

Received December 28, 2020, accepted January 11, 2021, date of publication January 18, 2021, date of current version January 25, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3051954

# Integrated Design of Fault Estimation and Fault-Tolerant Control for Closed-Loop Systems With Uncertain Short Delay

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This work was supported in part by the Postgraduate Research and Practice Innovation Program of Jiangsu Province under Grant KYCX19\_2061, and in part by the Nantong Basic Research Project under Grant JC2018119.

**ABSTRACT** In this paper, an integrated design scheme of fault estimation (FE) and fault-tolerant control (FTC) is presented for a closed-loop system with uncertain short delay. Firstly, a closed-loop system with uncertain short delay is modeled as a system with uncertain parameters, and a bi-directional robustness interaction of uncertainty between the observer and the control system is analyzed. Then a fault estimation observer and a fault-tolerant controller are constructed, and the integrated design problem is converted into a robust control problem for the augmented system under  $H_\infty$  performance index. Furthermore, the parameters of the fault estimation observer and fault-tolerant controller are solved by Lyapunov function and relaxation methods. Finally, the effectiveness and superiority of the proposed method are verified by an aircraft simulation.

**INDEX TERMS** Uncertain short delay, integrated design, FE, FTC.

## I. INTRODUCTION

Modern industrial systems are becoming more complex and larger, and the probability of their failures is also increasing. Even minor failures, if not detected and effectively addressed in time, will propagate and evolve into catastrophic accidents. Therefore, it is particularly important to study how to effectively conduct fault detection and isolation (FDI), fault estimation (FE) as well as fault-tolerant control (FTC) [1]–[4].

Modern industrial systems are usually characterized by model uncertainty, system nonlinearity, strong coupling of multiple variables, etc. In order to realize high precision control, closed-loop control becomes an inevitable choice [5]–[7]. At present, most fault diagnosis studies are carried out for open-loop system without considering the influence of feedback control [8], as the reasons follow: 1) Fault diagnosis of open-loop system is relatively simple. Because there is no feedback control in open-loop system, and fault diagnosis performance cannot be suppressed. Once a fault occurs, the output of open-loop system will deviate from the

expected values evidently; 2) Under certain circumstances, fault diagnosis of open-loop system and closed-loop system are equivalent, which makes people mistakenly believe that the fault diagnosis method of open-loop system can be directly applied to closed-loop system for a long time. In fact, the introduction of the feedback control usually makes the system more robust to external disturbances, so when the fault is in the early stage or the amplitude is small, the impact may be masked by control signals. Furthermore, feedback may lead to fault propagation in the system, as well as abnormal signals. Therefore, the fault diagnosis methods for open-loop system may not be applicable and need to be redesigned in closed-loop system [9]. Unfortunately, fault diagnosis studies in closed-loop system are still limited. In [8], a simulation of a three-capacity water tank system with model uncertainty was studied, which proved that open-loop fault diagnosis method degrades or even disabled the system performance. In [10], the impact of feedback control on faulted induction machine behavior was presented, and the diagnostic indexes usually used for open-loop operation were no longer effective. In [11], considering the additive and multiplicative faults in the system, the influence of closed-loop control on

The associate editor coordinating the review of this manuscript and approving it for publication was Youqing Wang<sup>1</sup>.

fault diagnosis was analyzed by simulation experiments in open-loop system and closed-loop system respectively. The residual signals of closed-loop system and open-loop system are almost equal when the additive faults occur, but for the multiplicative faults, the residual signals of both systems are proportional to each other. In [12], it was shown that feedback control can make fault isolation more difficult.

Furthermore, owing to the advantages of lower cost, easy maintenance and simplicity in installation, the networked control system (NCS) are widely applied in many fields, such as unmanned aerial vehicle (UAV) [13], [14]. The communication between the components of UAV is realized through the network, which may lead to time delay caused by the limitation of network bandwidth and channel capacity. Furthermore, when the UAV system is disturbed by external faults or changed by the working state, the signal measured by sensors cannot be fed back to the controller and actuator timely, so the fault detection and control system cannot respond accurately and timely. All these lead to the degradation of system performance and even failure [15], [16]. It is usually assumed that the delay is fixed or random under known probability distribution characteristics in the existing analysis and comprehensive design of delay systems [17]–[20]. For uncertain delay with unknown statistical characteristics, researches on fault diagnosis of closed-loop systems are insufficient. In [21], the performance of time-delay networked control system with random faults was analyzed, but only the feedback control method based on system static output was given. In [22], network delay was defined as a bounded random number, on this basis, a robust FTC for actuator fault in discrete uncertain networked control systems was designed.

