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On Spectrum-Efficient Routing in Interference-Limited Full-Duplex Multihop Wireless Networks

MOHAMED SAAD[1], (Senior Member, IEEE), AND SAEED ABDALLAH[2], (Member, IEEE)

[1]Department of Computer Engineering, University of Sharjah, Sharjah, United Arab Emirates
[2]Department of Electrical Engineering, University of Sharjah, Sharjah, United Arab Emirates

Corresponding author: Mohamed Saad (msaad@sharjah.ac.ae)

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ABSTRACT The problem of finding the route with maximum end-to-end spectral efficiency in a multihop wireless network has been subject to considerable interest in the recent literature. It has been established that the problem is polynomially solvable in networks that employ time division multiple access (TDMA) or frequency division multiple access (FDMA). The main characteristic of TDMA and FDMA is being interference-free. Motivated by the advances in full-duplex (FD) communications and self-interference (SI) cancelation, this paper considers the problem for *interference-limited* wireless networks, in which nodes can send and receive at the same time and using the same frequency, but *all* links may interfere with each other. The contribution of this paper is three-fold. We formulate the problem as a mixed integer non-linear programming problem, and provide two equivalent re-formulations that provide more insight into the problem structure. Moreover, to the best of our knowledge for the first time, we provide a *formal* proof that the problem is *NP-complete* in the interference-limited case. Our proof uses reduction from a more general link scheduling problem subject to signal-to-interference-plus-noise-ratio (SINR) constraints. Finally, using an algorithm based on searching over paths while pruning the allowed number of hops, we provide a detailed numerical study that illustrates the tradeoffs between interference-limited FD relaying and TDMA half-duplex (HD) relaying. Our results indicate that *sub-optimal* interference-limited FD relaying leads to up to 2.69 times higher spectral efficiencies as compared to *optimal* TDMA HD relaying. Moreover, the lower the signal-to-noise ratio (SNR) regime, the larger the network size and/or the higher the SI capabilities, the higher the spectral efficiency gain due to FD (over HD) relaying.

INDEX TERMS Multihop wireless networks, spectrum-efficient routing, full-duplex, interference-limited, self-interference.

I. INTRODUCTION

Multihop wireless networks consist of a set of wireless devices that communicate with each other over multiple wireless hops, with participating nodes collaboratively relaying ongoing traffic. Wireless multihop communication is a promising technology for various state-of-the-art applications, such as wireless backhaul networks interconnecting small-cell base stations in fifth-generation (5G) systems [1], and interconnected Internet-of-Things (IoT) devices [2].

The end-to-end *spectral efficiency* (in bps/Hz) of a communication route is defined as the data rate that can be achieved over the route per unit bandwidth. Since the channel bandwidth is a limited resource, this paper addresses the problem of finding the communication route with maximum end-to-end spectral efficiency. Moreover, motivated by the advances in full-duplex (FD) communications [3], [4] and self-interference (SI) cancelation [7], this study focuses on *interference-limited* wireless networks, in which wireless devices can send and receive at the same time and using the same frequency band. In fact, FD has been considered as a promising technology for 5G systems. See, e.g., [5], [6]. The use of FD and SI cancellation can potentially further improve the spectral efficiency. However, this comes at the cost that a transmitting link causes interference to *all* other existing links, and vice versa.

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The problem of finding the source-destination routing path with maximum end-end-spectral efficiency in wireless networks that employ time division multiple access (TDMA) has been subject to considerable interest in the literature [8]–[11]. In particular, the authors of [8] introduce the problem, but notice that shortest path algorithms cannot be used because the resulting routing metric is not isotonic. See, e.g., [12] and [13]. Thus, they propose efficient, yet sub-optimal algorithms to solve the problem. The study in [9] closes the algorithmic gap by proposing the first *provably* optimal polynomial-time algorithm for the problem. The study in [10] presents a reduced complexity algorithm for the same TDMA routing problem. The work in [11] provides an extension to the case of multiple simultaneous source-destination pairs. Furthermore, the study in [14] considers the problem of jointly finding a routing path and allocating transmit powers to the wireless devices along the obtained path such that the sum-power (respectively, maximum power) is minimized, and a spectral efficiency target is satisfied. Again, provably optimal polynomial-time algorithms are presented for the TDMA case. The study in [10] extends the results (and provably optimal polynomial-time algorithms) to networks that employ frequency division multiple access (FDMA).

The main characteristic of TDMA and FDMA systems (considered in the above studies) is being *interference-free*. In contrast, this study focuses on the case that all links of the selected path transmit at the same time, and using the same frequency band. This is motivated by the advances in FD communications. However, the added difficulty is the *interference-limited* regime resulting from simultaneous link transmissions, including SI. Studies that address FD and/or SI cancellation in mutihop wireless networks do exist. See, e.g., [15]–[20]. In particular, the study in [15] assesses the throughput gain in multihop networks due to FD transmissions. However, it focuses on the scheduling of end-to-end flows on known paths. So in contrast to this paper, there is no routing aspect. The study in [16] quantifies the capacity gain of FD as compared to half-duplex (HD) communications. To this end, the authors provide an asymptotic study on large networks using tools from stochastic geometry. The capacity is derived focusing on a typical link in the network. In other words, there is also no routing aspect. The study in [17] addresses the joint problem of routing and power allocation in FD multihop wireless networks. However, the study assumes that intra-route interference is cancelled by the use of Markov Block Coding/Sliding Window Decoding. Therefore, and in contrast to this paper, there is no interference aspect. The study in [18] assesses the benefits of FD over HD communications in multihop wireless networks. However, the study considers a linear network in which there is one fixed path connecting the source and destination nodes. In other words, there is also no routing aspect. The study in [19] also considers joint routing and power allocation in FD wireless networks, but under a *one-hop interference* assumption. In particular, it is assumed that interference on a link comes only from the neighboring link and from SI. In contrast, however, this paper considers the most general interference case, in which *all* links interfere with each other. Finally, the study in [20] addresses the problem of joint routing and power allocation in FD wireless network under a *full* interference model, in which all links interfere with each other. However, the main focus of the study, in terms of problem formulation and algorithmic development, is for power allocation assuming a fixed path. The routing problem is then solved by incorporating the power allocation step in a Dijkstra shortest path procedure. Although the authors notice that the full-duplex routing problem cannot be solved using shortest path algorithms, the modified Dijkstra algorithm is still adopted as a sub-optimal heuristic.

