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Time-Varying DOA Tracking Algorithm Based on Generalized Labeled Multi-Bernoulli

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ABSTRACT Direction of arrival (DOA) tracking for multi-sources is a hot issue in array signal processing. To deal with the problem that sources DOA and their number are time-varying, a DOA tracking algorithm based on Generalized Labeled Multi-Bernoulli (GLMB) filter is proposed. Since the measurement value has only one set of data, the measurement association mapping (MAM) does not match, which leads to deviations in the GLMB filter update step. In this regard, we used the estimated sources number of the previous time step as the measurement number of the current time step, and successfully achieved MAM matching. Subsequently, particle filtering is used to approximate the posterior distribution of DOA, where the particle likelihood function can be calculated by the multi-signal classification (MUSIC) spatial spectrum function. In addition, by exponentially weighting the likelihood function, the number of particles in the high likelihood region of the posterior distribution increases, which makes the GLMB filter pruning and merging operations more effective. Simulation results show that the method is better than the probability hypothesis density DOA (PHD-DOA) algorithm in tracking state sources and estimating the number of targets.

INDEX TERMS DOA tracking, particle filtering, generalized labeled multi-Bernoulli filter, MUSIC spatial spectrum.

I. INTRODUCTION

Array signal processing technology plays an increasingly important role in mobile location, sonar system, fault location and bio-medicine. The direction of arrival (DOA) estimation problem for narrowband sources with the same carrier frequency has been extensively studied in the past few decades [1], [2]. Array high resolution DOA estimation and tracking method is an effective way to solve practical problems such as radar low elevation tracking. The earliest super-resolution DOA estimation methods are the well-known Multiple Signal Classification (MUSIC) [3], Estimating Signal Parameter via Rotational Invariance Techniques (ESPRIT) [4], Maximum Likelihood (ML) [5], etc. The traditional estimation algorithms assumes that the sources is stationary during the observation time, the correlation characteristics of these adjacent time steps cannot be considered. In fact, the source is moving, and the correlation between adjacent time steps is close. If the multiple moving sources is still estimated by the static DOA estimation, the calculation load will increase and it is not convenient to realize real-time tracking. As the accuracy of moving sources is required more and more, it is meaningful to realize DOA tracking of time-varying moving sources.

There are mainly subspace update algorithm and state filter tracking algorithm in dynamic DOA tracking methods. The former is represented by Bin Yang's projection approximation subspace tracking (PAST) algorithm and projection approximation subspace tracking with deflation (PASTd) algorithm [6], which take the projection of eigensubspace as the solution of unconstrained optimization problem. It has the advantages of low computational complexity and fast convergence speed. However, this type of subspace update methods needs to perform spatial recursion on the premise of determining the number of sources, and can only solve small-scale changes in the source state. In addition, it cannot solve the time-varying situation when the number of sources are change. The latter for particle filtering (PF) is a typical representative of state-based filtering algorithm, which has

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been widely applied to target tracking problems [7]–[10]. In [8], a moving target azimuth tracking and beamforming algorithm based on PF for the time-varying problem of sources is proposed, which realizing the integration of DOA tracking and signal reception. However, the method can only track the motion sources whose number are fixed during the obseration time. In [9], the author used the quaternion MUSIC algorithm to improve the likelihood function of particle filter, realizing the real-time tracking of angle for DOA estimation of acoustic vector array when the moving target is single. In [10], aiming at DOA tracking of maneuvering targets, the PF likelihood function by using the spectral function of MUSIC algorithm is improved. These above mentioned methods in [9], [10] are all studies on DOA tracking of moving single target, but they cannot solve the problem of moving multiple targets. Therefore, it is significant to study a solution to the problem that the sources number and its state change over time.

Since particle filtering cannot solve the problem of multi-target tracking, Mahler introduced the concept of random finite set (RFS) [11]. Among them, probability hypothesis density (PHD) [12] recursive method timely propagates the posterior intensity of target. Cardinality PHD (CPHD) [13] recursion is a generalization of PHD recursion, which combines the propagation of posterior intensity and distribution of posterior cardinality. In [14], the author proposed an improved CPHD method, which can effective tracking of time-varying DOA in impulse noise environment by carrying out auxiliary step size for persistent particles and adapting importance distribution for new particles. The measurement involved in [14] is phased array image observation, which is inconsistent with the signal receiving model we considered. In [15], the superposition measurement model is used to directly merge the measured values into the approximate CPHD filter of track before detection (TBD), which has solved the problem of multi-targets DOA tracking. Although this method can solve the problem of time-varying sources, the receiving model is different from the considered in this paper. If this methods are used to ULA, the expected effect of the algorithm will not be achieved due to the measurement association mismatch.

