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Design Optimization of Water Distribution Networks through a Novel Differential Evolution

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ABSTRACT Optimization algorithms have proven to be a useful tool in different areas of water resource management including water distribution network (WDN), which may be modelled as a large scale, combinatorial problem subject to constraints with the typical objective of minimizing the cost. In this study, a fuzzy C-means adaptive differential evolution (FCADE) is proposed to solve three well-known WDN problems: the two-loop distribution network problem, the Hanoi distribution network problem, and New York tunnel distribution network problem. FCADE is integrated with a well-known simulation software EPANET to manage the pressure constraints requirement. A comparative analysis of results through various performance metrics indicate the competence of FCADE for solving complex WDN problems.

INDEX TERMS Differential evolution, optimization, water clustering, EPANET.

NOMENCLATURE

C_T	Cost of the Network
C_i	unit cost of the i^{th} pipe
L_i	length of the i^{th} pipe
Q_{ext}	External demand
Q_{in}	Inflows
Q_{out}	Outflows
ΔH_i	Head lose in pipe i
ω	Numerical comparison constant
R	Roughness coefficient of pipe i
α, β	Regression coefficients
B	Total number of loops
H_k	Nodal head of node k
$H_{\min}$	Minimum pressure limit
$H_{\max}$	Maximum pressure limits
NN	Total number of nodes
d_i	diameter of the pipe i
D	Set of diameters for commercially available pipes
G_c	Current generation
G_l	Generation limit
C_p	Penalty cost function
U	Unit pressure penalty constant
s	Power for the penalty function
X	Set of Individuals
n	Number of Individuals
V	Number of clusters centers
c	Number of Clusters
U_0	Initial membership matrix
U	Fuzzy membership matrix
μ_{ki}	fuzzy membership of k^{th} cluster i^{th} individual
d_{ki}	Euclidean distance between i^{th} data and k^{th} cluster center.
m	Level of cluster fuzziness
v_k	k^{th} cluster center
J	Objective function for FCM
β	Termination criterion for FCM
S^G	Set of the distributed population
X_j^G	D-dimensional vector
NP	Size of the population
X_{lower}	Lower bound
X_{upper}	Upper bound
Y	Resultant position vector
F	Scaling constant
X_{ipos}	Position vector of the current individual
$Bestsol_{pos}$	Position vector of the member with minimum objective function value present in the entire population

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$X_{nhd1pos}$	Position vector of the random member present in the same cluster as a current individual known as neighbor1
$X_{nhd2pos}$	Position vector of the random member present in the same cluster as the current individual known as neighbor2
X_{rand1_pos}	Position vector of the random member present in the population
X_{rand2_pos}	Position vector of the random member present in the population, different from X_{rand1_pos}
X_{rand3_pos}	Position vector of the random member present in the population, different from X_{rand1_pos} and X_{rand2_pos}

I. INTRODUCTION

Network distribution plays an important role in various disciplines including power systems [1], [2], supply chain management [3], [4], computer network systems [5], [6], etc.

In the case of water distribution systems, which is the focus of the present study, a network system, also known as Water Distribution Network (WDN), refers to a combination of hydraulic control elements like pipes and tanks connected through a finite set of nodes, representing reservoirs or tanks, to transport the desired amount of water from one point (source node) to another (consumer node). New systems are developed from time to time and the existing ones are upgraded to meet the water requirements of the consumers consequently making the WDN an integral part of the infrastructure development. The significance of a WDN system optimization can also be witnessed through the vast literature of more than five decades.

In the present study, the literature review is done for a looped distribution network design optimization problem where the objective is to determine the set of commercially available diameters that will minimize the cost. The resulting model is a non-linear, discrete, combinatorial optimization problem, for which the solution approaches may be broadly classified as classical and non-classical. Classical here refers to the traditional mathematical programming based optimization methods which depend a lot on the general mathematical properties of the given problem, while non-classical refers to the newly developed metaheuristics algorithms that are usually stochastic in nature and are not dominated by the nature of the problem.

Initial studies in this direction advocated the application of linear and non-linear programming approaches for obtaining the optimal solution. However, with the advent of powerful personal computers, metaheuristics emerged as a prominent area of research. A brief overview of the application of these two approaches is provided in the subsequent sections.

A. MATHEMATICAL PROGRAMMING METHODS FOR WDN

One of the earliest instances of the application of mathematical programming techniques on WDN was anticipated by Alperovits and Shamir in 1977 [7]. They proposed an iterative linear programming gradient (LPG) method for solving such problems through a hierarchical decomposition. They also executed post-optimality analysis to acquire the information necessary to calculate the gradient of the total cost with respect to changes in the flow distribution.

Quindry *et al.*, 1981 [8] used linear programming for planning and designing of looped WDN. They proposed an iterative technique incorporating linear programming and a gradient procedure to obtain a local optimal solution. They demonstrated their method on the New York City water supply system and showed a potential cost saving of 13 %, through their method. Fujiwara *et al.*, 1987 [9] modified The LPG method of Alperovits and Shamir [7] in terms of search direction and step size. They proposed a quasi-Newton search direction in place of steepest descent and used backtracking to determine the step size in contrast to the fixed step size as proposed by Alpervotis and Shamir [7]. Kessler and Shamir, 1989 [10] further modified LPG to solve WDN. They altered LPG in each stage and reformulated the constraint set independently from the set of loops and paths and tried to improve the search by using the projected gradient method. A non-linear extension to the model proposed in [7] was given by Fujiwara and Khang, 1990 [11], who proposed a two-phase, iterative method for solving WDN problems. In phase 1, the non-linear programming gradient method was implemented to specify the flow distribution and pumping heads as decision variables. While, in phase 2, the link head losses of the local optimal solution obtained in the first phase were fixed, and the subsequent problem, concave in nature, was solved for the pumping heads and link flows; and the first phase was restarted to obtain a better local optimal solution. The process was repeated till no further improvement was obtained. Varma *et al.*, 1997 [12] introduced a reduced successive quadratic programming method for WDNs. They eliminated the hydraulic constraints using graph theory to obtain a reduced optimization model, which was solved through successive quadratic programming method.

Nonetheless, one shortcoming of linear and non-linear programming techniques is that in order to apply these techniques, many times the model has to be either linearized or simplified for their possible implementation, sometimes leading to solutions of split-pipe or continuous-diameter design. Often, the solutions obtained are unrealistic and impractical. For a more detailed review of split-pipe and continuous-diameter approaches, the interested reader may refer to Walski, 1985 [13] and Lansey and Mays, 1989 [14].

Consequently, techniques like heuristics and metaheuristics started gaining popularity due to their flexibility and ease of use for a wide variety of problems and their features like independence from the auxiliary nature of the problem and stochastic (probabilistic) behavior. A brief description of the implementation of Metaheuristics for WDN is given in the next subsection.

B. METAHEURISTICS FOR WDN

One of the earliest applications of metaheuristic algorithms in the case of WDN is that of Genetic Algorithms (GA) in the late 1980s by Goldberg and Kuo in 1987 [15], who used it for pipeline optimization and showed that they were able to achieve near-optimal solutions in a reasonably small time. The paradigm shift towards such algorithms was clear by the mid-nineties and several metaheuristics (existing as well as newly developed) were suggested to solve WDN. In the next paragraph, selected references are given:

1) APPLICATIONS OF GA AND ITS VARIANTS

Simpson *et al.*, 1994 [16] implemented binary GA to solve the network earlier studied by Gessler (1985) [17] and computed the results for a different combination of pipes. They also compared the results with the non-linear optimization method and gave some points of comparison between the two concluding that non-linear optimization is good for small size problems but GA may be beneficial for larger systems as it deals with a set of solutions. Savic and Walters, 1997 [18] proposed a computer model called GANET (Genetic Algorithm for least-cost pipe network design) for solving WDN problems and demonstrated its application on the two-loop network, Hanoi Network, and New York water system. They adopted the standard GA with grey encoding for their model and compared the results with the previously existing methods.

Walters *et al.*, 1999 [19] implemented structured messy GA (SMGA), proposed by Halhal *et al.*,1997 [20] for the optimization of WDNs rehabilitation. They expanded the network model and used pumping installations and storage tanks as variables. They treated the problem as a multi-objective model with cost and benefits as two objectives. Dandy *et al.*, 1996 [21] modified the standard GA by incorporating into it a power scaling fitness function and creeping mutation operator for New York tunnel network optimization. They showed the competency of the proposed algorithm by comparing the results with the traditional GA. Prasad *et al.*, 2003 [22] implemented NSGA for solving WDNs with two objectives, minimizing the network cost and maximization the reliability measure. They considered the well-known two-loop network; the Hanoi network for their study.

Two loop network and Hanoi water distribution network were also studied by Gencoglu, 2007 [23] using GA. Reca *et al.*, 2008 [24] solved the WDNs using GA. The algorithm was first tested on medium size benchmark network and was then validated on large-scaled water distribution networks. After that, Reca *et al.*, 2018 [25] proposed a bounded GA to evaluate the optimal cost of the two-loop and Hanoi Network. A search space reduction approach was combined with GA to design WDNs.

