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Fast Wideband Monostatic Scattering Analysis Method Combining Macro-Block Characteristic Basis Function Method With Improved FIR for Freestanding Large-Scale Finite Periodic Arrays

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ABSTRACT An efficient numerical method for the wideband scattering analysis of the freestanding large-scale finite periodic array is presented using the macro block-characteristic basis function method (MB-CBFM) and the improved frequency-independent reaction (FIR). In the MB-CBFM, the blocks (unit cells) are divided into some types of macro blocks. By analyzing the subarray with the macro blocks, the CBFs for the macro blocks will be obtained. The blocks in the same macro block share the same CBFs. Substituting the CBFs into the moment matrix equation, a reduced matrix equation whose size does not depend on the number of the blocks can be obtained and handled utilizing direct methods. When the MB-CBFM is applied for wideband analysis, the matrix needs to be recomputed. The reduced matrix can be efficiently generated by using the improved FIR, in which the matrix elements are expressed as the product of the geometrical elements and the phase factor. The geometrical elements are computed one time and reused. Only the phase factor needs to be recomputed. Several numerical examples are carried out. Simulation results show that the results of the proposed algorithm agree well with those of other numerical methods. The total CPU time is significantly reduced.

INDEX TERMS Finite periodic array, macro block-characteristic basis function, improved frequency-independent reaction, scattering, wideband.

I. INTRODUCTION

The method of moments (MoM) is an accurate numerical method and has been used to analyze electromagnetic problems for decades [1]. The resultant matrix is dense. When applied to analyze an electrically large object, both huge memory and very long CPU time are needed. Hence, it is unsuitable to analyze the large-scale finite periodic arrays. The finite periodic array can be analyzed as the infinite one [2]. However, the induced currents on the edge unit cells are not very accurate [3].

The fast algorithms are suitable for analysis of the large-scale arrays, i.e., the multilevel fast multipole method (MLFMM) [4], the adaptive integral method (AIM) [5], etc. By using the MLFMM and the AIM, the computational complexity is reduced to $O(N\log N)$, where N is the number of unknowns. Several basis function methods, i.e., the macro basis function method [6], the characteristic basis function method (CBFM) [7], [8], the synthetic basis function method [9], the subentire-domain (SED) basis

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function method [10], the macro block-CBF method (MB-CBFM) [11], etc., are also suitable for analysis of large-scale arrays, by which the number of unknowns is decreased. In the CBFM, the original object is divided into B blocks. The $B^2 \times B^2$ reduced matrix is obtained [7]. In the SED basis function method, the SED basis functions for cells are solved by analyzing a subarray problem [10]. The $N_c \times N_c$ matrix equation is obtained, where N_c is the number of unit cells. For the large-scale array, the iterative methods have to be applied. The CPU time depends on the condition number. In the MB-CBFM, the blocks (unit cells) are classified as P types of macro blocks [11]. The CBFs are solved firstly. Then, the $PM \times PM$ reduced matrix can be obtained and resolved using a direct method, where M is the number of the elementary basis functions defined on block. The solution time is not affected by the condition number. The CPU time and the memory cost are $O(N)$ [11]. These methods can accelerate the single-frequency simulation. For the frequency sweep, the matrices need to be recalculated. For the MB-CBFM, the computation of the reduced matrix is still very time consuming.

Several efficient frequency sweep techniques have been proposed, i.e., the interpolation [12]–[14], the asymptotic waveform estimation (AWE) [15], the frequency-independent reaction (FIR) [16], etc. For the interpolation, the matrix elements must vary slowly with frequency. To this end, the wide band usually has to be divided into several sub-bands. In [14], the wide scattering was analyzed by using the CBFM and the interpolation. In [17], the ASED basis function was combined with the interpolation to analyze the electromagnetic scattering by the finite periodic arrays. For the AWE [15], the accuracy will be reduced when the frequency is not near the center frequency. In the FIR [16], the impedance element is approximated as the series summation with $Q_{\max}+1$ terms, in which every term is the product of the geometrical elements and the phase factor. The former are computed and saved once, while the latter is recomputed. This method is suitable for the wideband response analysis without the frequency-band division. The accuracy can also be tuned. The memory cost of this technique is $(2Q_{\max} + 5)N^2$ [16]. Therefore, it is unsuitable for electrically large objects if is not combined with other techniques. In [18], it was combined with the higher-order basis function for analysis of the electrically large objects. In [19], the combination of the ASED basis function method with the FIR was applied for wideband analysis of the finite periodic arrays. The improved FIR has been presented in our recent work [20], in which the far-field matrix elements are computed using the concept of the frequency-independent reaction and the equivalent dipole model. In the new version, less geometrical elements are needed. Thus, it is more efficient than the conventional FIR. At the same time, the memory cost is also reduced.

