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A Novel Nonlinear Expanded Dominance Relation Based Evolutionary Algorithm for Many-Objective Optimization Problems

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ABSTRACT Multi-objective optimization problems exist widely in scientific research and engineering applications. With the number of objectives increasing, the proportion of non-dominated individuals in the population of many-objective optimization problems increases sharply, resulting in a reduction of convergence pressure of the traditional multi-objective optimization algorithms. In some cases, the optimal solutions may be located in the special regions, such as many discrete regions and the regions with very few feasible solutions. In this case, the existing nonlinear expanded evolutionary algorithm can not find the true Pareto fronts. To address the limitation, a novel nonlinear expanded dominance relation based many-objective evolutionary algorithm is proposed to handle many-objective optimization problems. Experimental results show that compared with the state of art algorithms, the proposed algorithm is effective for DTLZs, in terms of IGD, PD and GD metrics.

INDEX TERMS Many-objective optimization, nonlinear expanded dominance relation, evolution algorithm.

I. INTRODUCTION

Many-objective optimization problem (MaOP) refers to the multi-objective optimization problem with more than three objectives. For example, in the airport scheduling problem, in order to solve the allocation of aircraft seats more efficiently, many factors need to be considered: the number of passengers, the time of taking off and landing, the choice of gate, the path of ground taxiing, weather factors, etc. Moreover, many-objective optimization problem also exists in scientific research and engineering applications including automatic control, portfolio and decision-making, job shop scheduling, biomedicine, image processing and data mining [1]–[6]. Recently,a growing number of experts and scholars have participated in the research of multi-objective optimization algorithm. Using evolutionary algorithms(EAs) have been proved to be an effective method for solving multi-objective optimization problems (MOPs), because it can obtain a set of well-convergent and well-distributed solutions after one run. In addition, MOEAs can easily solve complex optimization problems which are difficult to be solved by traditional optimization methods.

However, as the number of objectives grows, the proportion of non-dominated individuals in the population of MaOPs increases sharply, resulting in a reduction of convergence pressure of multi-objective optimization evolutionary algorithms (MOEAs). So all the superior solutions in the population can not be distinguished by the dominance-based selection criterion. Over the past few years, quite a bit of many-objective evolutionary algorithms (MaOEAs) have been proposed to solve MaOPs, which can be roughly divided into the following methods.

The first method is to generate a series of reference points or vectors before evolution. The aim is to promote the convergence and diversity of MaOEAs. In [7], the algorithm using reference-point based non-dominated sorting approach (NSGA-III) emphasizes population members which are non-dominated yet close to the supplied reference points. With the increase of the number of objectives, the convergence pressure will decrease when there are more dominant solutions. In [8], the algorithm generates a set of reference vectors to decompose the original MOPs into single-objective sub-problems and to elucidate user preferences to target a

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preferred subset of the whole PF(pareto fronts). In [9], the algorithm uses the reference vectors provided by the process based on incremental learning to select the potential solutions. One of the limitations is how to generate a set of uniformly distributed reference points or vectors.

The second method is based on decomposition. For instance, MOEA/D [10] decomposes m objectives into a set of single-objective optimization problems and each subproblem is constrained by corresponding reference vectors in its neighborhood. MOEA/D is heavily dependent on whether the reference vectors match the shape of the PF. In [11], MOEA/D-PaS proposes a simple method called Pareto adaptive scalarizing (PaS) approximation to improve the performance. However, it is difficult to express the Pareto front with the number of objectives increasing. In [12], MOEA/D-AED uses adaptive Epsilon dominance (AED) for controlling the diversity of EP in MOEA/D. The results by MOEA/D-AED are not satisfactory in dealing with Pareto fronts with extreme convexity. The limitation of these algorithm is the large computational complexity coming from the decomposition technique.

The third method is the indicator based MaOEAs. An indicator is a parameter to measure the performance of MaOEAs. Indicator-based algorithms [13]–[18] select individuals by indicators. In [17], an IGD indicator based many-objective algorithm is proposed to solve many-objective optimization problems. In [19], an algorithm for fast hypervolume based MaOEA is proposed to handle many-objective optimization problems. In [20], a GD indicator based algorithm is proposed to solve MaOPs. In [16], an improved metaheuristic based on the R2 indicator MaOEA is proposed to solve many-objective optimization problems. In the real-world problem, the optimal solutions may be located in the special regions, such as many discrete regions and the regions with very few feasible solutions. The limitation is the existing non-linear expanded evolutionary algorithm can not find the true Pareto fronts in this case.

The fourth method is based on designing a new sorting method for the solutions in the population to improve the selection pressure. In [22], a vector angle-based many-objective evolutionary algorithm (VaEA) is proposed. VaEA uses maximum-vector-angle-first principle and worse-elimination principle to guarantee the uniformity of solution set. In [23], a Pareto-based many-objective evolutionary algorithm using space partitioning selection and angle-based truncation (SPSAT) to enhance convergence and diversity. The volume dominance strategy is to differentiate the advantages and disadvantages by comparing the volume size of the individual and the reference point in [24]. In this paper, the method of controlling the dominating region of the solution is extended or contracted by modifying the value of the objective function through the setting of formula. The setting of parameter S determines whether to expand or contract the dominant region of the solution.

Recently, a promising region based EMO algorithm (PREA) [25] is proposed to evaluate the fitness value of each individual in the current population by using the ratio based indicator with infinite norm. The proposed algorithm selects the solutions with the best fitness values to define a promising region in the objective space. The ratio based indicator with infinite norm is good at eliminating outliers in the population and protecting the boundary points of the PF.

However, the main limitations of the above algorithms are: (1) the existing non-linear expanded evolutionary algorithm can not find the true Pareto fronts in many discrete regions or other special regions; (2) many existing algorithms have large computational complexity; (3) too many parameters in new sorting methods.

A. GOALS

To address the above issues, the goal of the paper is to develop a novel nonlinear expanded dominance relation based many-objective evolutionary algorithm for MOPs. We expect the proposed algorithm to expand the area of domination and slow down the rising speed of the proportion of non dominated solutions in the population. To achieve the goal, we propose two new techniques. Firstly, a novel nonlinear extended dominating (NED) relation is used to expand the dominating regions, and to select some points in the special regions, such as these solutions located in the discrete regions, sparse regions, small feasible regions and so on. Secondly, a novel nonlinear extended dominating (NED) relation still needs a mechanism to ensure its diversity while promoting convergence pressure. The niching method is used to enhance the diversity of Pareto optimal solutions.

B. ORGANIZATION

The reminder of the paper is outlined as follows. Section II introduce related work. Section III describes the proposed algorithm. Section IV shows the experiment results and analysis. Section V presents the conclusions and future work directions.

II. RELATED WORK

A. MANY-OBJECTIVE OPTIMIZATION CONCEPTS AND DEFINITIONS

The mathematical definition of a many-objective optimization problem (MaOP) can be defined as follows:

$$\begin{cases} \min \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x})) & (m \geq 3) \\ s.t. & \boldsymbol{x} \in \boldsymbol{X} \end{cases} \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2, \cdots, x_n) \in \boldsymbol{X} \subset \boldsymbol{R}^n$ is a feasible solution in the decision space, f_i $(i = 1, 2, \cdots, m)$ is the i-th minimized objective function. There are four significant definitions of MaOPs as follows:

Pareto Dominance: A solution $\boldsymbol{x}$ dominates $\boldsymbol{y}$ (denoted by $\boldsymbol{x} \prec \boldsymbol{y}$) if

$$\begin{aligned} &\forall i \in 1, 2, \cdots, m : f_i(\boldsymbol{x}) \leq f_i(\boldsymbol{y}) \text{ and} \\ &\exists i \in 1, 2, \cdots, m : f_i(\boldsymbol{x}) < f_i(\boldsymbol{y}) \end{aligned} \tag{2}$$

Pareto Optimal: A solution $\boldsymbol{x}$ is said to be Pareto optimal if it is not dominated by any other solutions.

Pareto Optimal Solutions (PS): PS is a set of all Pareto optimal solutions.

$$\text{PS} = \{\boldsymbol{x} \in \boldsymbol{X} \mid \boldsymbol{x} \text{ is Pareto optimal}\}$$

Pareto Front (PF): PF is the surface of all Pareto optimal solutions corresponding to objective function vectors.

$$\text{PF} = \{\boldsymbol{f}(\boldsymbol{x}) \mid \boldsymbol{x} \text{ is Pareto optimal}\}$$

B. NSGA-III [7]

NSGA-III is proposed by Deb *et al.* This algorithm is a high-dimensional multi-objective NSGA-II based on the reference point. It provides a new idea for solving high-order multi-objective optimization algorithm. Compared with NSGA II algorithm, NSGA-III and NSGA-II have similar framework. The difference between them is mainly due to the change of selection mechanism. NSGA-II mainly relies on crowding degree for sorting, which obviously has no obvious effect in high-dimensional target space. NSGA-III adjusts the crowding degree ranking greatly and maintains the diversity of population by introducing widely distributed reference points.

In [7], NSGA-III uses the method of constructing weights by boundary crossing proposed by Das and Dennis, and put the reference point on a standardized hyperplane. However, this method has a serious drawback, that is, the size of the partition number P will affect the generation of intermediate points. If you want to generate intermediate points, the number of reference points will increase sharply when the target dimension is large. Therefore, in order to avoid this situation, NSGA-III is proposed with a two-level reference point generation method. By adding an inner layer, it can ensure the generation of intermediate points. Moreover, it does not cause too many reference points.

The main limitation is how to define a set of reference points. If the reference points are not defined accurately,the Pareto fronts should be totally wrong.

C. ANALYSIS OF EXISTING EXPANDING DOMINATING AREA BASED MaOEAs

One of the famous expending algorithm called $\alpha-$ domination [29], which permits a solution $\boldsymbol{x}$ dominates $\boldsymbol{y}$ if:

$$\forall i \in 1, 2, \cdots, m : g_i(\boldsymbol{x}, \boldsymbol{y}) \leq 0 \wedge \exists i \in 1, 2, \cdots, m : g_i(\boldsymbol{x}, \boldsymbol{y}) < 0 \quad (3)$$

$$g_i(\boldsymbol{x}, \boldsymbol{y}) = f_i(\boldsymbol{x}) - f_i(\boldsymbol{y}) + \sum_{j=1, j\neq i}^{m} \alpha_{ij} \left(f_j(\boldsymbol{x}) - f_j(\boldsymbol{y})\right) \quad (4)$$

where α is set to $1/3$.

