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Research on the Inverse Problem of Reliability Evaluation–Model and Algorithm

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ABSTRACT Due to the high dependence of economic and social development on power systems, the demand for reliable operation of power systems is increasing. Considering the popularity and widespread installation of smart meters, accurate system/node reliability indexes can be obtained. The inverse problem of reliability evaluation (IPRE) refers to the use of known system/node reliability indexes to obtain component reliability parameters. In this paper, a novel method of solving the IPRE is proposed. First, based on a nonsequential Monte Carlo (NSMC) method, analytical expressions for system reliability indexes in terms of component reliability parameters are derived, and then, the nonlinear equations of the IPRE are constructed. Second, a high-order polynomial approximation based on the conjugate gradient algorithm is used to calculate the unknown component reliability parameters, and the results are compared with those obtained using traditional neural networks method. Finally, a continuation method is used to correct the errors of the obtained component reliability parameters. Three cases, namely, the IEEE 1979 Reliability Test System (IEEE RTS-79), the Roy Billinton Test System (RBTS) and the Chuanyu power system in Southwest China, are used to test the method proposed in this paper to verify its feasibility and accuracy.

INDEX TERMS Inverse problem of reliability evaluation, nonsequential Monte Carlo method, high-order polynomial approximation, continuation method.

I. INTRODUCTION

With the increasing proportion of complex power electronic equipment in power systems, the reliable and secure operation of such systems has been greatly challenged [1], [2]. Finding the weak links in a power system for further investment and reinforcement is an effective means of improving the reliability of the system. Accurate component reliability parameters can serve as the basis for guaranteeing the secure operation of such a system under the N-1 scenario. However, if the component reliability parameters are incorrect, it may be difficult to balance the economy and safety of the system [3].

Component reliability parameters are usually obtained via point estimation from historical statistical data [4], [5]. If the historical statistical data are inaccurate, the estimated component reliability parameters may be incorrect. Unfortunately, considering the shortage of component outage data and the subjective nature of manual records, the available component failure statistics are often inaccurate, hindering efforts to realize precise investments in power system planning [6]. Therefore, it is very important to accurately calculate or correct the component reliability parameters.

The concept of power system reliability evaluation was clearly defined in [3] in terms of the quantitative assessment and risk analysis of random fault events affecting a large power grid. The concept of the inverse problem of reliability evaluation (IPRE) was first proposed in [7]; the objective of this problem is to use known system/node reliability indexes such as the loss of load probability (*LOLP*), the loss of load frequency (*LOLF*) and the expected energy not supplied (*EENS*) to obtain the unknown component reliability parameters and then strengthen components with low reliability to improve the reliability of the whole system. With the increasing popularity of smart meters, it has become feasible to

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obtain accurate node/system data, which can lay a foundation for solving the IPRE.

At present, studies on the IPRE are rare. The existing relevant studies mainly include investigations of reliability parameter correction [8]–[10] and reliability parameter optimization [11], [12]. However, relevant research on the establishment of accurate mathematical models and the realization of accurate calculations is still lacking.

The IPRE is constructed as a set of nonlinear equations that are analytical expressions for system reliability indexes in terms of component reliability parameters. In [7], the nonlinear equations of the IPRE were established based on the enumeration method. However, the applicability of the enumeration method is limited by the scale of the system, and the computational burden is often enormous for a large-scale power system. In [13], a mathematical model of the IPRE based on a sequential Monte Carlo method was proposed. However, the sampling rules of the sequential Monte Carlo method are complex and require a large amount of original data, resulting in a high time cost.

Traditional methods excessively depend on the initial values and have poor accuracy when solving the nonlinear equations of the IPRE. The Newton method and the Gauss–Seidel method have been widely used in power flow calculation [14], [15]. However, the mathematical model of the IPRE is highly nonlinear, and improper initial values will lead to nonconvergence. Artificial neural networks are effective and popular methods of nonlinear approximation [16], but their performance is highly dependent on the selection of the training samples, which may lead to concerns about the credibility of the approximation results.

Considering the accuracy requirements of the IPRE and the shortcomings of traditional algorithms, this paper proposes a novel method of solving the IPRE. The contributions of this paper are as follows:

1. First, analytical expressions for the IPRE are derived based on a nonsequential Monte Carlo (NSMC) method.

2. By means of a high-order polynomial approximation method, a set of high-order polynomials equivalent to the nonlinear equations of the IPRE is derived, and the conjugate gradient algorithm is used to calculate the fitting coefficients, yielding solutions with improved accuracy compared with those obtained using traditional methods.

3. A continuation method is proposed to modify some of the component reliability parameters with large errors in order to obtain accurate final results.

The remainder of this paper is organized as follows. In Section II, analytical expressions for the IPRE are derived based on an NSMC method. Section III introduces the high-order polynomial approximation method based on the conjugate gradient algorithm. In Section IV, the continuation method for correcting the deviations in the values of some component reliability parameters is presented. Section V presents the use of three test systems, namely, the IEEE 1979 Reliability Test System (IEEE RTS-79), the Roy Billinton Test System (RBTS) and the Chuanyu power grid, to verify the accuracy and feasibility of the proposed method. Section VI provides a summary of the whole paper.

