Received December 30, 2020, accepted January 10, 2021, date of publication January 14, 2021, date of current version January 21, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3051719

# Spectrum Sensing With Non-Gaussian Noise Over Multi-Path Fading Channels Towards Smart Cities With IoT

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This work was supported in part by the Aeronautical Science Foundation under Grant 2019ZH0T7001, and in part by the Scientific Research Foundation of Xi'an Aeronautical University under Grant 2019KY0207.

**ABSTRACT** As the limited communication spectrum can not meet the demand of the exponential growth of intelligent connected devices in the internet of things(IoT) and typical smart city applications, in this paper, we propose a tractable spectrum sensing method based on Rao detection over non-Gaussian noise, such as generalized Gaussian noise(GGN), Gaussian mixture noise(GMN) and symmetric alpha-stable distribution (S $\alpha$ S) noise, multi-path fading channels environment to alleviate the issue of spectrum scarcity. In this method, there are unknown parameters in the multi-path fading channels. When the probability density function (P.D.F.) of non-Gaussian noise has a closed-form expression, the spectrum sensing method based on Rao detection is used. Otherwise the P.D.F. for S $\alpha$ S noise is estimated firstly by using non-parametric kernel estimation method, which addresses the issue that S $\alpha$ S noise has no closed-form P.D.F. expression, and then the performance of spectrum sensing is derived based on the theory of Rao detection in multi-path fading channels over typical smart city applications. Simulation results show that the accuracy of estimated P.D.F. for S $\alpha$ S noise and the performance of spectrum sensing under different  $\alpha$  values over indoor, outdoor, and vehicle fading channels environment.

**INDEX TERMS** Spectrum sensing, non-Gaussian noise, multi-path fading channels, Rao detection, smart cities.

#### I. INTRODUCTION

Wireless communications play an increasingly prominent role in the internet of things(IoT) and typical smart city applications, since they are able to provide ubiquitous and transparent service [1]–[4]. The IoT will be able to connect countless devices, which can range from toothbrushes and lights to people and animals, from cars and homes to smart offices. It will therefore have a major impact on many aspects of smart cities, such as employment, healthcare and transportation. Both the IoT technology and the development of smart cities need to realize the perception, collection and interaction of large amounts of data through wireless communication.

The associate editor coordinating the review of this manuscript and approving it for publication was Kun Wang<sup>(D)</sup>.

With the exponential growth of the number of smart connected devices, and more spectrum resources will be needed. However, spectrum resources are limited and do not increase with the increase of connected devices. The limited availability of the communication spectrum therefore is one of the challenges which hinder the massive deployment of smart city based IoT systems [5], [6]. With the development of the IoT communication paradigm, the scarcity of spectrum resources will become more serious [7]. At the same time, due to various reasons, a large range of frequency band remain under-utilized. Cognitive radio network (CRN) techniques are expected to leverage the under-utilized frequency band to resolve the issue of spectrum scarcity for the billions of connected devices. The CRN allows primary users (PUs) and secondary users (SUs) to share spectrum resources. The PUs have priority while the SUs can temporarily occupy spectrum

resources. To ensure the PU's priority and the efficiency of quality of service (QoS), the interference between the PUs and SUs should be minimized.

Spectrum sensing is the fundamental in cognitive radio networks for sharing spectrum resource to alleviate the problems of spectrum scarcity [8]-[10]. Spectrum sensing results directly affect whether harmful interference will occur between PUs and PUs. When the spectrum sensing result is correct, the SUs will not cause harmful interference to the PUs; otherwise, the communication quality of the PUs will be seriously affected. The performance of spectrum sensing is closely related to the complex electromagnetic environment [11]–[15]. Non-Gaussian noise and multi-path fading channels are two important factors that affect the performance of spectrum sensing. Specially, non-Gaussian noise is composed of multi-user interference, lightning noise, sea cluttering noise, or low-frequency atmospheric noise, which generally presents the characteristics of "spike impulse". In practice, the problem is more challenging as we need to sense the PUs signals affected by non-Gaussian noise and multi-path fading channels [16]. As described in [17], [18], when the presence of non-Gaussian noise occurs, due to the heavy tailed characteristic of its probability density function (P.D.F.) and/or amplitude fading, the performance of the optimized spectrum spectrum may decrease dramatically.

