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Distributed Predictive Control of Multi-Agent Systems Based on Error Upper Bound Approach

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ABSTRACT In this paper, a novel distributed predictive control (DMPC) method for multi-agent systems based on error upper bounds is proposed. To reduce the communication burden, the error upper bound condition between the subsystem and the neighbor subsystems is calculated by introducing the min-max function from the local state error of neighbouring subsystems. Additionally, an improved coupling constraints to describe the relationship between the neighbouring subsystems is introduced. Then, the proposed DMPC algorithm with kinds of the constraints is given, including the terminal cost, the terminal set and the terminal controller. Furthermore, the feasibility of the proposed DMPC algorithm is analyzed and the stability conditions of multi-agent systems are derived. Finally, a numerical example is given to verify the effectiveness of the method.

INDEX TERMS Distributed model predictive control, multi-agent systems, error upper bound.

I. INTRODUCTION

In recent years, with more and more in-depth research on multi-agent systems, it has been widely applied in military operations [1], [2], transportation [3], mobile sensor network [4], micro-grid [5] and many other fields. Therefore, the control problem of multi-agent systems has also become a research focus. In this scenario, Yang *et al.* studied the problem of consistency control of multi-agent systems [6]–[8]. In [6], [7], Yang *et al.* proposed a model-free distributed control method, and a new optimal control protocols for the distributed output synchronization problem of multi-agent systems was addressed in [8]. For the consistency issues of nonlinear systems, Liang *et al.* addressed the adaptive event-triggered neural control problem for nonaffine pure-feedback nonlinear multi-agent systems with dynamic disturbance, unmodeled dynamics, and dead-zone input [9] and the distributed observer-based event-triggered bipartite tracking control problem for stochastic nonlinear multi-agent systems with input saturation [10] separately, which can effectively reduce the communication burden. In the above

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papers, agents only exchange current information to achieve the objectives of cooperation, but have little knowledge of themselves and their neighbor agents in the future behavior. Due to the ability to effectively deal with constraints and enable agents to estimate the future behaviors of neighbors, distributed predictive control has become a widely used method for multi-agent systems. Wei *et al.* proposed a model predictive control method [11]–[13], which used the information between the agent systems in the future moment to solve the control problems, it can not only get the optimal state of the agent systems in the future, but also optimize the related cost function. However, the constraints between multi-agent systems are ignored. Although Zhi *et al.* came up with a way to synchronously update the state of the multi-agent systems, and set compatible constraints [14], [15], the cost and coupling constraint between multi-agent systems was not considered. Qian *et al.* [16] emphasized that each agent updated the control input in sequence, in this way, the cost and coupling constraint can be satisfied to a certain extent, while, the overall amounts of communication between controllers is quite large. Later, Jian *et al.* proposed a new control method to reduce the communication workload, where entire system only updated the state of one agent at each moment, and

other neighborhood used feasible solutions. While, the solution obtained after updating using this method was not optimal [17], [18].

In this work, a distributed model predictive control based on the error upper bound for multi-agent systems is studied. The main contributions of this article are as follows: (1) Different from [19], a local neighborhood error is adopted to construct a cost function, and then the error upper bound condition for the DMPC design is introduced by the min-max function characteristics to reduce the communication burden. (2) By utilizing the 2-norm form, the coupling constraint [20] is improved to represent the relationship between neighbouring subsystems. (3) Based on the condition related to the compatible constraints, the proposed DMPC algorithm guarantees the closed-loop system is asymptotically stable.

This article is organized as follows. The problem is formulated in Section II, In Section III, distributed predictive control for multi-agent systems is given, Section IV shows simulation examples to illustrate the effectiveness of the obtained results. Conclusions are discussed in Section V.

Notations: R^n and R^m represent n -dimensional and m -dimensional real space. $R^{n \times n}$ denote $n \times n$ -dimensional real matrix, $I_{1:N_a}$ is the set of integers from 1 to N_a , the sequence $x(a), x(a + 1), \dots, x(b)$ is denoted by $x(a : b)$, and $\lambda_{\min}(T)$, $\lambda_{\max}(T)$ represent the minimum and maximum eigenvalues of the positive definite matrix T . $s^T Q s$ is represented by $\|s\|_Q^2$ for a vector $s \in R^n$ and a positive definite matrix $Q \in R^{n \times n}$. For $X \subseteq R^n$, $Y \subseteq R^n$, $X \oplus Y = \{x + y | x \in X, y \in Y\}$ is the Minkowski sum, and $X \ominus Y = \{t \in R^n | \{t\} + Y \subseteq X\}$ is Pontryagin difference. b^T , b^{-1} are respectively used to represent the transpose and inverse of the matrix b .

II. PROBLEM FORMATION

Considering a multi-agent systems with multiple subsystems, the dynamic model of the i -th ($\forall i \in I_{1:N_a}$) subsystem [21] is as follow

$$x_i(k + 1) = Ax_i(k) + Bu_i(k) \quad (1)$$

with $x_i(k) \in R^n$ and $u_i(k) \in R^m$ representing the state and control input of the agent i at time k , and input constraints $u_i(k) \in U_i$, where $U_i \subseteq R^m$ is the compact set that contains the origin. $A \in R^{n \times n}$ and $B \in R^{n \times m}$ are constant matrices with the appropriate dimensions.

The network topology of all agents is represented by an undirected graph $\zeta = (\nu, \zeta, \mathcal{A})$, where $\nu = \{1, 2, \dots, N\}$ is the set of agents and $\zeta \subseteq ((i, j) \in \nu \times \nu | i \neq j)$ is the edge set. The element a_{ij} of the adjacency matrix \mathcal{A} is represented such that $a_{ij} \geq 0 \Leftrightarrow (i, j) \in \zeta$. Especially, it is noted that $a_{ij} = a_{ji}$ always holds in an undirected graph.

