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Entanglement-Based Competition Resolution in Distributed Systems

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ABSTRACT Accessing scarce resources in distributed systems where entities are not centrally coordinated, whether roebucks are fighting for a doe or WiFi users are competing for a common radio channel, is associated with moderate efficiency and proves to be a challenging problem. In this study, we present our hypothesis on shared entanglement and how such a process can improve resource access in macroscopically distributed systems. Furthermore, the required entangled states can be established in a distributed manner.

INDEX TERMS Distributed systems, medium access control (MAC), quantum computing, quantum entanglement, slotted-ALOHA, unitary transformation.

I. INTRODUCTION

Nature developed appropriate solutions to handle scenarios in which individuals compete for resources. The resolution process involves random behavior to avoid conflict, which often results in injuries. This basic principle was adapted for human-made artificial systems; for example, in distributed networks, entities compete for limited resources, such as computing time and channel usage. In the famous random access protocols (ALOHA) system [1] and its successor 802.11 based WiFi [2], [3] and sensor networks [4], [5] users attempt to access a common single radio channel with a certain probability. The fairness of the competition depends on the strategy used to determine these access issues. Although fairness is provided if the number of users is known, the efficiency of distributed systems remains low due to the random nature of strategies.

Since being proposed by Feynman in 1985, the principles of quantum mechanics in computing and communication systems have been strengthened. In the two main streams, experimental quantum computers [6], [7] raise the expectations that computationally complex problems can be solved quickly, while quantum key distribution (QKD) systems have reached the commercialization phase [8], [9]. QKD implementations can also be assembled from individual components that are available on the market [10]. Another important usage of quantum computing systems is codification, as presented in [11]–[13].

Notably, competition resolution can also be achieved via a central intelligence. For instance, in cellular networks, the base station coordinates the connected entities, providing medium access to them [14]–[16]. Medium access control can be extended to the quantum layer [17].

With the benefit of quantum computing, in this paper, we focus on a common problem: resource access in distributed systems is known to have very low efficiency, i.e., the majority of attempts fail because of colliding entities and resources remain unused when people back off. This problem can be avoided by a coordinating entity that schedules node attempts. However, scenarios exist in which such centralized solutions cannot be applied, even though centralization significantly improves the system's vulnerability. If the controlling node is injured or destroyed, the system would break down.

Quantum mechanical phenomena, known as entanglements, cannot deliver information between distant locations. However, as we demonstrate in the following sections, entanglements can coordinate entities in a distributed manner such that they behave as if a dedicated coordinator is carefully watching them. This process efficiently improves resource access while maintaining the level of reliability.

II. THE SLOTTED-ALOHA REFERENCE PROTOCOL

As a reference, a short overview of the Slotted-ALOHA resource access method used in modeling distributed systems

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is provided [1]. The protocol itself has been widely analyzed under different conditions [18]–[20]. Here, a short summary of the protocol is presented. In slotted-Aloha, the time axis consists of equal-length periods for resource access and intact synchronization among users. Each user i accesses the resource at the beginning of the slot with probability p_i and remains silent with probability $q = 1-p_i$. If at least two users attempt to access the resource in the same slot, a collision occurs and the slot is lost or unused. This loss also occurs if all the nodes remain silent. Resource access is successful only if a single user makes an attempt. Assuming n players operate independently of one another and use the same strategy, i.e., they are cooperative, then the optimization is made from an individual user perspective; therefore, the normalized number of successful resource attempts per slot is represented by a Bernoulli random variable S^i with success probability

$$q_i = p_i(1-p_i)^{(n-1)} \tag{1}$$

and the corresponding expected value is

$$E(S_i) = q_i. \tag{2}$$

We aim to maximize resource access for all users

$$\frac{dE(S_i)}{dp_i} = 0 \Rightarrow p_i = \frac{1}{n} \Rightarrow \max_{p_i} E(S_i) = \frac{1}{n}\left(1-\frac{1}{n}\right)^{n-1} \tag{3}$$

so $S = \overset{n}{\underset{i=1}{*}} S_i$. We then have

$$\max_{p_i} E(S_i) = n\frac{1}{n}\left(1-\frac{1}{n}\right)^{n-1} = \left(1-\frac{1}{n}\right)^{n-1}. \tag{4}$$

The optimal value $p_i = \frac{1}{n}$ means that to maximize resource access, users must be familiar with the total number of players. We then show that as n goes to infinity, we obtain

$$\begin{aligned}\lim_{n\to\infty} \max_{p_i} E(S_i) &= \lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{n-1} \\ &= \lim_{n\to\infty}\frac{1}{1-\frac{1}{n}}\left(1-\frac{1}{n}\right)^{n} = \frac{1}{e}.\end{aligned} \tag{5}$$

If we approach the problem from a game theory perspective, i.e., players optimize individual throughput with the same strategy, then the result $p_i = \frac{1}{n}$ leads to the Pareto optimum of the game since none of the users can increase their payload without hurting another player. For an easier comparison of the proposed quantum entanglement-based method, we emphasize that from an information theoretic perspective, the efficiency of resource access can be measured as the channel capacity of the virtual Slotted-ALOHA channel. At each time point, a slot can be regarded as a channel that is fed by resource attempts, given that collision refers to noise in the channel, as its output is successful resource access. Considering (4) and (5), the capacity varies from $1/2$ to $1/e$ when the number of users goes from $n = 2$ to infinity. Independently of the manner of interpretation, we can conclude that the optimal strategy strongly depends on accurate knowledge of the number of users present in the system. Unfortunately, this information is not available in distributed systems, which substantially decreases the performance.