The other expending algorithm is called the controlling dominance area of solutions (CDAS) method [30], which expands the dominating area of a solution $\boldsymbol{x}$ by modifying the objective values:

$$f_i'(\boldsymbol{x}) = \frac{\|\boldsymbol{f}(\boldsymbol{x})\| \sin(\omega_i + S\pi)}{\sin(S\pi)}, \quad i = 1, 2, \cdots, m \quad (5)$$

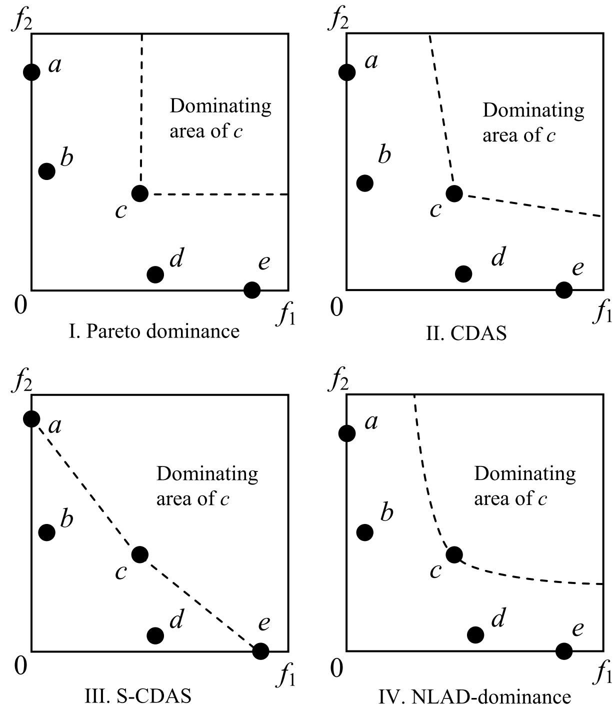

FIGURE 1. The dominating area of the expanding dominance algorithms.

where $\|\cdot\|$ denotes the L_2 norm, ω_i denotes the angle between $f_i(\boldsymbol{x})$ and $\boldsymbol{f}(\mathbf{x})$, and $S \in [0.25, 0.5]$ is a parameter.

In order to eliminate the parameter S, a modification of CDAS called self-controlling dominance area of solutions (S-CDAS) method [31] is proposed, which adaptively determines the expanding degree of a solution $\boldsymbol{x}$ according to the extreme solutions:

$$f_i'(\boldsymbol{x}) = \frac{\|\boldsymbol{f}(\boldsymbol{x})\| \sin(\omega_i + \varphi_i)}{\sin(\varphi_i)}, \quad i = 1, 2, \cdots, m \quad (6)$$

$$\varphi_i = \arcsin \frac{\|\boldsymbol{f}(\boldsymbol{x})\| \sin(\omega_i)}{\|\boldsymbol{f}(\boldsymbol{x}) - p_i\|} \quad (7)$$

where φ_i and p_i is the extreme solution with respect to the i-th axis in the population.

In [32], in order to further increase the pressure of the environmental selection, the strategy called NLAD-dominated is proposed to solve the problem, which modifies the objective values:

$$f_i'(\boldsymbol{x}) = \alpha_i \cdot f_i^3(x) + \sum_{j=1, j\neq i}^{m} f_j(x), \quad i = 1, 2, \cdots, m \quad (8)$$

where α is set to $1/3$.

Fig. 1 takes a bi-objective minimization problem to illustrate the dominating area of the expanding dominance algorithms.

As is shown in Fig. 1, a, b, c, d, and e are five non-dominance solutions. CDAS extends the dominating area of the solutions outward at the same angle. The dominance relation can promote the convergence, but individual b will dominate a and c after expansion (individual d dominates c and e

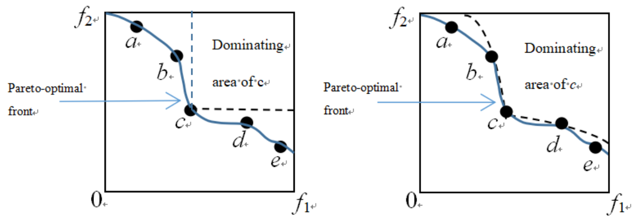

FIGURE 2. The emergence of new domination.

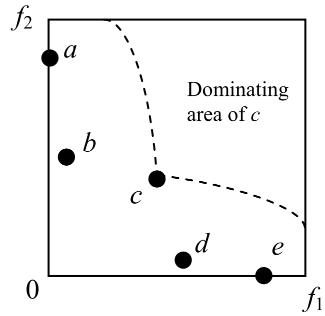

FIGURE 3. The dominating area of the solution expanded by NED.

similarly). This may cause the optimal front of the population to shrink in the evolution process, thus losing the breadth and diversity. As is shown in Fig. 1(III), the dominating area of the solution in S-CDAS always intersects with the extreme points in the population, which guarantees the wideness of the Pareto front. However, through this dominance relationship, individual c will be dominated by b and d, which could not guarantee diversity. NLAD-dominance relation cannot solve the above problems.

III. PROPOSED ALGORITHM

For a feasible solution, it can dominate those solutions that are slightly better in a few goals but significantly worse in most other goals, and the growth rate of the expanded area should increase gradually. Meanwhile, NSGA-III only uses a set of reference points without considering some potential points in engineering problems. As is shown in Fig. 2, in some engineering problems, points a-e are not dominated with each other according to Pareto dominance in NSGA-III, but the number of non-dominated solutions will increase greatly with the increase of the number of objectives. In some cases, it is possible that one of the points is the optimal solution in the evolutionary process and is not found in a special region. At this time, we hope to solve it through a new extended dominance relationship. In order to enhance the performance of NSGA-III in these respects, a new nonlinear extended dominating relationship is proposed.

A. A NOVEL NONLINEAR EXPANDED DOMINANCE RELATION

Let $\boldsymbol{F} = \{\boldsymbol{F}_1, \boldsymbol{F}_2, \cdots, \boldsymbol{F}_n\}$ be the function vectors of the n individuals in the population. First, in order to unify the order of magnitude, the objective values of individuals are normalized:

$$f_i'(\boldsymbol{x}) = \frac{f_i(\boldsymbol{x}) - f_i^{\min}}{f_i^{\max} - f_i^{\min}}, \quad i = 1, 2, \cdots, m \tag{9}$$

where $f_i^{\max}$ is the maximum value of the i-th objective function in $\boldsymbol{F}_i$ and $f_i^{\min}$ is the minimum value.

Then, the novel nonlinear expanded domination (NED) relation extends the domination area of the solutions by the following formula:

$$f_i''(\boldsymbol{x}) = f_i'(\boldsymbol{x}) - \left(1 - \|\boldsymbol{f}'(\boldsymbol{x})\| \sin\omega_i\right)^H, \quad i = 1, 2, \cdots, m \tag{10}$$

where $\boldsymbol{f}'(\boldsymbol{x})$ is the normalized function vector of $\boldsymbol{f}(\boldsymbol{x})$, $\|\boldsymbol{f}'(\boldsymbol{x})\|$ denotes the L_2 norm and ω_i denotes the angle between $f_i'(\boldsymbol{x})$ and $\boldsymbol{f}'(\boldsymbol{x})$. H is a parameter whose value is greater than 0. The dominating area of the solution expanded by NED is shown in Fig. 3.

As is shown in Fig. 3, the growth rate of the area dominated by individual c increases gradually from f_2^c (the value of the 2-th objective function of point c)to $f_2^{\max}$, as well as from f_1^c (the value of the 2-th objective function of point c) to $f_1^{\max}$. This is consistent with our motivation for the proposed algorithm.

However, while NED promotes the convergence pressure, it still needs a mechanism to ensure diversity. Fortunately, the niching method is an effective method to improve diversity, and many niching methods based MaOEAs [26]–[28] have been proposed to verify their effectiveness. In [26], the niche size is set by calculating the angle between solutions. The minimum angle between an individual and other individuals is calculated by the following formula:

$$\theta_i = \min_{j\neq i}\left\{\arccos\frac{\boldsymbol{F}_i \cdot \boldsymbol{F}_j}{\|\boldsymbol{F}_i\|\,\|\boldsymbol{F}_j\|}\right\}, \quad i = 1, 2, \cdots, 2n \tag{11}$$

After crossover and mutation, the selection operation is to select n individuals out of $2n$ individuals to enter the next generation. The author sets the niche size as the n-th minimum element in $\{\theta_1, \theta_2, \cdots, \theta_{2n}\}$.

In this paper, we use the angle between individuals to set the niche size to divide individuals. But in order to have at least two solutions in the same niche, we set the size of the niche as $\tilde{\theta} = \max\{\theta_1, \theta_2, \cdots, \theta_{2n}\}$.

B. OVERALL ALGORITHM

According to the novel nonlinear expanded dominance relation mentioned above, a solution $\boldsymbol{x}$ dominates $\boldsymbol{y}$ if:

$$\begin{aligned} &\forall i \in 1, 2, \cdots, m : f''(\boldsymbol{x})_i \le f_i''(\boldsymbol{y}), \\ &\exists i \in 1, 2, \cdots, m : f''(\boldsymbol{x})_i < f_i''(\boldsymbol{y}), \text{ and} \\ &\theta(\boldsymbol{x}, \boldsymbol{y}) \le \tilde{\theta} \end{aligned} \tag{12}$$

We apply the new dominance relationship to NSGA-III [7] (we call the novel algorithm NSGAIII-NED) to enhance the non-dominance sorting. The overall algorithm is outlined in Algorithm 1. The simulated binary crossover [46] and polynomial mutation are employed to generate offspring.

Z is a set of uniformly generate reference points in NSGA-III for environmental selection, which is obtained by the method proposed in [33]. The offspring is generated by crossover process and mutation process. In the next section, we compare the proposed algorithm with the state of the art algorithms to evaluate the performance.