In light of the above, the contribution of this paper can be summarized as follows.

- Given an interference-limited multihop wireless network with FD capabilities, we formulate the problem of finding the source-destination routing path with maximum spectral efficiency as a mixed integer non-linear programming problem. We provide two equivalent re-formulations that give more insight into the problem structure.
- To the best of our knowledge for the first time, we devise a *formal* proof that the decision version of the problem is *NP-complete* in the interference-limited case. This implies that the optimization version of the problem is *NP-hard*. Our proof uses reduction from a more general link scheduling problem subject to signal-to-interference-plus-noise-ratio (SINR) constraints.
- Using an algorithm based on searching over paths while pruning the allowed number of hops, we provide a detailed numerical study that illustrates the tradeoffs between interference-limited FD relaying and TDMA HD relaying. In particular, we illustrate that FD relaying is able to achieve double the spectral efficiency of HD relaying (or more), especially in large networks, high SI cancellation capabilities and/or in the low signal-to-noise-ratio (SNR) regime. To the best of knowledge, these are the first results that assess FD over HD due to routing alone.

The remainder of this paper is organized as follows. Section II provides a formal definition of the problem, and gives insightful re-formulations. The NP-completeness proof is presented in Section III, together with the pruned hop-count search algorithm. Numerical examples and results are presented in Section IV. Section V concludes the paper.

II. PROBLEM DEFINITION

In this section, we present a mathematical formulation for the problem of finding the source-destination routing path with maximum spectral efficiency in an interference-limited multihop wireless network with FD capabilities. The notation used in this paper is summarized in Table 1.

A multihop wireless network is modeled as a graph $G = (V, E)$, where V is the set of nodes (i.e., wireless devices)

TABLE 1. Notation.

Symbol	Definition
$G = (V, E)$	multihop wireless network represented as a graph
V	set of nodes
E	set of links
$l \in E$	signifies a link in the network
$N = \|V\|$	number of nodes
$M = \|E\|$	number of links
P	transmit power of all wireless devices (nodes)
L	signifies a path in the network, and consists of a sequence of links
$SINR_l$	signal-to-interference-plus-noise ratio at the receiver of link l
G_{ll}	path gain from the transmitter of link l to the intended receiver of link l
G_{kl}	path gain from the transmitter of link k to the receiver of link l
$\tilde{G}(i, j)$	path gain from node i to node j, where $G_{kl} = \tilde{G}(Tx(k), Rx(l))$
$Tx(l)$	transmitter node of link l
$Rx(l)$	receiver node of link l
N_0	background (additive white gaussian) noise power
R_L	end-to-end spectral efficiency of path L
s	source node
d	destination node
$\mathcal{L}_{sd}$	set of all paths connecting node s to node d
δ	scalar variable, equal to $\min_{l\in L}\log(1 + \frac{PG_{ll}}{\sum_{k\in L:k\neq l} PG_{kl}+N_0})$
γ	scalar variable, equal to $2^{\delta} - 1$
x_l	binary variable that indicates whether or not link l is selected as part of the solution
a_{vl}	binary parameter that indicates whether or not link l is entering node v
b_{vl}	binary parameter that indicates whether or not link l is leaving node v

and E is the set of wireless links. We let $l \in E$ signify a link in the network. We also let $N = |V|$ and $M = |E|$ denote the number of nodes and links, respectively. Following [8], [9], we consider the setting in which all transmit devices are constrained by the same symbol-wise average transmit power P, and assume that all devices transmit with power P when transmitting. A possible justification for this assumption is that nodes in *infrastructure* wireless mesh networks are mostly immobile and connected with abundant power supplies. Moreover, and following [15] and [20], the wireless devices are assumed to have single antenna FD radios capable of transmitting and receiving at the same time and using the same frequency. The design of such FD radios has been addressed in [4]. A source node (wireless device) requests to communicate with a destination node (wireless device). We consider the case that this communication is realized by finding a multihop path from the source to the destination, where other nodes on the path operate as relays. An illustrative example of such multihop wireless networks is depicted in Fig. 1. The figure illustrates a source node communicating with a destination node via a path with four hops. All relay nodes along the path can simultaneously transmit and receive using the same frequency, but the wireless links interfere with each other.

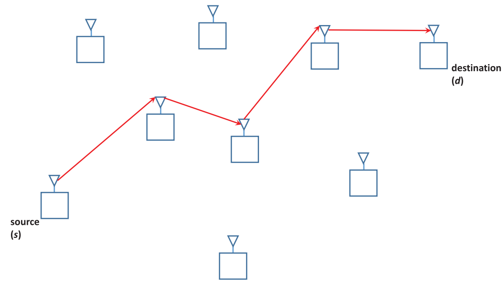

FIGURE 1. Illustrative example of a wireless multihop network.

Let L be an active path in the network, and let $l \in L$ be a link on that path. In the FD regime, wireless devices can transmit at the same time, and using the same frequency band. Hence, the SINR at the receiver of link l can be expressed as:

$$SINR_l = \frac{PG_{ll}}{\sum_{k\in L:k\neq l} PG_{kl} + N_0}, \tag{1}$$

where G_{ll} is the path gain from the transmitter of link l to its intended receiver, G_{kl} is the path gain from the transmitter of link k to the receiver of link l, and N_0 is the background (additive white gaussian) noise power. Note that the term G_{kl} captures also the SI cancellation factor. In particular, if the transmitter of link k is the same as the receiver of link l (i.e., when links l followed by k are two successive links on path L), then G_{kl} captures the SI cancellation factor at the receiver of link l. In other words, PG_{kl} becomes the residual SI at the receiver of link l. This allows for a compact formulation, without having to use separate notation for the SI cancellation factor.