In addition, assume that N_i is the set of neighbor agents of agent i , and $|N_i|$ is the number of neighboring agents. If agent i and agent j are neighboring agents, then they can communicate with each other. Therefore, the control input $u_i(k)$ of the agent i can be described by

$$u_i(k) = u_{ii}(k) + \sum_{j \in \{c_{i,k} \cup \{i\}\}} K x_j(k) + \sum_{j \in \{N_i \setminus c_{i,k} \cup \{i\}\}} K x_j(k),$$

where $C_{i,k} \subseteq N_i$ is the set of neighbor agents that cooperate with agent i at time k , and $|C_{i,k}|$ represents the number of agents in the set. To calculate the error upper bound, a local error for each agent i at time k is defined as follow

$$\delta_i(k) = \sum_{j \in N_i} a_{ij}(x_i(k) - x_j(k)), \quad (2)$$

where $\delta_i(k) \in R^n$. By the new notation $m(k) = -B \sum_{j \in N_i} a_{ij} u_j(k)$, $\bar{B} = B \sum_{j \in N_i} a_{ij}$, and combining the above formulas (1)-(2), the dynamics of the local neighborhood error is given by

$$\delta_i(k + 1) = A\delta_i(k) + \bar{B}u_i(k) + m(k). \quad (3)$$

To facilitate the DMPC design, the following assumptions and lemmas are introduced.

Lemma 1 [22]: Assume that W_i^t and W_j^t respectively are the state sets of age i and age j . For all $t = 1, \dots, N$, $\forall i \in I_{1:N_a}$, the following state set relations are established

$$x_i(k + t | k + 1) - x_i(k + t | k) \in W_i^t, \quad (4)$$

$$X_i^t = X_i^{t-1} \ominus W_i^t, U_i^t = U_i^{t-1} \ominus KW_j^t, \quad \forall j \in N_i \cup \{i\}. \quad (5)$$

Assumption 1 [23]: (1) Assume that terminal function $\delta_i(k + T)$ is a Lyapunov function for each agent $i \in I_{1:N_a}$, and $P_i, Q_{ii}, R_{ii} \in R^{n \times n}$ are positive definite matrices, then there is a control input $u_i(k + T)$ such that

$$\begin{aligned} & \|\delta_i(k + T + 1)\|_{P_i}^2 - \|\delta_i(k + T)\|_{P_i}^2 \\ & \leq -\|\delta_i(k + T)\|_{Q_{ii}}^2 - \|u_i(k + T)\|_{R_{ii}}^2. \end{aligned} \quad (6)$$

(2) For $i \in I_{1:N_a}$, there exists a control input $u_i(\tau)$ such that every error sequence $\tilde{\delta}_i(\tau)$ compatible with control input $u_i(\tau)$ for all $\tau = k - L, \dots, k$, in the sense holds

$$\tilde{\delta}_i(1 + \tau) = A\tilde{\delta}_i(\tau) + \bar{B}u_i(\tau) + m(\tau). \quad (7)$$

Remark 1: The terminal cost $\|\delta_i(k + T)\|_{Q_{ii}}^2$ in (6) is regarded as a Lyapunov function for the close-loop system, which will gradually decrease with the system trajectory when the system has an appropriate control input $u_i(k + T)$. The compatibility means that the error sequence $\tilde{\delta}_i(\tau)$ and control input $u_i(\tau)$ can work together to make dynamics equations (7) be valid.

Assumption 2 [24], [25]: (1) For the the i -th subsystem (1) with control input $u_i(k) = Kx_i(k) \in U_i$, the terminal set $X_i^f \subseteq X_i$ is a positive invariant set, if there exist all $x_i(k) \in X_i^f$ such that $x_i(k + 1) \in X_i^f$.

(2) If set x_i^f, x_i^{f+1} and Δx_i^f are the elements in the terminal set X_i^f , then the following relationship is satisfied

$$x_i^f \oplus \Delta x_i^f \subseteq x_i^{f+1}. \quad (8)$$

(3) If there exists a state $x_i(k) \in X_i^{N-1}$, then $u_i(k) = Kx_i(k) \in U_i^{N-1}$.

Assumption 3 [26]: (1) For $\forall j \in C_{i,k}, \forall i \in I_{1:N_a}$, if the terminal controller $u_i(k + N | k) = K \sum_{j \in N_i} a_{ij} \varepsilon_i(k + N | k)$ is satisfied, then the cost of terminal is $\sum_{j \in N_i} a_{ij} \|\varepsilon_i(k + N | k)\|_P^2$,

and terminal set is $X_i^f = \{ \mathbf{x}_i | \sum_{j \in N_i} a_{ij} \|\varepsilon_i\|_P^2 \leq \varphi \}$, where φ is a scalar.

(2) If the system has an optimal solution at time k , the assumed control input at time $k + 1$ is defined as follow

$$\begin{aligned} & \hat{u}_i(k + t | k + 1) \\ &= \begin{cases} u_i^*(k + t | k), & t = 1, \dots, N - 1 \\ K(x_i^*(k + N | k), x_j^*(k + N | k)), & t = N \end{cases} \end{aligned} \quad (9)$$

(3) For all $i \in I_{1:N_a}$, $j \in C_{i,k}$, there exist the terminal control law $u_i(k + N | k) = K \sum_{j \in N_i} a_{ij} \varepsilon_i(k + N | k)$, and terminal cost function $\sum_{j \in N_i} a_{ij} \|\varepsilon_i(k + N | k)\|_P^2$ such that the following formula holds

$$\begin{aligned} & \sum_{j \in N_i} a_{ij} (\|\varepsilon_i(k + N + 1 | k)\|_P^2 - \|\varepsilon_i(k + N | k)\|_P^2) \\ & \leq - \sum_{j \in N_i} a_{ij} \|\varepsilon_i(k + N | k)\|_{Q_i}^2 - \|u_i(k + N | k)\|_{R_i}^2. \end{aligned} \quad (10)$$

In this paper, by introducing a new judgment condition, a new DMPC algorithm will be presented, and then the state of the agents asymptotically achieve consensus. The detailed presentation will be given in the next section.

III. DISTRIBUTED PREDICTIVE CONTROL FOR MULTI-AGENT SYSTEMS

In this section, a new distributed predictive control algorithm is designed by introducing an error upper bound condition considering the influence of neighbors. Beside this, the conditions for stability and feasibility are also designed. The detailed control scheme is unfolded below.

A. ERROR UPPER BOUND CONDITION

In this subsection, the error upper bound as a judgment condition is calculated for reducing communication cost, the calculation method is reported as follows.