III. THE QUANTUM ENTANGLEMENT COMPETITION RESOLUTION METHOD

First, we consider the Slotted-ALOHA system with 2 nodes, as it is assumed that the users previously shared an entangled Bell pair.

$$|\beta_{10}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}. \tag{6}$$

If both users measure their own qubit on a computational basis $\{|0\rangle;\ |1\rangle\}$, they obtain opposite values, regarded as tokens. The user who finds $|1\rangle$ is allowed to send his packet during the next time slot, while the other user must remain silent, thereby avoiding a collision. Due to the postulates of quantum mechanics, measurements selected among random users with uniform probability amplitudes keep this competition resolution method uniform and fair, statistically guaranteeing the same number of resources.

To generalize this concept to n nodes, the Bell pair $|\beta_{10}\rangle$ is modified to a specific entangled state called a $|w\rangle$ N qubit of length

$$|w_n\rangle = \frac{1}{\sqrt{n}}\sum_{i=1}^{N}|2^{(n-i)}\rangle. \tag{7}$$

The entanglement we generate to fit our algorithm is a special one. For the generation of the entangled pairs we use Bell pairs, but by no means is that a generalization of Bell pairs. We suppose that such a state $|w_n\rangle$ (W-State)has already been shared among the nodes, that each of the nodes performs the measurement on the qubit, and that one of the n different states is selected collectively. The binary form of state $|2^i\rangle$ contains only one 1 in the ith bit position, yielding access to the resource in the next slot, while any other bit will be 0, which forces the corresponding user to remain silent.

The uniform probability amplitude distribution of this state offers the same access probability $1/n$ for all users. Certain users may require priority over others, which demands higher access probability; therefore, the generalized form of $|w_n\rangle$ is introduced

$$|\tilde{w}_n\rangle = \sum_{i=1}^{n}\varphi_{n,2^{(n-i)}}|2^{(n-i)}\rangle. \tag{8}$$

The probability that node i will access the common resource when n nodes are present in the system is strongly related to the probability amplitude

$$p_{n,i} = \left|\varphi_{n,2^{(n-i)}}\right|^2. \tag{9}$$

Clearly, all probabilities must fulfill the following condition:

$$\sum_{i=1}^{n} p_{n,i} = 1. \tag{10}$$

Generally, if a special node - called coordinator - is present in the system to generate and distribute the qubits, $|\tilde{w}_n\rangle$ resources will be accessed in each time slot without collision. Furthermore, in that case, no superposed states are required. The coordinator node randomly generates a power of 2 in binary form and distributes the bits of the resulting binary vector.

This approach does not work in a distributed environment where nodes are uncoordinated, i.e., no coordinator node is available. Nodes can communicate only with neighbors and typically only one neighbor at each step. We assume that two nodes have already agreed to share a bit pair 01. A newly arriving node would likely to modify this vector to 001, where the 1 in the 3rd position guarantees channel access. To achieve this goal, one node must communicate with two other nodes, which is extremely inefficient in the case of n nodes.

To overcome this limitation, we show how the classic binary vector compares to quantum state $|\tilde{w}_n\rangle$, which is generated if the newly arriving user communicates with only one node in the community.

IV. DISTRIBUTED GENERATION OF THE MULTIPARTY ENTANGLED STATE

In this paper we assume that both classical and quantum channels are idealistic. Also we assume that all devices work as expected. Before going into details lets see a three party example of what this algorithm is capable of.

A. EXAMPLE OF COMPETITION RESOLUTION IN A DISTRIBUTED ENVIRONMENT

This short example of how the communication occurs in our distributed system will be described in details later. Let us suppose that Alice and Bob are communicating and already part of a distributed system. Eve wants to join the system. First Eve needs to gain a part of an entangled system. After bearing one part of an entangled quantum system the distributed system is prepared for communication. The first step is a measurement on the entangled pairs. In step 2 the quantum layer controls the classical layer to obtain optimal performance based on the result of step 1.. Classical communication occurs in step 3.

The number of nodes in distributed systems is continuously changing. New nodes are switched on or arrive in the geographical area (see Fig. 4) as other nodes are switched off or leave the area. To keep state $|\tilde{w}_n\rangle$ updated, each node interested in resource access possesses one of its qubits, but no other user is entangled with it. We introduce the JOIN and LEAVE operators in this section. Furthermore, the probability TRANSFER operator is constructed to guarantee fair resource access among nodes.

B. ADDING A NEW NODE TO THE SYSTEM: THE JOIN OPERATOR

When the system contains only two nodes, the appropriate coordinating state is Bell state $|\beta_{01}\rangle$

$$|\beta_{01}\rangle = |w_2\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}. \tag{11}$$

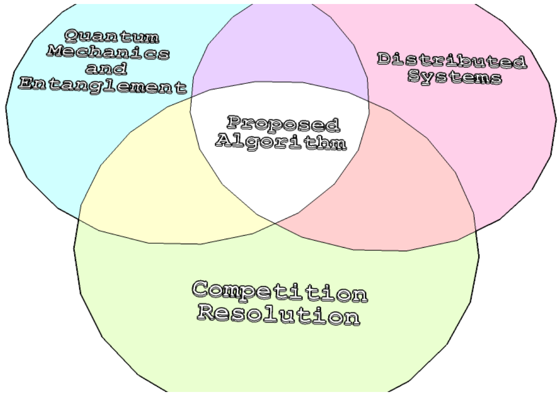

FIGURE 1. Utilization of proposed algorithm.

