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Global Prescribed-Time Controller Design for *p*-Normal Switched Nonlinear Systems With Dead-Zone Inputs

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ABSTRACT The global stabilization controller design is addressed in this paper for a kind of uncertain switched nonlinear systems (SNSs) in p-normal form. Notably, the study possess two significant features: the system under investigation is subject to the dead-zone inputs, and the system states are expected to be driven to zero in prescribed finite time. To do this, a novel time-varying scaling transformation (TVST), which can be used to change the original prescribed-time stabilization into the asymptotic stabilization of transformed one, is first recommended. Then, with the aid of the common Lyapunov function (CLF) technique, the state feedback stabilization within prescribed-time of SNSs is established by using the recursive idea. Finally, the effectiveness of given control method is confirmed by simulation results of a liquid-level system.

INDEX TERMS Switched nonlinear systems, p-normal form, dead-zone inputs, time-varying scaling transformation, prescribed-time stabilization.

I. INTRODUCTION

Switched nonlinear systems (SNSs), as a kind important hybrid dynamic systems, have received widespread attention thanks to their significant values in practical applications, such as power systems, mechanical systems, aircraft and traffic systems [1]–[3]. Roughly speaking, two switching types, that is, constrained switchings (CSs) and arbitrary switchings (ASs), are considered in the literature. In contrast with stable under CSs, stable under ASs of switched systems is more desirable owing to its theoretical and practical significance. It was affirmed in [4] that the existing of a common Lyapunov function(CLF) for all subsystems is enough to assure the whole switched system being asymptotically stable under ASs. With the aid of this fact, great progress has been made in the asymptotic stabilizing/tracking control of SNSs, see [5]–[14] and the references therein.

On the other hand, pursuing finite-time convergence has become an active research area in recent years because the finite-time stable system owns the nice properties of not only faster response but also better robustness and disturbance rejection. Since the breakthrough work that the Lyapunov finite-time stability theorem of nonlinear systems was established in [15], fruitful achievements have been achieved [16]–[28], just mention a few. Of particular note is that the settling time functions acquired in the aforementioned results rely on initial system conditions. This renders that the settling time may increase unacceptably large as the magnitude of initial conditions becomes large. To overcome this weakness, Andrieu *et al.* in [29] developed the notion of fixed-time stability, which demands that the associated settling time function is regardless of initial system conditions. Under the new framework of fixed-time stability, various works have appeared to address control designs of linear/nonlinear systems. Roughly, the existing approaches on such control designs can be classified into two classes: the bi-limit homogeneous approach [30], [31] and the Lyapunov-based approach [32]–[38]. It is worth pointing out that, in the bi-limit homogeneous approach, the upper bound of the settling time (UBST) function exists but is unknown.

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In the Lyapunov-based approach, (namely, by constructing a positive definite and proper function U to satisfy $\dot{U} \leq -aU^{\beta} - bU^{\eta}$ for some real numbers $a > 0$, $b > 0$, $0 < \beta < 1$, $\eta > 1$, one obtains $T \leq 1/(a - a\beta) + 1/(b\eta - b)$), although its UBST is bounded, it is difficult or even impossible to be prespecified discretionarily according to requirements [39], [40]. This is mostly because the settling time function derived from the Lyapunov-based approach currently relies on a few design parameters, whose choice is not easy to satisfy pregiven settling time.

Nevertheless, prespecifiable settling time is wanted by many practice applications, e.g., missile guidance [41]. Based on scaling the state by a function that increases unboundedly toward the terminal time, a computationally singular controller was proposed to solve prescribed-time regulation of Brunovsky systems in [42]. The extension of this technique was further refined in [43], where a time-varying scaling transformation (TVST) was recommended to overcome the computationally singular problem and the solution to prescribed-time stabilization of SNSs was given. Note that the proposed methodology in [43] requires that the powers of the studied system are strictly equal to 1, which limits its applications because of many practical systems (e.g, the liquid-level system [33]) described by the p-normal SNSs. Moreover, another common drawback of the above-mentioned papers are that the effect of the dead-zone inputs are ignored. However, many actual systems are usually inevitably subject to input dead-zone nonlinearity during operation due to physical limitations of devices. Such undesirable property may significantly degrade the system's performance [44]–[48]. Therefore, the interesting question naturally arises: *For a p-normal SNS with dead-zone inputs, is it possible to design a controller to achieve the prescribed-time stabilization under ASs? If possible, how can one design it?*

This paper tries to solve the problem of global prescribed-time stabilization under ASs for a kind of p-normal SNSs with dead-zone inputs and give an affirmative answer to above question. The significant contributions are highlighted as: (i) Fully considering practical system requirements, global prescribed-time stabilization problem of SNSs with dead-zone inputs is firstly addressed. (ii) A novel TVST is suggested to change the original prescribed-time stabilization problem into the problem of asymptotic stabilization of transformed one. (iii) Under the weaker restricted condition on characterizing system growth, a systematic design method is proposed by delicately utilizing the CLF technique. (iv) As an application of the proposed theoretical result, the problem of prescribed-time control with dead-zone input for a liquid-level system is solved.

The notations adopted in this paper are fairly standard. Specifically, for a vector $z = (z_1, \ldots, z_n)^T \in \mathbb{R}^n$, denote $\bar{z}_m = (z_1, \ldots, z_m)^T \in \mathbb{R}^m$, $m = 1, \ldots, n$, and the function $\lceil z \rceil^{\gamma}$ is defined as $\lceil z \rceil^{\gamma} = \text{sign}(z)|z|^{\gamma}$ where $\text{sign}(\cdot)$ is the signum function.

The rest of the paper is organised as follows. In Section II, the problem formulation and preliminaries of this paper are introduced. In Section III, details on the controller design are presented, followed by the rigorous stability analysis of the CLS. In Section, a practical example of the liquid-level system is provided with simulation studies to validate the efficiency of the proposed method. Finally, the paper is concluded in Section IV.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. PROBLEM FORMULATION

Consider a kind of p-normal SNSs with dead-zone inputs given by

$$\begin{cases} \dot{z}_1 = h_{1\sigma(t)} \lceil z_2 \rceil^{q_{1\sigma(t)}} + g_{1\sigma(t)}(z_1), \\ \dot{z}_2 = h_{2\sigma(t)} \lceil z_3 \rceil^{q_{2\sigma(t)}} + g_{2\sigma(t)}(\bar{z}_2), \\ \quad \vdots \\ \dot{z}_n = h_{n\sigma(t)} \left\lceil D_{\sigma(t)}(u_{\sigma(t)}) \right\rceil^{q_{n\sigma(t)}} + g_{n\sigma(t)}(\bar{z}_n), \end{cases} \tag{1}$$

where $\bar{z}_i \in \mathbb{R}^i$ is the system state (vector) and $\sigma(t) : [0, +\infty) \to \mathbb{S} = \{1, 2, \ldots, s\}$ is the piecewise continuous switching signal. For each $k \in \mathbb{S}$, $h_{ik} \in \mathbb{R}$ is the coefficient of the kth subsystem. For any $k \in \mathbb{S}$ and $i = 1, \ldots, n$, $g_{ik} : \mathbb{R}^i \to \mathbb{R}$ are continuous functions satisfying $g_{ik}(0) = 0$, $q_{ik} \in \mathbb{R}^+$ are the power orders, $u_k \in \mathbb{R}$ are the control inputs, and D_k denote the dead-zone input nonlinearities which can be described as

$$D_k(u_k) = \begin{cases} m_k(t)(u_k - b_k(t)), & u_k \geq b_k(t), \\ 0, & -b_k(t) < u_k < b_k(t), \\ m_k(t)(u_k + b_k(t)), & u_k \leq -b_k(t), \end{cases} \tag{2}$$

where $m_k(t)$ and $b_k(t)$ represent the slopes and the breakpoints of the dead-zone characteristic, respectively. Moreover, it is further supposed that the state of switched system (1) does not jump at the switching instants.

Remark 1: System (1) is indeed a general SNS because its powers p_i's are allowed to take values continuously in interval $(0, +\infty)$. As a result, it includes the extensively studied strict-feedback one as a special case. The practical interest of studying such system can be justified via the example of liquid-level system [33], which cannot be global prescribed-time stabilized by using the existing control methods.

Assumption 1: For given numbers q_{ik}, there exist smooth functions $\varphi_{ik} \geq 0$ and constants $\tau_k \geq 1$ such that

$$|g_{ik}(\bar{z}_i)| \leq \varphi_{ik}(\bar{z}_i) \sum_{j=1}^{i} |z_j|^{\frac{\lambda_i - \tau_k}{\lambda_j}}, \tag{3}$$

where $\lambda_{n+1} = 1$, $q_{ik}\lambda_{i+1} = \lambda_i - \tau_k > 0$, $i = 1, \ldots, n$ and $k \in \mathbb{S}$.

Assumption 2: For $i = 1, \ldots, n$ and $k \in \mathbb{S}$, there exist constants $\underline{h}_{ik} > 0$ and $\overline{h}_{ik} > 0$ such that $\underline{h}_{ik} \leq h_{ik} \leq \overline{h}_{ik}$.

Assumption 3: For any $k \in \mathbb{S}$, there exist constants $\underline{m}$ and $\overline{b}$ such that $0 < \underline{m} \leq m_k(t)$ and $0 < b_k(t) \leq \overline{b}$.