In this work, we combine the MB-CBFM with the improved FIR to compute wideband electromagnetic scattering from the finite periodic arrays. The number of unknowns can be significantly reduced using the MB-CBFM.

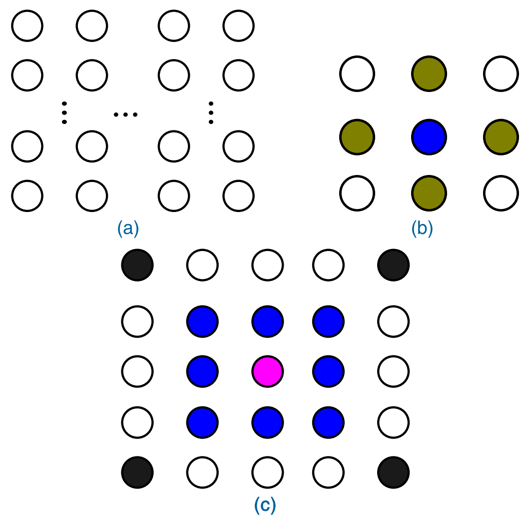

FIGURE 1. (a) Finite periodic array with N_c unit cells (blocks). (b) subarray with 9 blocks; (c) subarray with 25 blocks.

The frequency sweep can be accelerated using the improved FIR. In Section II, the MB-CBFM and the improved FIR are briefly reviewed. In Section III, numerical examples are implemented to test the proposed method. In Section IV, some conclusions are drawn.

II. FORMULATION

A. MB-CBFM

Figure 1a shows the finite periodic array with N_c unit cells (blocks). Only the perfectly electric conducting blocks are considered. Each block is discretized with the triangles. There are M RWG basis functions [21] on each block. The electric field integral equation (EFIE) is used to formulate the problem. Applying the conventional MoM with Galerkin's procedure to solve the EFIE, one has [1], [21]

$$ZI = V \tag{1}$$

with the matrix element

$$z_{tb} = \frac{1}{j\omega\varepsilon_0}\int_s\int_{s'} g(\boldsymbol{r},\boldsymbol{r}')\left[k^2\boldsymbol{f}_t(\boldsymbol{r})\cdot\boldsymbol{f}_b(\boldsymbol{r}') - \left(\nabla\cdot\boldsymbol{f}_t(\boldsymbol{r})\right)\left(\nabla'\cdot\boldsymbol{f}_b(\boldsymbol{r}')\right)\right]dS'dS \tag{2}$$

where Z and V are the impedance matrix of size $N_cM \times N_cM$, and the excitation vector of size $N_cM \times 1$, respectively. I is the current vector of size $N_cM \times 1$ to be solved. ω and ε_0 are the angular frequency and the permittivity of the free space, respectively. k is the wavenumber. $\boldsymbol{f}_t(\boldsymbol{r})$ and $\boldsymbol{f}_b(\boldsymbol{r}')$ are the testing and the basis functions, respectively. $g(\boldsymbol{r}, \boldsymbol{r}') = \exp(-jkR)/4\pi R$ is the Green's function, where $R = |\boldsymbol{r} - \boldsymbol{r}'|$.