II. ANALYTICAL MODEL OF THE IPRE BASED ON A NSMC METHOD

Based on the conditional probability criterion, unknown component reliability parameters can be extracted via an NSMC process; then, an analytical model of the system reliability indexes in terms of the unknown component reliability parameters can be established. Suppose that each component has two states, namely, the normal state and the fault state, and that the fault states of different components are independent of each other. For N_C components with unknown parameters, there are a total of $M = 2^{N_C}$ possible combined states. Let F_j denote the j-th of these M combined states. These M states are mutually exclusive and constitute a complete event group in the state space.

The formula for calculating the probability $P(F_j)$ of the combined state F_j of the components with unknown parameters is expressed as shown in (1):

$$P(F_j) = \prod_{g\in\Lambda_{j,u}} A_g \prod_{h\in\Lambda_{j,d}} U_h \tag{1}$$

$$\text{where } A_g = \frac{\mu_g}{\mu_g+\lambda_g},\ U_h = \frac{\lambda_h}{\mu_h+\lambda_h} \tag{2}$$

Here, A_g and U_h are the availability rate of component g and the unavailability rate of component h, respectively, and $\Lambda_{j,u}$ and $\Lambda_{j,d}$ are the sets of components with unknown parameters that are in the normal state and the fault state, respectively. For fault events generated through enumeration, the system index *EENS* can be obtained as shown in (3):

$$EENS = \sum_{s\in\Omega} H(s)P(s) \tag{3}$$

where H(s) represents the loss of load and Ω represents the set of all load-loss events in the system. Then, based on the combined state of the components with unknown parameters, (3) can be

$$EENS = \sum_{s\in\Omega\cap\pi_1} H(s)P(s) + \sum_{s\in\Omega\cap\pi_2} H(s)P(s) + \cdots + \sum_{s\in\Omega\cap\pi_M} H(s)P(s) \tag{4}$$

In (4), π_j represents the set of component fault events for which the associated combined state of the components with unknown parameters is F_j. Considering the full probability and conditional probability formulas, (4) can be rewritten as shown in (5):

$$EENS = P(F_1)\sum_{s\in\Omega\cap\pi_1} H(s)P(s|F_1) + \cdots + P(F_M)\sum_{s\in\Omega\cap\pi_M} H(s)P(s|F_M) \tag{5}$$

where

$$L_j = \sum_{s\in\Omega\cap\pi_j} H(s)P(s|F_j),\quad j = 1, 2, \ldots, M \tag{6}$$

(5) is the analytical expression for *EENS* obtained through event enumeration. For failure events generated via the NSMC approach, the corresponding *EENS* expression is shown in (7):

$$EENS = \frac{1}{N_{samp}} \sum_{s\in\Psi} H(s) \tag{7}$$

where N_{samp} represents the total number of samples drawn and ψ represents the set of all load-loss events generated via sampling.

Similar to the derivation process for the analytical *EENS* model based on the state enumeration method, (7) can be divided into a sum of M terms:

$$EENS = \frac{1}{N_{samp}} \left(\sum_{s\in\Psi\cap\pi_1} H(s) + \sum_{s\in\Psi\cap\pi_2} H(s) + \cdots + \sum_{s\in\Psi\cap\pi_M} H(s) \right) \tag{8}$$

Since (8) is a sample-based estimate of the *EENS* index, theoretically, when the number of samples is sufficiently large, the *EENS* values obtained via the state enumeration and NSMC methods should be equal. By combining (3) and (7), (9) can be obtained:

$$\frac{1}{N_{samp}} \sum_{s\in\Psi} H(s) = \sum_{s\in\Omega} H(s)P(s) \tag{9}$$

Similarly, by comparing the terms on the right-hand sides of (5) and (9), (10) can be obtained:

$$P(F_j)L_j = \frac{1}{N_{samp}} \sum_{s\in\Psi\cap\pi_j} H(s), \quad j = 1, 2, \ldots, M \tag{10}$$

$$L_j = \frac{1}{P(F_j)} \times \frac{1}{N_{samp}} \times \sum_{s\in\Psi\cap\pi_j} H(s), \quad j = 1, 2, \ldots, M \tag{11}$$

(11) is the calculation formula for L_j based on the NSMC method. In summary, the analytical expression for the *EENS* index based on the NSMC process is given in (12):

$$EENS = \sum_{j=1}^{M} P(F_j)L_j = \sum_{j=1}^{M} \Big(\prod_{g\in\Lambda_{j,u}} A_g \prod_{h\in\Lambda_{j,d}} U_h \Big) L_j \tag{12}$$

Similarly, an expression for *LOLF* can be written as shown in (13) based on the enumeration method, and (13) can then be divided into a sum of M terms as shown in (14).

