $\longrightarrow$ (left) Fused silica: 2.98 mm $\times$ 2.98 mm $\times$ 50 mm and (right) Al <sub>2</sub> O <sub>3</sub> : 2.95 mm $\times$ 2.95 mm $\times$ 50 mm.											
Frequency	Previous	s method Proposed metho		d method	Relative	Frequency	Previous method		Proposed method		Relative
[GHz]	$\varepsilon'_r$	$\mu_r'$	$\varepsilon'_r$	$\mu'_r$	difference	[GHz]	$\varepsilon'_r$	$\mu'_r$	$\varepsilon'_r$	$\mu'_r$	difference
0.71		1.001		0.994	0.70%	0.71		1.013		0.991	2.22 %
1.07		1.000	]	0.999	0.10 %	1.07		1.002		0.999	0.30 %
1.95	2 8 2 2	1.003	2 0 2 2	1.000	0.30 %	1.95	0.056	1.008	0.056	1.000	0.80~%
2.86	3.022	1.006	3.822	1.000	0.60 %	2.86	9.050	1.019	9.050	1.001	1.80 %
3.78	$\pm 0.02$	1.011	1 ±0.02	1.000	1.10 %	3.78	$\pm 0.03$	1.033	$\pm 0.03$	1.001	3.20 %
4.71		1.017		1.000	1.70 %	4.71		1.054		1.004	4.98 %
5.64		1.024		1.000	2.40 %	5.64		1.078		1.005	7.26 %
6.58		1.033		1.000	3.30 %	6.58		1.109		1.007	10.8 %



FIGURE 5. Block diagram of the experimental setup.

the values calculated with the MMT is less than 0.06%, which means the field and shape factors can also be calculated accurately.

Besides, both the measured and simulated Q-factor have been attached in Table 2. What can be seen is that the two sample holes introduced on the wide side of the cavity did not cause a significant drop in the Q value. In addition, because the high-frequency conductor loss is more sensitive to the surface roughness of the conductor wall and the electrical continuity of the contact surface, the difference between the measured quality factor and the simulated value increases as the number of modes (frequencies) increases.

## V. MEASUREMENT AND RESULT

Figure 5 shows a block diagram of the experimental setup. The resonant cavity consists of a double-ridged waveguide section terminated at both ends with conducting plates. Two coupling loops through small circular holes (2.0 mm in diameter) in the center of the two endplates provide coupling into the cavity. Then the cavity can be connected to the two ports of the Vector Network Analyzer (VNA) by coaxial cables. And the sample can be inserted into the cavity through two small square holes.



FIGURE 6. Photograph of the apparatus for measuring complex permittivity and complex permeability.

The photograph of the experimental setup is shown in Figure 6. The VNA (Agilent E8363B) has been calibrated and connected to the test cavity. The resonance parameters of the cavity before and after loading the specimen can be measured. Besides, the sample can be fixed on the two-dimensional moving platform, and it can be moved to the middle of the hole precisely along the arrow direction by adjusting two knobs.

First, the proposed approach was verified by measuring the permittivity and permeability values of several reference dielectric materials (PTFE, Fused silica, and Al<sub>2</sub>O<sub>3</sub>). The results are shown in Figure 7. The calculated permittivity and permeability values with and without considering the mutual interference of electric and magnetic field are shown in Table 3. Meanwhile, these three kinds of reference dielectric materials were tested using the split-cavity method [28] at 8.4GHz, and the complex permittivity of PTFE, Fused silica, and Al<sub>2</sub>O<sub>3</sub> was measured to be 2.062-j0.00054, 3.827-j0.00045, and 9.078-j0.0027, respectively. As can be seen from Figure 7 and Table 3, the permittivity gathered with the proposed perturbation method are consistent with those obtained by the split resonator method, and the test accuracy of permeability has been improved by introducing the iterative method.