In the MB-CBFM, the blocks are classified as P ($P = (2n + 1)^2$, $n = 0, 1, \ldots$) kinds of the macro blocks. Each macro block includes Q blocks (Q may differ for different macro block). Consider that a uniform plane wave normally illuminates the object. If mutual couplings are considered, there are 9 ($n = 1$ case), 25 ($n = 2$ case) or more macro blocks as shown in Figs. 1 (b) and (c). The CBFs of the macro blocks need to be determined firstly. Analyze the subarray with P macro blocks by solving the matrix equation:

$$\begin{bmatrix} Z_{11}^{sub} & Z_{12}^{sub} & \cdots & Z_{1P}^{sub} \\ Z_{21}^{sub} & Z_{12}^{sub} & \cdots & Z_{2P}^{sub} \\ \vdots & & & \\ Z_{P1}^{sub} & \cdots & & Z_{PP}^{sub} \end{bmatrix} \begin{bmatrix} J_1^{sub} \\ J_2^{sub} \\ \vdots \\ J_P^{sub} \end{bmatrix} = \begin{bmatrix} V_1^{sub} \\ V_2^{sub} \\ \vdots \\ V_P^{sub} \end{bmatrix} \tag{3}$$

where $Z_{ii}^{sub}(i = 1, 2, \ldots, P)$ is the self-impedance matrix of the ith macro block in the subarray. $Z_{ij}^{sub}(i \neq j)$ is the mutual impedance matrix between the ith and the jth macro blocks. J_i^{sub} is the current vector of the ith macro block. The mth CBF of the ith macro block is [11]

$$J_{im}^{mb} = \left(x_{im}^T J_i^{sub}\right) x_{im}, \quad m = 1, 2, \ldots, M \tag{4}$$

where the superscript 'T'' denotes the transpose. x_{im} is the mth eigenvector of size $M \times 1$ of Z_{ii}^{sub}. Because the self-matrix is symmetric, its M eigenvectors are orthogonal. Using the CBFs, the current vector of the ith macro block can be expressed as [11]

$$I_i^{mb} = \sum_{m=1}^{M} \alpha_{im} J_{im}^{mb} \tag{5}$$

where α_{im} is the unknown coefficient.

The CBFs of the blocks in the same macro block are identical. Thus, the CBFs of all blocks can be obtained by analyzing the subarray problem. Thus, the current vector of the blocks belonging to the ith macro block is

$$\left(I_{i1}, I_{i2}, \ldots, I_{iQ(i)}\right) = I_i^{mb} \tag{6}$$

where $Q(i)$ is the number of the blocks in the ith macro block.

Substituting (6) into (1), one has [11]

$$\sum_{p=1}^{P} \sum_{m=1}^{M} \mathbf{u}_{pm} \alpha_{pm} = V \tag{7}$$

where

$$\mathbf{u}_{pm} = \left[[Z_{1p}] J_{pm}^{mb}, [Z_{2p}] J_{pm}^{mb}, \ldots, [Z_{N_c p}] J_{pm}^{mb}\right]^T \tag{8}$$

where $[Z_{ip}]$ (i = 1,2,..., N_c) denotes the $N_c M \times M$ mutual impedance matrix between the ith block and the pth macro block. The matrix in (7) of size $N_c M \times PM$ is overdetermined. The $PM \times PM$ matrix can be obtained using the least square method.

B. EFFICIENT COMPUTATION OF Z USING THE IMPROVED FIR

The MB-CBFM is the frequency-based technique. Recomputation of the matrix is required for the frequency sweep if no other methods are used. The impedance matrix can be efficiently generated by using the improved FIR. The impedance elements are divided into two categories: the near-region and the far-region ones. They are calculated using the conventional FIR [16], and the improved one [20], respectively.

The near-region impedance element z_{tb} is approximated as [16]

$$z_{tb} \approx \frac{\phi}{4\pi j\omega\varepsilon_0} \sum_{q=0}^{Q_{\max}} k^q \left(k^2 A_{tb}^q - B_{tb}^q\right) \tag{9}$$

with

$$A_{tb}^q = \frac{(-j)^q}{q!} \int_S \int_{S'} \frac{(R - R_{tb})^q}{R} \boldsymbol{f}_t(\boldsymbol{r}) \cdot \boldsymbol{f}_b(\boldsymbol{r}') dS' dS \tag{10}$$

$$B_{tb}^q = \frac{(-j)^q}{q!} \int_S \int_{S'} \frac{(R - R_{tb})^q}{R} \left(\nabla \cdot \boldsymbol{f}_t(\boldsymbol{r})\right) \times \left(\nabla' \cdot \boldsymbol{f}_b(\boldsymbol{r})\right) dS' dS \tag{11}$$

where R_{tb} is the distance between the centers of the tth and the bth RWG edges. A_{tb}^q, $B_{tb}^q(q = 0, 1,\ldots, Q_{\max})$, and R_{tb} depend on the geometry. $\phi = \exp(-jkR_{\text{tb}})$ is the phase factor. $Q_{\max}$ is set to 4 for wideband scattering [16].