For the non-Gaussian noise, the symmetric alpha-stable distribution (S $\alpha$ S) noise model is widely used for its good description of non-Gaussian noise [19]-[22], and it can match Gaussian or non-Gaussian noise by choosing different characteristic exponents  $\alpha$  (0 <  $\alpha \leq 2$ ). Due to no closed-form expression for P.D.F. of  $S\alpha S$  except for three special cases, i.e., Gaussian, Cauchy and Pearson distribution, many spectrum sensing methods cannot be developed in case of  $S\alpha S$ noise. Some existing works have solved spectrum sensing problems in the presence of  $S\alpha S$  noise. For example, X.M. Zhu et al. used high order cumulant based on fractional lower order moments (FLOM) with orders  $p_m \in (0, \alpha_m \leq 2)$ for spectrum sensing in [23], where  $\alpha_m$  denotes the characteristic exponent. S. Ma used eigenvalue matrix to achieve spectrum sensing with known PU signal cyclic frequency in [24]. H.G. Kang et al. realized spectrum sensing based on Cauchy detector, and the sensing performance is better than that of linear detector when the prior knowledge of noise dispersion parameter,  $\gamma$ , is known in [25]. However, in practice these prior information may not be readily available.

Fading channel is also an important factor affecting the sensing performance [26], [27]. Spectrum sensing performance has been extensively studied in different traditional fading channel environments, such as Rayleigh [28], Rician and Nakagami-*m* [29]. However, these research are based on known P.D.F. of noise and fading channel parameters, such as  $\kappa$ ,  $\mu$  and *m*, and so on. When the P.D.F. of the noise is unknown, the spectrum sensing performance based on the

above method will not be directly obtained in such fading channels.

To solve the spectrum sensing problem with unknown parameters of the noise and fading channel, [30] studied the spectrum sensing performance based on generalized log-likelihood ratio test (GLRT) where the unknown parameters were replaced by the estimated values, such as the noise variance and the gain of fading channels, and so on. However, GLRT needs to estimate the unknown parameters under  $H_0$  and  $H_1$ . The classical Rao detection is an approximate form of GLRT, where the maximum likelihood estimation of unknown parameters in case of  $H_0$  is only needed, which reduces the computational complexity.

In this paper, we develop a novel spectrum sensing method based on the theory of Rao detection over non-non-Gaussian noise multi-path fading channels environment. Due to the adaptive kernel method is better than the classical fixed kernel method in revealing the data features at the tails of the distribution, the adaptive kernel function estimation method is used to approximate the P.D.F. of the S $\alpha$ S noise. Moreover, the Rao detection theory is used to realize spectrum sensing in order to reduce the computational complexity of the proposed method. Based on big data statistics, the decision statistics and the progressive spectrum sensing performance are obtained in multi-path fading channels. To verify the effectiveness of the proposed method, we compare the spectrum sensing performance under three multi-path fading channels environments, such as indoor, outdoor and vehicle. Simulation results demonstrate that the applicability of the proposed method.

The rest of this paper is organized as follows. The system model and signal model are presented in Section II. The P.D.F. estimation of S $\alpha$ S noise based on non-parametric window estimation method and Rao detection with Non-Gaussian Noise is expressed in Section III. The progressive spectrum sensing performance over multi-path fading channls is derived in Section IV. Section V shows the numerical examples to verify the derived result and the estimation performance. Finally in Section VI we conclude the main results of this paper.

# **II. SYSTEM AND SIGNAL MODEL**

### A. SYSTEM MODEL

Fig. 1 is the system model of CRN and shows a spectrum sensing scenario with the coexistence of PUs and SUs, which share spectrum resources over S $\alpha$ S noise multi-path fading channels without harmful interference. The primary network consists of a PU-Tx communicating with a PU-Rxs, whereas the secondary network consists of a SU-Tx serving a SU-Rxs. The wireless transmission channel consists of *L* fading channels, which are caused by reflection, diffraction, scattering and multipath propagation of electromagnetic waves. The red dotted line indicates that harmful wireless interference may be generated between PUs and SUs.



FIGURE 1. System model of cognitive radio network.

# **B. SIGNAL MODEL**

In the CR, define x(n),  $n = 1, \dots, N$ , as the data sample received by the SU. We assume that the number of fading channels is *L*, the amplitude fading of the *l*-th fading channel is  $a_l$ , and the phase delay of the *l*-th fading channel at the *n*th sampling time is  $\tau_{nl}$ . s(n) is the deterministic signal sample transmitted by the PU and  $s(n - \tau_{nl})$  is the PU signal sample transmitted through the multi-path fading channels. w(n) is the additive S $\alpha$ S noise sample. We also assume that the signal components and noise are independent of each other. The statistical hypotheses of the PU's presence or absence can be formulated as

$$H_0: x(n) = w(n)$$
  

$$H_1: x(n) = \sum_{l=1}^{L} a_l s(n - \tau_{nl}) + w(n), \quad n = 1, \cdots, N,$$
(1)

where  $H_0$  is the event that PU does not exist, and  $H_1$  denotes the event of the existence of PU.