Successful implementation of GA for solving WDN problems, encouraged researchers to try other metaheuristics as well for which the selected references are given in the next paragraph.

2) APPLICATIONS OF SIMULATED ANNEALING

Cunha and Sousa, 1999 [26] implemented a single solution based stochastic optimizer simulated annealing [SA] to solve WDNs. after that, Cunha and Sousa, 2001, [27] applied SA on gravitational looped water distribution network to find the least cost of the network. Recently, Marques *et al.*, 2018 [28] proposed an enhanced SA algorithm for solving a multi-objective flexible design water distribution network having four objectives as: minimize pressure deficit; minimize undelivered demand, minimize cost and minimize carbon emissions.

3) APPLICATIONS OF PARTICLE SWARM OPTIMIZATION (PSO) AND ITS VARIANTS

Suribabu & Neelakantan, 2006 [29] proposed a combination of PSO + EPANET, called PSONET for solving well-known WDN problems: two-loop network and Hanoi network and showed that PSONET outperformed the other algorithms like GA and SA. Montalvo *et al.*, 2008 [30] presented a discrete variant of PSO and demonstrated its reasonable performance for solving WDNs: Hanoi and New York City water distribution. Later, Montalvo *et al.*, 2010 [31] incorporated self-adaptive features in PSO and evaluated its performance on Hanoi and New York city EDNs. The results however, were same as the ones obtained through discrete PSO and other evolutionary methods.

Ezzeldin *et al.*, 2013 [32] proposed an integer discrete variant of PSO for WDNs. They proposed a new boundary conditions called billiard boundary condition to improve the sensitivity of PSO towards its parameters. They applied the proposed variant on a two-loop and two source network to minimize the total cost.

Surco *et al.*, 2018 [33] proposed a discrete variant of PSO for WDNs and implemented it for solving on the two-loop network, Hanoi Network, and a network model proposed in [34].

4) APPLICATIONS OF DIFFERENTIAL EVOLUTION (DE) AND ITS VARIANTS

Suribabu, 2009 [35] developed a simulation-based optimization model by combining EPANET and DE and named the proposed variant as DENET, for WDNs. He considered four case studies: Two loop network, Hanoi network, New York tunnel, and New York water supply system and showed that DENET performs satisfactorily for such problems. Zheng *et al.*, 2013 [36] proposed Self- Adaptive Differential Evolution (SaDE) having adaptive parameters and demonstrated its performance on four well-known WDNs: Balerma network, Hanoi network, New York tunnel, and double New York tunnel.

5) APPLICATIONS OF ANT COLONY OPTIMIZATION AND ITS VARIANTS

Maier *et al.*, 2003 [37] advocated the application of ACO for solving WDN problems and implemented it on the 14 pipe network model given by Simpson, 1994 [16] and the New York City water Supply Tunnel problem. They showed that the results obtained through ACO were better than the ones obtained through GA. Zecchin *et al.*, 2006 [38] proposed two variants of ant colony optimization algorithms: Ant System (AS) and Max-Min Ant System (MMAS), for determining the optimal design of Hanoi and New York tunnel networks.

6) MISCELLANEOUS METAHEURISTICS ALGORITHMS FOR WDNs

Several other metaheuristics besides GA, PSO, DE, ACO have been used for solving WDNs. selected references are: Eusuff and Lansey [39] proposed the use of Shuffled Frog Leaping (SLA) in 2004 for solving WDNs. they integrated it with EPANET and called the resulting method SFLANET. They implemented it on the New York tunnel water distribution problem.

Cunha and Ribeiro, 2004 [40] applied Tabu Search on five different WDNs models to determine the optimal pipe diameters and least cost of the WDNs. They compared the results with GA and SA and showed that with some modifications, the Tabu Search algorithm, performs better than the other two algorithms.

In 2006, Haddad *et al.* [41] proposed a new heuristic approach Harmony search (HS) and in 2006, Geem [42] demonstrated the performance of HS for determining the optimal design of five WDNs: Two loop network, Hanoi network, New York tunnel, GoYang network and bakRyun network [43]. Comparison of results with other meta-heuristic algorithms showed an improvement of about 0.28-10.26% in terms of cost.

Min-Der *et al.*, 2004 [44] used Scattered Search (SS) [45] for three water distribution networks: Two loop network, Hanoi network and New York Tunnel Network and Mohan & Jinesh, 2010 [46] implemented Honey Bee Mating Algorithm (HMBO) developed by Haddad *et al.* [41] for two-loop and Hanoi network. Sadollah *et al.*, 2014 [47] implemented the Water Cycle Algorithm (WCA) developed by Eskandar *et al.*, 2012 [48], to solve the Balerma water distribution network. The algorithm was coupled with the hydraulic simulator, EPANET for dealing with the constraints. Moosavian & Roodsari, 2014 [49] suggested the application of Soccer League Competition (SLC) WDNs. they applied SLC, with some modifications, on Two loop network, Hanoi network, New York tunnel network and compared the results with several other metaheuristics like PSO, DE, SS, etc. and indicated that SLC outperforms the other algorithms in most of the cases. In [50], Moosavian & Jaefarzadeh, 2014 applied Shuffled Complex Evolution (SCE) developed by Q. Duan, 1993 [51] for hydraulic analysis of WDN problems and implemented it on 4 numerical examples of WDN.

7) HYBRID APPROACHES FOR WDNs

Instances of hybrid variants, where a metaheuristics algorithm is combined with some classical method or with some other metaheuristics approach, are also available in the literature for solving the WDNs problems. Some examples are: Geem, 2009 [52] proposed a hybrid version of harmony search (HS) and particle swarm optimization (PSO) for four WDNs: Two loop network, Hanoi network, New York tunnel, Balerma network.

Tolson *et al.*, 2009 [53] hybridized Discrete Dynamically Dimensioned Search (HD-DDS) is proposed for minimum design cost of WDNs.

Bolognesi *et al.*, 2010 [54] proposed a novel algorithm named Genetic Heritage Evolution by Stochastic Transmission (GHEST) and combined it with EPANET 2. They applied their algorithm on three well-known WDNs namely two-loop network, Hanoi Network, and New York tunnel network. Sedki & Ouazar, 2012 [55] proposed a hybrid version of PSO and DE and linked it with the hydraulic simulator, EPANET for the optimal design of WDNs. The performance of the proposed hybrid PSO+DE tested on three well-known water distribution networks the two-loop network, the Hanoi network, and the New York Tunnels network. The results when compared with other metaheuristics like GA, PSO, etc. indicated the efficiency of the proposed method. More recently, Elshaboury *et al.*, 2020 [56], proposed a hybrid PSO and GA variant for the rehabilitation of the water distribution network and validated the efficiency of the hybrid model on the Shaker Al-Bahery [57] water distribution network. Pankaj *et al.*, 2020 [58] proposed a self-adaptive variant of the cuckoo search for single and multi-objective problems and implemented the algorithms for determining the optimal design of WDN.

A list of meta-heuristics algorithms and networks to which they have been applied are given in Table 1.

C. PRESENT STUDY

In the present study, the focus is on Differential Evolution (DE) mainly because of the following reason: DE is similar to GA in terms of operators and was proposed almost at the same time as that of PSO but its potential for solving WDN problems is relatively less explored in comparison to the other two. Through this article have tried to demonstrate that by making certain modifications in the structure of DE, it can be a good alternative for solving related to WDN.

The basic structure of DE has certain inherent drawbacks like lack of diversity while generating the mutant vectors and insufficient use of the knowledge produced during evolution. These two factors often lead to the premature convergence of DE [59]– [61].

In this article, we have implemented an adaptive variant of DE, called FCADE, in which the population is distributed into two clusters through a fuzzy C-means algorithm and mutation strategies are decided in an adaptive manner along with the crossover parameter. Dividing the population into clusters help in exploring the search space in a better manner, while adaptability makes FCADE improves its resilience.

Besides providing an overview of the mathematical programming methods and Metaheuristics applied for solving WDN problems, the main contributions of the present study are summarized below:

TABLE 1. Metaheuristic approach in wdn.