There is also a method to model each target into Bernoulli RFS with the characteristics of existence probability and target probability density function, which is called multi-target multi-Bernoulli (MeMBer) filter [16]. In recent years, the MeMBer filter has been successfully applied in array signal processing [17]–[19]. Choppala P B *et al.* studied the Bayesian multi-target tracking problem based on phased array sensors, and solved the multi-target tracking problem by using MUSIC spatial spectrum estimation as the likelihood function in the multi-Bernoulli update step [17]. The MeMBer filter to track the azimuth tracking of two targets for the acoustic vector sensors is proposed in [18], and the simulation results proved the robustness of the MeMBer. In [19], the author adopted UT-MeMBer filter to solve the time-varying DOA problem in impulse noise, and replaced the calculation of likelihood function with MUSIC spatial spectrum. The simulation results showed that the time-varying crossover sources DOA could also be well tracked. But this method has a long running time when the author apply the UT algorithm to MeMBer. In addition, the tracking performance based on MeMBer filtering will significantly reduce in the case of low SNR (such as SNR $= -5$dB).

In [20], [21], the labeled multi-Bernoulli (LMB) RFS is proposed. Vo *et al.* combined multi-hypothesis tracking (MHT) with LMB RFS theory and proposed a generalized labeled MeMBer (GLMB) method, and presented the Sequential Monte Carlo (SMC) and Gaussian mixture implementation strategy. By assigning labels to newborn particles and calculating the labels for survival particles, the time of birth and death of the target can be effectively captured. Each iteration of the filter involves an update operation and a prediction operation, both of which result in a weighted sum of multi-target exponentials with a large number of terms. To truncate these sums, ranked assignments and K-shortest path algorithms are used to determine the most important items in the update and forecast, respectively, without enumerating all items.

Based on the above analysis, a particle filter DOA tracking method based on GLMB filter is proposed, called GLMB-DOA. The advantage of this algorithm is that it can estimate the crossover sources accurately and effectively when the number and motion state of targets are time-varying. The proposed algorithm has the following advantages:

- We use the number of sources estimated in the previous time step to represent the number of measurements at the current time step, which can achieve the purpose of measurements association mapping (MAM) between measurements and sources;
- We use the similar function between MUSIC spatial pseudo-spectrum and particle filter, and take the MUSIC spatial pseudo-spectrum as the particle likelihood function of the proposed algorithm, which can effectively calculate the particle weight of the posterior distribution;
- Since the likelihood function of particles is too flat in the case of low SNR, we further weighted the likelihood function of particles exponentially, which alleviates the flattening problem of the likelihood function and makes the weight of particles more discriminative. Thus the pruning and merging of the proposed algorithm more effective than before.

The paper is organized as follows. In Section II-*A*, the problem of the DOA tracking in uniform linear radar array is described, Multi-sources Bayesian theory and GLMB filter are provided in section II-*B* and II-*C*, respectively. The proposed strategy and the GLMB-DOA particle filter for DOA tracking algorithm is given in Section III. Then we show our simulation results in Section IV and conclusion in Section V.

Notations: We use upper-case (lower-case) bold characters to represent matrices (vectors). $(\cdot)^T$ and $(\cdot)^H$ stand for the transpose and conjugate transpose, respectively. $diag(\cdot)$ denotes the diagonal matrices. $\mathbf{I}_N$ is a $N \times N$ identity matrix and $\mathbb{E}$ denotes the expectation operator. $|\cdot|$ denotes the module of a complex value.

II. BACKGROUND

The signal model is a uniform linear array (ULA), and its receiving model can be represented by mathematical expression. We introduce the received model in II-A, multi-target Bayesian theory and GLMB filter are provided in II-B and II-C, respectively.

A. SIGNAL MODEL

Consider the case of $P(k)$ narrow far field signals $s_p(k), p = 1, 2, \ldots P(k)$ with different DOA $\theta_1, \theta_2, \ldots, \theta_{P(k)}$ arriving at a ULA with M sensors at discrete time k. The DOA of the p th source can be written as $\theta_{p(k)}$. The received signal of the arrays in [1] can be expressed as

$$\mathbf{Z}(k) = \mathbf{A}(\theta)\mathbf{S}(k) + \mathbf{N}(k) \tag{1}$$

with

- $\mathbf{N}_{M\times 1}(k) = [n_1(k), n_2(k), \ldots, n_M(k)]^T$ represents the Guassian white noise vector, which is not correlated with signals.
- $\mathbf{Z}_{M\times 1}(k) = [\mathbf{z}_1(k), \mathbf{z}_2(k), \ldots, \mathbf{z}_M(k)]^T$ denotes the measurement.
- $\mathbf{S}_{P\times 1}(k) = \left[s_1(k), s_2(k), \ldots, s_{P(k)}(k)\right]^T$ denotes the vector of sources.
- $\mathbf{A}_{M\times P}(\theta) = \left[\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}\left(\theta_{P(k)}\right)\right]^T$ is the direction matrix.

and

$$\mathbf{a}\left(\theta_p\right) = \left[1, e^{-j\frac{2\pi}{\lambda}d\sin\theta_{p(k)}}, \ldots, e^{-j\frac{2\pi}{\lambda}(M-1)d\sin\theta_{p(k)}}\right] \tag{2}$$