The spectral efficiency of an arbitrary path L in the network is defined as the bandwidth-normalized end-to-end data rate [8]. Using the well-known Shannon capacity formula, it is straightforward to see that the spectral efficiency R_L of path L can be expressed as follows:

$$R_L = \min_{l\in L} \log(1 + \frac{PG_{ll}}{\sum_{k\in L:k\neq l} PG_{kl} + N_0}). \tag{2}$$

Note that the minimum function in (2) results from the fact that the end-to-end path data rate is a bottle-neck quantity, i.e., the end-to-end data rate of path L is the smallest link data rate along the path.

Given a source-destination pair of nodes $(s, d) \in V \times V$, the problem of finding a path from s to d with maximum spectral efficiency can be formulated as the following optimization problem:

$$\max_{L\in\mathcal{L}_{sd}} \min_{l\in L} \log(1 + \frac{PG_{ll}}{\sum_{k\in L:k\neq l} PG_{kl} + N_0}), \tag{3}$$

where $\mathcal{L}_{sd}$ is the set of all paths connecting node s to node d. Note that the objective in (3) is to maximize the spectral efficiency as given by (2), while the constraint is to ensure the selected path indeed connects nodes s and d.

A. PROBLEM RE-FORMULATION

It is clearly seen that the problem formulation (3) is a non-linear integer problem, which is very hard to solve. However, to gain more insight into the problem, especially its NP-completeness, we re-formulate the FD spectrum-efficient routing problem as follows. We introduce the new variable

$$\delta = \min_{l\in L} \log(1 + \frac{PG_{ll}}{\sum_{k\in L:k\neq l} PG_{kl} + N_0}). \tag{4}$$

Note that the right-hand side of (4) is precisely the objective function to be maximized in (3). Therefore, problem (3) can be re-formulated as a problem of maximizing δ, subject to the constraint that the resulting path joins s and d (i.e., $L \in \mathcal{L}_{sd}$), in addition to (4) as a new constraint. It is easily seen that the latter can be re-written as set of per-link constraints of the form $\log(1 + \frac{PG_{ll}}{\sum_{k\in L:k\neq l} PG_{kl}+N_0}) \geq \delta$, for each link $l \in L$. In short, and using (4) as a substitution, problem (3) can be expressed as:

$$\max_{L,\delta} \delta \tag{5a}$$

$$\text{s.t. } \log(1 + \frac{PG_{ll}}{\sum_{k\in L:k\neq l} PG_{kl} + N_0}) \geq \delta, \quad \forall l \in L \tag{5b}$$

$$L \in \mathcal{L}_{sd}. \tag{5c}$$

Again, note that the per-link set of constraints (5b) are equivalent to (4).

By raising both sides of (5b) to the power of 2, constraint (5b) becomes equivalent to

$$\frac{PG_{ll}}{\sum_{k\in L:k\neq l} PG_{kl} + N_0} \geq 2^{\delta} - 1, \quad \forall l \in L. \tag{6}$$

Furthermore, due to the monotonicity of the 2^x function, maximizing δ is equivalent to maximizing $(2^{\delta} - 1)$. Therefore, problem (5) is equivalent to the problem of maximizing $(2^{\delta} - 1)$, subject to the constraints (6) and (5c). Consequently, and using $\gamma = 2^{\delta} - 1$ as substitution, problem (5) is, in fact, equivalent to:

$$\max_{L,\gamma} \gamma \tag{7a}$$

$$\text{s.t. } \frac{PG_{ll}}{\sum_{k\in L:k\neq l} PG_{kl} + N_0} \geq \gamma, \quad \forall l \in L \tag{7b}$$

$$L \in \mathcal{L}_{sd}. \tag{7c}$$

In light of the re-formulation (7) of the original problem given by (3), the FD spectrum-efficient problem at hand is in fact a problem of *finding a set of links in the network, such that each link can be realized at the **maximum possible** SINR threshold γ, and such that the selected links constitute a path from node s to node d*. Note that re-formulation (7) is a non-linear mixed-integer problem. Constraint set (7b) is clearly non-linear. Moreover, the SINR threshold γ is a continuous variable, while the path L is a discrete variable.

B. EXPLICIT FORMULATION

The equivalent formulations (3) and (7) are succinct problem formulations that aim at finding the path L with maximum end-to-end spectral efficiency, among all paths $\mathcal{L}_{sd}$ connecting nodes s and d. Here, we provide another equivalent formulation, which uses *explicit* link decision variables. In particular, we use the binary variables $(x_l : l \in E)$, where

$$x_l = \begin{cases} 1, & \text{link } l \text{ is selected} \\ 0, & \text{otherwise.} \end{cases}$$

Moreover, given the (directed) network graph $G = (V, E)$, it is straightforward to express the node-link incidence matrices $(a_{vl} : v \in V,\ l \in E)$ and $(b_{vl} : v \in V,\ l \in E)$, where

$$a_{vl} = \begin{cases} 1, & \text{link } l \text{ is leaving node } v \\ 0, & \text{otherwise,} \end{cases}$$

and

$$b_{vl} = \begin{cases} 1, & \text{link } l \text{ is entering node } v \\ 0, & \text{otherwise.} \end{cases}$$

Now, problem (7) can be re-written as:

$$\max_{\{x_l:l\in E\},\gamma} \gamma \tag{8a}$$

$$\text{s.t. } \frac{PG_{ll}}{\sum_{k\in E:k\neq l} x_k PG_{kl} + N_0} \geq x_l \cdot \gamma, \quad \forall l \in E \tag{8b}$$

$$\sum_{l\in E} x_l a_{vl} - \sum_{l\in E} x_l b_{vl} = \begin{cases} 1, & v = s \\ -1, & v = d \\ 0, & \forall v \in V\backslash\{s, d\} \end{cases} \tag{8c}$$

$$x_l \in \{0, 1\}, \quad \forall l \in E. \tag{8d}$$

Note that the objective (8a) maximizes the SINR threshold γ, as in (7a). The constraints (8b) ensure that the SINR of every selected link l is at least as large as the threshold γ, which is equivalent to (7b). Moreover, the constraints (8c) are flow conservation constraints, which ensure that, among the selected links, exactly one link leaves the source s, exactly one link enters the destination node d, and for all other nodes the number of links entering is equal to the number of links leaving. In other words, (8c) ensures that the selected links constitute a connected path from s to d, as in the succinct form (7c). Finally, constraints (8d) enforce that the decision (link selection) variables x_l take only the values 1 or 0.