$$LOLF = \sum_{s\in\Omega} P(s)R(s) \tag{13}$$

$$LOLF = \sum_{s\in\Omega\cap\pi_1} P(s)R(s) + \ldots + \sum_{s\in\Omega\cap\pi_M} P(s)R(s) \tag{14}$$

Based on the NSMC method, the expression for *LOLF* is as follows:

$$LOLF = \frac{1}{N_{samp}} \sum_{s\in\psi} R(s) = \frac{1}{N_{samp}} \Big[\sum_{s\in\psi\cap\pi_1} R(s) + \ldots + \sum_{s\in\psi\cap\pi_M} R(s) \Big] \tag{15}$$

In (15), $R(s)$ represents the sum of the transfer rates of the system components, and R_j is the sum of the transfer rates of these components when the combined state of the components with unknown parameters is F_j, which can be obtained by comparing (16-a, b and c):

$$\frac{1}{N_{samp}} \sum_{s\in\psi\cap\pi_j} R(s) = \sum_{s\in\Omega\cap\pi_j} P(s)R(s) \tag{16-a}$$

$$\frac{N_{s\in\psi\cap\pi_j}}{N_{samp}} R_j = P(F_j)R_j \sum_{s\in\Omega\cap\pi_j} P(s|F_j) \tag{16-b}$$

$$\frac{1}{N_{samp}} \sum_{s\in\psi\cap\pi_j} R^o(s) = P(F_j) \sum_{s\in\Omega\cap\pi_j} P(s|F_j)R^o(s) \tag{16-c}$$

Here, $R^o(s)$ represents the sum of the transfer rates of the components with known parameters for system event s. From (16), the analytical expression for *LPLF* based on the NSMC method can be obtained as shown in (17):

$$LOLF = \sum_{j=1}^{M} [P(F_j)R_j T_{1,j} + P(F_j)T_{2,j}] \tag{17}$$

where

$$R_J = \sum_{g\in\Lambda j,u} -\lambda_g + \sum_{h\in\Lambda j,d} \mu_h \tag{18-a}$$

$$T_{1,j} = \sum_{s\in\Omega\cap\pi_j} P(s|F_j) = \frac{N_{s\in\psi\cap\pi_j}}{N_{samp}P(F_j)} \tag{18-b}$$

$$T_{2,j} = \sum_{s\in\Omega\cap\pi_j} P(s|F_j)R^o(s) = \frac{\sum_{s\in\psi\cap\pi_j} R^o(s)}{N_{samp}P(F_j)} \tag{18-c}$$

The derivation process for the system index *LOLP* is similar to that for *EENS*. The analytical expression for *LOLP* is given in (19):

$$LOLP = \sum_{j=1}^{M} P(F_j)K_j = \sum_{j=1}^{M} \Big(\prod_{g\in\Lambda_{j,u}} A_g \prod_{h\in\Lambda_{j,d}} U_h \Big) K_j \tag{19}$$

where

$$K_j = \frac{N_{s\in\psi\cap\pi_j}}{N_{samp}P(F_j)} \tag{20}$$

(12), (17) and (19) are the analytical expressions for the reliability indexes in terms of the unknown reliability parameters of the components. Based on these analytical expressions, the system reliability indexes can be quickly obtained, and a large number of accurate data samples can be used to derive high-order polynomial approximations.

III. METHOD FOR OBTAINING UNKNOWN COMPONENT RELIABILITY PARAMETERS BASED ON HIGH-ORDER POLYNOMIAL APPROXIMATION

When solving the nonlinear equations of the IPRE, traditional numerical methods excessively depend on the initial values, which may lead to nonconvergence. In this paper, a high-order polynomial approximation approach is used

to solve the IPRE. Suppose that the component reliability parameters are regarded as the independent variables x_i, the system reliability indexes are regarded as the dependent variables y, and the approximation coefficients are denoted by a with appropriate subscripts; then, the nonlinear equations of the IPRE can be approximated in the form of the following n-order polynomial (21):

$$\begin{aligned} y = {} & a_{10} + a_{11}x_1 + a_{12}x_1^2 + \ldots + a_{1n}x_1^n \\ & + \ldots a_{j0} + a_{j1}x_j + a_{j2}x_j^2 + \ldots + a_{jn}x_j^n \\ & + \sum_{f,g,p,q=1}^{f,g=j;p,q=n} a_k x_f^p \ldots x_g^q \end{aligned} \tag{21}$$

In the least squares method, the objective is to minimize the sum of the squares of the errors between the fitted values and the actual values, as shown in (22):

$$\min \varphi(x) = \|G\Delta a - H\|^2 \tag{22}$$

In (22), a_0 is the initial value of a, and $G = \nabla y(a, x)$, $\Delta a = a - a_0$, and $H = y - y(a_0, x)$. Then, (23) can be obtained.

$$G^T G\Delta a - G^T H = 0 \tag{23}$$

The above equation (23) is called the normal equation of (22) in mathematical terminology. Based on (23), the fitting coefficient can be obtained through an iterative solution process as shown in (24-a and b):

$$\Delta a = (G^T G)^{-1} G^T H \tag{24-a}$$

$$a = a_0 + \Delta a \tag{24-b}$$

From (24-a and b), it can be found that the initial value of the fitting coefficient α will affect the final result, and an unsuitable value of α may lead to fitting deviations or even failure. To avoid the occurrence of the above situation, this paper takes the fitting coefficient α as the search direction D, and a one-dimensional search is carried out along the direction D.