$\mathbf{a}\left(\theta_p\right)$ is the steering vector with λ denoting the wavelength of the carrier, and d is the array spacing. $\mathbf{S}(k)$ and $\mathbf{N}(k)$ are independent of each other. According to [1], the measurement probability density function (PDF) can be expressed as

$$\mathbf{Z}_k \sim \mathcal{CN}(0, \mathbf{R}_k) \tag{3}$$

where $\mathcal{CN}(\cdot|\mu, \Sigma)$ represents a multivariate complex Gaussian distribution with a mean value μ and a covariance matrix Σ, and the measurement covariance matrix $\mathbf{R}_k$ is

$$\mathbf{R}_k = \mathbb{E}\left\{\mathbf{Z}_k\mathbf{Z}_k^H\right\} = \mathbf{A}(\theta)\mathbf{R}_s\mathbf{A}^H(\theta) + \mathbf{Q} \tag{4}$$

where $\mathbf{Q} = \mathbb{E}\left[\mathbf{N}_k\mathbf{N}_k^H\right] = diag\left\{\sigma^2\mathbf{I}_M\right\} \in \mathbb{C}^{M\times M}$, $\mathbf{R}_s$ is the signal covariance matrix, and $\mathbf{R}_k$ in [2] can be approximated as

$$\mathbf{R}_k \approx \frac{1}{L}\mathbf{Z}_k\mathbf{Z}_k^H \tag{5}$$

where L is the number of snapshot. From [17], the MUSIC spectral peak search function is

$$P_{\text{MUSIC}}\left(\theta_p\right) = \frac{1}{a\left(\theta_p\right)^H\mathbf{U}\mathbf{U}^H a\left(\theta_p\right)} \tag{6}$$

$\mathbf{U}$ denotes the noise subspace, and can be obtained by the eigenvalue decomposition of (5). The DOA estimation can easily be obtained by implementing a search over the potential $\hat{\theta}_p$, which maximizes the output of (7), given as

$$\hat{\theta}_p = \arg\max_{\theta\in[-\pi,\pi]}\left|P_{\text{MUSIC}}\left(\theta_p\right)\right| \tag{7}$$

On this basis, we use the MUSIC spatial spectral function as the likelihood function of GLMB particle filter, and proposes the GLMB-DOA tracking algorithm. This algorithm can directly filter the current target considering the correlation between adjacent time steps, which is described in the next section.

B. MULTI-TARGET BAYESIAN THEORY

Assume that the state of the source at time k is $x_k = \left[\theta_k, \dot{\theta}_k\right]^T$, where θ_k is the DOA angle and moves at $\dot{\theta}_k$ (rad/s). The state and number of sources are varied at time k, which can be described by RFS. The source state set in multi-target tracking can be seen as an RFS, namely

$$\mathbf{X}_k = \left\{x_{k,1}, \cdots, x_{k,P(k)}\right\} \tag{8}$$

where $\mathbf{X}_k$ represents a set of sources at time k, and it may represent one source or multiple sources or nulls. $\mathbf{Z}_k$ denotes a superimposed information set generated by all sources received by the ULA at time k.

Single-target Bayesian filtering can be extended to multi-target tracking by modeling the above source states and measured values. The single target PDF $p_{k|k}(x_k|\mathbf{Z}_{1:k})$ is replace by the joint multi-target posterior $p_{k|k}(\mathbf{X}_k|\mathbf{Z}_{1:k})$. The Bayes' joint filter recurses in two stages: prediction and update. The prediction and update at time $k-1$ in [17] is

- prediction:

$$p_{k|k-1}(\mathbf{X}_k|\mathbf{Z}_{1:k-1}) = \int f_{k|k-1}(\mathbf{X}_k|\mathbf{X}_{k-1})\, p_{k-1|k-1}(\mathbf{X}_{k-1}|\mathbf{Z}_{1:k-1})\,\delta\mathbf{X}_{k-1} \tag{9}$$

- update:

$$p_{k|k}(\mathbf{X}_k|\mathbf{Z}_{1:k}) = \frac{g(\mathbf{Z}_k|\mathbf{X}_k)\, p_{k|k-1}(\mathbf{X}_k|\mathbf{Z}_{1:k-1})}{\int g(\mathbf{Z}_k|\mathbf{X}_k)\, p_{k|k-1}(\mathbf{X}_k|\mathbf{Z}_{1:k-1})\,\delta\mathbf{X}_k} \tag{10}$$

where $\delta\mathbf{X}$ denotes the set integral and $\mathbf{Z}_{1:k-1}$ represents all the measurement sets up to time $k-1$. $g(\mathbf{Z}_k|\mathbf{X}_k)$ is a multi-target joint likelihood function and $f_{k|k-1}(\cdot|\cdot)$ is a multi-target state transition probability density function. $p_{k|k-1}(\mathbf{X}_k|\mathbf{Z}_{1:k-1})$ represents the multi-target joint prediction probability density and $p_{k|k}(\mathbf{X}_k|\mathbf{Z}_{1:k})$ is the multi-target joint posterior probability density function.