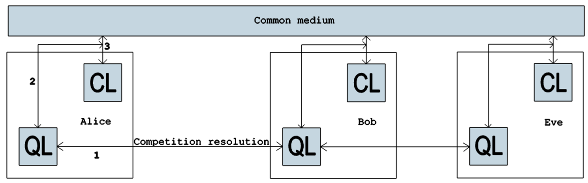

FIGURE 2. Cooperation schematics of classical and quantum channels.

This state can be generated as seen in [21]. The allowance of a nonuniform probability amplitude distribution (11) can generally be written as

$$|\tilde{w}_2\rangle = \varphi_{2,1}|0\overset{2.}{1}\rangle + \varphi_{2,2}|\overset{1.}{1}0\rangle = \begin{bmatrix} 0 \\ \varphi_{2,1} \\ \varphi_{2,2} \\ 0 \end{bmatrix}, \tag{12}$$

where 1. and 2. over the lines refer to the nodes shown in Fig. 4. Since the system is distributed, when a new node wants to join, the node must meet only one of the members (called the 'interface node') of the system.

When a new node successfully joins, the coordination state is

$$|\tilde{w}_3\rangle = \varphi_{3,1}|00\overset{3.}{1}\rangle + \varphi_{3,2}|0\overset{2.}{1}0\rangle + \varphi_{3,4}|\overset{1.}{1}00\rangle. \tag{13}$$

In compliance with the postulates of quantum mechanics, we must derive a unitary transform J that generates $|\tilde{w}_3\rangle$

$$(I \otimes J)\, |\tilde{w}_2\rangle|0\rangle = |\tilde{w}_3\rangle. \tag{14}$$

By solving this inhomogeneous linear equation system, we derive unitary matrix J in the following form

$$J = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -J_{33} & J_{23} & 0 \\ 0 & J_{23} & J_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{15}$$

and the condition

$$|J_{23}|^2 + |J_{33}|^2 = 1. \tag{16}$$

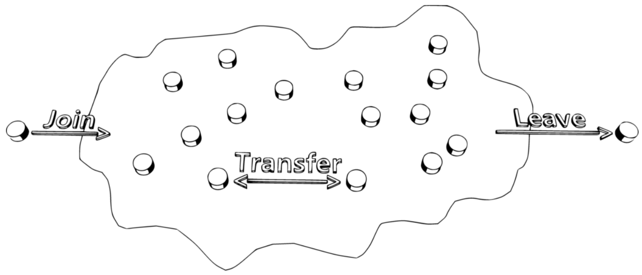

FIGURE 3. Operators in the system: Switched-on nodes or new nodes arriving to the area can JOIN the medium access system to obtain access grants. Leaving or switched-off nodes have to LEAVE the system to return their access probabilities to the community. Finally, active nodes can adjust their access possibilities by TRANSFERring a certain portion of their probabilities.

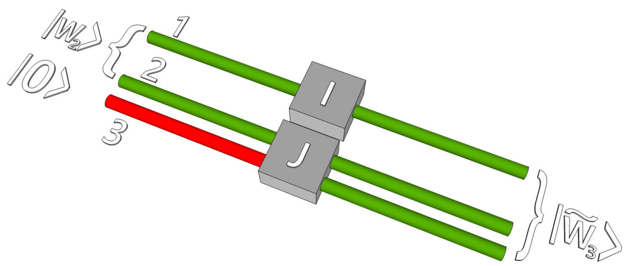

FIGURE 4. New node joins the system: node1 and node2 have already formed a medium access system; node3 connects to node2 and performs the JOIN operation to become part of the system.

must be satisfied. At this point, it is worth emphasizing that both parameters in (16) can be used freely as long as they produce a unitary matrix.

After a new node joins, the system is in the state

$$|\tilde{w}_3\rangle = \varphi_{2,1}J_{23}|00\overset{3.}{1}\rangle + \varphi_{2,1}J_{33}|0\overset{2.}{1}0\rangle + \varphi_{2,2}|\overset{1.}{1}00\rangle. \quad (17)$$

By means of a projective measurement in the computational basis states, we obtain the access probabilities for the 3 users, as shown in [22]

$$\begin{aligned} p_{3,1} &= |\varphi_{2,2}|^2, \\ p_{3,2} &= |\varphi_{2,1}|^2|J_{33}|^2, \\ p_{3,3} &= |\varphi_{2,1}|^2|J_{23}|^2, \end{aligned} \quad (18)$$

from which

$$\begin{aligned} p_{3,2} + p_{3,3} &= |\varphi_{2,1}|^2|J_{23}|^2 + |\varphi_{2,1}|^2|J_{33}|^2 \\ &= |\varphi_{2,1}|^2(|J_{23}|^2 + |J_{23}|^2). \end{aligned} \quad (19)$$

We write the measurement probabilities for $|\tilde{w}_2\rangle$ from (12) in the form of

$$\begin{aligned} p_{2,1} &= |\varphi_{2,2}|^2, \\ p_{2,2} &= |\varphi_{2,1}|^2. \end{aligned} \quad (20)$$

Substituting (20) into (19), we establish the relation among the access probabilities of the 2- and 3-user systems

$$p_{3,2} + p_{3,3} = |\varphi_{2,1}|^2 = p_{2,2}. \quad (21)$$

Therefore, a new node will receive access probability from only the interface node, i.e., from the node to which it attaches. In other words, a node with "questionable ethics" cannot distribute other nodes' probabilities. This restriction is equivalent to the speed of light limits of information distribution.