Algorithm 1 A Nonlinear Expanded Dominance Relation

$P_0 \leftarrow$ Randomly initialize the population
$Z \leftarrow$ Uniformly generated reference points
$Z^{\min} = \left(Z_1^{\min}, Z_2^{\min}, \cdots, Z_m^{\min}\right)$
$t \leftarrow 0$
while the termination condition is not satisfied do
  Offspring $_t \leftarrow$ Generate offspring from P_t
  Normalize the function vectors of $P_0 \cup \text{Offspring}_t$ using (formula 9)
  Modify the objective values by NED using (formula 10)
  Calculate the niche size $\tilde{\theta}$ using (formula 11)
  Update $Z_{\min}$: $Z_j^{\min} = \min\left\{Z_j^{\min}, Offspring_{tj}^{\min}\right\}$
  Non-dominance sorting the individuals using (formula 12)
  $P_{t+1} \leftarrow$ Environmental Selection of NSGA-III
  $t \leftarrow t+1$
end

The environment selection strategy used in this paper is consistent with that in NSGA-III. According to Formula 12, different non-domination levels (L1, L2 and so on) can be obtained. Then,starting from L1,each non-domination level is selected one at a time to construct a new population St until the size of St is equal to N or exceeds N for the first time. In [7], the algorithm is implemented through five main steps: determining the reference point, standardizing the target space, association operation and environment selection operation.

For environment selection operation, this is the operation of selecting k individuals from the critical level L_l (the lth level) to join the next parent Pt+1. After the association operation, the following situations may occur. Firstly, a reference point is associated with one or more individuals. Secondly, no individual is associated with it. In [7], the number of population members is counted from Pt + 1 associated with each reference point. This unique count is denoted as θ_j for the jth reference point. The niche-preserving operation proposed in NSGA-III is devised as follows. First, the reference point set $J_{\min}$ having minimum θ_j is identified. In the case of multiple such reference points, a reference point belonging to Jmin is randomly selected. If $\theta_j = 0$, there is no point associated with it in the set Pt+1. Then there are two situations in L_l. If there are one or more individuals associated with L_l, the nearest individual is associated with FL and the individual is added to Pt + 1. If there is no individual associated with it in L_l, the reference point will not be considered in the remaining operations. If $\theta_j \geq 1$, there is an individual associated with the reference point in Pt + 1. If there is an individual associated with it in L_l, randomly select one and add the individual to Pt + 1. Repeat these procedure until the size of Pt + 1 is equal to N.

IV. EXPERIMENTAL RESULTS AND ANALYSIS

To evaluate the performance of the proposed algorithm, we compare NSGAIII-NED with the state of the art algorithms. We use the platform PLATEMO [34] to conduct our experiments. All algorithms are implemented on Intel (R) core (TM) i5 CPU (1.60ghz, 1.80GHz) and 16GB memory configuration PC. The remained experimental settings are as follows.

A. BENCHMARKS AND PERFORMANCE METRICS

All algorithms are tested on the widely used DTLZ test suite [35], WFG toolkit [36] and MaF test problems [37]. The properties of all the test problems are summarized in Table 1.

TABLE 1. The properties of correlation test function in experiment.

Problem	Properties
DTLZ1	Linear, multimodal
DTLZ2	Concave, unimodal
DTLZ3	Concave, multimodal
DTLZ4	Concave, biased
DTLZ5	Concave
DTLZ6	Concave, biased
WFG1	Sharp tails
WFG2	Disconnected
WFG3	Degenerate
WFG4	Concave
MaF1	Inverted
MaF3	Convex
MaF6	Degenerate

DTLZ was first proposed by Deb et al in 2002. According to the different difficulty settings, two test functions were added to the original seven functions in 2005 to form DTLZ test suite. DTLZ is a continuous problem suite, which can be extended to any number of targets (m >= 2) and can have any number of variables (n >= m). Because the number of variables and the number of objectives are easy to control, DTLZ is widely used in multi-objective optimization problems as a standard test function. An m-objective test problem in DTLZ has $k + m - 1$ decision variables. k is set to 5 for DTLZ1, and 10 forDTLZ2-DTLZ6.

Another popular continuous test suite WFG is scalable to any number of objectives and decision variables. Compared with DTLZ, the variables of DTLZ are separable, so the complexity is not high, while WFG is more complex and more challenging. The attributes of WFG include separability or indivisibility, unimodality or multimodality, convex or non-convex PF shape, unbiased parameter or biased parameter. In order to compare as many as possible, this paper considers WFG1-4. One test function is a problem with regular PF, the other is a problem with irregular PF. According to the suggestion by Huband *et al.* [36], the parameters k and l are set to $2 \times (m - 1)$ and 20, respectively.

MaF is a benchmark function suite proposed in 2017 to promote research on evolutionary many-objective optimization. Fifteen benchmark functions are designed in MaF with diverse properties that provide a good representation of various real-world scenarios. Due to space limitations, three representative problems with different characteristics are selected to evaluate the algorithms.

Three performance metrics, IGD [38], GD [43] and PD [39], are used to evaluate the performance of the compared algorithms. IGD metric is an indicator to measure the convergence and diversity of an solution set. The smaller the IGD value is, the better the performance is. PD metric is an indicator to measure the diversity of an solution set. The larger the PD value, the better the diversity. The generation distance index GD is designed to measure the convergence of the solution set. It uses the proximity distance to obtain the GD value. For individuals in the population, if individuals are selected according to the neighborhood distance from small to large, the value of GD is the minimum. The smaller the GD value, the better the convergence of the solution set.

B. GENETIC OPERATORS

The simulated binary crossover and polynomial mutation are employed to generate offspring. Specifically, the probability of crossover is set to 1 and the probability of mutation is set to 1/n. The distribution indexes of crossover and mutation are set to 20.

C. PARAMETER SETTINGS

In [16], for MOMBI-II the author recommends that parameter α is set to 0.5 and Parameter ε is set to 0.001. All the test problems with 5, 10, 15 objectives are considered. The population size is set to 100 for all problems. Each algorithm runs 30 times independently for each problem. The maximal number of function evaluations is set to 50, 000. Besides, the Mann-Whitney-Wilcoxon Rank-Sum test [40] with a 5% significance level is used as the statistical approach to compare the mean IGD with other MaOEAs. "+", "−" and "=" indicate that the result is significantly better, significantly worse and statistically similar to that obtained by the proposed algorithm, respectively.

D. STATE OF THE ART ALGORITHMS

In order to verify the performance of NSGAIII-NED, the state of the art algorithms NSGA-III [7], MOEAD/D-PaS [11], MOMBI-II [16] and S-CDAS [16] are selected to compare with the proposed algorithm.

(1) NSGA-III [7]: NSGA-III uses a set of reference points to generate reference vectors. In order to maintain diversity, those candidate solutions close to the reference vectors are selected for the next generation.

(2) MOEAD/D-Pas [11]: MOEA/D-Pas analyzes a family of frequently used scalarizing methods, the L_p methods, and shows that the p value is crucial to balance the selective pressure toward the Pareto optimal. Then, a simple yet effective method called Pareto adaptive scalarizing (PaS) approximation is proposed to approximate the optimal p value.

(3) MOMBI-II [16]: MOMBI-II is an indicator based algorithm. MOMBI-II presents an improved version of an MOEA based on the R2 indicator, which uses the scalar function and the statistical information of the population close to the real Pareto optimal frontier to optimize the algorithm.

(4) S-CDAS [16]: S-CDAS is a modification of CDAS. In S-CDAS, the algorithm self-controls dominance area for each solution without the need of an external parameter. S-CDAS considers convergence and diversity and realizes a fine grained ranking that is different from conventional CDAS.

(5) PREA [25]: PREA is a promising-region based evolutionary many-objective algorithm with the ratio based indicator. In PREA, a promising region is identified in the objective space using the ratio based indicator with infinite norm. To ensure the diversity of population, a strategy based on the parallel distance is introduced to select individuals in the promising region.

(6) AR-MOEA [45]: ARMOEA is an adaptive evolutionary algorithm based on reference point.The algorithm framework based on the enhanced IGD index is adopted, and a set of points uniformly generated on the simplex is used as the reference point set for calculating the enhanced IGD index.

(7) VaEA [22]: In VaEA, the convergence and diversity are considered, and the maximum vector angle first principle is used to ensure the uniformity of the solution set. By using the worse elimination principle, the solution set with poor convergence is replaced by other solutions.

E. EXPERIMENT RESULTS

The mean values and the standard deviations of IGD results for six test functions (DTLZ1−DTLZ6) are listed in Table 2. For each test problem, the cells with the best and the second-best IGD values are labeled with a dark grey colored and a light grey colored background, respectively.

As can be seen from the Table2, our algorithm obtains the best IGD value in one of the eighteen problems and the second-best value in six problems. As shown in Table3, the proposed NSGAIII-NED (ours) only gets the best GD value in one problem. NSGAIII-NED achieves the best PD value in four problems and the second-best PD value in two problems in Table4.

NSGAIII-NED achieves the best PD value on all problems of DTLZ1. On DTLZ1 with 10 objectives the proposed algorithm achieve the best IGD value. When solving the linear and multimodal problems like DTLZ1, the proposed niching method can retain the better individuals in different regions and enrich the population diversity.

The Pareto front of DTLZ2 problem is concave and unimodal. The ideal PF generated by the proposed algorithm is a set of points uniformly distributed on the hyperplane. When calculating the adjacent distance of individuals, the error is large, which may lead to the result of the proposed

TABLE 2. Statistical results of the mean IGD values obtained by the state of the art algorithms for DTLZ.

