IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received December 3, 2020, accepted December 28, 2020, date of publication January 4, 2021, date of current version February 4, 2021.

Digital Object Identifier 10.1109/ACCESS.2020.3049017

Injection Coding With Spectral Shaping Capability

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ABSTRACT In digital transmission systems that involve pulse amplitude modulation (PAM), channel coding is useful for spectral shaping by concentrating the frequency energy of the transmitted information towards a predetermined range of the frequency spectrum, or making it have low power content at such frequencies, so as to suit the frequency characteristics of the communication channel. We thus present a permutation coding system with injections that can exhibit spectral shaping at rational sub-multiples of the symbol frequency. We also present a mathematical expression that allows for the prediction of low energy positions in the codebook's spectrum. Upper and lower bounds for the low energy spectrum were also determined. Due to the way the injections are introduced, it gives the scheme an advantage of achieving higher symbol rate, when compared with conventional permutation coding systems.

INDEX TERMS Frequency spectrum, injection, PAM, permutation codes, power spectral density, pulse amplitude modulation, spectral shaping, spectrum nulls.

I. INTRODUCTION

Injection coding is a coding scheme whose construction and characteristics are similar to those of permutation coding (PC). It however has an advantage of offering larger cardinality than PC, if they both have the same codeword length. Also, if an injection coding and PC of the same cardinality are considered, the former has the advantage of higher symbol rate than the latter. Advantages of injection coding over PC is reported in [1], [2]. While non-binary PC has been extensively studied in the literature [3]–[9], non-binary injection coding has not been very much reported.

If high speed digital transmissions, as in optical fiber channel and LAN systems are desired, multilevel pulse amplitude modulation (PAM) is one modulation scheme to consider [10]. This is because it is capable of reducing the symbol rate and bandwidth of the modulated signal since it requires lower switching rate [11], [12]. Also, since PAM is characterized with bi-polar constellations, one can take advantage of manipulating these channel sequences in such a way that low frequency energy is achieved at certain frequency sub-multiples. With this, spectral shaping is said to be achieved.

The associate editor coordinating the review of this manuscript and approving it for publication was Yufeng Wang.

Codebooks that are capable of spectral shaping can be used to overcome some communications problem like zero frequency components and can be used to match specific transmission, multiplexing or storage system design requirements. In [10], the authors defined a distance-preserving mapping algorithm that can find the right non-binary sequence, which is in turn able to generate another non-binary sequences, which when concatenated with a PAM system, spectral shaping can be achieved. In this article, we shall be considering a similar approach, but in our case, injections shall be introduced in the system. More so, we shall present mathematical expressions for predicting low energy levels, as well as their lower and upper bounds, in the spectrum of the proposed injection coding system.

With a view to achieving high rate spectral shaping systems, it is pertinent to make use of a coding technique that can simultaneously achieve these two attributes without introducing complexity in the system. Since existing studies on spectral shaping permutation coding schemes have not considered the possibility of achieving high symbol rates, this thus motivated us to investigate the injection coding technique, which is endowed with high symbol rate capabilities, and its spectral attributes.

Therefore, our contribution in this work is to make injection coding exhibit spectral shaping capabilities at certain predicted rational sub-multiples of frequency, and attain higher symbol rates at the same time. An expression which can be used to determine the best choice of symbol set to be injected in the codebook is derived as well. As far as we are aware, this is the first time such a scheme and expression are reported in the history of injection coding.

Applications such as magnetic and optical recording systems are usually designed by combining two separate schemes which are an error-correction coding scheme and a modulation scheme [13]. To optimise the recording system, the coding and modulation are designed to be of high rate, low complexity, and be able to minimise the negative effects of AC coupling that can result into inter-symbol interference and error propagation [14]–[18]. To mitigate the effects of AC coupling, coding schemes that can perform spectral shaping are needed. This is possible by exploiting unique properties in the frequency spectra of such coding schemes so as to suit the system in which they are used [11]. Also, reports have shown that PAM symbols can be manipulated to achieve spectral shaping by the use of PCs [10], [19], [20]. Therefore, the proposed injection coding scheme can be adopted for such purposes, since it can attain high rates and be manipulated such that spectrum shaping is achieved, when combined with a PAM system. Also, owing to the fact that injection coding has relatively similar characteristics with PCs, it can be employed in other applications such as power line communications, where PCs are involved [1], [2]. Although the scope of this work is limited to coding and modulation, if pulse-shaping is needed in the system, it can be applied after the PAM symbols mapping.