A cost function about local neighborhood error can be defined as

$$\begin{aligned} & J_{ii}(\delta_i(k), u_i(k - L : k + T - 1)) \\ &= \sum_{\tau=k}^{k+T-1} \|\delta_i(\tau)\|_{Q_{ii}}^2 + \|\delta_i(k + T)\|_{P_i}^2 + \sum_{\tau=k-L}^{k+T-1} \|u_i(\tau)\|_{R_{ii}}^2 \end{aligned} \quad (11)$$

where P_i , Q_{ii} and R_{ii} are positive definite weighted matrices, L is the length of the past horizon, and T is the length of the future horizon. Based on (11), the optimization problem of the form is obtained as

$$J_{ii}^*(k) = \max_{\delta_i} J_{ii}(\delta_i, \mathbf{u}_i, \tilde{\mathbf{u}}_i^*) = \min_{\tilde{\mathbf{u}}_i} J_{ii}(\tilde{\delta}_i^*, \mathbf{u}_i, \tilde{\mathbf{u}}_i) \quad (12)$$

and for $\tau = k - L, \dots, k$

$$\tilde{\delta}_i(1 + \tau | k) = A\tilde{\delta}_i(\tau | k) + \bar{B}u_i(\tau) + m(k) \quad (13)$$

and for $\tau = k + 1, \dots, k + T - 1$

$$\tilde{\delta}_i(1 + \tau | k) = A\tilde{\delta}_i(\tau | k) + \bar{B}\tilde{u}_i(\tau | k) + m(k) \quad (14)$$

where $\tilde{\delta}_i = \tilde{\delta}_i(k | k)$, $\tilde{\delta}_i^* = \tilde{\delta}_i^*(k | k)$, $\mathbf{u}_i = u_i(k - L : k)$, $\tilde{\mathbf{u}}_i = \tilde{u}_i(k + 1 : k + T - 1 | k)$, $\tilde{\mathbf{u}}_i^* = \tilde{u}_i^*(k + 1 : k + T - 1 | k)$, $\tilde{\delta}_i^*$ and $\tilde{\mathbf{u}}_i^*$ are the corresponding optimal sequences respectively. $\tilde{\delta}_i(1 + \tau | k)$ and $\tilde{u}_i(\tau | k)$ are denoted as predicted values of the future states and control inputs.

Theorem 1: If the Assumption 1 is satisfied, then the following error upper bound condition holds

$$\begin{aligned} \|\delta_i(k)\|_{Q_{ii}}^2 & \leq J_{ii}^*(L) - \sum_{\tau=L}^{k-1} \|\hat{\delta}_i(\tau)\|_{Q_{ii}}^2 + \sum_{\tau=0}^{k-L-1} \|\hat{u}_i(\tau)\|_{R_{ii}}^2 \\ & - \sum_{\tau=k-L}^k \|u_i(\tau)\|_{R_{ii}}^2 \end{aligned} \quad (15)$$

Proof: With the help of [23], by letting $\delta_i(k + T) = \tilde{\delta}_i^*(k + T | k + 1)$, there exists a control input $\hat{u}_i(k + T) \in U_i$ satisfying

$$\begin{aligned} & \|\tilde{\delta}_i^*(k + T + 1 | k + 1)\|_{P_i}^2 - \|\tilde{\delta}_i^*(k + T | k + 1)\|_{P_i}^2 \\ & \leq - \|\tilde{\delta}_i^*(k + T | k + 1)\|_{Q_{ii}}^2 - \|\hat{u}_i(k + T)\|_{R_{ii}}^2 \end{aligned} \quad (16)$$

then, it is concluded from (7) that there exists a actual state vector $\hat{\delta}_i(k)$ satisfying $\tilde{\delta}_i^*(k + 1 | k + 1) = A\hat{\delta}_i(k) + \bar{B}u_i(k) + m(k)$. Combining (7) and (12) with control input $\tilde{u}_i(\tau | k + 1)$ for all $\tau = k + 1, \dots, k + T - 1$, leads to

$$J_{ii}^*(k + 1) = \min_{\tilde{\mathbf{u}}_i} J_{ii}(A\hat{\delta}_i(k) + \bar{B}u_i(k) + m(k), \mathbf{u}_i, \tilde{u}_i(\tau | k + 1))$$

where $\mathbf{u}_i = u_i(k - L + 1 : k)$. Next, by defining $\tilde{u}_i(\tau | k + 1) = \tilde{u}_i^*(\tau | k)$, $\tilde{u}_i(k + T | k + 1) = \hat{u}_i(k + T)$, it has

$$J_{ii}^*(k + 1) \leq J_{ii}(A\hat{\delta}_i(k) + \bar{B}u_i(k) + m(k), \hat{u}_i(k + T), \tilde{u}_i^*(\tau | k), u_i(k - L + 1 : k)) \quad (17)$$

Similarly, by setting $\tilde{\delta}_i = \hat{\delta}_i(k)$ for all $\tau = k + 1, \dots, k + T - 1$, it has

$$J_{ii}^*(k) \geq J_{ii}(\hat{\delta}_i(k), u_i(k - L : k), \tilde{u}_i^*(\tau | k)) \quad (18)$$

Furthermore, combining (17) and (18) leads to

$$\begin{aligned} & J_{ii}^*(k + 1) - J_{ii}^*(k) \\ & \leq J_{ii}(A\hat{\delta}_i(k) + \bar{B}u_i(k) + m(k), u_i(k - L + 1 : k), \\ & \quad u_i^*(\tau | k), \hat{u}_i(k + T)) \\ & \quad - J_{ii}(\hat{\delta}_i(k), u_i^*(\tau | k), u_i(k - L : k)). \end{aligned} \quad (19)$$

By substituting (11) into (19), it gives

$$J_{ii}^*(k + 1) - J_{ii}^*(k) \leq - \|\hat{\delta}_i(k)\|_{Q_{ii}}^2 - \|\hat{u}_i(k - L)\|_{R_{ii}}^2 \quad (20)$$

and then (21) can be satisfied

$$J_{ii}^*(k) \leq J_{ii}^*(L) - \sum_{\tau=L}^{k-1} \|\hat{\delta}_i(\tau)\|_{Q_{ii}}^2 + \sum_{\tau=0}^{k-L-1} \|\hat{u}_i(\tau)\|_{R_{ii}}^2 \quad (21)$$

In addition, by letting $\tilde{\delta}_i = \tilde{\delta}_i(k)$, it can be concluded that

$$J_{ii}^*(k) = \max_{\tilde{\delta}_i} J_{ii}(\tilde{\delta}_i, \mathbf{u}_i, \tilde{\mathbf{u}}_i^*) \geq J_{ii}(\delta_i(k), u_i(k - L : k), \mathbf{0})$$

which can be changed into the following form

$$J_{ii}^*(k) \geq \|\delta_i(k)\|_{Q_{ii}}^2 + \sum_{\tau=k-L}^k \|u_i(\tau)\|_{R_{ii}}^2 \quad (22)$$

Finally, by combining (21) and (22), the (15) is achieved and then the proof is complete.

