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Multi-Valued Eigen-Processing for Isolating Multiple Sources With a Rectangular Array

MICHAEL D. COLLINS, (Member, IEEE), AND JOSEPH F. LINGEVITCH
U.S. Naval Research Laboratory, Washington, DC 20375, USA
Corresponding author: Michael D. Collins (michael.collins@nrl.navy.mil)

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ABSTRACT The multi-valued Bartlett (MVB) processor is useful for determining the locations of multiple acoustic sources in the ocean [J. Acoust. Soc. Am. 97, 235–241 (1995)]. This approach was originally applied to a vertical line array of hydrophones. The application to a rectangular array is explored here. The MVB processor is an eigen-processor that is based on the eigenvectors of the covariance matrix. It is multi-valued in the sense that an ambiguity surface is constructed for each member of a subset of the eigenvectors that correspond to the largest eigenvalues. The motivation for the approach is the fact that energy from different sources tends to partition into different eigenvectors. One of the advantages of the MVB processor on a rectangular array is that it is possible to determine if the partitioning is favorable without computing replica fields, which is often the most time-consuming task of matched-field processing computations. Examples are presented to illustrate the capabilities and limitations of the approach.

INDEX TERMS Ocean acoustics, matched-field processing, beamforming, eigen-processing, source localization, replica fields, multi-valued Bartlett processor, rectangular array.

I. INTRODUCTION

A considerable amount of information may exist in a data set that is collected with an $m \times n$ rectangular array of acoustic receivers, where the horizontal dimension m and vertical dimension n are both substantially greater than one. It seems unlikely that a signal-processing technique that was originally developed for a one-dimensional array would be capable of realizing the full potential of a two-dimensional array. A signal-processing technique that is designed specifically for rectangular arrays is considered here for a matched-field processing [1]–[3] scenario in which the array is placed in an ocean acoustic waveguide with one of its axes oriented vertically (the approach is also applicable to other problems in acoustics and other fields). Some of the advantages of the approach are illustrated for test cases involving multiple moving sources.

Various single-valued ambiguity functions have been developed, including the Bartlett processor, the maximum likelihood and maximum entropy methods, the MUSIC method [4], and the environmentally tolerant processor [5].

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There are cases for which some of these approaches appear to have advantages, but they are merely different ways of quantifying the correlation between test solutions and data. Like the MUSIC method, the multi-valued Bartlett (MVB) processor [6] is based on the eigenvectors of the covariance matrix, but this approach is multi-valued in the sense that multiple ambiguity surfaces are constructed, one for each member of a subset of the eigenvectors. The motivation for the MVB processor is the fact that energy from different sources tends to partition into different eigenvectors. After the signals from different sources have been isolated from each other into different eigenvectors, the MVB processor exploits this partitioning by analyzing each eigenvector separately. In a single-valued ambiguity surface, it may be difficult to assess the local maxima and decide which (if any) of them correspond to sources rather than sidelobes. The MVB processor does not require a subjective assessment of local maxima; the main peak in each ambiguity surface is the only one that is considered.

As multiple sources move through a waveguide, there may be ambiguous points at which the acoustic fields from two or more sources are correlated on the array. At such points, the partitioning will be unfavorable, and the MVB processor will

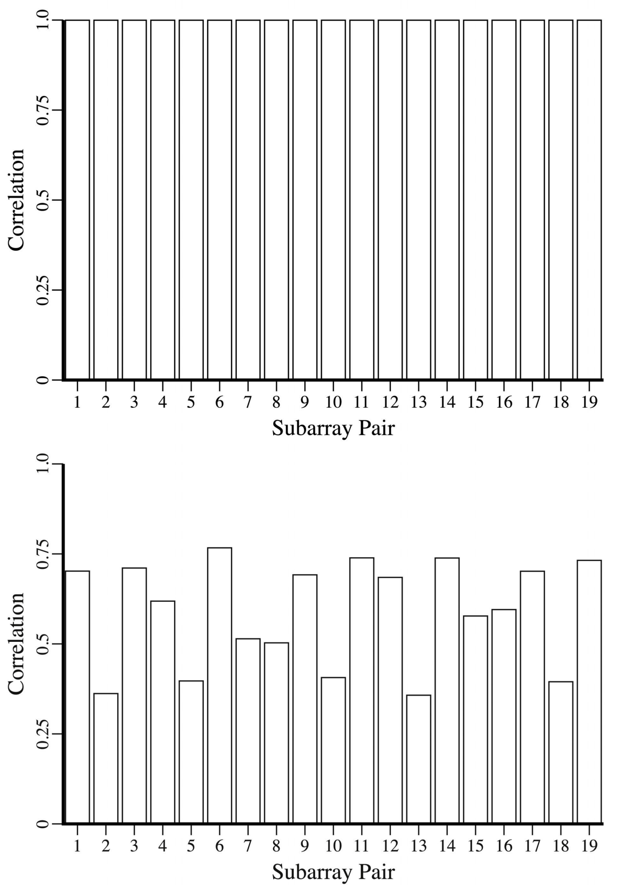

FIGURE 1. Results for example A. Complex pressure correlations between adjacent vertical subarray pairs. The subarrays are highly correlated when only one source is turned on (top). The subarrays are not correlated when three sources are turned on (bottom).