FE and FTC are usually designed separately. However, due to the uncertainty caused by time delay, FE and FTC influence each other, which results in an integrated design of FE/FTC. The main contributions of the paper are

- 1) The influence of uncertainty caused by time delay on fault diagnosis performance is analyzed, considering its bi-directional robustness interaction between the observer and the control system, the integrated design of FE/FTC is proposed.
- 2) The integrated design problem is formulated as solving a robust control problem based on observer under an  $H_\infty$  performance index, the Lyapunov functions and relaxation methods are used to solve the design parameters simultaneously.

The paper is organized as follows. In Section II, the mathematical model of a closed-loop system with uncertain short delay is established, and the influence of uncertainty caused by time delay on fault diagnosis performance is analyzed. Then the integrated design of FE and FTC is carried out in Section III. Finally, the effectiveness and superiority of the proposed method are verified by simulation of the morphing aircraft under the longitudinal short period motion in Section IV, followed by some concluding remarks in Section V.

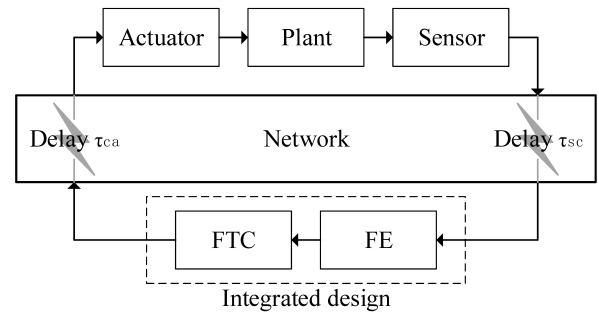


FIGURE 1. Closed-loop systems with uncertain short delay.

## II. SYSTEM DESCRIPTION AND PRELIMINARIES

### A. SYSTEM DESCRIPTION

In this section, a closed-loop control system with time delay is discussed as shown in Fig. 1, where  $\tau_{sc}$  is the time delay between sensors and controllers,  $\tau_{ca}$  is the time delay between controllers and actuators. The state equations of the control system are

$$\begin{cases} \dot{x}(t) = A_c x(t) + B_c u(t) + D_{cf} f(t) + D_{cd} d(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where  $x \in \mathbb{R}^n$  denotes the state vector,  $u \in \mathbb{R}^m$  denotes the input vector,  $y \in \mathbb{R}^l$  denotes the measured output vector,  $f \in \mathbb{R}^q$  represents the fault vector (the fault may occur in the plant, actuators, and sensors), and  $d \in \mathbb{R}^p$  represents the external disturbance. The matrices  $A_c$ ,  $B_c$ ,  $C$ ,  $D_{cf}$  and  $D_{cd}$  have appropriate dimensions.

*Assumption 1:* The sensor is time-driven, and the controller and actuator are event-driven.

*Assumption 2:* The total time delay  $\tau_k = \tau_{sc} + \tau_{ca}$  of the control system is a short time delay, which occurs between the controller and the actuator, and  $\tau_k$  is time-varying and bounded.

The control system (1) is dispersed at the sampling period  $h$  [23], and the equations are

$$\begin{cases} x(k+1) = Ax(k) + B_0(\tau(k))u(k) + B_1(\tau(k))u(k-1) \\ \quad + D_f f(k) + D_d d(k) \\ y(k) = Cx(k) \end{cases} \quad (2)$$

where  $A = e^{A_c h}$ ,  $B_0(\tau(k)) = \int_0^{h-\tau(k)} e^{A_c t} dt \cdot B_c$ ,  $B_1(\tau(k)) = \int_{h-\tau(k)}^h e^{A_c t} dt \cdot B_c$ ,  $D_f = \int_0^h e^{A_c t} dt \cdot D_{cf}$  and  $D_d = \int_0^h e^{A_c t} dt \cdot D_{cd}$ . Since the time delay  $\tau(k)$  is time-varying,  $B_0(\tau(k))$  and  $B_1(\tau(k))$  are parameter uncertainty matrices. Therefore, the generalized controlled plant of the closed-loop control system is constructed as a discrete linear system with uncertain time-varying parameters. Let

$$\begin{aligned} B_0(\tau(k)) &= \int_0^{h-\tau(k)} e^{A_c t} dt \cdot B_c \\ &= \int_0^{h-\bar{\tau}(k)} e^{A_c t} dt \cdot B_c + \int_{h-\bar{\tau}(k)}^{h-\tau(k)} e^{A_c t} dt \cdot B_c \\ &= B_0 + DF(\tau(k))E \end{aligned}$$

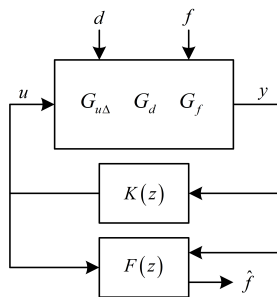


FIGURE 2. Diagram for fault estimation of an uncertain system.