Algorithm	Authors	Network Type
Iterative Gradient Method	Alperovits and Shamir (1977) [7]	Two Loop Network
Linear Programming	Quindry *et al.*, (1981) [8]; Fujiwara *et al.*, (1987) [9]; Kessler and Shamir, (1989) [10]; Fujiwara and Khang, (1990) [11]	Two Loop Network, Hanoi Network
Non-Linear Programming	Fujiwara and Khang, (1990) [11]; Varma *et al.*, (1997) [12]	Two Loop Network, Hanoi Network
Genetic Algorithm (GA)	Goldberg and Kuo in (1987) [15] Simpson *et al.*, (1994) [16]; Savic and Walters, (1997) [18]; Walters et al., (1999) [19]; Dandy *et. al.*, (1996) [21]; Prasad *et al.*, (2004) [22]; Gencoglu (2007) [23]; Reca *et al.*, (2008) [24]	Two Loop Network, New York Network, Hanoi Network, Balerma Network
Bounded Genetic Algorithm (B-GA)	Reca *et al.* (2017) [25]	Two Loop Network, Hanoi Network
Simulated Annealing (SA)	Reca et al. (2008) [24]; Cunha and Sousa (1999) [26], [27]; Marques *et al.*, (2018) [28]	Two Loop Network, Hanoi Network
Particle Swarm Optimization (PSO)	Suribabu & Neelakantan (2006) [29]; Montalvo *et al.*, (2008) [30]; Montalvo *et al.*, (2010) [31]; Ezzeldin *et al.* (2014) [32]	Two Loop Network, Hanoi Network, New York Tunnel
Modified Particle Swarm Optimization (Mod. PSO)	Surco *et al.* (2018) [33]	Two Loop Network, Hanoi Network, Balerma Network
Differential Evolution (DE)	Suribabu (2009) [35]	Two Loop Network, Hanoi Network, New York Network, 14-pipe Network

TABLE 1. *(Continued.)* Metaheuristic approach in wdn.

Self-Adaptive Differential Evolution (SADE)	Zheng *et al.* (2013) [36]	New York Network, Hanoi Network, Balerma Network, Double New York Network
Artificial Bee Colony (ABC)	Maier *et al.*, (2003) [37]; Zecchin *et al.*, (2006) [38]	New York Network, Hanoi Network
Shuffled Frog Leaping (SFL)	Eusuff and Lansey (2004) [39]	Two Loop Network, Hanoi Network, New York Network
Tabu Search (TS)	Cunha & Ribeiro (2004) [40]; Reca *et al.* (2008) [24]	Two Loop Network, Hanoi Network, New York Network
Harmony Search (HS)	Geem (2006) [42]	Two Loop Network, Hanoi Network, New York Network, GoYang water distribution network, BakRyun water distribution network
Scatter Search (SS)	Min-Der Lin *et al.* (2007) [44]	Two Loop Network, Hanoi Network, New York Network
Honey Bee Mating Algorithm (HBMA)	Mohan & Jinesh (2010) [46]	Two Loop Network, Hanoi Network
Water Cycle Algorithm (WCA)	Sadollah *et al.* (2014) [47]	Balerma Network
Soccer League Competition (SLC)	Moosavian & Roodsari (2014) [49]	Two Loop Network, Hanoi Network, New York Network
Shuffled Complex Evolution (SCE)	Moosavian & Jaefarzadeh (2014) [50]	Numerical example
Harmony Search-Particle Swarm Optimization (HS-PSO)	Geem (2009) [52]	Two Loop Network, New York Network, Hanoi Network, Balerma Network
Hybrid Discrete Dynamically Dimensioned Search (HD-DDS)	Tolson *et al.* (2009) [53]	New York Network, Hanoi Network, Balerma Network
Genetic Heritage Evolution by Stochastic Transmission	Bolognesi *et al.* (2010) [54]	Two Loop Network, New York Network,

TABLE 1. *(Continued.)* **Metaheuristic approach in wdn.**

(GHEST)		Hanoi Network, Balerma Network
Hybrid Particle swarm with Differential Evolution (PSO-DE)	Sedki & Ouazar (2012) [55]	Two Loop Network, Hanoi Network, New York Network
GA and PSO	Nehal *et al.* (2020) [56]	Shaker Al-Bahery, Erypt
Self-Adaptive Cuckoo Search (SACSA)	Pankaj *et al.* (2020) [58]	Two Loop Network, Hanoi Network, Telangana Network
Artificial Bee Colony (ABC)	Yilmaz *et al.* (2020) [75]	Two Loop Network, Hanoi Network, New York Tunnel

- Experimental analysis of FCADE on a test set of CEC 2005 benchmark problems [62] and its comparison with other adaptive Metaheuristics.
- Implementation of FCADE, a novel fuzzy adaptive DE variant for solving the three well known, complex WDN problems: Two Loop Network, Hanoi Network, and New York Tunnel Network.
- A meticulous empirical analysis of FCADE by comparing it with fourteen other Metaheuristics for Two Loop Network; thirteen other Metaheuristics for Hanoi Network and New York Tunnel Network, which included previously tested as well as newly proposed algorithms, on the basis of different performance metrics.

D. STRUCTURE OF THIS ARTICLE

The entire study is discussed through seven sections. In the first section, a literature review highlighting the implementation of different meta-heuristic algorithms for solving WDN problems is given. In the second section key features of the selected WDN are provided. In the third and the fourth sections, the mathematical formulation and the methodology implemented are discussed respectively. In the fifth section, experimental settings are given and the results are discussed in the sixth section. Finally, in the seventh section concluding remarks on the basis of the present study are provided.

II. CASE STUDIES

Three network designs selected for the present work are two loop network, Hanoi Network, and New York Tunnel network. Due to their complex structure, these networks have been a focus of attention for researchers and have been studied from time to time under different scenarios. In this section, the characteristic features of these networks are explained in brief and the corresponding diagrams and relevant data are provided.

TABLE 2. **Node & pipe cost data for two-loop network.**

Node Data			Cost Data for Pipes	
Node	Demand (m^3/h)	Ground Level (m)	Diameter (Inches)	Cost (Units)
1	-1120	210	1	2
2	100	150	2	5
3	100	160	3	8
4	120	155	4	11
5	270	150	6	16
6	330	165	8	23
7	200	160	10	32
			12	50
			14	60
			16	90
			18	130
			20	170
			22	300
			24	550

A. NETWORK 1 - TWO LOOP WATER DISTRIBUTION NETWORK

This is one of the oldest hypothetical piped network structures available in the literature. It was proposed by Alperovits and Shamir [7] in 1977 and since then it has been studied by many researchers ([7], [20], [29], [33], [16], [46], [63]) from time to time. It is 7 nodes - 8 pipes, gravity-fed single reservoir system having a 210-m fixed head. The network pipes, combined together through two loops, are 1000m long and have a Hazen–Williams coefficient of 130. The minimum pressure head required is 30 m. Design restrictions for the network are due to the availability of 14 pipe sizes leading to a number of possible combinations as $14^8 = 1.48*10^9$.

The input data [7] for this network is provided in Table 2, and the network diagram of the two-loop network is given in Figure 1.

B. NETWORK 2 - HANOI WATER DISTRIBUTION NETWORK

Hanoi network was first discussed by Fujiwara and Khang [11] in 1990. This again is a well-known network studied by various researchers ([11], [26], [30], [16], [44], [55]) due to its complex structure. This is a gravity-fed network and the reservoir is located at an elevation of 100 m and for all nodes, the ground elevation is zero. The length of the network pipes is 39.4 km, and the roughness coefficient is 130. The minimum pressure head required is

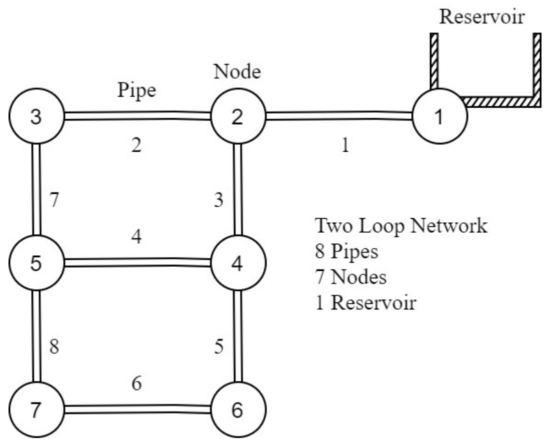

FIGURE 1. Two loop network.

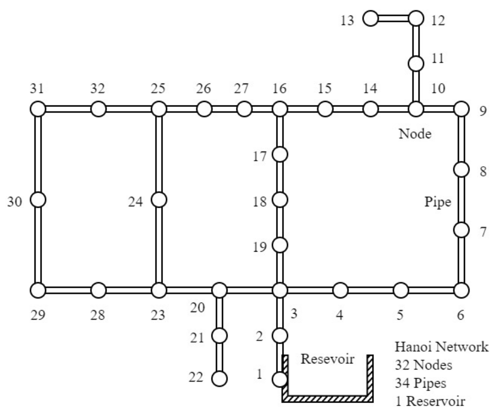

FIGURE 2. Hanoi network.

30 m. 34 pipes and 32 nodes of this network are combined through 3 loops. The availability of commercial pipes is restricted to 6 therefore the number of possible combinations is calculated as $6^{34} = 2.865*10^{26}$. Relevant data [11] of the Hanoi network is given in Tables 3 and 4. A network diagram for the Hanoi network is provided in Figure 2.