Again, the FD spectral-efficient routing problem as re-formulated by (8) is a non-linear mixed-integer problem. Moreover, the problem combines the flow-conservation constraints (from network flow problems is graphs) with SINR constraints (from wireless communications). The difficulty of the flow-conservation constraints stems from the integrality of the (link selection) variables involved, while the difficulty of the SINR constraints originates from their non-linearity and non-convexity. Furthermore, the overall problem is heavily constrained. For a network with N nodes and M links, problem (8) has $(N + M)$ constraints.

The above discussion provides intuitive arguments on the difficulty of the problem at hand. In the following section, however, we will provide a formal proof that the decision version of problem (7), which is equivalent to formulations (3) and (8), is NP-complete. This implies that the optimization problem (7) itself is NP-hard. See, e.g., [21].

III. NP-COMPLETENESS

The *decision problem* which corresponds to the FD spectrum-efficient routing problem (7) can be expressed as follows.

- **FD spectrum-efficient routing decision problem:** *Is it possible to find a set of links, such that each link can be realized at an SINR of at least γ, and such that the links make a path from s to d?*

We establish the NP-completeness of the FD spectrum-efficient routing by providing a reduction from the following, more general link scheduling problem. See, e.g., [22] and [1].

- **Link scheduling decision problem:** *Is it possible to find a set of at least k links, such that each link can be realized at an SINR of at least γ?*

The above link scheduling problem is known to be NP-complete, where the proof has been established in [22].

The main result follows.

Theorem 1: The decision version of the FD spectrum-efficient routing as given by (7) is NP-complete.

Proof: A decision problem is NP-complete if the following can be proven [21]:

- The problem is in the class of non-deterministic polynomial (NP) problems. A problem is in NP, if a provided solution can be verified to be a correct solution (i.e., yes-instance) to the decsion problem in polynomial-time.
- An instance of a problem known to be NP-complete can be reduced to the problem at hand in polynomial-time.

To show that the FD spectrum-efficient routing is in NP, assume that a set of links L is given as a candidate solution to a yes-instance of the problem. Clearly, it can be verified in polynomial-time that the links in L constitute a path from s to d. Moreover, the SINR at the receiver of every link $l \in L$ can also be computed in polynomial-time. In particular, the SINR as given by (1) contains a summation over all links in L. Since adding n numbers has a complexity of $O(n)$, we conclude that verifying that the SINR at the receiver of every link $l \in L$ exceeds γ can be accomplished in polynomial-time. This concludes that the FD spectrum-efficient routing is in NP.

Now, we provide a reduction from the NP-complete link scheduling problem to the FD spectrum-efficient routing problem at hand. In particular, we will proceed with the proof by restriction [21], to show that our FD spectrum-efficient routing problem contains link scheduling (known to be NP-complete) as a special case. This is done by showing that our FD spectrum-efficient routing problem with restricted inputs is equivalent to link scheduling, which proves that the *more general* FD spectrum-efficient routing is also NP-complete.

Consider the FD spectrum-efficient routing problem, were the input network is restricted to a class of linear networks with $k-1$ nodes and parallel links connecting every two neighboring nodes, as in the example shown in Fig. 2. We let node 1 be the source and node $k-1$ be the destination. We also use the restriction that the SINR threshold $\gamma > 1$, and that the path gains of parallel links are equal. For example, the path gains for links $l_{1,2}$ and $l'_{1,2}$ are both equal to G_1. Therefore, if both $l_{1,2}$ and $l'_{1,2}$ are selected in the solution to the routing problem, then $SINR(l_{1,2}) = SINR(l'_{1,2}) \leq \frac{PG_1}{PG_1+N_0} < 1$. Hence, the SINR threshold of $\gamma > 1$ will not be satisfied. Consequently, only one link of every set of parallel links can be selected in the optimal solution to the routing problem. Therefore, any selected path from node 1 to node $k-1$ contains always precisely k links. Now, let path L be a solution to a yes-instance of the FD spectrum-efficient routing decision problem. In other words, path L connects nodes 1 and $k-1$, as well as satisfies the SINR threshold γ on every link. Since path L must have exactly k links, path L is also a solution to the link scheduling problem of deciding whether or not it is possible to find a set of at least k links, such that each link can be realized at an SINR of at least γ.

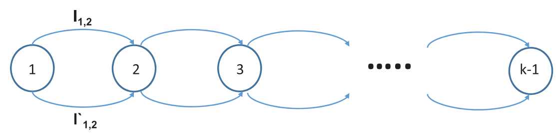

FIGURE 2. Network used for reducing "link scheduling" to "FD spectrum-efficient routing".

Conversely, let the set L of links be a solution to a yes-instance of the link scheduling problem. In other words, the set L has at least k links, as well as satisfies the SINR threshold γ on every link. Again, since the set L of links cannot have any parallel links, so L must contain exactly k links. Therefore, L must constitute a path from node 1 to node $k-1$, because any set of *exactly* k links (with no parallel links) forms a path from node 1 to node $k-1$. Consequently, the set L of links is also a solution to the FD spectrum-efficient routing problem. In conclusion, there is a one-to-one correspondence between solutions to the routing problem and solutions to the link scheduling problem. Combining this with the fact that FD spectrum-efficient routing is in NP completes the proof. ■

A. PRUNED HOP-COUNT PATH SEARCH

The NP-completeness of the decision problem (as established in Theorem 1) implies that the optimization version of the FD spectrum-efficient routing problem as given (7) is NP-hard. See, e.g., [21]. This, in turn, implies that polynomial-time algorithms that provide exact optimal solutions to (7) do *not* exist. This justifies the use of heuristics. In our numerical study, we use a heuristic based on searching over all paths with at most k hops. Then, the path with maximum spectral efficiency, as computed by (2), is returned.

In particular, we use $k = 4$, and compute a pruned set of paths with at most four hops.

