IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received December 9, 2020, accepted December 24, 2020, date of publication January 8, 2021, date of current version January 20, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3050198

Distributed Robust Dimensionality Reduction Fusion Estimation Under DoS Attacks and Uncertain Covariances

MENG XU[1], YUCHEN ZHANG[2], DONGMEI ZHANG[1], AND BO CHEN[2,3], (Member, IEEE)
[1]Department of Mathematics, Zhejiang University of Technology, Hangzhou 310023, China
[2]Department of Automation, Zhejiang University of Technology, Hangzhou 310023, China
[3]Institute of Cyberspace Security, Zhejiang University of Technology, Hangzhou 310023, China

Corresponding author: Dongmei Zhang (meidzh@zjut.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant 61973277, and in part by the Zhejiang Provincial Natural Science Foundation of China under Grant LR20F030004.

ABSTRACT This paper is concerned with the networked distributed fusion estimation problem under denial-of-service (DoS) attacks, where the noise covariances are unknown but bounded, and the distribution information of DoS attacks is not required to be known. Based on the dimensionality reduction and compensation model, the local Kalman filter (LKF) with unknown covariances is designed by the maximum and minimum robust estimation criterion, while the distributed fusion Kalman filter (DFKF) is derived from the optimal weighted fusion criterion. Moreover, the robustness of the developed DFKF is also analyzed in the presence of DoS attacks. Finally, an illustrative example is exploited to demonstrate the effectiveness of the proposed methods.

INDEX TERMS Robust fusion, Kalman filter, unknown covariances, bandwidth constraints, DoS attacks.

I. INTRODUCTION

With the development of sensors and computer technology, the multi-sensor fusion estimation (MFE) problem has attracted considerable interest, due primarily to its extensive applications in various fields including moving target tracking [1], signal processing [2] and industrial monitor [3]. Most often, the major focus of the MFE is how to effectively use the information contained in various kinds of data to better estimate system states or parameters. Besides, lots of fusion estimation methods have been proposed to improve the reliability and robustness of estimators [2]–[6]. Generally, the distributed fusion structures, compared with centralized ones, are more reliable, robust and fault-tolerant to some extent [7]–[9]. Moreover, the distributed fusion Kalman filter (DFKF) method can significantly reduce the amount of calculation and communication burdens [10], [11].

The recent years have witnessed a surge of research interest in networked multi-sensor fusion estimation (NMFE) problems [12]–[19], where the measurement information can be sent to the fusion center (FC) via the wired or wireless communication networks. However, due to the bandwidth constraints, only limited information can be carried by communication network for each unit of time, and thus the FC can only receive a limited number of sensor messages. Therefore, information loss is inevitable, which may lead to the estimated performance degradation of the fusion estimation. To overcome the problem of bandwidth constraints, several methods have been developed, including quantification method [20]–[22] and dimensionality reduction method [23]–[28]. As the literature in [25]–[28] pointed out, the dimensionality reduction method shows more advantages in solving the bandwidth constraints problem for high-dimensional state.

At the same time, the transmission channel is exposed to the network environment when the information of each sensor is transmitted from the measurement node to the FC. Once the accuracy and the reliability of the estimation are affected by cyber attacks, the precise control for system stability will be difficult to achieve. Several cyber attacks have been addressed, such as DoS attacks and deception attacks [29], [30]. The DoS attacks is the most reachable attacks [24] and dangerous. A famous example includes the regional power office systems in Ukraine [31], which suffered from highly disrupted malware attacks and caused large-scale regional blackouts for hours.

The problem of remote state estimation under DoS attacks has been well investigated, where attacks exist in the communication channel between the sensors and the remote estimator. Concretely, from the perspective of an attacker, a dynamic

The associate editor coordinating the review of this manuscript and approving it for publication was Ze Ji.

attack power allocation algorithm was proposed to reduce the estimation performance in [32]. Further, the construction of an optimal attack schedule [33] was studied for the attack under energy constraints, where the purpose of the attacker is to maximize the expected mean value of the estimation error. On the other hand, in view of the stability problem of the estimation system under DoS attacks, the defender proposed a secure packet coding method to protect the communication channel against DoS attacks by compensating the previous partial loss of packets [34]. Under the constraints of energy budget and the asymmetry of information between attackers and defenders, a game method [35], [36] was proposed to simulate the decision-making process of both parties, and the optimal strategy was obtained to improve the performance of the system. Most often, DoS attacks can affect the integrity of data, and further cause performance degradation in remote estimators. A common and direct method to mitigate the situation is to use compensation strategies [37] when data is intercepted. For example, the data successfully transmitted last time, or alternatively the prediction of current lost data, are feasible to compensate for partial lost information. With the compensation strategies applied to estimation problem under DoS attacks, the designed estimator will be more robust to attacks and can recover from attacks to satisfactory estimation performance faster. Meanwhile, the problem of NMFE was also proposed to further improve the system performance under DoS attacks. Different solutions have been proposed to solve the bandwidth constraint problems for fusion estimation. An optimal recursive Kalman filter was designed by using dimensionality reduction strategy and an effective attack strategy was proposed in [24]–[28] and [37]. An event-triggered control algorithm [30] was proposed and then adopted to reduce the computational burdens [38], where the impulsive system approach was used to prove the stability of the system.

More recently, some researchers have been paying attention to the security fusion estimation of networked multi-sensor systems. By exploring the attacked system, a novel filtering learning algorithm [39] and a game strategy [40] were proposed to solve the problem of abnormal observation distorted information in the FC. While the method of system switching [41] and the rate conversion of multi-rate sensors [29] were used to study the problem of filtering. Some strategies were proposed to prevent the networked fusion systems from being attacked, sufficient criterion of the bounded state estimation was pointed to solve the fusion estimation problem for time-delay and random disturbance systems in [42].

Due to the wireless communication network congestion between the sink nodes and the remote estimators, the transmission channel is more vulnerable to DoS attacks. However, the problem of distributed dimensionality reduction fusion estimation problem under DoS attacks has rarely been studied by researchers, which motivate the present research to generate. On the other hand, the Gaussian assumption with known noise covariance for the celebrated Kalman filter is hard to be satisfied in practical situations. The literature [43] dealt with systems under unknown noise covariances such that the corresponding Kalman filter has some nice form, but it does not guarantee the robustness of the designed Kalman filter. Therefore, we concern ourselves with robust filter design problem with known upper bounds of noise covariances, that is the actual noise variance is unknown but bounded to us.

This paper deals with the DFKF problem for networked system architecture with the communication constraints, DoS attacks and uncertain convariances. Specifically, the sink node is responsible for receiving the measurements from sensors and transmitting them to the remote estimator via communication network, while the remote estimator calculates the LKF, and sent to the FC for the distributed fusion estimation. Since only partial sensor measurement of each sink node can be transmitted to the remote estimator due to bandwidth constraints, and thus the dimensionality reduction strategy is applied to reduce the communication traffic. Meanwhile, the measurement data packets may be lost in case of DoS attacks. Though the dimensionality reduction fusion estimation problem under bandwidth constrains and DoS attacks were discussed in our previous work [24] and [26], the above works focused on the communication uncertainties between the local estimates and the FC, while the uncertain communication problem in this paper is from the sensor measurements that may suffer DoS attacks and bandwidth constraints. In this case, the place to reducing the dimensionality and the design of compensation strategy are different from [24] and [26]. The contribution of this paper can be summarized as follows: i) A unified compensation model is proposed to reduce the information loss caused by DoS attacks and bandwidth constraints; ii) A recursively DFKF is designed in the presence of uncertain covariances, and the robustness is guaranteed by establishing a positive semidefinite decision problem and can be solved by the Lyapunov equation.

Notations: $\mathbb{R}^n$ represents the n-dimensional real Euclidean space. '$\circ$' means the Hadamard product. 'I' is an identity matrix, while $\text{col}\{\cdot\}$ stands for a block column matrix, and $\text{diag}\{\cdot\}$ denotes a block diagonal matrix. $\text{prob}\{X\}$ is the occurrence probability of the event X, while X^T represents transpose of matrix X. $E\{\cdot\}$ is the mathematical expectation, and $x \perp y$ denotes that x and y are orthogonal vectors.