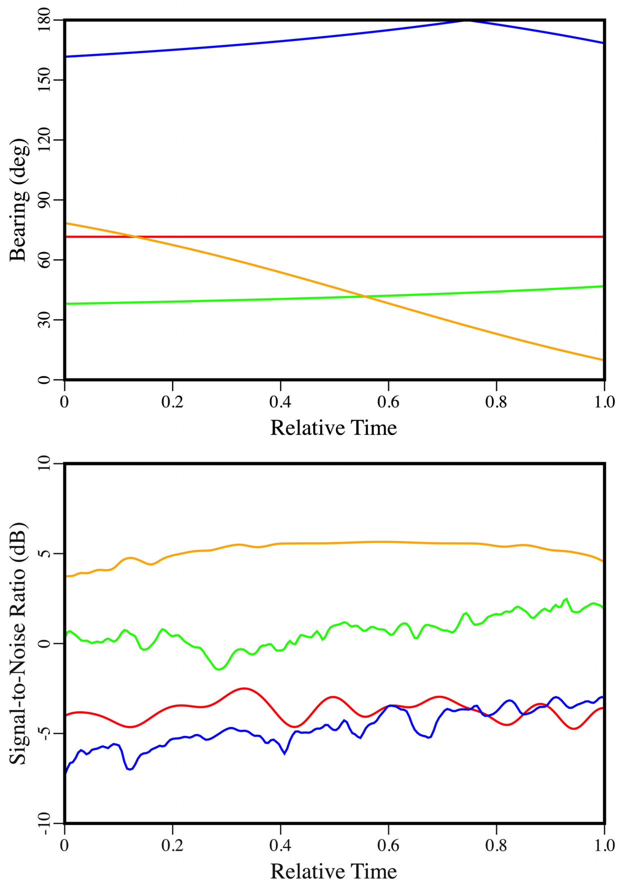

FIGURE 2. Results for example B (red = source 1, green = source 2, blue = source 3, orange = source 4). The bearings (top) cross at two locations (the absolute value of bearing is shown since negative bearings are ambiguous on the array). There are multiple crossings of two of the SNR curves (bottom).

break down. As illustrated in Fig. 2 of [6], however, it may be possible to determine the paths of the sources by waiting for them to move through points at which the partitioning is favorable. With a vertical line array, this process typically involves the computation of a large number of replica fields (solutions of the wave equation) for comparison with the data. With a rectangular array, it is possible to determine when the partitioning is favorable (i.e., when the signals from different sources have been isolated from each other) without generating any replica fields. It can be a substantial advantage to avoid generating replica fields, a task that requires computations that may not be practical and environmental information (e.g., sound speed and bathymetry) that may not be available.

II. THE MVB PROCESSOR

The data from the array are used to estimate the covariance matrix K [1–3], which may be expressed in terms of the normalized eigenvectors $\hat{x}_j$ and eigenvalues λ_j as follows:

$$K = \sum_{j=1}^{N} K_j, \tag{1}$$

$$K_j = \lambda_j \hat{x}_j \hat{x}_j^*, \tag{2}$$

where $\lambda_j \geqslant \lambda_{j+1}$ and $N = m \times n$ is the number of receivers in the array. Eigen-processing techniques are based on the fact that signals from discrete point sources tend to partition into different eigenvectors, while signals from distributed noise sources, such as surface-generated ambient noise [7], tend to be distributed among the eigenvectors. The MVB processor,

$$B_j = \hat{u}^* K_j \hat{u} \tag{3}$$

is based on the strategy of taking advantage of the partitioning by beamforming on each eigenvector separately, where the replica vector $\hat{u}$ corresponds to the field on the array due to a source at a test location (or in a test direction for beamforming). An ambiguity surface is constructed by evaluating B_j over a region of test locations. When the partitioning is favorable, the energy from a discrete source will correspond approximately to one of the $\hat{x}_j$, and there will be a peak in B_j at the location of the source. Summing over j, we obtain the Bartlett processor,