Remark 2: Assumption 1 can be seen as a new homogeneous-growth-like condition because λ_i's defined in it are radically different from the conventional ones used in [18]–[22], [30] where they are recursively defined by $\lambda_1 = 1$, $q_{ik}\lambda_{i+1} = \lambda_i - \tau_k \geq 0$, $i = 1, \ldots, n$. Moreover, it should be pointed out that, it is also fairly common and reasonable in engineering practice by imposing the control coefficients and the unknown dead-zone parameters bounded in Assumption 2 and 3 [45], [46].

B. PRELIMINARIES

Consider the nonlinear system

$$\dot{z} = g(z), \quad z(0) = z_0 \in \mathbb{R}^n \tag{4}$$

where $g : \mathbb{R}^n \to \mathbb{R}^n$ is a (discontinuous) nonlinear vector field that satisfies $\phi(0) = 0$. Moreover, all solutions of system (4) are understood in the Filippov sense [49].

Definition 1 [31]: The origin of system (4) is globally finite-time stable if it is globally asymptotically stable and for any $z_0 \in \mathbb{R}^n$, there is a settling time function $T : \mathbb{R}^n \setminus \{0\} \to (0, \infty)$, which makes every solution $z(t, z_0)$ of (4) satisfying $z(t, z_0) = 0, \forall t \geq T(z_0)$.

Definition 2 [31]: The origin of system (4) is globally fixed-time stable if it is globally finite-time stable and its settling-time function $T(z_0)$ is bounded, that is, there exists a positive constant $T_{\max}$ such that $T(z_0) \leq T_{\max}, \forall z_0 \in \mathbb{R}$.

Definition 3: The origin of system (4) is globally prescribed-time stable if it is globally fixed-time stable and for any prescribed finite time $T_c > 0$, there exists some tunable designing parameters $\theta \in \mathbb{R}^l$ such that $T(z_0) \leq T_c, \forall z_0 \in \mathbb{R}$.

Lemma 1 [50]: For $\zeta \in \mathbb{R}$, $\eta \in \mathbb{R}$, and a constant $q \geq 1$, the following inequalities hold: (i)$|\zeta + \eta|^q \leq 2^{q-1}|\zeta^q + \eta^q|$; (ii)$(|\zeta| + |\eta|)^{1/q} \leq |\zeta|^{1/q} + |\eta|^{1/q} \leq 2^{(q-1)/q}(|\zeta| + |\eta|)^{1/q}$.

Lemma 2 [50]: For positive constants c, d and real-valued function $\delta(\zeta, \eta) > 0$, $|\zeta|^c|\eta|^d \leq \frac{c}{c+d}\delta(\zeta, \eta)|\zeta|^{c+d} + \frac{d}{c+d}\delta^{-c/d}(\zeta, \eta)|\eta|^{c+d}$.

Lemma 3 [51]: If $0 < q \leq 1$ and $a > 0$, then for any $\zeta, \eta \in \mathbb{R}$, one has $|\lceil\zeta\rceil^{aq} - \lceil\eta\rceil^{aq}| \leq 2^{1-q}|\lceil\zeta\rceil^a - \lceil\eta\rceil^a|^q$.

III. PRESCRIBED-TIME STABILIZATION

In this section, a constructive design mechanism of common stabilizer of switched system (1) is established for any given finite settling time $T_c > 0$ under ASs. The design consists of defining such common stabilizer as a piecewise controller. Specially, when $t \in [0, T_c)$ we first construct a non-autonomous controller to drive the state tending to and reaching the origin regardless of the initial conditions within T_c, thereafter we design an autonomous controller to keep the state at the origin.

A. CONTROLLER DESIGN FOR THE CASE OF $t \in [0, T_c)$

Inspired by recent works [38], [39], to shift the original prescribed-time stabilization to the framework of asymptotic stabilization, we introduce the following novel TVST:

$$\zeta_i = \Gamma^{\lambda_i} z_i, \quad i = 1, \ldots, n, \; D_k(v_k) = \Gamma^{\lambda_{n+1}} D_k(u_k), \tag{5}$$

where

$$\Gamma = \frac{T_c}{T_c - t}. \tag{6}$$

Remark 2: It is clear that $\Gamma(\cdot)$ is monotonically increasing on $[0, T_c)$ and satisfies $\Gamma(0) = 1$, $\Gamma(T_c) = +\infty$.

With the aid of (5), switched system (1) can be reinterpreted as

$$\begin{cases} \dot{\zeta}_1 = \Gamma^{\tau_{\sigma(t)}} \left(h_{1\sigma(t)} \lceil \zeta_2 \rceil^{q_{1\sigma(t)}} + \bar{g}_{1\sigma(t)}(\zeta_1)\right), \\ \dot{\zeta}_2 = \Gamma^{\tau_{\sigma(t)}} \left(h_{2\sigma(t)} \lceil \zeta_3 \rceil^{q_{2\sigma(t)}} + \bar{g}_{2\sigma(t)}(\bar{\zeta}_2)\right), \\ \quad \vdots \\ \dot{\zeta}_n = \Gamma^{\tau_{\sigma(t)}} \left(h_{n\sigma(t)} \left\lceil D_{\sigma(t)}(v_{\sigma(t)})\right\rceil^{q_{n\sigma(t)}} + \bar{g}_{n\sigma(t)}(\bar{\zeta}_n)\right), \end{cases} \tag{7}$$

where

$$\bar{g}_{i\sigma(t)}(\bar{\zeta}_i) = \lambda_i \zeta_i \frac{\dot{\Gamma}}{\Gamma^{1+\tau_{\sigma(t)}}} + \Gamma^{\lambda_i - \tau_{\sigma(t)}} g_{i\sigma(t)}(\bar{z}_i), \quad i = 1, \ldots, n, \tag{8}$$

for which we have

Proposition 1: For $i = 1, \ldots, n$ and $k \in \mathbb{S}$, there are smooth functions $\bar{\varphi}_{ik}(\bar{\zeta}_i) \geq 0$ such that

$$|\bar{g}_{ik}(\bar{\zeta}_i)| \leq \bar{\varphi}_{ik}(\bar{\zeta}_i) \sum_{j=1}^{i} |\zeta_j|^{\frac{\lambda_i - \tau_k}{\lambda_j}}. \tag{9}$$

Proof: From the definition of Γ in (6), one has $\dot{\Gamma} = \Gamma^2/T_c$. This further together with (5) and Assumption 1 renders

$$\begin{aligned} |\bar{g}_{ik}(\bar{\zeta}_i)| &= \left|\zeta_i \frac{\lambda_i \dot{\Gamma}}{\Gamma^{1+\tau_k}} + \Gamma^{\lambda_i - \tau_k} g_{ik}(\bar{z}_i)\right| \\ &\leq \lambda_i \Gamma^{1-\tau_k} |\zeta_i| + \left|\Gamma^{\lambda_i - \tau_k} \varphi_{ik} \sum_{j=1}^{i} |z_j|^{\frac{\lambda_i - \tau_k}{\lambda_j}}\right| \\ &\leq \lambda_i \Gamma^{1-\tau_k} |\zeta_i| + \varphi_{ik} \sum_{j=1}^{i} \left|\zeta_j\right|^{\frac{\lambda_i - \tau_k}{\lambda_j}}. \end{aligned} \tag{10}$$

By noting that $\Gamma \geq 1$ for all $t \in [0, T_c)$, one can find the smooth functions $\bar{\varphi}_{ik}(\bar{\zeta}_i) \geq \lambda_i |\zeta_i|^{\tau_k/\lambda_i} + \varphi_{ik}$ such that this proposition is true.

In the sequel, a state feedback asymptotic stabilizing controller of switched system (7) is constructed for the case of $t \in [0, T_c)$ by using the CLF-based recursive technique.

Step 1. Let $\rho \geq \max_{1 \leq i \leq n}\{\lambda_i\}$ being a real number and select the common Lyapunov function V_1 for this step as

$$V_1 = W_1 = \int_0^{\zeta_1} \left\lceil \lceil s \rceil^{\frac{\rho}{\lambda_1}} - 0 \right\rceil^{\frac{2\rho - \lambda_1}{\rho}} ds. \tag{11}$$

Then, for each subsystem k of switched system (7), applying Assumption 1 and (9) gives

$$\begin{aligned} \dot{V}_1 &= \Gamma^{\tau_k} \lceil \pi_1 \rceil^{\frac{2\rho - \lambda_1}{\rho}} \left(h_{1k} \lceil \zeta_2 \rceil^{q_{1k}} + \bar{g}_{1k}\right) \\ &\leq \Gamma^{\tau_k} \Bigg(\lceil \pi_1 \rceil^{\frac{2\rho - \lambda_1}{\rho}} h_{1k}(\zeta_2 \rceil^{q_{1k}} - \lceil \zeta_2^* \rceil^{q_{1k}}) \\ &\quad + h_{1k} \lceil \pi_1 \rceil^{\frac{2\rho - \lambda_1}{\rho}} \lceil \zeta_2^* \rceil^{q_{1k}} + |\pi_1|^{\frac{2\rho - \tau_k}{\rho}} \bar{\varphi}_{1k} \Bigg). \end{aligned} \tag{12}$$

where $\pi_1 = \lceil \zeta_1 \rceil^{\frac{\rho}{\lambda_1}}$ and ζ_2^* is the virtual controller of ζ_2 to be specified.