II. PROBLEM FORMULATION

Consider the framework of NMFE under DoS attacks in Fig. 1, where the physical process is described as discrete time-varying state-space model:

$$x(t+1) = A(t)x(t) + B(t)w(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the current state of the process, $w(t) \in \mathbb{R}^r$ is the process noise, $A(t)$ and $B(t)$ are time-varying matrices with appropriate dimensions. System (1) is observed by L groups of sensors, and the measurement of each sensor group is collected by its sink node, i.e.,

$$y_k(t) = C_k(t)x(t) + v_k(t)(k = 1, \cdots, L) \tag{2}$$

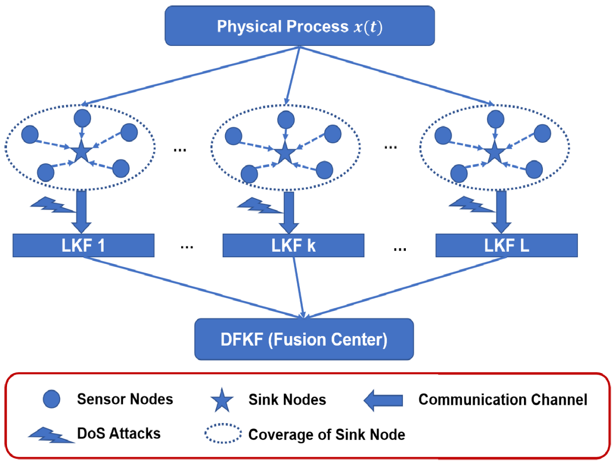


FIGURE 1. **The framework of NMFE under DoS attacks.**

where $y_k(t) = \text{col}\{y_k^1(t), \cdots, y_k^{m_k}(t)\}$ is the measurement of the k-th sink node, $v_k(t) = \text{col}\{v_k^1(t), \cdots, v_k^{m_k}(t)\}$ is the measurement noise for the k-th sink node and $v_k(t) \in \mathbb{R}^{m_k}$. $C_k(t) = \text{col}\{C_k^1(t), \cdots, C_k^{m_k}(t)\}$ is the augmented measurement matrix for the k-th sink node. Here, $w(t)$ and $v_k(t)$ are uncorrelated zero-mean Gaussian white noises, which satisfies

$$E\left\{\begin{bmatrix} w^T(t) & v_k^T(t)\end{bmatrix}^T \begin{bmatrix} w^T(t_1) & v_j^T(t_1)\end{bmatrix}\right\} = \text{diag}\{Q_w(t), Q_{v_k}(t)\delta_{kj}\}\delta_{tt_1} \quad (3)$$

where the noise covariances $Q_w(t)$ and $Q_{v_k}(t)$ are unknown but bounded, i.e.,

$$Q_w(t) \leq \bar{Q}_w(t), Q_{v_k}(t) \leq \bar{Q}_{v_k}(t)(k = 1, \cdots, L, \forall t) \quad (4)$$

where the upper bounds $\bar{Q}_w(t)$ and $\bar{Q}_{v_k}(t)$ are known in advance. Furthermore, m_k is the number of sensors in the k-th sink node. The dynamics of the measurement equation for each sensor in (2) can be described as

$$y_k^i(t)=C_k^i(t)x(t)+v_k^i(t)(i=1, \cdots, m_k, k=1, \cdots, L) \quad (5)$$

where $y_k^i(t)$ is the measurement from i-th sensor, $v_k^i(t) \in \mathbb{R}$ is the measurement noise, and $C_k^i(t)$ is the measurement matrix. It is assumed that the initial state $x(0)$ is a random variable uncorrelated to $w(t)$ and $v_k(t)$, where $E[x(0)] = \mu_0$ and its actual covariance denoted by P_0 is unknown but bounded by

$$P_0 \leq \bar{P}_0 \quad (6)$$

When all the components of $y_k(t)$ are transmitted via the communication networks with limited communication capacity, it will suffer from lots of network induced problems, such as packet loss, disorder and time delay. Resorting to the dimensionality reduction idea in [23], only r_k $(1 \leq r_k < m_k)$ elements of the measurement $y_k(t)$ are chosen to be transmitted from k-th sink node to the FC at each instant. $y_{e,k}(t)$ is the output of each sink node after the dimensionality reduction, and each component of $y_{e,k}(t)$ is selected from $y_k^i(t)(i = 1, \cdots, m_k)$. It is obvious that $y_{e,k}(t)$ does not have to be equal to $y_k(t)$. There have Δ_k options of the component of $y_{e,k}(t)$, where

$$\Delta_k = \frac{m_k!}{r_k!(m_k - r_k)!} \quad (7)$$

Once the selection strategy is finished at each instant, the output of $y_{e,k}(t)$ will be transmitted from k-th sink node to the FC via communication networks. To model the selection process for the dimensionality reduction strategy, the binary-valued white random sequences $\sigma_k^l(t) \in \{0, 1\}, \forall l \in O_k \triangleq \{1, \cdots, \Delta_k\}$ are introduced, where $\sigma_k^l(t) = 1(l \in O_k)$ indicates that the l-th case of Δ_k is selected to reduce the dimension of measurement in the sink node, otherwise $\sigma_k^l(t) = 0$. Let $\alpha_k^l = \text{prob}\{\sigma_k^l(t) = 1\}(l \in O_k)$. It is obvious that $\sum_{l=1}^{\Delta_k} \alpha_k^l(t) = 1$. Since only one possible case can be selected at each instant and $E[\sigma_k^l(t)] = \alpha_k^l, \forall l \in O_k$, one has $E[\sigma_k^l(t)\sigma_{k_1}^{l_1}(t)] = \begin{cases} \alpha_k^l, l = l_1, k = k_1 \\ 0, \quad \text{otherwise} \end{cases}$

To mathematically describe the relationship of $y_{e,k}(t)$ and $y_k(t)$, we define the selection matrix of the l-th case to be:

$$\Pi_k^l(t) = \text{diag}\{\xi_k^1(t), \cdots, \xi_k^{m_k}(t)\}(k = 1, \cdots, L) \quad (8)$$

where $\xi_k^i(t) \in \{0, 1\}$ is the binary variables. $\xi_k^i(t) = 1$ means the i-th component of $y_k(t)$ is allowed to be transmitted, otherwise $\xi_k^i(t) = 0$.

$$\sum_{i=1}^{m_k} \xi_k^i(t) = r_k(k = 1, \cdots, L) \quad (9)$$

It follows from (2) - (9) that $y_{e,k}(t)$ can be written as

$$y_{e,k}(t) = \Theta_k(t)y_k(t) \quad (10)$$

where $\Theta_k(t) = \sum_{l=1}^{\Delta_k} \sigma_k^l(t)\Pi_k^l(t)$.

As mentioned before, DoS attacks can be easier to be successfully executed by the adversary in the communication networks. When $y_{e,k}(t)$ is transmitted to the remote estimators, the attacker may launch DoS attacks on the channel between the sink nodes and the remote estimators such that the remote estimators cannot receive $y_{e,k}(t)$ at a particular time. Let $\eta_k(t) \in \{0, 1\}$ denote whether the adversary launches a DoS attack or not at time t, and it is assumed that $\eta_k(t)$ is the indicator value (i.e., 0 or 1 at each time). Notice that the value of $\eta_k(t)$ can be determined for estimator design. Since data packets are marked with the time-stamp method [27] before transmission, and thus the remote estimator can easily detect DoS attacks by checking the time-stamp information.

Let $\bar{y}_{e,k}(t)$ denote input of the remote estimator from the k-th sink node at each instant. When $\eta_k(t) = 0$, the k-th communication channel is jammed by DoS attacks, then the input of the remote estimator is proposed to be

$$\bar{y}_{e,k}(t) = \bar{y}_{e,k}(t - 1) \quad (11)$$

Otherwise, $\eta_k(t) = 1$, which means that $y_{e,k}(t)$ is successfully transmitted to the remote estimators. To compensate for the information loss caused by dimensionality reduction strategy, $\bar{y}_{e,k}(t)$ is modeled as

$$\bar{y}_{e,k}(t) = y_{e,k}(t) + (I - \Theta_k(t))\bar{y}_{e,k}(t-1) \tag{12}$$

Moreover, it follows from (10) - (12) that the unified form of $\bar{y}_{e,k}(t)$ is formulated by

$$\bar{y}_{e,k}(t) = \eta_k(t) y_{e,k}(t) + (I - \eta_k(t)\Theta_k(t))\bar{y}_{e,k}(t-1) \tag{13}$$

Define $X_{a,k}(t) \triangleq \mathrm{col}\{x(t), \bar{y}_{e,k}(t-1)\}$ and $W_{a,k}(t) \triangleq \mathrm{col}\{w(t), v_k(t)\}$, it then follows from (1) and (13) that the compensation systems under DoS attacks and bandwidth constraints are modeled by:

$$X_{a,k}(t+1) = \Phi_k(t) X_{a,k}(t) + \Gamma_k(t) W_{a,k}(t) \tag{14}$$

$$\bar{y}_{e,k}(t) = \Psi_k(t) X_{a,k}(t) + \eta_k(t)\Theta_k(t) v_k(t) (k = 1, \cdots, L) \tag{15}$$

where

$$\begin{cases} \Phi_k(t) = \begin{bmatrix} A(t) & 0 \\ \eta_k(t)\Theta_k(t)C_k(t) & I - \eta_k(t)\Theta_k(t) \end{bmatrix} \\ \Gamma_k(t) = \mathrm{diag}\{B(t), \eta_k(t)\Theta_k(t)\} \\ \Psi_k(t) = \begin{bmatrix} \eta_k(t)\Theta_k(t)C_k(t) & I - \eta_k(t)\Theta_k(t) \end{bmatrix} \end{cases}$$

Definition 1 [5]: For a system with uncertain noise covariances, if the existence of uncertain noise covariances satisfies (4) and (6), so that for all admissible uncertainties, the corresponding filtering estimation error covariance has the minimum upper bound or the maximum lower bound, then the actual Kalman filter is said to have guaranteed performance robustness.

Consequently, the aims of this paper are described as follows:

- Based on (4) and (6), design recursive DFKF for the compensation systems (14) and (15) under DoS attacks, bandwidth constraints and uncertain covariances;
- According to all admissible actual covariances satisfying (4) and (6), prove that the designed DFKF is robust, i.e., it has a minimum upper bound.