For the convenience of calculation, we express (1) in the form of matrix. Therefore, (1) can be expressed as a binary hypothesis test problem based on linear model, that is

$$H_0: x = w$$
  
$$H_1: x = Sa + w,$$
 (2)

where  $x = [x (1), \dots, x (N)]^T$  is the received signal vector, S is a  $N \times L$  (N > L) dimensional observation matrix and its rank is L,  $[S]_{nl} = s(n - \tau_{nl})$ .  $a = [a_1, \dots, a_L]^T$  is an unknown parameter vector.  $w = [w(1), \dots, w(N)]^T$  is the noise vector and the P.D.F. of its elements is p(w).

Since the closed-formed P.D.F. of  $S\alpha S$  noise is difficult to obtain, the characteristic function is used to describe its statistics as follows [19]

$$\phi(z) = \exp\left\{j\mu z - \gamma |z|^{\alpha} \left[1 + j\beta \operatorname{sign}(z) \Omega(z, \alpha)\right]\right\}, \quad (3)$$

where  $\alpha$  (0 <  $\alpha \le 2$ ) is characteristic exponent,  $\gamma$  denotes dispersion parameter,  $\beta$  (-1  $\le \beta \le 1$ ) is symmetrical

parameter, sign(z) represents a sign function [19], and  $\Omega(z, \alpha)$  is

$$\Omega(z,\alpha) = \begin{cases} \tan(\pi\alpha/2), & \alpha \neq 1, \\ (2/\pi) \log|z|, & \alpha = 1. \end{cases}$$
(4)

Note that, when  $\alpha = 2$  and  $\beta = 0$ , the S $\alpha$ S distribution reduces to the Gaussian distribution with mean  $\mu$  and variance  $2\gamma$ . Especially, the S $\alpha$ S distribution is Cauchy distribution when  $\alpha = 1$  and  $\beta = 0$ , while it is Pearson distribution when  $\alpha = 0.5$  and  $\beta = -1$ .

# III. SPECTRUM SENSING METHOD BASED ON RAO DETECTION

#### A. P.D.F. OF GENERALIZED GAUSSIAN NOISE

Generalized Gaussian distribution, also known as exponential distribution, including Gaussian, Laplace and uniform distribution. The P.D.F. of generalized Gaussian noise is defined as

$$p(w) = \frac{c_1(\beta)}{\sqrt{\sigma^2}} \exp\left(-c_2(\beta) \left|\frac{w}{\sqrt{\sigma^2}}\right|^{\frac{2}{1+\beta}}\right), \quad (5)$$

where 
$$\beta > -1$$
,  $c_1(\beta) = \frac{\Gamma^{\frac{1}{2}}(\frac{3}{2}(1+\beta))}{(1+\beta)\Gamma^{\frac{3}{2}}(\frac{1}{2}(1+\beta))}$  and  $c_2(\beta) =$ 

 $\begin{bmatrix} \frac{\Gamma\left(\frac{3}{2}(1+\beta)\right)}{\Gamma\left(\frac{1}{2}(1+\beta)\right)} \end{bmatrix}^{\frac{1}{1+\beta}}, \Gamma(x) \text{ is the gamma function, that is} \\ \Gamma(x) = \int_{0}^{\infty} u^{x-1} \exp(-u) du.$ 

Note that, the generalized Gaussian distribution is Gaussian distribution when  $\beta = 0$ ; and it is Laplace distribution when  $\beta = 1$ ; while  $\beta \rightarrow -1$ , the distribution tends to be uniform.