When applying the FIR, the geometrical elements are computed once and saved before the frequency sweep. The phase factor ϕ is recomputed each frequency. Substituting the precomputed geometrical elements and the phase factor into (9), the near-region elements can be computed.

The far-region elements are computed using the improved FIR that is based on the equivalent dipole model [22], [23]. The plus-minus triangle pair can be equivalent to the electric dipole when the distance between the testing and the basis functions is greater than the limit value. In this work, the average length of the triangle sides is $0.1\lambda_{\min}$, where $\lambda_{\min}$ is the minimum wavelength within the frequency range. The limit value is $0.3\lambda_{\min}$ [20]. Using the equivalent dipole model, the far-region element z_{tb} can be rewritten as [20]

$$z_{tb} \approx \eta \frac{e^{-jkD_{tb}}}{4\pi D_{tb}} \Big[jk\left(\boldsymbol{M}_b - \boldsymbol{m}_b\right) \cdot \boldsymbol{m}_t + \left(3\boldsymbol{M}_b - \boldsymbol{m}_b\right) \cdot \boldsymbol{m}_t \left(\frac{1}{D_{tb}} + \frac{1}{jkD_{tb}^2}\right)\Big] \tag{12}$$

where

$$\boldsymbol{M}_b = \left(\boldsymbol{D}_{tb} \cdot \boldsymbol{m}_b\right) \boldsymbol{D}_{tb} / D_{tb}^2 \tag{13}$$

$$\boldsymbol{m}_{b/t} = l_{b/t} \left(\boldsymbol{r}_{b/t}^{c-} - \boldsymbol{r}_{b/t}^{c+}\right) \tag{14}$$

where η is the wave impedance of the free space. $\boldsymbol{D}_{tb} = \boldsymbol{r}_{db} - \boldsymbol{r}_{dt}$, where $\boldsymbol{r}_{db}$, $\boldsymbol{r}_{dt}$ are the coordinates of the centers of the bth and the tth dipoles, respectively. D_{tb} is the length of $\boldsymbol{D}_{tb}$. $\boldsymbol{m}_{b/t}$ is the moment of the bth/tth dipole [23]. $l_{b/t}$ is the length of the bth/tth RWG edges. $\boldsymbol{r}_{b/t}^{c+}$ and $\boldsymbol{r}_{b/t}^{c-}$ are the centers of the bth/tth plus and minus triangles, respectively.

$\boldsymbol{M}_b$, $\boldsymbol{m}_b$, $\boldsymbol{m}_t$, and D_{tb} depend on the geometry. $\exp(-jkD_{\text{tb}})$ is the phase factor. When applying the improved FIR, these three geometrical elements $(\boldsymbol{M}_b - \boldsymbol{m}_b)\cdot\boldsymbol{m}_t$, $(3\boldsymbol{M}_b - \boldsymbol{m}_b)\cdot\boldsymbol{m}_t$ and D_{tb} should be computed and stored firstly. The phase factor is recomputed every frequency. Using these geometrical elements and the phase factor, the far-region elements can be efficiently calculated. Comparing with the conventional

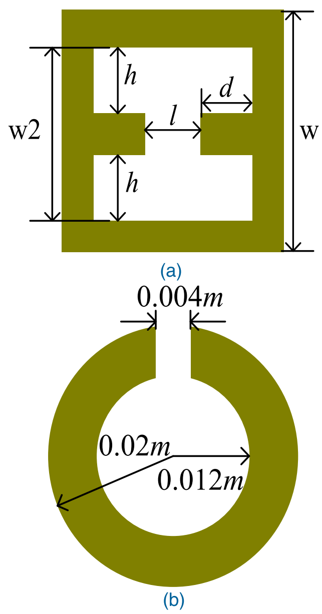

FIGURE 2. Geometry of the unit cells. (a) the stepped-impedance (SI) loop resonator. w1 = 0.04*m*, w2 = 0.02*m*, *h* = 0.005*m*, *d* = 0.008*m*, *l* = 0.004*m*; (b) the circular split ring resonator (SRR).