C. NETWORK 3 - NEW YORK TUNNEL NETWORK (NYTN)

NYTN was first discussed by Schaake [64] in 1969. Its structure is shown in Fig. 3. It is a famous network studied by many researchers ([31], [16], [63]– [65]) due to its complex structure. This is also a gravity-fed single reservoir network system having a fixed head of 300-ft. The number of nodes and pipes in this network are 20 and 21, respectively. It is a one loop system, i.e. the nodes and pipes are united through a single loop. Hazen-William constant for this system is 100. The number of possible combinations for this network is $16^{21} = 1.93*10^{25}$, due to the commercial restriction of the availability of 16 pipes. The network diagram for New York Tunnel is given in Figure 3 and the relevant input data [64] is given in Table 5 and Table 6.

TABLE 3. Node and link data for Hanoi network.

Link	Length (m)	Demand (m^3/h)	Node	Minimum Head required (m)
1	100	19940	1	100
2	1350	19050	2	30
3	900	8705	3	30
4	1150	8575	4	30
5	1450	7850	5	30
6	450	6845	6	30
7	850	5495	7	30
8	850	4945	8	30
9	800	4420	9	30
10	950	2000	10	30
11	1200	1500	11	30
12	3500	940	12	30
13	800	1895	13	30
14	500	1280	14	30
15	550	1000	15	30
16	2730	1105	16	30
17	1750	1970	17	30
18	800	3315	18	30
19	400	3375	19	30
20	2200	6120	20	30
21	1500	1415	21	30
22	500	485	22	30
23	2650	3430	23	30
24	1230	1320	24	30
25	1300	500	25	30
26	850	525	26	30
27	300	1425	27	30
28	750	1795	28	30
29	1500	1065	29	30
30	2000	775	30	30
31	1600	415	31	30
32	150	55	32	30
33	860	50		
34	950	855		

D. COMPLEXITY OF NETWORKS

The characteristics of the three networks are given in Table 7. Out of the three, the Hanoi network has the highest numbers

TABLE 4. Cost data for pipe for Hanoi network.

Diameter(inches)	Diameter (mm)	Unit Cost(unit/m)
12	304.8	45.73
16	406.4	70.4
20	508	98.38
24	609.6	129.3
30	762	180.8
40	1016	278.3

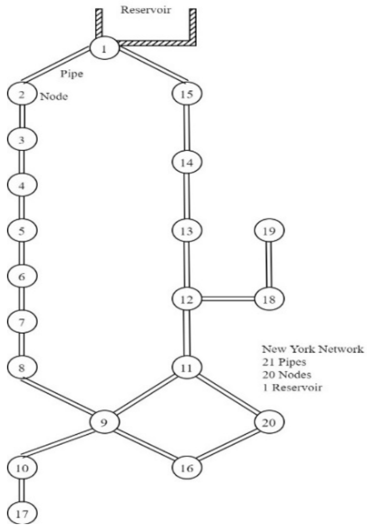

FIGURE 3. New york tunnel network.

TABLE 5. Nodal data for NYC water supply tunnels.

Node	Demand (ft^3/s)	Minimum Total Head (ft)
1	Reservoir(-2017.5)	300
2	92.4	255
3	92.4	255
4	88.2	255
5	88.2	255
6	88.2	255
7	88.2	255
8	88.2	255
9	170	255
10	1	255
11	170	255
12	117.1	255
13	117.1	255
14	92.4	255
15	92.4	255
16	170	260
17	57.5	272.8
18	117.1	255
19	117.1	255
20	170	255

of nodes and pipes. The order of complexity for the Hanoi network is 10^{26}; for New York Tunnel network, it is 10^{25} making it the second most difficult problem out of the three selected problems.

III. MATHEMATICAL FORMULATION

From an optimization point of view, a WDN has a typical non-linear, mixed-integer combinatorial model. Discreteness in the model is due to the restrictions on commercially available discrete diameters while non-linearity is due to the equations representing hydraulic behavior like energy conservation laws.

The objective of the design problem may be stated as follows: to identify the combination of diameters from a given set of discrete commercial diameters, such that the lowest cost is obtained for the network. The objective function represented in eq. 1 is a function of pipe length L_i and cost C_i.

$$\text{Min } C_T = \sum\nolimits_{i=1}^{N} C_i \times L_i \tag{1}$$

where C_T represents the network cost C_i represents the unit cost of the i^{th} pipe and L_i denotes the length of the i^{th} pipe; while N is the total number of pipes in the network.

The related constraints problems are:

- Hydraulic constraints – for managing the mass and energy conservation equations.
- Pressure constraints – for maintaining an appropriate pressure which is neither high nor low.
- Available size for pipe diameters constraints - to ensure the design restrictions that the pipe diameters belong to the set of the diameters of commercially available pipes.

Mathematically these constraints may be defined with the help of equation (2) – equation (6).

A. HYDRAULIC CONSTRAINTS

1) MASS CONSERVATION CONSTRAINT

$$\sum Q_{in} - \sum Q_{out} = Q_{ext} \tag{2}$$

The above restriction helps in ensuring that the external demand (Q_{ext}) at each junction node is met by calculating the difference between inflows (Q_{in}) and outflows (Q_{out}).

TABLE 6. Pipe data for NYC water supply tunnels.

Pipe	Start Node	End Node	Length feet	Existing Diameter (Inches)
1	1	2	11600	180
2	2	3	19800	180
3	3	4	7300	180
4	4	5	8300	180
5	5	6	8600	180
6	6	7	19100	180
7	7	8	9600	132
8	8	9	12500	132
9	9	10	9600	180
10	11	9	11200	204
11	12	11	14500	204
12	13	12	12200	204
13	14	13	24100	204
14	15	14	21100	204
15	1	15	15500	204
16	10	17	26400	72
17	12	18	31200	72
18	18	19	24000	60
19	11	20	14400	60
20	20	16	38400	60
21	9	16	26400	72

TABLE 7. Characteristic features of the networks.

Characteristic	Network 1	Network 2	Network 3
Gravity Fed	Yes	Yes	Yes
Nodes	7	32	20
Pipes	8	34	21
Available pipes	14	6	16
Loop	2	3	1
Complexity (combinations)	$1.48*10^{9}$	$2.865*10^{26}$	$1.93*10^{25}$

2) ENERGY CONSERVATION CONSTRAINT

$$\sum\nolimits_{i \in b} \Delta H_i = 0, b = 1, 2, \ldots, B \tag{3}$$

$$or\, \Delta H_i = \omega \times \left({}^{L_i}\!/_{R_i^{\alpha} \times D_i^{\beta}} \right) \times Q_i \times \left| Q_i^{\alpha - 1} \right| \tag{4}$$

where ΔH_i represents the head lose in the pipe i, $\omega = 10.667$, is the numerical comparison constant. R is the roughness coefficient of pipe i. α and β are regression coefficients with value 1.852 and 4.871 respectively. B denotes the total number of loops.

B. PRESSURE CONSTRAINTS

$$H_{\min} \leq H_k \leq H_{\max}, \quad k = 1, 2, \ldots, NN \tag{5}$$

where H_k represents the nodal heads; $H_{\min}$ and $H_{\max}$ are the minimum and maximum pressure limits respectively; NN represents the total number of nodes.

C. PIPE DIAMETER SIZE AVAILABILITY CONSTRAINTS

$$d_i \in D, \quad i = 1, 2, \ldots, N \tag{6}$$

where d_i is the diameter of the pipe i; and D denotes the set of diameters for commercially available pipes.

D. CONSTRAINT HANDLING

Hydraulic constraints are solved using hydraulic simulation software [66]. Mathematically, the hydraulic performance of a network can be converted into costs using the penalty function. Thus minimum nodal pressure constraints are converted into a penalty cost function [23].

$$C_p = \left(\sum_{k}^{NN} \begin{Bmatrix} \left| \sqrt[s]{H_{\min} + 1} - H_k \right|^2 * u; \Leftrightarrow H_k \leq 0 \\ \left| (H_{\min} + 1) - H_k \right|^s * u; \Leftrightarrow 0 < H_k < H_{\min} \\ 0; \Leftrightarrow H_{\min} \leq H_k \leq H_{\max} \\ \left| (H_{\max}) - H_k \right| * u; \Leftrightarrow H_{\max} < H_k \end{Bmatrix} \right)$$

$$\text{where } s = \left({}^{G_c}\!/_{G_l} \right) + 1; \tag{7}$$

The power s depends on the current generation G_c and Generation limit G_l

C_p = penalty cost function

U = unit pressure penalty constant, pre-defined.

s = power for the penalty function

So, the total cost (network cost and penalty cost) for construction of a network will be,

$$C = C_T + C_p \tag{8}$$

1) EPANET

EPANET is simulation software, intellectualized by USEPA [66], an American agency, which is specially designed to deal with problems related to water resources. It is easy to use software with the flexibility of getting integrated with common programming languages like MATLAB, C#, Visual Basic, Python, etc. In this study, EPANET MATLAB Toolkit is used.