# B. P.D.F. OF GAUSSIAN MIXED DISTRIBUTED NOISE

Gaussian mixed distributed noise is often used to describe man-made noise and interference caused by ultra-wideband systems in wireless channels, and is widely used in modeling non-Gaussian noise [20]. The P.D.F. of the binary Gaussian mixture distribution noise is

$$p(w) = \frac{1-\varepsilon}{\sqrt{2\pi\sigma_1^2}} \exp\left[-\frac{1}{2}\frac{w^2}{\sigma_1^2}\right] + \frac{\varepsilon}{\sqrt{2\pi\sigma_2^2}} \exp\left[-\frac{1}{2}\frac{w^2}{\sigma_2^2}\right].$$
(6)

where  $\varepsilon$  is a mixed parameter, and  $0 < \varepsilon < 1$ . One Gaussian distribution is a Gaussian random variable that obeys  $N(0, \sigma_1^2)$  distribution, and the other Gaussian distribution is a Gaussian random variable that obeys  $N(0, \sigma_2^2)$  distribution. Generally, when  $\sigma_2^2 \gg \sigma_1^2$  and  $\varepsilon \ll 1$ , the Gaussian noise with variance  $\sigma_2^2$  is used to describe sudden pulses or interferences with short duration and large changes in pulse amplitude, while the Gaussian distribution with variance  $\sigma_1^2$  plays a major role in the background noise. Due to the Gaussian distributed noises, the mean value E(w) is zero, the variance  $\sigma^2$  is  $(1 - \varepsilon)\sigma_1^2 + \varepsilon \sigma_2^2$ .

If  $\sigma_1^2$ ,  $\sigma_2^2$  and  $\varepsilon$  are known, the P.D.F. of the distribution is known. In fact, in general,  $\varepsilon$  is an unknown parameter and can be estimated by moments. Suppose when  $N \to \infty$ ,  $\frac{1}{N} \sum_{n=1}^{N} w^2 \to E(w^2)$  holds. The estimator  $\hat{\varepsilon}$  and variance of  $\varepsilon$ can be obtained as

 $\hat{\varepsilon} = \frac{\frac{1}{N} \sum_{n=1}^{N} w^2 - \sigma_1^2}{\sigma_2^2 - \sigma_1^2}$ (7)

and

$$var\left(\hat{\varepsilon}\right) = \frac{3\left(1-\varepsilon\right)\sigma_{1}^{4} + 3\varepsilon\sigma_{2}^{4} - \left[\left(1-\varepsilon\right)\sigma_{1}^{2} + \varepsilon\sigma_{2}^{2}\right]^{2}}{N\left(\sigma_{2}^{2} - \sigma_{1}^{2}\right)^{2}}.$$
(8)

It can be proved that when  $N \to \infty$ , the estimate is a uniform estimator, that is  $\hat{\varepsilon} \to \varepsilon$ . Therefore, we can think that the P.D.F. of Gaussian mixture noise can be calculated by the variance of two known Gaussian distribution noises.

# C. P.D.F. ESTIMATION OF Sas NOISE

In this section, a non-parametric window estimation method is used to estimate P.D.F. of  $S\alpha S$  noise. This method does not need to know the distribution of random variables which are to be estimated, and it can directly obtain the estimation of P.D.F. from samples. Furthermore, compared with the fixed-core method, the adaptive kernel method can clearly display the characteristic information of the data, and does not obscure the important features in the data on account of over-smooth [31].

Based on the S $\alpha$ S noise characteristics, such as symmetry, unimodality and severe tailing, the adaptive kernel estimation method is used to estimate the P.D.F. of S $\alpha$ S noise from the sample data. Assuming that the independent and identically distributed (IID) noise samples are  $w_m$ ,  $m = 1, \dots, M$ , the estimated value of the P.D.F. [31] is

$$\hat{p}_M(w) = \frac{1}{M} \sum_{m=1}^M \frac{1}{h\lambda_m} k\left(\frac{w - w_m}{h\lambda_m}\right),\tag{9}$$

where  $k(\cdot)$  is a kernel function, h is a global bandwidth, which is used to control the smoothness of the P.D.F. and the spread of the kernel function. The optimal value of h is  $0.79RM^{(-0.2)}$ . R is the interquartile range of the sample data, that is, the sample data is sorted from small to large, and the 1/4 quantile  $R_1$  and the 3/4 quantile  $R_2$  are obtained, then  $R = R_2 - R_1 \cdot \lambda_m$  is the local bandwidth, which is used to adapt to the local characteristics of the function, such as the severe tailing characteristic of S $\alpha$ S noise, which can be calculated by

$$\lambda_m = \left(\frac{p_{M,0}(w)}{\prod_{m=1}^M p_{M,0}(w)^{1/M}}\right)^{-1/2},\tag{10}$$

where  $p_{M,0}(w)$  can be obtained when  $\lambda_m = 1$ .