Problem	m	D	NSGAIII	MOEAD-PaS	MOMBI-II	PREA	SCDAS	AR-MOEA	VaEA	NSGAIII-NED (Ours)
DTLZ1	5	9	6.9246e-2 (1.54e-3) +	4.3890e+0 (8.16e+0) -	9.1335e-2 (2.46e-2) +	7.3754e-2 (2.58e-3) +	1.2823e-1 (7.03e-2) =	7.9519e-2 (4.60e-2) +	1.4356e-1 (6.15e-2) =	1.3816e-1 (7.03e-2)
	10	14	2.8137e-1 (1.45e-1) -	3.1663e+1 (1.03e+1) -	2.8790e-1 (2.89e-2) -	5.3124e-1 (3.05e-1) -	1.3327e-1 (3.56e-3) +	1.9435e-1 (1.05e-1) =	4.9956e-1 (2.65e-1) -	1.7036e-1 (1.06e-2)
	15	19	3.9500e-1 (1.66e-1) =	4.0988e+1 (1.04e+1) -	3.9889e-1 (1.26e-1) =	2.6845e+0 (1.65e+0) -	1.4736e-1 (2.30e-3) +	2.3398e-1 (1.28e-1) +	6.3928e-1 (4.74e-1) =	6.7362e-1 (7.76e-1)
DTLZ2	5	14	2.1228e-1 (5.89e-5) +	3.0536e-1 (3.62e-2) -	2.1781e-1 (1.45e-3) +	2.1621e-1 (1.39e-3) +	7.0468e-1 (2.29e-1) -	2.1247e-1 (1.53e-4) +	2.0697e-1 (1.48e-3) +	2.3152e-1 (4.98e-3)
	10	19	5.5544e-1 (5.97e-2) +	1.2473e+0 (2.52e-2) -	7.1652e-1 (1.22e-1) -	4.8913e-1 (3.42e-3) +	1.0104e+0 (3.28e-1) -	5.0648e-1 (4.92e-3) +	4.9191e-1 (2.30e-3) +	6.1235e-1 (2.17e-2)
	15	24	7.5331e-1 (1.74e-2) +	1.5496e+0 (3.22e-1) -	1.0990e+0 (4.61e-2) -	6.1279e-1 (4.50e-3) +	1.1010e+0 (3.02e-1) -	6.7242e-1 (2.36e-2) +	6.2672e-1 (1.23e-2) +	7.6515e-1 (1.65e-2)
DTLZ3	5	14	1.2469e+0 (1.30e+0) -	3.1791e+1 (2.58e+1) -	2.5691e-1 (4.41e-2) +	4.1239e-1 (3.01e-1) =	6.3871e-1 (4.48e-1) -	7.0695e-1 (8.26e-1) -	2.6008e+0 (1.62e+0) -	3.2243e-1 (4.49e-2)
	10	19	7.4951e+0 (6.06e+0) -	1.5120e+2 (7.34e+1) -	1.1440e+0 (2.32e-1) -	8.4011e+0 (3.74e+0) -	4.0587e+1 (2.55e+1) -	2.2129e+0 (1.66e+0) -	2.2816e+1 (8.83e+0) -	8.2354e-1 (2.85e-1)
	15	24	4.8492e+0 (2.77e+0) +	2.2293e+2 (1.63e+1) -	1.2691e+0 (2.33e-1) +	3.7599e+1 (1.78e+1) =	3.4350e+1 (2.66e+1) =	1.5480e+0 (9.47e-1) +	2.1572e+1 (9.32e+0) +	3.4946e+1 (1.40e+1)
DTLZ4	5	14	2.9821e-1 (1.21e-1) =	3.5706e-1 (7.36e-2) -	2.9482e-1 (1.01e-1) =	4.6703e-1 (2.09e-1) -	6.5138e-1 (1.61e-1) -	3.0667e-1 (1.26e-1) =	2.0981e-1 (1.78e-3) +	2.5239e-1 (6.26e-2)
	10	19	5.9495e-1 (5.75e-2) =	7.4173e-1 (1.01e-1) -	6.6527e-1 (7.00e-2) -	5.5119e-1 (5.40e-2) +	5.8609e-1 (1.33e-2) =	5.1024e-1 (4.05e-3) +	4.9628e-1 (5.05e-3) +	5.9316e-1 (3.49e-2)
	15	24	7.7041e-1 (1.86e-2) +	1.3236e+0 (8.10e-2) -	1.0020e+0 (8.31e-2) -	6.3681e-1 (1.22e-2) +	7.1349e-1 (1.34e-2) +	7.2534e-1 (1.59e-2) +	6.2566e-1 (5.13e-3) +	7.9004e-1 (1.66e-2)
DTLZ5	5	14	1.1027e-1 (3.64e-2) =	1.7618e-1 (1.49e-2) -	1.2960e-1 (8.20e-3) -	1.1565e-1 (2.65e-2) =	7.1001e-1 (4.69e-2) -	7.9982e-2 (1.38e-2) +	1.3486e-1 (3.52e-2) -	1.0653e-1 (2.66e-2)
	10	19	2.4206e-1 (7.98e-2) =	2.8122e+0 (2.20e-1) -	6.9069e-1 (3.94e-2) -	3.5579e-1 (1.03e-1) -	7.1072e-1 (3.76e-2) -	1.1511e-1 (2.35e-2) +	5.0899e-1 (1.65e-1) -	2.0335e-1 (4.85e-2)
	15	24	2.9562e-1 (5.99e-2) =	2.9560e+0 (3.43e-6) -	7.0638e-1 (2.32e-2) -	3.7958e-1 (1.04e-1) -	7.3401e-1 (2.73e-2) -	1.2440e-1 (4.62e-2) +	6.0006e-1 (1.26e-1) -	3.1093e-1 (1.97e-2)
DTLZ6	5	14	2.6288e-1 (6.58e-2) -	2.3552e-1 (2.48e-2) -	2.0539e-1 (5.95e-4) -	2.6226e-1 (4.92e-2) -	7.4216e-1 (2.11e-1) -	9.4289e-2 (3.17e-2) +	3.5457e-1 (1.46e-1) -	1.4261e-1 (4.02e-2)
	10	19	1.4246e+0 (8.14e-1) -	9.8774e+0 (1.11e+0) -	6.7263e-1 (9.05e-2) =	1.4153e+0 (4.60e-1) -	1.8444e+0 (8.04e-1) -	1.2234e-1 (2.36e-2) +	2.4141e+0 (6.44e-1) -	5.0282e-1 (3.79e-1)
	15	24	1.2972e+0 (5.97e-1) -	1.0400e+1 (2.51e-6) -	7.2423e-1 (4.22e-2) =	2.5123e+0 (9.27e-1) -	2.3567e+0 (1.15e+0) -	1.4419e-1 (4.37e-2) +	1.5388e+0 (4.81e-1) -	8.9708e-1 (5.34e-1)
+/-/=			6/6/6	0/18/0	4/10/4	6/9/3	3/12/3	14/2/2	7/9/2	

"+", "–", "=" indicate that the result is significantly better, significantly worse and statistically similar to that obtained by NSGAIII-NED, respectively. The standard deviation is in bracket.

algorithm on DTLZ2 problem not quite satisfactory. On DTLZ2 with 10 and 15 objectives, PREA both gets the best IGD value. NSGAIII-NED performs worse than PREA. The reason is that a strategy based on the parallel distance is introduced to select individuals in the promising region in PREA. This confers major advantage on PREA in maintaining the diversity of final results.

Additionally, NSGAIII-NED performs the best on DTLZ3 with 10 objectives. Except DTLZ2, the proposed algorithm gets at least one best or second-best IGD value on all concave test instances. For example, the DTLZ4 problems with 5 objectives and 14 decision variables, NSGA-III's result is 0.2982, MOEAD-PaS's result is 0.3571, MOMBI-II's result is 0.2948, PREA's result is 0.4670, S-CDAS's result is 0.6514, AR-MOEA's result is 0.3067, VaEA's result is 0.2098, NSGAIII-NED's result (ours) is 0.2524 which gets the second-best value. On DTLZ5 with 5 or 10 objectives, our algorithm has the second-best IGD values. Not only that, our algorithm gets the same number of the second-best IGD values on DTLZ6.

As shown in Table3, we can find that AR-MOEA and MOMBI-II get better GD values than other algorithms on most problems. The advantage of AR-MOEA in GD value has been analyzed. MOMBI-II obtains better GD value because of scalar function and statistical information of population approaching true PFs. VaEA performs better than other algorithms on most problems on GD values. The reason is that the effect of VaEA's principle is better in environmental selection.

As seen in Table4, our algorithms gets at least one best or second-best PD value on DTLZ2-4. Nevertheless, the true PFs of DTLZ2-4 are concave. The true PFs of DTLZ2-4 is a little more complicated than that of DTLZ1, which has a terrible impact on the diversity of niche technologies. On DTLZ5 and DTLZ6, the effects are even worse.

TABLE 3. Statistical results of the mean GD values obtained by the state of the art algorithms for DTLZ.