Remark 1: Many dimensionality reduction methods have been proposed to solve the bandwidth constraint problem in [24]–[28], which allows only part components of estimated signal to be transmitted to adapt for the limited communication capacity. The un-transmitted components are compensated by one-step prediction. Different from these state dimensionality reduction methods, the un-transmitted measurement at time t is compensated by (12) (i.e., $\bar{y}_{e,k}(t)$) at time $t-1$. On the other hand, the defense strategy (13) is implemented against the DoS attacks in bandwidth constraints communication environment. That is, the signal received by the remote estimator at time $t-1$ is used as a compensator when DoS attacks caused packet dropout occurs at time t, i.e., $y_{e,k}(t) = \bar{y}_{e,k}(t-1)$ in (11). In what follows, we consider a simple estimated signal transmission example over the wireless network to get $y_{e,k}(t)$. Two sink nodes are used to monitor the system, and the dimensionality reduction strategy is employed before the measurements are transmitted via network. $L = 2$, $m_k = 3$, $r_1 = 1$ and $r_2 = 2$, it follows from (7) and (8) that

$$\begin{cases} \Delta_1 = 3, \Pi_1^1(t) = \mathrm{diag}\{1, 0, 0\}, \Pi_1^2(t) = \mathrm{diag}\{0, 1, 0\}, \\ \Pi_1^3(t) = \mathrm{diag}\{0, 0, 1\}; \Delta_2 = 3, \Pi_2^1(t) = \mathrm{diag}\{1, 1, 0\}, \\ \Pi_2^2(t) = \mathrm{diag}\{1, 0, 1\}, \Pi_2^3(t) = \mathrm{diag}\{0, 1, 1\}. \end{cases} \tag{16}$$

Notice that $\Pi_1^l(t)$ and $\Pi_2^l(t)$ represent different selection strategies. For example, $\Pi_1^1(t) = \mathrm{diag}\{1, 0, 0\}$ in (16) indicates that the first component of $y_1(t)$ is allowed to be transmitted the FC, but the second and thrid components of $y_1(t)$ are lost.

Remark 2: The idea of dimensionality reduction strategy in this paper is different from the strategies in [24] and [26], mainly reflected in two aspects. First, the places to reduce the dimensionality are different. Due to the constrained communication channel between each sink node and the FC, the dimensionality reduction strategy in [24] and [26] is implemented when each local estimate is sent to the FC. However, the strategy in this paper is implemented before the measurement signals are sent to sink nodes. Second, the design of compensation strategy is distinct from these literature. The compensation model in [24] and [26] is used for compensating estimation signals, while the compensation model built in this paper is for compensating measurement signals.

Remark 3: Most of existing DoS attacks assumptions need distributed information of attacks. For example, the occurrence of attacks is assumed to follow a Markov transition probability distribution [33], or the maximum number of attacks during a separated time period [45] is assumed to be given. These assumptions are sometimes not available in practical situations, and the introduced parameters (i.e., attack statistics, transition probability and maximum number of attacks) are hard to be determined in advance. Instead, the distribution information of DoS attacks is not needed in this paper. The occurrence of attacks (i.e., $\eta_k(t)$) is the only value that needs to be determined for estimator design. From a practical point of view, data packets can be marked with the time-stamp before transmission, and thus the remote estimator can detect DoS attacks easily by receiving the time-stamp information of the data. Therefore, the distribution information of DoS attacks is not required to be known.

III. MAIN RESULTS

A. LOCAL ROBUST ESTIMATION UNDER DoS ATTACKS

Before presenting the main results, some preliminary results are displayed. Taking expectations on $\Phi_k(t)$, $\Gamma_k(t)$, $\Psi_k(t)$, $\Theta_k(t)$ and denoting $\Phi_k^e(t) = E[\Phi_k(t)]$, $\Gamma_k^e(t) = E[\Gamma_k(t)]$, $\Psi_k^e(t) = E[\Psi_k(t)]$ and $\Theta_k^e(t) = E[\Theta_k(t)]$, one has

$$\begin{cases} \Phi_k^e(t) = \begin{bmatrix} A(t) & 0 \\ \eta_k(t)\Theta_k^e(t)C_k(t) & I - \eta_k(t)\Theta_k^e(t) \end{bmatrix} \\ \Gamma_k^e(t) = \mathrm{diag}\{B(t), \eta_k(t)\Theta_k^e(t)\} \\ \Psi_k^e(t) = \begin{bmatrix} \eta_k(t)\Theta_k^e(t)C_k(t) & I - \eta_k(t)\Theta_k^e(t) \end{bmatrix} \\ \Theta_k^e(t) = \sum_{l=0}^{\Delta_k} \alpha_k^l \Pi_k^l(t) \end{cases}$$

Some statistical information about the noise is given by

$$\begin{cases} E\left\{\begin{bmatrix} W_{a,k}^T(t) & v_k^T(t)\end{bmatrix}^T\begin{bmatrix} W_{a,j}^T(t_1) & v_j^T(t_1)\end{bmatrix}\right\} \\ \quad = \begin{bmatrix} Q_{W_a}(t) & S_k(t) \\ S_k^T(t) & Q_{v_k}(t)\delta_{kj}\delta_{tt_1}\end{bmatrix} \\ Q_{W_a}(t) = E\left[W_{a,k}(t)W_{a,j}^T(t_1)\right] \\ \quad = \mathrm{diag}\{Q_w(t), Q_{v_k}(t)\delta_{kj}\}\delta_{tt_1} \\ S_k(t) \ = E[W_{a,k}(t)v_j^T(t_1)] = \mathrm{col}\{0, Q_{v_k}(t)\delta_{kj}\delta_{tt_1}\} \end{cases} \tag{17}$$

According to Eq. (4), we can obtain $Q_{W_a}(t) \leq \bar{Q}_{W_a}(t)$, $S_k(t) \leq \bar{S}_k(t)$, where $\bar{Q}_{W_a}(t) = \mathrm{diag}\{\bar{Q}_w(t), \bar{Q}_{v_k}(t)\delta_{kj}\}\delta_{tt_1}$ and $\bar{S}_k(t) = \mathrm{col}\{0, \bar{Q}_{v_k}(t)\delta_{kj}\delta_{tt_1}\}$.

Obviously, the augmented noise $W_{a,k}(t) \in \mathbb{R}^{r+m_k}$ and the measurement noise $v_k(t) \in \mathbb{R}^{m_k}$ are correlated Gaussian white noises with zero means. In this case, the compensation systems (14) and (15) need to be transformed into a new system with uncorrelated noises, and the detailed steps are as follows:

Adding a zero item to the right hand side of (14) yields

$$\begin{aligned} X_{a,k}(t+1) = \Phi_k(t)X_{a,k}(t) + \Gamma_k(t)W_{a,k}(t) + J_k(t)[\bar{y}_{e,k}(t) \\ - \Psi_k(t)X_{a,k}(t) - \eta_k(t)\Theta_k(t)v_k(t)] \end{aligned} \tag{18}$$

Define $Z_k(t) \triangleq \Phi_k(t) - J_k(t)\Psi_k(t)$, and $\omega_k(t) \triangleq \Gamma_k(t)W_{a,k}(t) - \eta_k(t)J_k(t)\Theta_k(t)v_k(t)$. $J_k(t)$ can be viewed as an undetermined matrix. Thus $X_{a,k}(t+1)$ can be organized as

$$X_{a,k}(t+1) = Z_k(t)X_{a,k}(t) + \omega_k(t) + J_k(t)\bar{y}_{e,k}(t) \tag{19}$$

To make $\omega_k(t)$ be a zero-mean white noise which is uncorrelated to $v_k(t)$ i.e.,

$$\begin{aligned} E[\omega_k(t)v_k^T(t)] &= \Gamma_k^e(t)S_k(t) - \eta_k(t)J_k(t)\Theta_k^e(t)Q_{v_k}(t) \\ &= 0 \end{aligned} \tag{20}$$

and then the specific expression of $J_k(t)$ is

$$J_k(t)=\begin{cases}\Gamma_k^e(t)S_k(t)Q_{v_k}^{-1}(t)[\Theta_k^e(t)]^{-1}, & \eta_k(t)=1 \\ \text{matrix with appropriate dimension}, & \eta_k(t)=0\end{cases} \tag{21}$$

At same time, $\bar{J}_k(t)$ can be achieved by the upper bounds $\bar{Q}_{v_k}(t)$ and $\bar{S}_k(t)$

$$\bar{J}_k(t)=\begin{cases}\Gamma_k^e(t)\bar{S}_k(t)\bar{Q}_{v_k}^{-1}(t)[\Theta_k^e(t)]^{-1}, & \eta_k(t)=1 \\ \text{matrix with appropriate dimension}, & \eta_k(t)=0\end{cases} \tag{22}$$

Obviously, the state transition matrix and the process noise have also been introduced in the systems. Noticed that the covariance of the white noise $\omega_k(t)$ can be computed by

$$\begin{aligned} Q_\omega(t) &= E[\omega_k(t)\omega_k^T(t)] \\ &= \Gamma_k^e(t)[Q_{W_a}(t) - \eta_k(t)S_k(t)Q_{v_k}^{-1}(t)S_k^T(t)][\Gamma_k^e(t)]^T \end{aligned} \tag{23}$$