Take the common ζ_2^* for each subsystem $k \in \mathbb{S}$ as

$$\zeta_2^* = -\lceil \pi_1 \rfloor^{\frac{\lambda_2}{\rho}} \beta_1^{\frac{\lambda_2}{\rho}}(\zeta_1), \tag{13}$$

where

$$\beta_1(\zeta_1) \geq \max_{k\in\mathbb{S}} \left(\frac{n+\bar{\varphi}_{1k}}{\underline{h}_{1k}}\right)^{\frac{\rho}{q_{1k}\lambda_2}}, \tag{14}$$

is a smooth function independent of k, and substituting (13), (14) into (12), one has

$$\begin{aligned}\dot{V}_1 &\leq -n\Gamma^{\tau_k}|\pi_1|^{\frac{2\rho-\tau_k}{\rho}} \\ &\quad + \Gamma^{\tau_k} h_{1k}\lceil \pi_1 \rfloor^{\frac{2\rho-\lambda_1}{\rho}} \left(\lceil \zeta_2 \rfloor^{q_{1k}} - \lceil \zeta_2^* \rfloor^{q_{1k}}\right).\end{aligned} \tag{15}$$

Step 2. Define $\pi_2 = \lceil \zeta_2 \rfloor^{\frac{\rho}{\lambda_2}} - \lceil \zeta_2^* \rfloor^{\frac{\rho}{\lambda_2}}$ and take the common Lyapunov function $V_2 = V_1 + W_2$ with

$$W_2 = \int_{\zeta_2^*}^{\zeta_2} \left\lceil \lceil s \rfloor^{\frac{\rho}{\lambda_2}} - \lceil \zeta_2^* \rfloor^{\frac{\rho}{\lambda_2}} \right\rfloor^{\frac{2\rho-\lambda_2}{\rho}} ds. \tag{16}$$

From the fact that

$$\begin{cases} \dfrac{\partial W_2}{\partial \zeta_2} = \lceil \pi_2 \rfloor^{\frac{2\rho-\lambda_2}{\rho}}, \\ \dfrac{\partial W_2}{\partial \zeta_1} = -\dfrac{2\rho-\lambda_2}{\rho} \dfrac{\partial\left(\lceil \zeta_2^* \rfloor^{\frac{\rho}{\lambda_2}}\right)}{\partial \zeta_1} \\ \qquad \times \displaystyle\int_{\zeta_2^*}^{\zeta_2} \left|\lceil s \rfloor^{\frac{\rho}{\lambda_2}} - \lceil \zeta_2^* \rfloor^{\frac{\rho}{\lambda_2}}\right|^{\frac{\rho-\lambda_2}{\rho}} ds, \end{cases} \tag{17}$$

a direct calculation gives

$$\begin{aligned}\dot{V}_2 &\leq -n\Gamma^{\tau_k}|\pi_1|^{\frac{2\rho-\tau_k}{\rho}} \\ &\quad + \Gamma^{\tau_k} h_{1k}\lceil \pi_1 \rfloor^{\frac{2\rho-\lambda_1}{\rho}} \left(\lceil \zeta_2 \rfloor^{q_{1k}} - \lceil \zeta_2^* \rfloor^{q_{1k}}\right) \\ &\quad + \Gamma^{\tau_k}\left(\frac{\partial W_2}{\partial \zeta_1}(h_{1k}\lceil \zeta_2 \rfloor^{q_{1k}} + \bar{g}_{1k}) \right.\\ &\quad + h_{2k}\lceil \pi_2 \rfloor^{\frac{2\rho-\lambda_2}{\rho}} \left(\lceil \zeta_3 \rfloor^{q_{2k}} - \lceil \zeta_3^* \rfloor^{q_{2k}}\right) \\ &\quad \left. + h_{2k}\lceil \pi_2 \rfloor^{\frac{2\rho-\lambda_2}{\rho}} \lceil \zeta_3^* \rfloor^{q_{2k}} + \lceil \pi_2 \rfloor^{\frac{2\rho-\lambda_2}{\rho}} \bar{g}_{2k}\right),\end{aligned} \tag{18}$$

where ζ_3^* is the virtual controller to be designed later. To proceed, we need to give the upper bound estimates for some terms of (18).

First, based on the definitions of π_j and ζ_j^* ($j = 1, 2$) and Lemma 3, one has

$$\begin{aligned}\left|\lceil \zeta_2 \rfloor^{q_{1k}} - \lceil \zeta_2^* \rfloor^{q_{1k}}\right| &= \left|\left(\lceil \zeta_2 \rfloor^{\frac{\rho}{\lambda_2}}\right)^{\frac{\lambda_2 q_{1k}}{\rho}} - \left(\lceil \zeta_2^* \rfloor^{\frac{\rho}{\lambda_2}}\right)^{\frac{\lambda_2 q_{1k}}{\rho}}\right| \\ &\leq 2^{1-\frac{\lambda_2 q_{1k}}{\rho}} \left|\lceil \zeta_2 \rfloor^{\frac{\rho}{\lambda_2}} - \lceil \zeta_2^* \rfloor^{\frac{\rho}{\lambda_2}}\right|^{\frac{\lambda_2 q_{1k}}{\rho}} \\ &= 2^{1-\frac{\lambda_2 q_{1k}}{\rho}} |\pi_2|^{\frac{\lambda_2 q_{1k}}{\rho}}.\end{aligned} \tag{19}$$

Thus, it is from (19), Assumption 2 and Lemma 2 obtained that

$$\begin{aligned}& h_{1k}\lceil \pi_1 \rfloor^{\frac{2\rho-\lambda_1}{\rho}} \left(\lceil \zeta_2 \rfloor^{q_{1k}} - \lceil \zeta_2^* \rfloor^{q_{1k}}\right) \\ &\leq 2^{1-\frac{\lambda_2 q_{1k}}{\rho}} \bar{h}_{1k}|\pi_1|^{\frac{2\rho-\lambda_1}{\rho}} |\pi_2|^{\frac{\lambda_2 q_{1k}}{\rho}} \\ &\leq \frac{1}{3}|\pi_1|^{\frac{2\rho-\tau_k}{\rho}} + |\pi_2|^{\frac{2\rho-\tau_k}{\rho}} \varpi_{21k},\end{aligned} \tag{20}$$

where $\varpi_{21k} \geq 0$ is a smooth function.

Secondly, from (9) and Lemma 1, one can get

$$\begin{aligned}|\bar{g}_{2k}| &\leq \bar{\varphi}_{2k}\left(|\zeta_1|^{\frac{\lambda_2-\tau_k}{\lambda_1}} + |\zeta_2|^{\frac{\lambda_2-\tau_k}{\lambda_2}}\right) \\ &\leq \bar{\varphi}_{2k}\left(|\pi_1|^{\frac{\lambda_2-\tau_k}{\rho}} + |\pi_2|^{\frac{\lambda_2-\tau_k}{\rho}} + \beta_1^{\frac{\lambda_2-\tau_k}{\rho}}|\pi_1|^{\frac{\lambda_2-\tau_k}{\rho}}\right) \\ &\leq \tilde{\varphi}_{2k}\left(|\pi_1|^{\frac{\lambda_2-\tau_k}{\rho}} + |\pi_2|^{\frac{\lambda_2-\tau_k}{\rho}}\right),\end{aligned} \tag{21}$$

where $\tilde{\varphi}_{2k} = \left(1 + \beta_1^{\frac{\lambda_2-\tau_k}{\rho}}\right)\bar{\varphi}_{2k} \geq 0$ is a smooth function.

Using (21) and Lemma 2 produces

$$\begin{aligned}\lceil \pi_2 \rfloor^{\frac{2\rho-\lambda_2}{\rho}} \bar{g}_{2k} &\leq \lceil \pi_2 \rfloor^{\frac{2\rho-\lambda_2}{\rho}} \tilde{\varphi}_{2k}\left(|\pi_1|^{\frac{\lambda_2-\tau_k}{\rho}} + |\pi_2|^{\frac{\lambda_2-\tau_k}{\rho}}\right) \\ &\leq \frac{1}{3}|\pi_1|^{\frac{2\rho-\tau_k}{\rho}} + |\pi_2|^{\frac{2\rho-\tau_k}{\rho}} \varpi_{22k}.\end{aligned} \tag{22}$$

where $\varpi_{22k} \geq 0$ is a smooth function.

Finally, note that

$$\begin{aligned}&\frac{2\rho-\lambda_2}{\rho}\int_{\zeta_2^*}^{\zeta_2} \left|\lceil s \rfloor^{\frac{\rho}{\lambda_2}} - \lceil \zeta_2^* \rfloor^{\frac{\rho}{\lambda_2}}\right|^{\frac{\rho-\lambda_2}{\rho}} ds \\ &\leq \frac{2\rho-\lambda_2}{\rho}|\pi_2|^{\frac{\rho-\lambda_2}{\rho}}|\zeta_2 - \zeta_2^*| \\ &\leq \frac{2\rho-\lambda_2}{\rho}2^{1-\frac{\lambda_2}{\rho}}|\pi_2|,\end{aligned} \tag{23}$$

and

$$\begin{aligned}\left|\frac{\partial\left(\lceil \zeta_2^* \rfloor^{\frac{\rho}{\lambda_2}}\right)}{\partial \zeta_1}\right| &= \left|\frac{\partial(\beta_1\lceil \pi_1 \rfloor)}{\partial \zeta_1}\right| \\ &\leq \left|\frac{\partial \beta_1}{\partial \zeta_1}\right||\pi_1| + \frac{\rho}{\lambda_1}\beta_1|\pi_1|^{\frac{\rho-\lambda_1}{\rho}} \\ &\leq |\pi_1|^{\frac{\rho-\lambda_1}{\rho}}\gamma_2,\end{aligned} \tag{24}$$

where $\gamma_2 \geq 0$ is a smooth function.