C. GLMB FILTER

The GLMB filter, which is a class of optimal Bayesian filters emerged from RFS framework. The GLMB filter mainly obtains the weighted components through the MAM between the measurement data and the sources. Mahler's Finite Set

Statistics (FISST) provides a powerful and practical mathematical tool for processing RFS [11], based on the concept of integral, in this paper we shall no distinguish a FISST density and a probability density. The inner product notation is $\langle f, g\rangle \triangleq \int f(x)g(x)\,dx$ and the following multi-object exponential nation $h^X \triangleq \prod_{x\in X} h(x)$, where h is a real-valued function, with $h^{\emptyset} = 1$ by convention. A generalization of the Kroneker delta that takes arbitrary arguments in [21], by

$$\delta_Y(X) = \begin{cases} 1, & \text{if } X = Y \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

and the inclusion function, a generalization of the indicator function, by

$$1_Y(X) = \begin{cases} 1, & \text{if } X \subseteq Y \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

We also write $1_Y(x)$ in place of $1_Y(\{x\})$ when $X = \{x\}$. In the multi-target tracking problem, the filter method based on the traditional RFS theory cannot provide accurate target tracking, so Vo introduced the labeled multi-Bernoulli (LMB) RFS, and proposed the GLMB filter. Random variables in LMB RFS are unordered, and labeling each element in LMB RFS is an effective way to solve this problem. In the Bayesian recursion process, the posterior probability density of GLMB RFS has the characteristic of keeping the same form, which can avoid the complicated integral solution process in the traditional algorithm. The posterior probability density in [21] can be expressed as

$$\pi(\mathbf{X}) = \Delta(\mathbf{X}) \sum_{(I,\xi)\in\mathcal{F}(\mathbb{L})\times\Xi} \omega^{(I,\xi)} \delta_I[\mathcal{L}(\mathbf{X})][p(\cdot;\xi)]^X \tag{13}$$

Each element in LMB RFS is added with a corresponding labeled and can be represented as

$$\mathbf{X} = \{(x_i, l_i)\}_{i=1,2,\ldots,|X|} \tag{14}$$

where $x_i \in \Omega$ represents the single target state and Ω denotes the target state space, $l \in \mathbb{L}$ is a label independent of the target state, and $\mathbb{L}$ is a discrete label space. $|\cdot|$ represents the cardinality of the set, the function $\Delta(\mathbf{X}) = \delta_{|\mathbf{X}|}(|\mathcal{L}(\mathbf{X})|)$ is called the distinct label indicator. $\mathcal{L}(X) = \{\mathcal{L}(x,l)\,|(x,l)\in\mathbf{X}\}$ is the label mapping, Ξ denotes a history of the discrete association space. Each pair (I,ξ) represents an association hypothesis, and $\omega^{(I,\xi)}$ is the corresponding weight. $p(x,l;\xi)$ is the single-objective probability distribution, and satisfied: $[p(\cdot;\xi)]^X = \prod_{(x,l)\in X} p(x,l;\xi)$ and $\int p(x,l;\xi)\,dx = 1$. Now assuming that the multi-object posterior probability density at time step $k-1$ is in the form of equation (13), the prediction of GLMB can be expressed as follows:

$$\pi_+(\mathbf{X}) = \Delta(\mathbf{X}) \sum_{(I_+,\xi)\in\mathcal{F}(\mathbb{L}_+)\times\Xi} \omega_+^{(I_+,\xi)} \delta_{I_+}[\mathcal{L}(\mathbf{X})]\,[p_+(\cdot;\xi)]^X \tag{15}$$

where

$$\omega_+^{(I_+,\xi)} = \omega_B(I_+\cap\mathbb{B})\,\omega_S^{\xi}(I_+\cap\mathbb{L}) \tag{16}$$

$$\omega_S^{\xi}(L) = \sum_{I_+\in\mathcal{F}(L)} 1_{I_+}(L)\left[\tilde{P}_S(\cdot;\xi)\right]^L \left[1-\tilde{P}_S(\cdot;\xi)\right]^{I_+-L} \omega^{(I,\xi)} \tag{17}$$

$$\tilde{P}_S(\cdot,l;\xi) = \langle p(\cdot,l;\xi), P_S(\cdot,l)\rangle \tag{18}$$

$$p_+(\cdot;\xi) = 1_{\mathbb{L}}(l)\,\frac{\langle P_S(\cdot,l) f(x|\cdot,l), p(\cdot,l;\xi)\rangle}{\tilde{P}_S(\cdot,l;\xi)} + 1_{\mathbb{B}}(l)\,p_B(x,l) \tag{19}$$

$\mathbb{B}$ is the new target labeled space and $\mathbb{L}\cap\mathbb{B} = \emptyset$, $p_B(x,l)$ is the probability distribution of the new target. $f(x|\cdot,l)$ is the Markov state transition function, and the labeled of the predicted survival target remains unchanged. $P_S(\cdot,l)$ is the probability of survival of the target. The update of GLMB in [21] can be expressed as follows:

$$\pi(\mathbf{X}) \propto \Delta(\mathbf{X}) \sum_{(I_+,\xi)\in\mathcal{F}(\mathbb{L}_+)\times\Xi,\theta\in\Theta} \omega^{(I,\xi,\theta)}(\mathbf{Z}) \times\delta_I[\mathcal{L}(\mathbf{X})]\,[p(\cdot;\xi,\theta(l))]^{\mathbf{X}} \tag{20}$$