The path with one hop is simply the direct link from s to d. Paths with two hops are those of the form (s, i, d). Thus, computing all paths with two hops is equivalent to enumerating all possible intermediate nodes i, other than s and d. This has a complexity of $(N-2) = O(N)$. Paths with three hops are of the form (s, i, j, d). Therefore, computing all paths with three hops is equivalent to enumerating all possible ordered intermediate sets of two nodes (i, j), other than s and d. This has a complexity of $(N-2)(N-3) = O(N^2)$. Similarly, enumerating all paths with four hops of the form (s, i, j, k, d) is equivalent to enumerating all possible ordered intermediate sets of three nodes (i, j, k), other than s and d. The latter has a complexity of $(N-2)(N-3)(N-4) = O(N^3)$. Putting everything together, computing all paths with at most four hops has an overall complexity of $O(N^3)$.

B. EXAMPLE

We present an example to illustrate the conceptual tradeoff between the presented FD routing approach (using pruned hop-count search), as compared to HD TDMA relaying, as well as simply routing along the direct link from source to destination. Consider the 5-node network of Fig. 3, where the nodes are placed in a 100×100 two-dimensional area. The coordinates of nodes 1 through 5 are (0,0), (86.14,65.78), (10.22,58.55), (96.35,84.49), (100,100), respectively. The network is fully connected, i.e., there is a link from each node to every other node. For clarity, however, not all links are shown. The network SNR (P/N_0) is set to 70 dB, and the SI cancellation factor is set to -80 dB for all nodes. We also consider node 1 to be the source, and node 5 to be the destination.

In Section II, we used the notation G_{kl} to denote the path gain from the *transmitter* of link k to the *receiver* of link l. This definition implies that G_{kl} can be thought of as $G_{kl} = \tilde{G}(Tx(k), Rx(l))$, where $Tx(\cdot)$ and $Rx(\cdot)$ denote the transmitter node and receiver node of a link, respectively. Since for any link l, the transmitter $Tx(l)$ and the receiver $Rx(l)$ are simply nodes in the network, it is straightforward to see that the path gains can be equivalently defined between *node pairs*. For the network of Fig. 3, the path gains (in dB) between node pairs, $\tilde{\boldsymbol{G}} = \left(\tilde{G}(i,j) : (i,j) \in V \times V\right)$, are given as follows:

$$\tilde{\boldsymbol{G}} = \begin{pmatrix} -80.0 & -40.39 & -24.82 & -36.21 & -43.07 \\ -35.8 & -80.0 & -32.19 & -11.64 & -26.34 \\ -26.79 & -32.83 & -80.0 & -32.53 & -37.83 \\ -40.83 & -14.14 & -35.78 & -80.0 & -6.54 \\ -45.44 & -15.23 & -35.04 & -1.63 & -80.0 \end{pmatrix}.$$

Note that the diagonal entries represent the SI cancellation factors from a node to itself. Finding the path gains between *link pairs* G_{kl} from the above (node pairs path gain) matrix $\tilde{\boldsymbol{G}}$ is straightforward. For example let l be the link between nodes (1,2) and k be the link between nodes (3,4). Then $G_{ll} = \tilde{G}(1, 2)$, and $G_{kl} = \tilde{G}(3, 2)$. As another example,

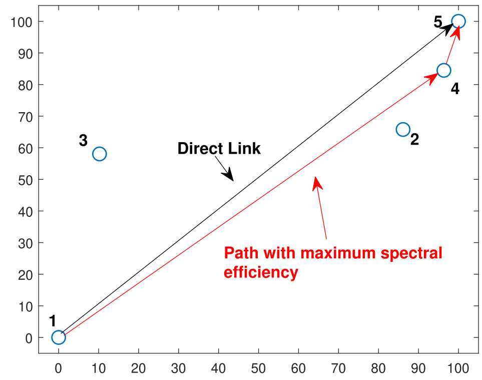

FIGURE 3. Example 5-node network.

let l be the link between nodes (1,2) and k be the link between nodes (2,4), then $G_{ll} = \tilde{G}(1, 2)$, and $G_{kl} = \tilde{G}(2, 2)$.

It is worth noting that any path in a 5-node network has at most 4 hops. Therefore, the presented pruned search over *all* paths with at most 4 hops is guaranteed to give the *exact* optimal solution for a 5-node network. Running the FD pruned hop-count path search algorithm, results in that the path with maximum spectral efficiency is path 1-4-5. Using (2), the spectral efficiency of this path is calculated as $\min\left\{\log(1+\frac{P\tilde{G}(1,4)}{P\tilde{G}(4,4)+N_0}, \log(1+\frac{P\tilde{G}(4,5)}{P\tilde{G}(1,5)+N_0}\right\} = 11.09$. Note that in the latter equation, P/N_0 and all path gains are substituted as ratios, not in dB. If the direct link 1-5 is used for transmission, the achieved spectral efficiency will be $\log(1+\frac{P\tilde{G}(1,5)}{N_0}) = 8.95$. Consequently, using the path 1-4-5 in the FD mode leads to 23.89% improvement in spectral efficiency as compared to simply using the direct link from source to destination. Furthermore, consider that the same path 1-4-5 will be used for transmission in HD TDMA mode. This implies that each one of the two links 1-4 and 4-5 will be activated *separately*, half of the time each. This implies that the HD TDMA spectral efficiency of path 1-4-5 can be calculated as $\min\left\{\frac{1}{2}\log(1+\frac{P\tilde{G}(1,4)}{N_0}), \frac{1}{2}\log(1+\frac{P\tilde{G}(4,5)}{N_0})\right\} = 5.61$. In other words, using the path 1-4-5 in FD (yet interference-limited) mode leads to an improvement in spectral efficiency of 97.55% as compared to using the same path in HD TDMA mode. The main reason is that FD allows for using both links 1-4 and 4-5 at the same time. In short, for the above network and path gain values, operating the *interfering* links 1-4 and 4-5 in the FD mode (i.e., at the same time and frequency) yields significantly higher spectral efficiency, as compared to operating each of the links 1-4 and 4-5 in HD TDMA mode (i.e., in separate time slots).