In addition, the conjugate gradient algorithm is used to improve the computational efficiency. A new functional is defined in this paper, as shown in (25):

$$L(D) = \frac{1}{2}[GD, D] - [H, D] \tag{25}$$

where $[x, y]$ denotes the inner product operation, i.e., $[x, y] = x_1y_1 + x_2y_2 + x_3y_3 + \ldots + x_ny_n$.

The gradient of the objective function $L(D)$ can be expressed as follows:

$$\nabla L(D) = GD - H \tag{26}$$

Let $GD - H = -\gamma$, where γ represents the residual on the negative gradient of $L(D)$. Therefore, s search along the direction D can yield (27) and (28):

$$a = a_0 + \omega_{\min} D \tag{27}$$

$$\omega_{\min} : L(a_0 + \omega D) = \min \tag{28}$$

By substituting (27) into (25), (29) can be obtained. Then, equation (30) can be found.

$$L(a_0 + \omega D) = \frac{1}{2}[G(a_0 + \omega D), a_0 + \omega D] - [H, a_0 + \omega D] \tag{29}$$

$$\frac{\partial L(a_0 + \omega D)}{\partial \omega} = \omega[GD, D] - [\gamma, D] \tag{30}$$

The minimum iteration step $\omega_{\min}$ is calculated as shown in (31):

$$\omega_{\min} = \frac{[\gamma, D]}{[GD, D]} \tag{31}$$

(25)-(31) constitute the mathematical description of the conjugate gradient algorithm. In the k-th step of iteration, the following equations apply:

$$a_{k+1} = a_k + \omega_k D_k \tag{32}$$

where

$$\omega_k : L = \min(a_k + \omega_k D_k) \tag{33-a}$$

$$\omega_k = \frac{\nabla L(a_k)^T D_k}{D_k^T G D_k} \tag{33-b}$$

$$\gamma = \frac{\Delta L(a_{k+1})^T G D_k}{D_k^T G D_k} \tag{33-c}$$

$$D_{k+1} = -\nabla L(a_{k+1}) + \gamma_k D_k \tag{33-d}$$

Based on the above theory, the algorithm flow of the conjugate gradient algorithm in the high-order nonlinear polynomial approximation model is shown in TABLE 1.

TABLE 1. Flow of the conjugate gradient algorithm.

STEP 1	Select the appropriate high-order polynomial (21) and the corresponding initial value of each approximation coefficient a_0
STEP 2	Based on the given initial value, calculate the initial search direction $D_0 = -\nabla y(a_0, x) = \gamma$
STEP 3	Calculate the iteration step size ω such that $\omega_k = \frac{[\gamma^{(k-1)}, \gamma^{(k-1)}]}{[GD^k, D^k]}$ is satisfied
STEP 4	Search in a new direction and obtain $a^{(k)} = a^{(k-1)} + \omega_k D^{(k)}$
STEP 5	Calculate the new residual $\gamma^{(k)} = \gamma^{(k-1)} - \omega_k GD^{(k)}$
STEP 6	If $\|\nabla y(a_{k+1})\| \le \varepsilon$, output the approximate solution; otherwise, return to Step 2
	END

IV. CONTINUATION METHOD FOR MODIFYING THE NUMERICAL SOLUTION TO THE IPRE

The method proposed in Section III can be used to approximately obtain the component reliability parameters. In this section, considering the need for more accurate component reliability parameters in some engineering cases, a continuation method is proposed to modify component reliability parameters with large deviations. The results obtained via the

high-order polynomial approximation method are used as the initial solutions to the nonlinear equations of IPRE, and then, the deviations are iteratively modified via the continuation method.

A. TECHNIQUE AND THEORY OF THE CONTINUATION METHOD

To make the iterative sequence $\{\mathbf{x}_k\}$ converge to the true value $\mathbf{x}^*$, traditional methods of solving nonlinear equations, such as the Newton method, require that the initial value $\mathbf{x}_0$ and the true value $\mathbf{x}^*$ are sufficiently close to each other. However, it is difficult to find an initial value for the iterative solution process that satisfies the requirements for practical calculation, especially when solving equations with a high degree of nonlinearity. In contrast, a continuation method has the advantages of a strong convergence ability and independence of the initial values. Therefore, in this paper, a continuation method is used to modify the deviations of the solutions to the IPRE.

