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Discrete BP Polar Decoder Using Information Bottleneck Method

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ABSTRACT Polar code is one of the channel codes and is used in the 5th generation of mobile communication system (5G). This encoding scheme is based on the operation of channel polarization, and it is possible to achieve the capacity of arbitrary binary-input symmetric discrete memoryless channels. Compared with Turbo codes and LDPC (Low-Density Parity-Check) codes, the implementation of the encoder and decoder is easier. BP (Belief Propagation) decoding, one of the decoding methods of the polar codes, can be performed at high speed because it can decode in parallel. However, the disadvantages of the BP decoding are the hardware and computational complexities. The technique of quantization can be used to reduce complexities of hardware and calculation. One of the quantization methods is the information bottleneck method, which allows an observation variable to compressed one while trying to preserve the mutual information shared with a relevant variable. As a novel approach, the information bottleneck method is used in the design of quantizers for the BP decoding of LDPC codes. In this paper, we propose a discrete BP polar decoder that can use only unsigned integers in the decoding process by using the information bottleneck method. Thus, we can replace complex calculations of BP decoding with simple lookup tables. We also investigate the minimum bit width for quantization with negligible degradation and the suboptimal E_b/N_0 for designing lookup tables, where E_b and N_0 denote energy per bit and noise power density, respectively. The simulation results show that the proposed method can achieve almost the same error correcting capability compared with the BP decoding without compression in the range of low E_b/N_0. Besides, we show that the proposed decoder can compress both channel outputs and BP messages with small loss compared with the uniform quantization decoder.

INDEX TERMS Channel coding, polar codes, BP decoding, information bottleneck method.

I. INTRODUCTION

Polar codes are attracting much attention as codes that achieve symmetric channel capacity in binary input discrete memoryless channels [1]. The 5th generation of mobile communication system (5G) adopts the polar codes as the code of the control channel. As a feature, the implementation of polar codes is easier [2] than that of Turbo codes [3] and Low-Density Parity-Check (LDPC) codes [4]. Moreover, in some situation, polar codes achieve better error correcting capability compared to the LDPC codes and Turbo codes [5]. The major decoding methods for polar codes are SC decoding [1] and BP decoding [6]. The SC decoder successively estimates the transmitted bits. As a modified version of the SC decoder, SCL decoder stores a fixed number of likelihood decoding paths [7]. The error correcting capability of the SCL decoder is better than that of the other decoders. However, SC and SCL decoders have low throughout because of sequential decoding. In addition, if an error occurs in the early stage of the decoding process, errors propagate to the later decoding process. On the other hand, BP decoder propagates messages which indicate the likelihood of the transmitted bit in parallel. Due to its structure, BP decoder can decode at high speed. The drawbacks of the BP decoding algorithm are the error correcting capability and the difficulty in hardware implementation and the complexity in calculation. In this paper, we focus on reducing the hardware and calculation complexities.

Some papers tackle these problems. In [8], Simplified Belief Propagation (SBP) is proposed, which reduces the

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calculation of message updates by replacing the so-called frozen nodes with prior probabilities. The method proposed in [9] can reduce the number of calculation in real numbers and memories by ignoring some messages passed on the factor graph of BP decoding. From a point of view other than calculation and hardware complexities, [10] and [11] can simplify the BP decoding. However, in [10] and [11], all messages are calculated in real numbers. A long bit representation is necessary for good decoding performance, however it is one of the factors leading to hardware complexity. Min-sum decoding for polar codes [12] is a hardware-friendly method, however, its decoding performance is not good. The purpose of this paper is to reduce the hardware and computational complexities without degrading the decoding performance as much as possible.

Quantization can be used to reduce these complexities. As one of the effective methods to reduce the quantization loss, we use the information bottleneck method [13]. It is a generic clustering framework in the field of machine learning. This method compresses an observation variable to a quantized one while attempting to preserve the mutual information shared with a relevant random variable. This method is used in the fields of image processing [14], [15] and machine learning [16]. This method is also used for several researches in the wireless communications [17]–[21].

In the conventional approaches, the information bottleneck method is applied to the SC/SCL decoder of polar codes [19]–[21]. However, there is no research applying this method to the polar BP decoding.

As a novel approach, [22] proposes the quantizer using this method and applies it to the decoding of the LDPC codes. In general, implementation of quantization for the reduction of the hardware complexity leads to the degradation of the decoding performance [23]. From a different perspective, a hardware-friendly decoding method so called min-sum decoding of LDPC codes exists [24]. However, the min-sum decoder of LDPC codes also results in a degradation of the decoding performance. On the other hand, in [22], the discrete LDPC decoder is shown to outperform the min-sum decoder. In this paper, we consider applying this approach to the BP decoding of polar codes. The BP decoding of polar codes has four types of messages, which leads to increase of the number of the joint distribution needed to design the decoder using the information bottleneck method. In this paper, we tackle this problem and design the decoder for polar codes. Quantization by the quantizer using the information bottleneck method makes the decoding process discretized, then we refer to the decoder as the discrete polar decoder.

This paper proposes a discrete BP polar decoder using only unsigned integers in the BP decoding process. The discrete BP polar decoder was proposed in [25]. The difference between this paper and [25] is that we provide more evaluation of the proposed method. Specifically, we add the simulation for different block lengths and compare the proposed method with other decoders to emphasize the effectiveness of the quantizer designed by the information bottleneck method. This paper compares the BER (Bit Error Rate) performance of BP decoder, discrete BP polar decoder, and min-sum decoder. The simulation results show that the proposed decoder can achieve almost the same BER as the BP decoding without quantization in the range of low SNR. The main contributions of this paper are summarized as follows.

1) The information bottleneck method is used for the BP decoding of polar codes. By using this method, real numbers used in the BP decoding are quantized into unsigned integers. We call this proposed decoder the discrete BP polar decoder. The discrete BP polar decoder uses only unsigned integers in decoding process, which can reduce the complexities of hardware and calculation.
2) To prove the effectivity of the information bottleneck method for the quantization loss, we design another discrete decoder which is a fusion of uniform quantization and lookup table. In this discrete decoder, uniform quantization is used to quantize the channel output and propagation messages for BP decoding, and the lookup table is used to replace the computational blocks. Simulation results show that the proposed method can quantize the real numbers into unsigned integers with less loss than uniform quantization decoder.
3) We quantize the floating-point information in LLR domain BP decoding with some quantization bits to implement BP decoder of polar codes, and analyze the quantization bits achieving the same BER performance as that of the floating-point decoder. This analysis proves that the proposed decoder can reduce the hardware complexities.

The reminder of this paper is organized as follows. In Section II, we introduce the polar codes, the BP decoding and the information bottleneck method. In Section III we present the novel research about discrete LDPC decoder including the sequential information bottleneck method. In Section IV, we describe the proposed decoder in detail. Section V shows the simulation results of several decoders and emphasizes the advantages of the proposed decoder by comparing those with other decoders. Finally, Section VI concludes the paper.

II. PRELIMINARIES

We introduce the polar codes, the BP decoding and the information bottleneck method in this section.

