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A Novel Equivalent Input Disturbance-Based Adaptive Sliding Mode Control for Singularly Perturbed Systems

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ABSTRACT This paper develops a novel equivalent input disturbance (EID)-based adaptive sliding mode control (SMC) method for singularly perturbed systems (SPSs). Firstly, the block diagonalization approach is introduced to decompose the full-order SPSs exactly, and slow and fast subsystems are obtained by solving the upper and lower triangular matrices individually. Secondly, an EID is constructed to estimate the unknown disturbances with the observer gain and error system convergence analyzed. Then, depending on the decoupled reduced-order system models, a Lyapunov equation-based solution is adopted to construct a composite sliding surface. Finally, combined with the EID estimation, an adaptive SMC law is proposed to compensate the adverse effect of disturbances and the reachability condition is proven. The presented control strategy is free of any priori disturbances information while the satisfactory system performance can be guaranteed. Simulation results on two examples illustrate its superiority over the existing methods.

INDEX TERMS Singularly perturbed systems, equivalent input disturbance, adaptive sliding mode control, block diagonalization approach.

I. INTRODUCTION

Multi-time-scale property widely exists in many large-scale industrial applications, such as advanced heavy water reactors, neuron systems, motor systems and electric circuits [1]–[4]. In stability analysis and controller design for these systems, there inevitably confront high dimensionality and ill-conditioned numerical issues [5], which may be caused by the existence of capacitances, inductances or other parasitic small parameters. To deal with the above issues, the systems are usually modeled as singularly perturbed systems (SPSs), where the separation degree between the fast and slow modes is indicated by a singular perturbation parameter [6]. To analyze the SPSs, singular perturbation theory has been extensively studied (see [1]–[6] and the references therein).

Unmodeled dynamics, external disturbances and parameter perturbations characterized by the lumped disturbances existed in many practical situations may extremely deteriorate the system performance, which poses challenges to the control for nonlinear systems [7]–[10]. Many researchers have paid attention to explore new approaches to attenuate the adverse effects of disturbances for a variety of systems, such as networked switched systems [11], [12], robotic manipulators [13], Markov jump systems [14], [15] and so on. Benefiting from simple concept, fast response, powerful robustness and especially insensitivity to the disturbances, sliding mode control (SMC) has been extensively researched in nonlinear control [16]–[19]. Generally speaking, the SMC design procedure is mainly comprised of two components [20]. A suitable sliding surface is required, and thus the system dynamics are strictly restricted onto it during the sliding mode [21]. Meanwhile, during the reaching mode, the designed SMC law should drive the sliding surface to zero in finite time, and consistently maintain the system state variables on it for all subsequent time [22]. The Lyapunov stability theorem and linear matrix inequality technique was employed

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in [23] to design a SMC strategy. A considerable number of disturbance-rejection approaches, such as the disturbance observer (DO) [10] and the extended state observer (ESO)-based method [24], implicitly generate an equivalent disturbance estimation through the same channel as the control input [25]. Motivated by this, She *et.al* [26] proposed an equivalent input disturbance (EID) approach and conducted theoretical analysis. EID-based compensation method can perfectly suppress any kind of disturbance [27], and only the information of control input and the system output is required to produce the EID estimation in this approach. It is worth mentioning that the strong robustness against disturbances can be guaranteed by incorporating the observer estimation-based feedforward compensation into the conventional SMC feedback, straightforwardly.

Although there have some excellent literatures about SMC approaches for normal systems, directly applying these above mentioned strategies to SPSs usually leads to ill-conditioned numerical problems. Therefore, some scholars are dedicated to investigating the SMC for SPSs (see [28]–[36] and the references therein). According to singular perturbation theory, the original full-order SPSs can be approximately decoupled into a boundary layer system and a quasi-steady equation [28], which individually correspond to the reaching mode dynamics and sliding mode dynamics. A novel DO was presented and the integral SMC gain was determined by applying the classical H_∞ control theory in [29]. The passivity-based integral SMC for uncertain SPSs had been addressed in [30], where the controller gain was determined by solving a set of linear matrix inequalities (LMIs). Furthermore, preserving the passivity and internally exponential stability for SPSs, a less conservative ε-bound estimation algorithm was derived in [31]. In addition, Liu *et.al* [32] proposed a H_∞ observer-based SMC for the SPSs with input nonlinearity, and the resulting sliding mode dynamics was input-to-state stable with respect to the observer error. By employing a more general storage function [33], an adaptive integral SMC incorporating the DO estimation was designed for uncertain SPSs with disturbances. For the nonlinear Takagi-Sugeno (T-S) fuzzy SPSs, Wang *et.al* [34] proposed an integral fuzzy sliding surface in a convex optimization framework [35]. Although the ill-conditioned numerical problem is well solved in the aforementioned literatures without the system decomposition, the high dimensionality still exists in the analysis and design process for SPSs. To solve this issue, Zhou *et.al* [36] decomposed the full-order SPSs into two redeced-order subsystems, then provided a state feedback control method and a DO-based integral SMC method for them respectively. SMC plays an excellent role in suppressing the adverse effects of disturbances for SPSs, but these existing methods are usually based on the prior information that the disturbances meet certain boundedness condition.

Inspired by the discussion above, a novel SMC method is proposed in this paper to suppress the adverse effects of disturbances on SPSs. An EID is presented to estimate the disturbances first. Then, the Lyapunov equation-based composite sliding surface is derived and an adaptive SMC law in combination with the EID estimation is designed. Finally, the reachability condition is guaranteed and the system stability is analyzed. The main contributions of this paper can be summarized as follows.

1) The singular perturbation parameter ε is fully taken into account during the construction of state observer and sliding surface to handle the ill-conditioned numerical problems.
2) The block diagonalization approach for exactly decomposing the original SPSs into two subsystems is presented and the decoupled dynamics are incorporated in the composite sliding surface.
3) Combined with EID estimation, the proposed adaptive SMC can effectively compensate the disturbance effect and guarantee satisfactory system performance.
4) The proposed control strategy is free of any priori disturbances information and is applicable for different disturbances.

The resting sections of this paper are arranged as follows. Section II gives problem statement and preliminaries. Section III presents main results. Section IV shows simulation results. Section V is a conclusion. The future research prospect is presented in Section VI.