Similarly, its upper bound covariance can be written as

$$\bar{Q}_\omega(t) = \Gamma_k^e(t)[\bar{Q}_{W_a}(t) - \eta_k(t)\bar{S}_k(t)\bar{Q}_{v_k}^{-1}(t)\bar{S}_k^T(t)][\Gamma_k^e(t)]^T \tag{24}$$

Theorem 1: For the compensation systems (14) and (15), the conservative optimal recursive Kalman filter is given by

$$\begin{cases} \hat{X}_{a,k}(t) &= \bar{Z}_k(t-1)\hat{X}_{a,k}(t-1) \\ &\quad +\bar{J}_k(t-1)\bar{y}_{e,k}(t-1) + \bar{K}_k(t)\varepsilon_k(t) \\ \varepsilon_k(t) &= \bar{y}_{e,k}(t) - \Psi_k(t)\hat{X}_{a,k}(t|t-1) \\ \bar{K}_k(t) &= \bar{P}_k(t|t-1)[\Psi_k^e(t)]^T\bar{Q}_{\varepsilon_k}^{-1}(t) \\ \bar{Q}_{\varepsilon_k}(t) &= \Psi_k^e(t)\bar{P}_k(t|t-1)[\Psi_k^e(t)]^T \\ &\quad +\eta_k(t)\Theta_k^e(t)\bar{Q}_{v_k}(t)[\eta_k(t)\Theta_k^e(t)]^T \\ \bar{P}_k(t|t) &= \bar{H}_k^e(t)\bar{P}_k(t-1|t-1)[\bar{H}_k^e(t)]^T \\ &\quad +\bar{N}_k^e(t)\bar{Q}_\omega(t-1)[\bar{N}_k^e(t)]^T \\ &\quad +\eta_k(t)\bar{K}_k(t)\Theta_k^e(t)\bar{Q}_{v_k}(t)[\eta_k(t)\bar{K}_k(t)\Theta_k^e(t)]^T \end{cases} \tag{25}$$

where $\hat{X}_{a,k}(t)$ is the LKF, $\bar{Q}_{\varepsilon_k}(t)$ is the covariance matrix of innovation $\varepsilon_k(t)$, $\bar{K}_k(t)$ is the filtering gain matrix, $\bar{Z}_k(t-1) = \Phi_k(t-1) - \bar{J}_k(t-1)\Psi_k(t-1)$ is the conservative state transition matrix, and $\bar{P}_k(t|t)$ is the estimation error covariance matrix with the initial values $\hat{X}_{a,k}(0) = \mathrm{col}\{\mu_0, 0\}$, $\bar{P}_k(0|0) = \mathrm{diag}\{\bar{P}_0, 0\}$.

Proof: Replace $J_k(t)$ in (18) as $\bar{J}_k(t)$ in (22), and then take the projective operation [44] to derive the relationship between filtering and prediction filtering

$$\hat{X}_{a,k}(t|t-1)=\bar{Z}_k(t-1)\hat{X}_{a,k}(t-1)+\bar{J}_k(t-1)\bar{y}_{e,k}(t-1) \tag{26}$$

Define prediction filtering error as: $\tilde{X}_{a,k}(t|t-1) \triangleq X_{a,k}(t) - \hat{X}_{a,k}(t|t-1)$. From (19) and (26), the prediction filtering error is expressed by

$$\tilde{X}_{a,k}(t|t-1) = \bar{Z}_k(t-1)\tilde{X}_{a,k}(t-1) + \bar{\omega}_k(t-1) \tag{27}$$

where $\bar{\omega}_k(t-1) = \Gamma_k(t-1)W_{a,k}(t-1) - \eta_k(t-1)\bar{J}_k(t-1)\Theta_k(t-1)v_k(t-1)$. Therefore, the prediction estimation error covariance is calculated by

$$\begin{aligned} \bar{P}_k(t|t-1) &= E[\tilde{X}_{a,k}(t|t-1)\tilde{X}_{a,k}^T(t|t-1)] \\ &= \bar{Z}_k^e(t-1)\bar{P}_k(t-1|t-1)[\bar{Z}_k^e(t-1)]^T \\ &\quad + \bar{Q}_\omega(t-1) \end{aligned} \tag{28}$$

where $\bar{Z}_k^e(t) = E[\bar{Z}_k(t)] = \Phi_k^e(t) - \bar{J}_k(t)\Psi_k^e(t)$. From the recursive projection formula, the filtering can be written as

$$\hat{X}_{a,k}(t) = \hat{X}_{a,k}(t|t-1) + \bar{K}_k(t)\varepsilon_k(t) \tag{29}$$

where $\varepsilon_k(t)$ is defined in (25) and $\bar{K}_k(t) = E[X_{a,k}(t)\varepsilon_k(t)]\bar{Q}_{\varepsilon_k}^{-1}(t)$. The one-step prediction of (15) is

$$\hat{y}_{e,k}(t|t-1) = \Psi_k(t)\hat{X}_{a,k}(t|t-1) \tag{30}$$

It follows from (15) and (30) that

$$\varepsilon_k(t) = \Psi_k(t)\tilde{X}_{a,k}(t|t-1) + \eta_k(t)\Theta_k(t)v_k(t) \tag{31}$$

and

$$E[X_{a,k}(t)\varepsilon_k^T(t)] = \bar{P}_k(t|t-1)[\Psi_k^e(t)]^T \tag{32}$$

The innovation covariance matrix $\bar{Q}_{\varepsilon_k}(t) = E[\varepsilon_k(t)\varepsilon_k^T(t)]$ is given by

$$\begin{aligned}\bar{Q}_{\varepsilon_k}(t) = \Psi_k^e(t)\bar{P}_k(t|t-1)[\Psi_k^e(t)]^T \\ + \eta_k(t)\Theta_k^e(t)\bar{Q}_{v_k}(t)[\eta_k(t)\Theta_k^e(t)]^T \end{aligned} \quad (33)$$

From (26), (29), (32) and (33), $\hat{X}_{a,k}(t)$ is expressed as

$$\begin{aligned}\hat{X}_{a,k}(t) = \bar{Z}_k(t-1)\hat{X}_{a,k}(t-1) \\ + \bar{J}_k(t-1)\bar{y}_{e,k}(t-1) + \bar{K}_k(t)\varepsilon_k(t) \end{aligned} \quad (34)$$

where $\bar{K}_k(t) = \bar{P}_k(t|t-1)[\Psi_k^e(t)]^T\bar{Q}_{\varepsilon_k}^{-1}(t)$. Note that $\tilde{X}_{a,k}(t-1)\perp\bar{\omega}_k(t-1)$, $\tilde{X}_k(t-1)\perp v_k(t)$, and $v_k(t)\perp\bar{\omega}_k(t-1)$. Therefore, it follows from (19), (31) and (34) that

$$\begin{aligned}\tilde{X}_{a,k}(t) = \bar{H}_k(t)\tilde{X}_{a,k}(t-1) + \bar{N}_k(t)\bar{\omega}_k(t-1) \\ - \eta_k(t)\bar{K}_k(t)\Theta_k(t)v_k(t) \end{aligned} \quad (35)$$

where $\bar{N}_k(t) = I_{n+m_k} - \bar{K}_k(t)\Psi_k(t)$, $\bar{H}_k(t) = \bar{N}_k(t)\bar{Z}_k(t-1)$. Define $\bar{N}_k^e(t) \triangleq E[\bar{N}_k(t)] = I_{n+m_k} - \bar{K}_k(t)\Psi_k^e(t)$ and $\bar{H}_k^e(t) \triangleq E[\bar{H}_k(t)] = \bar{N}_k^e(t)\bar{Z}_k^e(t-1)$. The Lyapunov equation of the conservative estimation error covariance can be rewritten as

$$\begin{aligned}\bar{P}_k(t|t) = \bar{H}_k^e(t)\bar{P}_k(t-1|t-1)[\bar{H}_k^e(t)]^T \\ + \bar{N}_k^e(t)\bar{Q}_\omega(t-1)[\bar{N}_k^e(t)]^T \\ + \eta_k(t)\bar{K}_k(t)\Theta_k^e(t)\bar{Q}_{v_k}(t)[\eta_k(t)\bar{K}_k(t)\Theta_k^e(t)]^T \end{aligned} \quad (36)$$

This completes the proof.

Remark 4: Notice that the noise-correlated system is transformed into a new system where the process noise and the measurement noise are uncorrelated to each other. Its purpose is to make the calculation easier in this paper. With the upper bounds of actual covariances in (4) and (6), Theorem 1 provides a group of recursive equations for designing the optimal LKF. Furthermore, the one-step predictor is yielded as a byproduct. Since $X_{a,k}(t) = \mathrm{col}\{x(t), \bar{y}_{e,k}(t-1)\}$, the optimal LKF of the state $x(t)$ can be obtained by $\hat{x}_k(t|t) = \begin{bmatrix} I_n & 0\end{bmatrix}\hat{X}_{a,k}(t)$.