A. CODING

[1] proposes the polar codes, which are based on channel polarization. Let the block length be N and the number of information bits be K, then the channel polarization transforms N independent copies of a channel into polarized channels. The most reliable K channels are used to send the information bits and the rest of the $N - K$ channels are set to the fixed bits (typically 0). This fixed bit is called a frozen bit. In polar codes, the generator matrix G can be written as

follows

$$G = F^{\otimes n}. \tag{1}$$

$F = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $n = \log_2 N$, $F^{\otimes n} = F \otimes F^{\otimes(n-1)}$, and $\otimes$ expresses the operation of XOR. The relation between the code sequence x_1^N and the information sequence u_1^N can be expressed as:

$$x_1^N = u_1^N G. \tag{2}$$

The set of the bit indices used to send the frozen bits is called a frozen set. For example, when information bits are [0, 1, 1, 1] and the frozen set is $\mathcal{F} = \{0, 2, 3, 4\}$, the information sequence is $u_1^N = [0, 1, 0, 0, 0, 1, 1, 1]$. As a result, we can obtain the code sequence $x_1^N = [0, 1, 0, 1, 1, 0, 0, 1]$.

To choose the most reliable K out of N channels, we have to estimate the channel capacities or error rate, however, it is not easy to estimate them easily and accurately. Several papers propose the methods estimating channel capacities or error rate. [1] uses the Bhattacharyya parameters to calculate the upper bounds of error rate. [1] proposes a simulation-based method to evaluate the channel quality in terms of the error rate. [26] proposes the method using the transition probability matrices (TPMs) of channels to estimate the error rate. The method, called Gaussian Approximation (GA), is the method estimating the log likelihood ratio (LLR) at intermediate stage as Gaussian variables [27]. The method using the information bottleneck method exits [28]. In this paper, we use GA due to its simplicity.

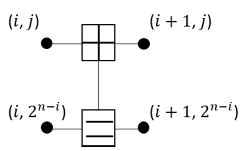

FIGURE 1. The computational block of polar codes. $n = \log_2 N$ and the node stores the left and right messages. i and j denote the column and row number of the node, respectively.

B. BP DECODING ALGORITHM

BP algorithm of the polar codes is performed on the factor graph containing computational blocks like Figs. 1, 2. The BP decoder propagates the messages indicating the likelihood on this factor graph and estimates the transmitted bits. In Fig. 2, (i, j) is the number of nodes holding the messages. This factor graph of the polar codes has $N/2 \times \log_2 N$ computational blocks. In each computational block, these calculations are carried out.

$$\begin{cases} L_{i,j}^{(k+1)} = f(L_{i+1,j}^{(k)}, L_{i+1,j+2^{n-i}}^{(k)} + R_{i,j+2^{n-i}}^{(k)}) \\ L_{i,j+2^{n-i}}^{(k+1)} = L_{i+1,j+2^{n-i}}^{(k)} + f(L_{i+1,j}^{(k)}, R_{i,j}^{(k)}) \\ R_{i+1,j}^{(k+1)} = f(R_{i,j}^{(k)}, L_{i+1,j+2^{n-i}}^{(k)} + R_{i+1,j+2^{n-i}}^{(k)}) \\ R_{i+1,j+2^{n-i}}^{(k+1)} = R_{i,j+2^{n-i}}^{(k)} + f(R_{i,j}^{(k)}, L_{i+1,j}^{(k)}) \end{cases} \tag{3}$$

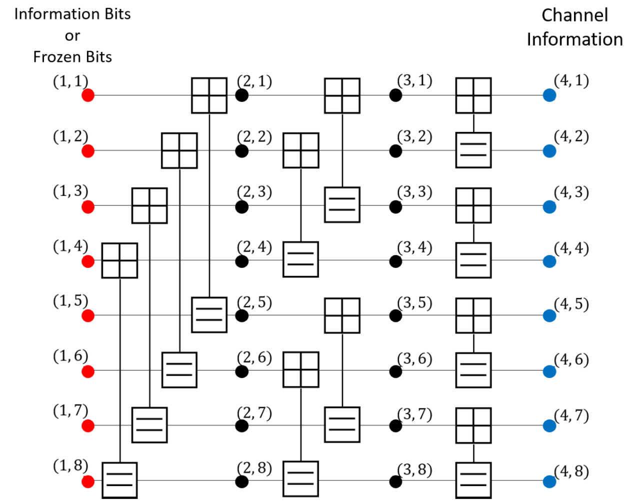

FIGURE 2. The factor graph of $N = 8$ polar codes. The red circles and blue circles denote transmitted and received nodes, respectively.

where k is an iteration number, and

$$f(x, y) \triangleq \log \frac{\cosh (x+y)/2}{\cosh (x-y)/2} \tag{4}$$
$$\approx sgn(x)sgn(y)\min(|x|, |y|). \tag{5}$$

L is the message propagated from right to left and it is called the left message. Similarly, R is the message propagated from left to right and it is called the right message. In the decoding process, at first, the right most nodes receive channel outputs and calculate LLRs in each node.

$$L_{n+1,j} = \ln \frac{P(y|x=0)}{P(y|x=1)} \tag{6}$$

Next, the left messages are generated by eq. (3). These left messages are passed from right to left in the factor graph, then the right messages are calculated after the $(1, j)$ left messages are generated. The initialization of the right message in the left most node is

$$R_{1,j} = \ln \frac{P(u=0)}{P(u=1)} = \infty \; (\textit{if frozen bit}). \tag{7}$$

To avoid bias, the values of the other right messages are set to 0 at the beginning of the decoding process. BP decoding repeats message propagation several times while using the information of frozen bits and channel outputs, thereby marginal probabilities of each bit can be obtained. As mentioned above, the factor graph has $N/2 \times \log_2 N$ computational blocks. The calculation in real numbers is carried out in this computational block. This and eq. (4) lead to high computational and hardware complexities. Although the min-sum decoding [12] denoted by eq. (5) is hardware-friendly, the drawback is the low error correcting performance [29].

C. INFORMATION BOTTLENECK METHOD

The information bottleneck method [13] is an unsupervised clustering framework in the machine learning field.

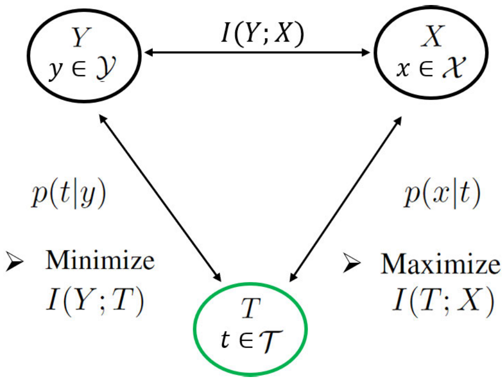

FIGURE 3. The relationship between different variables in the information bottleneck method [13].

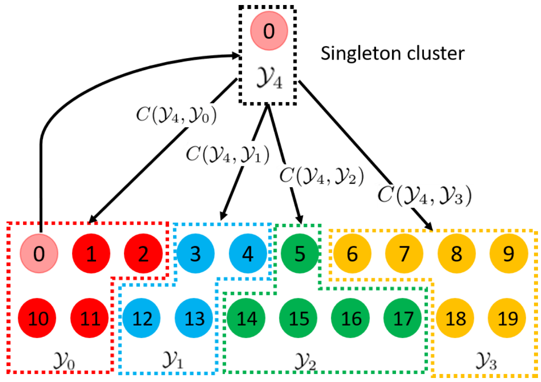

FIGURE 4. The procedure of the sequential information bottleneck method.