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Since the Gaussian kernel function is a commonly used kernel function and has been widely used, we also chooses the Gaussian kernel function and substitutes it into (9). Then we have

$$\hat{p}_{M}(w) = \frac{1}{\sqrt{2\pi}} \frac{1}{M} \sum_{m=1}^{M} \frac{1}{h\lambda_{m}} exp\left(-\frac{1}{2} \frac{(w-w_{m})^{2}}{(h\lambda_{m})^{2}}\right).$$
 (11)

In order to guarantee the symmetry property of P.D.F., we denote the estimation of P.D.F. as

$$\hat{p}(w) = \left(\hat{p}_M(w) + \hat{p}_M(-w)\right)/2.$$
 (12)

It can be proved that when  $M \to \infty$ , (13) is established, where p(w) is the P.D.F. of the noise when  $\alpha$  is equal to 0.5, 1 and 2, respectively.

$$\Pr\left\{\sup_{w} \left| \hat{p}(w) - p(w) \right| \to 0 \right\} = 1.$$
(13)

#### D. DECISION STATISTIC BASED ON RAO DETECTION

Based on the theory of Rao detection, when the fading channel amplitude parameter vector a is unknown, the decision statistic is

$$T_R(x) = \frac{\frac{\partial \ln p(x;a,H_1)}{\partial a} \Big|_{a=0}^T \frac{\partial \ln p(x;a,H_1)}{\partial a} \Big|_{a=0}}{I(a)|_{a=0}} \frac{\partial \ln p(x;a,H_1)}{\partial a} \Big|_{a=0} \stackrel{H_1}{\underset{H_0}{\leq}} \eta_R,$$
(14)

where  $p(x; a, H_1)$  is the conditional P.D.F. of x in case of  $H_1$ .  $I(a)|_{a=0}$  is the Fisher information matrix.  $\eta_R$  denotes the detection threshold, which can be calculated from the given false alarm probability  $(P_f)$ .

Based on the estimated value of P.D.F. for S $\alpha$ S noise  $\hat{p}(w)$ , according to (1), we have

$$\hat{p}(x; a, H_1) = \prod_{n=1}^{N} \hat{p}\left(x(n) - \sum_{l=1}^{L} a_l s(n - \tau_{nl})\right), \quad (15)$$

where  $w(n) = x(n) - \sum_{l=1}^{L} a_l s(n - \tau_{nl})$ . Taking the logarithm on both sides in (15) and calculating the derivative of an arbitrarily chosen element  $a_l$  in a, we have

$$\frac{\ln \hat{p}(x; a, H_1)}{\partial a_l} = \sum_{n=1}^N \left( -\frac{\frac{d\hat{p}(w(n))}{dw}}{\hat{p}(w(n))} s(n - \tau_{nl}) \right)$$
$$= \sum_{n=1}^N \hat{g}(w(n)) s(n - \tau_{nl}), \quad (16)$$

where

$$\hat{g}(w(n)) = -\frac{\frac{d\hat{p}(w(n))}{dw}}{\hat{p}(w(n))}.$$
 (17)

According to (12), we can get

$$\frac{d\hat{p}(w(n))}{dw} = \frac{1}{2} \left( \frac{d\hat{p}_M(w(n))}{dw} + \frac{d\hat{p}_M(-w(n))}{dw} \right), \quad (18)$$

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where

$$\frac{d\hat{p}_{M}(w(n))}{dw} = \frac{1}{M} \sum_{m=1}^{M} \left[ \frac{1}{\sqrt{2\pi}} \frac{1}{h\lambda_{m}} exp\left( -\frac{1}{2} \frac{(w-x_{m})^{2}}{(h\lambda_{m})^{2}} \right) \left( \frac{x_{m}-w}{(h\lambda_{m})^{2}} \right) \right].$$
(19)

According to (17) and (18),  $\hat{g}(w(n))$  is obtained as follows

$$\hat{g}(w(n)) = -\frac{\frac{d\hat{p}(w(n))}{dw}}{\hat{p}(w(n))} = -\frac{\frac{d\hat{p}_{M}(w(n))}{dw} + \frac{d\hat{p}_{M}(-w(n))}{dw}}{2\hat{p}(w(n))},$$
(20)

when a = 0, w(n) = x(n), then,  $\hat{g}(x(n))$  is

$$\hat{g}(x(n)) = -\frac{\frac{d\hat{p}_M(x(n))}{dx(n)} + \frac{d\hat{p}_M(-x(n))}{dx(n)}}{2\hat{p}(x(n))}.$$
(21)