Therefore, in the light of (21), (23), (24) and Lemma 2, one concludes that

$$\begin{aligned}&\frac{\partial W_2}{\partial \zeta_1}(h_{1k}\lceil \zeta_2 \rfloor^{q_{1k}} + \bar{g}_{1k}) \\ &\leq \frac{2\rho-\lambda_2}{\rho}\int_{\zeta_2^*}^{\zeta_2} \left|\lceil s \rfloor^{\frac{\rho}{\lambda_2}} - \lceil \zeta_2^* \rfloor^{\frac{\rho}{\lambda_2}}\right|^{\frac{\rho-\lambda_2}{\rho}} ds \\ &\quad \times \left|\frac{\partial\left(\lceil \zeta_2^* \rfloor^{\frac{\rho}{\lambda_2}}\right)}{\partial \zeta_1}\right|(h_{1k}\lceil \zeta_2 \rfloor^{q_{1k}} + \bar{g}_{1k}) \\ &\leq \frac{1}{3}|\pi_1|^{\frac{2\rho-\tau_k}{\rho}} + |\pi_2|^{\frac{2\rho-\tau_k}{\rho}}\varpi_{23k},\end{aligned} \tag{25}$$

where $\varpi_{23k} \geq 0$ is a smooth function.

Substituting (20), (22) and (25) into (19) yields

$$\dot{V}_2 \leq -(n-1)\Gamma^{\tau_k}|\pi_1|^{\frac{2\rho-\tau_k}{\rho}}$$

$$+\Gamma^{\tau_k} h_{2k}\lceil\pi_2\rceil^{\frac{2\rho-\lambda_2}{\rho}}\left(\lceil\zeta_3\rceil^{q_{2k}}-\lceil\zeta_3^*\rceil^{q_{2k}}\right)$$
$$+\Gamma^{\tau_k}\left(h_{2k}\lceil\pi_2\rceil^{\frac{2\rho-\lambda_2}{\rho}}\lceil\zeta_3^*\rceil^{q_{2k}}\right.$$
$$\left.+(\varpi_{21k}+\varpi_{22k}+\varpi_{23k})|\pi_2|^{\frac{2\rho-\tau_k}{\rho}}\right). \quad (26)$$

Then, for each subsystem $k\in\mathbb{S}$, one can design the common virtual controller

$$\zeta_3^*=-\lceil\pi_2\rceil^{\frac{\lambda_3}{\rho}}\beta_2^{\frac{\lambda_3}{\rho}}(\bar{\zeta}_2), \quad (27)$$

where the smooth function β_2 is independent of k and satisfies

$$\beta_2(\bar{\zeta}_2)\geq\max_{k\in\mathbb{S}}\left(\frac{n-1+\varpi_{21k}+\varpi_{22k}+\varpi_{23k}}{\underline{h}_{2k}}\right)^{\frac{\rho}{q_{2k}\lambda_3}}, \quad (28)$$

such that

$$\dot{V}_2\leq-(n-1)\Gamma^{\tau_k}\left(|\pi_1|^{\frac{2\rho-\tau_k}{\rho}}+|\pi_2|^{\frac{2\rho-\tau_k}{\rho}}\right)$$
$$+\Gamma^{\tau_k}h_{2k}\lceil\pi_2\rceil^{\frac{2\rho-\lambda_2}{\rho}}\left(\lceil\zeta_3\rceil^{q_{2k}}-\lceil\zeta_3^*\rceil^{q_{2k}}\right). \quad (29)$$

Step i ($i=3,\ldots,n$)**.** In this step, the following property is obtained.

Proposition 2: Suppose at step $i-1$, there are a C^1 common Lyapunov function V_{i-1}, and a row of C^0 common virtual controllers $\zeta_1^*,\ldots,\zeta_i^*$ defined by

$$\begin{aligned}
\zeta_1^*&=0, & \pi_1&=\lceil\zeta_1\rceil^{\frac{\rho}{\lambda_1}}-\lceil\zeta_1^*\rceil^{\frac{\rho}{\lambda_1}},\\
\zeta_2^*&=-\lceil\pi_1\rceil^{\frac{\lambda_2}{\rho}}\beta_1^{\frac{\lambda_2}{\rho}}(\zeta_1), & \pi_2&=\lceil\zeta_2\rceil^{\frac{\rho}{\lambda_2}}-\lceil\zeta_2^*\rceil^{\frac{\rho}{\lambda_2}},\\
&\vdots & &\vdots\\
\zeta_i^*&=-\lceil\pi_{i-1}\rceil^{\frac{\lambda_i}{\rho}}\beta_{i-1}^{\frac{\lambda_i}{\rho}}(\bar{\zeta}_{i-1}), & \pi_i&=\lceil\zeta_i\rceil^{\frac{\rho}{\lambda_i}}-\lceil\zeta_i^*\rceil^{\frac{\rho}{\lambda_i}}, \quad (30)
\end{aligned}$$

with $\beta_1>0,\ldots,\beta_{i-1}>0$, being smooth and independent of k, such that

$$\dot{V}_{i-1}\leq-(n-i+2)\Gamma^{\tau_k}\sum_{j=1}^{i-1}|\pi_j|^{\frac{2\rho-\tau_k}{\rho}}+\Gamma^{\tau_k}h_{i-1,k}\lceil\pi_{i-1}\rceil^{\frac{2\rho-\lambda_{i-1}}{\rho}}$$
$$\times\left(\lceil\zeta_i\rceil^{q_{i-1,k}}-\lceil\zeta_i^*\rceil^{q_{i-1,k}}\right). \quad (31)$$

Then the *ith* common Lyapunov function $V_i=V_{i-1}+W_i$ with

$$W_i=\int_{\zeta_i^*}^{\zeta_i}\left\lceil\lceil s\rceil^{\frac{\rho}{\lambda_i}}-\lceil\zeta_i^*\rceil^{\frac{\rho}{\lambda_i}}\right\rceil^{\frac{2\rho-\lambda_i}{\rho}}ds, \quad (32)$$

is C^1, positive definite and proper, and there is a C^0 common state feedback controller

$$\zeta_{i+1}^*=-\beta_i^{\frac{\lambda_{i+1}}{\rho}}(\bar{\zeta}_i)\lceil\pi_i\rceil^{\frac{\lambda_{i+1}}{\rho}}, \quad (33)$$

such that

$$\dot{V}_i\leq-(n-i+1)\Gamma^{\tau_k}\sum_{j=1}^{i}|\pi_j|^{\frac{2\rho-\tau_k}{\rho}}$$
$$+\Gamma^{\tau_k}h_{ik}\lceil\pi_i\rceil^{\frac{2\rho-\lambda_i}{\rho}}\left(\lceil\zeta_{i+1}\rceil^{q_{ik}}-\lceil\zeta_{i+1}^*\rceil^{q_{ik}}\right). \quad (34)$$

Proof: See the Appendix.

Step n. Choose

$$V_n=\sum_{j=1}^{n}W_j=\sum_{j=1}^{n}\int_{\zeta_j^*}^{\zeta_j}\left\lceil\lceil s\rceil^{\frac{\rho}{\lambda_j}}-\lceil\zeta_j^*\rceil^{\frac{\rho}{\lambda_j}}\right\rceil^{\frac{2\rho-\lambda_j}{\rho}}ds. \quad (35)$$

Then, the previous inductive steps indicate that there exists an expected common dead-zone output

$$\zeta_{n+1}^*=-\lceil\pi_n\rceil^{\frac{\lambda_{n+1}}{\rho}}\beta_n^{\frac{\lambda_{n+1}}{\rho}}(\bar{\zeta}_n), \quad (36)$$

such that

$$\begin{aligned}
\dot{V}_n&\leq-\Gamma^{\tau_k}\sum_{j=1}^{n}|\pi_j|^{\frac{2\rho-\tau_k}{\rho}}\\
&\quad+\Gamma^{\tau_k}h_{nk}\lceil\pi_n\rceil^{\frac{2\rho-\lambda_n}{\rho}}\left(\lceil D_k(v_k)\rceil^{q_{nk}}-\lceil\zeta_{n+1}^*\rceil^{q_{nk}}\right)\\
&=-\Gamma^{\tau_k}\sum_{j=1}^{n}|\pi_j|^{\frac{2\rho-\tau_k}{\rho}}\\
&\quad+\Gamma^{\tau_k}h_{nk}\lceil\pi_n\rceil^{\frac{2\rho-\lambda_n}{\rho}}\\
&\quad\times\left(\lceil\Gamma^{\lambda_{n+1}}D_k(u_k)\rceil^{q_{nk}}-\lceil\zeta_{n+1}^*\rceil^{q_{nk}}\right). \quad (37)
\end{aligned}$$

Thus, from Assumption 3, one can design the common state feedback control u as

$$u=u_{com}=\begin{cases}\dfrac{\zeta_{n+1}^*}{\underline{m}\Gamma^{\lambda_{n+1}}}+\bar{b}, & \zeta_{n+1}^*>0,\\ 0, & \zeta_{n+1}^*=0,\\ \dfrac{\zeta_{n+1}^*}{\underline{m}\Gamma^{\lambda_{n+1}}}-\bar{b}, & \zeta_{n+1}^*<0,\end{cases} \quad (38)$$

which renders for any $k\in\mathbb{S}$, the inequality (39), as shown at the bottom of the next page holds. By noting $-\lceil\pi_n\rceil^{\frac{2\rho-\lambda_n}{\rho}}\lceil\zeta_{n+1}^*\rceil^{q_{nk}}\geq0$, one gets

$$\dot{V}_n\leq-\Gamma^{\tau_k}\sum_{j=1}^{n}|\pi_j|^{\frac{2\rho-\tau_k}{\rho}}\leq-\sum_{j=1}^{n}|\pi_j|^{\frac{2\rho-\tau_k}{\rho}}. \quad (40)$$

As a result, the following result is obtained.