II. PROBLEM STATEMENT AND PRELIMINARIES

In this paper, the following linear time-invariant SPSs with disturbances is studied

$$\begin{cases} E(\varepsilon)\,\dot{\psi}(t) = A\psi(t) + Bu(t) + Df(t) \\ y(t) = C\psi(t) \end{cases} \tag{1}$$

where

$$E(\varepsilon) = \begin{bmatrix} I_{n_1} & 0 \\ 0 & \varepsilon I_{n_2} \end{bmatrix}, \quad \psi(t) = \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}$$
$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$
$$C = I_{n_1+n_2}, \quad D = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

where I is the identity matrix, $\varepsilon > 0$ is the singular perturbation parameter, $\psi(t) \in R^n$ is the system state vector ($n = n_1 + n_2$), while $x(t) \in R^{n_1}$ and $z(t) \in R^{n_2}$ are the slow-time and fast-time state variables, $u(t) \in R^m$ is the control input ($m \leq n$), $y(t) \in R^p$ is the output of plant, $f(t) \in R^q$ represents the unknown disturbances.

Assumption 1: Matrix A_{22} is invertible and B is of full column rank, *i.e.*, $rank(A_{22}) = n_2$ and $rank(B) = m$.

Assumption 2: The pairs (A_0, B_0) and (A_{22}, B_2) are controllable, where $A_0 = A_{11} - A_{12}A_{22}^{-1}A_{21}$ and $B_0 = B_1 - A_{12}A_{22}^{-1}B_2$.

Remark 1: Although the ill-conditioned numerical problem can be solved based on full-order SPSs, the high dimensionality still exists in analysis and design of SPSs. According to the singular perturbation theory [6], the full-order SPSs dynamics can be approximated by the dynamics of lower-order quasi-steady-state (slow subsystem) and boundary layer system (fast subsystem). Besides, Assumption 2 allows us to design the control laws for two decomposed subsystems, separately. By using the decomposition method, the high dimensionality and ill-conditioned numerical problems can be avoided simultaneously.

Definition 1 [26]: Let the control input be $u(t) = 0$. If the output of the system (1) under the disturbance $f(t)$ is the same as that influenced by the disturbance $f_e(t)$. The disturbance $f_e(t)$ is defined as an EID of the disturbance $f(t)$, which enters the system through the same channel as the control input.

Based on the eigenvalue placement technique and Assumption 2, there exist state feedback gains K_0 and K_2 such that slow and fast subsystems are stable by letting $A_0 + B_0K_0$ and $A_{22}+B_2K_2$ be both asymptotically stable. Then we can design the following composite control law

$$u(t) = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + v(t) \tag{2}$$

where $K_1 = \left(I_m + K_2A_{22}^{-1}B_2\right)K_0 + K_2A_{22}^{-1}A_{21}$, and $u(t)$ is the virtual control input whose detailed expression will be determined later.

Substituting the composite control law (2) into the full-order system (1), we have

$$\begin{bmatrix} \dot{x}(t) \\ \varepsilon\dot{z}(t) \end{bmatrix} = \begin{bmatrix} A_{11}+B_1K_1 & A_{12}+B_1K_2 \\ A_{21}+B_2K_1 & A_{22}+B_2K_2 \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} v(t) + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} f(t)$$

which indicates that

$$\begin{bmatrix} \dot{x}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ \frac{T_{21}}{\varepsilon} & \frac{T_{22}}{\varepsilon} \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ \frac{B_2}{\varepsilon} \end{bmatrix} v(t) + \begin{bmatrix} D_1 \\ \frac{D_2}{\varepsilon} \end{bmatrix} f(t) \tag{3}$$

where

$$T_{11} = A_{11} + B_1K_1, \quad T_{12} = A_{12} + B_1K_2$$
$$T_{21} = A_{21} + B_2K_1, \quad T_{22} = A_{22} + B_2K_2$$

The following two steps are necessary to transform the system (3) into the block diagonal form.

1) Upper triangular form

Introduce the following new state variables:

$$\eta(t) = Lx(t) + z(t) \tag{4}$$

where $L \in R^{n_2\times n_1}$ is the upper transformation matrix.

Combining (3) with (4) yields

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\eta}(t) \end{bmatrix} = \begin{bmatrix} T_{11} - T_{12}L & T_{12} \\ \frac{T_{21}}{\varepsilon} - \frac{T_{22}}{\varepsilon}L + L(T_{11} - T_{12}L) & \frac{T_{22}}{\varepsilon} + LT_{12} \end{bmatrix} \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ \frac{B_2}{\varepsilon} + LB_1 \end{bmatrix} v(t) + \begin{bmatrix} D_1 \\ \frac{D_2}{\varepsilon} + LD_1 \end{bmatrix} f(t) \tag{5}$$

To obtain the upper triangular form, let

$$R(L) = \frac{T_{21}}{\varepsilon} - \frac{T_{22}}{\varepsilon}L + L(T_{11} - T_{12}L) = 0 \tag{6}$$

and system (5) can be reduced to

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\eta}(t) \end{bmatrix} = \begin{bmatrix} A_s & T_{12} \\ 0 & \frac{A_f}{\varepsilon} \end{bmatrix} \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ \frac{B_f}{\varepsilon} \end{bmatrix} v(t) + \begin{bmatrix} D_1 \\ \frac{D_f}{\varepsilon} \end{bmatrix} f(t) \tag{7}$$

where

$$A_s = T_{11} - T_{12}L, \quad A_f = T_{22} + \varepsilon LT_{12}$$
$$B_f = B_2 + \varepsilon LB_1, \quad D_f = D_2 + \varepsilon LD_1$$

2) Lower triangular form

Another transformation of variables is introduced

$$\xi(t) = x(t) - \varepsilon H\eta(t) \tag{8}$$

where $H \in R^{n_1\times n_2}$ is the lower transformation matrix.