Similarly,

$$B_1(\tau(k)) = B_1 - DF(\tau(k))E$$

where  $\bar{\tau}(k) = (\tau_{\min} + \tau_{\max})/2$ ,  $B_0 = \int_0^{h-\bar{\tau}(k)} e^{A_c t} dt \cdot B_c$ ,  $B_1 = \int_{h-\bar{\tau}(k)}^h e^{A_c t} dt \cdot B_c$ ,  $D = I$ ,  $F(\tau(k)) = \int_{h-\bar{\tau}(k)}^h e^{A_c t} dt$  and  $E = B_c$ .

System model (2) can be rewritten as the following system

$$\begin{cases} x(k+1) = Ax(k) + (B_0 + \Delta\tau(k))u(k) \\ \quad + (B_1 - \Delta\tau(k))u(k-1) + D_d d(k) + D_f f(k) \\ y(k) = Cx(k) \end{cases} \quad (3)$$

where  $\Delta\tau(k) = DF(\tau(k))E$ .

Lemma 1 [24]: Consider the uncertain item  $F(\tau(k))$ , it satisfies

$$\sigma_{\max} F(\tau(k)) \leq \delta, \forall k$$

where

$$\delta = \frac{e^{\sigma_{\max}(A)(h-\tau_{\min}(k))} - e^{\sigma_{\max}(A)(h-\bar{\tau})}}{\sigma_{\max}(A)}$$

Lemma 2 [25]: Given matrices  $G$ ,  $D$ , and  $E$  of compatible dimensions with  $G$  symmetric, then

$$G + DF(\tau(k))E + D^T F(\tau(k))^T E^T < 0$$

holds for all  $F(\tau(k))$  satisfying  $\sigma_{\max} F(\tau(k)) \leq \delta$  if and only if there exists a constant  $\lambda > 0$  such that

$$G + \delta^2 \lambda D D^T + \frac{1}{\lambda} E E^T < 0$$

### B. ANALYSIS OF FAULT DIAGNOSIS PERFORMANCE OF UNCERTAIN SYSTEMS

In this section, the influence of uncertainty caused by time delay on fault diagnosis performance is analyzed. In this connection, system (3) is written as the following transfer function

$$y(z) = G_{u\Delta}(z)u(z) + G_f(z)f(z) + G_d(z)d(z) \quad (4)$$

where  $G_{u\Delta}(z) = Cz^{-1}(zI - A)^{-1}(zB_0(\Delta) + B_1(\Delta))$ ,  $G_f(z) = C(zI - A)^{-1}D_f$ ,  $G_d(z) = C(zI - A)^{-1}D_d$ .

For the uncertain system (4), its fault estimation structure is shown in Fig. 2. where  $F(z)$  and  $K(z)$  denote the fault

estimator and the feedback controller respectively,  $u(z) = K(z)y(z)$ , and substituting this into (4) yields

$$y = (I - G_{u\Delta}K)^{-1}G_d d + (I - G_{u\Delta}K)^{-1}G_f f$$

The fault estimation error is then given by

$$\begin{aligned} e_{close}(z) &= f - \hat{f} = f - F(z)(y - G_u u) \\ &= f - F(z)(I - G_u K)y \\ &= \left[ I - F(z)(I - G_u K)(I - G_{u\Delta}K)^{-1}G_f \right] f \\ &\quad - F(z)(I - G_u K)(I - G_{u\Delta}K)^{-1}G_d d \end{aligned}$$

where  $G_u(z) = Cz^{-1}(zI - A)^{-1}(zB_0 + B_1)$ . Obviously, the system uncertainty  $\Delta$ , disturbance  $d$ , and fault  $f$  affect the fault estimation performance.

According to the above analysis, there exists a bi-directional robustness interaction between the observer and the control system in (3), which breaks down the Separation Principle. Thus it is necessary and important to develop the integrated design of FE/FTC.

### III. INTEGRATED DESIGN OF FE AND FTC

FE and FTC unit are usually designed separately in the active fault tolerant control systems [26]. Considering the bi-directional robustness interaction caused by short delay on fault estimation observer and fault tolerant controller, the integrated fault estimation observer is designed as follows

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + B_0 u(k) + B_1 u(k-1) + D_f \hat{f}(k) \\ \quad + L(y(k) - \hat{y}(k)) \\ \hat{y} = C\hat{x}(k) \\ \hat{f}(k+1) = \hat{f}(k) + M(y(k) - \hat{y}(k)) \end{cases} \quad (5)$$

where  $\hat{x}(k) \in \mathbb{R}^n$  denotes the estimated state vector,  $\hat{f}(k) \in \mathbb{R}^q$  denotes the estimated fault vector,  $L \in \mathbb{R}^{n \times m}$  and  $M \in \mathbb{R}^{q \times m}$  denote the matrices to be designed.