According to (16), we can get

$$\frac{\partial \ln \hat{p}\left(x; a, \sigma_{w}^{2}\right)}{\partial a_{l}}\bigg|_{a=0} = \sum_{n=1}^{N} \hat{g}\left(x\left(n\right)\right) s\left(n - \tau_{nl}\right). \quad (22)$$

Let  $y = [\hat{g}(x(1)), \dots, \hat{g}(x(N))]^T$ ,  $S_l$  is the *l*-th column element of  $\overline{S}$ , then

$$\frac{\partial \ln \hat{p}(x; a, \sigma_w^2)}{\partial a} \bigg|_{a=0} = [S_1^T y \quad S_2^T y \quad \cdots \quad S_L^T y]^T = S^T y. \quad (23)$$

According to the definition and diagonal nature of Fisher information, we know that  $[I(a)]_{lk}$  is a function of  $\frac{\partial \ln \hat{p}(x;a,H_1)}{\partial a_l}$ , and

$$\begin{split} &[I(a)]_{lk} \\ &= E\left[\frac{\partial \ln \hat{p}\left(x; \, a, \, H_{1}\right)}{\partial a_{l}} \frac{\partial \ln \hat{p}\left(x; \, a, \, H_{1}\right)}{\partial a_{k}}\right] \\ &= E\left[\sum_{n=1}^{N} \hat{g}\left(w_{n}\left(n\right)\right) s\left(n - \tau_{nl}\right) \sum_{m=1}^{N} \hat{g}\left(w_{m}\left(m\right)\right) s\left(n - \tau_{mk}\right)\right] \\ &= \sum_{n=1}^{N} \sum_{m=1}^{N} E\left[\hat{g}\left(w_{n}\right) \hat{g}\left(w_{m}\right)\right] s\left(n - \tau_{nl}\right) s\left(n - \tau_{mk}\right), \\ &\quad (l, \, k = 1, \, \cdots, \, L), \end{split}$$
(24)

where  $w_n = w_n(n) = x(n) - \sum_{l=1}^{L} a_l s(n - \tau_{nl})$  and  $w_m =$  $w_m(m) = x(m) - \sum_{l=1}^{L} a_l s(m - \tau_{ml}).$ 

Since each *x* (*n*) is independent, for  $m \neq n$ ,

$$E\left[\hat{g}\left(w_{n}\right)\hat{g}\left(w_{m}\right)\right] = E\left[\hat{g}\left(w_{n}\right)\right]E\left[\hat{g}\left(w_{m}\right)\right], \quad (25)$$

and

$$E\left[\hat{g}\left(w_{n}\right)\right] = -\int_{-\infty}^{\infty} \frac{\frac{d\hat{p}\left(w_{n}\right)}{dw_{n}}}{\hat{p}\left(w_{n}\right)}\hat{p}\left(w_{n}\right)dx\left(n\right)$$

$$= -\int_{-\infty}^{\infty} \frac{d\hat{p}(w_n)}{dw_n} dx (n)$$
$$= -\int_{-\infty}^{\infty} \frac{d\hat{p}(w_n)}{dw_n} dw_n = 0.$$
(26)

For m = n, we have

$$[I(a)]_{lk} = \sum_{n=1}^{N} E\left[\hat{g}^{2}(w_{n})\right] s\left(n - \tau_{nl}\right) s\left(n - \tau_{nk}\right), \quad (27)$$

where

$$E\left[\hat{g}^{2}(w_{n})\right] = \int_{-\infty}^{\infty} \hat{g}^{2}(w_{n})\hat{p}(w_{n})dx(n)$$
$$= \int_{-\infty}^{\infty} \frac{\left(\frac{d\hat{p}(w_{n})}{dw_{n}}\right)^{2}}{\hat{p}(w_{n})}dx(n)$$
$$= \int_{-\infty}^{\infty} \frac{\left(\frac{d\hat{p}(w_{n})}{dw_{n}}\right)^{2}}{\hat{p}(w_{n})}dw_{n} = z(a). \quad (28)$$

Thus, we can get

$$[I(a)]_{lk} = z(a) \sum_{n=1}^{N} s(n - \tau_{nl}) s(n - \tau_{nk}) = z(a) S^{T} S.$$
(29)

Then the Fisher information matrix is

$$I(a)|_{a=0} = z(a)|_{a=0}S^{T}S,$$
(30)

where

$$z(a) = \int_{-\infty}^{+\infty} \frac{\left(\frac{d\hat{p}(w)}{dw}\right)^2}{\hat{p}(w)} dw.$$
 (31)

The procedure of spectrum sensing based on Rao detection with  $S\alpha S$  distribution noise over multi-path fading channels is summarized in Algorithm 1.