Substituting (8) into (7) yields

$$\begin{bmatrix} \dot{\xi}(t) \\ \dot{\eta}(t) \end{bmatrix} = \begin{bmatrix} A_s & \varepsilon A_sH - HA_f + T_{12} \\ 0 & \frac{A_f}{\varepsilon} \end{bmatrix} \cdot \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} B_1 - HB_f \\ \frac{B_f}{\varepsilon} \end{bmatrix} v(t) + \begin{bmatrix} D_1 - HD_f \\ \frac{D_f}{\varepsilon} \end{bmatrix} f(t) \tag{9}$$

To obtain the lower triangular form, let

$$R(H) = \varepsilon A_sH - HA_f + T_{12} = 0 \tag{10}$$

and thus system (9) can be simplified as

$$\begin{bmatrix} \dot{\xi}(t) \\ \dot{\eta}(t) \end{bmatrix} = \begin{bmatrix} A_s & 0 \\ 0 & \frac{A_f}{\varepsilon} \end{bmatrix} \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} B_s \\ \frac{B_f}{\varepsilon} \end{bmatrix} v(t) + \begin{bmatrix} D_s \\ \frac{D_f}{\varepsilon} \end{bmatrix} f(t)$$

where

$$B_s = B_1 - HB_f, \quad D_s = D_1 - HD_f$$

By combining (4) and (8), we introduce the following Chang transformation [37]

$$\varphi(t) = M(\varepsilon)\psi(t)$$

where

$$\varphi(t) = \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix}, \quad M(\varepsilon) = \begin{bmatrix} I_{n_1} - \varepsilon HL & -\varepsilon H \\ L & I_{n_2} \end{bmatrix}$$

and H, L are the solutions to the following algebraic equations

$$T_{21} - T_{22}L + \varepsilon LT_{11} - \varepsilon LT_{12}L = 0$$
$$\varepsilon(T_{11} - T_{12}L)H - H(T_{22} + \varepsilon LT_{12}) + T_{12} = 0 \tag{11}$$

According to the fixed-point recursive algorithm [36], we can derive the solutions of the equation (11) and yield

$$L^{(i+1)} = T_{22}^{-1}\left[T_{21} + \varepsilon L^{(i)}T_{11} - \varepsilon L^{(i)}T_{12}L^{(i)}\right]$$
$$H^{(j+1)} = \left[\varepsilon(T_{11} - T_{12}L)H^{(j)} + T_{12}\right](T_{22} + \varepsilon LT_{12})^{-1}$$

where $L^{(0)} = T_{22}^{-1}T_{21}$ and $H^{(0)} = T_{12}T_{22}^{-1}$.

Hence, the state equation of the SPS (1) can be approximated by the equations of the following exact reduced-order subsystems

$$E(\varepsilon)\dot{\varphi}(t) = \bar{A}\varphi(t) + \bar{B}v(t) + \bar{D}f(t) \tag{12}$$

where

$$\bar{A} = \begin{bmatrix} A_s & 0 \\ 0 & A_f \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_s \\ B_f \end{bmatrix}, \bar{D} = \begin{bmatrix} D_s \\ D_f \end{bmatrix}$$

III. MAIN RESULTS

In this section, we will design an EID for SPS (1). The resulting estimation is incorporated into the Lyapunov approach-based adaptive SMC law. System stability and reachability condition are also analyzed.

A. EID DESIGN AND ANALYSIS

Under Definition 1, the SPS (1) can be rewritten as

$$\begin{cases} E(\varepsilon)\dot{\psi}(t) = A\psi(t) + B\left[u(t) + f_e(t)\right] \\ y(t) = C\psi(t) \end{cases} \tag{13}$$

For the system (13), construct the following state observer

$$\begin{cases} E(\varepsilon)\dot{\hat{\psi}}(t) = A\hat{\psi}(t) + Bu_0(t) + G\left[y(t) - \hat{y}(t)\right] \\ \hat{y}(t) = C\hat{\psi}(t) \end{cases} \tag{14}$$

where $\hat{\psi}(t)$ is the estimate value of $\psi(t)$, $u_0(t)$ is the nominal control input in the absence of disturbances, and G is the observer gain to be determined later.

Remark 2: It is noticed that the state observer constructed in (14) takes the singular perturbation structure $E(\varepsilon)$ of SPSs into account, which is obviously different from the ones for normal systems. Such a consideration can effectively avoid the ill-conditioned numerical problems that may occur in the subsequent derivation of sliding surface and adaptive SMC law.

Define the estimation error of the state variable

$$\tilde{\psi}(t) = \psi(t) - \hat{\psi}(t) \tag{15}$$

Substituting (15) into (13) yields

$$E(\varepsilon)\dot{\hat{\psi}}(t) = A\hat{\psi}(t) + Bu(t) + \left[Bf_e(t) + A\tilde{\psi}(t) - E(\varepsilon)\dot{\tilde{\psi}}(t)\right] \tag{16}$$

and it is reasonable to assume that there exists a control input $\tilde{f}_e(t)$ that satisfies

$$A\tilde{\psi}(t) - E(\varepsilon)\dot{\tilde{\psi}}(t) = -B\tilde{f}_e(t) \tag{17}$$

Meanwhile, we define the estimate error of the EID as

$$\tilde{f}_e(t) = f_e(t) - \hat{f}_e(t) \tag{18}$$

where $\hat{f}_e(t)$ is the estimation of $f_e(t)$.

Then, substituting (17) and (18) into (16) yields

$$E(\varepsilon)\dot{\hat{\psi}}(t) = A\hat{\psi}(t) + B\left[u(t) + \hat{f}_e(t)\right] \tag{19}$$

By combining (14) and (19), the following equation can be concluded

$$B\left[u(t) + \hat{f}_e(t)\right] = Bu_0(t) + G\left[y(t) - \hat{y}(t)\right]$$

then $\hat{f}_e(t)$ can be obtained as

$$\hat{f}_e(t) = u_0(t) - u(t) + B^{+}G\left[y(t) - \hat{y}(t)\right] \tag{20}$$

where B^{+} is the Moore-Penrose pseudoinverse matrix of B, $B^{+} = \left(B^TB\right)^{-1}B^T$.