Problem	m	D	NSGAIII	MOEAD-PaS	MOMBI-II	PREA	SCDAS	AR-MOEA	VaEA	NSGAIII-NED (Ours)
DTLZ1	5	9	2.1246e-3 (1.56e-3) =	4.7624e+0 (5.32e+0) -	1.6834e-3 (7.66e-5) +	1.4479e-3 (2.04e-4) =	1.8125e-3 (4.15e-5) +	3.1192e-3 (7.05e-3) =	9.3186e-1 (8.62e-1) -	5.2070e-2 (1.64e-1)
	10	14	7.5004e-1 (1.21e+0) =	2.1845e+1 (7.73e+0) -	6.5256e-3 (1.64e-3) +	1.1952e-1 (9.62e-2) +	4.9301e-3 (1.56e-4) +	1.5954e-2 (2.39e-2) +	2.3992e+0 (9.46e-1) -	4.5013e-1 (1.07e+0)
	15	19	1.5354e+0 (1.85e+0) +	2.8956e+1 (8.12e+0) =	1.1020e-2 (1.87e-2) +	6.8460e+0 (2.83e+0) +	6.1669e-3 (1.19e-4) +	2.5277e-2 (4.01e-2) +	3.7860e+0 (9.67e-1) +	2.4231e+1 (9.37e+0)
DTLZ2	5	14	5.4376e-3 (2.27e-5) +	3.7691e-3 (1.81e-4) +	5.0188e-3 (1.44e-4) +	4.7702e-3 (1.65e-4) +	5.1245e-3 (2.69e-3) =	4.6571e-3 (6.63e-5) +	5.2656e-3 (1.79e-4) +	5.8532e-3 (3.13e-4)
	10	19	8.0664e-3 (4.20e-3) +	5.1587e-3 (2.26e-2) +	1.0707e-2 (2.59e-3) +	1.5436e-2 (6.38e-4) -	1.2374e-1 (7.85e-2) -	5.8100e-3 (8.74e-4) +	1.7877e-2 (6.86e-4) -	1.3987e-2 (2.91e-3)
	15	24	2.8525e-2 (4.70e-3) =	1.1759e-1 (1.27e-1) =	1.9452e-2 (7.86e-3) +	2.5733e-2 (2.01e-3) +	1.5842e-1 (8.05e-2) -	1.0250e-2 (1.50e-3) +	2.8201e-2 (1.85e-3) +	3.0176e-2 (3.59e-3)
DTLZ3	5	14	6.1003e-1 (1.01e+0) -	1.2047e+1 (4.49e+0) -	4.9249e-3 (5.57e-4) =	1.2676e-1 (4.30e-1) +	1.5030e+0 (1.19e+0) -	6.6970e-2 (9.01e-2) +	2.6893e+0 (1.63e+0) -	1.9545e-1 (5.93e-1)
	10	19	7.9412e+0 (4.96e+0) -	4.6820e+1 (4.44e+1) -	2.8824e-2 (4.53e-2) +	1.3448e+0 (6.53e-1) +	1.0874e+2 (5.55e+1) -	5.1106e-1 (6.04e-1) =	1.9607e+1 (4.73e+0) -	2.2600e+0 (7.07e+0)
	15	24	1.9618e+1 (6.30e+0) +	9.3490e+1 (5.34e+1) +	2.9345e-2 (5.76e-2) +	3.0892e+1 (1.17e+1) +	1.1839e+2 (5.02e+1) +	2.0748e-1 (2.49e-1) +	5.8825e+1 (8.08e+0) +	3.5741e+2 (1.19e+2)
DTLZ4	5	14	5.0766e-3 (4.80e-4) +	3.3882e-3 (1.64e-4) +	4.7837e-3 (2.68e-4) +	4.0156e-3 (1.04e-3) +	5.8308e-3 (4.20e-3) =	4.4789e-3 (2.74e-4) +	5.3195e-3 (2.48e-4) +	5.5796e-3 (2.50e-4)
	10	19	9.6072e-3 (3.16e-3) +	4.0619e-3 (2.22e-3) +	9.1489e-3 (1.34e-3) +	1.4242e-2 (6.74e-4) +	1.2347e-1 (5.32e-3) -	6.9890e-3 (5.43e-4) +	1.7917e-2 (1.15e-3) =	1.7570e-2 (2.53e-3)
	15	24	2.6862e-2 (5.10e-3) =	1.2658e-2 (2.75e-2) +	2.3553e-2 (7.02e-3) =	1.5091e-2 (1.76e-3) +	1.3985e-1 (4.79e-3) -	1.1596e-2 (1.98e-3) +	2.7906e-2 (1.26e-3) =	2.6631e-2 (4.05e-3)
DTLZ5	5	14	1.0041e-1 (7.59e-3) +	2.2955e-1 (3.73e-2) -	1.9234e-1 (1.54e-2) -	1.3727e-1 (5.60e-3) -	1.4778e-1 (4.08e-3) -	1.1198e-1 (3.46e-3) +	1.3214e-1 (5.32e-3) =	1.2839e-1 (2.54e-2)
	10	19	1.4618e-1 (4.90e-2) +	3.0605e-1 (1.06e-2) +	5.0552e-6 (6.41e-6) +	2.0750e-1 (8.12e-3) +	1.7605e-1 (2.12e-2) +	1.2067e-1 (1.24e-2) +	2.2327e-1 (6.74e-3) +	4.0672e-1 (7.45e-2)
	15	24	2.8907e-1 (4.90e-2) +	4.5722e-1 (2.40e-3) +	4.6594e-6 (1.18e-6) +	2.1875e-1 (8.34e-3) +	2.0765e-1 (2.03e-2) +	7.9632e-2 (1.21e-2) +	2.3668e-1 (6.18e-3) +	6.1517e-1 (3.38e-1)
DTLZ6	5	14	3.2761e-1 (3.98e-2) -	6.2518e-1 (1.48e-2) -	6.3617e-1 (3.55e-2) -	3.8680e-1 (1.79e-2) -	5.6662e-1 (3.97e-2) -	2.6617e-1 (2.94e-2) -	3.0872e-1 (3.00e-2) -	3.9681e-2 (4.82e-2)
	10	19	4.9067e-1 (7.83e-2) +	1.2238e+0 (5.55e-2) -	1.3608e-4 (5.92e-4) +	7.2727e-1 (3.44e-2) +	8.1686e-1 (1.31e-1) =	2.2898e-1 (7.13e-2) +	9.0698e-1 (2.48e-2) =	1.0161e+0 (3.17e-1)
	15	24	1.1023e+0 (1.92e-1) +	2.6697e+0 (2.51e+0) -	3.7992e-4 (1.16e-3) +	7.6729e-1 (4.59e-2) +	8.4454e-1 (1.68e-1) +	1.1443e-1 (9.28e-2) +	9.1528e-1 (1.69e-2) +	1.5205e+0 (3.94e-1)
+/-/=			11/3/4	8/8/2	14/2/2	14/3/1	7/8/3	15/1/2	8/6/4	

"+", "−", "=" indicate that the result is significantly better, significantly worse and statistically similar to that obtained by NSGAIII-NED, respectively. The standard deviation is in bracket.

In summary, no one algorithm can solve all problems, and no one algorithm can get the best values on all indicators [44]. NSGAIII-NED performs better than other algorithms in some problems of DTLZ. We also conclude that the proposed NSGAIII-NED performs worse than AR-MOEA or VaEA on regular problems with fifteen objectives. AR-MOEA is an adaptive evolutionary algorithm based on reference point. It can automatically adjust the distribution of the reference point set according to the shape of the current population in the target space, so that the reference point set can adapt to the different shape of the PFs. Therefore, the convergence of AR-MOEA is better than that of the proposed NSGAIII-NED.

The maximum-vector-angle-first principle has been used in VaEA for environmental selection. This principle aims to select the best solution in terms of the maximum vector angle, and add it into the population to construct the new population. So the diversity is enriched greatly by the principle. However, the framework of NSGAIII-NED is similar to that of NSGAIII. The reference points used in NSGAIII are almost evenly distributed on the regular PFs of those conventional instances. Moreover, NED lead to the decrease of population diversity for conventional problems, and the proposed niching method works very well in slightly complicated multimodal problem, but not very well in complex multimodal problems due to the lack of adaptive adjustment.

Meanwhile, when comparing the IGD values of WFG1-4 in Table 5, except AR-MOEA and VaEA, we can notice that PREA performs better than other algorithms including the proposed NSGAIII-NED. The reason is that, compared with the other algorithms' solutions, the solutions obtained by the proposed PREA approach are closer to the PF of those problems. Besides,the diversity of final results of PREA is better than that of other algorithms except AR-MOEA and VaEA. This point is proved by Table 4.

TABLE 4. Statistical results of the mean PD values obtained by the state of the art algorithms for DTLZ.

Problem	m	D	NSGAIII	MOEAD-PaS	MOMBI-II	PREA	SCDAS	AR-MOEA	VaEA	NSGAIII-NED (Ours)
DTLZ1	5	9	3.2238e+6 (1.44e+6) =	4.5443e+6 (4.69e+6) =	2.3698e+6 (4.14e+5) -	2.1240e+6 (4.91e+5) -	6.4274e+6 (1.60e+6) =	3.2726e+6 (6.97e+5) -	7.5424e+6 (4.57e+6) =	8.5222e+6 (1.44e+7)
	10	14	1.2844e+9 (9.24e+8) -	7.6947e+8 (1.11e+9) -	2.4933e+7 (3.91e+7) -	2.2685e+9 (1.62e+9) =	7.3557e+9 (7.89e+8) -	1.3197e+9 (4.99e+8) -	7.5946e+9 (6.14e+9) -	1.3361e+10 (3.10e+10)
	15	19	2.3904e+10 (3.19e+10) -	3.5213e+9 (9.31e+9) -	7.4836e+7 (9.21e+7) -	2.3018e+12 (3.24e+12) -	3.1796e+11 (2.66e+10) -	7.1816e+10 (2.71e+10) -	9.1458e+11 (7.63e+11) -	1.3712e+13 (5.85e+12)
DTLZ2	5	14	5.2405e+6 (4.69e+5) -	2.7431e+6 (6.73e+5) -	7.0275e+6 (1.01e+6) -	7.5093e+6 (9.62e+5) -	2.0739e+4 (1.52e+4) -	6.9031e+6 (6.04e+5) -	2.0252e+7 (1.96e+6) +	9.2995e+6 (1.78e+6)
	10	19	1.8999e+9 (1.14e+9) +	1.0167e-2 (3.53e-2) -	3.0518e+8 (4.18e+8) -	5.1546e+8 (2.33e+8) =	1.2842e+9 (2.02e+9) =	2.7158e+9 (7.29e+8) +	5.8964e+9 (1.37e+9) +	5.1530e+8 (5.61e+8)
	15	24	1.2459e+11 (3.81e+10) +	1.7885e+1 (7.65e+1) -	1.0568e+8 (2.32e+8) -	4.5753e+10 (2.14e+10) =	8.8462e+10 (1.53e+11) +	2.2817e+11 (5.47e+10) +	3.0779e+11 (7.16e+10) +	4.1780e+10 (3.76e+10)
DTLZ3	5	14	2.8014e+7 (2.27e+7) +	2.8436e+7 (3.73e+7) =	4.3156e+6 (1.31e+6) =	7.8412e+6 (4.03e+6) +	8.8822e+5 (1.47e+6) -	1.9712e+7 (9.63e+6) +	6.5403e+7 (5.09e+7) +	3.9880e+6 (2.21e+6)
	10	19	9.3497e+9 (8.39e+9) +	8.0606e+1 (1.22e+2) -	1.4660e+7 (2.00e+7) -	2.4012e+10 (1.86e+10) +	1.4101e+9 (2.28e+9) =	4.0857e+9 (2.80e+9) +	3.7611e+11 (1.61e+11) +	2.9744e+9 (1.30e+10)
	15	24	1.6894e+11 (2.20e+11) -	5.9158e+5 (2.48e+6) -	3.2690e+7 (4.76e+7) -	1.6196e+13 (1.16e+13) -	1.3480e+12 (2.63e+12) -	1.7446e+11 (1.61e+11) -	9.7588e+13 (2.80e+13) -	1.8366e+14 (8.04e+13)
DTLZ4	5	14	4.3482e+6 (2.53e+6) -	1.7387e+5 (1.31e+5) -	3.0763e+6 (1.73e+6) -	2.2895e+6 (2.67e+6) -	3.5494e+4 (8.24e+4) -	5.5728e+6 (3.32e+6) =	1.5257e+7 (2.46e+6) +	6.4662e+6 (2.11e+6)
	10	19	7.4154e+8 (1.18e+9) =	2.3791e+4 (1.85e+4) -	3.2820e+7 (4.81e+7) -	3.2906e+7 (4.05e+7) -	6.5677e+8 (5.80e+8) +	8.8812e+8 (4.91e+8) +	1.0476e+9 (4.58e+8) +	7.1213e+7 (6.34e+7)
	15	24	3.9294e+9 (1.03e+10) +	1.1193e+2 (5.00e+2) -	5.0539e+7 (1.16e+8) -	1.8711e+7 (2.34e+7) -	3.8423e+9 (3.09e+9) +	9.9672e+8 (1.02e+9) +	1.7242e+10 (7.32e+9) +	1.7208e+8 (2.55e+8)
DTLZ5	5	14	2.7775e+7 (3.29e+6) +	7.4147e+6 (1.92e+6) -	1.4225e+7 (1.71e+6) -	3.6350e+7 (2.00e+6) +	1.4267e+5 (2.21e+5) -	3.9473e+7 (2.93e+6) +	5.0455e+7 (3.25e+6) +	1.7852e+7 (3.72e+6)
	10	19	1.5945e+10 (2.20e+9) +	8.5332e-1 (1.51e+0) -	1.0230e+8 (8.42e+7) -	3.6527e+10 (2.86e+9) +	6.8264e+7 (7.86e+7) -	2.3529e+10 (2.61e+9) +	7.1319e+10 (4.06e+9) +	1.3527e+10 (3.45e+9)
	15	24	2.4186e+11 (8.26e+10) =	9.1955e-6 (2.86e-5) -	1.8771e+9 (1.31e+9) -	1.4149e+12 (1.41e+11) +	5.3017e+8 (1.40e+9) -	3.1284e+11 (5.31e+10) +	2.8359e+12 (2.09e+11) +	2.3112e+11 (1.52e+11)
DTLZ6	5	14	3.8573e+7 (5.62e+6) +	6.4119e+6 (6.62e+5) -	1.3276e+7 (1.66e+6) +	5.1362e+7 (3.25e+6) +	3.8015e+5 (1.41e+6) -	4.5922e+7 (2.88e+6) +	6.1753e+7 (5.08e+6) +	8.5699e+6 (2.56e+6)
	10	19	3.7898e+10 (7.01e+9) =	2.0757e+0 (5.16e+0) -	1.6220e+8 (2.64e+8) -	7.7897e+10 (1.01e+10) +	3.9778e+8 (1.78e+9) -	2.4328e+10 (5.54e+9) =	1.9364e+11 (2.39e+10) +	3.4809e+10 (2.29e+10)
	15	24	9.0398e+11 (3.03e+11) =	2.8480e-6 (1.12e-5) -	1.0903e+9 (2.76e+9) -	2.7769e+12 (2.84e+11) +	3.3977e+10 (1.05e+11) -	2.4357e+11 (1.57e+11) -	8.9973e+12 (1.16e+12) +	1.0932e+12 (4.32e+11)
+/-/=			8/5/5	0/16/2	1/16/1	8/7/3	2/12/4	10/5/3	14/3/1	