Define conservative or actual Kalman estimation errors as $\tilde{X}_{a,k}(t) \triangleq X_{a,k}(t) - \hat{X}_{a,k}(t)$, where $X_{a,k}(t)$ is conservative state in (18). From (15), (18), (22), (31) and (34), it is easy to obtain the state estimation error equation (i.e., Eq. (35)). According to the calculation formula of the covariance matrix $\bar{P}_k(t|t) = E[\tilde{X}_{a,k}(t)\tilde{X}_{a,k}^T(t)]$, combined with Eq. (35), the Lyapunov equation of the conservative estimation error covariance is obtained (i.e., Eq. (36)).

Remark 5: Note that the matrix inverse operation in a Kalman filter is the major computation burden, and its computational complexity is $O(\pi^3)$, where π is the dimensionality of the estimated state [46]. The centralized filter augments all ρ measurements and needs a computational complexity of $O(\rho^3 \times \pi^3)$, while the distributed fusion Kalman filter computers ρ local filters in parallel such that the computational complexity is reduced to $O(\rho \times \pi^3)$. Similarly, the matrix inverse computation of $\bar{Q}_{\varepsilon_k}(t)$ in the formulation (25) is the major computation burden. Since the dimension of $\bar{Q}_{\varepsilon_k}(t)$ is r_k, the computational complexity of the proposed conservative recursive Kalman filter is $O(L \times r_k^3)$.

B. ROBUST WEIGHTED FUSION ESTIMATION UNDER DoS ATTACKS

Lemma 1 (See [16]): If the $n \times n$ dimensional matrix B is a positive semidefinite matrix, i.e., $B \geq 0$, thus the $nL \times nL$ dimensional matrix B is regarded as a positive semidefinite matrix, i.e.,

$$\mathrm{B} = \begin{bmatrix} B & \cdots & B \\ \vdots & \ddots & \vdots \\ B & \cdots & B \end{bmatrix} \geq 0 \quad (37)$$

Lemma 2 (See [4]): According to the unbiased linear minimum variance (ULMV) criteria, the conservative optimal DFKF is expressed as

$$\hat{x}_d(t|t) = \sum_{k=1}^{L} \bar{\Omega}_k^d(t|t)\hat{x}_k(t|t) \quad (38)$$

where the local optimal state estimator of the k-th sink node is given by $\hat{x}_k(t|t) = M\hat{X}_{a,k}(t)(k = 1, \cdots, L)$, $M = \begin{bmatrix} I_n & 0 \end{bmatrix}$. The row vector of the optimal matrix weighted coefficient is defined by $\bar{\Omega}_d(t|t) = [\bar{\Omega}_1^d(t|t), \cdots, \bar{\Omega}_L^d(t|t)]$, which is calculated by $\bar{\Omega}_d(t|t) = (\beta^T\bar{P}_f^{-1}(t|t)\beta)^{-1}\beta^T\bar{P}_f^{-1}(t|t)$, where $\beta = \mathrm{col}\{I_n, \cdots, I_n\}$. The $nL \times nL$ dimensional augmented covariance $\bar{P}_f(t|t)$ is defind as

$$\bar{P}_f(t|t) = (\bar{P}_{W_{kj}}(t|t))_{nL\times nL}(k, j = 1, \cdots, L) \quad (39)$$

where $\bar{P}_{W_{kj}}(t|t) = M\bar{P}_{kj}(t|t)M^T$, $\bar{P}_{kj}(t|t) = E[\tilde{X}_{a,k}(t)\tilde{X}_{a,j}^T(t)]$ is the conservative local estimation error cross-covariance. Then the conservative estimation error cross-covariance of the k-th and the j-th sink nodes can be calculated as

$$\begin{aligned}\bar{P}_{kj}(t|t) = \bar{H}_k^e(t)\bar{P}_{kj}(t-1|t-1)[\bar{H}_j^e(t)]^T \\ + \bar{N}_k^e(t)\bar{Q}_\omega(t-1)[\bar{N}_j^e(t)]^T (k \neq j) \end{aligned} \quad (40)$$

with the initial value $\bar{P}_{kj}(0|0) = \bar{P}_k(0|0)$. Therefore, the conservative fusion estimation error covariance can be computed by

$$\bar{P}_d(t|t) = \bar{\Omega}_d(t|t)\bar{P}_f(t|t)\bar{\Omega}_d^T(t|t) \quad (41)$$

According to Lemma 2, the actual fusion estimation error covariance is written as

$$P_d(t|t) = \Omega_d(t|t)P_f(t|t)\Omega_d^T(t|t) \quad (42)$$

where $\Omega_d(t|t) = (\beta^T P_f^{-1}(t|t)\beta)^{-1}\beta^T P_f^{-1}(t|t)$. The augmented covariance $P_f(t|t)$ is obtained by

$$P_f(t|t) = (P_{W_{kj}}(t|t))_{nL\times nL} \quad (43)$$

where $P_{W_{kj}}(t|t) = MP_{kj}(t|t)M^T$. The actual estimation error covariance and the actual estimation error cross-covariance are written as

$$\begin{aligned}P_k(t|t) = H_k^e(t)P_k(t-1|t-1)[H_k^e(t)]^T \\ + N_k^e(t)Q_\omega(t-1)[N_k^e(t)]^T \\ + \eta_k(t)K_k(t)\Theta_k^e(t)Q_{v_k}(t)[\eta_k(t)K_k(t)\Theta_k^e(t)]^T \end{aligned} \quad (44)$$

$$P_{kj}(t|t) = H_k^e(t)P_{kj}(t-1|t-1)[H_j^e(t)]^T + N_k^e(t)Q_\omega(t-1)[N_j^e(t)]^T (k \neq j) \tag{45}$$

where the initial value $P_{kj}(0|0) = P_k(0|0) = \bar{P}_{kj}(0|0)$.

Theorem 2: For the compensation systems (14) and (15), the actual DFKF is a guaranteed robust DFKF with the upper bound covariances $\bar{Q}_w(t)$, $\bar{Q}_{v_k}(t)$ and $\bar{P}_0$. Then for all admissible actual covariances $Q_w(t)$, $Q_{v_k}(t)$ and P_0 satisfying (4) and (6), one has

$$P_d(t|t) \leq \bar{P}_d(t|t) \tag{46}$$

where $\bar{P}_d(t|t)$ is the minimum upper bound for all admissible uncertain noise covariances.

Proof: By Lemma 2, the global Lyapunov equation can be written as

$$\bar{P}_f(t|t) = M\bar{P}_\lambda(t|t)M^T \tag{47}$$

$$\bar{P}_\lambda(t|t) = \bar{H}(t)\bar{P}_\lambda(t-1|t-1)\bar{H}^T(t) + \bar{N}(t)\bar{Q}_\lambda(t-1)\bar{N}^T(t) + \Xi(t)\circ\bar{K}(t)\Sigma(t)\bar{R}(t)\Sigma^T(t)\bar{K}^T(t) \tag{48}$$

where

$$\begin{cases} \bar{H}(t) = \mathrm{diag}\{\bar{H}_1^e(t), \cdots, \bar{H}_L^e(t)\} \\ \bar{N}(t) = \mathrm{diag}\{\bar{N}_1^e(t), \cdots, \bar{N}_L^e(t)\} \\ \bar{K}(t) = \mathrm{diag}\{\bar{K}_1(t), \cdots, \bar{K}_L(t)\} \\ \Xi(t) = \mathrm{diag}\{\eta_1(t)\eta_1(t)I_{m_1}, \cdots, \eta_L(t)\eta_L(t)I_{m_L}\} \\ \Sigma(t) \ = \mathrm{diag}\{\Theta_1^e(t), \cdots, \Theta_L^e(t)\} \\ \bar{R}(t) = \mathrm{diag}\{\bar{Q}_{v_1}(t)I_1, \cdots, \bar{Q}_{v_L}(t)I_L\} \\ \bar{Q}_\lambda(t-1) = \begin{bmatrix} \bar{Q}_\omega(t-1) & \cdots & \bar{Q}_\omega(t-1) \\ \vdots & \ddots & \vdots \\ \bar{Q}_\omega(t-1) & \cdots & \bar{Q}_\omega(t-1) \end{bmatrix} \end{cases}$$

Similarly, it follows from (47) and (48) that the global actual fusion estimation error covariance is calculated by

$$P_f(t|t) = MP_\lambda(t|t)M^T \tag{49}$$

$$P_\lambda(t|t) = H(t)P_\lambda(t-1|t-1)H^T(t) + N(t)Q_\lambda(t-1)N^T(t) + \Xi(t)\circ K(t)\Sigma(t)\,R(t)\Sigma^T(t)K^T(t) \tag{50}$$

where

$$\begin{cases} H(t) = \mathrm{diag}\{H_1^e(t), \cdots, H_L^e(t)\} \\ N(t) = \mathrm{diag}\{N_1^e(t), \cdots, N_L^e(t)\} \\ K(t) = \mathrm{diag}\{K_1(t), \cdots, K_L(t)\} \\ R(t) = \mathrm{diag}\{Q_{v_1}(t)I_1, \cdots, Q_{v_L}(t)I_L\} \\ Q_\lambda(t-1) = \begin{bmatrix} Q_\omega(t-1) & \cdots & Q_\omega(t-1) \\ \vdots & \ddots & \vdots \\ Q_\omega(t-1) & \cdots & Q_\omega(t-1) \end{bmatrix} \end{cases}$$