Lemma 3 [27]: Assuming that  $\text{rank}(B_0, F) = \text{rank}(B_0)$ , there exists a matrix  $B_0^\dagger \in \mathbb{R}^{m \times n}$  satisfying the following equation

$$(I - B_0 B_0^\dagger)F = 0$$

A fault compensation controller based on output feedback is designed as follows

$$u(k) = K_1 y(k) + K_2 u(k-1) + K_3 \hat{f}(k) \quad (6)$$

where  $K_1 \in \mathbb{R}^{q \times n}$  is the matrix to be designed. According to Lemma 3, the matrix  $K_2$  and  $K_3$  are given by

$$K_2 = -B_0^\dagger B_1, K_3 = -B_0^\dagger D_f$$

By substituting (6) into (3) and (5), integrated design of FE and FTC can be described as

$$\begin{cases} x(k+1) = Ax(k) + B_0(K_1y(k) + K_2u(k-1) + K_3f(k)) \\ \quad + B_1u(k-1) + \Delta\tau(k)(u(k) - u(k-1)) \\ \quad + D_d d(k) + D_f f(k) \\ \hat{x}(k+1) = A\hat{x}(k) + B_0(K_1y(k) + K_2u(k-1) + K_3f(k)) \\ \quad + B_1u(k-1) + D_f(k)\hat{f}(k) + L(y(k) - \hat{y}(k)) \\ \hat{y}(k) = \hat{C}\hat{x}(k) \\ \hat{f}(k+1) = \hat{f}(k) + M(y(k) - \hat{y}(k)) \end{cases} \quad (7)$$

Define the state estimation error vector  $e_x(k) = x(k) - \hat{x}(k)$  and the fault estimation error vector  $e_f(k) = f(k) - \hat{f}(k)$ , then (7) can be rewritten as

$$\begin{cases} x(k+1) = (A + B_0K_1C)x(k) + D_f e_f(k) \\ \quad + \Delta\tau(k)(u(k) - u(k-1)) + D_d d(k) \\ e_x(k+1) = (A - LC)e_x(k) + \Delta\tau(k)(u(k) - u(k-1)) \\ \quad + D_f e_f(k) + D_d d(k) \\ e_f(k+1) = e_f(k) - M C e_x(k) + \Delta f(k) \\ y = Cx(k) \end{cases} \quad (8)$$

where  $\Delta f(k) = f(k+1) - f(k)$ .

Define  $d_\tau(k) = \Delta\tau(k)\Delta u(k)$ , where  $\Delta u(k) = u(k) - u(k-1)$ . Now, let the augmented state vector

$$\zeta(k) = \begin{bmatrix} x^T(k) & e_x^T(k) & e_f^T(k) \end{bmatrix}^T$$

and the augmented generalized disturbance vector

$$w(k) = \begin{bmatrix} d(k) & d_\tau(k) & \Delta f(k) \end{bmatrix}^T$$

Then, from (8), the following augmented system is obtained

$$\begin{cases} \zeta(k+1) = \bar{A}\zeta(k) + \bar{B}w(k) \\ e_f(k) = \bar{C}_f \zeta(k) \\ y(k) = \bar{C}\zeta(k) \end{cases} \quad (9)$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A + B_0K_1C & 0 & D_f \\ 0 & A - LC & D_f \\ 0 & -MC & I_q \end{bmatrix} \\ \bar{B} &= \begin{bmatrix} D_d & I_m & 0 \\ D_d & I_m & 0 \\ 0 & 0 & I_q \end{bmatrix} \\ \bar{C}_f &= \begin{bmatrix} 0 & 0 & I_q \end{bmatrix} \\ \bar{C} &= \begin{bmatrix} C & 0 & 0 \end{bmatrix} \end{aligned}$$

*Remark:*  $\Delta u(k) = u(k) - u(k-1)$  is considered as a part of the external interference in (9), for the variation of control input is generally bounded at adjoining times and the boundedness of the input signal is ensured by the saturation characteristics of actuators. Thus the assumption is reasonable in actual situation.