Theorem 1: For switched system (1) with Assumptions 2.1-2.3, the common state feedback controller u consisting of (30), (36) and (38) drives the state of the CLS to zero within prescribed finite time $T_c>0$ under ASs.

Proof: Proposition 3.2 indicates that V_n is positive definite and proper. Therefore, by (40) and Lemma 4.3 in [52], there are class $\mathcal{K}_\infty$ functions η_1, η_2 and η_3 such that

$$\eta_1(|\zeta|)\leq V_n(\zeta)\leq\eta_2(|\zeta|), \quad (41)$$
$$\dot{V}_n\leq-\eta_3(|\zeta|), \quad (42)$$

which means that $\zeta(t)$ on $[0,T_c)$ is globally asymptotically convergent and bounded.

On the other side, the TVST (5) gives

$$z_i(t)=\Gamma^{-\lambda_i}(t)\zeta_i(t)=\left(\frac{T_c-t}{T_c}\right)^{\lambda_i}\zeta_i(t),\quad i=1,\ldots,n. \quad (43)$$

Consequently, one further has

$$\lim_{t\to T_c} z_i(t)=\lim_{t\to T_c}\left(\frac{T_c-t}{T_c}\right)^{\lambda_i}\zeta_i(t)=0,\quad i=1,\ldots,n. \quad (44)$$

Thus, the proof is completed.

B. CONTROLLER DESIGN FOR $t \geq T_c$ AND MAIN RESULT

Since the common state feedback controller has been designed in the above subsection to drive the state of the CLS to the origin in prescribed finite time $T_c > 0$ under ASs. Consequently, next we are only to design a controller such that the state is maintained at the origin for $t \geq T_c$.

By the solution properties of existence and continuation, one easily obtained that $z(T_c) = 0$. Moreover, observe that the original system (1) and the transformed system (7) have the similar structure except the time-varying control coefficient Γ^{τ_k}. Therefore, by letting $\Gamma = 1$, one can design a new common state feedback controller u consisting of (30), (36) and (38), to maintain the state at the origin for $t \geq T_c$.

Till now, it is completed the design of prescribed-time stabilizer for switched system (1). Accordingly the following theorem is stated to sum up the main results of this paper.

Theorem 2: Considering switched system (1) with Assumptions 2.1–2.3, if the common state feedback controller

$$u = u_{com} = \begin{cases} \dfrac{\zeta^*_{n+1}}{\underline{m}\Gamma^{\lambda_{n+1}}} + \bar{b}, & \zeta^*_{n+1} > 0, \\ 0, & \zeta^*_{n+1} = 0, \\ \dfrac{\zeta^*_{n+1}}{\underline{m}\Gamma^{\lambda_{n+1}}} - \bar{b}, & \zeta^*_{n+1} < 0, \end{cases} \quad (45)$$

with

$$\Gamma_1 = \begin{cases} \dfrac{T_c}{T_c - t}, & 0 \leq t < T_c, \\ 1, & t \geq T_c, \end{cases} \quad (46)$$

$$\zeta^*_{n+1} = -\lceil \pi_n \rfloor^{\frac{\lambda_{n+1}}{\rho}} \beta_n^{\frac{\lambda_{n+1}}{\rho}}(\bar{\zeta}_n), \quad (47)$$

is applied, then the origin of the CLS is globally prescribed-time stable under ASs.

Proof: From the monotonous growth property of $\Gamma(t) = T_c/(T_c - t)$ and the asymptotical convergent of $\zeta(t)$ for all $t \in [0, T_c)$, one has

$$|z(t)| \leq |\zeta(t)| \leq |\zeta(0)| = |z(0)|. \quad (48)$$

This together with $z(t) = 0$ for any $t \in [T_c, +\infty)$ renders

$$|z(t)| \leq |z(0)|, \quad t \geq 0, \quad (49)$$

in other words, the origin of the CLS is globally Lyapunov stable.

With this and the global prescribed-time convergent of the CLS given in Theorem 1 in mind, it is straightforward from Definition 3 that the origin of the CLS is globally prescribed-time stable. This completes the proof.

Remark 3: To further stress the contributions of this paper, we offer some comparison with the existing related results in TABLE 1.

TABLE 1. Qualitative comparison with the existing related results.

References	Power orders	Involving input dead-zone	Convergence time
[19]	$p_{ik} \geq 1$	No	Finite-time
[33]	$p_{ik} > 0$	No	Fixed-time
[43]	$p_{ik} = 1$	No	Prescribed-time
This paper	$p_{ik} > 0$	Yes	Prescribed-time

IV. AN APPLICATION EXAMPLE

To verify the applicability of the proposed control scheme, we consider a liquid-level system the dynamics of which are represented by

$$\begin{aligned} C_1\dot{H}_1 &= Q_1 \\ C_2\dot{H}_2 &= Q - Q_1 - Q_2 \\ Q_1 &= \begin{cases} \alpha_1\sqrt{2g|H_2 - H_1|}, & H_2 \geq H_1, \\ -\alpha_2\sqrt{2g|H_2 - H_1|}, & H_2 < H_1, \end{cases} \\ Q_2 &= \alpha_3\sqrt{2gH_2}. \end{aligned} \quad (50)$$

where the physical meanings of system parameters can be found in [33].

By introducing the variable change

$$z_1 = H_1 - H, \quad z_2 = H_2 - H_1, \; u = \frac{Q}{C_2} - \frac{\alpha_3\sqrt{2gH}}{C_2}, \quad (51)$$

and taking into the presence of the input dead-zone nonlinearity account, the dynamics of (50) can be modelled as the following switched p-normal form:

$$\begin{aligned} \dot{z}_1 &= h_{1\sigma(t)}\lceil z_2 \rfloor^{\frac{1}{2}}, \\ \dot{z}_2 &= D(u) + g_{2\sigma(t)}(\bar{z}_2), \end{aligned} \quad (52)$$

where $\sigma(t) : [0, +\infty) \to \{1, 2\}$, $h_{1\sigma(t)} = \frac{\alpha_{\sigma(t)}\sqrt{2g}}{C_1}$ and $g_{2\sigma(t)}(\bar{z}_2) = -\frac{C_1}{C_2}h_{1\sigma(t)}\lceil z_2 \rfloor^{\frac{1}{2}} - \frac{\alpha_3\sqrt{2g}}{C_2}\lceil z_1 + z_2 + H \rfloor^{\frac{1}{2}} + \frac{\alpha_3\sqrt{2g}}{C_2}\lceil H \rfloor^{\frac{1}{2}}$, D denotes the output of dead-zone input nonlinearity described by (2) with $m_k = 1 + 0.2\sin t$,

$$\lceil \Gamma^{\lambda_{n+1}} D_k(u_k) \rfloor^{q_{nk}} - \lceil \zeta^*_{n+1} \rfloor^{q_{nk}} = \begin{cases} \left(\Gamma^{\lambda_{n+1}} m_k \left(\dfrac{\zeta^*_{n+1}}{\underline{m}\Gamma^{\lambda_{n+1}}} + \bar{b} - b_k\right)\right)^{q_{nk}} - \zeta^{*q_{nk}}_{n+1} \geq 0, & \zeta^*_{n+1} > 0, \\ 0, & \zeta^*_{n+1} = 0, \\ -\left(-\Gamma^{\lambda_{n+1}} m_k \left(\dfrac{\zeta^*_{n+1}}{\underline{m}\Gamma^{\lambda_{n+1}}} - \bar{b} + b_k\right)\right)^{q_{nk}} + (-\zeta^*_{n+1})^{q_{nk}} \leq 0, & \zeta^*_{n+1} < 0. \end{cases} \quad (39)$$

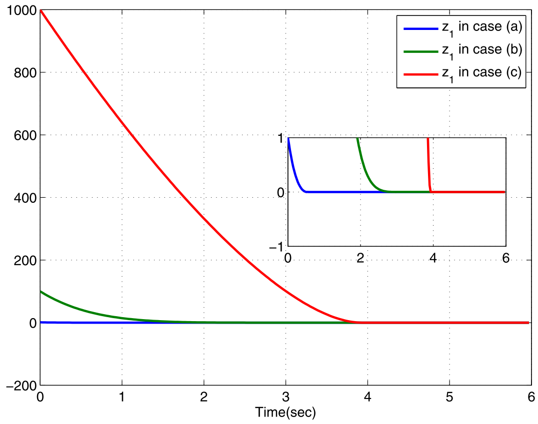

FIGURE 1. Trajectories of state z_1 under different initial conditions of case (a), case (b) and case (c).