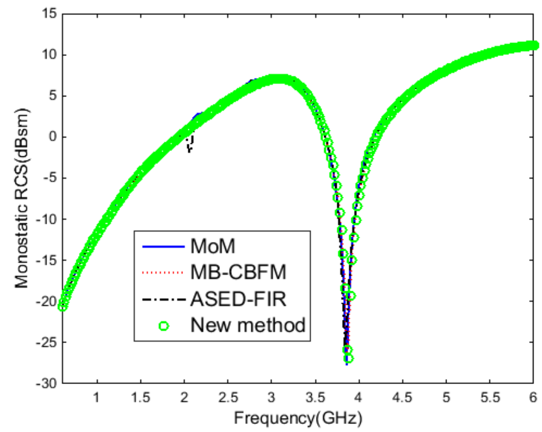

FIGURE 3. Wideband monostatic RCSs of the 5 × 5 SI loop resonator array obtained using different techniques.

TABLE 1. Generation time of Z per frequency for different methods (Unit: s).

Method	Generation time
MB-CBFM	15.490
Proposed method	0.402

TABLE 2. Total solution time of the SI loop resonator arrays (Unit: s).

Method	Total CPU time
ASED-FIR	821.89
Proposed technique	1362.55

FIR, the improved one is more efficient because it has less geometrical elements [20].

The percentage of the near-region elements is β for the reduced matrix in the MB-CBFM. The memory cost of the near/far-region geometrical elements are $O(3\beta(Q_{\max}+1)N)$, and $O(3(1-\beta)N)$, respectively. The total memory cost is $3(\beta Q_{\max}+1)O(N)$. For the large-scale array, β is very small. The memory cost is approximately equal to $3O(N)$. The memory cost of the proposed technique, including the impedance matrix, is $4O(N)$. It is still proportional to the number of unknowns.

For more details of the FIR and the improved version, the readers can refer to [16] and [20].

III. NUMERICAL RESULTS

In this section, some numerical examples are carried out. Wideband monostatic scatterings by the finite periodic arrays are calculated by using the proposed technique. To compare the accuracy, the results obtained using the MB-CBFM and the conventional MoM are also provided. The arrays located in the *xoy*-plane are with the *x*-polarized plane wave incidence. The incidence angle is $\theta^i = \varphi^i = 0$. The gap between the cells is 0.01 *m*. The frequency increment is 0.02 GHz. For the far-field analysis, accurate results can be obtained by solving the subarray with nine blocks for MB-CBFM [11]. Therefore, nine macro blocks are considered. All examples are implemented on the personal computer with the Intel Core i7-6700K processor 4.0 GHz and 32 GB of RAM.

Firstly, the 5 × 5 stepped-impedance (SI) loop resonator array shown in Fig. 2a is computed. There are 299 RWG basis functions on unit cell. The total number of unknowns is 7475 for the entire array. The wideband monostatic radar cross sections (RCSs) ranging from 0.6 to 6GHz are shown in Fig. 3. It can be seen that the results of the proposed technique agree well with those of the conventional MoM and the MB-CBFM. The results of the ASED-FIR are also given and agree well with other techniques at most frequencies. There is a small discrepancy around 2.1GHz.

Generation time of impedance matrix Z per frequency in the MB-CBFM and the proposed technique is shown in Tab. 1. The periodicity is considered. It can be observed that generation time is significantly reduced using the proposed technique. In the MB-CBFM, the matrix element needs to be recomputed. In the proposed technique, the geometrical elements are computed once and reused. The phase factors are recomputed. Therefore, the matrix can be efficiently generated. The total solution times for the ASED-FIR and the proposed method are shown in Tab. 2. The former is more efficient than the latter for this example. In the ASED-FIR, the 25 × 25 matrix equation needs to be solved for each frequency while the 2691 × 2691 matrix equation has to be handled in the proposed technique. For small array, the ASED-FIR may be more efficient than the proposed technique if accurate results can be obtained using the former. As mentioned above, the results of the proposed technique are