where

$$\omega^{(I,\xi,\theta)}(Z) \propto [\mu_Z(\cdot;\xi,\theta(l))]^I \omega_+^{(I_+,\xi)} \tag{21}$$

$$\mu_Z(\cdot,l;\xi,\theta(l)) = \langle p_+(l;\xi), \psi_Z(\cdot,l;\theta(l))\rangle \tag{22}$$

$$p(\cdot;\xi,\theta(l)) = \frac{p_+(\cdot;\xi)\,\psi_Z(\cdot,l;\theta(l))}{\mu_Z(\cdot,l;\xi,\theta(l))} \tag{23}$$

$$\psi_Z(\cdot,l;\theta(l)) = \begin{cases} p_D(x,l)\,g\left(z_{\theta(l)}|x,l\right), & \theta(l)>0 \\ 1-p_D(x,l), & \theta(l)=0 \end{cases} \tag{24}$$

$\theta \in \Theta$ is the target to MAM, $g\left(z_{\theta(l)}|x,l\right)$ is the measuring likelihood function, $p_D(x,l)$ is the probability of target detection, and $1_{\mathbb{L}}(l)$ is the indicator function.

III. THE PROPOSED STRATEGY AND ALGORITHM

The calculation of the likelihood function is a critical step in the update step of the GLMB algorithm, whose size determines the weight of particles. In this paper, we use MUSIC pseudo-spectral function instead of particle filter likelihood function, which is described in detail in III-*A*, and GLMB particle filter algorithm steps will be described in III-*B*.

A. THE LIKELIHOOD FUNCTION

Since the measurement data received by the array signal is a fusion data, and the target is multiple sources, bias will appear when the GLMB filter matches the MAM between measurement and sources. In order to achieve the purpose of MAM, we use the estimated number of signal sources at the previous time step to represent the number of measured values at the current time. Suppose the estimated number of sources at time $k-1$ is $\tilde{P}(k-1)$. Then we use $m = \tilde{P}(k-1)$, and $P(k=1) = 1$. i.e., m is the number of current measurements, which will play an important role in GLMB update strategy.

The MUSIC spatial spectrum function is given in (6), but there is a disadvantage that the number of sources must be known in advance before the MUSIC algorithm can be used. In this article, we use the estimated number of sources at the previous time step instead of the number of targets at the current time. After performing eigenvalue decomposition (EVD) of (5), the eigenvectors corresponding to the smallest $M-\tilde{P}(k-1)$ eigenvalues are represented as

$$\mathbf{U}_n = \left[\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_{M-\tilde{P}(k-1)}\right] \tag{25}$$

$\mathbf{U}_n$ contains the noise eigenvectors, which are listed as noise subspace. Ideally, $\tilde{P}(k-1)$ should be equal to the actual number of sources $P(k)$. The likelihood function can be expressed as

$$\begin{aligned} g\left(z_{\theta(l)} | x, l\right) &= g\left(\mathbf{Z}_k | x_{k|k-1}^{(l)}\right) = P_{\mathrm{MUSIC}}\left(\mathbf{C} x_{k|k-1}^{(l)}\right) \\ &= \left| \frac{1}{\mathbf{a}\left(\mathbf{C} x_{k|k-1}^{(l)}\right)^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{a}\left(\mathbf{C} x_{k|k-1}^{(l)}\right)} \right|^{\zeta} \end{aligned} \tag{26}$$

$x_k = [\theta_k, \dot{\theta}_k]$ represents the state and speed of the source, $\mathbf{C} = [1, 0]$ represents the location information of the source, $\mathbf{a}(\cdot)$ is the direction vector, and $\zeta \in \mathbb{R}^+$ represents the exponential weighting factor. In general, the response of traditional MUSIC spatial spectrum beamformer is distorted and extended in lower SNR. Such peak distortion and spectrum spread will result in significant performance degradation of resampling step. After weighting exponentially, the number of particles close to the real state can be enhanced, so that the pruning and merging more effectively.

B. SMC IMPLEMENTATION OF GLMB-DOA

Assume that the multi-sources probability density parameter set at time $k-1$ is $\left\{\left(\mathbb{L}_{k-1,h}, \omega_{k-1,h}, P_{k-1,h}^{l}(x)\right)\right\}_{h=1}^{H_{k-1}}$, where $P_{k-1,h}^{l}(x)$ is approximated by a weighted particle set $\left\{\omega_{k-1}^{(l,j)}, x_{k-1}^{(l,j)}\right\}_{j=1}^{J_{k-1}^{l}}$. J_{k-1}^{l} is the total number of particles. The SMC implementation of GLMB-DOA for DOA tracking is given in TABLE 1