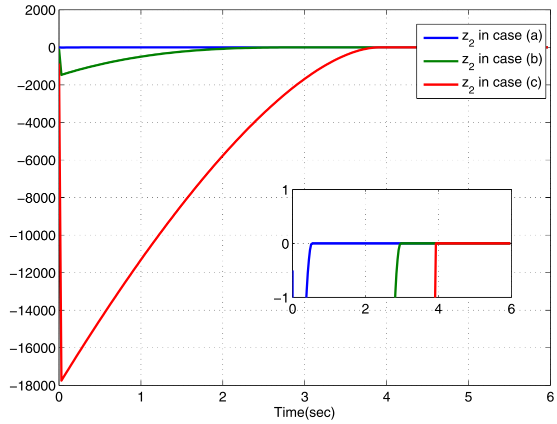

FIGURE 2. Trajectories of state z_2 under different initial conditions of case (a), case (b) and case (c).

$b_k = 0.4 + 0.1\cos t$ respectively. Based on Lemma 3, it is easily verified that

$$\begin{aligned} |g_{2k}| &\le \frac{C_1}{C_2} h_{1k}|z_2|^{\frac{1}{2}} + \frac{\alpha_3\sqrt{2g}}{C_2} \\ &\quad \times \left| \lceil z_1 + z_2 + H \rceil^{\frac{1}{2}} - \lceil H \rceil^{\frac{1}{2}} \right| \\ &\le \left(\frac{\alpha_k\sqrt{2g}}{C_2} + \frac{\alpha_3\sqrt{2g}}{C_2} \right) \left(|z_1|^{\frac{1}{2}} + |z_2|^{\frac{1}{2}} \right). \end{aligned} \tag{53}$$

As a result, Assumptions 2.1–2.3 hold for $\tau_1 = \tau_2 = 1$, $\lambda_1 = \lambda_2 = 2$, $\lambda_3 = 1$, $\underline{m} = 0.8$, $\overline{b} = 0.5$, $\underline{h}_{1k} = \overline{h}_{1k} = \underline{h}_{2k} = \overline{h}_{2k} = \frac{\alpha_k\sqrt{2g}}{C_1}$, $\varphi_{21} = \frac{\sqrt{2g}}{C_2}(\alpha_1 + \alpha_3)(1 + z_1^2)^{\frac{1}{2}}$ and $\varphi_{22} = \frac{\sqrt{2g}}{C_2}(\alpha_2 + \alpha_3)(1 + z_1^2)^{\frac{1}{2}}$.

Introducing $\zeta_i = \Gamma_1^{\lambda_i} z_i$, $i = 1, 2$ with

$$\Gamma_1 = \begin{cases} \dfrac{T_c}{T_c - t}, & 0 \le t < T_c, \\ 1, & t \ge T_c, \end{cases} \tag{54}$$

and taking $\rho = 2$, $\underline{h} = \min_{k=1,2}\{\underline{h}_{1k}, \underline{h}_{2k}\}$, $\overline{h} = \max\{\overline{h}_{1k}, \overline{h}_{2k}\}$ and $\bar{\varphi}_2 = \max_{k=1,2}(\lambda_2(1 + \zeta_2^2)^{\frac{1}{2}}/T_c + \varphi_{2k})$, according to Theorem 2 one can design a common state feedback controller

$$u = \begin{cases} \dfrac{\zeta_3^*}{\underline{m}\Gamma_1^{\lambda_3}} + \overline{b}, & \zeta_3^* > 0, \\ 0, & \zeta_3^* = 0, \\ \dfrac{\zeta_3^*}{\underline{m}\Gamma_1^{\lambda_3}} - \overline{b}, & \zeta_3^* < 0, \end{cases} \tag{55}$$

$$\zeta_3^* = -(1 + \psi_{21} + \psi_{22} + \psi_{23})\lceil \pi_2 \rceil^{\frac{1}{2}}, \tag{56}$$

with $\beta_1 = (2 + \frac{2}{T_c}(1 + \zeta_1^2)^{\frac{1}{2}})/\underline{h}$ if $t \in [0, T_c)$ and $\beta_1 = 2/\underline{h}$ if $t \in [T_c, +\infty)$, $\pi_2 = \zeta_2 - \zeta_2^*$, $\zeta_2^* = -\beta_1\zeta_1^2$, $\tilde{\varphi}_2 = (1 + \beta_1^{\frac{1}{2}})\bar{\varphi}_2$, $\psi_{21} = 3.7712\, d_1^{\frac{3}{2}}$, $\psi_{22} = 0.6667\tilde{\varphi}_2^{\frac{3}{2}} + \tilde{\varphi}_2$, $\psi_{23} = |\frac{\partial\zeta_2^*}{\partial\zeta_1}\overline{h}| + 0.6667|\frac{\partial\zeta_2^*}{\partial\zeta_1}|^3(\overline{h}\beta_1^{\frac{1}{2}} + \frac{2}{T_c}(1+\zeta_1^2)^{\frac{1}{2}})^3$, which renders the switched system (52) globally prescribed-time stable under ASs.

For the simplicity, select the system parameters as $H = 100$cm, $g = 9.8\text{m/s}^2$, $C_1 = C_2 = \sqrt{2g} = 4.427\text{cm}^2$,

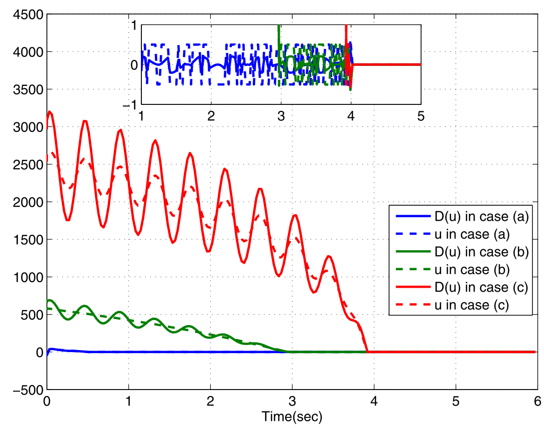

FIGURE 3. Trajectories of dead-zone input u and output $D(u)$ under different initial conditions of case (a), case (b) and case (c).

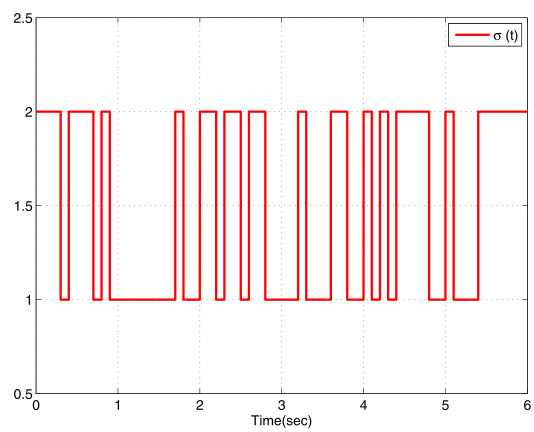

FIGURE 4. Switching signal $\sigma(t)$.

$\alpha_1 = 1.25\text{cm}^2$, $\alpha_2 = 1\text{cm}^2$ and $\alpha_3 = 0.25\text{cm}^2$ and the prescribed time as $T_c = 4s$. For different initial conditions: (a) $(z_1(0), z_2(0)) = (1, -0.5)$, (b) $(z_1(0), z_2(0)) = (100, -50)$ and (c) $(z_1(0), z_2(0)) = (1000, -500)$, FIGURES 1–3 are given to exhibit state trajectories of the closed-loop switched system (52)–(56), under a randomly produced switching signal exhibiting in FIGURE 4. It can be clearly observed that the convergence time of the switched system maintains below the prescribed time $4s$ in spite of the initial value increasing rapidly, which confirms the effectiveness of the control method.

V. CONCLUSION

In this paper, we discover a novel TVST which can change the original problem of prescribed-time stabilization into the asymptotic stabilization of transformed one, and provide a constructive solution to prescribed-time stabilization of SNSs in p-normal form. A practical application to prescribed-time control of a liquid-level system is provided to show the validity of the proposed method. In our future work, we will explore that how achieve the prescribed-time stabilization task by employing the switched Lyapunov function method for want of lower conservatism.

APPENDIX

Proof of Proposition 2: Firstly, some simple derivations give

$$\begin{cases} \dfrac{\partial W_i}{\partial \zeta_i} = \lceil \pi_i \rfloor^{\frac{2\rho-\lambda_i}{\rho}}, \\ \dfrac{\partial W_i}{\partial \zeta_j} = -\dfrac{2\rho-\lambda_i}{\rho} \dfrac{\partial \left(\lceil \zeta_i^* \rfloor^{\frac{\rho}{\lambda_i}}\right)}{\partial \zeta_j} \\ \times \displaystyle\int_{\zeta_i^*}^{\zeta_i} \left| \lceil s \rfloor^{\frac{\rho}{\lambda_i}} - \lceil \zeta_i^* \rfloor^{\frac{\rho}{\lambda_i}} \right|^{\frac{\rho-\lambda_i}{\rho}} ds, \end{cases} \tag{57}$$

for $j = 1, \ldots, i-1$. By $\rho \geq \max_{1\leq i\leq n}\{\lambda_i\}$ and $\beta_j(\cdot)$ being smooth, it is obvious that W_i, and also V_i is C^1.