Remark 3: By comparing the equation (15) with (18), it can be concluded that the difference between the actual state $\psi(t)$ and the estimated state $\hat{\psi}(t)$ (i.e., $\tilde{\psi}(t)$) has the same mechanism as $\tilde{f}_e(t)$. In addition, the EID is generated in real time using the system output and its estimation, which can be attributed to the effect of disturbances on system state.

Remark 4: It is worth mentioning that the EID estimator (20) employs the information of state observer (14) rather than the inverse dynamics of the plant. Besides, the realization of estimation does not require the differential of system outputs or priori information on the disturbances. Therefore, the proposed method has a wider application scope.

From the equation (17), it is easy to understand that $\tilde{f}_e(t)$ trends to zero as long as the dynamic of $\tilde{\psi}(t)$ is stable. In order to analyze the observer system stability, it is reasonable to assume that $f(t) = 0$ and the system (13) is equivalent to

$$\begin{cases} E(\varepsilon)\dot{\psi}(t) = A\psi(t) + Bu_0(t) \\ y(t) = C\psi(t) \end{cases} \tag{21}$$

Combining (14), (15) and (21) yields

$$E(\varepsilon)\dot{\tilde{\psi}}(t) = (A - GC)\tilde{\psi}(t)$$

Thus, the observer error system is stable as long as $A - GC$ is a Hurwitz matrix.

B. ADAPTIVE SMC DESIGN AND ANALYSIS

Under Assumption 2 and control law (2), it can be concluded that the system matrix $\bar{A}$ in (12) is asymptotically stable. That is to say, for given positive definite symmetric matrices $Q_s \in R^{n_1\times n_1}$ and $Q_f \in R^{n_2\times n_2}$, there exist $P_s \in R^{n_1\times n_1}$ and $P_f \in R^{n_2\times n_2}$ such that the following algebraic Lyapunov equation is satisfied

$$\bar{A}^TP + P\bar{A} = -Q$$

where $Q = diag\left\{Q_s, Q_f\right\}$, and $P = diag\left\{P_s, P_f\right\}$.

Then a novel sliding surface for the SPS (12) can be designed as

$$S(t) = \bar{B}^TPE(\varepsilon)\varphi(t) \tag{22}$$

When the sliding mode is achieved, the equivalent control method is usually adopted to analyze the sliding motion equation [38]. The system stability is analyzed in the following theorem.

Theorem 1: During the sliding mode, the system (12) is asymptotically stable on the sliding surface (22).

Proof: It is well known that when the controlled system enters sliding mode, we have

$$S(t) = \bar{B}^T PE(\varepsilon)\varphi(t) = 0$$

Choosing a Lyapunov function candidate as

$$V_1(t) = \varphi^T(t)PE^2(\varepsilon)\varphi(t)$$

Taking the time-derivative of $V_1(t)$ along the system (12) leads to

$$\begin{aligned}\dot{V}_1(t) &= [E(\varepsilon)\dot{\varphi}(t)]^T PE(\varepsilon)\varphi(t)\\ &\quad +\varphi^T(t)PE(\varepsilon)[E(\varepsilon)\dot{\varphi}(t)]\\ &= \varphi^T(t)\left[\bar{A}^T PE(\varepsilon) + PE(\varepsilon)\bar{A}\right]\varphi(t)\\ &\quad +2[v(t)+f_e(t)]^T \bar{B}^T PE(\varepsilon)\varphi(t)\\ &= -\varphi^T(t)E(\varepsilon)Q\varphi(t) + 2[v(t)+f_e(t)]^T S(t) < 0\end{aligned}$$

where the EID relationship is $\bar{D}f(t) = \bar{B}f_e(t)$.

The above inequality indicates that the system (12) is asymptotically stable when the state trajectory is restricted to the sliding surface (22). This completes the proof. ■

Remark 5: For the external disturbances, we can not explicitly extract the fast-time disturbance component aside from the slow-time portion [28]. As a result, it is not reasonable to achieve two individual sliding dynamics because the time-scale attribute does not hold for disturbances. Therefore, the Lyapunov approach is employed to construct the composite sliding surface (22), which dependents on the decoupled reduced-order system models.

Transform the sliding surface (22) into the original coordinates

$$\begin{aligned}S(t) &= \bar{B}^T PE(\varepsilon)\varphi(t)\\ &= \bar{B}^T PE(\varepsilon)M(\varepsilon)\psi(t)\\ &= \left[S_1\ S_2\right]E(\varepsilon)\psi(t)\end{aligned}$$

where

$$\begin{aligned}S_1 &= B_s^T P_s\left(I_{n_1} - \varepsilon HL\right) + \varepsilon B_f^T P_f L\\ S_2 &= B_f^T P_f - B_s^T P_s H\end{aligned}$$

By incorporating an appropriate reaching law, an adaptive SMC law can be designed as

$$\begin{aligned}u_0(t) = \left\{\left[S_1\ S_2\right]B\right\}^{-1}&\left\{\Gamma S(t) - \sigma\frac{S(t)}{\|S(t)\|}\right.\\ &\left. -\left\|\left[S_1\ S_2\right]B\right\|\hat{\chi}(t)\frac{S(t)}{\|S(t)\|} - \left[S_1\ S_2\right]A\psi(t)\right\}\end{aligned}$$

where Γ is chosen as a negative constant, $\|\cdot\|$ denotes Euclidean norm. The EID estimate error satisfies $\left\|\tilde{f}_e(t)\right\| \le \chi$, χ and σ are two positive constant, meanwhile

$$\dot{\hat{\chi}}(t) = \gamma\left\|\left[S_1\ S_2\right]B\right\|\|S(t)\|$$

where $\gamma > 0$ is a design parameter, and error value $\tilde{\chi}(t) = \chi - \hat{\chi}(t)$.

Thus, combined with the estimation of the EID, the SMC control law can be derived as

$$u(t) = u_0(t) - \hat{f}_e(t) \tag{23}$$

Remark 6: Disturbances are usually required to satisfy certain boundedness conditions in the existing disturbance suppression results [11], [12], [14]. An integral SMC was constructed in [15] for stochastic singular semi-Markov jump systems, but the bound of nonlinearity is required to be known. Our proposed adaptive SMC law $u_0(t)$ is free of any priori disturbance information, and thus the constraints on the upper bounds of disturbances and their derivatives are completely released. The proposed SMC is available for suppressing various kinds of disturbances.