Now, the main work of the paper is an integrated design for FE in (5) and FTC in (6) to make the augmented system (9)

stable, as well as minimize the influence of disturbance  $w(k)$  on fault estimation error  $e_f(k)$  and output  $y(k)$ . It is that there exist constants  $\gamma_z > 0$  and  $\gamma_e > 0$ , the matrices  $L, M$  and  $K_1$  should be designed to satisfy the following conditions

- 1) The augmented system in (9) is stable.
- 2)  $y(k)$  is robust to  $w(k)$ , and the following  $H_\infty$  performance index is satisfied.

$$y^T(k)y(k) \leq \gamma_z^2 w^T(k)w(k) \quad (10)$$

- 3) And  $e_f(k)$  is robust to  $w(k)$  and the following  $H_\infty$  performance index is satisfied.

$$e_f^T(k)e_f(k) \leq \gamma_e^2 w^T(k)w(k) \quad (11)$$

*Lemma 4* [28]: The following conditions are equivalent

- 1) There exists a positive definite symmetric matrix  $P > 0$  such that

$$A^T P A - P < 0$$

- 2) There exists a positive definite symmetric matrix  $P > 0$  and a matrix  $G$  such that

$$\begin{bmatrix} -P & A^T G \\ G^T A & -G - G^T + P \end{bmatrix} < 0$$

*Theorem 1:* Given constants  $\gamma_z > 0$ ,  $\gamma_e > 0$  and  $\eta > 1$ , if there exist positive definite symmetric matrices  $P_t, Q_t, G_t$  ( $t = 1, 2, 3$ ) and matrices  $\hat{G}_1, X, Y, Z$ , the following LMIs hold

$$\begin{aligned} \begin{bmatrix} -\eta I_n & G_1^T B_0 - B_0 \hat{G}_1 \\ * & -\eta I_r \end{bmatrix} < 0 \\ \begin{bmatrix} \Xi_e & \Gamma \\ * & \Psi_e \end{bmatrix} < 0 \\ \begin{bmatrix} \Xi_z & \Gamma \\ * & \Psi_z \end{bmatrix} < 0 \end{aligned}$$

where

$$\begin{aligned} \Xi_e &= \begin{bmatrix} P_1 - He(G_1) & 0 & 0 \\ 0 & P_2 - He(G_2) & 0 \\ 0 & 0 & P_3 - He(G_3) \end{bmatrix} \\ \Psi_e &= \begin{bmatrix} P_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_3 & 0 & 0 & 0 & I_q \\ 0 & 0 & 0 & -\gamma_e^2 I_p & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\gamma_e^2 I_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma_e^2 I_q & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -I_q \end{bmatrix} \\ \Psi_z &= \begin{bmatrix} Q_1 & 0 & 0 & 0 & 0 & 0 & C^T \\ 0 & Q_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma_z^2 I_p & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\gamma_z^2 I_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma_z^2 I_q & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -I_q \end{bmatrix} \\ \Xi_z &= \begin{bmatrix} Q_1 - He(G_1) & 0 & 0 \\ 0 & Q_2 - He(G_2) & 0 \\ 0 & 0 & Q_3 - He(G_3) \end{bmatrix} \end{aligned}$$

Then, the augmented system in (9) satisfies the  $H_\infty$  performance indices in (10) and (11). Further, the gains  $L$ ,  $M$  and  $K_1$  are given

$$\begin{cases} K_1 = \hat{G}_1^{-1} X \\ L = (G_2^T)^{-1} Y \\ M = (G_3^T)^{-1} Z \end{cases} \quad (12)$$

*Proof:* First, the fault estimation performance of the system is considered. Define the Lyapunov function

$$V_e(\zeta(k)) = \zeta^T(k)P(k)\zeta(k)$$

where  $P(k)$  is a positive definite symmetric matrix. According to (9), the following expression is obtained

$$\begin{aligned} &V_e(\zeta(k+1)) - V_e(\zeta(k) + e_f^T(k)e_f(k) - \gamma_e^2 w^T(k)w(k)) \\ &= \zeta^T(k+1)P\zeta(k+1) - \zeta^T(k)P\zeta(k) + \zeta^T(k)\bar{C}_f^T \bar{C}_f \zeta(k) \\ &\quad - \gamma_e^2 w^T(k)w(k) \\ &= (\bar{A}\zeta(k) + \bar{B}w(k))^T P(\bar{A}\zeta(k) + \bar{B}w(k)) \\ &\quad - \zeta^T(k)P\zeta(k) + \zeta^T(k)\bar{C}_f^T \bar{C}_f \zeta(k) - \gamma_e^2 w^T(k)w(k) \\ &= \zeta^T(k) (\bar{A}^T P \bar{A} - P + \bar{C}_f^T \bar{C}_f) \zeta(k) + \zeta^T(k) \bar{A}^T P \bar{B} w(k) \\ &\quad + w^T(k) \bar{B}^T P \bar{A} \zeta(k) + w^T(k) (-\gamma_e^2 + \bar{B}^T P \bar{B}) w(k) \\ &= \begin{bmatrix} \zeta(k) \\ w(k) \end{bmatrix}^T \Omega_1 \begin{bmatrix} \zeta(k) \\ w(k) \end{bmatrix} \end{aligned}$$

where

$$\Omega_1 = \begin{bmatrix} \bar{A}^T P \bar{A} - P + \bar{C}_f^T \bar{C}_f & \bar{A}^T P \bar{B} \\ \bar{B}^T P \bar{A} & -\gamma_e^2 + \bar{B}^T P \bar{B} \end{bmatrix}$$