Subtracting (49) from (47), we denote $\tilde{P}_f(t|t) \triangleq \bar{P}_f(t|t) - P_f(t|t)$, $\tilde{P}_\lambda(t|t) \triangleq \bar{P}_\lambda(t|t) - P_\lambda(t|t)$. Thus the Lyapunov equation is given by

$$\tilde{P}_f(t|t) = M\tilde{P}_\lambda(t|t)M^T \tag{51}$$

$$\begin{aligned}\tilde{P}_\lambda(t|t) = {} & \bar{H}(t)\bar{P}_\lambda(t-1|t-1)\bar{H}^T(t) - H(t) \\ & \times P_\lambda(t-1|t-1)H^T(t) + \bar{N}(t)\bar{Q}_\lambda(t-1)\bar{N}^T(t) \\ & - N(t)Q_\lambda(t-1)N^T(t) + \Xi(t)\circ[\bar{K}(t)\Sigma(t)\bar{R}(t) \\ & \times \Sigma^T(t)\bar{K}^T(t) - K(t)\Sigma(t)R(t)\Sigma^T(t)K^T(t)]\end{aligned} \tag{52}$$

It follows from (24), (23) and (4) that $\bar{Q}_\omega(t) - Q_\omega(t) \geq 0$ and $\bar{Q}_{v_k}(t) - Q_{v_k}(t) \geq 0$. By Lemma 1, $\bar{Q}_\lambda(t-1) - Q_\lambda(t-1) \geq 0$ and $\bar{R}(t) - R(t) \geq 0$ are obtained. Let $P_{kj}(0|0) = P(0)$, $\bar{P}_{kj}(0|0) = \bar{P}(0)$. And then $P_{W_{kj}}(0|0) = MP_{kj}(0|0)M^T = P_0$, $\bar{P}_{W_{kj}}(0|0) = \bar{P}_0(k, j = 1, \cdots, L)$. The initial value of the augmented matrix can be obtained by (47) and (48), which means that

$$\begin{cases} \bar{P}_f(0|0) = \begin{bmatrix} \bar{P}_0 & \cdots & \bar{P}_0 \\ \vdots & \ddots & \vdots \\ \bar{P}_0 & \cdots & \bar{P}_0 \end{bmatrix} \\ \bar{P}_\lambda(0|0) = \begin{bmatrix} \bar{P}(0) & \cdots & \bar{P}(0) \\ \vdots & \ddots & \vdots \\ \bar{P}(0) & \cdots & \bar{P}(0) \end{bmatrix} \end{cases}$$

The actual initial value can be obtained in the same way. It follows from (6) that $\bar{P}_0 - P_0 \geq 0$, then one has $\bar{P}_f(0|0) - P_f(0|0) \geq 0$. Therefore, after a few simple steps on (52), $\tilde{P}_f(0|0) = \bar{P}_f(0|0) - P_f(0|0) \geq 0$, then $\tilde{P}_\lambda(0|0) = \bar{P}_\lambda(0|0) - P_\lambda(0|0) \geq 0$. From (52), one has $\tilde{P}_\lambda(1|1) \geq 0$, then $\tilde{P}_f(1|1) \geq 0$. By mathematical induction, $\tilde{P}_\lambda(t|t) \geq 0$, one has $\tilde{P}_f(t|t) \geq 0$, which implies

$$\bar{P}_f(t|t) - P_f(t|t) \geq 0 \tag{53}$$

It follows from (41), (42) and (53) that the inequality can be calculated by

$$\bar{P}_d(t|t) - P_d(t|t) \geq 0 \tag{54}$$

Obviously, one has $P_d(t|t) \leq \bar{P}_d(t|t)$. Therefore, (46) is true for all admission uncertain covariance $Q_w(t)$, $Q_{v_k}(t)$ and P_0 satisfing (4) and (6). Suppose that $\bar{P}_d^*(t|t)$ is an arbitrary upper bound estimation error covariance matrix for $P_d(t|t)$. When $Q_w(t) = \bar{Q}_w(t)$, $Q_{v_k}(t) = \bar{Q}_{v_k}(t)$ and $P_0 = \bar{P}_0$, then (4) and (6) are satisfied. Thus, $P_d(t|t)$ satisfies $P_d(t|t) \leq \bar{P}_d^*(t|t)$. In addition, it is known that $\tilde{P}_f(t|t) = 0$ for any time $t \geq 0$, then $\bar{P}_d(t|t) = P_d(t|t)$. Therefore, $\bar{P}_d(t|t) \leq \bar{P}_d^*(t|t)$, which means that $\bar{P}_d(t|t)$ is the minimum upper bound covariance of the actual fusion estimation error covariance $P_d(t|t)$. In this case, the actual DFKF is robust according to Definition 1. This completes the proof.

Remark 6: For the compensation systems (14) and (15) under DoS attacks and bandwidth constraints, our objective is to design the robust DFKF by weighting the local state estimators with weighting matrices $\bar{\Omega}_d(t|t)$ and $\Omega_d(t|t)$, and then obtain the conservative and actual weighted fusion Kalman estimation error covariances, respectively. In this sense, actual state estimation error covariances $P_d(t|t)$ yielded by all admissible uncertain noise covariances $Q_w(t)$, $Q_{v_k}(t)$ and P_0 satisfying (4) and (6) have the corresponding minimal upper bound $\bar{P}_d(t|t)$. Therefore, the robustness of the DFKF can be guaranteed in this paper.

IV. SIMULATION EXAMPLES

Consider the systems (1) and (2) with the following parameters:

$$A(t) = \begin{bmatrix} 0.9 & 0.1 & 0 & 0 \\ 0 & 0.25 & 0 & \cos(t-1) \\ 0 & 0 & 0.46 & 0 \\ 0 & 0.67 & 0 & \sin^2(t-1) \end{bmatrix}$$

$$B(t) = \begin{bmatrix} 0.1+\sin(t^3) & 1 & 1 & 0.02 \times \frac{t}{2} \end{bmatrix}^T$$

$$C_1(t) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} C_2(t) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

The system noises of $w(t)$, $v_1(t)$ and $v_2(t)$ are uncorrelated zero-mean Gaussian white noises. To show that the conservative error covariance $\bar{P}_d(t|t)$ is an upper bound of actual error covariance $P_d(t|t)$, three groups of experiments are carried out with different actual noise covariances, where the upper bounds of actual noise covariances in these groups are identical. Concretely, $Q_{w,1} = 0.8$, $Q_{v_{1,1}} = \mathrm{diag}\{0.4, 0.7, 0.8\}$, $Q_{v_{2,1}} = \mathrm{diag}\{0.3, 0.54, 0.8\}$ for the first group, $Q_{w,2} = 0.1$, $Q_{v_{1,2}} = \mathrm{diag}\{0.1, 0.5, 0.3\}$, $Q_{v_{2,2}} = \mathrm{diag}\{0.1, 0.14, 0.18\}$ for the second group, $Q_{w,3} = 0.05$, $Q_{v_{1,3}} = \mathrm{diag}\{0.05, 0.09, 0.07\}$, $Q_{v_{2,3}} = \mathrm{diag}\{0.03, 0.05, 0.08\}$ for the third group, and their corresponding upper bounds are set to be $\bar{Q}_w = 1$, $\bar{Q}_{v_1} = \mathrm{diag}\{0.6, 0.9, 1.0\}$, $\bar{Q}_{v_2} = \mathrm{diag}\{0.46, 0.68, 0.9\}$. The initial states are generated by a random variable with $\mu_0 = 0$ and $P_0 = I_4$, and the corresponding known upper bound is assumed to be $\bar{P}_0 = \mathrm{diag}\{1.12, 1.18, 1.05, 1.09\}$.

The dimensionality reduction strategy is employed in this example with $r_1 = 2$ and $r_2 = 1$. It means that only two components of the measurement in the first sink node and one component of the measurement in the second sink can be transmitted to the estimator. Moreover, the numbers of the selected components correspond to sink node are $\Delta_1 = \Delta_2 = 3$. On the other hand, the transmitted measurement may suffer from DoS attacks, where the energy of the attack is always finite during a period of time. Therefore, the maximum numbers of consecutive DoS attacks are 3 and 1 in each transmission channel.

Fig. 2, Fig. 3 and Fig. 4 are obtained from the actual noise covariances and their upper bounds of the first group. A clear trajectory comparison is performed in Fig. 2 to verify the effectiveness of the DFKF, where the LKF $\hat{x}_k(t|t)$ calculated by Theorem 1, the DFKF $\hat{x}_d(t|t)$ yielded by Lemma 2 and the state $x(t)$. It can be seen that the designed DFKF estimates the real state better. The Monte Carlo method is used to calculate the mean square errors (MSEs) with an average of 100 runs. Subsequently, Fig. 3 shows the MSEs results of the LKF and the DFKF. As can be observed, the fusion filter outperforms each local estimator in estimation accuracy, which is consistent with the advantages of the fusion method. Furthermore,

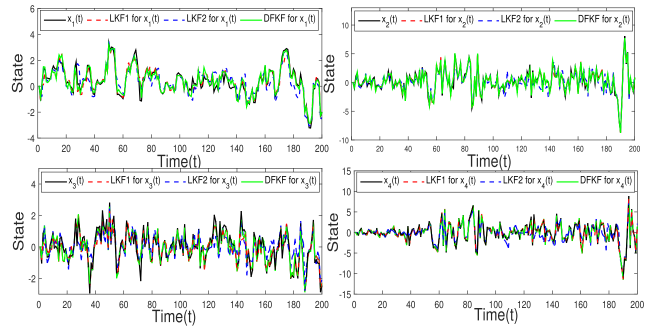


FIGURE 2. Trajectories of each components of the state $x(t)$, the LKFs $\hat{x}(t|t)$ and DFKF $\hat{x}_d(t|t)$.