Theorem 2: With the SMC law (23), the reachability condition can be guaranteed, that is, the system state variables will be globally driven onto the sliding surface in finite time.

Proof: Taking the time-derivative of the sliding surface yields

$$\begin{aligned}\dot{S}(t) &= \left[S_1\ S_2\right]E(\varepsilon)\dot{\psi}(t)\\ &= \left[S_1\ S_2\right]A\psi(t)\\ &\quad + \left[S_1\ S_2\right]B\left[u_0(t) + \tilde{f}_e(t)\right]\end{aligned} \tag{24}$$

Define the following Lyapunov function candidate

$$V_2(t) = \frac{1}{2}S^T(t)S(t) + \frac{1}{2\gamma}\tilde{\chi}^2(t) \tag{25}$$

Then, taking the derivative of $V_2(t)$ with respect to t and considering (23), (24) yield

$$\begin{aligned}\dot{V}_2(t) &= S^T(t)\dot{S}(t) - \frac{1}{\gamma}\tilde{\chi}(t)\dot{\hat{\chi}}(t)\\ &= S^T(t)\left\{\left[S_1\ S_2\right]A\psi(t)\right.\\ &\quad \left.+\left[S_1\ S_2\right]B\left[u_0(t)+\tilde{f}_e(t)\right]\right\}\\ &\quad -\tilde{\chi}(t)\left\|\left[S_1\ S_2\right]B\right\|\|S(t)\|\\ &\le \Gamma\|S(t)\|^2 - \sigma\|S(t)\| + \left\|\left[S_1\ S_2\right]B\right\|\\ &\quad \cdot\|S(t)\|\left\{\left\|\tilde{f}_e(t)\right\| - \left[\hat{\chi}(t) + \tilde{\chi}(t)\right]\right\}\\ &\le -\sigma\|S(t)\| + \left\|\left[S_1\ S_2\right]B\right\|\|S(t)\|\\ &\quad \cdot\left[\left\|\tilde{f}_e(t)\right\| - \chi\right]\\ &\le -\sigma\|S(t)\|\end{aligned} \tag{26}$$

where $\|S(t)\|^2 = S^T(t)S(t)$, and the above inequality implies that $\dot{V}_2(t) \le 0$. Thus, the sliding surface (22) can be attained in a finite time. This completes the proof. ■

Due to the high frequency noise and switching mechanism in $u_0(t)$, the EID estimation procedure may be affected.

In view of this, we can introduce a low pass filter (LPF)

$$\hat{F}_e(t) = \frac{\omega_f}{s+\omega_f}\hat{f}_e(t) \tag{27}$$

where ω_f is the cutoff angular frequency of LPF.

Thus, by combining (23) and (27), the synthesized adaptive SMC law can be derived

$$u(t) = u_0(t) - \hat{F}_e(t) \tag{28}$$

The control block diagram of the proposed EID-based adaptive SMC for SPSs with disturbances is shown in Fig. 1.

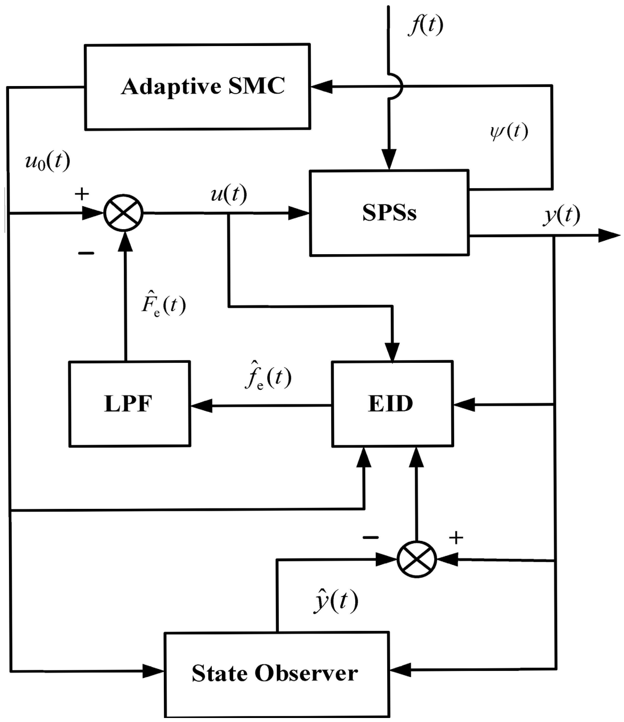

FIGURE 1. The EID-based adaptive SMC for SPSs with disturbances.

Remark 7: It can seen from Fig. 1 that the proposed control strategy is mainly composed of adaptive SMC, LPF, EID and state observer. The corresponding parameters are illustrated as follows: G is the observer gain to be determined such that $A - GC$ is a Hurwitz matrix, Γ in the SMC law is a negative constant which may influence the reaching speed of sliding surface, σ is selected to guarantee reachability condition, γ determines the convergence rate of the adaptive law and ω_f is the cutoff angular frequency of LPF. These parameters should be designed so that satisfactory system performance can be obtained.

Remark 8: The proposed adaptive SMC is able to guarantee the robustness against disturbances by combining the SMC feedback with the EID based-feedforward compensation, which is different from the SMC strategy in [28]. As a result, the systems under control possess better performance such as less overshoot and steady-state error, which will be illustrated in the next section.

IV. NUMERICAL EXAMPLE

In this section, we consider two examples to demonstrate the advantages of the proposed method over the existing results. Example 1 is borrowed from [28], and Example 2 is compared with the fast terminal sliding mode control proposed in [13].

A. EXAMPLE 1

In this subsection, the magnetic tape control system with disturbances is studied, whose mathematical model can be found in [28]. The system matrices and parameters are as follows.

$$A_{11} = \begin{bmatrix} 0 & 0.4 \\ 0 & 0 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 & 0 \\ 0.345 & 0 \end{bmatrix}$$
$$A_{21} = \begin{bmatrix} 0 & -0.524 \\ 0 & 0 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -0.465 & 0.262 \\ 0 & -1 \end{bmatrix}$$
$$B_1 = D_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad B_2 = D_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$C = I_4$, $\varepsilon = 0.1$, and $\psi(0) = \begin{bmatrix} -2 & 3 & -4 & 1 \end{bmatrix}^T$.