This method uses three random variables, an observed variable $\mathcal{Y}$, a relevant one $\mathcal{X}$, and a compression one $\mathcal{T}$. Fig. 3 shows the relationship of three variables, where $I(X;Y) = \sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} p(x,y)\log\frac{p(x,y)}{p(x)p(y)}$. We use the Lagrange method to find a suitable conditional distribution $p(t|y)$ that maximizes $I(T;X)$ or minimizes $I(Y;T)$. Using the trade-off parameter β, we use the following equation [13]:

$$\mathcal{L}\{p(t|y)\} = I(Y;T) - \beta I(T;X). \tag{8}$$

In both the decoder in [22] and the proposed decoder, we consider $\beta \to \infty$, which means maximizing $I(T;X)$ and preserving the relevant information. In the information bottleneck algorithm, inputs are $p(x,y)$, $|\mathcal{T}|$ and β, then, outputs are $p(t|y)$, $p(x|t)$ and $p(t)$ as shown in [13]. Moreover, the information bottleneck algorithm also gives $p(x,t)$ due to $p(x,t) = p(x|t)p(t)$. The mapping function $p(t|y)$ is important because the quantization loss depends on how we determine it. There are several applications and extensions of the information bottleneck algorithm as summarized in [30]. We will focus on the iterative information bottleneck algorithm [13], [16] and show it in the next section.

III. NOVEL APPROACH FOR BP DECODING OF LDPC CODES

A. SEQUENTIAL INFORMATION BOTTLENECK METHOD

The sequential information bottleneck method is proposed in [16], and is used for unsupervised document classification. [22] and this paper use it for the quantizer design. We assume the following event spaces of the observed and compression variables

$$\mathcal{Y} = \{0, 1, \ldots, |\mathcal{Y}| - 1\}, \tag{9}$$
$$\mathcal{T} = \{0, 1, \ldots, |\mathcal{T}| - 1\}. \tag{10}$$

The process of the sequential information bottleneck method is shown in Fig. 4. This is the example for $|\mathcal{Y}| = 20$ and $|\mathcal{T}| = 4$. At first, the sequential information bottleneck algorithm classifies $|\mathcal{Y}|$ elements into $|\mathcal{T}|$ clusters randomly. In Fig. 4, 20 elements $y \in \mathcal{Y}$ are categorized into $\mathcal{Y}_t$ ($t \in \mathcal{T}$) clusters. This relationship of y and t is represented by the mapping function $p(t|y)$.

Next, the algorithm extracts one element y from $\mathcal{Y}_t$ (In Fig. 4, $t = 0$, $y = 0$) and makes a singleton cluster $\mathcal{Y}_{|\mathcal{T}|}$ ($\mathcal{Y}_4$) which has only one element. This manipulation changes the mapping function $p(y|t)$. This algorithm merges the singleton cluster into the original cluster according to the cost function $C(\mathcal{Y}_{|\mathcal{T}|}, \mathcal{Y}_t)$. This cost is calculated for each cluster $\mathcal{Y}_t$ and is given in [16], [30] as follows

$$C(\mathcal{Y}_{|\mathcal{T}|}, \mathcal{Y}_t) = (p(y) + p(t)) \cdot JS(p(x|y), p(x|t)), \tag{11}$$

where $JS(p,q)$ denote the Jensen-Shannon divergence defined as

$$JS(p,q) = \pi_1 D_{KL}(p|\bar{p}) + \pi_2 D_{KL}(q|\bar{p}), \tag{12}$$

where, $D_{KL}\{p(x)|q(x)\} = \sum_{x\in\mathcal{X}} p(x)\log\frac{p(x)}{q(x)}$. We adapt the notation in [16],

$$\{p, q\} \equiv \{p(x|y), p(x|t)\} \tag{13}$$
$$\{\pi_1, \pi_2\} \equiv \left\{\frac{p(y)}{p(y)+p(t)}, \frac{p(t)}{p(y)+p(t)}\right\} \tag{14}$$
$$\bar{p} = \pi_1 p(x|y) + \pi_2 p(x|t). \tag{15}$$

The algorithm iterates the extraction and merges an element for all $y \in \mathcal{Y}$ until the clusters are unchanged, which gives the final mapping function $p(t|y)$.

B. QUANTIZER DESIGN FOR DISCRETE DECODER

Since the information bottleneck method can set the compression variable $\mathcal{T}$ arbitrarily, [22] (and our paper) use the set of unsigned integers, which enables design of the discrete LDPC decoders. We refer to the decoder using only unsigned integers in decoding process as a discrete decoder. In [22], $\mathcal{Y}$ and $\mathcal{X}$ are the set of received values and transmitted bits, respectively.

[22] proposes the design of the quantizer that has the sub-optimum quantization boundaries in terms of mutual information comparable to the optimum ones [31]. We need the channel output quantizer to design the lookup tables, and it is mandatory for the discrete LDPC decoders and our proposed decoder. This quantizer is constructed by using the modified sequential information bottleneck algorithm shown

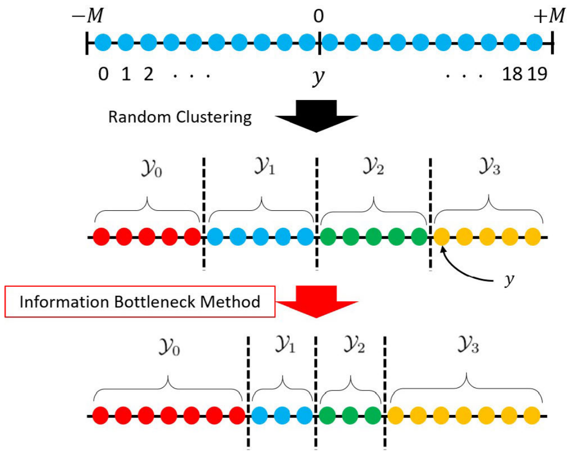

FIGURE 5. The quantization of channel outputs. In this example, channel outputs are discretized into $|\mathcal{Y}| = 20$ values and these are clustered into $|\mathcal{T}| = 4$ clusters.

in [22]: Algorithm 1. We call this algorithm the quantizer design algorithm and briefly describe it. At first, channel outputs are discretized as shown in Fig. 5. In this example, channel outputs in the range $[-M, +M]$ are discretized into $|\mathcal{Y}| = 20$ unsigned integers. Next, these unsigned integers are clustered randomly. In the middle figure of Fig. 5, 20 unsigned integers are clustered into 4 clusters. Here, when y is a received value and x is a transmitted bit, the joint probability density function (pdf) $p(x, y)$ is denoted as follows in the case of an AWGN channel:

$$p(x, y) = \frac{1}{2\sqrt{2\pi}\sigma} \exp\left(-\frac{|y - s(x)|^2}{2\sigma^2}\right). \tag{16}$$

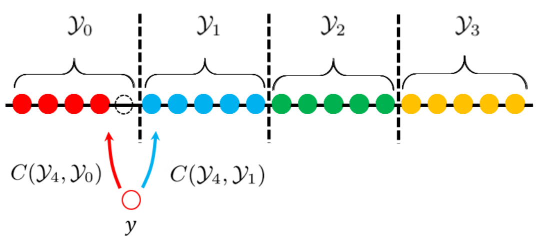

FIGURE 6. The procedure of the modified sequential information bottleneck method.