$$B = \sum_{j=1}^{N} B_j = \hat{u}^* K \hat{u}. \tag{4}$$

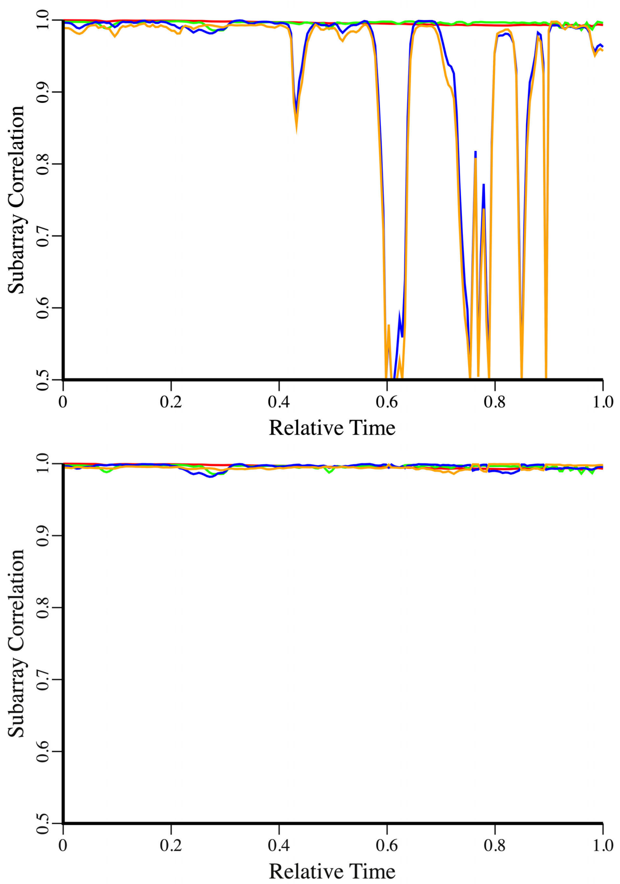

FIGURE 3. Results for example B (red = eigenvector 1, green = eigenvector 2, blue = eigenvector 3, orange = eigenvector 4). Subarray correlations for the eigenvectors corresponding to the four largest eigenvalues (top). For two of the eigenvectors, the subarrays are not correlated near locations where there are small differences between the first and third SNR curves in Fig. 2. The breakdowns in correlation are eliminated when $\hat{X}_3$ and $\hat{X}_4$ are used in place of $\hat{x}_3$ and $\hat{x}_4$ (bottom).

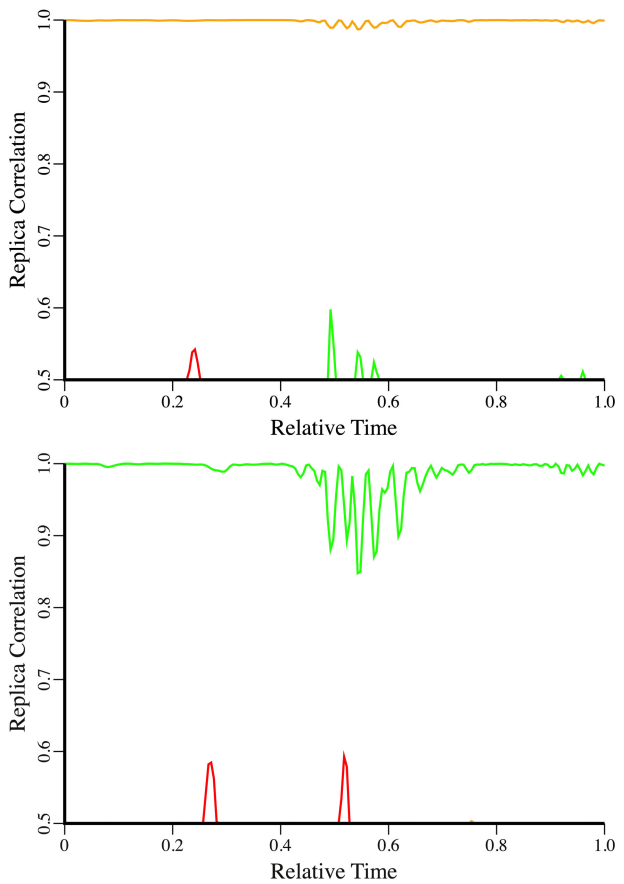

FIGURE 4. Results for example B (red = source 1, green = source 2, blue = source 3, orange = source 4). Correlations between vertical subarrays of the first (top) and second (bottom) eigenvectors and replica fields that correspond to the correct source locations. The correlations are degraded near the point where the second and fourth sources cross in bearing in Fig. 2.

We consider a problem involving white noise and several discrete point sources that may radiate at different levels, with

$$K = \sigma I + \sum_j \bar{p}_j \bar{p}_j^*, \tag{5}$$

where σ is the white noise level and the ith entry of the signal vector $\bar{p}_j$ is the complex pressure due to the jth source at the ith receiver in the array. By definition, the signal-to-noise ratio (SNR) of the jth source on the array is

$$\mathrm{SNR}_j = 10\log_{10}\left(\frac{\mathrm{Tr}\left(\bar{p}_j\bar{p}_j^*\right)}{N\sigma}\right). \tag{6}$$

If some of the point sources are regarded as noise, the definition of SNR_j may be modified to include the traces of those contributions in the denominator of Eq. (6). If the $\bar{p}_j$ are mutually orthogonal, they correspond to eigenvectors of K, and the partitioning is exact. Signal processing approaches are based on the fact that the $\bar{p}_j$ corresponding to two randomly chosen source locations are likely to be weakly correlated, in which case the partitioning may be favorable. The partitioning breaks down when two of the $\bar{p}_j$ are correlated, but the MVB processor may be effective for moving sources if the partitioning is favorable at a sufficient number of points along the paths of the sources. The approach may also break down when there is only a small difference between two of the eigenvalues, which may occur when there is a small difference between two of the SNR_j.