We let R_J denote the ratio of access probability the interface node shares with the newly joining node:

$$R_J = \frac{p_{3,2}}{p_{3,3}}. \quad (22)$$

For instance, $R = 2$ means that the interface node will keep $2/3$ of its access probability and will give $1/3$ to the joining node.

We determine coefficients J_{23}, J_{33} as functions of the a priori input parameters of $|\tilde{w}_2\rangle$ and the ratio R_J. Substituting (18) into definition (22), it follows that

$$R_J = \frac{|J_{23}|^2}{|J_{33}|^2}. \quad (23)$$

From (18) and applying the unit length of rows and columns of matrix J, we determine how to set up transform J to achieve the desired distribution of access probabilities

$$J_{23} = \sqrt{\frac{1}{R_J + 1}}, \quad J_{33} = \sqrt{\frac{R_J}{R_J + 1}}. \quad (24)$$

The matrix J is as follows:

$$J = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\sqrt{\frac{R_J}{R_J+1}} & \sqrt{\frac{1}{R_J+1}} & 0 \\ 0 & \sqrt{\frac{1}{R_J+1}} & \sqrt{\frac{R_J}{R_J+1}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (25)$$

C. DISCONNECTING: THE LEAVE OPERATOR

When a node leaves the system, the node must pass its access rights (probability) to the common media of an interface node. We must find a transformation that breaks the entanglement between the exiting entity and the rest of the system, returning the measurement probability to the interface node:

$$(I \otimes L)|w_3\rangle = |\tilde{w}_2\rangle|0\rangle. \quad (26)$$

Independent of the number of steps needed to produce $|\tilde{w}_3\rangle$ from $|\tilde{w}_2\rangle$, we can determine one single virtual JOIN operator J that would generate $|\tilde{w}_3\rangle$ from $|\tilde{w}_2\rangle$. Since the evolution of the system obeys unitary operators, it can also be reversed. To leave the distributed network, a node must apply the LEAVE operator L, which is the inverse or, equivalently, adjoint (the conjugate transpose) of the JOIN operator J, i.e., $L = J^{-1} = J^\dagger$. Disconnection is shown in Fig. 5.

In possession of $|\tilde{w}_2\rangle$ and $|\tilde{w}_3\rangle$, we can solve the linear equation system (14) to obtain operator J in the form of (25).

J is Hermitian since it has only real elements and is symmetrical in terms of reflection along the main diagonal; thus, it follows that

$$L = J^{-1} = J^\dagger = J. \quad (27)$$

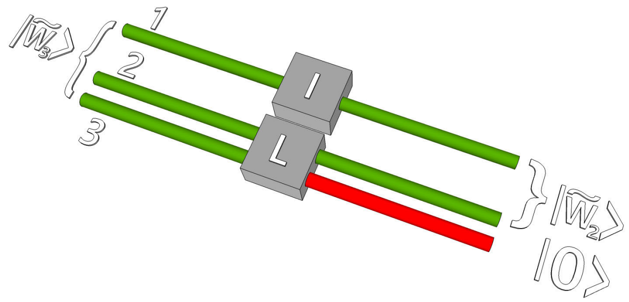

FIGURE 5. A node leaves the system: node1, node2 and node3 have already formed a medium access system, and node3 decides to disconnect. Therefore, node3 returns his access probability to node2 by performing the LEAVE operation.

Matrix L can be written as

$$J = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\sqrt{\dfrac{R_L}{R_L+1}} & \sqrt{\dfrac{1}{R_L+1}} & 0 \\ 0 & \sqrt{\dfrac{1}{R_L+1}} & \sqrt{\dfrac{R_L}{R_L+1}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (28)$$

where $R_L = R_J$ refers to the ratio defined in (22).

To calculate the coefficients of operator L, the leaving node must be familiar with the possessed probability that will be returned to the interface node; this interface node must modify its memory variable and store its access probability according to the value obtained from the exiting node.

D. ADJUSTING PROBABILITIES: THE TRANSFER OPERATOR

In some scenarios, transferring probabilities among nodes in the system is practical, if not actually necessary. To resolve this problem, a third functionality is required for a resource access point to allow nodes to adjust their access probabilities. During probability TRANSFER, two nodes exchange probabilities, e.g., a node possessing more probability gives its excess probability to another node. We call this operator probability TRANSFER and denote it by T.

We must emphasize that if the LEAVE operator is constructed with inappropriate parameters, then only a portion of the entire access probability is returned to the interface node, which will remain in the system with modified probability.

To determine the matrix of operator T, we consider that each node is able to update its own access probability during any of the three operations. In this way, all the probabilities are known when n nodes are present in the system $p_{n,i}, i \in [0, n-1]$. After a new node joins, $n+1$ nodes are present with probabilities $p_{n+1,i}, i \in [0, n]$. Clearly, the new node may receive access probability from any node in the system.