σ^2 is a noise variance and if $x = 1$, $s(x) = +1$; otherwise $s(x) = -1$. This $p(x, y)$ is regarded as an input of the quantizer. By using this $p(x, y)$, the cost function C is calculated in the boundaries of each cluster. The modified sequential algorithm differs from the original one in that it only considers adjacent clusters when calculating the cost function as shown in Fig. 6. The quantizer draws and merges the discretized channel outputs in accordance with this cost function C, which changes the boundaries of the quantization. By repeating this operation until the boundaries remain unchanged, we can obtain the mapping function $p(t|y)$ as in the bottom of Fig. 5. This $p(t|y)$ has quantization regions

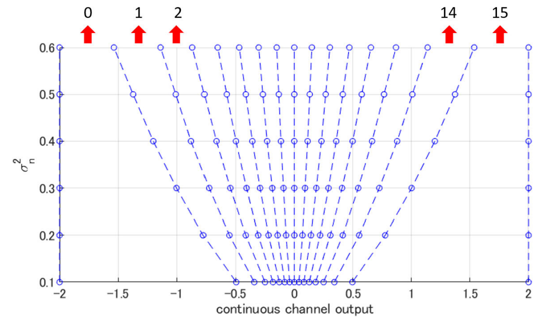

FIGURE 7. Quantization boundaries of the channel output by the quantizer using the information bottleneck method.

that maximize $I(T; X)$ for the output cardinality $\mathcal{T}$, because it considers the Lagrangian letting $\beta \to \infty$. Fig. 7 shows the example of the quantization boundaries obtained by the quantizer using the information bottleneck method. In this example, we use $M = 2$, $|\mathcal{T}| = 16$, $|\mathcal{Y}| = 2000$ and adapt BPSK (Binary Phase Shift Keying) and an AWGN channel. As shown in this figure, the channel output in the leftmost region is assigned to an integer 0 and the channel output in the second leftmost region is assigned to an integer 1.

Furthermore, since the information bottleneck method provides a posteriori probability $p(x|t)$ as a side effect, we can calculate following

$$L = \log \frac{Pr(x = 0|t)}{Pr(x = 1|t)}. \tag{17}$$

This quantizer is important for the discrete LDPC decoder and is used in the proposed decoder. More detailed information about this quantizer is provided in [22]. [22] uses this quantizer to replace the check and variable node operations of LDPC codes with lookup tables. In this paper, we use this quantizer to replace the computational block of polar BP decoding with lookup tables.

IV. PROPOSED METHOD

The drawbacks of the polar BP decoding with eq. (4) are the computational and hardware complexities. Therefore, in this paper, we attempt to reduce these complexities by applying the idea of the novel approach to the BP decoding of the polar codes.

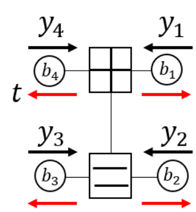

FIGURE 8. The relationship of each message in the computational block of polar BP decoding.

Fig. 8 shows the computational block of polar codes to assign the variables to messages needed for the calculation.

In the BP decoding of polar codes, the calculation of one message uses three messages. In Fig. 8, y denotes the left or right message propagated on the factor graph and t is the target message that we want to calculate. The $p(x, t)$ about the upper left message required as input to the quantizer design algorithm is calculated as follows.

$$p(b, \tilde{y}_4) = \sum_{\substack{(b_1,b_2,b_3):\\ b=b_1 \oplus b_2, b_2=b_3}} p(b_1, y_1)p(b_2, y_2)p(b_3, y_3). \tag{18}$$

$\tilde{y}_4 = [y_1, y_2, y_3]$ and $b_k = 0$ *or* 1 $(k = 1, 2, 3, 4)$. The quantizer design algorithm supplies the quantization boundaries of upper left messages, which make it possible to construct the lookup table to generate t. Similarly, $p(x, t)$ of the lower left, lower right, and upper right message need the following calculation respectively to construct the lookup tables for each target message, where $\tilde{y}_3 = [y_1, y_2, y_4]$, $\tilde{y}_2 = [y_1, y_3, y_4]$, and $\tilde{y}_1 = [y_2, y_3, y_4]$.

$$p(b, \tilde{y}_3) = \sum_{\substack{(b_1,b_2,b_4):\\ b=b_1 \oplus b_4, b=b_2}} p(b_1, y_1)p(b_2, y_2)p(b_4, y_4), \tag{19}$$

$$p(b, \tilde{y}_2) = \sum_{\substack{(b_1,b_3,b_4):\\ b=b_1 \oplus b_4, b=b_3}} p(b_1, y_1)p(b_3, y_3)p(b_4, y_4), \tag{20}$$

$$p(b, \tilde{y}_1) = \sum_{\substack{(b_2,b_3,b_4):\\ b=b_3 \oplus b_4, b_2=b_3}} p(b_2, y_2)p(b_3, y_3)p(b_4, y_4). \tag{21}$$

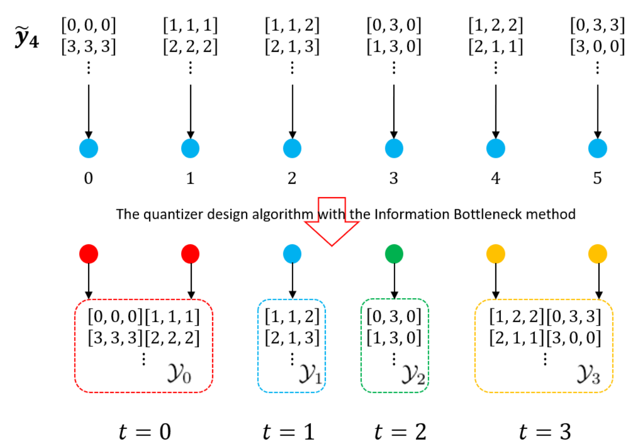

FIGURE 9. The design of lookup table. In this example, $|\mathcal{T}| = 4$.

For example, we show the process of the design of a lookup table for updating the upper left message. The aim is to generate the message t from the other messages $\tilde{y}_4$. At first, the joint input distribution is calculated from eq. (18). In eq. (18), $p(b_1, y_1)$ and $p(b_2, y_2)$ are given from the channel output quantizer if this computational block is located at the rightmost point of Fig. 1. Otherwise, $p(b_1, y_1)$ and $p(b_2, y_2)$ are obtained from the other designs of the lookup tables as well as $p(b_3, y_3)$. The LLRs are calculated from the joint input distribution, and $\tilde{y}_4$ is ordered by the LLRs and grouped by the same LLRs like Fig. 9. Next, unsigned integers are assigned to each group and by applying the quantizer design algorithm [22], $|\mathcal{T}|$ new unsigned integers are allocated to each group like Fig. 9. These are the outputs of the lookup table for each combination $\tilde{y}_4$ instead of the computational block.