III. APPLICATION TO RECTANGULAR ARRAYS

The MVB processor is not limited to a particular array geometry. It may be applied to a rectangular array in the same way it is applied to a one-dimensional array, but the extra dimension makes it possible to obtain some information without (or before) generating replica fields. The first step (which is not possible with a vertical line array) is to determine if the partitioning is favorable by comparing vertical subarrays. This approach is based on the following expression for the acoustic pressure due to a single source:

$$p\left(r, z, \theta\right) = p_0\left(r, z, \theta\right)\exp\left(ik_0 r\right), \tag{7}$$

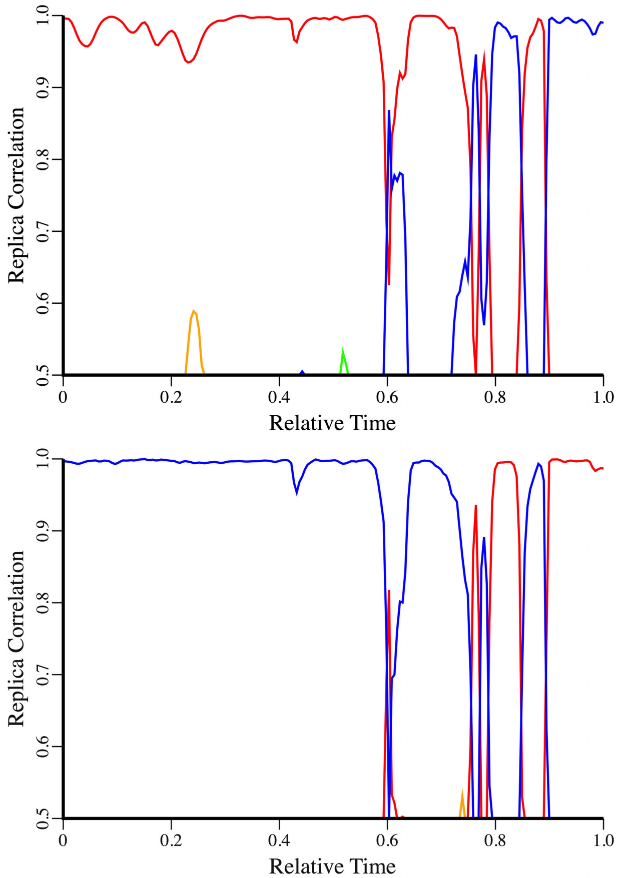

FIGURE 5. **Results for example B (red = source 1, green = source 2, blue = source 3, orange = source 4). Correlations between vertical subarrays of the third (top) and fourth (bottom) eigenvectors and replica fields that correspond to the correct source locations. The correlations break down near locations where there are small differences between the first and third SNR curves in Fig. 2. For the third eigenvector, the correlation is degraded near the point where the first and fourth sources cross in bearing in Fig. 2.**

where the source is located at $r = 0$, k_0 is a representative wave number, and p_0 varies gradually in the horizontal directions. We define the normalized vector $\hat{x}_{i,j}$, which contains the entries of $\hat{x}_j$ that correspond to the ith vertical subarray, and the correlation between vertical subarrays,

$$\gamma_j = \frac{1}{m-1}\left|\sum_{i=1}^{m-1} \hat{x}_{i,j}^* \hat{x}_{i+1,j}\right|. \quad (8)$$

It follows from Eq. (7) and the assumption on p_0 that $\gamma_j \cong 1$ when the partitioning is favorable.

We consider 100 Hz examples in an 800 m deep ocean in which the sound speed is 1500 m/s. In the sediment, the sound speed is 1700 m/s, the density is 1.5 times the density of water, and the attenuation is 0.5 dB per wavelength. The array has a receiver spacing of 7.5 m (half wavelength) in both directions. In a coordinate system that is associated with the array, the receivers are located in the plane $y = 0$, the array is centered at $x = 0$, the ocean surface is at $z = 0$, and the top row of receivers is at $z = 50$ m. The bearing is defined such that its