The LEAVE and JOIN operators can be applied sequentially to create the TRANSFER operator, as shown in Fig. 6., i.e., one of the probability exchanging nodes gives its entire probability to the other, which returns a portion of the probability when the first node joins. Thus, node2 operates as an interface node. This approach models the transfer operation and helps to calculate the matrix of T

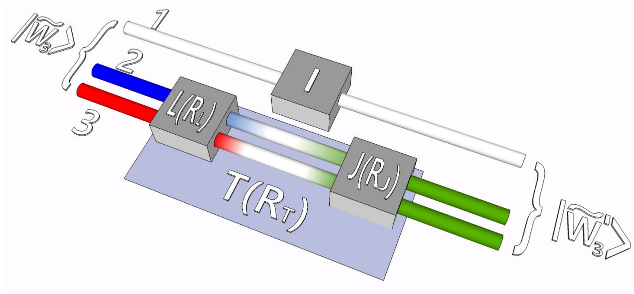

FIGURE 6. Application of the TRANSFER operator: node1, node2 and node3 have already formed a medium access system. Node2 and node3 decide to exchange access probabilities to smooth the overall probability distribution; therefore, they perform the TRANSFER operation.

$$T(R_T) = J(R_J)L(R_L), \quad (29)$$

where the ratio R_T represents the ratio of the probability amplitude of node3 in (13) before and after the probability transfer:

$$R_T = \frac{|\varphi_{3,1}|^2}{|\varphi'_{3,1}|^2}. \quad (30)$$

We assume that the measurement probability transfer occurs from node2 to node3.

From (29), we seek $T = JL$ in the following form:

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & T_{22} & -T_{23} & 0 \\ 0 & T_{23} & T_{22} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (31)$$

Using (29), (30) and (31), the post-transfer state $|\tilde{w_3}'\rangle$ is expressed as

$$|\tilde{w_3}'\rangle = (I \otimes T(R_T))|\tilde{w_3}\rangle = \begin{bmatrix} 0 \\ T_{22}\varphi_{3,1} - T_{23}\varphi_{3,2} \\ T_{23}\varphi_{3,1} + T_{22}\varphi_{3,2} \\ 0 \\ \varphi_{3,4} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (32)$$

Reducing the linear equation system yields

$$\begin{aligned} T_{22}\varphi_{3,1} - T_{23}\varphi_{3,2} &= \sqrt{R_T}\varphi_{3,1}, \\ T_{23}\varphi_{3,1} + T_{22}\varphi_{3,2} &= \sqrt{|\varphi_{3,1}|^2 + |\varphi_{3,2}|^2 - R_T}\varphi_{3,2}. \end{aligned} \quad (33)$$

From (33), the coefficients of matrix T can be obtained

$$\begin{aligned} T_{22} &= \frac{\sqrt{R_T}\varphi_{3,1} + U_{23}\varphi_{3,2}}{\varphi_{3,1}}, \\ T_{23} &= \frac{\varphi_{3,1}\varphi_{3,2}(\sqrt{|\varphi_{3,1}|^2 + |\varphi_{3,2}|^2 - R_T} - \sqrt{R_T})}{(\varphi_{3,1}^2 + \varphi_{3,2}^2)}. \end{aligned} \quad (34)$$

E. GENERALIZATION OF OPERATORS

For the sake of plausible discussion, the JOIN, LEAVE and TRANSFER operators have been introduced for a system containing three nodes in the previous sections. Now, we present how these results can be easily extended to larger systems.

The matrices of these operators have a special structure

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & U_{22} & U_{23} & 0 \\ 0 & U_{32} & U_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{35}$$

as one can observe a special consequence when $I \otimes U$ is applied to input $|\tilde{w}_N\rangle$.

$$\begin{aligned}
&(I \otimes U)|\tilde{w_N}\rangle \\
&= \begin{bmatrix}
1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & U_{22} & U_{23} & 0 & & 0 & 0 & 0 & 0 \\
0 & U_{32} & U_{33} & 0 & & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & & 0 & 0 & 0 & 0 \\
\vdots & & & & \ddots & & & & \vdots \\
0 & 0 & 0 & 0 & & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & & 0 & U_{22} & U_{23} & 0 \\
0 & 0 & 0 & 0 & & 0 & U_{32} & U_{33} & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1
\end{bmatrix} \\
&\times \begin{bmatrix} 0 \\ \varphi_{N,0} \\ \varphi_{N,1} \\ 0 \\ \vdots \\ \varphi_{N,N-1} \\ 0 \\ 0 \\ 0 \end{bmatrix}
= \begin{bmatrix} 0 \\ U_{22}\varphi_{N,0} + U_{23}\varphi_{N,1} \\ U_{32}\varphi_{N,0} + U_{33}\varphi_{N,1} \\ 0 \\ \vdots \\ \varphi_{N,N-1} \\ 0 \\ 0 \\ 0 \end{bmatrix} = |\tilde{w}'_N\rangle.
\end{aligned} \tag{36}$$

The column vector on the right-hand side of (36) highlights that only the probability amplitudes of the last two wires (nodes) are affected by operator U and are connected to it independently of whether $|\tilde{w}_N\rangle$ is in an entangled state.

Based on this observation, since the sequence of nodes can be chosen logically, the system can be virtually arranged so that the actual operation always acts on the last two wires, as depicted in Fig. 9. Therefore, the number of nodes in the system is extended to any number while the results obtained for the 3-node system remain valid.