"+", "–", "=" indicate that the result is significantly better, significantly worse and statistically similar to that obtained by NSGAIII-NED, respectively. The standard deviations is in the bracket.

For WFG, we choose four problem to make tests. Different problems have different PFs. The PFs of WFG1-3 are irregular. The PFs of WFG4-9 are regular. So we choose WFG4 at random. As shown in Table 5, the proposed algorithm only achieves the second-best IGD value on WFG3 with 5 objectives. The result show that the proposed algorithm has its advantage on degenerate problem with a small number of objectives. On WFG1-4, the results of proposed algorithms are not better than AR-MOEA, VaEA and PREA. PREA performs the best on WFG3 with 5 and 10 objectives. It achieves the second-best IGD value for the same problem with 15 objectives.

During the deletion process, the proposed diversity maintenance mechanism in PREA always eliminates the one with the smaller fitness value between the two closest individuals, and this may be helpful for the obtained results be well distributed on the PF. It is difficult for our algorithm to achieve this effect.

What is more, the proposed NSGAIII-NED obtains relatively poor performance on this test suite compared with other recent algorithms, especially compared with VaEA. There are some reasons as followed. On the one hand, the PFs of the WFG test problems are irregular, divisible, multimodal, or mixed, and are scaled at different ranges in each target. The good distribution of the solution can not be guaranteed by a set of well distributed weight vectors. For example, if the true PFs of problems are not degenerate, some weight vectors will be not related to any Pareto-optimal solution. On the other hand, the reason why VaEA performs better in some WFG test problems may be that the maximum-vector-angle-first principle can not only guarantee the width of the search area, but also dynamically adjust the search direction of the whole group.

For MaF, we choose three problems to represent three different situations. It can be seen from Table 5 that NSGAIII-NED gets the second-best IGD value in three of

TABLE 5. Statistical results of the IGD values obtained by the state of the art algorithms for WFG AND MAF.

Problem	m	D	NSGAIII	MOEAD-PaS	MOMBI-II	PREA	SCDAS	AR-MOEA	VaEA	NSGAIII-NED (Ours)
WFG1	5	14	7.2838e-1 (7.38e-2) +	2.1303e+0 (1.30e-1) -	6.7002e-1 (1.28e-1) +	6.2323e-1 (2.89e-2) +	1.3418e+0 (1.14e-1) -	7.1810e-1 (4.58e-2) +	1.0358e+0 (1.22e-1) -	8.0489e-1 (1.00e-1)
	10	19	1.6272e+0 (1.42e-1) +	3.0380e+0 (1.80e-1) -	2.1952e+0 (2.21e-1) -	1.7198e+0 (8.63e-2) =	2.0321e+0 (8.86e-2) -	1.4949e+0 (6.66e-2) +	2.1765e+0 (1.31e-1) -	1.7690e+0 (1.55e-1)
	15	24	2.4920e+0 (2.45e-1) +	7.0500e+0 (7.88e+0) -	1.3308e+1 (9.89e+0) -	2.4008e+0 (1.15e-1) +	2.5881e+0 (4.79e-2) +	2.3665e+0 (9.43e-2) +	2.9537e+0 (2.38e-1) +	3.6119e+0 (4.80e-1)
WFG2	5	14	4.9925e-1 (5.21e-3) +	9.0546e-1 (6.38e-2) -	6.3248e-1 (3.05e-2) +	6.5868e-1 (3.16e-2) +	1.7113e+0 (2.29e-2) -	4.9674e-1 (5.18e-3) +	5.0074e-1 (9.47e-3) +	7.4953e-1 (5.15e-2)
	10	19	1.6052e+0 (1.70e-1) +	1.7040e+1 (3.33e+0) -	5.3514e+0 (1.68e+0) -	1.8313e+0 (8.23e-2) =	2.2721e+0 (1.49e-2) -	1.3705e+0 (4.37e-2) +	1.2595e+0 (3.33e-2) +	1.8363e+0 (1.29e-1)
	15	24	3.0022e+0 (6.83e-1) -	2.9315e+1 (4.36e-2) -	2.0528e+1 (4.55e+0) -	2.5349e+0 (1.57e-1) +	2.7726e+0 (1.30e-2) -	2.0680e+0 (7.77e-2) +	1.8276e+0 (5.71e-2) +	2.7014e+0 (3.32e-1)
WFG3	5	14	6.2675e-1 (8.89e-2) =	7.1836e-1 (1.08e-1) -	6.2807e-1 (9.58e-2) =	5.1230e-1 (7.49e-2) +	3.0109e+0 (1.40e+0) -	7.2664e-1 (3.66e-2) -	6.4361e-1 (7.84e-2) =	6.1003e-1 (4.72e-2)
	10	19	2.6926e+0 (5.93e-1) +	1.1189e+1 (6.64e-5) -	1.0508e+1 (3.53e-2) -	2.1643e+0 (3.41e-1) +	1.0777e+1 (1.22e-1) -	3.3155e+0 (2.14e-1) -	2.2606e+0 (2.49e-1) +	3.0875e+0 (6.54e-1)
	15	24	4.7333e+0 (4.45e-1) +	1.6953e+1 (1.12e-3) -	1.6579e+1 (7.59e-2) -	4.3766e+0 (6.79e-1) +	1.7301e+1 (1.15e-1) -	6.3114e+0 (6.93e-1) =	3.7121e+0 (6.07e-1) +	6.3141e+0 (1.05e+0)
WFG4	5	14	1.2416e+0 (8.82e-2) +	2.0873e+0 (1.04e-1) -	1.5148e+0 (9.08e-2) -	1.2301e+0 (1.66e-2) +	6.0000e+0 (1.42e-1) -	1.2244e+0 (1.78e-3) +	1.1863e+0 (9.78e-3) +	1.4495e+0 (6.16e-2)
	10	19	5.9906e+0 (1.53e-1) +	1.8218e+1 (4.27e-1) -	1.1057e+1 (1.08e+0) -	5.0687e+0 (6.30e-2) +	1.1953e+1 (8.74e-2) -	6.0007e+0 (1.14e-1) +	4.9787e+0 (4.65e-2) +	6.2277e+0 (2.87e-1)
	15	24	1.2107e+1 (4.16e-1) =	2.9078e+1 (8.90e-1) -	2.5854e+1 (1.61e+0) -	8.9555e+0 (1.21e-1) +	1.7377e+1 (8.39e-1) -	1.1976e+1 (2.83e-1) +	8.7611e+0 (9.17e-2) +	1.2141e+1 (4.30e-1)
MaF1	5	14	2.3665e-1 (2.68e-2) +	1.9583e-1 (4.37e-3) +	2.2764e-1 (8.54e-4) +	1.5153e-1 (4.45e-3) +	1.6894e-1 (4.30e-3) +	1.5266e-1 (2.56e-3) +	1.3664e-1 (1.40e-3) +	4.4077e-1 (7.49e-1)
	10	19	3.1943e-1 (6.05e-3) +	3.5466e-1 (1.09e-2) +	3.8909e-1 (1.26e-2) +	3.2333e-1 (1.21e-2) +	3.2464e-1 (8.29e-3) +	3.0012e-1 (7.36e-3) +	2.7052e-1 (2.48e-3) +	4.5885e-1 (8.77e-2)
	15	24	3.8510e-1 (1.82e-2) +	3.8152e-1 (7.90e-3) +	4.0773e-1 (1.40e-2) +	4.0668e-1 (1.57e-2) +	3.8628e-1 (1.45e-2) +	4.3827e-1 (3.47e-2) +	3.1549e-1 (3.69e-3) +	5.8051e-1 (9.00e-2)
MaF3	5	14	4.2775e+0 (6.14e+0) -	6.8940e+3 (9.37e+3) -	1.6289e-1 (2.64e-1) +	4.2493e-1 (1.21e+0) -	6.1631e-1 (7.59e-1) -	3.4475e-1 (1.03e+0) -	5.6610e+1 (1.43e+2) -	3.3099e-1 (4.55e-1)
	10	19	1.0838e+4 (2.48e+4) -	8.9429e+7 (4.90e+8) -	3.9260e-1 (3.98e-1) +	4.0358e+2 (5.34e+2) -	1.6341e+5 (2.27e+5) -	1.3689e+1 (2.03e+1) =	3.5587e+4 (5.77e+4) -	3.9723e-1 (7.18e-1)
	15	24	1.0928e+3 (2.50e+3) =	4.4363e+4 (4.59e+4) -	4.4557e-1 (1.02e-1) +	2.3222e+3 (1.59e+3) -	2.1911e+5 (2.35e+5) -	1.5827e+1 (2.75e+1) +	3.4443e+4 (8.95e+4) -	2.0603e+3 (7.88e+3)
MaF6	5	14	5.8770e-2 (1.73e-2) +	1.2311e-1 (2.09e-2) -	1.4014e-1 (2.06e-2) -	4.6343e-3 (1.05e-4) +	6.8058e-1 (1.14e-1) -	5.2840e-3 (1.40e-4) +	5.4432e-3 (3.48e-4) +	9.9148e-2 (2.93e-2)
	10	19	3.4177e-1 (1.19e-1) =	1.4785e+0 (1.21e+0) -	7.3652e-1 (1.41e-2) -	3.9694e-2 (1.08e-1) +	8.7117e-1 (2.38e-1) -	1.2689e-1 (1.43e-1) +	6.2059e-1 (2.53e-1) -	3.3510e-1 (1.12e-1)
	15	24	3.6209e-1 (1.57e-1) =	3.8201e+1 (9.14e+1) -	7.3953e-1 (6.77e-3) -	3.8780e-1 (1.13e-1) -	8.5682e-1 (8.52e-2) -	1.0777e-1 (1.28e-1) +	5.8743e-1 (1.72e-1) -	3.1566e-1 (1.12e-1)
+/-/=			13/3/5	3/18/0	8/12/1	15/4/2	4/17/0	16/3/2	13/7/1	