Different from the results of the reference [23], the error upper bound condition (15) is introduced, related to the past values of the cost function $J_{ii}^*(L)$ and control inputs $u_i(\tau)$.

Remark 2: It should be noted from (15) that the error upper bound is considered as a judgement condition. Only when the state error between the subsystem and its neighboring subsystems violates the error upper bound condition, the neighboring subsystems can be activated and participate in the solution of the following DMPC algorithm, which reduces the communication burden between the controllers of subsystems.

B. A NEW DMPC DESIGN

Based on the above analysis of the error upper bound condition, the new DMPC scheme is proposed in this subsection. Firstly, the cost function of state sequence $\mathbf{x}_i(\cdot | \mathbf{k})$ and an input sequence $\mathbf{u}_i(\cdot | \mathbf{k})$ with the predictive horizon N for each agent i is defined as

$$\begin{aligned} J_i(k) &= \sum_{t=0}^{N-1} \sum_{j \in N_i} a_{ij} \|x_i(k+t|k) - x_j(k+t|k)\|_{Q_i}^2 \\ &\quad + \sum_{j \in N_i} a_{ij} \|x_i(k+N|k) - x_j(k+N|k)\|_P^2 \\ &\quad + \|u_i(k+t|k)\|_{R_i}^2 \\ &= \sum_{t=0}^{N-1} \sum_{j \in N_i} a_{ij} \|\varepsilon_i(k+t|k)\|_{Q_i}^2 + \|u_i(k+t|k)\|_{R_i}^2 \\ &\quad + \sum_{j \in N_i} a_{ij} \|\varepsilon_i(k+N|k)\|_P^2 \end{aligned}$$

and the cost function of whole system is denoted as

$$J(k) = \sum_{i=1}^{N_a} J_i(k) \quad (23)$$

where $\mathbf{x}_i(\cdot | \mathbf{k}) = \{x_i(k|k), x_i(k+1|k), \dots, x_i(k+N|k)\}$ and $\mathbf{u}_i(\cdot | \mathbf{k}) = \{u_i(k|k), u_i(k+1|k), \dots, u_i(k+N|k)\}$. Q_i, R_i, P are positive definite weighted matrices, and $\varepsilon_i(k+t|k)$ represents the state deviation between the agent i and its neighboring agents. In the process of solving the optimization problem, the assumed predictive states of agents are transmitted to its neighboring agents, hence, the standard DMPC optimization problem is given by

$$J_i^*(k) = \min_{u_i(k+t|k)} J_i(k, x_i(k), \hat{x}_j(k), u_i(k)) \quad (24)$$

$$s.t. : \quad i \in I_{1:N_a}, \quad j \in C_{i,k}, \quad t = 0, \dots, N-1$$

$$x_i(k+t|k) \in X_i^t \quad (25a)$$

$$u_i(k+t|k) \in U_i^t \quad (25b)$$

$$x_i(k+N|k) \in x_i^f \subset X_i \quad (25c)$$

$$\|x_i^*(k+t|k) - \hat{x}_i(k+t|k)\| \leq \xi_i(k) \quad (25d)$$

$$\begin{aligned} x_i(k+t+1|k) &= Ax_i(k+t|k) \\ &\quad + Bu_i(k+t|k) \end{aligned} \quad (25e)$$

$$X_i^t = \left\{ \left\{ X_i^0 \right\}, \left\{ X_i^1 \right\}, \dots, \left\{ X_i^{N-1} \right\} \right\} \subseteq X_i \quad (25f)$$

$$U_i^t = \left\{ \left\{ U_i^0 \right\}, \left\{ U_i^1 \right\}, \dots, \left\{ U_i^{N-1} \right\} \right\} \subseteq U_i \quad (25g)$$

$$X_i^f = \left\{ \left\{ x_i^f \right\}, \left\{ \Delta x_i^f \right\}, \left\{ x_i^{f+1} \right\} \right\} \subseteq X_i \quad (25h)$$

$$Q_{ij} = \sum_{j \in N_i} a_{ij} \left\| \sum_{i \in 1+|N_i|} \beta_{ij} \varepsilon_i \right\|^2 \leq q_{ij} \quad (25i)$$

Different from the reference [20] where the coupling constraint was a simple description about the relationships between neighbouring agents, in this paper, a new description of the coupling constraint (25i) is constructed, which takes the 2-norm form to make states in terminal set meet the constraint and which is also one of the conditions to ensure the feasibility of the optimization problem of DMPC. However, before solving the optimization problem (24), the issues about terminal cost, terminal set and terminal controller should be considered. To this end, the following theorem is given.

Theorem 2: Suppose that Assumption 3 holds. If there exist matrices φ and P such that the following conditions (26-27) are satisfied, then X_i^f for all agent $i \in I_{1:N_a}$ is a positive definite invariant set. And the states in the terminal set also satisfy the coupling constraint (25i).

$$\begin{bmatrix} X & * & * & * \\ AX + BY & X & * & * \\ Q_i^{1/2} X & 0 & I & * \\ R_i^{1/2} Y & 0 & 0 & I \end{bmatrix} \geq 0 \quad (26)$$

$$\begin{aligned} \varphi &= \min \lambda_{\min}(P) \times \frac{q_{ij}}{2 \times (1 + |N_i|)} \\ &\quad \times |\beta_{ij}|^2 \end{aligned} \quad (27)$$

Proof: Firstly, the formula (10) can be written as

$$\begin{aligned} \sum_{j \in N_i} a_{ij} [\varepsilon_i(k+N|k)]^T ((A+BK)^T P(A+BK) - P \\ + Q_i + K^T R_i K) \varepsilon_i(k+N|k) \leq 0. \end{aligned} \quad (28)$$

By letting $X = P^{-1} > 0, Y = KX$ [27], and multiplying P^{-1} on the left and right sides of the (28), the inequality (26) is obtained. In this case, the terminal set is denoted as $X_i^f = \left\{ \mathbf{x}_i | \sum_{j \in N_i} a_{ij} \|\varepsilon_i\|_P^2 \leq \varphi \right\}$ under the corresponding input $u_i(k+N|k) = K \sum_{j \in N_i} a_{ij} \varepsilon_j(k+N|k)$.