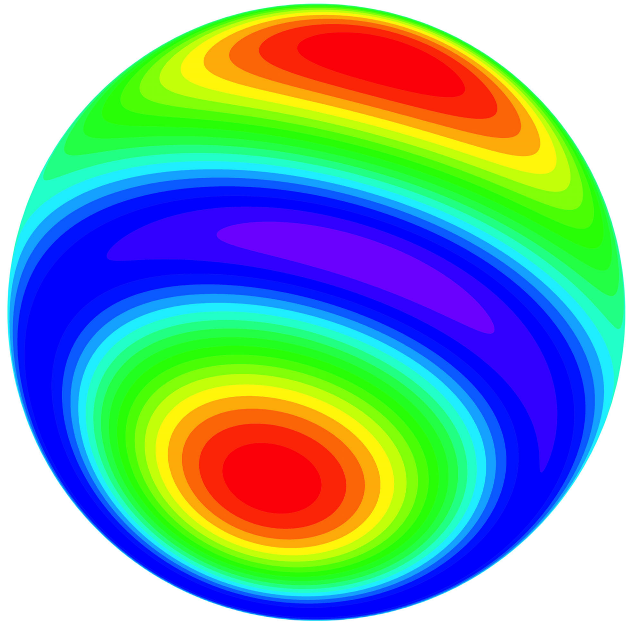

FIGURE 6. **Results for example B. Correlation of the vertical subarrays of $\hat{X}_3$ as a function of the coefficient of $\hat{x}_3$ in Eq. (9) over the unit disk of the complex plane. Red corresponds to high correlation, blue corresponds to low correlation, and the origin is at the center.**

tangent is y/x and 90 deg is broadside to the array; due to the inherent ambiguity on a planar array, results appearing in the figures are displayed in terms of the absolute value of the bearing.

The purpose of example A is to illustrate the difference between favorable and unfavorable partitioning for a problem involving a 20×40 array. Three point sources that transmit at the same level are located at the following (x, y, z) positions (in km): $(5, 15, 0.16)$, $(16, -12.5, 0.11)$, and $(-18, 6, 0.5)$. Appearing in Fig. 1 are the correlations between adjacent vertical subarrays for two cases. When only one source is turned on, the subarrays are highly correlated as would be expected from Eq. (7). When all three sources are turned on, the acoustic field on the array consists of energy propagating from three different directions, Eq. (7) is not valid, and the subarrays are not correlated; this example is representative of the signature of unfavorable partitioning.

The purpose of example B is to illustrate some of the capabilities and limitations of the MVB processor. This example involves a 10×40 array and four moving sources that transmit at different levels. The sources are initially located at the following positions: $(5, 15, 0.16)$, $(16, -12.5, 0.11)$, $(-18, 6, 0.5)$ and $(1, -5, 0.15)$; they move at constant speed along linear paths to the following final positions: $(5, 15, 0.36)$, $(8, -8.5, 0.11)$, $(-10, -2, 0.5)$ and $(4.5, -0.8, 0.15)$. Note that the first source moves vertically. In terms of the coefficient of the source term in the wave equation, the relative transmission levels are 0.6, 1.0, 0.5, and 0.75. Appearing in Fig. 2 are the bearing and SNR for each of the sources as they move along their tracks. Appearing in the top part of Fig. 3 are the subarray correlations in

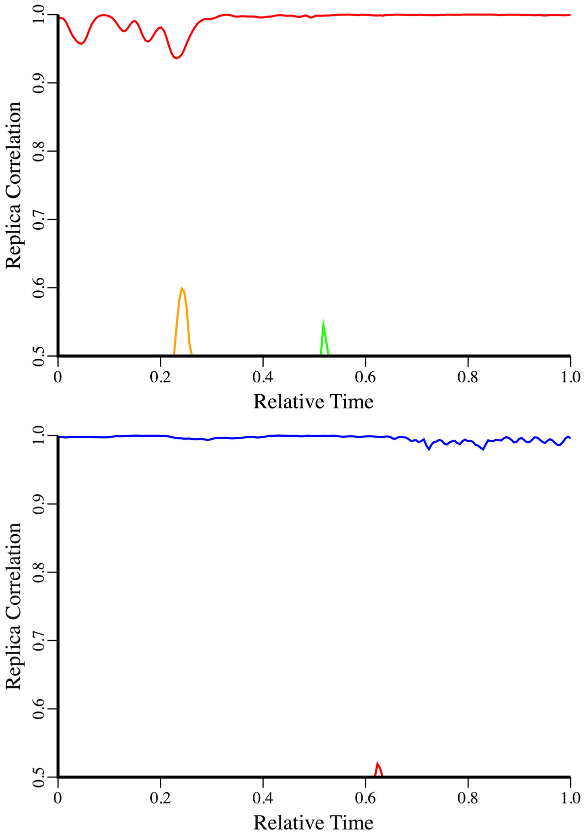

FIGURE 7. **Results for example B (red = source 1, green = source 2, blue = source 3, orange = source 4). Similar to the results in Fig. 5 but with $\hat{X}_3$ and $\hat{X}_4$ used in place of $\hat{x}_3$ and $\hat{x}_4$. Correlations between vertical subarrays of the third (top) and fourth (bottom) eigenvectors and replica fields that correspond to the correct source locations. There is high correlation along much of the tracks, and the breakdowns near the SNR crossing points have been eliminated.**