"+", "–", "=" indicate that the result is significantly better, significantly worse and statistically similar to that obtained by NSGAIII-NED, respectively. The standard deviations is in the bracket.

nine problems. On MaF3, NSGAIII-NED performs better than other algorithms except MOMBI-II. The PFs of MaF3 are convex. When PF is convex, the reference weight vector provided in advance causes the population to gather in the intermediate region, such as the solution obtained by NSGAIII. Since all the preset reference weight vectors intersect with PF of MaF3, the angle based adaptive method adopted in AR-MOEA can easily be mistaken as the ideal reference weight vector without adjustment. Hence, the population also are crowded in the middle of PF. However, NSGAIII-NED uses the NED strategy and niche technology to find out the non-dominated solutions on convex PF and retain the diversity of the population. MOMBI-II uses the scalar function and the statistical information of the population close to the real Pareto optimal frontier to optimize the algorithm, which is helpful to obtain good results.

In order to visualize the performance, we also compare the Pareto front of these algorithms. For DTLZ4 with 5 objectives and 14 decision variables,the true PF and the Pareto front comparisons are shown in Fig. 4 by parallel coordinates.

As is shown in Fig. 4, MOEAD-PaS, MOMBI-II and S-CDAS converge to a small number of optimal solutions. From this we can see that the horizontal axis represents the objective dimension, and the vertical axis represents

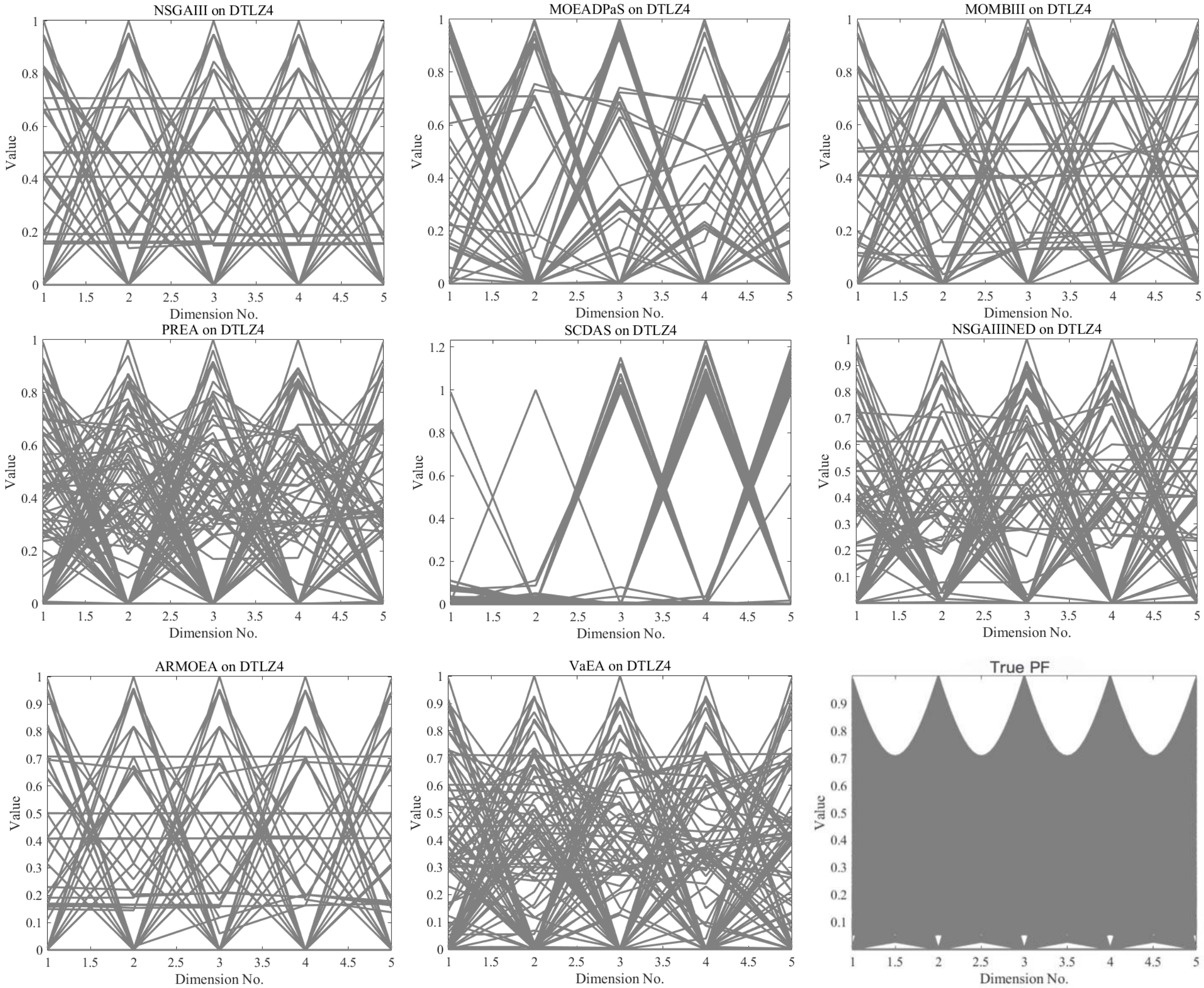

FIGURE 4. The Pareto front comparisons for NSGAIII-NED on DTLZ with 5 objectives.

the objective function value. In Figure 4, the true Pareto front of DTLZ4 problem is given. And the optimal frontier obtained by each algorithm also is given. Combined with Figure 4, we can analyze the convergence, coverage and uniformity [41], [42].

First of all, from the perspective of convergence, the target value range of NSGA-III, MOEAD/D-Pas, MOMBI-II, PREA, AR-MOEA, VaEA and NSGAIII-NED are all in the range of 0-1, that are consistent with the corresponding target value range of true Pareto front, but the target value of S-CDAS in the fourth and fifth dimension is significantly higher than 1. That means many individuals of NSGA-III can not converge, and they have some distances to the Pareto front.

From the perspective of coverage, the overall coverage area of S-CDAS and MOEAD-PaS are significantly less than that of other algorithms, and its coverage is more locally concentrated. Although all the algorithms cover on all 10 objectives, the solutions of other algorithms can not cover a large area except NSGAIII-NED, VaEA and PREA in the region between 0.5-0.7. So their coverage are better than other algorithms.

From the explanation in [41], [42], the uniform distribution in the parallel coordinates always imply the uniform solution sets, and a bad uniformity in the parallel coordinates does not mean bad-distributed solution sets. We can see that the solutions obtained by NSGA-III, MOMBI-II, and AR-MOEA distribute similarly. The solutions obtained by NSGAIII-NED, PREA, and VaEA also distribute similarly.

To sum up, the optimal frontier obtained by combining the comparison algorithm covers the proportion and distribution effect on the real optimal frontier. It can be concluded that solutions obtained by VaEA, PREA and NSGAIII-NED are close to the True PF. The experimental results show the effectiveness of the proposed algorithm.

According to the theory (no free lunch theorems for optimization [44]), no algorithm can make all indicators good, and no algorithm can solve all problems well. The above experiments show that the proposed algorithm is effective in solving most of DTLZ problems.

TABLE 6. The t-test results of the IGD value obtained by comparisons between NSGAIII-NED and other algorithms.

Problem	m	NSGAIII	MOEAD-PaS	MOMBI-II	PREA
DTLZ1	5	1	1	1	1
	10	1	1	1	1
	15	0	1	0	1
DTLZ2	5	1	1	1	1
	10	1	1	1	1
	15	1	1	1	1
DTLZ3	5	1	1	1	0
	10	1	1	1	1
	15	1	1	1	0
DTLZ4	5	0	1	0	1
	10	0	1	1	1
	15	1	1	1	1
DTLZ5	5	0	1	1	0
	10	0	1	1	1
	15	0	1	1	1
DTLZ6	5	1	1	1	1
	10	1	1	0	1
	15	1	1	0	1

Problem	m	SCDAS	AR-MOEA	VaEA
DTLZ1	5	0	1	0
	10	1	0	1
	15	1	1	0
DTLZ2	5	1	1	1
	10	1	1	1
	15	1	1	1
DTLZ3	5	1	1	1
	10	1	1	1
	15	0	1	1
DTLZ4	5	1	0	1
	10	0	1	1
	15	1	1	1
DTLZ5	5	1	1	1
	10	1	1	1
	15	1	1	1
DTLZ6	5	1	1	1
	10	1	1	1
	15	1	1	1

TABLE 7. The Statistical results of the IGD values obtained by the different value of H.