According to Schur complement lemma, then the following inequation can be obtained

$$V_e(s(k+1)) - V_e(s(k) + e_f^T(k)e_f(k) - \gamma_e^2 w^T(k)w(k)) < 0$$

According to Lemma 4 and Schur complement lemma,  $\Omega_1$  can be converted to

$$\begin{bmatrix} -P & P\bar{A} & P\bar{B} & 0 \\ \bar{A}^T P & -P & 0 & \bar{C}_f^T \\ \bar{B}^T P & 0 & -\gamma_e^2 I & 0 \\ 0 & \bar{C}_f & 0 & -I \end{bmatrix} < 0 \quad (13)$$

Now, we can conclude that the augmented system in (9) is stable and the fault estimation error  $e_f(k)$  is robust to disturbance  $w(k)$ .

Next, the fault-tolerant control performance of the system is proved. Define the Lyapunov function

$$V_z(s(k)) = \zeta^T(k)Q(k)\zeta(k)$$

Similar to the above deduction, the following inequation can be obtained

$$\begin{aligned} &V_z(\zeta(k+1)) - V_z(\zeta(k) + y^T(k)y(k) - \gamma_z^2 w^T(k)w(k)) \\ &= \begin{bmatrix} \zeta(k) \\ w(k) \end{bmatrix}^T \Omega_2 \begin{bmatrix} \zeta(k) \\ w(k) \end{bmatrix} < 0 \end{aligned}$$

where

$$\Omega_2 = \begin{bmatrix} \bar{A}^T Q \bar{A} - Q + \bar{C}^T \bar{C} & \bar{A}^T Q \bar{B} \\ \bar{B}^T Q \bar{A} & -\gamma_z^2 + \bar{B}^T P \bar{B} \end{bmatrix}$$

According to Lemma 4 and Schur complement lemma,  $\Omega_2$  can be converted to

$$\begin{bmatrix} -Q & Q\bar{A} & Q\bar{B} & 0 \\ \bar{A}^T Q & -Q & 0 & \bar{C}^T \\ \bar{B}^T Q & 0 & -\gamma_z^2 I & 0 \\ 0 & \bar{C} & 0 & -I \end{bmatrix} < 0 \quad (14)$$

Suppose that  $\Lambda = \text{diag}\{G^T P^{-1}, I, I, I\}$  and (13) is multiplied by  $\Lambda$  on the left and on the right, and the following equation can be obtained

$$\begin{bmatrix} -G^T P^{-1} G & G^T \bar{A} & G^T \bar{B} & 0 \\ \bar{A}^T G & -P & 0 & \bar{C}_f^T \\ \bar{B}^T G & 0 & -\gamma_e^2 I & 0 \\ 0 & \bar{C}_f & 0 & -I \end{bmatrix} < 0 \quad (15)$$

Since the matrices  $P$  and  $G$  are positive definite symmetric matrices, it can be obtained that

$$(P - G)^T P^{-1} (P - G) \geq 0$$

which can be expanded to

$$G^T P^{-1} G \geq -P + G + G^T$$

Define  $He(X) = X^T + X$  and (15) can be converted to

$$\begin{bmatrix} P - He(G) & G^T \bar{A} & G^T \bar{B} & 0 \\ \bar{A}^T G & -P & 0 & \bar{C}_f^T \\ \bar{B}^T G & 0 & -\gamma_e^2 I & 0 \\ 0 & \bar{C}_f & 0 & -I \end{bmatrix} < 0 \quad (16)$$

Similarly, inequation (14) can be converted to

$$\begin{bmatrix} Q - He(G) & G^T \bar{A} & G^T \bar{B} & 0 \\ \bar{A}^T G & -Q & 0 & \bar{C}^T \\ \bar{B}^T G & 0 & -\gamma_z^2 I & 0 \\ 0 & \bar{C} & 0 & -I \end{bmatrix} < 0 \quad (17)$$