In the continuation method, a continuation parameter t is introduced into the considered problem, where $t \in [0, 1]$. A cluster of mappings $\boldsymbol{H}(\mathbf{x}, t) : D \times [0, 1] \subset \boldsymbol{R}^{n+1} \rightarrow \boldsymbol{R}^n$ is constructed, and the original mapping $\boldsymbol{F}$ is embedded into the mapping cluster $\boldsymbol{H}$, so that $\forall \mathbf{x} \in D$, $\boldsymbol{H}$ satisfies the following conditions (34):

$$\boldsymbol{H}(\mathbf{x}, 0) = \boldsymbol{F}_0(\mathbf{x}), \boldsymbol{H}(\mathbf{x}, 1) = \boldsymbol{F}(\mathbf{x}) \tag{34}$$

where $\mathbf{x}_0$ is the solution to the equation $\boldsymbol{F}_0(\mathbf{x}) = \mathbf{0}$, which can be easily obtained, and the equation $\boldsymbol{H}(\mathbf{x}, 1) = \mathbf{0}$ represents the original nonlinear equations. Thus, the problem is transformed into the problem of solving equation (35). (35) is called a homotopic equation, and the function $\mathbf{x}(t)$ is the solution function of equation system (35).

$$\boldsymbol{H}(\mathbf{x}, t) = \mathbf{0}, t \in [0, 1], \quad \mathbf{x} \in D \tag{35}$$

The core of the continuation method is that the domain of the continuation parameter $t \in [0, 1]$ is divided into N segments, where N is the number of continuation steps.

$$\boldsymbol{H}_j = \boldsymbol{H}(\mathbf{x}, t_j) = \mathbf{0}, 0 = t_0 < \ldots < t_j < \ldots < t_N = 1 \tag{36}$$

Taking the solution $\mathbf{x}_j^*$ as the initial value for the next equation $\boldsymbol{H}_{j+1} = \mathbf{0}$, if $t_{j+1} - t_j$ is sufficiently small, then $\mathbf{x}_j^*$ can be regarded as an approximate solution for $\mathbf{x}_{j+1}^*$, meaning that the iterative process has converged. Thus, $\mathbf{x}_j^*$ is the final solution to the original equations.

B. ITERATIVE FORMULAS FOR AND FLOW OF THE CONTINUATION METHOD

The aim of the continuation method is to provide good initial values for a conventional algorithm to solve the nonlinear equations. Therefore, it is necessary to take the solution obtained via the continuation method as the initial values and then use another conventional algorithm, such as the Newton method, to continue the iterative process until the solution finally converges to the true values. If the number of continuation steps is N, then the iterative formulas for the continuation method are as shown in (37):

$$\begin{cases} x^1 = x^0 - h[J(x^0) + aI]^{-1}F(x^0) \\ x^{k+1/2} = x^k + \dfrac{1}{2}(x^k - x^{k-1}) \\ x^{k+1} = x^k - h[J(x^{k+1/2}) + a(1 - t_{k+1/2}^3)I]^{-1} \times [F(x^0) \\ -3at_{k+1/2}^2(x^{k+1/2} - x^0)], k = 1, \cdots, N-1 \end{cases} \tag{37}$$

After the first iteration of the continuation method, the Newton method can be used to continue the iteration:

$$x^{k+1} = x^k - [J(x^k)]^{-1}F(x^k)k = N, N+1, \cdots \tag{38}$$

where $h = 1/N, t_{k+1/2} = (k + 1/2)h$, and $J(x)$ is the Jacobian matrix of the mapping $\boldsymbol{F}(x)$. a is called the nonsingular control parameter, which is used to deal with the singular case of the matrix $J(x)$ during the iterative process, and I is the identity matrix with dimensions of $n \times n$. When using (37) for iterative calculation, the singular problem for the matrix $J(x)$ can be solved by automatically controlling the nonsingular parameter a. TABLE 2 shows the algorithm flow of the continuation method.

TABLE 2. Flow of the continuation method.

STEP 1	Input the initial value x^0, the minimum step size h_0, the number of cycles M, the accuracy requirements ε_1 and ε_2, and $I = 0$
STEP 2	Set a=0
STEP 3	Set $c = x^0$ and calculate $F(x^0)$, $\|F(x^0)\|$, $J(x^0)$, h=0.1, ε_N=$\|F(x^0)\|$, and k=1
STEP 4	Calculate $[J(x^0)+aI]^{-1}$; if it does not exist, then go to Step 15
STEP 5	Calculate x^1 using (37) and record $\|H(x^1,t_1,a)\|$ as H_1; if $H_1>\varepsilon_N h$ or $h>h_0$, then return to Step 4, or proceed to Step 6 otherwise
STEP 6	When $k\geq1$, calculate x^{k+1} using (37); if $[J(x^{k+1/2})+a(1-(t_{k+1/2})^3)I]^{-1}$ does not exist, then go to Step 15, or proceed to Step 7 otherwise
STEP 7	Set k=k+1; if $k<N$, then return to Step 6, or proceed to Step 8 otherwise
STEP 8	Calculate $F(x^N)$; if $\|F(x^N)\|<\varepsilon_N$, then proceed to Step 9, or go to Step 14 otherwise
STEP 9	Set $x^0 = x^N$ and k=0
STEP 10	Calculate x^{k+1} using (37); if $[J(x^k)]^{-1}$ does not exist, then go to Step 15, or set k=k+1 otherwise; if $\|F(x^{k+1})\| > \|F(x^k)\|$, then go to Step 12, or proceed to Step 11 otherwise
STEP 11	If $k<2$, then return to Step 9; otherwise, proceed to Step 12
STEP 12	If $\|x^{k+1} - x^k\|<\|x^k - x^{k-1}\|$, then proceed to Step 13; otherwise, go to Step 14
STEP 13	If $\|x^{k+1} - x^k\|\leq\varepsilon_1\|x^k - x^{k-1}\|$ or $\|F(x^{k+1})\|\leq\varepsilon_2$, then go to Step 17; otherwise, return to Step 10
STEP 14	Set x^0=x^k and $I = I + 1$; if $I \leq M$, then return to Step 2, or go to Step 16 otherwise
STEP 15	Set x^0=c, $a = a + \Delta a$, and $I = I + 1$; if $I \leq M$, then return to Step 3, or proceed to Step 16 otherwise
STEP 16	Abnormal end
STEP 17	**END**