TABLE 1. An Example of the lookup table for the upper left message.

y_1	y_2	y_3	t
0	0	0	0
2	2	2	0
3	3	3	0
⋮	⋮	⋮	⋮
1	2	2	3
3	0	0	3

As a result, the lookup table is configured like Table 1 for example. As this lookup table, the upper left message t is generated using 3 unsigned messages y_1, y_2, y_3. Moreover, as mentioned before, the information bottleneck algorithm also gives $p(x|t)$ and $p(t)$, which enables the calculation of the joint input distribution for the next lookup table.

The design of the lookup tables for the other messages are configured in the same way. All the computational blocks are replaced by the lookup tables in the same order as in the BP decoding.

At last, the initialization also changes from that of conventional BP decoding. The conventional BP decoding initializes the right messages according to eqs. (6), (7) in section II, however the decoding processes are done with all unsigned integers in the proposed decoder by using the information bottleneck method. Therefore, the initialization of the right messages in the proposed method becomes

$$R_{1,j} = 0, \quad (\textit{if frozen bit}). \tag{22}$$

The other messages are initialized randomly to $|\mathcal{T}|/2 - 1$, $|\mathcal{T}|/2$ in order not to be biased.

As a result, the discrete BP polar decoder can treat only unsigned integer in the decoding process.

A. DESIGN SNR/PARAMETER SELECTION

It is preferable to create the lookup table offline rather than online because the lookup table design is computationally extensive. Channel outputs change for each E_b/N_0, so it is optimum to design the lookup tables for each value of E_b/N_0 (in fact, it is not optimum for each value of E_b/N_0. See Section V.), where E_b and N_0 denote energy per bit and noise power density, respectively. However, this is also computationally expensive, then we use only one particular E_b/N_0 for the sake of simplicity. The proposed decoder uses mismatched (we refer to this E_b/N_0 as a design E_b/N_0) E_b/N_0 to the actual condition in most cases. Therefore, it is important how we decide the design E_b/N_0 that achieves good error correcting performance with mismatched E_b/N_0.

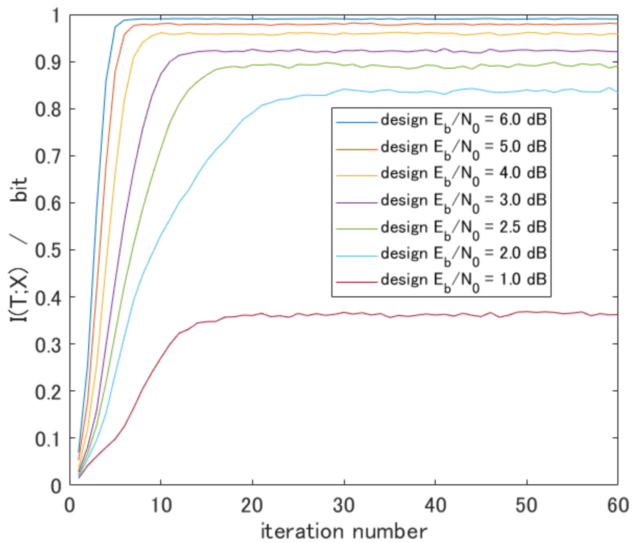

FIGURE 10. Pursuing the mutual information $I(T;X)$ with $|\mathcal{T}| = 16$ over decoding iterations. E_b/N_0 is from 1.0 dB to 6.0 dB.

In the quantizer design algorithm using the information bottleneck method, we use the joint pdf $p(x,t)$. By using $p(x,t)$, we can calculate mutual information $I(T;X)$. This mutual information represents the reliability of each bit, so we pursue the mutual information growth to obtain a suitable design E_b/N_0. Fig. 10 shows the mutual information for iteration number in various E_b/N_0. We use $p(x,t)$ at the decision level and evaluate the average of mutual information. Fig. 10 shows that $I(T;X)$ gets close to 1 in each iteration and when the design E_b/N_0 is greater than 2.5 dB, $I(T;X)$ gets close to 1 in early iteration. Actually, the design $E_b/N_0 =$ 3.0 dB shows a larger $I(T;X)$, however, it becomes constant in a small iteration (iteration number $=$ 10). This feature degrades the error correcting performance of the proposed decoder due to the gap between the designed and actual mutual information used in the simulations. We want to make the mutual information converge at 50 iterations because we use the iteration number 50 in the simulations. This is because we want to terminate the iteration in high reliability and an extra iteration is unfavorable in terms of computational cost. Though $I(T;X)$ in a smaller design E_b/N_0 may become constant at 50 iterations, it can not reach near 1. In this situation, the decoder underestimates the ability of the iterative decoding, which degrades the error correcting performance of the proposed decoder. On the other hand, the mutual information reaches 1 in too early stage in high design E_b/N_0. In this situation, the decoder overestimates the ability of the iterative decoding. The actual value of the mutual information in the simulation varies with the E_b/N_0 like Fig. 10. Based on these facts, we use $E_b/N_0 = 2.5$ dB to design the lookup tables and the quantization of channel outputs in the proposed decoder. In fact, the mutual information at $E_b/N_0 = 2.5$ dB and $E_b/N_0 = 3.0$ dB are not much different. We decided to use $E_b/N_0 = 2.5$ dB for design E_b/N_0 based on the simulation results.

V. SIMULATION RESULTS

In this section, we show the performance of the channel output quantizer and discrete BP polar decoders, and we compare the complexity of the proposed decoder with that of the min-sum decoder [12]. Besides, we design another discrete decoder which is a fusion of the uniform quantization and the lookup table design to prove that the proposed decoder can quantize the information with less loss by using the quantizer designed by the information bottleneck method. In this discrete decoder, uniform quantization is used to quantize the channel output and propagation messages for BP decoding, and the lookup table is used to replace the computational blocks. We also analyze the conventional BP decoding and find the minimum quantization bits which achieves the same BER performance as the floating-point.

TABLE 2. Simulation parameters.

block length	128, 256
code rate	1/2
modulation	BPSK
channel	AWGN
code construction	GA (Gaussian Approximation)
parameter for GA	$\sigma = 0.9$
$\lvert\mathcal{T}\rvert$	4, 8, 16
iteration number	50

A. BER PERFORMANCE OF THE PROPOSED DECODER

The simulation parameters are listed in Table 2. To investigate the performance of the quantizer, we compare the BER performance of the min-sum decoder without quantization and the min-sum decoder with $q = \log_2 |\mathcal{T}|$ bit quantization of only channel outputs. In this decoder, we use eq. (17) for the calculation of the LLR and feed it to the min-sum decoder. Fig. 11 shows the error correcting performance of the min-sum decoder without quantization and that of the min-sum decoder with quantization level $|\mathcal{T}| = 4, 8, 16$. The larger the $|\mathcal{T}|$ becomes, the better BER performance the decoder achieves. When $|\mathcal{T}| = 16$, the BER of min-sum decoder with quantization is almost the same as that of the min-sum decoder without quantization. Therefore, we use a channel quantizer with quantization level $|\mathcal{T}| = 16$ in the discrete polar decoders.