IV. NUMERICAL RESULTS

We consider multihop wireless networks with nodes scattered randomly in a 100×100 two-dimensional area. Without loss of generality, we assume that the source node s is located at the point $(0, 0)$, and the destination node d is located at the point $(100, 100)$. The x- and y-coordinates of all other nodes are independent random variables, uniformly distributed between 0 and 100, respectively. Without loss of generality, we assume that the network is fully-connected, i.e., there is a directed link between every pair of nodes. In fact, from an information-theoretic point of view, two nodes can always communicate at a sufficiently low rate [8]. The path gain from the transmitter of link k to the receiver of link l is given by $G_{kl} = c \cdot A_{kl} \cdot (d_{kl}/d_0)^{-4}$, where d_{kl} is the distance from the transmitter of link k to the receiver of link l, d_0 is the reference distance, A_{kl} is a log-normally distributed random variable (with 0-dB mean and 8-dB log-variance) that reflects shadowing, and c is a constant. Without loss of generality, and following [9], we set $d_0 = \min\{d_{kl}\}$ and $c = 1/max\{A_{kl}\}$. The resulting path gains have, thus, non-negative values smaller than one. We obtain each of our results by averaging over 10^4 random, and independent network scenarios. Each scenario is identified by its random node positions and path gains, generated as described above. Thus, each point in our result figures is averaged over 10^4 random network scenarios. Moreover, in our numerical results, the boundaries of the 95% confidence interval are less than 5% of the mean spectral efficiency.

To assess the performance of FD spectrum-efficient routing, we compare the *sub-optimal* pruned-hop-count search algorithm for FD spectrum-efficient routing against the following benchmarks:

1) **HD TDMA** relaying, using the routing algorithm in [10] as a benchmark. Note that the HD TDMA spectrum-efficient routing algorithm in [10] is *provably optimal*, and has a complexity of $O(N^3)$, which is the smallest possible complexity for this class of problems.
2) **Direct link** routing, i.e., routing along the direct link from s to d. It is straightforward to see that choosing the direct link from s to d has a constant complexity of $O(1)$, regardless of the size of the network.

Recall also that our pruned-hop-count search algorithm for FD spectrum-efficient routing has a worst-case complexity of $O(N^3)$.

First, we set the network SNR (P/N_0) to 70 dB, the SI cancellation factor to -80 dB, and vary the number of nodes N from 5 to 30. Fig. 4 depicts the spectral efficiencies of the paths obtained for interference-limited FD relaying, HD TDMA relaying and direct link routing, respectively. It is clearly seen that FD relaying leads to consistently higher spectral efficiencies as compared to HD TDMA and direct link routing, respectively. Moreover, the spectral efficiency gain due to FD increases with the increase in network size. For example, in 30-node networks, the spectral efficiency achieved with interference-limited FD relaying was 1.83 times that achieved with HD TDMA relaying, and 5.65 times that achieved with direct link routing. In other words, the *improvement* in spectral efficiency in 30-node networks due to FD relaying was 83% and 465% relative to HD TDMA and direct link routing, respectively. This is in spite of the fact that we use *sub-optimal* FD routing and *exact*

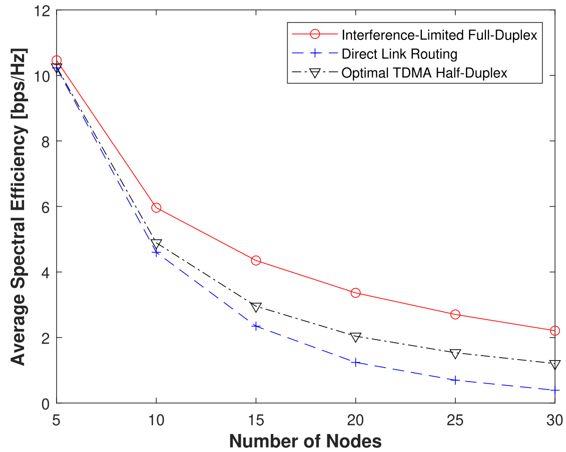

FIGURE 4. Spectral efficiency vs. the number of nodes N.

optimal HD routing. In particular, routing along the direct link may lead to inferior spectral efficiencies, especially if the direct link is too long, or if the quality (path gain) of the direct link is too low. Moreover, this result emphasizes that simultaneous FD transmissions (with SI cancellation capabilities) may lead to higher spectral efficiencies as compared to HD TDMA, in which each link transmits in a separate time slot. Fig. 5 depicts the number of hops of the optimal paths resulting from interference-limited FD relaying vs. HD TDMA relaying. The average number of hops in the resulting paths increases with the network size. However, the increase in hop-count somewhat flattens in the case of FD relaying, as compared to TDMA HD relaying. We conclude that, due to the interference limitation, FD relaying does not lead to using paths with excessively many hops, as compared to TDMA HD relaying. In fact, when the number of nodes N is 30, the average hop-count in the FD case is about 2.9, while the average hop-count in the TDMA HD case is about 4.

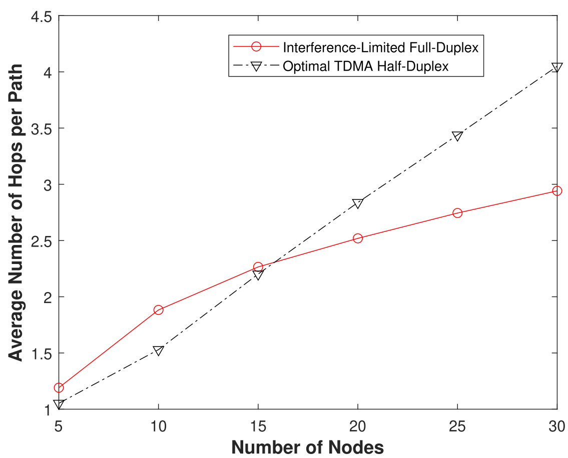

FIGURE 5. The number of hops of the paths obtained vs. the number of nodes N.