IV. METHODOLOGY

A. DIFFERENTIAL EVOLUTION

Differential Evolution (DE) is a well-known and well-researched metaheuristic for finding reliable solutions to real-world optimization problems. Conceptualized by Storn and Price [67], the first recorded article on DE was in the form of a technical report. The entire working of DE may be organized into two phases. The first phase is initialization during which a set of feasible solutions is generated in the feasible domain. The second phase of DE is that of evolution where the operator's mutation, crossover, and selection are activated to guide the population generated in the first phase till a predefined termination criterion is fulfilled.

Despite the competent performance of DE, like most of the population-based metaheuristics, it has certain inherent drawbacks like premature convergence, stagnation of population, etc. [68]. Another drawback is fine-tuning the values of control parameters like scaling factor (F), crossover rate (Cr) [59]– [61].

Through this article, two modifications are suggested in the basic structure of DE, to overcome or to minimize these shortcomings. First is C-means clustering in the initial phase for the better transition of points in the search space and second is adaptive control for F and Cr.

Before explaining the proposed algorithm in detail, two preliminary concepts used are described in brief:

1) CLUSTERING

Clustering is a widely used technique to group data into clusters where similar data are grouped together depending upon data properties. In general, cluster analysis refers to a broad spectrum of methods which try to subdivide a data set X into c subsets (clusters).

2) FUZZY C-MEANS ALGORITHM

Fuzzy C-means is a distinctive clustering algorithm where one data point can be a member of more than one group or cluster. It was proposed by Dunn [69] and was later improved by Bezdek [70].

The fuzzy C-means (FCM) algorithm works by assigning a membership to each individual in the population corresponding to each cluster center on the basis of the distance between the cluster center and the data point. The nearer the data to the cluster center, the higher will its membership value for that cluster. The summation of membership of each data point should be equal to one. Every individual in the population is bound to a cluster by means of a membership function, representing the fuzzy behavior of the algorithm.

Algorithmic 1: steps for Fuzzy C-means:

1. Let $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\}$ be the data set containing n individuals, present in the search space.

2. Fix the number of clusters as c, where $2 \leq c < n$.

3. Randomly initialize cluster center for c clusters represented as $\boldsymbol{V} = \{v_1, v_2, \ldots, v_c\}$. Create an initial membership matrix $\boldsymbol{U}_0$ (where '$\boldsymbol{U} = (\mu_{ij})_{n*c}$' is the fuzzy membership matrix for iteration 0) using a pseudo-random initialization strategy.

4. Calculate the fuzzy membership μ_{ij} using the formula:

$$\mu_{ij} = \frac{1}{\sum_{j,k=1}^{c} \left({dij}/{d_{ik}} \right)^{\left(2/(m-1) \right)}}, \quad i = 1, 2, \ldots, n \tag{9}$$

where

- $d_{ij} = \|x_i - v_j\|$
- $d_{ik} = \|x_i - v_k\|$
- m is the level of cluster fuzziness having a value between 0 and ∞.

5. Calculate the fuzzy center using the following formula:

$$v_j = \frac{\sum_{i=1}^{n} \mu_{ij}^{m} \times x_i}{\sum_{i=1}^{n} \mu_{ij}^{m}}, \quad j = 1, 2, \ldots, c \tag{10}$$

where v_j represents j^{th} cluster center

6. Calculate the objective function $J(X, V)$:

$$J(X, V) = \sum_{i=1}^{n} \sum_{j=1}^{c} \mu_{ij}^{m} \left\| x_i - v_j \right\|^2 \tag{11}$$

where $\|x_i - v_j\|$ represents the Euclidean distance between i^{th} data and j^{th} cluster center.

Repeat steps 4-6, until $\left\| U^{itr+1} - U^{itr} \right\| < \beta$('$\beta$' is the termination criterion between [0, 1]) or minimum value of J (weighted Sum of Squared Error) is achieved (close to zero).

B. FUZZY C-MEANS ADAPTIVE DIFFERENTIAL EVOLUTION

The proposed FCADE algorithm is based on two concepts (I) segregating the initial population on the basis of fuzzy C-means clustering (II) using adaptive mutation and crossover operators.

The main steps of FCADE(s) are as follows:

1) INITIALIZATION

A set of the uniformly distributed population is generated as follows:

Let $\boldsymbol{S}^G = \left\{ \boldsymbol{X}_j^G : j = 1, 2, \ldots, NP \right\}$ be the population at any generation G, NP denotes the size of the population. Here, $\boldsymbol{X}_j^G$ denotes a D-dimensional vector as $\boldsymbol{X}_j^G = \left\{ x_{1,j}^G, x_{2,j}^G, \ldots, x_{D,j}^G \right\}$ and $rand(0, 1)$ is a uniformly distributed random number between 0 and 1.

$$\boldsymbol{X}_j^G = X_{lower} + (X_{upper} - X_{lower}) \times rand(0, 1) \tag{12}$$

2) CLUSTERING

After the completion of step (a); cluster centers are generated and the membership matrix $\boldsymbol{U}_0$ is randomly initialized for each population member. Individuals are then segregated into different clusters as per their membership value, i.e. an individual $'i'$ with maximum membership value for the cluster $'j'\{(\mu_{ij}, \{\forall j \in [1, 2, \ldots, c]$ *and* $\forall i \in [1, 2, \ldots, n]\})$ is assigned to the cluster $'j'$.

The center of $'c'$ (number of clusters) clusters and the membership value of each individual (X_i^G) is updated (using eq-10 and eq-9 respectively) for each generation G until a stopping criterion as defined in step (7), Algorithm1 (Fuzzy C-means Algorithm), is achieved. Thus for each generation, individuals are reassigned to different clusters according to the updated membership matrix $\boldsymbol{U}_G$.

3) MUTATION

In FCADE, the mutation strategy for each individual is decided according to the cluster to which the individual belongs. If the current individual belongs to the same cluster as that of the best solution individual, then the following mutation strategy is followed:

$$\boldsymbol{Y} = \left\{ \begin{array}{l} (X_{ipos} + F \times (Bestsol_{pos} - X_{ipos}) + F \\ \times (X_{nhd1pos} - X_{nhd2ipos})), \\ \text{if } X \text{ is in best individual's cluster} \\ X_{rand1\_pos} + F \times (X_{rand2\_pos} - X_{rand3\_pos}), \text{ Otherwise} \end{array} \right\} \tag{13}$$

4) CROSS-OVER

The trial vector $\boldsymbol{T}_j^G = \left\{t_{1,j}^G, t_{2,j}^G, \ldots, t_{D,j}^G\right\}$ is generated as

$$t_{i,j}^G = \begin{pmatrix} v_{i,j}^G \ if\ rand_j \leq Cr \\ x_{i,j}^G \ Otherwise \end{pmatrix} \tag{14}$$

Cross-over rate(Cr) is updated as follows for each generation G:

$$Cr = \begin{pmatrix} CRu,\ where\ CRu = CRu + rand^{*}0.1\ if\ f(T) < f(X) \\ CRl,\ where\ CRu = CRu + rand^{*}0.1\ \ Otherwise \end{pmatrix}$$

where

$f(T)$: Objective function value for trial vector T.

$f(X)$: Objective function value for the current individual X.

CRl : Lower limit for Crossover rate set as 0.1

CRu : Upper limit for Crossover rate - $CRu = 1 - CRl$

The adaptive cross-over rate helps in better exploration of search space as with every generation the percentage of the newly generated population differ and hence provides the possibility to expand the searching domain for a probable optimal solution for a problem.

5) SELECTION

In this operation, a comparison is done between the target vector $\boldsymbol{X}_j$ and trial vector $\boldsymbol{T}_j$ according to their fitness value. The one having better fitness survives to the next generation. This operation is performed as:

$$\boldsymbol{X}_j^{G+1} = \begin{pmatrix} T_j^G \text{ if } f(T_j^G) \leq f(X_j^G) \\ X_j^G \text{ Otherwise} \end{pmatrix} \tag{15}$$

Algorithm FCADE Algorithm

Begin

Step 1: Initialization: Randomly initialize the population (target vectors) and set pipe diameters
Step 2: Run EPANET and compute nodal pressure head. If pressure constraints violate, the penalty cost computed using Eq. (7) and added in the total cost in Eq. (8)
Step 3: Evaluate the cost of the network
Step 4: Set the number of the clusters for the problem as 2 and Initialize the membership value of each individual in the population for each cluster.
Step 5: while ($t < MaxGen$) **do**
  Update the membership value and center position using Eq. (9) and Eq. (10) respectively.
  for each individual
    Update set cluster no and cluster head to which the individual
  end for
  for each cluster **do**
    Identify the individual with the minimum objective function value
  end for
  for each individual
    if the individual belongs to the same cluster as that of the best solution **then**
      Update Y using Eq. (13)
    else
      Update Y using Eq. (13)
    end if
    Perform crossover using Eq. (14) to obtain a trial vector
    Evaluate the objective function for the newly generated trial vector.
    Perform a tournament selection between trial and target vector using Eq. (15)
  End for
  Update population as pipe diameters and run EPANET
  $t = t+1$
Step 6: End while
Step 7: Output the optimal pipe diameters and optimal cost of the network