Figs. 12 and 13 show the BER of proposed discrete BP polar decoder. Fig. 12 shows the BER when the block length is 128, and Fig. 13 shows the BER when the block length is 256. In both the figures, the proposed discrete BP polar decoder achieves almost the same BER performance as the BP decoding in the range of low E_b/N_0. In the range of high E_b/N_0, the influence of the quantization in both channel outputs and updating messages degrades the BER of the proposed decoder, because the discrete BP polar decoder uses only $|\mathcal{T}|$ unsigned integers.

The proposed decoder is investigated in more detail in terms of the design E_b/N_0 and the number of iterations.

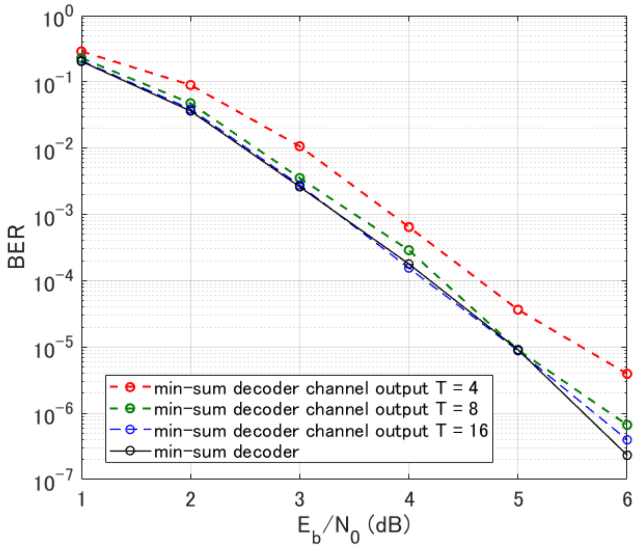

FIGURE 11. The BER performance of the decoders whose channel outputs are quantized using the quantizer designed by the information bottleneck method with different quantization bits. The iteration number of all decoders is 50. The block length is 256.

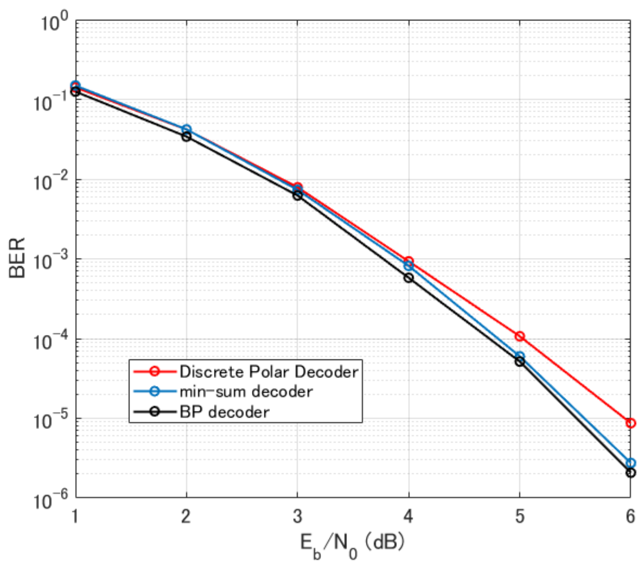

FIGURE 12. The BER performance of the proposed decoder and the BP decoder with and without quantization of the channel outputs. The iteration number of all decoders is 50. The block length is 128 and the code rate is 1/2.

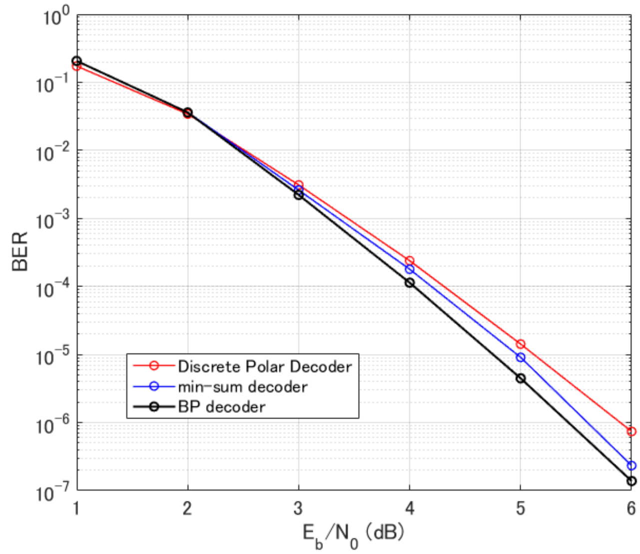

FIGURE 13. The BER performance of the proposed decoder and the BP decoder with and without the quantization of the channel outputs. The iteration number of all decoders is 50. The block length is 256.

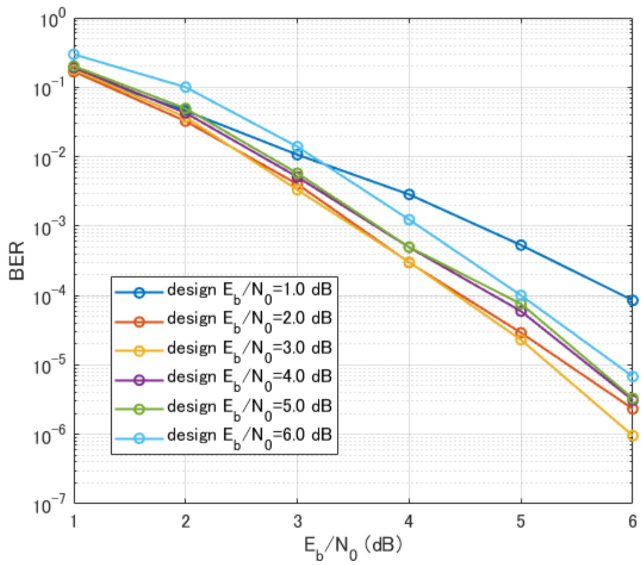

FIGURE 14. The BER of the proposed decoder for design E_b/N_0 from 1 to 6 dB.

Fig. 14 shows the BER of the proposed decoder with varying the design E_b/N_0. In the range of low E_b/N_0 (1, 2, 3 dB), the decoder designed for each design E_b/N_0 achieves the best BER performance. However, in the range of high E_b/N_0 (4, 5, 6 dB), the decoder designed for each design E_b/N_0 does not achieve the best BER performance. This may be because the error correcting performance of polar codes depends largely on the code construction.