Furthermore, for all $i \in \{1 + |N_i|\}, j \in N_j$, based on the ideal of the reference [19], the terminal set X_i^f indicates

$$\sum_{j \in N_i} a_{ij} \|\varepsilon_i\|_P^2 \leq \min \lambda_{\min}(P) \times \frac{q_{ij}}{2 \times (1 + |N_i|) \times |\beta_{ij}|^2}$$

Relying on the above formulas, it has

$$\frac{2|\beta_{ij}|^2 \sum_{j \in N_i} a_{ij} \|\varepsilon_i\|_P^2}{\lambda_{\min}(P)} \leq \frac{q_{ij}}{1 + |N_i|}$$

and

$$2 \sum_{j \in N_i} a_{ij} \|\beta_{ij} \times \varepsilon_i\|^2 \leq \frac{q_{ij}}{1 + |N_i|}$$

Furthermore, it gives

$$2 \sum_{i \in 1+|N_i|} \sum_{j \in N_i} a_{ij} \|\beta_{ij} \times \varepsilon_i\|^2 \leq q_{ij}$$

Finally, the above formulas can be changed into

$$\sum_{j \in N_i} a_{ij} \left\| \sum_{i \in 1+|N_i|} \beta_{ij} \times \varepsilon_i \right\|^2 \leq q_{ij}$$

which implies that the states in terminal set X_i^f satisfy the coupling constraint. This proof is complete.

As for the optimization problem (24), the nonlinear programming solver can be adopted, because of the nonlinear coupling constraint (25i). In addition, different from the other methods such as adaptive controls [28], [29] which can not be effectively handle the kinds of the constraints, the DMPC algorithm proposed in this paper is an online optimization algorithm which can deal with the constraints online. Then, the implementation of the proposed DMPC approach is summarized as follows.

Algorithm

Require: X_i^f , P and K

Initialization $x_i(0)$ and $k = 0$

Repeat

- (1): If the condition (15) is not satisfied, then $C_{i,k} = C_{i,k} \cup \{j\}$
 - (2): For $t \in [0, N - 1]$, calculate $\hat{u}_i(k + t|k)$ and $\hat{x}_i(k + t|k)$.
 - (3): Send $\hat{x}_i(k + t|k)$ to the neighboring agents j and receive $\hat{x}_j(k + t|k)$ from neighboring agent j
 - (4): Calculate $\Psi_i(k)$ and $\omega_i(k)$ according to (29) and (30).
 - (5): For $\forall j \in N_i$, and $t \in [0, N - 1]$, the measured state $x_j(k)$ is sent to the controller to solve the optimization problem(24), and then the optimal solution $u_i(k) = u_i^*(k|k)$.
 - (6): Otherwise $C_{i,k} = C_{i,k}$, then go to step (2).
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Remark 3: Whether the subsystem communicates with the neighboring subsystems is judged according to the error upper bound condition, therefore, the assumed state information of agent i is required to exchange with its neighbor agent j . That is, agent i receives its own information and the relative local information $(x_i, x_j, x_i - x_j)$.

Remark 4: It was shown from the references [30], [31] that the theory about the finite-time stability is usually utilized to derive an upper limit on the system convergence time, then the system can converge in a finite-time under the control strategy. However, the finite-time consistency problem of multi-agent systems is not considered, only the asymptotical consistency problem is studied in this paper, because of some

difficult in the combination between the DMPC algorithm and the finite-time strategy. In the future work, the proposed DMPC method combining the finite-time stability will be extended to the the consistency problem of multi-agent systems.

C. FEASIBILITY AND STABILITY ANALYSIS

In this section, the feasibility of the above algorithm will be verified, and then the results on the feasibility of the proposed algorithm and the closed-loop stability of multi-agent systems are derived in the following theorems.

Theorem 3: Suppose that Assumptions 2 and 3 hold. If the initial state $x_i(0)$ is chosen to be a solution of the optimization problem (24), and X_i^f is terminal set, then the optimization problem admits feasible solutions for all $k \geq 0$.

Proof: For $t \in [1, N]$, $i \in I_{1:N_a}$, by the fact that Lemma 1 and state constraints in (25), it has

$$x_i(k + t|k + 1) \in x_i(k + t|k) \oplus W_i^t.$$

Furthermore, together with (5), the following formulas holds

$$x_i(k + t|k + 1) \in (X_i^{t-1} \ominus W_i^t) \oplus W_i^t \subseteq X_i^{t-1} \subseteq X_i.$$

Similarly, the following result can be obtained

$$x_i(k + N|k + 1) \in X_i^{N-1}.$$

As for the control input at time k , it has

$$u_i(k + t|k) = u_{ii}(k + t|k) + \sum_{j \in \{c_{i,k} \cup \{i\}\}}^K x_j(k + t|k) + \sum_{j \in \{N_i \setminus c_{i,k} \cup \{i\}\}}^K x_j(k + t|k).$$

At time $k + 1$, it can be proved that

$$u_i(k + t|k + 1) = u_{ii}(k + t|k + 1) + \sum_{j \in \{c_{i,k} \cup \{i\}\}}^K x_j(k + t|k + 1) + \sum_{j \in \{N_i \setminus c_{i,k} \cup \{i\}\}}^K x_j(k + t|k + 1).$$

Since $u_i(k + t|k) \in U_i^t$, then it follows

$$u_i(k + t|k + 1) \in (U_i^{t-1} \ominus KW_j^t) \oplus KW_j^t, \quad j \in N_i \cup \{i\}$$

and $u_i(k + t|k + 1) \in U_i^{t-1}$. When Assumption 2 holds, it has

$$u_i(k + N|k + 1) \in U_i^{N-1} \subseteq U_i$$

and

$$x_i(k + N|k + 1) \in x_i(k + N|k) \oplus W_i^N \subseteq x_i^{f+1}.$$

Additionally, the control input $u_i(k + N|k + 1) \in U_i$ enforces $x_i(k + N + 1|k + 1) \in x_i^{f+1}$. By induction, the optimization problem (24) admits feasible solutions for all $k \geq 0$.

Next, the stability conditions of the closed-loop system are analyzed. With the help of [32], the following new variables are defined:

$$\Psi_i(k) = \max \|\hat{x}_i(k + t|k) - \hat{x}_j(k + t|k)\| \quad (29)$$

which represents the maximum value of the assumed trajectory deviation between agent i and neighboring agent j at time k , and

$$\omega_i(k) = \min \|\hat{x}_i(k+t|k) - \hat{x}_j(k+t|k)\| \quad (30)$$

which represents the minimum value of the assumed trajectory deviation between agent i and neighboring agent j at time k , and

$$\Delta_1(k) = (2(\xi_i(k) + \Psi_i(k)) \times \xi_j(k) + \xi_i^2(k)) \times ((N-1)\lambda_{\max}(Q_i) + \lambda_{\max}(P)) \quad (31)$$

$$\Delta_2(k) = \lambda_{\min}(Q_i)(\omega_i(k) - \xi_i^2(k)) \quad (32)$$

which are presented here to ensure the stability of the system. Then, a detailed statement for the stability results of multi-agent systems are presented.