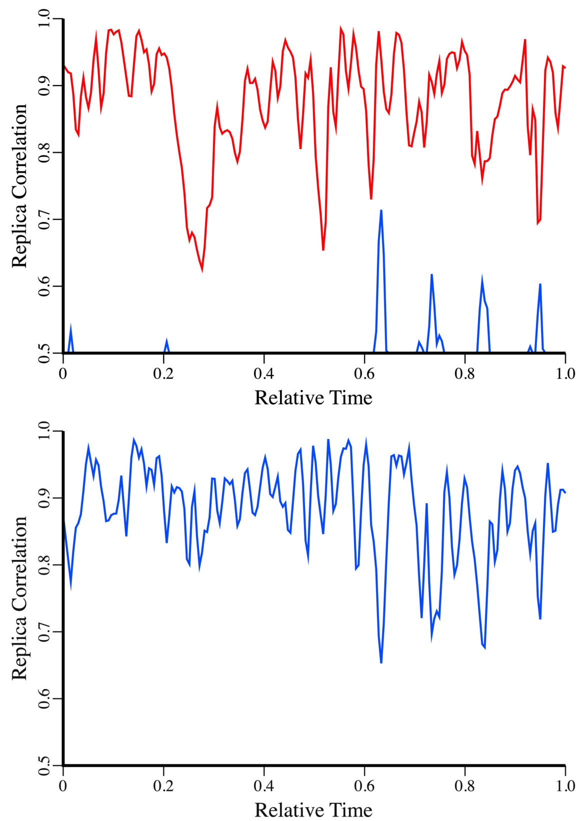

FIGURE 8. **Results for example B (red = source 1, green = source 2, blue = source 3, orange = source 4). Similar to the results in Fig. 7 but with the MVB processor applied to a vertical subarray. There is high correlation along much of the tracks in Fig. 7, but there are only a few isolated points of high correlation for the one-dimensional array.**

Eq. (8) for the eigenvectors that correspond to the four largest eigenvalues. The subarrays are highly correlated along most of the tracks for the first two eigenvectors, but they break down at some points for the other two eigenvectors.

After obtaining the eigenvectors on the full array, it may be possible to perform matched-field processing with a single vertical subarray; we obtain replica correlations by computing the correlation between a vertical subarray of an eigenvector and the replica fields that correspond to the correct source locations; ambiguity surfaces would be constructed in practice, but the objective here is to determine how accurately the sources are isolated. Appearing in Fig. 4 are the replica correlations for the first and second eigenvectors. The high correlation along most of the tracks is an indication of favorable partitioning in which the signals from the second and fourth sources are successfully isolated. The degradations near the middle of the tracks in Fig. 4 correspond to a location at which the sources cross in bearing; near that point, the advantages of the horizontal aperture of the array are reduced for that pair of sources. Appearing in Fig. 5 are the replica correlations for the third and fourth eigenvectors. The correlation is high at some points, it breaks down at several locations, and there are cross-over points; the third eigenvector initially corresponds to the first source, but it eventually crosses over to the third source; the fourth eigenvector initially corresponds to the third source, but it eventually crosses over to the first source. The breakdowns occur near crossing points of the SNR curves for the first and third sources in Fig. 2; at those points, the corresponding eigenvalues are nearly equal. In the top part of Fig. 5, there is a degradation near the point where the first and fourth sources cross in bearing in the top part of Fig. 2.

When the partitioning into separate eigenvectors breaks down at a point where two of the SNR curves are nearly equal, it may still be possible to isolate the signal vectors that correspond to those curves by applying an additional step. For this case, the signal vectors may partition into orthogonal linear combinations of eigenvectors (that correspond to a pair of nearly equal eigenvalues) of the form,

$$\hat{X}_i = \alpha e^{i\phi}\hat{x}_i + \sqrt{1-\alpha^2}\hat{x}_{i+1}, \quad (9)$$

$$\hat{X}_{i+1} = -\sqrt{1-\alpha^2}e^{i\phi}\hat{x}_i + \alpha\hat{x}_{i+1}, \quad (10)$$

where $\left|\alpha e^{i\phi}\right| \leqslant 1$ and the coefficients of the second terms are forced to be real (with no loss of generality). If the signal vectors are nearly orthogonal, they may be isolated by maximizing the correlation between the vertical subarrays of $\hat{X}_i$. For example B, the correlation between the vertical subarrays