V. CASE STUDY

In this section, three test systems, namely, the IEEE RTS-79, the RBTS and the Chuanyu power system in Southwest China, are used to verify the feasibility and accuracy

of the method proposed in this paper. The annual load curve of the IEEE RTS-79 is used for each test system, and the NSMC method is used to calculate the reliability indexes of the systems and nodes.

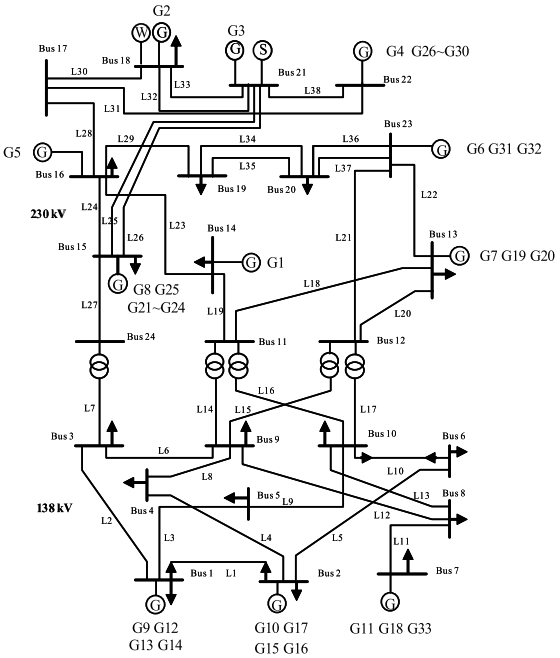

FIGURE 1. Topology of the IEEE RTS-79.

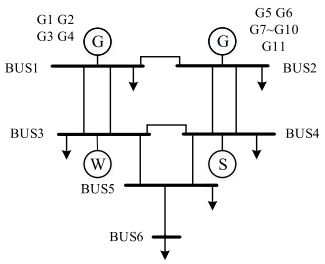

FIGURE 2. Topology of the RBTS.

As shown in Fig. 1 and Fig. 2, the IEEE RTS-79 consists of 33 units, 33 transmission lines and 5 transformer branches. The peak load is 2850 MW, and the installed capacity is 3405 MW. The RBTS consists of 11 units and 9 transmission lines. The peak load is 185 MW, and the installed capacity is 240 MW. The Chuanyu power system consists of 64 units and 173 transmission lines (voltage levels of 500 kV and 220 kV), with a peak load of 9732 MW and an installed capacity of 10684 MW.

A. CASE A: ACCURACY OF HIGH-ORDER POLYNOMIAL APPROXIMATION BASED ON THE CONJUGATE GRADIENT ALGORITHM

In this section, four traditional machine learning algorithms, namely, a feedforward neural network with a single hidden layer (FNN(1)), a feedforward neural network with three hidden layers (FNN(3)), a radial basis function neural network (RBFNN) and a convolutional neural network (CNN), are used for comparison with the high-order polynomial approximation based on the conjugate gradient algorithm proposed in this paper. The FNNs were trained via the back-propagation process based on the Levenberg-Marquardt algorithm, and for the CNN, the Adam algorithm was used for training. To solve for n reliability parameters, the numbers of neurons in the three hidden layers of FFN(3) were set to $3n$, $2n$ and $1.5n$, respectively, from the first to the last layer. The CNN contained two convolutional layers and two fully connected layers. The numbers of neurons in the first and second fully connected layers were set to $3n$ and $2n$, respectively. The number of training epochs was set to 2000.

TABLE 3 describes the known system/node reliability indexes and the unknown component reliability parameters in the three test systems.

TABLE 3. Known and unknown parameters of the three test systems.