Synthesized magneto-electric materials $\longrightarrow$ 4H2N1T: 3.01 mm $\times$ 3.01 mm $\times$ 50 mm.								
Frequency	Previo	us method	Iteration method		Relative	Multi-state method		Relative
[GHz]	$\mu'_r$	$tan \delta_{\mu}$	$\mu'_r$	$tan \delta_{\mu}$	difference	$\mu'_r$	$tan \delta_{\mu}$	difference
0.71	2.031	1.43E-01	2.030	1.41E-01	0.04%	2.030	1.41E-01	0.07%
1.07	1.903	1.67E-01	1.899	1.67E-01	0.19%	1.896	1.66E-01	0.32%
1.95	1.708	2.49E-01	1.698	2.48E-01	0.59%	1.693	2.46E-01	0.88%
2.86	1.562	2.85E-01	1.543	2.84E-01	1.24%	1.537	2.81E-01	1.63%
3.78	1.481	2.97E-01	1.450	2.95E-01	2.16%	1.444	2.91E-01	2.57%
4.71	1.438	3.06E-01	1.391	3.02E-01	3.38%	1.389	3.01E-01	3.58%
5.64	1.400	2.98E-01	1.335	2.92E-01	4.85%	1.334	2.90E-01	4.94%
6.58	1.407	2.94E-01	1.320	3.16E-01	6.63%	1.293	3.14E-01	8.87%
Frequency	<i></i>	4 5	-	Relative	1 5	Relative		
[GHz]	$\varepsilon_r$	$\iota a n o_{\varepsilon}$	$\varepsilon_r$	$\iota a n o_{\varepsilon}$	difference	$\varepsilon_r$	$tano_{\varepsilon}$	difference
0.71	5.588	2.46E-02	5.588	2.46E-02	0.00%	5.587	2.46E-02	0.02%
1.5	5.595	2.41E-02	5.595	2.41E-02	0.00%	5.594	2.41E-02	0.01%
2.4	5.619	2.55E-02	5.618	2.54E-02	0.02%	5.618	2.54E-02	0.02%
3.32	5.638	2.65E-02	5.637	2.64E-02	0.02%	5.637	2.64E-02	0.03%
4.25	5.708	3.16E-02	5.706	3.14E-02	0.02%	5.706	3.14E-02	0.02%
5.18	5.702	3.04E-02	5.700	3.02E-02	0.03%	5.702	2.86E-02	0.00%
6.11	5.721	3.65E-02	5.719	3.60E-02	0.04%	5.723	3.12E-02	0.04%

 TABLE 4. Comparison of measured results with different methods for magneto-electric material.



**FIGURE 7.** Measured permittivity and permeability values of several dielectric materials (PTFE, fused silica, Al<sub>2</sub>O<sub>3</sub>).

Then the coaxial line method [3] and the improved perturbation method (the multi-state method) are applied to the measurement of a synthesized magneto-electric material sample, and the results are shown in Figure 8.

Also, the magneto-electric material sample data measured by the previous method, the iteration method, and the multi-state method are listed in Table 4 for comparison. And the relative differences between the results of the latter two proposed methods and the previous method are also listed in Table 4. One can see from Table 4, data gathered without considering electric and magnetic mutual interference are somewhat larger than the data considering the mutual interference, and the greater the mutual influence as the number of modes increases. Since the sample is placed perpendicular to the magnetic field and the dielectric constant value is several



**FIGURE 8.** Measured results of (a) real part of complex permittivity and permeability values and (b) tangent of loss angle of a synthesized magneto electric material sample using coaxial line method and improved perturbation method.

times of the permeability, it can be found from the results of a specific sample that in the center of the cavity, the electric field perturbations in the even mode have a great (> 6.6%) impact on the permeability measurement, while the magnetic field perturbation in the odd mode has little (< 0.1%) effect on the measurement results of the dielectric constant.

Besides, the relative difference between the results obtained by the iterative method and the multi-state method is smaller. All test results are consistent with the previous theoretical analysis.

#### **VI. CONCLUSION**

In this paper, the evolution of the perturbation theory formula and the corresponding assumptions are described in detail. It is well known that when measuring the complex permittivity or permeability, the sample will be placed at the position where the electric field or magnetic field are maximum, respectively, and the assumption is made that only a single field interacts with the sample. In most cases, this assumption always seems to be satisfied. However, we have shown that under certain conditions, the influence of this assumption should not be ignored, such as when the direction of the magnetic field or the electric field is perpendicular to the sample, the sample size is not small enough, or higher-order modes are used for measurement. Two methods based on perturbation, namely the iteration method and the multi-state approach are proposed to reduce the impact of the mutual interference, which have been proven to be direct and effective through theoretical analysis and practical experiments.

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