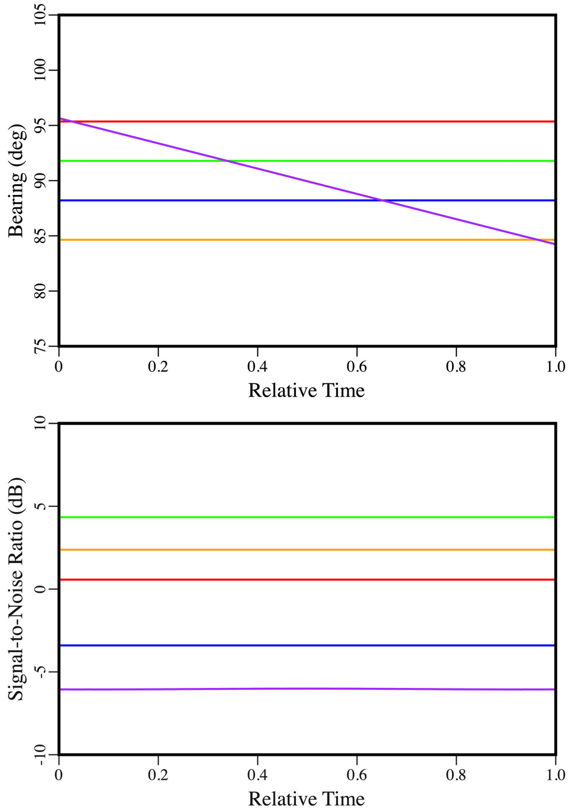

FIGURE 9. **Results for example C (red = source 1, green = source 2, blue = source 3, orange = source 4, purple = source 5). Bearings (top) and SNR (bottom) for the sources.**

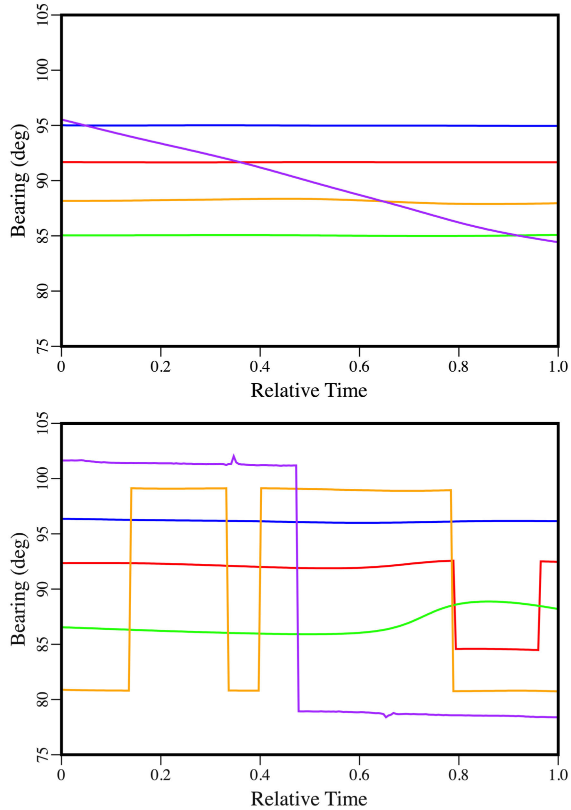

FIGURE 10. **Results for example C (red = eigenvector 1, green = eigenvector 2, blue = eigenvector 3, orange = eigenvector 4, purple = eigenvector 5). Estimates of the bearings of the sources obtained with the MVB processor for a rectangular array (top) and with a horizontal subarray (bottom). The tracks of all of the sources are recovered fairly well with the rectangular array. The tracking breaks down with a horizontal subarray.**

of $\hat{X}_3$ appears in Fig. 6 as a function of the coefficient of $\hat{x}_3$ in Eq. (9) at the relative time 0.625, where there is a small difference between two of the SNR curves in Fig. 2. Since the locations of the maxima in Fig. 6 are related according to the coefficients of $\hat{x}_i$ in Eqs. (9) and (10), it is necessary to locate only one of the maxima. Two sources of nearly equal SNR may be isolated by applying this approach on a rectangular array but not on a vertical line array. Applying this approach to example B, we determined $\hat{X}_3$ and $\hat{X}_4$ at each point along the tracks, used them in place of $\hat{x}_3$ and $\hat{x}_4$ in computing the subarray and replica correlations, and obtained the results appearing in the bottom part of Fig. 3 and both parts of Fig. 7; the SNR related breakdowns in the top part of Fig. 3 and both parts of Fig. 5 have been eliminated. Appearing in Fig. 8 are the replica correlations for the same case as in Fig. 7 but with all of the processing (including obtaining the eigenvectors) restricted to one of the vertical subarrays; the fact that the correlation is high only at a small number of isolated points is indicative of the advantages of the horizontal aperture of the rectangular array.

The purpose of example C is to illustrate the capability of determining the bearings of multiple sources for cases that cannot be handled with a horizontal line array. This example involves a 20×20 array and a source that moves along a track that passes behind four sources that are fixed at locations near broadside to the array. The fixed sources are located at the following positions: $(-0.75, 8, 0.16)$, $(-0.25, 8, 0.11)$, $(0.25, 8, 0.5)$, and $(0.75, 8, 0.15)$; the fifth source moves from $(-1.2, 12, 0.22)$ to $(1.2, 12, 0.22)$ at constant speed along a linear path. The transmission amplitudes are 0.6, 1.0, 0.4, 0.75, and 0.5. Appearing in Fig. 9 are the bearing and SNR of each of the five sources. The bearings appearing in the top part of Fig. 10 were obtained by first obtaining the eigenvectors for the full array and then applying plane-wave beamforming to one of the horizontal subarrays for each of the eigenvectors that correspond to the five largest eigenvalues; all five of the tracks are recovered. The bearings appearing in the bottom part of Fig. 10 were obtained by restricting all of the processing (including obtaining the eigenvectors) to one of the horizontal subarrays; none of the source tracks are recovered as this case is beyond the capabilities of a horizontal line array. With matched-field processing, it is possible to locate multiple sources that are located along the same bearing, but the results in Fig. 10 were obtained without generating any replica fields.