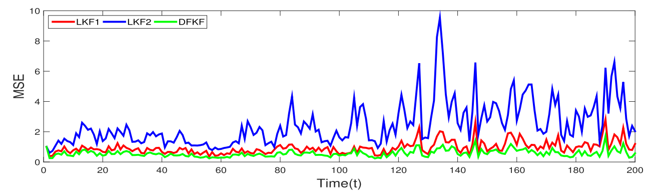


FIGURE 3. Mean square error of the LKFs and DFKF.

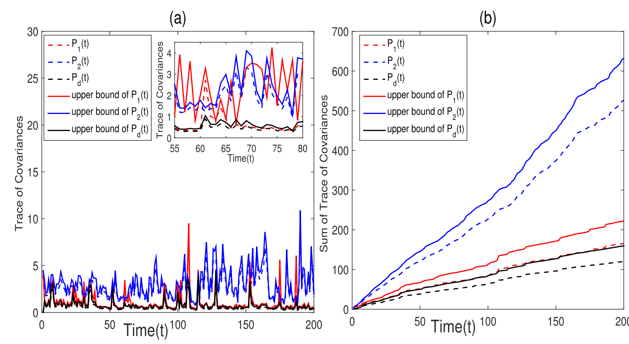


FIGURE 4. (a) The traces of the actual and conservative estimation error covariances; (b) the sum of traces of the actual and conservative estimation error covariances.

the comparison of traces of the actual and conservative estimation error covariance is illustrated in Fig. 4 (a) to evaluate the robustness of robust LKF/ DFKF, including the conservative local estimation error covariance $\bar{P}_k(t|t)$ yielded by Theorem 1, the conservative fusion estimation error covariance $\bar{P}_d(t|t)$, and their actual ones $P_k(t|t)$ and $P_d(t|t)$. The comparison result shows that the traces of their actual estimation error covariance are smaller than that of conservative estimation error covariance matrix. Furthermore, by the cumulative summations of the actual and conservative local/fusion estimation error covariance matrix traces, the trajectory comparison in Fig. 4 (b) shows the robustness of robust LKF/ DFKF. In order to analyze the robustness of distributed fusion estimators clearly, three groups of different actual noise covariances are first presented, and their actual fusion estimation error covariances $P_d(t|t)$ are obtained respectively. It can be seen from Fig. 5 that the increase of the actual noise covariance will result in large actual fusion estimation error covariance. Then they were compared with the conservative fusion estimation error covariance $\bar{P}_d(t|t)$, and it is tightly bounded with conservative fusion estimation error covariance

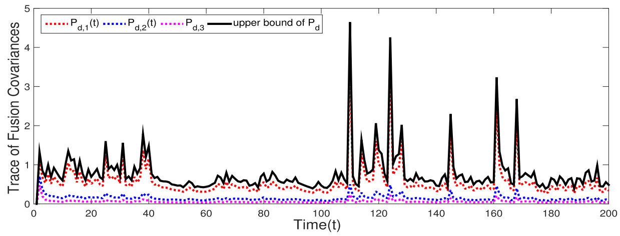


FIGURE 5. The traces of the actual and conservative fusion estimation error covariance.

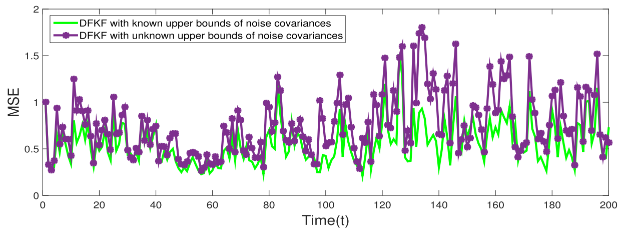


FIGURE 6. Comparison of the estimation performance of the DFKF with known and unknown upper bounds of the noise covariances.

and the conservative fusion estimation error covariance was the upper bound of the actual fusion estimation error covariances. It is obvious from the above analysis that the actual DFKF was a guaranteed robust DFKF, which has been proved by Theorem 2. Moreover, the performance of the proposed DFKF under known and unknown upper bounds of noise covariances is compared. The result can be seen from Fig. 6. It is observed that the performance of the proposed DFKF with known upper bounds of noise covariances is far more better.

V. CONCLUSION

This paper has studied the distributed robust fusion estimation problem for the NMFE systems under DoS attacks and uncertain convariances, where DoS attacks were characterized by a binary variable without distributed information. The dimensionality reduction strategy was utilized for network communication with limited bandwidth capacity, and the compensation strategy was proposed to keep estimated performance against both dimensionality reduction and DoS attacks. Based on the upper bounds of the noise covariances of the conservative system in the worst case, the conservative optimal LKF/DFKF was derived, which depends on the ULMV estimation criterion. Meanwhile, the robust LKF was obtained in the actual system, then the optimal LKF/DFKF was derived. Further, the problem was transformed into a positive semidefinite decision problem about the Lyapunov equation solution, thus the robustness of robust DFKF was strictly proved. Finally, an illustrative example was exploited to demonstrate the effectiveness of the proposed methods.

In practical applications, due to random disturbances, unmodeled dynamics and measurement errors, the system model is often approximate and uncertain. Therefore, how to design a distributed robust fusion estimator for uncertain time-varying systems in the presence of bandwidth constraints, unknown disturbances and DoS attacks will be our future work.

REFERENCES

[1] S. Sun, H. Lin, J. Ma, and X. Li, "Multi-sensor distributed fusion estimation with applications in networked systems: A review paper," *Inf. Fusion*, vol. 38, pp. 122–134, Nov. 2017.

[2] W.-J. Qi, P. Zhang, and Z.-L. Deng, "Robust sequential covariance intersection fusion Kalman filtering over multi-agent sensor networks with measurement delays and uncertain noise variances," *Acta Automatica Sinica*, vol. 40, no. 11, pp. 2632–2642, Nov. 2014.

[3] M. M. Rana, L. Li, and S. W. Su, "Cyber attack protection and control of microgrids," *IEEE/CAA J. Automatica Sinica*, vol. 5, no. 2, pp. 602–609, Mar. 2018.

[4] S.-L. Sun and Z.-L. Deng, "Multi-sensor optimal information fusion Kalman filter," *Automatica*, vol. 40, no. 6, pp. 1017–1023, Jun. 2004.

[5] W. Qi, P. Zhang, and Z. Deng, "Robust weighted fusion Kalman filters for multisensor time-varying systems with uncertain noise variances," *Signal Process.*, vol. 99, pp. 185–200, Jun. 2014.

[6] A. Assa and F. Janabi-Sharifi, "A Kalman filter-based framework for enhanced sensor fusion," *IEEE Sensors J.*, vol. 15, no. 6, pp. 3281–3292, Jun. 2015.

[7] J. Ma and S. Sun, "Centralized fusion estimators for multisensor systems with random sensor delays, multiple packet dropouts and uncertain observations," *IEEE Sensors J.*, vol. 13, no. 4, pp. 1228–1235, Apr. 2013.

[8] Q. Shen, J. Liu, X. Zhou, W. Qin, L. Wang, and Q. Wang, "Centralized fusion methods for multi-sensor system with bounded disturbances," *IEEE Access*, vol. 7, pp. 141612–141626, 2019.

[9] M. Rahmani and M. Abolhasani, "State estimation for stochastic time-varying multisensor systems with multiplicative noises: Centralized and decentralized data fusion," *Asian J. Control*, vol. 21, no. 4, pp. 1547–1555, Jul. 2019.

[10] B. Chen, W.-A. Zhang, and L. Yu, "Distributed fusion estimation with missing measurements, random transmission delays and packet dropouts," *IEEE Trans. Autom. Control*, vol. 59, no. 7, pp. 1961–1967, Jul. 2014.

[11] X. Xie, D. Yue, and J. H. Park, "Observer-based state estimation of discrete-time fuzzy systems based on a joint switching mechanism for adjacent instants," *IEEE Trans. Cybern.*, vol. 50, no. 8, pp. 3545–3555, Aug. 2020.

[12] G. Hao and S. Sun, "Distributed fusion filter for nonlinear multi-sensor systems with correlated noises," *IEEE Access*, vol. 8, pp. 39548–39560, 2020.

[13] W. Liu, G. Tao, Y. Fan, and G. Zhang, "Robust fusion steady-state filtering for multisensor networked systems with one-step random delay, missing measurements, and uncertain-variance multiplicative and additive white noises," *Int. J. Robust Nonlinear Control*, vol. 29, no. 14, pp. 4716–4754, Sep. 2019.

[14] C. Zhao, B. Xiang, and B. Chen, "Fusion estimation for discrete time-varying systems with bounded nonlinearities," *IEEE Access*, vol. 7, pp. 27097–27105, 2019.

[15] X. Wang, W. Liu, and Z. Deng, "Robust weighted fusion Kalman estimators for systems with multiplicative noises, missing measurements and uncertain-variance linearly correlated white noises," *Aerosp. Sci. Technol.*, vol. 68, pp. 331–344, Sep. 2017.