The pairs (A_0, B_0) are calculated as

$$A_0 = \begin{bmatrix} 0 & 0.4 \\ 0 & -0.3888 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ 0.1944 \end{bmatrix}$$

We can stabilize the slow and fast subsystems such that eigenvalues are placed as $eig(A_0 + B_0K_0) = \{-0.5, -1\}$ and $eig(A_{22} + B_2K_2) = \{-2, -3\}$ respectively. Thus, we can obtain the following feedback gain matrices

$$K_0 = \begin{bmatrix} -6.4305 & -5.7166 \end{bmatrix}$$
$$K_1 = \begin{bmatrix} -82.9738 & -90.4985 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} -14.852 & -3.535 \end{bmatrix}$$

The evolutions of recursive algorithm for solving H and L are shown in Figs. 2, where $\triangle H$ and $\triangle L$ denote the matrices error between the current value and that in the last iteration, respectively. It can be shown that the evolutions are uniformly convergent to their actual values when the numbers of iterations are $i = 12$ and $j = 7$. The corresponding calculation results are presented as follows.

$$L = \begin{bmatrix} 4.1789 & 4.8593 \\ 4.7427 & 4.153 \end{bmatrix}$$
$$R(L) = 10^{-7} \times \begin{bmatrix} -0.3712 & -0.309 \\ 0.3707 & 0.3069 \end{bmatrix}$$
$$H = \begin{bmatrix} 0.0095 & 0.0007 \\ -0.3399 & -0.0204 \end{bmatrix}$$
$$R(H) = 10^{-8} \times \begin{bmatrix} -0.0823 & -0.0074 \\ 0.2457 & 0.022 \end{bmatrix}$$

Set the expected eigenvalue as $eig(A - GC) = \{-10, -22.9075 \pm 2.4879j, -41\}$, then G can be obtained as

$$G = \begin{bmatrix} 10 & 4 & 0 & 0 \\ 0 & 20 & 3.45 & 0 \\ 0 & -5.24 & 25.35 & 2.62 \\ 0 & 0 & 0 & 40 \end{bmatrix}$$

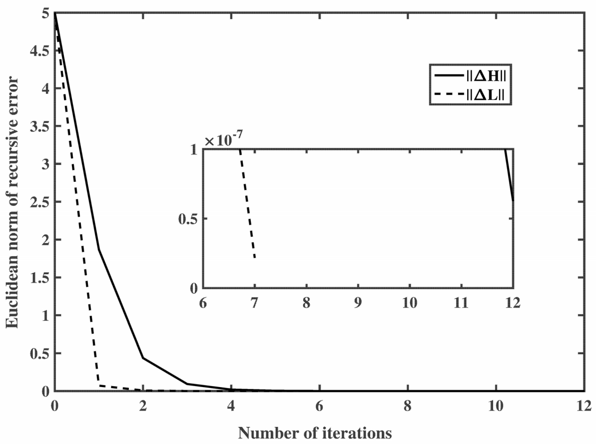

FIGURE 2. The evolutions of recursive algorithm for solving H and L.

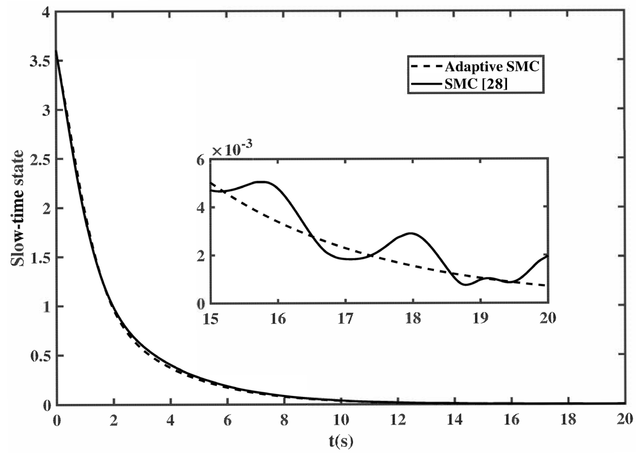

FIGURE 3. The norm of slow-time state variable.

Choose $Q = I_4$ and we can obtain the positive definite symmetric P as

$$P = \begin{bmatrix} 2.8268 & 0.3468 & 0 & 0 \\ 0.3468 & 0.3810 & 0 & 0 \\ 0 & 0 & 4.8156 & -0.0634 \\ 0 & 0 & -0.0634 & 0.1066 \end{bmatrix}$$

which generates the parameters of the constructed sliding surface as $S_1 = \begin{bmatrix} 0.030 & 0.0222 \end{bmatrix}$, $S_2 = \begin{bmatrix} -0.0609 & 0.1067 \end{bmatrix}$.

The developed control strategy can be realized by setting the parameters $\Gamma = -0.5337$, $\sigma = 0.10674$, $\gamma = 0.001$, $\omega_f = 100$.

The following two cases are presented to demonstrate the effectiveness of the EID-based adaptive SMC approach developed in this paper. Case I shows the advantage of the obtained method over the existing results given by [28]. Case II shows the wider applicability of the proposed controller design method.

1) CASE I: SINUSOIDAL DISTURBANCE

In this case, the *dominating slow dynamics approach* presented in [28] is borrowed to compare with the proposed method.

The sinusoidal disturbances $f(t)$ is

$$f(t) = 2\sin(3t)$$

The Euclidean norms of slow-time and fast-time state vectors are shown in Fig. 3 and Fig. 4, which implies that the closed-loop system under the proposed EID-based adaptive SMC possesses better performance than the results in [28], such as less overshoots and superior steady-state dynamics.