Define

$$G = \begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} G_1^T A + B_0 X C & 0 & G_1^T D_f & G_1^T D_d & G_1^T & 0 & 0 \\ 0 & G_2^T A - Y C & G_2^T D_f & G_2^T D_d & G_2^T & 0 & 0 \\ 0 & -Z C & G_3^T & 0 & 0 & G_3^T & 0 \end{bmatrix}$$

According to Schur complement lemma, inequation (16) and (17) can be converted to

$$\begin{bmatrix} \Xi_e & \Phi \\ * & \Psi_e \end{bmatrix} < 0$$

$$\begin{bmatrix} \Xi_z & \Phi \\ * & \Psi_z \end{bmatrix} < 0$$

where

$$\Phi = \begin{bmatrix} G_1^T A + G_1^T B_0 K_1 C & 0 & 0 \\ 0 & G_2^T A - G_2^T L C & -G_3^T M C \\ G_1^T D_f & G_2^T D_f & G_3^T \\ G_1^T D_d & G_2^T D_d & 0 \\ G_1^T & G_2^T & 0 \\ 0 & 0 & G_3^T \\ 0 & 0 & 0 \end{bmatrix}^T$$

According to the approaches in [29] and LMIs in Theorem 1, it can be obtained that

$$G_1^T B_0 \approx B_0 \hat{G}_1$$

Further, define  $X = \hat{G}_1 K_1$ ,  $Y = G_2^T L$ ,  $Z = G_3^T M$ , then the matrix  $\Phi$  is equivalent to the matrix  $\Gamma$ . Now, the proof of Theorem 1 is complete.

#### IV. SIMULATION EXAMPLE

In this section, the longitudinal short-period movement of the morphing aircraft in [30] is utilized to demonstrate the applicability and effectiveness of the proposed results.

Considering its strong nonlinearity, the dynamic model of the system is linearized at the equilibrium point with a flight altitude of 6000 meters, a sweep angle of 15 degree and the Mach number of 0.5, then the small linear perturbation model is as follows

$$\dot{x}(t) = \begin{bmatrix} -Z_\alpha & 1 \\ \bar{M}_\alpha - \bar{M}_{\dot{\alpha}} Z_\alpha & \bar{M}_q + \bar{M}_{\dot{\alpha}} \end{bmatrix} x(t) + \begin{bmatrix} -Z_{\delta e} \\ \bar{M}_{\delta e} - \bar{M}_{\dot{\alpha}} Z_{\delta e} \end{bmatrix} u(t)$$

where  $x = [\Delta\alpha \ \Delta q]^T$  denotes state vector,  $\alpha$  denotes attack angle,  $q$  denotes velocity of pitch angle, the control input  $u$  denotes the yaw angle of the elevator,  $Z_\alpha$ ,  $\bar{M}_\alpha$ ,  $\bar{M}_{\dot{\alpha}}$ ,  $\bar{M}_q$ ,  $Z_{\delta e}$  and  $\bar{M}_{\delta e}$  denote dynamics derivatives.

The discretization of the system with a sampling period of 1 seconds results in the following system matrices

$$A = \begin{bmatrix} 0.2759 & 0.1839 \\ 0.1379 & 0.0921 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0.2256 \\ 0.4234 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.1359 \\ 0.6671 \end{bmatrix}$$

$$C = [1 \ 0]$$

According to Lemma 1, it can be obtained

$$\delta = 0.3202, \quad E = [0 \ 1]^T, \quad D = I$$

Assume that the fault occurred in the yaw angle of the elevator at  $k = 20$  sampling time shown as follows

$$f(k) = \begin{cases} 2^\circ & k \geq 20 \\ 0^\circ & \text{else} \end{cases}$$

Choose wind disturbance [31] as the system disturbance, which can be described as

$$\begin{cases} s(k+1) = \begin{bmatrix} 0.9922 & 0.1247 \\ -0.1247 & 0.9922 \end{bmatrix} s(k) \\ d(k) = [0.1 \ 0] s(k) \end{cases}$$

where the initial value of  $s(k)$  is defined as  $[0.01 \ 0]^T$ . The disturbance matrix and the fault matrix are respectively given as

$$D_f = \begin{bmatrix} 0.2123 \\ 0.2169 \end{bmatrix}, \quad D_d = \begin{bmatrix} 0 \\ 0.2169 \end{bmatrix}$$

According to Theorem 1, the parameters of the fault estimation observer and fault tolerant controller can be obtained as

$$L = \begin{bmatrix} 0.0124 \\ -0.0008 \end{bmatrix}, \quad M = 0.8486, \quad K = -11.2657. \quad (18)$$

The performance of fault estimation in integrated design and separate design are compared in Fig. 3. The results show that the integrated design method can estimate the error signal more quickly.