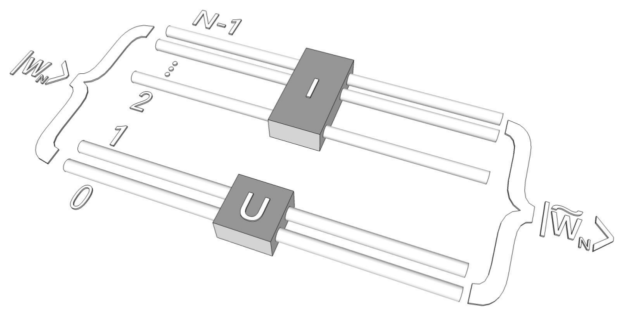

FIGURE 7. Generalization of operators.

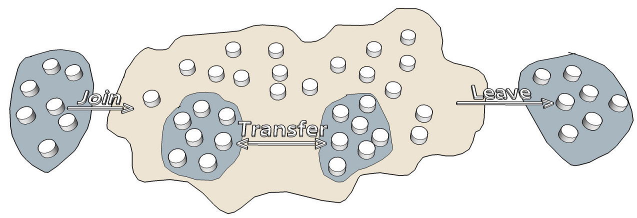

FIGURE 8. Operators acting on more nodes.

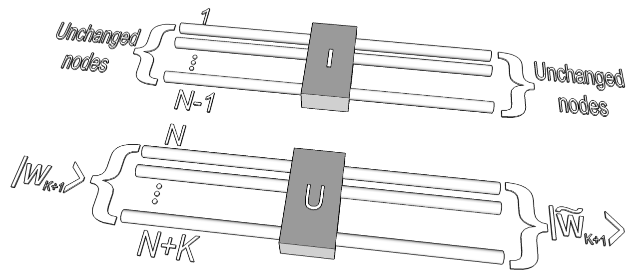

FIGURE 9. Generalized operator acting on K nodes.

To provide fairness in classically distributed systems, individual users may observe access attempts to a common resource. By detecting successful attempts, one can estimate other access probabilities, which are related to the corresponding priority. Clearly, this estimation is fairly coarse. In the case of collisions, identifiers cannot be detected. By contrast, the quantum-assisted MAC can solve this issue. During distributed $|w_N\rangle$ generation, nodes continuously update their access probabilities. Since no collision occurs, each access attempt is detected, and the entire access probability distribution of the system is known for all the nodes.

V. OPERATIONS ON MULTIPLE NODES

Previously, we considered operations on one joining or leaving node. What happens when numerous nodes would like to join or leave the system? For instance, when a group of students leave a seminar and go to another building together. We present how to construct operators that are capable of allowing more than one, i.e., (K) nodes, join (transfer or

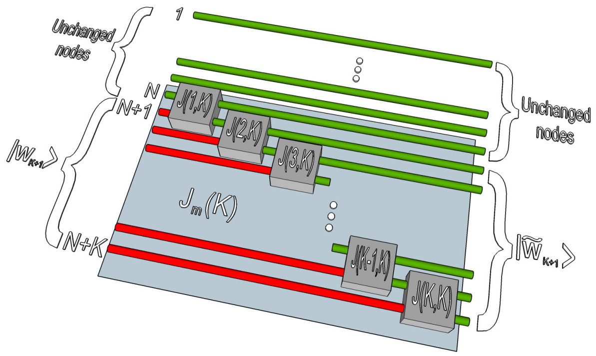

FIGURE 10. Construction method for the multi-JOIN operator for *K* nodes.

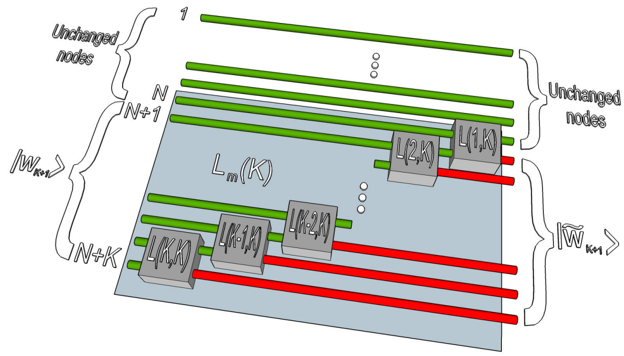

FIGURE 11. Construction method for the multi-LEAVE operator for *K* nodes.

leave). The system is assumed to initially include N nodes and the group that arrives consists of K nodes. To create a multi-operator we consider the following schematics.

To construct the matrix of operator U, the following inhomogeneous linear equation system must be solved, where the input and output superpositions are given as

$$U|w_{K+1}\rangle = |\tilde{w}_{K+1}\rangle. \tag{37}$$

Since U is unitary, i.e., $U^{-1} = U^{\dagger}$, the columns or rows of matrix U must form an orthonormal basis resulting in the conditions

$$\sum_{i=1}^{K} |u_{i,j}|^2 = 1, \tag{38}$$

where $u_{i,j}$ are the coefficients of U, and

$$\sum_{i=1}^{K} u_{i,j} u_{i,l} = 0 \tag{39}$$

for any $j, l \in [1, K]$ and $j \neq l$.

To avoid the exhausting calculations of the above equation system with conditions, we present a construction method that traces the problem back to the preciously discussed 2-qubit systems.

Our first example is the multi-JOIN operator $J_m(K)$, which allows K nodes to join the system in one step.