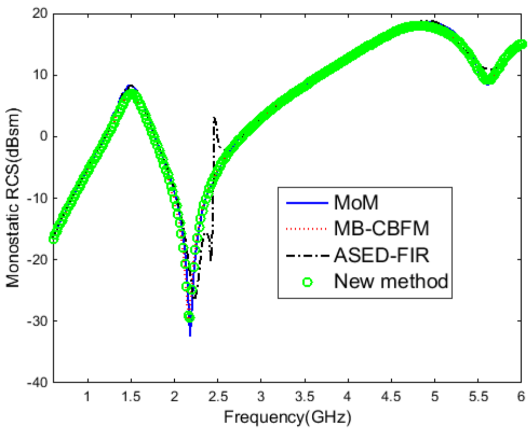

FIGURE 4. Wideband monostatic RCSs of the 8 × 8 circular SRR array computed using different methods.

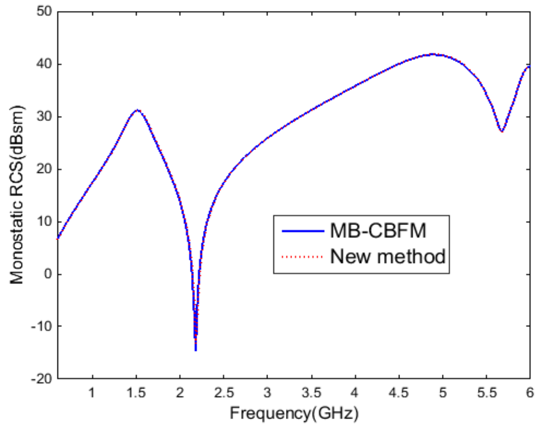

FIGURE 5. Wideband monostatic RCSs of the 30 × 30 circular SRR array.

TABLE 3. Generation time of impedance matrix Z per frequency using different techniques (Unit: s).

Method	Generation time
MB-CBFM	13.87
Proposed method	0.31

TABLE 4. CPU time of the 30 × 30 arrays (Unit: s).

	MB-CBFM	Proposed method
CPU time	82762.81	15601.04
Saving	-	81%

more accurate than those of the ASED-FIR around 2.1GHz. Therefore, the proposed technique has an advantage over the ASED-FIR in terms of the accuracy.

Secondly, the 8 × 8 circular split ring resonator (SRR) array shown in Fig. 2b is analyzed. Each cell has 157 RWG basis functions. The total number of unknowns is 10,048. The wideband monostatic RCSs obtained using different numerical methods are shown in Fig. 4. One can see that the results of the proposed technique agree well with those of the conventional MoM and the MB-CBFM. There is a considerable discrepancy between the results of the ASED-FIR and other techniques around 2.5GHz, which may be due to the inaccuracy of the ASED basis functions. To efficiently analyze the wideband RCS of the circular SRR arrays, it is better to apply the proposed technique.

Generation time of the impedance matrix Z per frequency using different methods are shown in Tab. 3. From Tab. 3, one can see that the time is also significantly decreased using the proposed technique.

Finally, the 30×30 circular SRR array with 141,300 RWG basis functions is analyzed. The wideband monostatic RCSs are shown in Fig. 5. One can see that good agreement is achieved again. The total CPU time for the MB-CBFM and the proposed method are shown in Tab. 4. The total solution time is reduced by 81% using the proposed technique. Therefore, the proposed technique is suitable to analyze the wideband scattering by the large-scale finite periodic arrays.

IV. CONCLUSION

The efficient algorithm combining the MB-CBFM with the improved FIR has been presented to analyze the wideband electromagnetic scattering from the large-scale finite periodic arrays. The number of unknowns is tremendously reduced by using the MB-CBFM. The reduced matrix equation can be handled using direct solvers even for the large-scale arrays. During the frequency sweep, the MB-CBFM's impedance matrices can be efficiently calculated by utilizing the improved FIR. Numerical simulation examples have demonstrated the accuracy and efficiency of the proposed algorithm. In this work, only the freestanding finite periodic arrays are analyzed. In the future work, the periodic arrays with the substrate will be considered.

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