[16] Y. Hu, Z. Jin, and Y. Wang, "State fusion estimation for networked stochastic hybrid systems with asynchronous sensors and multiple packet dropouts," *IEEE Access*, vol. 6, pp. 10402–10409, 2018.

[17] G.-Q. Duan and S.-L. Sun, "Self-tuning distributed fusion filter for multi-sensor systems subject to unknown model parameters and missing measurement rates," *IEEE Access*, vol. 6, pp. 61519–61528, 2018.

[18] B. Chen, W. Zhang, G. Hu, and L. Yu, "Networked fusion Kalman filtering with multiple uncertainties," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 51, no. 3, pp. 2232–2249, Jul. 2015.

[19] J. Wang, Y. Mao, Z. Li, J. Gao, and H. Liu, "Robust fusion estimation for multisensor uncertain systems with state delay based on data-driven communication strategy," *IEEE Access*, vol. 8, pp. 151888–151897, 2020.

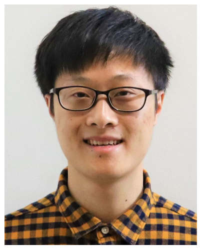
[20] W. Liu, F. Deng, J. Liang, and H. Liu, "Distributed average consensus in multi-agent networks with limited bandwidth and time-delays," *IEEE/CAA J. Automatica Sinica*, vol. 1, no. 2, pp. 193–203, Apr. 2014.

[21] J. Fang and H. Li, "Hyperplane-based vector quantization for distributed estimation in wireless sensor networks," *IEEE Trans. Inf. Theory*, vol. 55, no. 12, pp. 5679–5692, Dec. 2009.

[22] Z. Zhang, J. Li, and L. Liu, "Distributed state estimation and data fusion in wireless sensor networks using multi-level quantized innovation," *Sci. China Inf. Sci.*, vol. 59, no. 2, pp. 1–15, Feb. 2016.
[23] W.-A. Zhang, L. Yu, and G. Feng, "Optimal linear estimation for networked systems with communication constraints," *Automatica*, vol. 47, no. 9, pp. 1992–2000, Sep. 2011.
[24] B. Chen, D. W. C. Ho, W.-A. Zhang, and L. Yu, "Distributed dimensionality reduction fusion estimation for cyber-physical systems under DoS attacks," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 49, no. 2, pp. 455–468, Feb. 2019.
[25] B. Chen, G. Hu, D. W. C. Ho, and L. Yu, "Distributed covariance intersection fusion estimation for cyber-physical systems with communication constraints," *IEEE Trans. Autom. Control*, vol. 61, no. 12, pp. 4020–4026, Dec. 2016.
[26] B. Chen, W.-A. Zhang, L. Yu, G. Hu, and H. Song, "Distributed fusion estimation with communication bandwidth constraints," *IEEE Trans. Autom. Control*, vol. 60, no. 5, pp. 1398–1403, May 2015.
[27] B. Chen, D. W. C. Ho, G. Hu, and L. Yu, "Secure fusion estimation for bandwidth constrained cyber-physical systems under replay attacks," *IEEE Trans. Cybern.*, vol. 48, no. 6, pp. 1862–1876, Jun. 2018.
[28] B. Chen, L. Yu, W. A. Zhang, and H. X. Wang, "Distributed H_∞ fusion filtering with communication bandwidth constraints," *Signal Process.*, vol. 96, pp. 284–289, Mar. 2014.
[29] Z. Wang, D. Wang, B. Shen, and F. E. Alsaadi, "Centralized security-guaranteed filtering in multirate-sensor fusion under deception attacks," *J. Franklin Inst.*, vol. 355, no. 1, pp. 406–420, Jan. 2018.
[30] K. G. Vamvoudakis, "Event-triggered optimal adaptive control algorithm for continuous-time nonlinear systems," *IEEE/CAA J. Automatica Sinica*, vol. 1, no. 3, pp. 282–293, Jul. 2014.
[31] J. Gao, S. Chai, B. Zhang, and Y. Xia, "Research about DoS attack against ICPS," *Sensors*, vol. 19, no. 7, p. 1542, Mar. 2019.
[32] H. Zhang, Y. Qi, J. Wu, L. Fu, and L. He, "DoS attack energy management against remote state estimation," *IEEE Trans. Control Netw. Syst.*, vol. 5, no. 1, pp. 383–394, Mar. 2018.
[33] H. Zhang, P. Cheng, L. Shi, and J. Chen, "Optimal denial-of-service attack scheduling with energy constraint," *IEEE Trans. Autom. Control*, vol. 60, no. 11, pp. 3023–3028, Nov. 2015.
[34] H. Zhang, Y. F. Qi, H. Zhou, J. Zhang, and J. Sun, "Testing and defending methods against DoS attack in state estimation," *Asian J. Control*, vol. 19, no. 4, pp. 1288–1305, 2017.
[35] H. Yuan and Y. Xia, "Resilient strategy design for cyber-physical system under DoS attack over a multi-channel framework," *Inf. Sci.*, vols. 454–455, pp. 312–327, Jul. 2018.
[36] K. Ding, X. Ren, D. E. Quevedo, S. Dey, and L. Shi, "DoS attacks on remote state estimation with asymmetric information," *IEEE Trans. Control Netw. Syst.*, vol. 6, no. 2, pp. 653–666, Jun. 2019.
[37] B. Chen, L. Yu, D. W. C. Ho, and W. A. Zhang, "Networked fusion estimation under denial-of-service attacks," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 3835–3840, 2017.
[38] D. Ding, Z. Wang, and Q.-L. Han, "A set-membership approach to event-triggered filtering for general nonlinear systems over sensor networks," *IEEE Trans. Autom. Control*, vol. 65, no. 4, pp. 1792–1799, Apr. 2020.
[39] A. Chattopadhyay and U. Mitra, "Security against false data-injection attack in cyber-physical systems," *IEEE Trans. Control Netw. Syst.*, vol. 7, no. 2, pp. 1015–1027, Jun. 2020.
[40] J. Y. Lu and R. X. Niu, "A state estimation and malicious attack game in multi-sensor dynamic systems," in *Proc. Int. Conf. Inf. Fusion*, Jul. 2015, pp. 932–936.
[41] J. Chen, C. X. Dou, L. Xiao, and Z. Wang, "Fusion state estimation for power systems under DoS attacks: A switched system approach," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 49, no. 9, pp. 1679–1687, Aug. 2019.
[42] Y. Liu, Z. Wang, L. Ma, and F. E. Alsaadi, "A partial-nodes-based information fusion approach to state estimation for discrete-time delayed stochastic complex networks," *Inf. Fusion*, vol. 49, pp. 240–248, Sep. 2019.
[43] M. Nilsson, "Kalman filtering with unknown noise covariances," in *Proc. Reglermote*, 2006, p. 14.
[44] W.-J. Qi, P. Zhang, G.-H. Nie, and Z.-L. Deng, "Robust weighted fusion Kalman predictors with uncertain noise variances," *Digit. Signal Process.*, vol. 30, pp. 37–54, Jul. 2014.
[45] A. Cetinkaya, H. Ishii, and T. Hayakawa, "An overview on denial-of-service attacks in control systems: Attack models and security analyses," *Entropy*, vol. 21, no. 2, p. 210, Feb. 2019.
[46] K. S. Babu and K. Detroja, "Inverse free Kalman filter using approximate inverse of diagonally dominant matrices," *IEEE Control Syst. Lett.*, vol. 3, no. 1, pp. 120–125, Jan. 2019.

MENG XU received the B.S. degree in information and computing science from the Shengli College, China University of Petroleum, Shandong, China, in 2018, where she is currently pursuing the master's degree in science from the Zhejiang University of Technology, Zhejiang, China.

Her current research interests include robust distributed estimation and information fusion.



YUCHEN ZHANG received the B.E. degree in automation from the Zhejiang University of Technology, Hangzhou, China, in 2017, where he is currently pursuing the Ph.D. degree in control science and engineering.

His current research interests include distributed estimation of interconnected systems and fusion estimation of binary sensor systems.

DONGMEI ZHANG was born in Hangzhou, China. She received the M.D. and Ph.D. degrees in information engineering from the Zhejiang University of Technology, Hangzhou, in 2000 and 2008, respectively.

She is currently a Professor with the Department of Mathematics, Zhejiang University of Technology. Her research interests include networked control and information fusion.

BO CHEN (Member, IEEE) received the B.S. degree in information and computing science from the Jiangxi University of Science and Technology, Ganzhou, China, in 2008, and the Ph.D. degree in control theory and control engineering from the Zhejiang University of Technology, Hangzhou, China, in 2014.

From 2014 to 2015 and from 2017 to 2018, he was a Research Fellow with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. From 2015 to 2017, he was also a Postdoctoral Research Fellow with the Department of Mathematics, City University of Hong Kong, Hong Kong. He is currently a Professor with the Department of Automation, Zhejiang University of Technology. His current research interests include information fusion, distributed estimation and control, networked fusion systems, and secure estimation of cyber-physical systems.

Dr. Chen was a recipient of the Outstanding Thesis Award from the Chinese Association of Automation, in 2015.