Theorem 4: Suppose Assumption 3 holds, if there exists a condition $\Delta_1(k) \leq \Delta_2(k)$, and if optimization problem (24) is feasible by implementing $u_i(k) = u_i^*(k|k)$ at time $k = 0$, then multi-agent systems are asymptotically stable.

Proof: For all $t \in [1, N]$, the control inputs $\hat{u}_i(k+t|k+1)$ are feasible solution of (24) at time $k+1$. Furthermore, the resulting cost function is written as $\tilde{J}_i(k+1)$, and the cost function derived from the optimal control inputs $u_i^*(k+t|k+1)$ is denoted as $J_i^*(k+1)$. In that case, it can be inferred that

$$\sum_{i=1}^{Na} J_i^*(k+1) - \sum_{i=1}^{Na} J_i^*(k) \leq \sum_{i=1}^{Na} \tilde{J}_i(k+1) - \sum_{i=1}^{Na} J_i^*(k)$$

However it has that

$$\begin{aligned} & \sum_{i=1}^{Na} \tilde{J}_i(k+1) - \sum_{i=1}^{Na} J_i^*(k) \\ &= \sum_{i=1}^{Na} \sum_{t=0}^{N-2} \sum_{j \in N_i} a_{ij} \|\hat{\varepsilon}_i(k+t+1|k+1)\|_{Q_i}^2 \\ &+ \|\hat{u}_i(k+t+1|k+1)\|_{R_i}^2 - \sum_{i=1}^{Na} \sum_{j \in N_i} a_{ij} \|\varepsilon_i^*(k|k)\|_{Q_i}^2 \\ &+ \|u_i^*(k|k)\|_{R_i}^2 - \sum_{i=1}^{Na} \sum_{t=1}^{N-1} \sum_{j \in N_i} a_{ij} \|\varepsilon_i^*(k+t|k)\|_{Q_i}^2 \\ &+ \|u_i^*(k+t|k)\|_{R_i}^2 - \sum_{i=1}^{Na} \sum_{j \in N_i} a_{ij} \|\varepsilon_i^*(k+N|k)\|_P^2 \\ &+ \sum_{i=1}^{Na} \sum_{j \in N_i} a_{ij} \|\hat{\varepsilon}_i(k+N|k+1)\|_{Q_i}^2 \\ &+ \|\hat{u}_i(k+N|k+1)\|_{R_i}^2 \\ &+ \sum_{i=1}^{Na} \sum_{j \in N_i} a_{ij} \|\hat{\varepsilon}_i(k+N+1|k+1)\|_P^2 \\ &\leq \sum_{i=1}^{Na} \sum_{t=0}^{N-2} \sum_{j \in N_i} a_{ij} \|\hat{\varepsilon}_i(k+t+1|k+1)\|_{Q_i}^2 \end{aligned}$$

$$\begin{aligned} &+ \|\hat{u}_i(k+t+1|k+1)\|_{R_i}^2 - \sum_{i=1}^{Na} \sum_{j \in N_i} a_{ij} \|\varepsilon_i^*(k|k)\|_{Q_i}^2 \\ &+ \|u_i^*(k|k)\|_{R_i}^2 - \sum_{i=1}^{Na} \sum_{t=1}^{N-1} \sum_{j \in N_i} a_{ij} \|\varepsilon_i^*(k+t|k)\|_{Q_i}^2 \\ &+ \|u_i^*(k+t|k)\|_{R_i}^2 - \sum_{i=1}^{Na} \sum_{j \in N_i} a_{ij} \|\varepsilon_i^*(k+N|k)\|_P^2 \\ &+ \sum_{i=1}^{Na} \sum_{j \in N_i} a_{ij} \|\hat{\varepsilon}_i(k+N|k+1)\|_P^2 \\ &\leq \sum_{i=1}^{Na} \sum_{t=1}^{N-1} \sum_{j \in N_i} a_{ij} (\lambda_{\max}(Q_i) 2 (\|x_i^*(k+t|k) \\ &- \hat{x}_j(k+t|k)\| + \|\hat{x}_i(k+t|k) - \hat{x}_j(k+t|k)\|) \\ &\times (\|\varepsilon_j^*(k+t|k)\| + \|\varepsilon_j^*(k+t|k)\|^2)) \\ &+ \sum_{i=1}^{Na} \sum_{j \in N_i} a_{ij} \lambda_{\min}(Q_i) (\|\hat{x}_i(k|k) \\ &- \hat{x}_j(k|k)\| - \|x_i^*(k|k) - \hat{x}_j(k|k)\|^2) \\ &+ \sum_{i=1}^{Na} \sum_{j \in N_i} a_{ij} \lambda_{\max}(P) (\|\varepsilon_j^*(k+N|k)\|^2 \\ &+ \|\varepsilon_j^*(k+N|k)\| (2 (\|x_i^*(k+N|k) - \hat{x}_j(k+N|k)\| \\ &+ \|\hat{x}_i(k+N|k) - \hat{x}_j(k+N|k)\|))) \quad (33) \end{aligned}$$

Substituting (29-32) into (33), it leads to

$$\begin{aligned} & \sum_{i=1}^{Na} J_i^*(k+1) - \sum_{i=1}^{Na} J_i^*(k) \\ &\leq \sum_{i=1}^{Na} \sum_{j \in N_i} a_{ij} (2(\xi_i(k) + \Psi_i(k))\xi_j(k) + \xi_j^2(k)) \\ &\times (\lambda_{\max}(Q_i)(N-1) + \lambda_{\max}(p)) - \lambda_{\min}(Q_i) \\ &\times (\omega_i(k) - \xi_i^2(k)) \\ &\leq 0. \end{aligned}$$

It is worth noting that the above results demonstrate that the whole closed-loop system is asymptotic stability. In the proof procedure the compatibility constraint (25d) is related to the stability of the whole system.

IV. NUMERICAL EXAMPLE

To verify the effectiveness of the DMPC algorithm, a multi-agent system consisting of five inverted pendulum systems [33] is considered. The communication topology \mathcal{G} among agents is shown in Fig. 1.