Fig. 15 shows the BER of the proposed decoder with varying the number of iterations. The number of iterations varies from 10 to 50. In the range of middle E_b/N_0, as the number of iterations is reduced, the BER is slightly degraded. On the other hand, in the range of high E_b/N_0, the BER is almost the same except when the number of iterations is 10. As can be seen from the values of the mutual information in Fig. 10, 10 iterations are not enough to correct the error, which leads to significant deterioration of the BER. From 20 to 50 iterations are enough or may be excessive in the high E_b/N_0 region. The iterative decoding can reduce this extra iterations using some techniques [11]. Although not covered in this paper, these techniques can be applied in the proposed decoder and reduce the complexity further. In the BP decoding of the polar

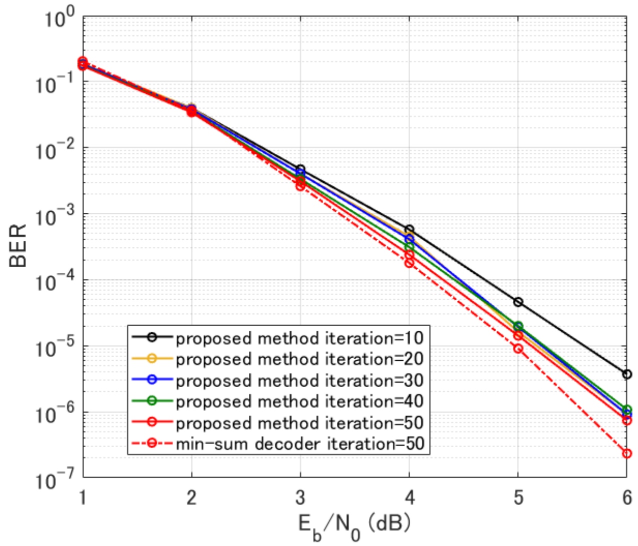

FIGURE 15. The BER of the proposed decoder for iteration number from 10 to 50.

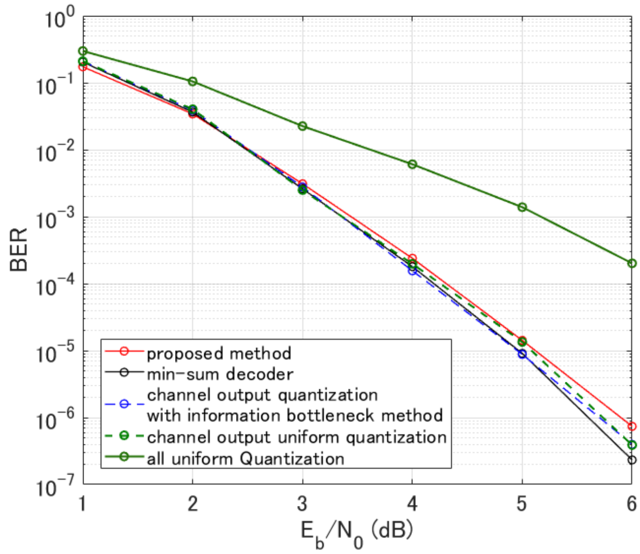

FIGURE 16. The BER comparison of the discrete decoder using quantization with the information bottleneck method and using uniform quantization. Both methods use $|\mathcal{T}| = 16$ i.e., 4 bits quantization.

codes, it is also the problem that the decoding performance is not as good as that of the SCL and the CRC-Aided-SCL decoders. There are some techniques [32], [33] for improving the decoding performances in the BP decoding and these also can be applied to our proposed decoder, i.e., the discrete BP polar decoder. Although the proposed decoder represents both the channel outputs and the BP messages as unsigned integers instead of real numbers, the decoding process itself does not change. Due to this fact, the proposed decoder can use some improving techniques and the proposed decoder will be able to improve the decoding performance and the decoding complexity further.

B. THE EFFECTS OF THE INFORMATION BOTTLENECK METHOD

As mentioned in section III, the quantizer designed by the information bottleneck method can quantize the observed variables into quantized variables while preserving the relevant information. To prove this, we design another discrete BP polar decoder that uses uniform quantization in both the channel outputs and BP messages. Since this decoder updates BP messages by lookup tables, it is a combination of the uniform quantization and the lookup table design. To construct the same conditions as the proposed decoder ($|\mathcal{T}| = 16$), the channel outputs and the BP messages are quantized into 16 unsigned integers uniformly. In Fig. 16, the BER of the proposed decoder using the information bottleneck method and the uniformly quantized decoder are compared. The green lines denote uniformly quantized decoder in terms of channel outputs only or both channel outputs and the BP messages. The BER of the uniformly quantized decoder is significantly degraded compared to the proposed decoder. On the other hand, the decoder that quantizes uniformly only channel outputs achieves the same BER as the decoder that quantizes channel outputs using the quantizer designed by the information bottleneck method. In general, the quantization loss of the uniform quantization is larger than that of the non-uniform quantization. Although the small loss is caused by uniform quantization in only channel outputs quantization, the iterative decoding can correct error. However, the BP messages of the polar codes are used iteratively in the decoding process. The decoder that quantizes both channel outputs and BP messages increases the quantization loss, which leads to great degradation of the BER. As a result, iterative decoding cannot correct the error and the BER of the uniformly quantized decoder is significantly degraded. In contrast, the proposed decoder can suppress the quantization loss in both channel outputs and BP messages. It is the benefit of the non-uniform quantizer designed by the information bottleneck method. In other words, the quantizer designed by the information bottleneck method can compress both channel outputs and BP messages while preserving the information of transmitted bits.

C. ANALYSIS OF THE QUANTIZATION BITS

While the previous subsection used a combination of uniform quantization and lookup tables as a comparison, in this subsection we use uniform quantization in conventional sense. In [34], they analyze quantization technology for the initial LLR and the path metric of the CRC-Aided-SCL decoder. In this paper, we analyze the quantization bits needed for the implementation of the BP decoding of polar codes without degradation. In other words, we perform uniform quantization in the LLR domain for polar BP decoding. Based on eqs. (16), (17), we can obtain the initial LLR.

$$LLR = ln\frac{\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(y-1)^2}{2\sigma^2}\right)}{\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(y+1)^2}{2\sigma^2}\right)} = \frac{2y}{\sigma^2} \tag{23}$$

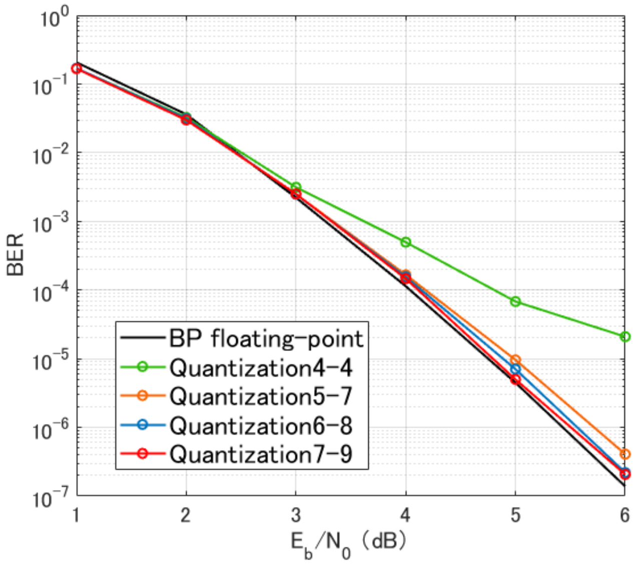

FIGURE 17. The quantization survey of the polar BP decoding.