Algorithm 1 The Procedure of Spectrum Sensing Based on Rao Detection With  $S\alpha S$  Distribution Noise Over Multi-Path Fading Channels

- 1: Calculate the local bandwidth  $\lambda_m$  according to (10).
- 2: Using the non-parametric adaptive kernel density estimation method, the estimated value of the P.D.F. of  $S\alpha S$ noise from the sample data by solving (11) and (12).
- 3: For a given  $P_f$ , the detection threshold value can be
- a. For a given *T*f, the detection difference value can be calculated as η<sub>R</sub> = Q<sup>-1</sup><sub>χ<sup>2</sup><sub>L</sub></sub>(P<sub>f</sub>).
  4: Using (23), we can get ∂ ln p̂(x;a,σ<sup>2</sup><sub>w</sub>)/∂a |<sub>a=0</sub>, and use it to replace ∂ ln p(x;a,H<sub>1</sub>)/∂a |<sub>a=0</sub> in the decision statistics in (14).
  5: Use (28) and (29) to calculate the [*I*(a)]<sub>lk</sub>, then let a = 0,
- use (30) and (31) to get the Fisher information matrix  $I(a)|_{a=0}$ .
- 6: Finally, we compare  $T_R(x)$  in (14) and  $\eta_R$ . When  $T_R(x)$  is greater than or equal to  $\eta_R$ , PU exists; otherwise, PU does not exist and SU can share the frequency.



**FIGURE 2.** Spectrum sensing performance versus different test channels for generalized Gaussian noise for  $\beta = 0$ .

# IV. PROGRESSIVE SPECTRUM SENSING PERFORMANCE BASED ON RAO DETECTION

Substituting (23) and (30) into (14), the decision statistic of Rao detection is

$$T_{R}(x) = (S^{T}y)^{T} (z(a) S^{T}S)^{-1} S^{T}y$$
  
=  $\frac{(S^{T}y)^{T} (S^{T}S)^{-1} S^{T}y}{z(a)}$ . (32)

when  $N \to \infty$ ,  $T_R(x)$  obeys the following distribution.

$$H_0: T_R(x) \sim \chi_L^2$$
  

$$H_1: T_R(x) \sim \chi_L^2(\psi), \qquad (33)$$

where  $\chi_L^2$  is the central Chi-square distribution with *L* degrees of freedom,  $\chi_L^2(\psi)$  is the noncentral Chi-square distribution with *L* degrees of freedom, and noncentrality parameter  $\psi$  is

$$\psi = a_1^T \mathbf{I}(a)|_{a=0} a_1 = z(a) a_1^T S^T S a_1, \qquad (34)$$

where  $a_1$  is the true value of a in the case of  $H_1$ . Therefore, the asymptotic spectrum sensing performance is as follows

$$P_f = Q_{\chi_L^2}(\eta_R)$$
  

$$P_d = Q_{\chi_I^2(\psi)}(\eta_R),$$
(35)

where  $P_d$  is the detection probability,  $Q_{\chi_L^2}(\eta_R) = \int_{\eta_R}^{\infty} p(w) dw$  is the right-tail probability for a  $\chi_L^2$  random variable.

Since the P.D.F. is determined by noise in the case of  $H_0$ , it is independent of unknown amplitude parameters of the multi-path fading channels. According to the Neyman-Pearson criterion, the detection threshold  $\eta_R$  can be calculated by a constant  $P_f$ , which is also called constant false alarm rate (CFAR). Therefore, in the process of simulation,  $P_f$  is set to a pre-specified value, and the detection threshold  $\eta_R$  is calculated according to the inverse of (35), that is  $\eta_R = Q_{\chi_I^2}^{-1}(P_f)$ , and then  $P_d$  is calculated.



**FIGURE 3.** Spectrum sensing performance versus different test channels for binary Gaussian mixture distribution noise for  $\sigma_1^2 = 1$ ,  $\sigma_2^2 = 4$  and  $\varepsilon = 0.5$ .

#### V. NUMERIC SIMULATION AND DISCUSSION

In this section, we show the simulation results and compare the estimation performance of the P.D.F. and the spectrum sensing performance of the proposed method. Without loss of generality, in simulations, the multi-path fading channels used are the three test channels in ITU-R M.1225, which are indoor, outdoor and vehicle test channels, and the number of fading channels is L = 6. The number of samples is 6000. The number of Monte Carlo experiments is 10000.