TABLE 1. **GLMB-DOA particle filter for DOA tracking.**

1.Input: $\left[\left\{\left\{\omega_{k-1}^{(l,j)}, x_{k-1}^{(l,j)}\right\}_{j=1}^{J_{k-1}^{l}}\right\}_{l=1}^{\mathbb{L}_{k-1}}, \{\mathbf{Z}_k^m\}_{m=1}^{\tilde{P}(k-1)}\right], \tilde{P}(1) = 1$
Prediction 2. Predict the state of time k using CV models: $x_{k\mid k-1}^{(l,j)} = \mathbf{F}_k x_{k-1}^{(l,j)} + v_k,$ where F_k is the state transition matrix, which is described in details in the section IV. 3. Update persistent particles at time k: $x_{p,k\mid k-1}^{(l,j)} = x_{k\mid k-1}^{(l,j)} . j = 1, \ldots, J_{k-1}^{l}.$ 4.Compute the weights at time k: $\omega_{p,k\mid k-1}^{(l,j)} = p_{S,k}\omega_{k-1}^{(l,j)}, l \in \mathbb{L}_{k-1}, j = 1, \ldots, J_{k-1}^{l}.$ 5.Construct a newborn target weighted particle: $x_{b,k\mid k-1}^{(l,j)} \sim p_{b,k}^{l}\left(x_k \mid \mathbf{Z}_{k-1}\right), j = 1, \ldots, J_{b,k}^{l},$ with $\omega_{b,k}^{(l,j)} \sim \omega_B(L), l \in \mathbb{B}, j = 1, \ldots, J_{b,k}^{l}.$ 6. Unite the weighted particle set: $\left(x_{k\mid k-1}^{(l,j)}, \omega_{k\mid k-1}^{(l,j)}\right)_{j=1}^{J_{k\mid k-1}^{l}}, l \in \mathbb{L}_+$ $= \left(x_{p,k-1}^{(l,j)}, \omega_{p,k-1}^{(l,j)}\right)_{j=1}^{J_{k-1}^{l}}, l \in \mathbb{L}_{k-1} \cup \left(x_{b,k}^{(l,j)}, \omega_{b,k}^{(l,j)}\right)_{j=1}^{J_{b,k}^{l}}, l \in \mathbb{B}$ where $J_{k\mid k-1} = J_{k-1} + J_{b,k}.$
Measurements Update 7. For each particle $x_{k\mid k-1}^{(l,j)}, j = 1, \ldots, J_{k\mid k-1}$ and each measurement $\mathbf{Z}_k^m$, the likelihood function $g\left(\mathbf{Z}_k^m \mid x_{k\mid k-1}^{(l,j)}\right)$ is calculated according to equation (26). 8.Compute (22): $\mu_Z(\cdot, l; \xi, \theta(l)) = \sum_{j=1}^{J_{k\mid k-1}^{l}} \omega_{k\mid k-1}^{(l.j)} \psi_Z\left(x_{k\mid k-1}^{(l,j)}, l; \theta(l)\right)$ 9.Compute weights $\omega_k^{(l,j)}$: $\omega_k^{(l,j)} = \frac{\omega_{k\mid k-1}^{(l.j)} \cdot \psi_Z\left(x_{k\mid k-1}^{(l,j)}, l; \theta(l)\right)}{\mu_Z(\cdot, l; \xi, \theta(l))}$
Capturing the cardinality according to [21] $\left(x_{k\mid k-1}^{(l,j)}, \omega_{k\mid k-1}^{(l,j)}\right)_{j=1}^{J_{k\mid k-1}}, l \in \mathbb{L}_+ \rightarrow \tilde{P}(k)$
Resampling, Pruning and Capping $\left(x_{k\mid k-1}^{(l,j)}, \omega_{k\mid k-1}^{(l,j)}\right)_{j=1}^{J_{k\mid k-1}}, l \in \mathbb{L}_+ \rightarrow \tilde{P}(k)$ 10. $\left(x_{k\mid k-1}^{(l,j)}, \omega_{k\mid k-1}^{(l,j)}\right)_{j=1}^{J_{k\mid k-1}}, l \in \mathbb{L}_+ \rightarrow \left(x_k^{(l,j)}, \omega_k^{(l,j)}\right)_{j=1}^{J_k}, l \in \mathbb{L}_k.$
11. **Output:** $\left\{x_k^{(l,j)}, \omega_k^{(l,j)}\right\}_{j=1}^{J_k^{l}}, l \in L_k, J_k^{l} = J_{k\mid k-1}^{l}$

Lines 2-6 of algorithm calculate the predicted particle weights and combine the new particle state to get the updated particles. The predicted probability density parameter set at time k is $\left\{\left(\mathbb{L}_{k|k-1,h}, \omega_{k|k-1,h}, x_{k|k-1,h}\right)\right\}_{h=1}^{H_{k|k-1}}$. The calculation formulas for the prediction labeled set $\mathbb{L}_{k|k-1,h}$, the prediction weights $\omega_{k|k-1,h}$, and the total number of truncated components $H_{k|k-1}$ are the same as in [21]. Steps 7-9 denote measurements update, the update particles and weights $\left(x_k^{(l,j)}, \omega_k^{(l,j)}\right)_{j=1}^{J_k^l}, l \in \mathbb{L}_k, J_k^l = J_{k|k-1}^l(1+m)$ at time k are obtained, where $m = \tilde{P}(k-1)$ is the number of estimated targets at time $k-1$. Then the multi-target posterior probability density parameter set at time k is $\left\{\left(\mathbb{L}_{k,h}, \omega_{k,h}, x_{k,h}\right)\right\}_{h=1}^{H_k}$. H_k is the total number of components after the second truncation. The detailed algorithm can be found in [21].