IEEE RTS-79		RBTS		CY	
KRI	UCP	KRI	UCP	KRI	UCP
$LOLP_{sys}$	μ_{G13}	$LOLP_{sys}$	λ_{G1}	$LOLP_{sys}$	μ_{G1}
$LOLF_{sys}$	μ_{G16}	$LOLF_{sys}$	λ_{G3}	$LOLF_{sys}$	μ_{G2}
$EENS_{sys}$	μ_{G18}	$EENS_{sys}$	λ_{G5}	$EENS_{sys}$	μ_{G3}
$LOLP_{15}$	λ_{L1}	$LOLP_{2}$	μ_{L1}	$EENS_{24}$	λ_{L1}
$LOLF_{15}$	λ_{L2}	$LOLF_{2}$	μ_{L3}	$LOLF_{6}$	λ_{L2}
$EENS_{15}$	λ_{L3}	$EENS_{2}$	μ_{L9}	$EENS_{7}$	λ_{L40}

'KRI' denotes known system/node reliability indexes, and 'UCP' denotes unknown component reliability parameters. The subscript 'sys' and subscript numbers denote system and node reliability indexes, respectively. The subscripts 'G' and 'L' denote generator units and transmission lines, respectively.

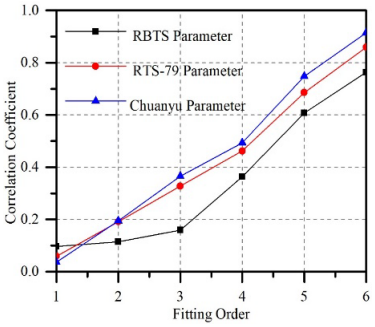

FIGURE 3. Relationship between fitting order and correlation coefficient.

When the high-order polynomial approximation method is used, a higher order will yield more accurate results. Fig. 3 shows the relationship between the fitting order N and the correlation coefficient R. As the value of the fitting order N increases, R approaches 1, indicating a better approximation effect.

TABLE 4. Calculation results for six reliability parameters in the ieee rts-79.

Unknown	True value	Calculation results				
		FNN (1)	FNN (3)	RBFNN	CNN	Approximation method
μ_{G13}	175.2	160.9	182.2	177.1	152.8	196.23
μ_{G16}	219	279.4	206.6	194.9	272.2	247.07
μ_{G18}	58.4	58.23	58.43	58.42	58.55	58.42
λ_{L1}	0.51	0.95	1.23	1.55	2.01	0.53
λ_{L2}	0.48	1.27	0.01	0.19	2.24	0.52
λ_{L3}	0.41	0.98	0.19	0.08	1.13	0.45
average error		71.2%	50.6%	59.4%	145.6%	7.6%

TABLE 5. Calculation results for six reliability parameters in the rbts.

Unknown	True value	Calculation results				
		FNN (1)	FNN (3)	RBFNN	CNN	Approximation method
λ_{G1}	6	6.02	6.00	5.99	6.01	6.01
λ_{G3}	5	5.01	4.98	5.00	5.02	4.98
λ_{G5}	3	2.99	3.00	3.00	2.98	3.01
μ_{L1}	876	1018.9	1035.0	1079.7	1036.0	905.1
μ_{L3}	876	881.90	886.26	985.31	1022.0	894.0
μ_{L9}	876	873.81	875.81	877.07	874.29	893.3
average error		3.0%	3.3%	6.0%	6.1%	2.1%

TABLE 6. Calculation results for six reliability parameters in the chuanyu test system.

Unknown	True value	Calculation results				
		FNN (1)	FNN (3)	RBFNN	CNN	Approximation method
μ_{G1}	411.27	459.75	411.33	391.72	501.23	453.01
μ_{G2}	195.54	204.73	195.36	156.86	264.24	201.54
μ_{G3}	411.27	458.31	409.93	311.11	485.65	418.66
λ_{L1}	1	0.999	1	1	1.211	0.990
λ_{L2}	1	0.998	0.999	0.894	1.256	1.015
λ_{L40}	1	1.001	1	1.001	1.412	1.002
average error		4.71%	1.03%	9.98%	2.94%	2.72%

TABLES 4, 5 and 6 show the calculation results for each test system obtained using the four traditional machine learning algorithms and the method proposed in this paper.

As shown in TABLES 4, 5 and 6, the high-order polynomial approximation method can yield more accurate results than any of the traditional machine learning algorithms.

To verify the feasibility and accuracy of the proposed method, the true values of the component reliability parameters in the three test systems were selected as reference values for comparison with the solutions obtained using the method proposed in this paper and the traditional methods. As shown in Fig. 4 and Fig. 5(a,b,c), the high-order polynomial approximation method proposed in this paper can effectively approximate the true values for the different systems, satisfying their practical operation requirements.