Next, we set the number of nodes N to 15, the SI cancellation factor to -80 dB and vary the network SNR (P/N_0)

from 40 to 100 dB. Fig. 6 depicts the spectral efficiencies of the paths obtained for interference-limited FD relaying, HD TDMA relaying and direct link routing, respectively. Again, the spectral efficiencies of the paths obtained using FD relaying are consistently higher than those obtained using HD TDMA relaying and direct link routing, respectively. Also, the spectral efficiency gain due to FD increases with the decrease of the network SNR (P/N_0). For example, when the network SNR is 40 dB, the spectral efficiency achieved with interference-limited FD relaying was 2.69 times that achieved with HD TDMA relaying, and 29 times that achieved with direct link routing. In other words, at a network SNR of 40 dB, the *improvement* in spectral efficiency due to FD relaying was 169% and 2800% relative to HD TDMA and direct link routing, respectively. However, when the network SNR (P/N_0) is sufficiently large, both interference-limited FD relaying and HD TDMA relaying result in paths with only one hop, i.e., the direct link from s to d. The explanation is straightforward. Consider for example the extreme case that the noise power N_0 approaches zero (which would result in an extremely high network SNR). Let the path gain of the direct link from s to d be denoted as G_{direct}. Then the spectral efficiency of the direct link $log(1 + PG_{direct}/N_0)$ would approach infinity. This high spectral efficiency cannot be achieved with an interference-limited multi-hop path (in the FD regime), or a multi-hop path where link transmissions occur in different time slots (in the HD TDMA regime). Fig. 7 depicts the number of hops of the optimal paths resulting from interference-limited FD relaying vs. HD TDMA relaying. Consistently with the results of Fig. 6, the hop-count of the resulting paths decreases as the network SNR increases. At sufficiently high network SNRs, e.g., when $P/N_0 = 100$ dB in our results, the path resulting from both FD relaying and HD TDMA relaying is simply the direct link (single-hop path) from s to d.

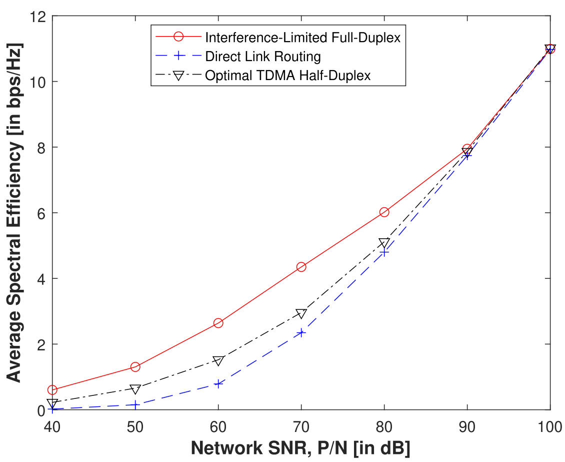

FIGURE 6. Spectral efficiency vs. the network SNR (P/N_0).

Now, we set the number of nodes N to 15, the network SNR (P/N_0) to 70 dB, and vary the SI cancellation factor from -20 to -100 dB. Fig. 8 depicts the spectral efficiencies

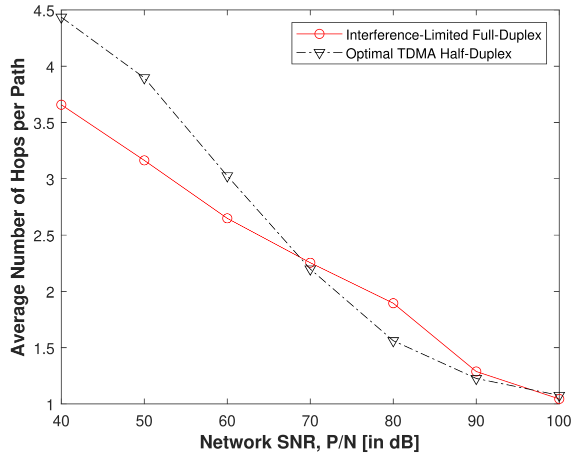

FIGURE 7. The number of hops of the paths obtained vs. the network SNR (P/N_0).

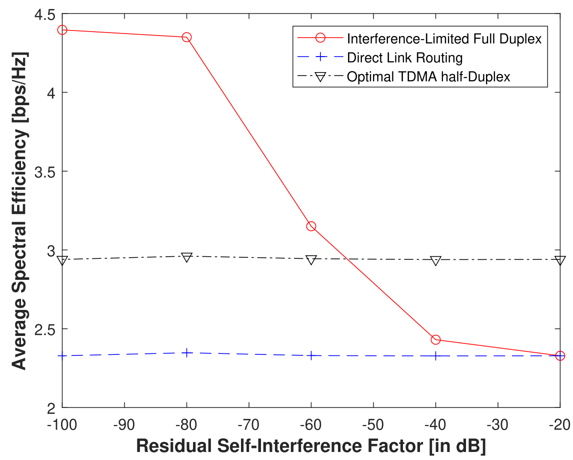

FIGURE 8. Spectral efficiency vs. the SI cancellation factor.

of the paths obtained for interference-limited FD relaying, HD TDMA relaying and direct link routing, respectively. It is clearly seen that with strong SI cancellation capabilities (i.e., with small residual SI power), FD relaying strongly outperforms HD TDMA relaying and direct link routing in terms of the achieved spectral efficiencies. For example, with a SI cancellation factor of -100 dB, the spectral efficiency achieved with interference-limited FD relaying was 1.5 times higher than that archived with HD TDMA relaying, and 1.89 times higher than that achieved with direct link routing. In other words, with a SI cancellation factor of -100 dB, the *improvement* in spectral efficiency due to FD relaying was 50% and 89% relative to HD TDMA and direct link routing, respectively. The spectral efficiency performance of FD relaying worsens as the SI cancellation capability worsens. With higher SI cancellation factors (i.e., with higher residual SI powers), the spectral efficiency resulting from FD relaying becomes even worse that HD TDMA relaying (which is not affected by SI), and approaches even that of direct link routing. In short, the benefit of interference-limited

FD relaying becomes more evident with strong SI cancellation capabilities. Otherwise, TDMA is a better option. Fig. 9 depicts the number of hops of the optimal paths resulting from interference-limited FD relaying vs. HD TDMA relaying. Consistently with the results of Fig. 8, the hop-count of the resulting FD paths decreases with the increase of the SI cancellation factor (i.e., with the increase of the residual SI power). FD relaying is only capable of exploring paths with more hops if the SI cancelation capabilities are sufficiently adequate.