As depicted in Fig. 10., the matrix of multi-JOIN operator $J_m(K)$ can be constructed from K two-by-two JOIN operators $J(l, K)$. On the bottom green wire is the node already included in the system. The arriving nodes receive transmission probability from this node. The coefficients of $J_m(K)$ can easily be acquired by the appropriate tensor product of the individual $J(l, K)$ and identity matrices.

The construction method for the multi-LEAVE operator, which supports K nodes leaving in one step, is similar to that of the multi-JOIN operator (see Fig. 11.). Individual 2-qubit $L(l, K)$ and identity matrices result in the matrix $L(K)$ by means of the tensor products.

With the definition of the multi-JOIN and multi-LEAVE operators, it is now possible to redistribute access probabilities among any number of nodes. The matrix of any multi-TRANSFER operator can be constructed by means of an appropriate combination of multi-LEAVE and multi-JOIN operators, similarly to the process for 2-qubit systems in Fig.6.

Possession of the probability distribution of the system priorities can be assigned to the nodes. Nodes with higher priorities do not keep their access probabilities when acting as interface nodes or giving a limited amount of access probability during the exchange.

VI. PERFORMANCE EVALUATION

Regarding performance evaluation Sotted Aloha was in our focus as it was one of the first operating distributed systems and is used even today as an appropriate reference for evaluation. The simulation results presented here were generated in a Matlab environment. The framework was developed by us to accomodate high number of simulations within a reasonable time frame. Simulation was executed for the quantum channel as competition resolution occurs there. The goal was to investigate how a distributed system would behave using the novel quantum protocol presented above. This can be used for instance to distributedly coordinate a slotted ALOHA system on a quantum channel (resulting in zero collision on the classical channel). Simulation methods were as follows. Each joining node selects an interface node with a uniform distribution. In the case of probability transfer, the nodes have selected pairs with uniform distribution, i.e., we assume that all nodes align with one another. The number of nodes in the system is set to 10 or 100 to evaluate the performance at different scales. The depicted access probabilities are averaged for 100,000 runs.

Since the proposed method avoids collisions and unused slots, we investigate from the individual user perspective, i.e., fairness and priorities will be discussed.

Initially, the network contains two nodes with uniform access probabilities. Each time a new node joins the system, it connects to any node with a uniform distribution and receives one-half of the interface node's access probability.

The first investigation focuses on a system with a small population. The simulation is run until the total number of entities in the system reaches 12. The access probability of

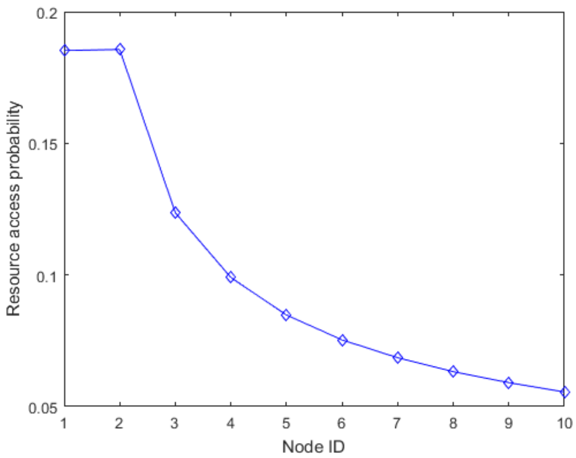

FIGURE 12. Common resource access probability for 10 users: successive JOIN operations result in a nonuniform probability distribution.

individual users is shown in Fig. 12. The x-axis lists the IDs of the nodes in the order in which the entities join the system.

Fig. 12. summarizes our expectations, i.e., medium access in its original form does not provide equal resource access time to the entities.

A. CONTINUOUS REBALANCING

Continuous rebalancing makes access fairer and more efficient, and we improve the operation with a new functionality. When a new node joins the system, the already joined nodes can use this time frame to exchange access probabilities via the TRANSFER operator. The previous simulation scenario is extended during each JOIN operation: all the nodes, except the new and interface node, perform probability TRANSFER with a uniformly selected partner, setting up the same access probability in pairs. We observe the trends by increasing the number of nodes in the system to 100. Fig. 13. highlights how the proposed improvement to the method is an efficient way to provide fair access to everyone, i.e., continuous rebalancing of the system's distribution provides a good extrapolation of the uniform distribution.

If one is interested in an asymptotically uniform access distribution, then the system may continue rebalancing for the defined number of iterations after all nodes have joined.

As depicted in Fig. 14, several additional rebalancing steps will bring the distribution appropriately close to a uniform distribution.

The sample standard deviation is an appropriate metric to measure the difference from the ideal uniform distribution

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N-1}} \tag{40}$$

where μ denotes the mean value of resource access probabilities, N is the number of nodes in the system, and X_i is the resource access probability of the ith node.

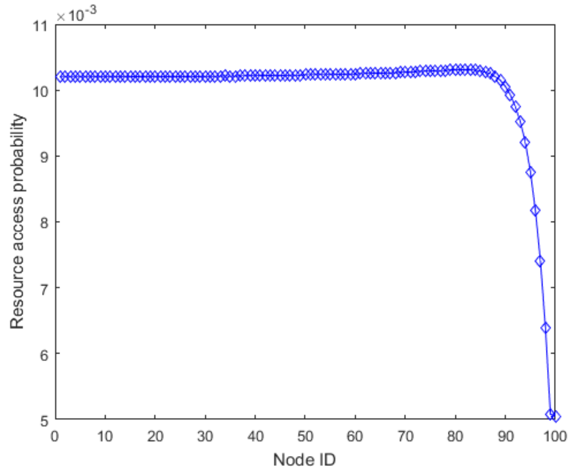

FIGURE 13. Continuous rebalancing for 100 users, averaged over 10000 runs. When nodes perform TRANSFER operations with their neighbors, the overall probability distribution tends toward a uniform distribution very efficiently.