Problem	m	H=1	H=2	H=2.5
DTLZ1	5	1.4587e-1 (6.64e-2)	1.2578e-1 (7.52e-2)	1.2079e-1 (6.11e-2)
	10	2.6352e-1 (6.24e-2)	1.7674e-1 (1.39e-2)	2.6388e-1 (1.48e-1)
	15	3.9844e-1 (4.93e-2)	8.8532e-1 (9.09e-1)	4.5555e-1 (1.81e-1)
DTLZ2	5	2.3508e-1 (5.03e-3)	2.3413e-1 (3.96e-3)	2.3098e-1 (3.85e-3)
	10	7.9185e-1 (8.88e-2)	6.1057e-1 (2.48e-2)	6.0705e-1 (4.20e-2)
	15	1.1640e+0 (3.78e-2)	7.9019e-1 (1.86e-2)	8.5675e-1 (2.68e-2)
DTLZ3	5	2.9979e-1 (2.05e-2)	3.2290e-1 (3.53e-2)	3.8188e+0 (3.21e+0)
	10	7.9193e-1 (1.91e-1)	1.0891e+0 (1.28e+0)	8.8114e+0 (5.65e+0)
	15	3.2646e+0 (3.39e+0)	3.7419e+1 (1.63e+1)	5.2118e+0 (3.23e+0)
DTLZ4	5	3.5735e-1 (1.18e-1)	3.3752e-1 (2.04e-1)	4.1811e-1 (1.64e-1)
	10	6.9800e-1 (6.41e-2)	6.0070e-1 (3.43e-2)	7.3601e-1 (1.02e-1)
	15	1.1318e+0 (4.75e-2)	7.9887e-1 (3.30e-2)	9.4626e-1 (2.69e-2)
WFG1	5	8.9643e-1 (1.78e-1)	7.6553e-1 (5.94e-2)	9.8880e-1 (2.10e-1)
	10	1.9054e+0 (1.19e-1)	1.7004e+0 (1.46e-1)	1.8062e+0 (1.65e-1)
	15	2.6581e+0 (1.06e-1)	3.4894e+0 (4.98e-1)	2.7214e+0 (1.67e-1)
WFG2	5	9.6607e-1 (8.38e-2)	7.5749e-1 (4.47e-2)	7.2119e-1 (5.93e-2)
	10	1.7629e+0 (9.70e-2)	1.8625e+0 (8.37e-2)	1.4625e+0 (1.15e-1)
	15	3.6261e+0 (2.28e+0)	2.6651e+0 (1.57e-1)	4.5189e+0 (1.77e+0)
WFG3	5	1.1652e+0 (4.20e-1)	6.0496e-1 (7.41e-2)	8.7160e-1 (1.56e-1)
	10	5.5923e+0 (1.43e+0)	3.2297e+0 (8.14e-1)	3.2307e+0 (3.68e-1)
	15	9.2902e+0 (2.23e+0)	6.4437e+0 (5.70e-1)	6.2394e+0 (7.09e-1)
WFG4	5	1.5704e+0 (1.21e-1)	1.4663e+0 (1.09e-1)	1.4199e+0 (5.73e-2)
	10	7.1608e+0 (8.66e-1)	6.2533e+0 (3.84e-1)	6.2716e+0 (3.53e-1)
	15	1.6921e+1 (1.15e+0)	1.2397e+1 (4.19e-1)	1.2742e+1 (3.19e-1)

In order to further discuss the performance of the algorithm, we conduct experiments for a small number of objectives, taking the three objectives of MaF1 as examples, and conduct correlation analysis. The non-dominated front of related algorithms is shown in Figure 5. We can find that the individual distribution of the VaEA population is more uniform, and it covers True PF better. The uniformity of PREA is worse than that of VaEA, but better than that of other algorithms. The population of AR-MOEA is mostly crowded in the middle area of PF. The population individuals of MOEADPaS and MOMBI-II are mostly concentrated in the border area, and there are fewer individuals in the middle of PF compared with other algorithms. Although the coverage of individuals in NSGA3 and NSGA3-NED is satisfactory in terms of overall PF, most individuals are crowded with each other and the population diversity is worse than VaEA, PREA, and AR-MOEA.

F. T-TEST

According to the relevant definition of t-test and its theoretical research, t-test is carried out on the mean value of IGD that belongs to each of other five algorithms and the NSGAIII-NED's mean value of IGD in Table 2, respectively. Firstly, the relevant hypothesis is established, that is, the mean value of the two groups of data is not significantly different Then the statistical significance level α is set as 0.05. The test operation is carried out in MATLAB, and the relevant results are shown in the table 4. 0 indicates that there is no significant difference between the two groups of data, and 1 indicates that the difference between the two groups of data is significant.

As is shown in Table 6, the results are consistent with the IGD results in Table 3. Therefore, our results are reliable on statistics.

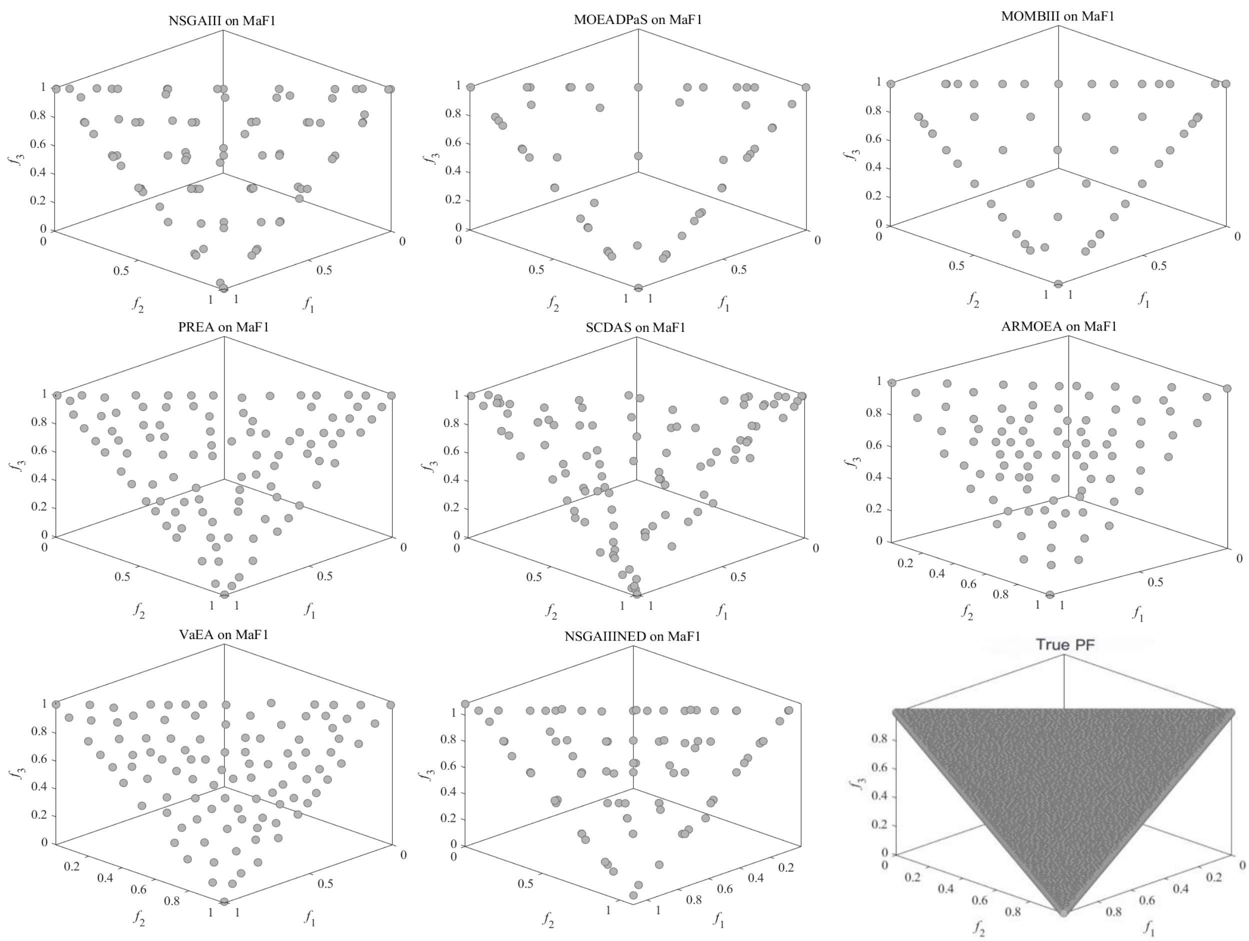

FIGURE 5. The Pareto front comparisons for NSGAIII-NED on MaF1 with 3 objectives.

G. PARAMETER ANALYSIS

In this subsection, we further analyse the effects of the different parameter settings, especially focusing on the value of H in formula 10. Table 7 shows that the results of the proposed algorithm are influenced by the value of parameter H in the DTLZ4 problem with different number of objectives and variables. The mean values are before the parentheses. The standard deviations are in the parentheses. Through a number of experiments, we choose three different values of H to represent different situations. Comparing the mean values and standard deviations of IGD obtained by our algorithm in corresponding cases, we can see that among the 24 groups of experimental data of all problems, twelve of the experimental results with $H = 2$ are the best, which is obviously more than the experimental results with other values. Therefore, we usually define the parameter H as 2.

V. CONCLUSION

For a feasible solution, it can dominate those solutions that are slightly better in a few goals but significantly worse in most other goals, and this dominance relation should be gradually expanded. For this motivation, we proposed a nonlinear expanded dominance relation based evolutionary algorithm for solving many-objective optimization problems in this paper. In order to solve the problem that diversity and wideness may become poor after expanding the dominating area, we used a niching method based on the angle between solutions in the operation of individual sorting.

To evaluate the performance of the proposed algorithm, experiments on DTLZ1-DTLZ6,WFG1-4, MaF1, MaF3 and MaF6 were carried out with the state of the art algorithm. The experimental results showed the effectiveness of the proposed algorithm. However, no one algorithm can solve all problems, and no one algorithm can get the best values on all indicators [44]. NSGAIII-NED also still exists some shortages. It does not get good values on WFG toolkit. In some test instances with complex PF, good results are not obtained. And there is also a further improvement in spatial diversity. In the near future, we will extend our work in these problems. Besides,we will try to apply NSGAIII-NED to the constrained many-objective optimization problems.

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