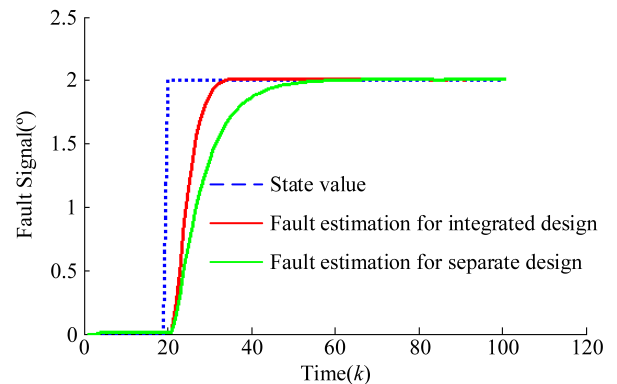
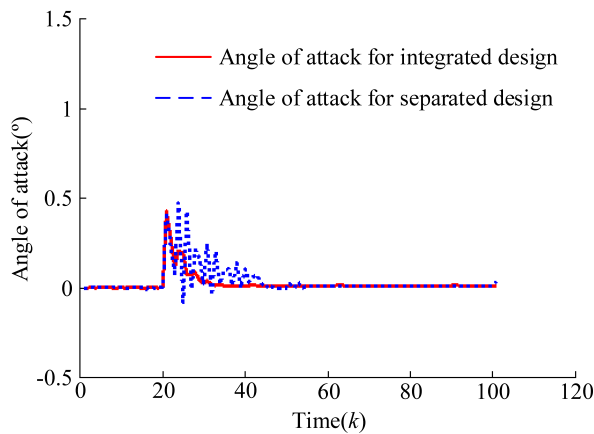
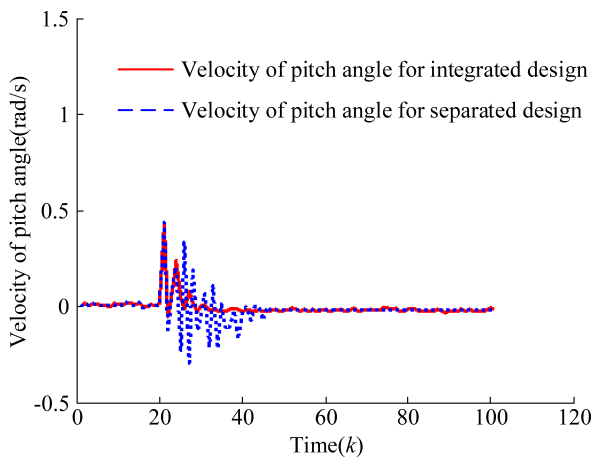


FIGURE 3. Performance of FE in integrated design and separate design.

The output response characteristics of the system under integrated design and separate design are respectively showed in Fig. 4 and Fig. 5, where Fig. 4 denotes the attack angle and Fig. 5 denotes the pitch angle rate. Obviously, the fault-tolerant controller obtained by the integrated design method enables the system to maintain better performance in case of failure. Furthermore, although the fault-tolerant controller designed according to the separation principle can also make the system stable when the fault occurs, the uncertainty of the



**FIGURE 4.** The response result of the Angle of attack when the fault occurs.



**FIGURE 5.** The response result of pitch Angle rate when the fault occurs.

system is magnified due to the propagation of the uncertainty between the observer and the controller, which causes the system take longer to restore stability.

## V. CONCLUSION

In this paper, integrated design of the optimal fault estimation and fault tolerant control for discrete linear systems with uncertain short delay are studied. Firstly, a closed-loop system with uncertain short delay is modeled as the parameter uncertainty system. Then the influence of uncertainty caused by time delay on fault diagnosis performance is analyzed, and there exists a bi-directional robustness interaction between the observer and the control system, the integrated design of FE/FTC is proposed. Furthermore, the controller is designed to compensate the delay signal and eliminate the bi-directional robustness interaction under  $H_\infty$  performance index, the relaxation method is used to solve the system parameters. Finally, a comparative simulation case between the proposed integrated design method and the separate design method is presented to prove the superiority of the integrated design method for fault diagnosis and fault tolerant

control. The extension of the proposed FE/FTC scheme to systems with uncertain long delay seems significant and challenging, which will be our main focus in the future work. In addition, packet loss usually exists in NCS and should be considered in the FE/FTC design, which is also a promising direction for future research.

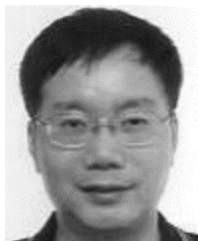
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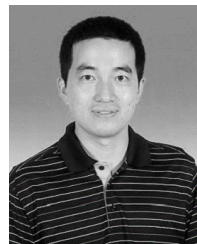
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