IV. SIMULATION RESULTS

Traditional DOA tracking needs to know the number of sources in advance, so they cannot solve the time-varying source tracking problem. On this basis, our proposed method GLMB-DOA does not need to know the number of targets in advance and can effectively track two time-varying sources. Since there is no method comparison for the DOA tracking problem under ULA, we use the existing methods for combined comparison.

- PHD-DOA: [12] introduced the probability hypothesis density (PHD) tracking algorithm, which is mainly the first moment of the recursive posterior probability. It is easier to correlate the PHD method with DOA tracking by using the measurement association mapping, which has been provided in section III. *A*.

In the following simulation experiments, the evaluation method of estimated performance is root mean square error

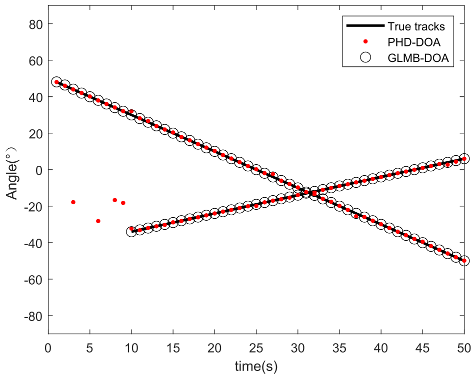

FIGURE 1. The real-time tracking, SNR = 10 dB, $L = 100$, $MC = 1$.

($RMSE$) and probability of convergence ($PROC$), defined as

$$RMSE = \frac{1}{P(k)}\sum_{p=1}^{P(k)}\frac{1}{MC}\sum_{j=1}^{MC}\left(\sqrt{\frac{1}{K}\sum_{k=1}^{K}\left(x_{kj}-\tilde{x}_{kj}\right)^2}\right) \quad (27)$$

$$PROC = \frac{1}{K}\sum\nolimits_{k=1}^{K}\sum\nolimits_{j=1}^{MC}1_{kj}/MC \times 100\% \quad (28)$$

where

$$1_{kj} = \begin{cases} 1, & \left|x_{kj}-\tilde{x}_{kj}\right| < \varepsilon \\ 0, & otherwise \end{cases} \quad (29)$$

x_{kj} and $\tilde{x}_{kj}$ are the true and estimated values in the simulation experiment at time k of the j th Monte Carlo, $P(k)$ is the number of sources at time k, and MC represents the number of monte carlo experiments. In the subsequent experimental simulation, we set $\varepsilon = 1$.

Suppose the source $x_k = \left[\theta_k(t), \dot{\theta}_k(t)\right]^T$ is moving in a constant velocity (CV) model

$$x_k = \mathbf{F}_k x_{k-1} + \mathbf{G} v_k \quad (30)$$

where the coefficient matrix $\mathbf{F}_k$ and $\mathbf{G}$ are respectively

$$\mathbf{F}_k = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix}; \quad \mathbf{G} = \begin{bmatrix} \Delta T^2/2 \\ \Delta T \end{bmatrix} \quad (31)$$

$\Delta T = 1s$ represents the sampling time interval, v_k is the noise term following the zero-mean Gaussian process. That is, $\mathbf{v}_k \sim N(0, \Sigma_k)$. The number of sources varies with time. Source 1 exists from 1 time step to the end of observation time, while source 2 appears from the 10th time step to 50, and the initial source targets are $x_1(0) = [50; -2]$; $x_2(0) = [-35; 1.0]$, $MC = 100$, $\zeta = 5$. Other experimental parameters: the number of array elements is $M = 10$, the array spacing $d = \lambda/2$, the observation time is $K = 50s$, the number of snapshot is $L = 100$, the survival probability and detection probability of source are assumed as constant $p_{s,k}(x_k) = 0.99$ and $p_{D,k}(x_k) = 0.98$, respectively.

The operating environment includes an Intel (R) Core (TM) i5-8500 CPU @ 3.00 GHz processor and a 64-bit operating system MATLAB 2014. In the GLMB-DOA and PHD-DOA prediction step, we assume that there are two new sources at each moment, which are uniformly distributed, and each new source produces 500 particles. Fig. 1 shows the source trajectory tracking graph when SNR = 10 dB and MC = 1. Both algorithms can effectively track the trajectory of targets, but PHD-DOA overestimates the number of targets. Fig. 1 shows that the PHD-DOA algorithm overestimates the number of targets. The PHD algorithm recurses the first moment of the posterior probability density function, which is a one-dimensional variable, so the tracking performance is weak. However, in the proposed GLMB algorithm, adding labels to the targets can get more accurately tracking performance. Compared with the PHD-DOA algorithm, the proposed algorithm has great performance advantages.