Second, similar to the one in [46] using the idea of the classified discussion, it can be showed

$$W_j \geq C_j |\zeta_j - \zeta_j^*|^{\frac{\rho-\lambda_j}{\rho}}, \tag{58}$$

is true for some constant $C_j > 0$.

Therefore, one has

$$V_i = V_{i-1} + W_i \geq V_{i-1} + C_i |\zeta_i - \zeta_i^*|^{\frac{\rho-\lambda_i}{\rho}}, \tag{59}$$

and thus V_i is positive definite and proper.

Finally, we prove inequality (34). From (31) and (57), it follows that

$$\begin{aligned} \dot{V}_i \leq& -(n-i+2)\Gamma^{\tau_k} \sum_{j=1}^{i-1} |\pi_j|^{\frac{2\rho-\tau_k}{\rho}} \\ &+ \Gamma^{\tau_k} \Bigg(h_{i-1,k} \lceil \pi_{i-1} \rfloor^{\frac{2\rho-\lambda_{i-1}}{\rho}} (\lceil \zeta_i \rfloor^{q_{i-1,k}} - \lceil \zeta_i^* \rfloor^{q_{i-1,k}}) \\ &+ h_{ik} \lceil \pi_i \rfloor^{\frac{2\rho-\lambda_i}{\rho}} \lceil \zeta_{i+1} \rfloor^{q_{ik}} + h_{ik} \lceil \pi_i \rfloor^{\frac{2\rho-\lambda_i}{\rho}} \bar{g}_{ik} \\ &+ \sum_{j=1}^{i-1} \frac{\partial W_i}{\partial \zeta_j} \left(h_{jk} \lceil \zeta_{j+1} \rfloor^{q_{jk}} + \bar{g}_{jk} \right) \Bigg). \end{aligned} \tag{60}$$

Along the same line as that in Step 2, the estimates of some terms of (60) based on Lemma 1–3 are given.

$$\begin{aligned} &h_{i-1,k} \lceil \pi_{i-1} \rfloor^{\frac{2\rho-\lambda_{i-1}}{\rho}} (\lceil \zeta_i \rfloor^{q_{i-1,k}} - \lceil \zeta_i^* \rfloor^{q_{i-1,k}}) \\ &\quad \leq \frac{1}{3} |\pi_{i-1}|^{\frac{2\rho-\tau_k}{\rho}} + |\pi_i|^{\frac{2\rho-\tau_k}{\rho}} \varpi_{i1k}, \end{aligned} \tag{61}$$

$$\lceil \pi_i \rfloor^{\frac{2\rho-\lambda_i}{\rho}} \bar{g}_{ik} \leq \frac{1}{3} \sum_{j=1}^{i-1} |\pi_j|^{\frac{2\rho-\tau_k}{\rho}} + |\pi_i|^{\frac{2\rho-\tau_k}{\rho}} \varpi_{i2k}, \tag{62}$$

$$\begin{aligned} &\sum_{j=1}^{i-1} \frac{\partial W_i}{\partial \zeta_j} \left(h_{jk} \lceil \zeta_{j+1} \rfloor^{q_{jk}} + \bar{g}_{jk} \right) \\ &\qquad \leq \frac{1}{3} \sum_{j=1}^{i-1} |\pi_j|^{\frac{2\rho-\tau_k}{\rho}} + |\pi_i|^{\frac{2\rho-\tau_k}{\rho}} \varpi_{i3k}, \end{aligned} \tag{63}$$

for some positive smooth functions ϖ_{ijk}, $j = 1, 2, 3$ and $k \in \mathbb{S}$.

Substituting (61)–(63) into (60), one has

$$\begin{aligned} \dot{V}_i \leq& -(n-i+1)\Gamma^{\tau_k} \sum_{j=1}^{i-1} |\pi_j|^{\frac{2\rho-\tau_k}{\rho}} \\ &+ \Gamma^{\tau_k} \Bigg(h_{ik} \lceil \pi_i \rfloor^{\frac{2\rho-\lambda_i}{\rho}} (\lceil \zeta_{i+1} \rfloor^{q_{ik}} - \lceil \zeta_{i+1}^* \rfloor^{q_{ik}}) \\ &+ h_{ik} \lceil \pi_i \rfloor^{\frac{2\rho-\lambda_i}{\rho}} \lceil \zeta_{i+1}^* \rfloor^{q_{ik}} \\ &+ |\pi_i|^{\frac{2\rho-\tau_k}{\rho}} (\varpi_{i1k} + \varpi_{i2k} + \varpi_{i3k}) \Bigg). \end{aligned} \tag{64}$$

Then, a common state feedback controller

$$\zeta_{i+1}^* = -\lceil \pi_i \rfloor^{\frac{\lambda_{i+1}}{\rho}} \beta_i^{\frac{\lambda_{i+1}}{\rho}} (\bar{\zeta}_i), \tag{65}$$

where $\beta_i(\cdot)$ is smooth and satisfies

$$\beta_i(\bar{\zeta}_i) \geq \max_{k\in\mathbb{S}} \left(\frac{n-i+1+\varpi_{i1k}+\varpi_{i2k}+\varpi_{i3k}}{\underline{h}_{ik}} \right)^{\frac{\rho}{q_{ik}\lambda_{i+1}}} \tag{66}$$

renders

$$\begin{aligned} \dot{V}_i \leq& -(n-i+1)\Gamma^{\tau_k} \sum_{j=1}^{i} |\pi_j|^{\frac{2\rho-\tau_k}{\rho}} \\ &+ \Gamma^{\tau_k} h_{ik} \lceil \pi_i \rfloor^{\frac{2\rho-\lambda_i}{\rho}} (\lceil \zeta_{i+1} \rfloor^{q_{ik}} - \lceil \zeta_{i+1}^* \rfloor^{q_{ik}}). \end{aligned} \tag{67}$$

This completes the proof.

REFERENCES

[1] H. Yang, B. Jiang, V. Cocquempot, and H. Zhang, "Stabilization of switched nonlinear systems with all unstable modes: Application to multi-agent systems," *IEEE Trans. Autom. Control*, vol. 56, no. 9, pp. 2230–2235, Sep. 2011.

[2] R. Ma, J. Fu, and T. Chai, "Dwell-time-based observer design for unknown input switched linear systems without requiring strong detectability of subsystems," *IEEE Trans. Autom. Control*, vol. 62, no. 8, pp. 2230–2235, Aug. 2017.

[3] D. Yang, G. Zong, and H. R. Karimi, "H_∞ refined antidisturbance control of switched LPV systems with application to aero-engine," *IEEE Trans. Ind. Electron.*, vol. 67, no. 4, pp. 3180–3190, Apr. 2020.

[4] D. Liberzon, *Switching in Systems and Control*. Boston, MA, USA: Brikhäuser, 2003.

[5] R. Ma and J. Zhao, "Backstepping design for global stabilization of switched nonlinear systems in lower triangular form under arbitrary switchings," *Automatica*, vol. 46, no. 11, pp. 1819–1823, Nov. 2010.

[6] X. Zhao, X. Zheng, B. Niu, and L. Liu, "Adaptive tracking control for a class of uncertain switched nonlinear systems," *Automatica*, vol. 52, pp. 185–191, Feb. 2015.

[7] R. Ma, Y. Liu, S. Zhao, M. Wang, and G. Zong, "Global stabilization design for switched power integrator triangular systems with different powers," *Nonlinear Anal., Hybrid Syst.*, vol. 15, pp. 74–85, Feb. 2015.

[8] X. Lin, X. Li, C.-C. Chen, and S. Li, "Smooth output feedback stabilization for a class of high-order switched nonlinear systems," *Nonlinear Anal., Hybrid Syst.*, vol. 29, pp. 34–53, Aug. 2018.

[9] X. Zhao, X. Wang, G. Zong, and X. Zheng, "Adaptive neural tracking control for switched high-order stochastic nonlinear systems," *IEEE Trans. Cybern.*, vol. 47, no. 10, pp. 3088–3099, Oct. 2017.

[10] S. Li, C. K. Ahn, and Z. Xiang, "Sampled-data adaptive output feedback fuzzy stabilization for switched nonlinear systems with asynchronous switching," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 1, pp. 200–205, Jan. 2019.

[11] X. Zhao, X. Wang, L. Ma, and G. Zong, "Fuzzy approximation based asymptotic tracking control for a class of uncertain switched nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 4, pp. 632–644, Apr. 2020.

[12] D. Cui, B. Niu, D. Yang, T. Hayat, and F. E. Alsaadi, "Adaptive neural fault-tolerant control for a class of stochastic switched nonlinear systems," *IEEE Access*, vol. 7, pp. 93219–93228, 2019.

[13] B. Niu, D. Wang, N. D. Alotaibi, and F. E. Alsaadi, "Adaptive neural state-feedback tracking control of stochastic nonlinear switched systems: An average dwell-time method," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 4, pp. 1076–1087, Apr. 2019.