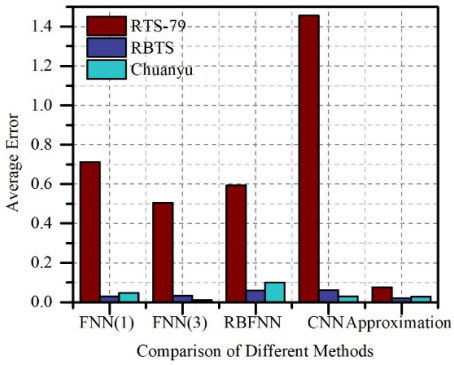

FIGURE 4. Average errors of different methods.

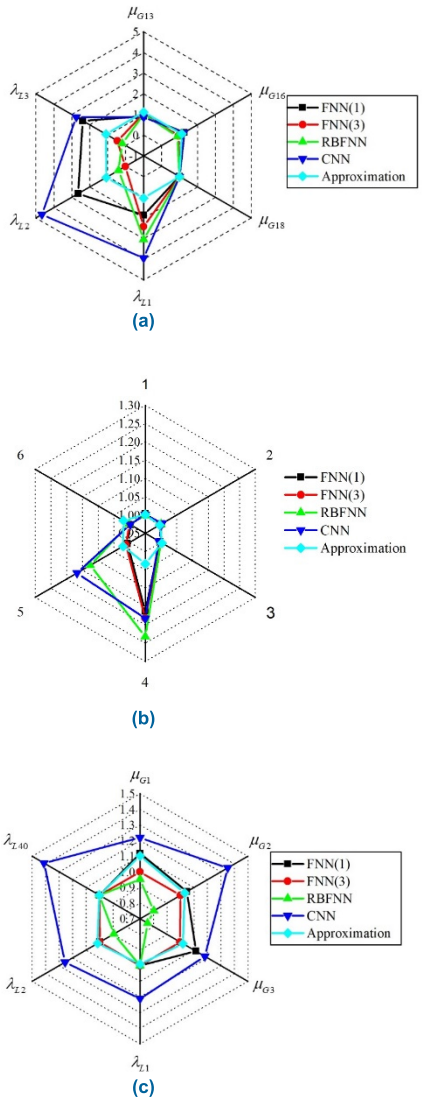

FIGURE 5. (a) Comparison of results obtained with different methods in the IEEE RTS-79. (b) Comparison of results obtained with different methods in the RBTS. (c) Comparison of results obtained with different methods in the Chuanyu power system.

B. CASE B: FEASIBILITY OF THE CONTINUATION METHOD

Considering the need for the true values of the component reliability parameters in certain engineering applications, this section demonstrates the use of the continuation method to correct errors in the solutions obtained with the high-order polynomial approximation method. TABLES 7, 8 and 9 show the modified results for the three test systems obtained using the continuation method. The efficiency and cost-time of continuation method also can be obviously observed from the results.

TABLE 7. Modified results obtained via the continuation method for the ieee RTS-79.

Unknown parameter	True value	Modified result	Number of iterations	Computing time
μ_{G13}	175.2	175.2	32	1.46s
μ_{G16}	219	219		
μ_{G18}	58.4	58.4		
λ_{L1}	0.51	0.51		
λ_{L2}	0.48	0.48		
λ_{L3}	0.41	0.41		

TABLE 8. Modified results obtained via the continuation method for the rbts.

Unknown parameter	True value	Modified result	Number of iterations	Computing time
λ_{G1}	6	6	29	1.09s
λ_{G3}	5	5		
λ_{G5}	3	3		
μ_{L1}	876	876		
μ_{L3}	876	876		
μ_{L9}	876	876		

TABLE 9. Modified results obtained via the continuation method for the chuanyu test system.

Unknown parameter	True value	Modified result	Number of iterations	Computing time
μ_{G1}	411.27	411.27	38	2.24s
μ_{G2}	195.54	195.54		
μ_{G3}	411.27	411.27		
λ_{L1}	1	1		
λ_{L2}	1	1		
λ_{L40}	1	1		

As shown in TABLES 7, 8 and 9, the errors of the obtained component reliability parameters can be corrected with the continuation method. By virtue of its strong convergence ability, an accurate solution to the IPRE can be realized.

Through an analysis of the above case studies, the following conclusions are drawn:

(1) Based on the NSMC method, analytical expressions for system/node reliability indexes and component reliability parameters can be derived. In accordance with these analytical expressions, a large number of data can be generated to serve as source data for high-order polynomial approximation.

(2) Compared with traditional machine learning algorithms, the proposed high-order polynomial approximation method based on the conjugate gradient algorithm offers a considerably improved solution accuracy.

(3) For component reliability parameters with large errors, the proposed continuation method is an effective means of correcting those errors to finally obtain an accurate solution to the IPRE.

VI. CONCLUSION

Considering the deficiencies of the existing research on the IPRE, this paper proposes a novel and feasible method of solving the IPRE. The main contributions of this paper are as follows: First, based on the NSMC method, analytical expressions for system reliability indexes and component reliability parameters are established. Second, approximate solutions to the nonlinear equations of the IPRE are obtained through high-order polynomial approximation. Finally, a continuation method is applied to correct errors. Case studies on three test systems are presented to prove the accuracy and feasibility of the proposed method.

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