IV. DISCUSSION

The MVB processor is a multi-valued eigen-processor for localizing multiple sources. Ambiguity surfaces are formed by beamforming on a subset of the eigenvectors of the covariance matrix that correspond to the largest eigenvalues. The main peak in each ambiguity surface provides an estimate for the location of one of the sources; there is no need to subjectively assess local maxima, which are not relevant. The MVB processor is based on the fact that energy from different point sources tends to partition into different eigenvectors. The approach may fail when the partitioning is not favorable for certain combinations of source locations, but it may be possible to determine the tracks of the sources if the partitioning is favorable at a sufficient number of points.

For the application of the MVB processor on a rectangular array, it is possible to determine when the partitioning is favorable; this step, which does not require the computation of any replica fields, is not possible on a vertical line array. With this capability, it is possible to rapidly search through a data set and determine when sources have been isolated. After this step has been completed, the eigenvector may be used as high SNR input into conventional beamforming and matched-field processing techniques for estimating bearing, range, and depth. An example involving a source moving behind four sources was used to illustrate the advantages of the vertical aperture of the array for estimating the bearings of sources; this example illustrates that improved performance (relative to the one-dimensional case) can be achieved without generating any replica fields.

An example involving four moving sources was used to illustrate some of the capabilities and limitations of the MVB processor on a rectangular array. With the extra dimension of the array, points at which the partitioning is not favorable occur less frequently than for the application of this approach on a vertical line array; if two sources are located along different bearings, the partitioning is likely to be favorable; if they are located along the same bearing, it is likely that the partitioning will be favorable at many points along its track. There may be degradations in the MVB processor when two sources cross in bearing. At such points, the benefits of the horizontal aperture of the array are reduced. At crossing points of the SNR curves, two eigenvalues may be nearly equal and the signals from two sources may correspond to linear combinations of the corresponding eigenvectors; in this case, the signals may be isolated by determining linear combinations of the eigenvectors that optimize the correlation of adjacent vertical subarrays.

Previous applications of eigen-processing techniques are based on the eigenvectors that correspond to a subset of the largest eigenvalues. However, it is conceivable that a weak source could be isolated in an eigenvector that ranks below (in terms of the magnitudes of the eigenvalues) some of the eigenvectors that represent distributed noise. With a vertical line array, it is likely that such an eigenvector would be overlooked. With a rectangular array, such an eigenvector could be discovered (by checking for correlation between the vertical subarrays) and then potentially used to localize the source. White noise was used for the examples presented here. There is a need for further testing using correlated noise, and the approach ultimately needs to be tested on data. In the original implementation and testing for the case of a vertical line array [6], the MVB processor was found to be effective for problems involving surface-generated ambient noise [7], a common type of correlated noise in the ocean that corresponds to sources that are distributed in azimuth. For the case of a rectangular array, the performance should be even better for such noise, which would be weakly correlated with the signal due to a discrete source that arrives from a particular direction.

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MICHAEL D. COLLINS (Member, IEEE) was born in Greenville, PA, in 1958. He received the B.S. degree in mathematics from the Massachusetts Institute of Technology, Cambridge, MA, in 1982, and the Ph.D. degree in applied mathematics from Northwestern University in Evanston, IL, in 1988.

Since 1985, he has been with the Naval Research Laboratory, Washington, DC, USA and the Stennis Space Center, MS. One of his areas of speciality is the parabolic equation techniques for solving non-separable wave propagation problems in ocean acoustics and seismology. He has published several articles on his privately funded research on the Ivory-billed Woodpecker. His research interests include wave propagation, inverse problems, and signal processing in ocean acoustics.

Dr. Collins is a member of the Society for Industrial and Applied Mathematics and the American Geophysical Union. He was a recipient of the R. Bruce Lindsay Award from the Acoustical Society of America and the A. B. Wood Medal from the Institute of Acoustics.

JOSEPH F. LINGEVITCH was born in Elmhurst, IL, in 1964. He received the B.S. degree in physics from Bradley University, Peoria, IL, in 1986, the M.S. degree in physics from the University of Illinois at Urbana-Champaign, in 1989, and the Ph.D. degree in applied mathematics from Northwestern University, Evanston, IL, in 1995. Since 1995, he has been with the Naval Research Laboratory, Washington, DC, USA. His research interests include acoustic propagation, signal processing, and inverse problems. He is a member of the Acoustical Society of America and the Skyline Soaring Club.