End

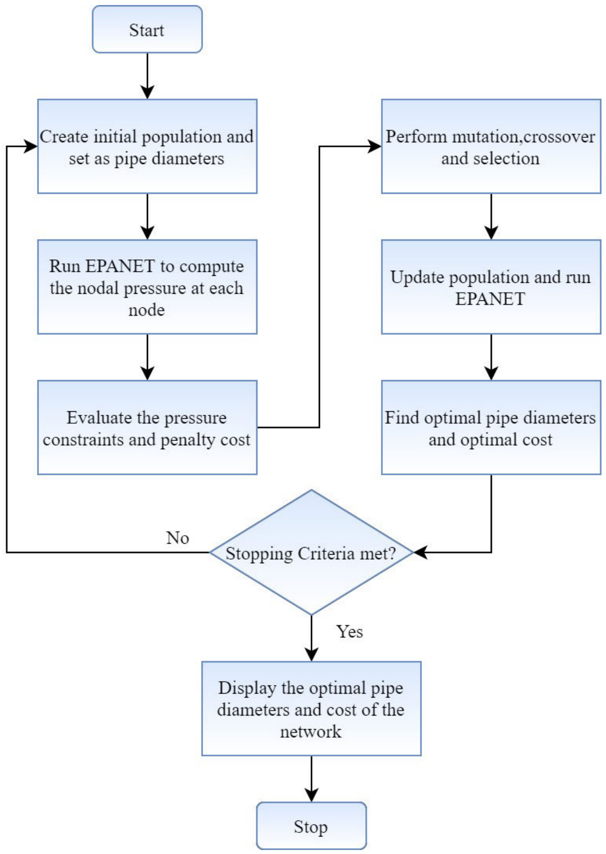

FIGURE 4. **Flowchart of the proposed methodology in WDNs.**

The performance of the proposed algorithm (FCADE) is analyzed on a set of selected unconstrained benchmark problems selected from the benchmark functions of CEC2005 [62], used for testing the efficiency of an optimization algorithm.

C. INTEGRATING FCADE WITH EPANET2.0

Integration of FCADE with EPANET2.0 is done to manage the hydraulic constraints. The initial population, also called the target vectors, for FCADE represents the diameter of the pipe. These are generated as real (floating point) numbers between the given lower and the upper limit for pipe diameter. Before processing these variables through EPANET2.0, these are converted to the nearest integer value.

The mass conversion constraint and energy conversion constraints are satisfied externally via EPANET and other constraints are satisfied within FCADE. The minimum nodal pressure constraints are converted into the penalty cost function. FCADE performs mutation and crossover to generate a trial vector (new pipe diameters, in decimal). The total cost of the trial vector is calculated after converting the new pipe diameters to the nearest commercial size. Selection is performed between the target vector and trial vector on the basis of their cost. Finally, the newly updated pipe diameters are selected for the next generation.

These steps are listed below and are illustrated in Figure 4.

Computational Steps

1. Generate initial population for FCADE representing the pipe diameters
2. Run EPANET to compute the nodal pressure head.
3. Evaluate the minimum pressure constraints and constraints violation.
4. Compute the penalty cost.
5. Perform mutation, crossover, and selection.
6. Update the population
7. Repeat the steps (2) - (6), until the termination criteria are satisfied.
8. Find the best cost and optimal pipe diameters.

V. EXPERIMENTAL SETTINGS

A. COMPUTATIONAL ENVIRONMENT

All listed algorithms are executed in a 64-bit operating system with window 10, Intel(R) Xeon(R)R) CPU E5-1650 v3 @ 3.50GHz processor, 16.00 GB RAM, and the tool of coding is MATLAB.

B. NUMERICAL BENCHMARK FUNCTIONS

To establish the versatility of FCADE on a wide range of problems, it is first tested on selected CEC2005 benchmark test functions [62]. This test suite consists of 25 commonly used benchmark functions categorized into four groups: 5 unimodal functions, 7 basic multi-modal functions, 2 expanded multi-modal functions, 11 hybrid composite functions.

C. PERFORMANCE METRICS

FCADE is compared with other algorithms on the basis of several performance metrics which include best and the average best objective function value, number of function evaluations (NFE), Convergence Graphs, and statistical analysis.

D. ALGORITHMS USED FOR COMPARISON

FCADE is first compared with four variants (jDE [68], SaDE [71], DE/rand/1 [67], DE/current-to-best/1 [67]) of DE on numerical benchmark problems.

For Two loop network, FCADE is compared with previously applied algorithms: GA [21], [22], [23]; SA [26], [24], [28]; SFL [39]; SCE [50]; PSO [32], [29]; DE [35]; HS [42]; ABC [63]; SS [44]; PSO +DE [55]; and SLC [49].

For Hanoi Network and New York tunnel network, FCADE is compared with previously applied algorithms: MMAS [38]; PSO [29]; HD-DDS [53]; GHEST [54]; SS [44]; DE [35]; SsDE [36]; SLCa [49]; SLCb [49]; and ABC [63] on WDNs.

Besides the above-listed algorithms, that have been used previously for solving the WDN problems, three recently proposed algorithms: hybrid Firefly Particle Swarm Optimization (FA-PSO) [72], Monarch Butterfly Optimization

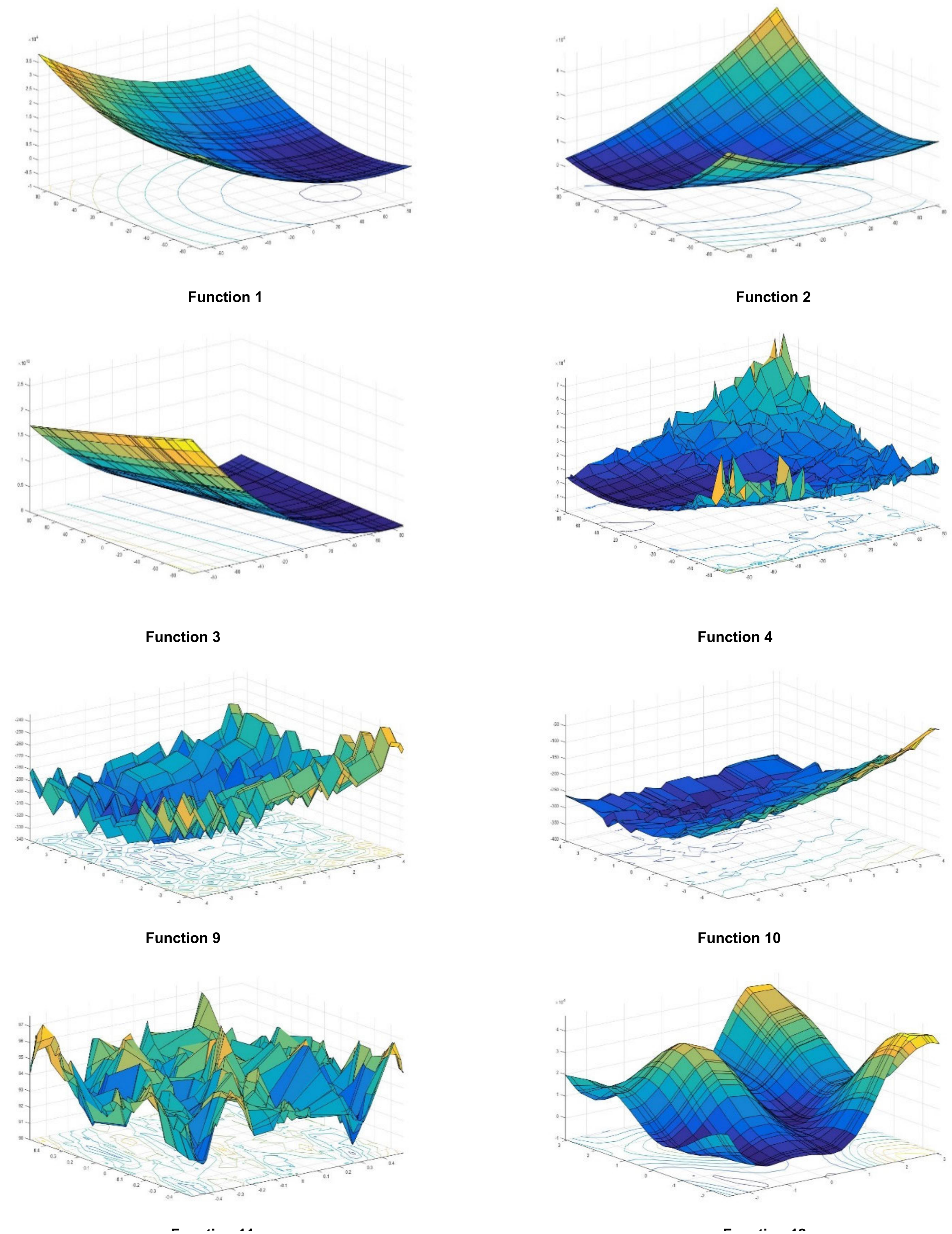

FIGURE 5. Some functions graph using FCADE.