In Section III, we assume that if $x = 1$, $s(x) = +1$; otherwise $s(x) = -1$. However, to make the description the same as [34], we use the opposite representation in this section i.e., if $x = 1$, $s(x) = -1$; otherwise $s(x) = +1$. We can calculate the cumulative probability distribution function by eq. (23).

$$P_r(LLR \leq r) = \frac{1}{2} + \frac{1}{4}\left(erf\left(\frac{\sigma^2 r - 2}{2\sqrt{2}\sigma}\right) + erf\left(\frac{\sigma^2 r + 2}{2\sqrt{2}\sigma}\right)\right) \quad (24)$$

Details of the calculations are given in [34], and as a result, we obtain the pdf of the initial LLR.

$$f(r) = \frac{1}{2\sqrt{2\pi}z}\left(\exp\left(-\frac{(r - z^2/2)^2}{2z^2}\right) + \exp\left(-\frac{(r + z^2/2)^2}{2z^2}\right)\right) \quad (25)$$

where $z = 2/\sigma$ and r denotes SNR. Consequently, the distribution of the initial LLR becomes Gaussian distribution $N(\pm 2/\sigma^2, 4/\sigma^2)$. Because of the 3σ principle of the Gaussian distribution, we should quantize uniformly the initial LLR in $[-2/\sigma^2 - 3 \times 2/\sigma, 2/\sigma^2 + 3 \times 2/\sigma]$. According to this range, we quantize the initial LLR and the BP messages (right and left messages). The quantization bits for BP messages should be more than that of the initial LLR, because BP messages are propagated iteratively and have great effects on the decoding performance. In the simulation, we use 2 bits more for the BP messages than that for the initial LLR. Fig. 17 shows the BER performance of the quantized BP decoder. "*a-b*" means that *a* bits are used for the quantization of the initial LLR and *b* bits are used for the quantization of the BP messages. "6-8" and "7-9" shows almost the same decoding performance as the floating-point BP decoder. It can be said that "6-8" quantization is the best in decoding performance and hardware complexities in the configuration of this polar code. Moreover, "4-4" represents the 4 bits quantization of the initial LLR and the BP messages respectively, which is the same configuration as the proposed decoder i.e., the discrete BP polar decoder. Fig. 18 shows the comparison of the proposed method and the uniformly quantized BP decoder. Although the proposed decoder and "4-4" quantization are using the same number of total bits for quantization, the proposed decoder achieves better BER than "4-4" quantization. Our proposed method can achieve almost the same BER performance as the "5-7" quantization decoder. This is because the quantizer constructed by the information bottleneck method can quantize real numbers including channel outputs and BP messages into unsigned integers with small loss.

TABLE 3. The computational complexity of the min-sum decoding [12] when the block length is *N*.

	'$*$'	'$+$'
Number of operations	$N/2 \times \log N \times 8$	$N/2 \times \log N \times 4$

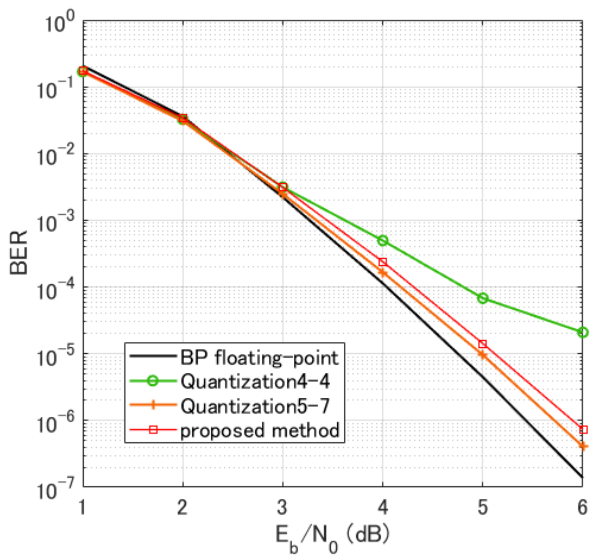

FIGURE 18. The comparison of the proposed method and the uniformly quantized LLR domain BP decoding.

D. COMPLEXITY

We briefly show the computational complexity of the min-sum decoding [12] based on eqs. (3) and (5). Eq. (5) has two operations of '$*$' and no '$+$', and eq. (3) has two '$*$' and one '$+$' for each line. As a result, each computational block on the factor graph of polar BP decoding has 8 '$*$' and 4 '$+$'. Furthermore, the factor graph of the polar BP decoding has totally $N/2 \times \log N$ computational blocks. Finally, we show the computational complexity of the min-sum decoding of the polar codes in the following table. Specifically, when the block length $N = 128$, the numbers of '$*$' and '$+$' become

3584 and 1792, respectively and when $N = 256$, the number of '$*$' and '$+$' become 8192 and 4096, respectively. Our proposed method can replace these calculations with only referring to the lookup table. From one perspective, it can be said to be a reduction in complexities of calculations. However, using the lookup table also increases the space complexities. Since the space complexity of lookup tables is closely related to the decoder design, a comparison of the space complexity between the use of lookup tables and the calculations of the conventional BP decoder is beyond the scope of this paper. In [35], variable node operations of LDPC decoder are replaced with the lookup tables. Moreover, [35] evaluates several complexities of the LDPC decoder and mentioned that the hardware complexities increased by the lookup tables are smaller than that decreased by the bit length reduction by the quantization. Decoding speed is also reduced by using the lookup table. Since the hardware complexity and decoding speed largely depend on the decoder architecture, it is beyond the scope of this paper. In fact, the min-sum decoding is hardware-friendly [36] and outperforms our proposal in error correcting capabilities as shown in Figs. 12, 13. Since the error correcting performance of polar codes is closely related to the code structure, we believe that it can be improved by optimizing the code structure for each design E_b/N_0 in future work.

E. ISSUES

In the proposed decoder, the lookup tables need much memory compared with the discrete LDPC decoder [22]. As mentioned in Section II, the factor graph has $N/2 \times \log_2 N$ computational blocks. In the proposed decoder, these computational blocks are replaced by the lookup tables and different lookup tables are needed for each message. The number of lookup tables becomes $N/2 \times \log_2 N \times 4$ in each iteration. Furthermore, the number of entries of each lookup table is $|\mathcal{T}| \times |\mathcal{T}| \times |\mathcal{T}|$. Namely, each lookup table has 4096 entries in the proposed decoder. For these reasons, the proposed decoder requires more memory than the discrete LDPC decoder. Note that, however, there is a tradeoff between the memory and the complexities in terms of hardware and computation.

VI. CONCLUSION

This paper proposes the discrete BP polar decoder which uses the quantizer designed by the information bottleneck method. The information bottleneck method is a generic clustering method in the machine learning fields and can compress a certain value into quantized values with small loss. The quantizer using this method preserves the information about the transmitted bits. The idea is that the updating of the messages is replaced by the lookup tables. The decoder uses only unsigned integers in decoding process, which makes it possible to call the decoder a discrete decoder. This paper also shows the effectivity of the information bottleneck method for the quantization loss by comparing with the uniform quantization decoder. In addition, we quantize the floating-point information in BP decoding with some quantization bits to implement BP decoder of polar codes, and analyze the quantization bits which achieve the same BER performance as that of the floating-point decoder. Simulation results show that the proposed discrete BP polar decoder shows almost the same BER (Bit Error Rate) performance as that of the BP (Belief Propagation) decoding without the quantization in the range of low E_b/N_0. Moreover, simulation results show that the proposed discrete BP polar decoder can reduce the hardware complexities compared with the uniform quantization decoder because it can be implemented with fewer bits in the same error correcting capability.

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