2) CASE II: APERIODIC DISTURBANCE

Without any modification of the proposed EID-based adaptive SMC, we consider the following external disturbance

$$f(t) = \frac{5}{1+t^2}$$

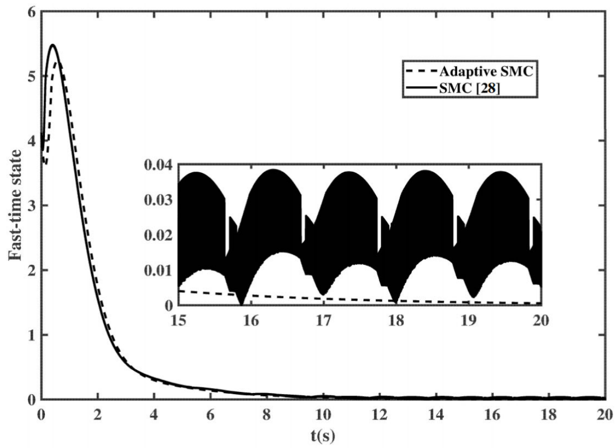

FIGURE 4. The norm of fast-time state variable.

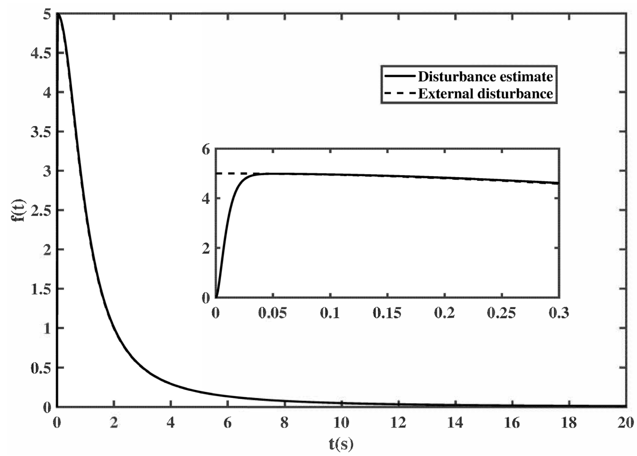

FIGURE 5. The estimate of the external disturbance.

The disturbance estimate is shown in Figs. 5, which indicates that the presented EID can estimate the external disturbance well. Finally, the evolutions of state vectors are presented in Fig. 6 and Fig. 7 respectively. It is easy to see that the

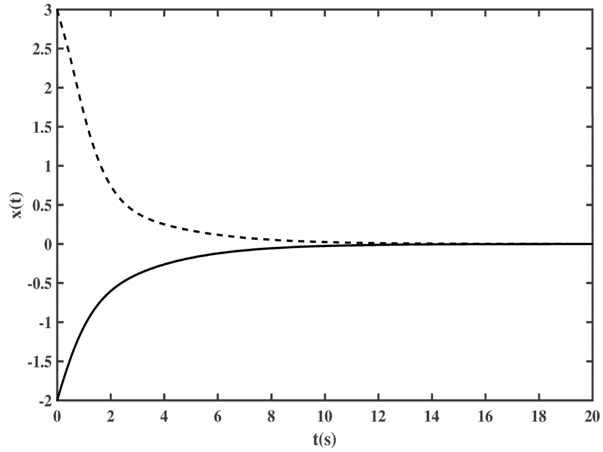

FIGURE 6. The evolutions of slow-time state vector.

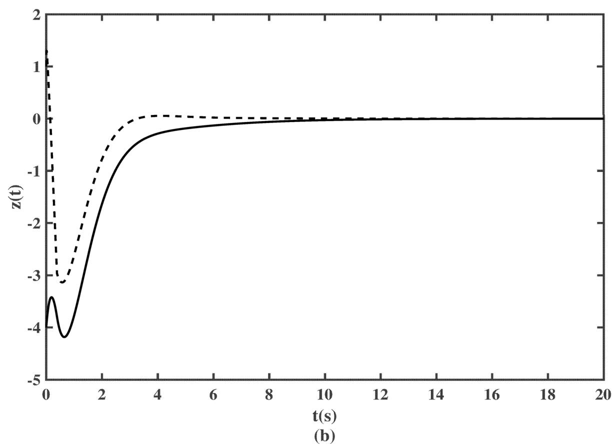

FIGURE 7. The variables of fast-time state vector.

closed-loop system is asymptotically stable and satisfactory system performance is guaranteed under the proposed control law.

B. EXAMPLE 2

In this subsection, we consider the following dynamic system

$$\begin{cases}\dot{q}(t) = -2q(t) + z(t) - 0.1u(t) \\ \dot{z}(t) = -20q(t) + 2z(t) + u(t) + 10f_L(t)\end{cases} \tag{29}$$

where $q(0) = 0$, $z(0) = 0$, and $f_L(t) = 0.1sin(20t)$. Then we can obtain the following expression from (29)

$$\ddot{q}(t) = -16q(t) + U(t) + 10f(t)$$

where $U(t) = 1.2u(t) - 0.1\dot{u}(t)$.

In the following, we will present the fast terminal sliding mode control (FTSMC) approach proposed in [13]. For convenience, we add subscripts "c" to the designed parameters and variables in the design procedure of the FTSMC. In order to construct the disturbance observer in [13], we need to introduce the auxiliary variable

$$\delta_c(t) = z_c(t) - \dot{q}_c(t)$$

with $z_c(t)$ expressed as

$$\begin{cases}\dot{z}_c(t) = \hat{D}(t) - 16q_c(t) + U_c(t) \\ \hat{D}(t) = -\varepsilon_{0c}\delta_c(t) - \varepsilon_{1c}sign\left[\delta_c(t)\right]\end{cases}$$

where $\hat{D}(t)$ is the lumped disturbance estimation.