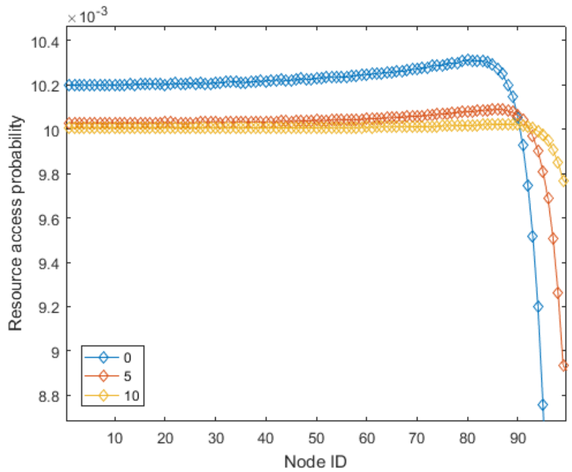

FIGURE 14. Rebalancing after all nodes have joined. When the number of nodes is stationary, nodes can continue the TRANSFER operation to achieve a better approximation of an overall uniform distribution.

In Fig. 15, the sample standard deviations are presented for 10 and 100 nodes as a function of the number of rebalancing steps after all nodes have joined the system. Rebalancing is shown to be an effective tool to ensure fairness.

The application of multi-node operators that act on K nodes, instead of 1 node, in a single step provides further improvement in the efficiency of the entanglement distribution proportional to the value of K.

The discussed simulation results are based on the assumption that any node can join another and that probability transfer can be performed between any two arbitrary nodes. In real scenarios, topological restrictions may restrict this freedom. However, this limitation does not hinder the operation, as only access probability rebalancing needs more time.

It is worth noting that setting up quantum links has extra cost. Upon real life implementation the extra cost of using a quantum channel can be lowered by the gain in performance increase of the classical channel.

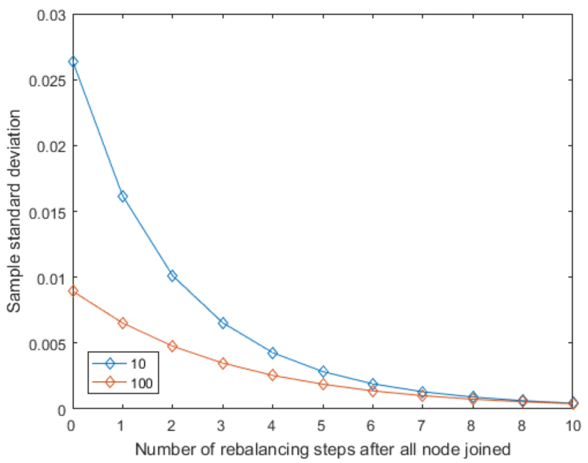

FIGURE 15. Sample standard deviation as a function of the number of rebalancing steps after all nodes have joined the system.

VII. CONCLUSION

State of the art distributed protocols faces channel attempts in random time instances. In the contrary distributed systems have been implemented for a while, in fact Slotted Aloha was on of the first ones. Quantum communication is just in its childhood, i.e., practically there are only 3 well-known protocols (superdense coding, teleportation and large set of quantum key distribution protocols, but these latter ones represent a very specific subset).

Superdense coding can be regarded as entanglement assisted communication of classical information over quantum channel. Teleportation means entanglement assisted communication of quantum information over classical channel.

Our new protocol opens a new track in quantum communications because it delivers classical information over classical channel using entanglement. This entanglement improves the efficiency of communication compared to the classical protocols. It means from information theory point of view that using entanglement may increase the classical capacity of a classical channel! Furthermore, both teleportation and superdense coding apply entangled Bell states while we presented that another family of entangled states - called W-states - can be exploited for communication.

The proposed quantum-assisted method outperforms Slotted-ALOHA and other classically distributed medium access solutions. This method avoids collisions and unused slots, i.e., it guarantees that 100% of the resource attempts will be successful via using quantum methods. In the case of classical solutions, this rate always remains below 100%. For Slotted-ALOHA, this value falls within the region $[0.5..1/e]$, depending on the number of users in an even application of the optimal access strategy.

Our method demonstrates that quantum mechanical effects can be exploited to substantially improve the efficiency of classical distributed systems. We anticipate our assay to be a starting point for more sophisticated and realistic methods. For example, imperfect storage of entangled particles introduces errors and may decrease the efficiency and individual resource access communities may merge into a larger community or be divided into smaller parts.

Another important novelty of the new protocol is related to its scalability. While the efficiency of classical methods depends on the number of users, the presented quantum MAC is invariant to this parameter: the method is always optimal. Although optimal solutions exist for classical distributions, MAC protocols and individual nodes are unaware of the required population size. Only rough estimates are made using exponential (back off) contention windows in WiFi systems [2], [3]. Therefore, the optimal performance, which is worse than that of the quantum-assisted method, can only be approximated.

Extension of the proposed basic protocol to systems with arbitrary random packet arrival times is in the focus of future research.

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