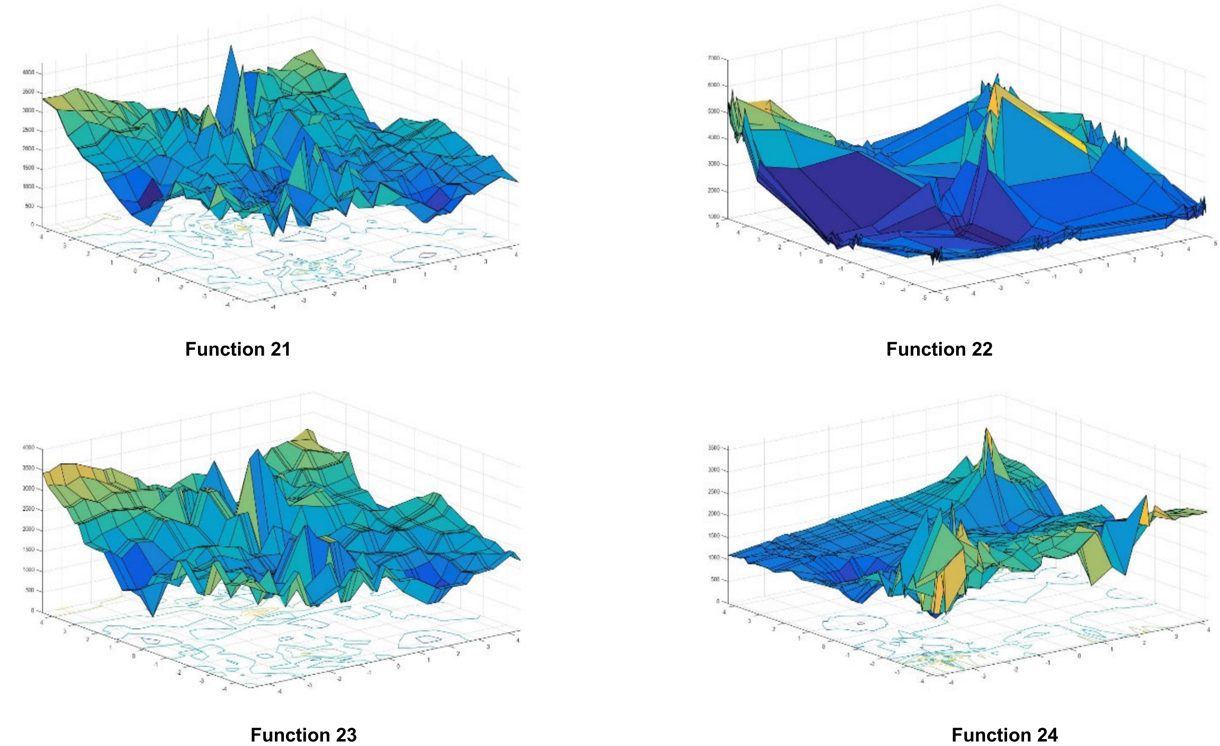

FIGURE 5. ***(Continued.)*** **Some functions graph using FCADE.**

(MBO) [73] and Slime Mould Algorithm (SMA) [74] are also used for comparison. The performance of these three algorithms has not been tested on WDN.

VI. RESULTS AND DISCUSSION

A. PERFORMANCE OF FCADE ON BENCHMARK FUNCTIONS

Performance of FCADE is first evaluated on CEC 2005 [62] benchmark functions for 10 dimensions. Table 8 shows the comparison of results obtained using FCADE and jDE, SaDE, and two classical variants of DE.

From the Table 8, it can be seen that FCADE performed better than jDE for 15 functions, SaDE for 11 functions, and beat both DE variants in 24 functions in terms of a mean error value. To further detect the significant differences between FCADE and the four competitors, Friedman's test was carried out. Friedman's test was implemented on SPSS software [75]. Table 9 summarizes the ranking of the five algorithms obtained through Friedman's test. As shown in Table 9, FCADE has the best ranking among the five algorithms on 25 test functions. Figure 5 depicts the graphs of the selected functions obtained using FCADE.

B. PERFORMANCE OF FCADE ON WDNS

The performance of FCADE is compared with previously used algorithms applied to the selected WDN problems. Also, according to the literature, as shown in Table 1, these problems have been popular among researchers for the application of various metaheuristics algorithms and can consequently be treated as test problems for WDN. All these problems vary in size and complexity according to the number of possible combinations, given in Table 7. The combinatorial nature of these problems and a small feasible region of the search space [38] poses a challenge for metaheuristics algorithms.

1) ANALYSIS OF NETWORK 1- TWO LOOP NETWORK

Table 10 shows a comparison of FCADE with other algorithms for Two Loop Network. The best cost obtained by all the algorithms is the same (= 419000), except for ABC, through which the cost is calculated as 429,000.

However, in terms of function evaluations, FCADE gave the best performance with NFE = 1732, which is around 15% better than the previously obtained best NFE (= 2051) by SLC.

The convergence graph of FCADE for Network 1 is provided in figure 6.

2) ANALYSIS OF NETWORK 2 - HANOI NETWORK

Performance analysis of Hanoi network is given in Table 11, from where it can be seen that all the algorithms gave similar best cost (= 6,081,087) except for MMAS and ABC for which the best cost obtained is 6,134,087 and 6,123,000 respectively, significantly higher than the cost obtained by other algorithms. In the case of average cost, FCADE and

TABLE 8. Comparison of mean error using FCADE with other DE variants.

Function	FCADE	jDE	SaDE	DE/rand	DE/best
F1	0.00E+00	0.00E+00	0.00E+00	3.78E-05	3.55E-05
F2	0.00E+00	6.01E-17	0.00E+00	4.98E-05	7.95E-07
F3	8.61E+03	4.15E-05	1.48E-25	3.14E+05	1.95E+05
F4	0.00E+00	2.36E-16	0.00E+00	6.14E-01	1.15E-02
F5	0.00E+00	2.40E-12	1.10E-07	2.70E+03	5.82E+02
F6	8.35E-06	8.37E-02	5.75E+00	5.06E+01	1.03E+01
F7	1.27E+03	1.27E+03	1.71E-02	1.66E+00	9.32E-01
F8	2.03E+01	2.04E+01	2.04E+01	3.59E+01	2.27E+01
F9	8.60E-01	0.00E+00	0.00E+00	5.60E+01	2.52E+01
F10	1.32E+01	1.17E+01	6.71E+00	1.81E+02	1.99E+02
F11	1.57E+00	5.39E+00	4.50E+00	3.82E+01	3.38E+01
F12	6.14E+00	5.82E+01	2.51E+03	9.05E+04	9.96E+04
F13	1.91E+00	4.25E-01	6.12E-01	4.13E+00	4.20E+00
F14	3.25E+00	3.31E+00	3.08E+00	1.34E+01	1.54E+01
F15	2.51E+02	2.27E+01	5.05E+01	4.90E+02	3.90E+02
F16	1.09E+02	1.10E+02	9.65E+01	2.85E+02	2.35E+02
F17	1.50E+02	1.30E+02	1.13E+02	3.10E+02	2.42E+02
F18	4.14E+02	5.40E+02	7.60E+02	9.21E+02	9.62E+02
F19	4.52E+02	5.80E+02	7.80E+02	9.53E+02	9.52E+02
F20	4.64E+02	6.80E+02	7.60E+02	9.24E+02	9.42E+02
F21	5.00E+02	5.28E+02	5.04E+02	5.91E+02	8.41E+02
F22	7.75E+02	7.63E+02	7.23E+02	9.65E+02	9.35E+02

TABLE 8. *(Continued.)* Comparison of mean error using FCADE with other DE variants.

F23	5.78E+02	6.14E+02	7.26E+02	6.65E+02	8.68E+02
F24	2.00E+02	2.00E+02	2.00E+02	3.30E+02	3.16E+02
F25	2.00E+02	1.74E+03	3.75E+02	9.88E+02	9.68E+02
+		15	11	24	24
-		7	10	1	1
≈		3	4	0	0

"+", "-", and "≈" denote that the performance of the corresponding algorithm is better than, worse than, and similar to that of FCADE, respectively.

TABLE 9. Ranking of FCADE, jDE, SaDE, DE/rand, and DE/best according to Friedman ranking test.