In Fig. 2, the mass of vehicle and pendulum represent M and m respectively. l is the distance from the rotation point of the pendulum to the center of gravity, x is the position of the car, the force given to the car in the x direction is denoted as u , and θ is the angle of the pendulum away from the vertical direction.

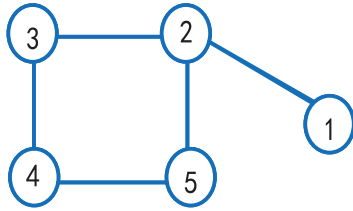


FIGURE 1. Connected topology ζ .

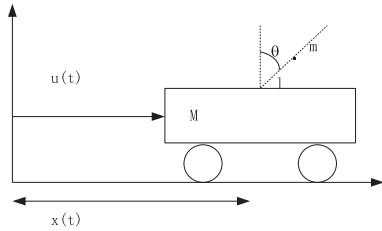


FIGURE 2. Inverted pendulum system.

In addition, presume that the pendulum is thought as a thin rod and its surface is smooth without friction. Therefore, through Newton's second law, the following kinematic model is obtained

$$\begin{aligned} (M + m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta &= u \\ ml\ddot{x}\cos\theta + \frac{4}{3ml^2}\ddot{\theta} - mgl\sin\theta &= 0 \end{aligned} \quad (34)$$

where g is the acceleration due to gravity, and the state variable $z = [z_1 \ z_2]^T = [\theta \ \dot{\theta}]^T$ of the system is selected. By linearizing the kinematic model (34) at the equilibrium point, the following continuous-time system model can be obtained that

$$\begin{aligned} \dot{z}(t) &= \begin{bmatrix} 0 & 1 \\ 3(M + m)g/l(4M + m) & 0 \end{bmatrix} z(t) \\ &+ \begin{bmatrix} 0 \\ -3/l(4M + m) \end{bmatrix} u(t) \end{aligned}$$

with $M = 8.0\text{kg}$, $m = 2.0\text{kg}$, $l = 0.5\text{m}$, $g = 9.8\text{m/s}^2$. When the sampling period is chosen as 1s, the discrete-time system model (35) is derived as

$$x(k + 1) = \begin{bmatrix} 1.0078 & 0.0301 \\ 0.5202 & 1.0078 \end{bmatrix} x(k) + \begin{bmatrix} -0.0001 \\ -0.0053 \end{bmatrix} u(k) \quad (35)$$

In the DMPC algorithm, the terminal feedback control law K and the weight matrix P in terminal cost from formula (26) with $Q_i = I$ and $R_i = 0.1$ can be computed as

$$\begin{aligned} K &= \begin{bmatrix} 221.8937 & 53.4029 \end{bmatrix} \\ P &= \begin{bmatrix} 9105.00 & 2177.59 \\ 2177.59 & 528.54 \end{bmatrix} \end{aligned}$$

Assume that the initial states are $x_1(0) = [0.6 \ -0.8]^T$, $x_2(0) = [0.8 \ -0.712]^T$, $x_3(0) = [-0.378 \ 0.641]^T$,

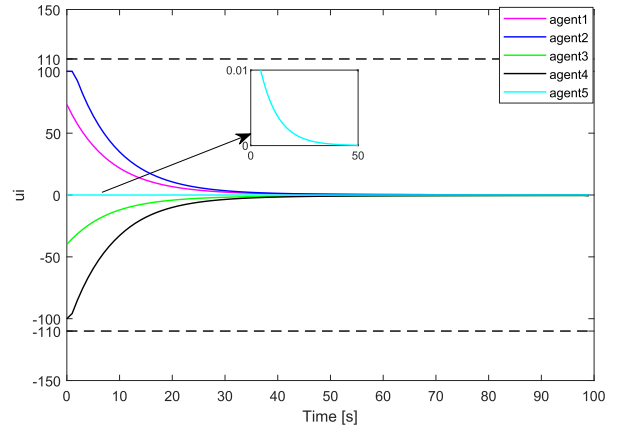


FIGURE 3. Control input trajectory of 5 agents.

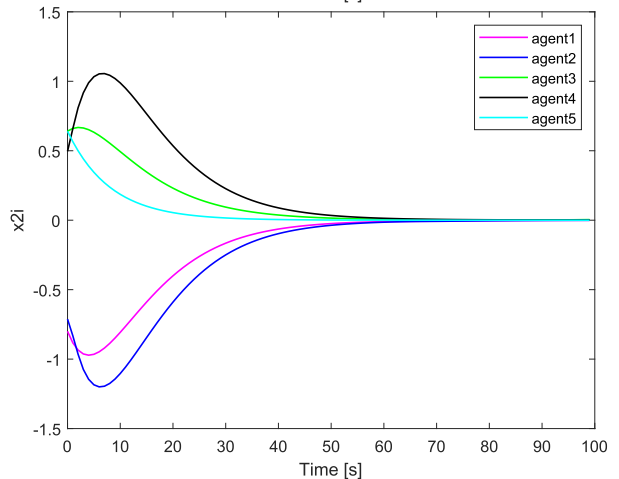
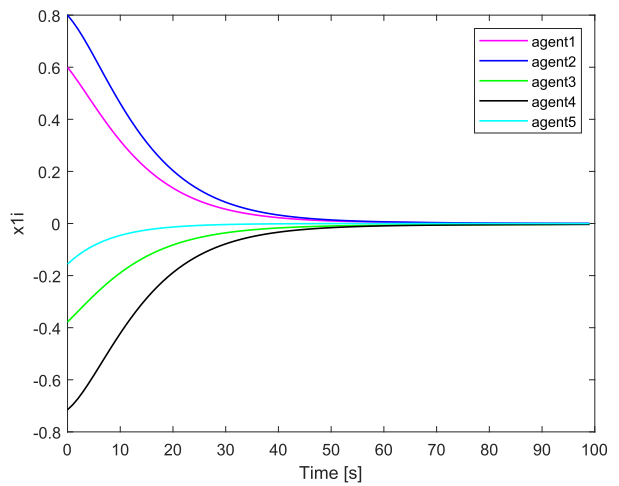


FIGURE 4. The state trajectory of 5 agents.

$x_4(0) = [-0.715 \ 0.496]^T$, $x_5(0) = [-0.156 \ 0.64]^T$, with input constraint $u_i(k) \in U_i = \{u_i(k) \in R^m \mid -110 \leq u_i(k) \leq 110\}$, and the prediction horizon taken as $N = 14$. The results of the experiment are presented in Figs.3-7.