For the desired position $q_d(t)$, the tracking error vector is defined as

$$e_c(t) = q_c(t) - q_d(t)$$

Then the surface variable and non-singular FTSMC manifold are designed as

$$\begin{cases}s_c(t) = e_c(t) - e_c(0)e^{-\varphi_c t} \\ \sigma_c(t) = s_c + \mu_{1c}s_c^{\gamma_c} + \mu_{2c}\dot{s}_c^{\eta_c}\end{cases}$$

where $s_c = s_c(t)$. Thus, the FSTMC law is

$$\begin{aligned}U_c(t) = &-\frac{1}{\eta_c\mu_{2c}}\dot{s}_c^{2-\eta_c} - \frac{\mu_{1c}\gamma_c}{\eta_c\mu_{2c}}s_c^{\gamma_c-1}\dot{s}_c^{2-\eta_c} + 16q_c(t) \\ &+\ddot{q}_d(t) - \hat{D}(t) + \varphi_c^2 e_c(0)e^{-\varphi_c t} + \dot{\delta}_c(t) \\ &-\frac{1}{\eta_c\mu_{2c}}\dot{s}_c^{1-\eta_c}\left\{\varepsilon_{0c}\sigma_c(t) - \alpha_c sign\left[\sigma_c(t)\right]\right\}\end{aligned}$$

and the parameters are selected as

$$\begin{aligned}\varepsilon_{0c} &= 10, \quad \varepsilon_{1c} = 1.5,\ \varphi_c = 2, \mu_{1c} = 0.1 \\ \mu_{2c} &= 1, \quad \eta_c = 1,\ \gamma_c = 1.5, \alpha_c = 1.5\end{aligned}$$

For the system (29), if we define the state variable as

$$x(t) = q(t) - q_d(t)$$

then the system (29) can be rewritten as

$$\begin{cases}\dot{x}(t) = -2x(t) + z(t) - 0.1u(t) - 2q_d(t) - \dot{q}_d(t) \\ \dot{z}(t) = -20x(t) + 2z(t) + u(t) + 10f_L(t) - 20q_d(t)\end{cases}$$

By setting the singular perturbation parameter $\varepsilon = 1/20$, the above system can be modeled as the SPS (1) with the following system matrices and parameters

$$A = \begin{bmatrix}-2 & 1 \\ -1 & 1/10\end{bmatrix}, \quad B = \begin{bmatrix}-1/10 & 0 \\ 0 & 1/20\end{bmatrix}$$
$$C = -I_2, \quad D = \begin{bmatrix}-1 & 0 \\ 0 & 1/2\end{bmatrix}$$
$$f(t) = \begin{bmatrix}f_1(t) \\ f_2(t)\end{bmatrix} = \begin{bmatrix}2q_d(t) + \dot{q}_d(t) \\ f_L(t) - 2q_d(t)\end{bmatrix}$$

Then we can calculate that $eig(A_0 + B_0K_0) = -1$ and $eig(A_{22} + B_2K_2) = -9.9$ by selecting feedback gains as $K_0 = \left[\,15\ 15\,\right]^T, K_2 = -\left[\,200\ 200\,\right]^T$, respectively. The evolutions of recursive algorithm for solving H and L are shown in Fig. 8, which illustrates the evolutions are convergent. The corresponding calculation results are presented as follows

$$\begin{aligned}L &= -2.51, \quad R(L) = -2.1263 \times 10^{-4} \\ H &= -1.6805, \quad R(H) = -1.7212 \times 10^{-7}\end{aligned}$$

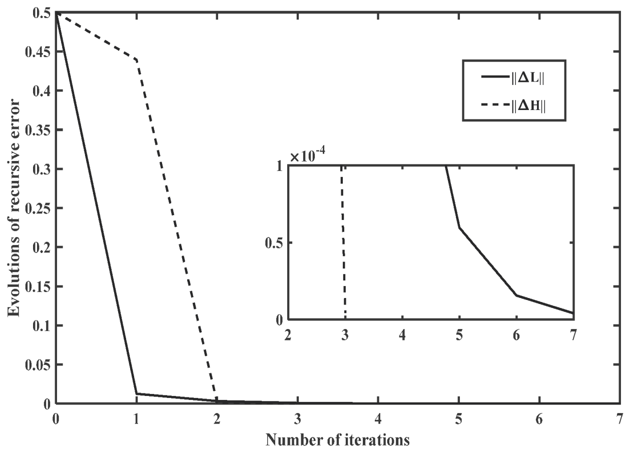

FIGURE 8. The evolutions of recursive algorithm for solving H and L.

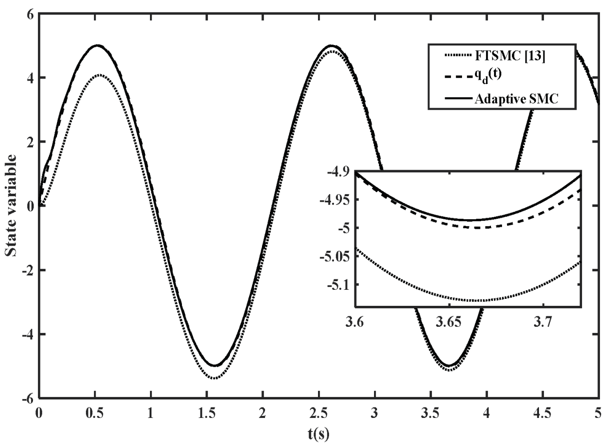

FIGURE 9. The evolutions of state variable.

By setting the expected eigenvalue $eig(A - GC) = \{-19.8314, -14.0686\}$ and $Q = 10I_2$, we can obtain matrices G and P as

$$G = \begin{bmatrix} 18 & 0 \\ 0 & 14 \end{bmatrix}, \quad P = \begin{bmatrix} 6.3312 & 0 \\ 0 & 0.3989 \end{bmatrix}$$

According to Remark 7, set the other parameters of the EID-based adaptive SMC as $\Gamma = -1.5, \sigma = 0.2, \gamma = 0.001, \omega_f = 100$.

When $q_d(t) = 5sin(3t)$, the evolutions of the proposed method and the FSTMC approach are shown in Fig. 9, which indicates that the controlled system under the proposed EID-based adaptive SMC has better tracking performance and stronger robustness against external disturbance than the results in [13]. In addition, the presented approach in this paper is free of the disturbance norm-bound information.

V. CONCLUSION

This paper desinged an EID-based adaptive SMC for SPSs. The ε-dependent state observer and the Lyapunov-based methods have been presented to construct the EID estimator and design the composite sliding surface respectively. The obtained adaptive SMC law can ensure that the closed-loop SPSs are asymptotically stable and the effects of disturbances can be effectively compensated without knowing any priori disturbance information.

VI. FUTURE RECOMMENDATION

The robust controller design for nonlinear systems with uncertainties will be more complex. It is worth trying to extend the proposed approach to suppress the adverse effects of the disturbances for nonlinear SPSs in the future.

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