[14] B. Niu, Y. Liu, W. Zhou, H. Li, P. Duan, and J. Li, "Multiple Lyapunov functions for adaptive neural tracking control of switched nonlinear nonlower-triangular systems," *IEEE Trans. Cybern.*, vol. 50, no. 5, pp. 1877–1886, May 2020.

[15] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751–766, Jan. 2000.

[16] X. Huang, W. Lin, and B. Yang, "Global finite-time stabilization of a class of uncertain nonlinear systems," *Automatica*, vol. 41, no. 5, pp. 881–888, May 2005.

[17] Z.-Y. Sun, L.-R. Xue, and K. Zhang, "A new approach to finite-time adaptive stabilization of high-order uncertain nonlinear system," *Automatica*, vol. 58, pp. 60–66, Aug. 2015.

[18] Y.-J. Liang, R. Ma, M. Wang, and J. Fu, "Global finite-time stabilisation of a class of switched nonlinear systems," *Int. J. Syst. Sci.*, vol. 46, no. 16, pp. 2897–2904, Dec. 2015.

[19] J. Fu, R. Ma, and T. Chai, "Global finite-time stabilization of a class of switched nonlinear systems with the powers of positive odd rational numbers," *Automatica*, vol. 54, pp. 360–373, Apr. 2015.

[20] M. Cai and Z. Xiang, "Global adaptive finite-time stabilization of a class of switched nonlinear systems with parametric uncertainty," *Int. J. Adapt. Control Signal Process.*, vol. 30, no. 7, pp. 1002–1018, Jul. 2016.

[21] J. Fu, R. Ma, and T. Chai, "Adaptive finite-time stabilization of a class of uncertain nonlinear systems via logic-based switchings," *IEEE Trans. Autom. Control*, vol. 62, no. 11, pp. 5998–6003, Nov. 2017.

[22] S. Sui, S. Tong, and C. L. P. Chen, "Finite-time filter decentralized control for nonstrict-feedback nonlinear large-scale systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3289–3300, Dec. 2018.

[23] S. Sui, C. L. P. Chen, and S. Tong, "Neural network filtering control design for nontriangular structure switched nonlinear systems in finite time," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 7, pp. 2153–2162, Jul. 2019.

[24] J. Wu, X. Yang, C. Zhang, and J. Li, "Adaptive finite-time control design for a class of uncertain nonlinearly parameterized switched systems," *IEEE Access*, vol. 7, pp. 95941–95949, 2019.

[25] S. Sui, C. L. P. Chen, S. Tong, and S. Feng, "Finite-time adaptive quantized control of stochastic nonlinear systems with input quantization: A broad learning system based identification method," *IEEE Trans. Ind. Electron.*, vol. 67, no. 10, pp. 8555–8565, Oct. 2020.

[26] G. Zong, H. Ren, and H. R. Karimi, "Event-triggered communication and annular finite-time H_∞ filtering for networked switched systems," *IEEE Trans. Cybern.*, vol. 51, no. 1, pp. 309–317, Jan. 2021, doi: 10.1109/TCYB.2020.3010917.

[27] H. Zhang, Y. Liu, J. Dai, and Y. Wang, "Command filter based adaptive fuzzy finite-time control for a class of uncertain nonlinear systems with hysteresis," *IEEE Trans. Fuzzy Syst.*, early access, Jun. 18, 2020, doi: 10.1109/TFUZZ.2020.3003499.

[28] H. Zhang, Y. Liu, and Y. Wang, "Observer-based finite-time adaptive fuzzy control for nontriangular nonlinear systems with full-state constraints," *IEEE Trans. Cybern.*, early access, Apr. 29, 2020, doi: 10.1109/TCYB.2020.2984791.

[29] V. Andrieu, L. Praly, and A. Astolfi, "Homogeneous approximation, recursive observer design, and output feedback," *SIAM J. Control Optim.*, vol. 47, no. 4, pp. 1814–1850, Jan. 2008.

[30] B. Tian, Z. Zuo, X. Yan, and H. Wang, "A fixed-time output feedback control scheme for double integrator systems," *Automatica*, vol. 80, pp. 17–24, Jun. 2017.

[31] A. Polyakov, "Nonlinear feedback design for fixed-time stabilization of linear control systems," *IEEE Trans. Autom. Control*, vol. 57, no. 8, pp. 2106–2110, Aug. 2012.

[32] Z. Zuo, D. Defoort, B. Tian, and Z. Ding, "Distributed consensus observer for multiagent systems with high-order integrator dynamics," *IEEE Trans. Autom. Control*, vol. 63, no. 2, pp. 563–570, Feb. 2018.

[33] F. Gao, Y. Wu, Z. Zhang, and Y. Liu, "Global fixed-time stabilization for a class of switched nonlinear systems with general powers and its application," *Nonlinear Anal., Hybrid Syst.*, vol. 31, pp. 56–68, Feb. 2019.

[34] B. Ning and Q.-L. Han, "Prescribed finite-time consensus tracking for multiagent systems with nonholonomic chained-form dynamics," *IEEE Trans. Autom. Control*, vol. 64, no. 4, pp. 1686–1693, Apr. 2019.

[35] F. Gao, C. Zhu, J. Huang, and Y. Wu, "Global fixed-time output feedback stabilization of perturbed planar nonlinear systems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, early access, Jul. 1, 2020, doi: 10.1109/TCSII.2020.3006195.

[36] F. Gao, H. Chen, J. Huang, and Y. Wu, "A general fixed-time observer for lower-triangular nonlinear systems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, early access, Nov. 20, 2020, doi: 10.1109/TCSII.2020.3039572.

[37] Y. Pan, P. Du, H. Xue, and H.-K. Lam, "Singularity-free fixed-time fuzzy control for robotic systems with user-defined performance," *IEEE Trans. Fuzzy Syst.*, early access, Jun. 3, 2020, doi: 10.1109/TFUZZ.2020.2999746.

[38] F. Gao, Y. Wu, J. Huang, and Y. Liu, "Output feedback stabilization within prescribed finite time of asymmetric time-varying constrained nonholonomic systems," *Int. J. Robust Nonlin. Control*, vol. 31, no. 2, pp. 427–446, 2021.

[39] Y. Cao, C. Wen, S. Tan, and Y. Song, "Prespecifiable fixed-time control for a class of uncertain nonlinear systems in strict-feedback form," *Int. J. Robust Nonlinear Control*, vol. 30, no. 3, pp. 1203–1222, Feb. 2020.

[40] H. Yao, F. Gao, J. Huang, and Y. Wu, "Global prescribed-time stabilization via time-scale transformation for switched nonlinear systems subject to switching rational powers," *Appl. Math. Comput.*, vol. 393, Mar. 2021, Art. no. 125766.

[41] P. Zarchan, *Tactical and Strategic Missile Guidance*. Reston, VA, USA: American Institute of Aeronautics and Astronautics (AIAA), 2007.

[42] Y. Song, Y. Wang, J. Holloway, and M. Krstic, "Time-varying feedback for regulation of normal-form nonlinear systems in prescribed finite time," *Automatica*, vol. 83, pp. 243–251, Sep. 2017.

[43] F. Gao, Y. Wu, and Z. Zhang, "Global fixed-time stabilization of switched nonlinear systems: A time-varying scaling transformation approach," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 66, no. 11, pp. 1890–1894, Nov. 2019.

[44] G. Tao and P. V. Kokotovic, "Adaptive control of plants with unknown dead-zones," *IEEE Trans. Autom. Control*, vol. 39, no. 1, pp. 59–68, Jan. 1994.

[45] C. Hua, Y. Li, and X. Guan, "Finite/fixed-time stabilization for nonlinear interconnected systems with dead-zone input," *IEEE Trans. Autom. Control*, vol. 62, no. 5, pp. 2554–2560, May 2017.

[46] F. Gao, Y. Wu, and Y. Liu, "Finite-time stabilization for a class of switched stochastic nonlinear systems with dead-zone input nonlinearities," *Int. J. Robust Nonlinear Control*, vol. 28, no. 9, pp. 3239–3257, Jun. 2018.

[47] H. Li, H. Sun, and L. Hou, "Adaptive fuzzy PI prescribed performance tracking control for switched nonlinear systems with dead-zone input and external disturbances," *IEEE Access*, vol. 8, pp. 143938–143949, 2020.

[48] H. Liang, G. Liu, H. Zhang, and T. Huang, "Neural-network-based event-triggered adaptive control of nonaffine nonlinear multiagent systems with dynamic uncertainties," *IEEE Trans. Neural Netw. Learn. Syst.*, early access, Jul. 14, 2020, doi: 10.1109/TNNLS.2020.3003950.

[49] A. F. Filippov, *Differential Equations With Discontinuous Right-Hand Side*. Dordrecht, The Netherlands: Kluwer, 1988.

[50] C. Qian and W. Lin, "A continuous feedback approach to global strong stabilization of nonlinear systems," *IEEE Trans. Autom. Control*, vol. 46, no. 7, pp. 1061–1079, Jul. 2001.

[51] S. Ding, W.-H. Chen, K. Mei, and D. J. Murray-Smith, "Disturbance observer design for nonlinear systems represented by input–output models," *IEEE Trans. Ind. Electron.*, vol. 67, no. 2, pp. 1222–1232, Feb. 2020.

[52] H. K. Khalil, *Nonlinear System*, 3rd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.

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