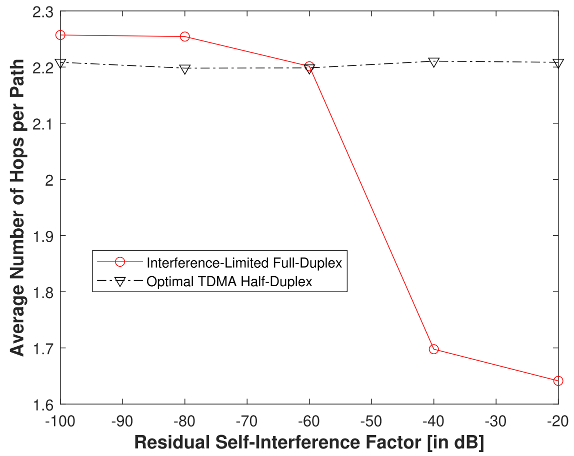

FIGURE 9. **The number of hops of the paths obtained vs. the SI cancellation factor.**

Finally, we compare the running time of the pruned-hop-count search algorithm for FD spectrum-efficient routing against that of the HD TDMA spectrum-efficient routing algorithm from [10]. Fig. 10 depicts the running time of both algorithms vs. the number of network nodes. Although both algorithms (FD pruned hop-count search and the HD TDMA algorithm from [10]) have the *same worst-case* computational complexity of $O(N^3)$, the HD TDMA algorithm enjoys a significantly lower *typical/average* running time. The reason is that the HD TDMA algorithm from [10] is based on the Bellman-Ford widest path algorithm, while the FD algorithm relies on exploring *all* paths with at most 4 hops.

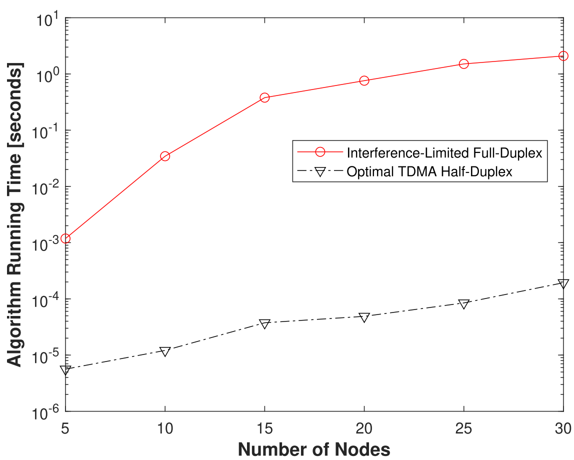

FIGURE 10. **Algorithm running time vs. the number of nodes.**

V. CONCLUSION

Motivated by the advances in full-duplex (FD) technologies and self-interference (SI) cancellation, this paper addresses the problem of finding the source-destination route with maximum end-to-end spectral efficiency in interference-limited multihop wireless networks that employ FD communications. The contribution of this paper is three-fold. We formulate the problem as a mixed integer non-linear programming problem, and provide two equivalent re-formulations that provide more insight into the problem structure. Moreover, to the best of our knowledge for the first time, we provide a *formal* proof that the problem is *NP-complete* in the interference-limited case. Our proof uses reduction from a more general link scheduling problem subject to signal-to-interference-plus-noise-ratio (SINR) constraints. Finally, using an algorithm based on searching over paths while pruning the allowed number of hops, we provide a detailed numerical study that illustrates the tradeoffs between interference-limited FD relaying and TDMA half-duplex (HD) relaying. Our results indicate that *sub-optimal* interference-limited FD relaying has up to 2.69 times higher spectral efficiencies as compared to *optimal* TDMA HD relaying. Moreover, the lower the signal-to-noise ratio (SNR) regime, the larger the network size and/or the higher the SI capabilities, the higher the spectral efficiency gain due to FD (over HD) relaying. Avenues for future research include developing lower complexity algorithms that do not sacrifice the optimality of the obtained solutions, as well as considering joint routing and wireless resource allocation/scheduling in the cases of multiple antennas and/or multiple user (source-destination) pairs.

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MOHAMED SAAD (Senior Member, IEEE) received the Ph.D. degree in electrical and computer engineering from McMaster University, Hamilton, Canada, in 2004.

He held research positions with the Department of Electrical and Computer Engineering, University of Toronto, Toronto, Canada, and with the Advanced Optimization Laboratory, Department of Computing and Software, McMaster University. He is currently an Associate Professor with the Department of Computer Engineering, University of Sharjah, United Arab Emirates. His research interests include networking, communications and optimization, with current activity focused on the optimal design of wireless and wired communication networks, and optimal network resource management. He was a recipient of the 2005-2006 Natural Sciences and Engineering Research Council of Canada (NSERC) Postdoctoral Fellowship, two best teaching awards by the IEEE Women in Engineering Society, University of Sharjah, in 2007 and 2009, the Best Paper Award in the IEEE Symposium on Computers and Communications, Riccione, Italy, in June 2010, the Annual Incentive Award for Distinguished Faculty Members, for excellence in research from the University of Sharjah, in April 2010 (university-wide). He is an Associate Editor for Frontiers in Communications and Networks.

SAEED ABDALLAH (Member, IEEE) received the B.Eng. degree in computer and communications engineering from the American University of Beirut, Beirut, Lebanon, in 2005, and the M.Sc. and Ph.D. degrees in electrical engineering from McGill University, Montreal, QC, Canada, in 2008 and 2013, respectively. He is currently an Assistant Professor with the Department of Electrical and Computer Engineering, University of Sharjah, Sharjah, United Arab Emirates. His research interests include signal processing for wireless communications, with special emphasis on relay networks, multicarrier systems, MIMO and massive MIMO systems, Wi-Fi systems, channel estimation/prediction, and adaptive modulation.