Figs. 3-4 respectively show the control input and state trajectories of the multi-agent systems. It can be seen that when the control strategy in this paper is utilized and the

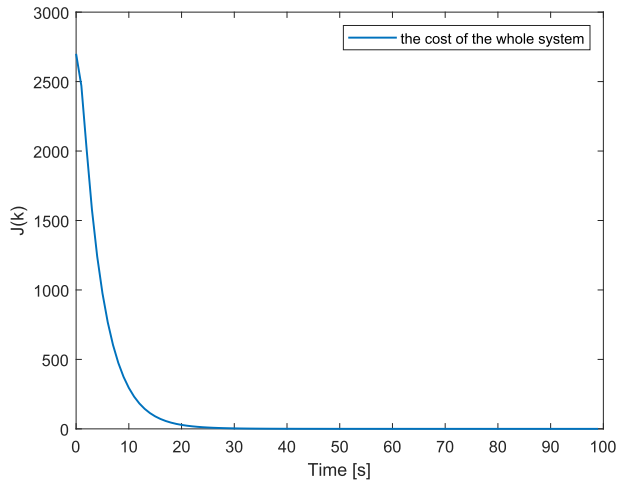


FIGURE 5. The cost of the whole system.

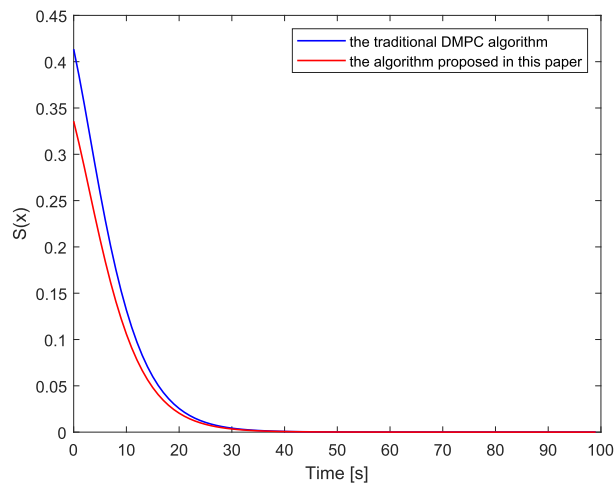


FIGURE 6. The trajectory of $S(x)$ under two different algorithms.

control input meet the constraint, the states of five agents starting from different positions can asymptotically reach a consistent state.

By solving the quadratic programming problem (24), the optimal control input and optimal state is obtained, and then the cost of the whole system computed by (23) monotonically decreases to 0, which means that the control object is achieved. The result is shown in Fig.5

In addition, for $i \in \{C_{i,k} \cup \{i\}\}$, the proposed DMPC algorithm is compared with traditional DMPC algorithm [19] in terms of convergence speed and optimization times. As well as [34], a function $S(x) = \sum \|x_i\|^2 / |\{C_{i,k} \cup \{i\}\}|$ is defined to represent the convergence speed. As shown in Fig.6, the trajectories of $S(x)$ decrease with both the algorithms in the same predictive time domain, however, the trajectory of $S(x)$ under the proposed DMPC algorithm with upper error bound converges faster.

Next, to further clarify the advantages of the proposed control strategy, the optimization times of the optimization problem for five agents using two algorithms are compared.

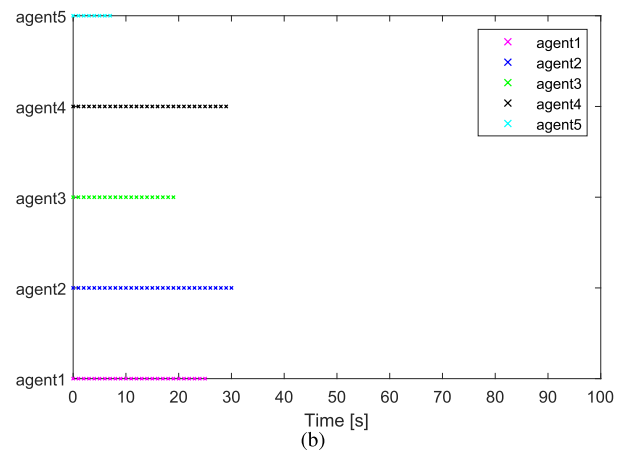
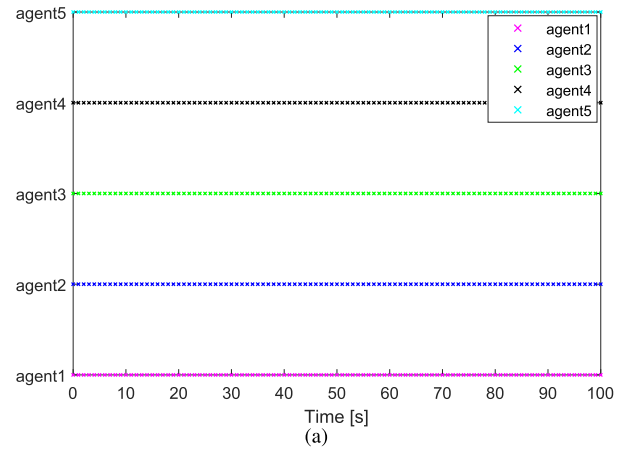


FIGURE 7. Optimization time of 5 agents under algorithm given in [19] and algorithm in this paper.

The result from the DMPC algorithm without the error upper condition is shown in Fig.7 (a), where each agent participates in the optimization solution through the whole predictive horizon. However, the results in Fig. 7(b) indicates that the agents only selectively participate in the optimization solution. Obviously, there is a significant advantage in the terms of the algorithm proposed in this paper, which also implies a significant reduction in computation burden and communication costs.

V. CONCLUSION

In this paper, a new DMPC scheme with an upper bound condition is developed for the consistency of multi-agent systems. Based on neighbor agents relative information, a upper bound condition is derived to reduce the communication burden. Moreover, the detailed conditions on ensuring the feasibility and closed-loop stability have been proposed after fully considering the constraints. Finally, simulation studies have verified the advantages of the proposed algorithm.

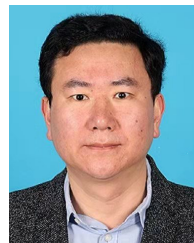
In our future study, we will consider the application of adaptive model predictive control in uncertain system [28], [29].

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