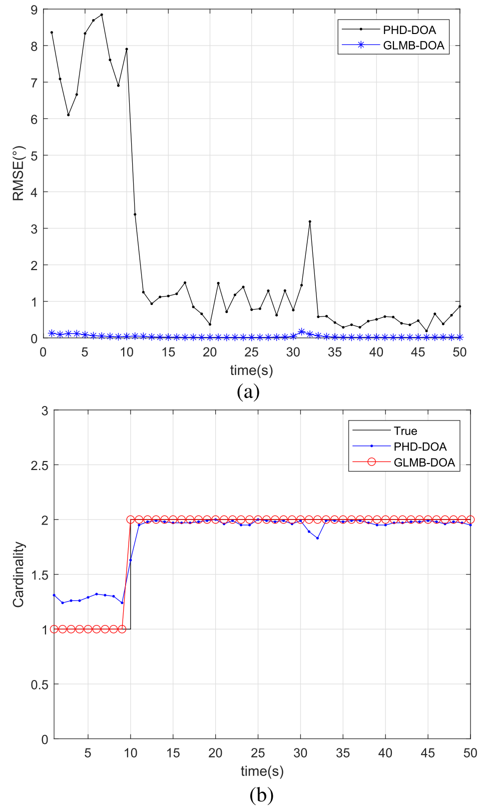

FIGURE 2. RMSE and cardinally estimation at MC = 100, SNR = 10 dB, $L = 100$. (a) RMSE. (b) Cardinally estimation.

Fig. 2 shows the cardinally estimation and RMSE for 100 MC runs, which the SNR = 10 dB. As can be seen from Fig. 2 (a), RMSE of GLMB-DOA is obviously better than PHD-DOA. PHD-DOA has obvious potential error before the first target appears 10 minutes later. It also suggests that labeling particles helps track new changes in the target. In Fig. 2 (b), GLMB-DOA can effectively estimate the number of targets after 100 MC runs, but the PHD-DOA estimation is not accurate. The cardinally of PHD-DOA in the first 10 time steps is estimated to be high.

Fig. 3 is the RMSE and PROC comparison diagram of the two methods versus different SNRs. It can be seen from Fig. 3 (a) that the RMSE error of GLMB-DOA has been

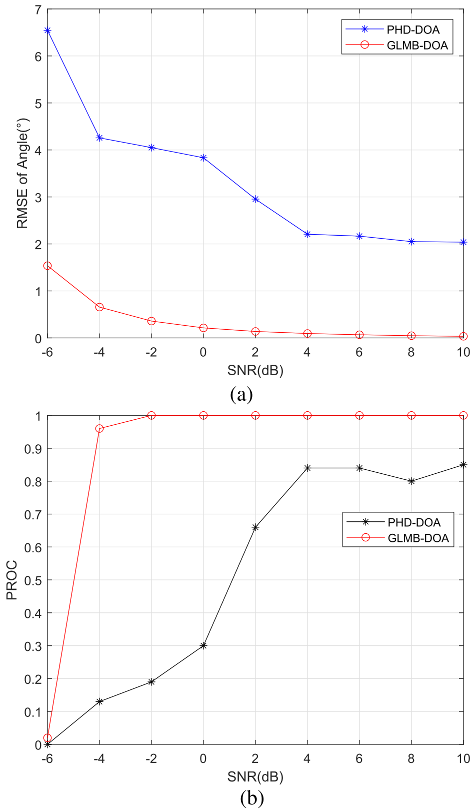

FIGURE 3. RMSE and PROC under different SNRs, MC = 100, $L = 100$. (a) RMSE. (b) PROC.

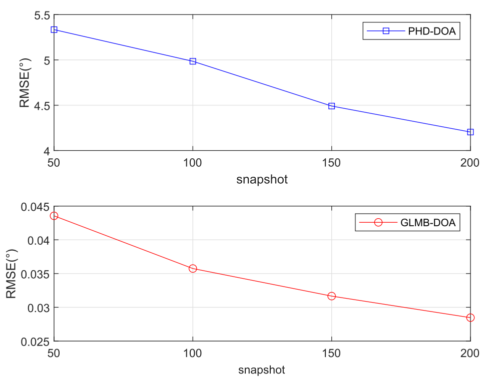

FIGURE 4. RMSE comparison between the two methods versus different snapshots.

reduced to less than 1° when SNR $>= -4$ dB, indicating that the proposed method in this paper can effectively solve the DOA tracking problem at low SNR. It can also be seen from Fig. 3 (b) that GLMB-DOA has a good PROC performance.

Fig. 4 is the RMSE comparison of the two methods of different snapshot. As can be seen from Fig. 4, with the increase of snapshots, the performance of both methods is improving. Comparatively speaking, the proposed algorithm has a lower RMSE under the same snapshots number, which shows the effectiveness of the proposed algorithm.

V. CONCLUSION

In this paper, a DOA tracking algorithm based on generalized labeled Multi-Bernoulli (GLMB) filter is proposed, and SMC implementation of the proposed method is given. We used the estimated number of sources at time $k-1$ to replace the number of measurements used at time k. In addition, the likelihood function can be calculated by MUSIC spatial spectrum function and weighted exponentially in the GLMB update step, then the number of particles in the high likelihood region increases, which makes pruning and merging effectively. The results show that compared with PHD-DOA tracking algorithm, GLMB-DOA algorithm can effectively deal with DOA tracking problem that sources DOAs are time-varying and crossing. But the disadvantage of this algorithm is that it is not effective in tracking more than two time-varying sources, and the follow-up work will be carried out here.

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