In order to verify the influence of the generalized Gaussian noise and multi-path channels on spectrum sensing performance, we conduct the simulation over three different test channels repectively and we observe the performance varying with test channel. In Fig.2, we can see that the proposed method can effectively realize spectrum sensing over generalized Gaussian noise and multi-path channels. In particular, the sensing performance is very close to the theoretical analysis value in the outdoor multi-path fading channels.

In order to verify the influence of the binary Gaussian mixture distribution noise and multi-path channels on spectrum sensing performance, we conduct the simulation over three different test channels repectively. We set  $\sigma_1^2 = 1$ ,  $\sigma_2^2 = 4$ and  $\varepsilon = 0.5$ . In Fig.3, We can see that this method can effectively realize spectrum sensing on binary Gaussian mixture noise and multi-path channels, and has the best sensing performance in outdoor multi-path fading channels.

Using (12) the P.D.F. of the noise is estimated, when M = 1000, the simulation result is shown in Fig.4. We selected three typical distributions, such as Gaussian, Cauchy and Pearson, and compared the approximate estimated values with the theoretical values of P.D.F.. We compare the P.D.F. calculated in [32] with the approximate estimated of P.D.F.. It can be seen from Fig.5 that when  $\alpha = 1$  and  $\alpha = 2$ , the proposed method is very close to the theoretical values of P.D.F. in [32], and the absolute error is less than 0.02, which demonstrates the accuracy of our proposed method, at the same time, proves that the proposed method is effective.



FIGURE 4. Estimation and comparison of P.D.F. for SaS noise.



FIGURE 5. Absolute Error of P.D.F. Estimation for SaS noise.

When  $\alpha = 0.5$ , near the origin, the proposed method has a certain error with the the theoretical values of P.D.F. in [32], the absolute error is less than 0.12; interestingly, at other points, the two are very close, and the absolute error is less than 0.02.

Without loss of generality, the performance of the proposed spectrum sensing method is evaluated by evaluating the receiver operating characteristic (ROC) curves, which illustrates the relationship between the  $P_d$  and  $P_f$ . Fig.6 to Fig.8 compare the spectrum sensing performance in  $S\alpha S$ noise multi-path fading channels environment under different  $\alpha$  values. It should be noted that Generalized Signal-to-Noise Ratio (GSNR) is used, and it is defined as  $GSNR \triangleq$  $10 \lg(P_s/\gamma) dB$ , where  $P_s$  and  $\gamma$  represent transmit power and the dispersion coefficient of  $S\alpha S$  noise, respectively. In the following simulations, GSNR = 12dB, L = 6. Due to  $\alpha$ is different, it corresponds to different non-Gaussian noises. The smaller  $\alpha$  is, the more obvious the spike of noise is, and the greater the impact on spectrum sensing performance. As can be seen from Fig.6, when  $\alpha = 2$ , the noise obeys the Gauss distribution, the spectrum sensing performance is



**FIGURE 6.** Spectrum sensing performance versus different  $\alpha$  values over indoor test channels.



**FIGURE 7.** Spectrum sensing performance versus different  $\alpha$  values over outdoor test channels.



**FIGURE 8.** Spectrum sensing performance versus different  $\alpha$  values over vehicle test channels.

the best, while when  $\alpha = 0.5$ , the noise obeys the Pearson distribution, and the spectrum sensing performance is the worst.

# VI. CONCLUSION

Smart connected devices in the IoT and typical smart city applications need a lot of spectrum to communicate. CR can effectively improve the spectrum utilization and alleviate the issue of spectrum scarcity. However, non-Gaussian noise and multi-path fading channels are important factors affecting spectrum sensing performance. In particular, there is no closed-form P.D.F. for  $S\alpha S$  noise. In this paper, we implemented the spectrum sensing technology in the S $\alpha$ S noise and multi-path fading channels environment, and analyzed the spectrum sensing performance. Firstly, the closed-form P.D.F. of  $S\alpha S$  noise is obtained by using the non-parametric adaptive kernel density estimation method. Secondly, the influence of multi-path fading channels on spectrum sensing performance is analyzed by using the Rao detection theory of unknown fading channel amplitude. Simulation results show that the proposed method can achieve better spectrum sensing in  $S\alpha S$  noise multi-path fading channels environment.

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