Algorithms	Mean Rank
FCADE	1.98
jDE	2.48
SaDE	2.10
DE_rand	4.34
DE_Best	4.10

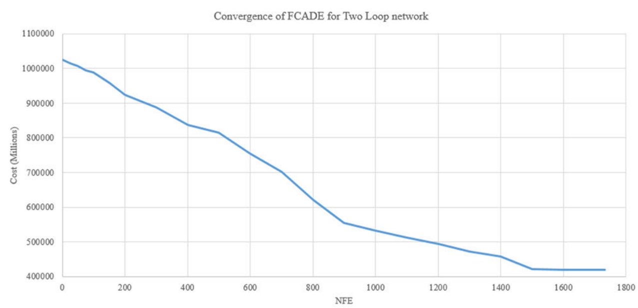

FIGURE 6. The convergence of FCADE in Network 1.

SLCb gave the best results ($= 6.081*10^6$). In terms of function evaluations, MMAS converged in NFE = 600.

FCADE converged to the best solution in 10,872 function evaluations which is almost 7 times lower than the NFE obtained by SLCb, the second-best algorithm in terms of NFE. The convergence rate of FCADE for Network 2 is shown in figure 7.

3) ANALYSIS OF NETWORK 3–NEW YORK TUNNEL NETWORK

Comparative analysis of FCADE vis-à-vis other algorithms is given in Table 12. Once again it can be seen that the average cost obtained by all the algorithms, including MMAS, is the

TABLE 10. Comparison of results of Network 1.

S.No.	Algorithm	Best cost ($)	Number of Function Evaluations
1	GA	419,000	65,000
2	SA	419,000	25,000
3	SFL	419,000	11,155
4	SCE	419,000	11,019
5	PSO	419,000	5138
6	DE	419,000	4750
7	HS	419,000	2891
8	ABC	429,000	2873
9	SS	419,000	3215
10	PSO+DE	419,000	3080
11	SLC	419,000	2051
12	FA-PSO	419,000	2591
13	MBO	419,000	2152
14	SMA	419,000	2097
15	FCADE	419,000	1732

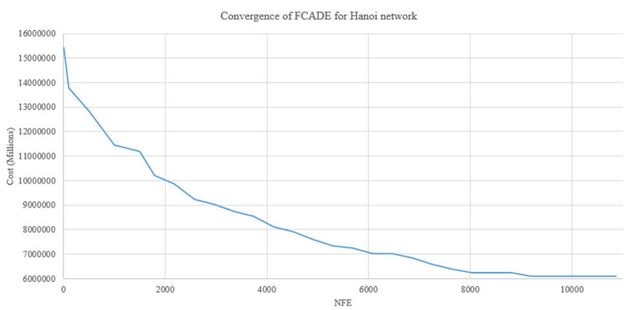

FIGURE 7. Convergence rate of FCADE in Network 2.

TABLE 11. Comparison of Results of Network 2.

S.No.	Algorithm	Best cost ($)	Average cost($)	Number of Function Evaluations
1	MMAS	6,134,087	6.38685	600
2	PSO	6,081,087	6.31	NA
3	HD-DDS	6,081,087	6.252	100000
4	GHEST	6,081,087	6.175	50,134
5	SS	6,081,087	NA	43,149
6	DE	6,081,087	NA	48,724
7	SsDE	6,081,087	6.09	60,532
8	SLCa	6,081,087	6.110	29,108
9	SLCb	6,081,087	6.081	71,789
10	ABC	6,123,000	NA	1,75,385
11	FA-PSO	6,081,087	6.081	30,127
12	MBO	6,081,087	6.250	31,708
13	SMA	6,081,087	6.141	35,337
14	FCADE	6,081,087	6.081	10,872

TABLE 12. Comparison of results of Network 3.

S.No.	Algorithm	Best cost Million ($)	Average cost($)	Number of Function Evaluations
1	MMAS	38.64	38.84	30700
2	PSO	38.64	NA	NA
3	HD-DDS	38.64	NA	47000
4	GHEST	38.64	NA	11464
5	SS	38.64	NA	57583
6	DE	38.64	NA	5494
7	SsDE	38.64	NA	6598
8	SLCa	38.64	38.81	7821
9	SLCb	38.64	38.64	15764
10	ABC	38.96	39.12	7530
11	FA-PSO	38.64	38.96	4793
12	MBO	38.64	38.75	4815
13	SMA	38.64	38.64	4460
14	FCADE	38.64	38.64	3631

same. Although, ABC in this case also, did not perform up to the mark like other algorithms. In terms of average best cost SCLb, SMA and FCADE were the best performers.

The lowest NFE (= 3631) is obtained by FCADE, which is around 18% less than the next lowest NFE obtained through SMA. The convergence graph of FCADE for Network 3 is depicted in figure 8.

An overall comparison of all the algorithms in terms of NFE for all the three WDNs is illustrated in Figure 9.

4) COMPARISON OF FCADE WITH MBO, SMA, AND FA-PSO

Besides comparing the performance of FCADE with the algorithms that have been used earlier on the three WDN problems, the authors also compared its performance with three recently proposed algorithms: Monarch Butterfly Optimization, Slime Mold Algorithm, and Hybrid Firefly and PSO algorithm. The results are given in Tables 10, 11, and 12 along with other algorithms. It was observed that, for Network 1, in terms of cost all the three algorithms performed equally good and gave the same result as that of FCADE. In terms of convergence, all the three algorithms gave the solution in less than 3000 iterations with SMA performing better than the other two algorithms. For Network 2 and

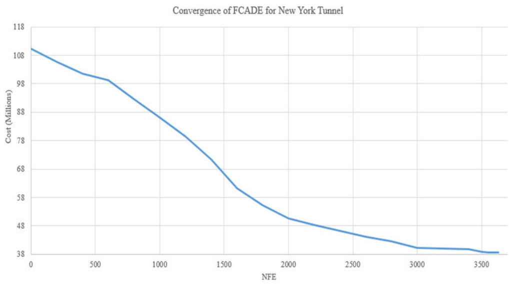

FIGURE 8. The convergence of FCADE in Network 3.

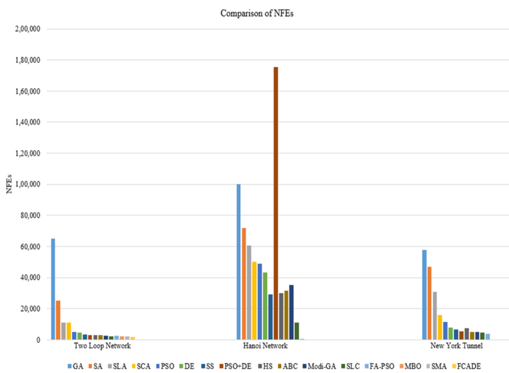

FIGURE 9. Comparison of NFEs of using all the listed algorithms.

Network 3 also, MBO, SMA, and FA-PSO performed either better or at par with the previously applied algorithms in terms of cost. However, FCADE still remained the top performer in terms of function evaluations.

VII. CONCLUSION

In this article, a Fuzzy C-means Adaptive Differential Evolution (FCADE) is proposed to optimize the total construction cost of three well-known WDN: Hanoi water distribution network; two-loop distribution network problem, and New York tunnel distribution network problem. The results are compared with several other Metaheuristics algorithms previously employed for solving these problems. Before applying FCADE for solving the abovementioned WDN problems, it is first validated on a set of 25 CEC2005 benchmark test suite.

Observations derived on the performance of FCADE are listed below:

1. For the CEC test suite, FCADE outperformed jDE in 15 functions, SaDE in 11 functions, and both the classical DE variants in 24 functions, and statistically, FCADE obtained the best rank among other DE variants taken for comparison.
2. In the case of WDNs, although the objective function cost was the same for all the algorithms barring MMAS and ABC for Networks 2 and 3.
3. The difference in the performance of algorithms was more prominent in terms of NFE, where FCADE emerged as a clear winner indicating it to be more computationally efficient in comparison to other algorithms.
4. Another noticeable argument in favor of FCADE is that when the complexity of the problem is increased, FCADE outperformed the other algorithms. For example, in the case of Hanoi WDN, where the number of possible combinations are $2.865*10^{26}$, FCADE gave the smallest cost for NFE which were around 84% lesser than the NFE of the best competitor algorithm (SLCa). This also indicates that FCADE is not sensitive towards the size or complexity of the problem.
5. FCADE performed better than the newly proposed metaheuristics like MBO, SMA and FA-PSO in terms of function evaluations.
6. The suggested modifications like clustering of the initial population and adapting the control parameters are generic in nature and can be implemented to any of the population-based metaheuristics to enhance its performance.

Thus, the overall performance of FCADE indicates that it has the capability to optimize various pipe networks with lesser computational efforts and may be considered as a suitable alternative optimizer for water distribution networks.

The present work may be extended for solving other WDN available in the literature and prepare a comprehensive study on the application of metaheuristics on different WDN having varying degrees of complexities. Secondly, the problem itself may be enhanced by considering the uncertainty of parameters or treating it as a multi-objective optimization problem.

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