In Section II, we give a brief description of spectral null codes and how they can be identified, after which we describe how PC codebooks can be designed to achieve spectral shaping. We then proceed to Section III, where PC with injections is discussed. The proposed injected PC with spectral shaping capability is detailed in Section IV, including the mathematical expressions that can identify its low spectrum energy and its lower and upper bounds. The evaluation approach and results are discussed in Section V followed by Section VI, where a concise conclusion to the article is provided.

II. SPECTRAL SHAPING

In the design of data transmission, multiplexing and storage, data-containing alphabets can be permuted to achieve spectral shaping. Let us define a codebook $\mathcal{C}$ as comprising of L permutation sequences $C_\tau = (c_0, c_1, \ldots, c_{M-1})$, each of length M, where $\tau = 1, 2, \ldots, L$ and $c_i \in \{0, 1, 2, \ldots, M-1\}$. Here, $C_1 \neq C_2 \neq \cdots \neq C_L$. This is called PC. If each element c_i in C_τ is mapped to its corresponding PAM or channel symbol x_i, the amplitude-phase frequency spectrum of the resulting discrete signal can be simply expressed as [21]

$$s_\tau = \sum_{i=0}^{M-1} x_i e^{-j2\pi fT(i+1)}, \tag{1}$$

where T is the duration of x_i.

For a codebook $\mathcal{C}$ with L number of sequences (i.e., codewords), we can show that its power spectral density function, derived from (1), is given by

$$S = \frac{1}{LM} \sum_{\tau=1}^{L} |s_\tau|^2. \tag{2}$$

The technique of spectral null codebooks started by designing a codeword C_τ with symbols $c_i \in \{0, 1\}$, which is an element of a general set $Q = \{0, 1\}$, whose corresponding PAM symbols are $x_i \in \{-1, 1\}$ [22]. To achieve spectral nulls (i.e., $S = 0$) at certain chosen frequencies, M is made to be a multiple of an integer G. Hence,

$$M = Gg, \tag{3}$$

where g is also an integer.

This implies that each codeword consists of symbols c_i grouped into G groups, with each group consisting of g symbols. If every codeword in $\mathcal{C}$ satisfies the expression [16], [21]

$$\begin{aligned} &A_0 = A_1 = \cdots = A_{G-1}, \quad \text{where} \\ &A_i = \sum_{n=0}^{g-1} x_{i+nG}, \quad i = 0, 1, \ldots, G-1, \end{aligned} \tag{4}$$

then the codebook will exhibit spectral nulls at the rational sub-multiples of frequency $1/G$. In other words, if $S = 0$ or has a low energy value at frequency multiples of $1/G$, the codebook is said to be a spectral shaping type. As such, spectral null codebooks are also regarded as spectral shaping codebooks. Let us consider the following binary spectral null codebook as an example, as shown at the bottom of the next page: where $G = 3$ and $g = 2$.

Here, symbol 0 is mapped to PAM symbol −1 and symbol 1 is mapped to +1. By applying (4) to codeword C_1, with corresponding PAM symbols X_1 : −1 −1 −1 +1 +1 +1, we have

$$\begin{aligned} A_0 &= x_0 + x_3 = -1 + -1 = -2, \\ A_1 &= x_1 + x_4 = -1 + -1 = -2 \text{ and} \\ A_2 &= x_2 + x_5 = -1 + -1 = -2. \end{aligned} \tag{6}$$

With this, we have that $A_0 = A_1 = A_2$. This scenario applies to all the other codewords $C_2, C_3, \ldots, C_8$. Hence, this codebook is expected to have spectral nulls at sub-multiple frequencies of $1/G = 1/3$. This is evident through its spectrum curve shown in Fig. 1, using a normalized frequency range.

Although the above example is for the case of a binary code, a similar approach is applicable to PCs. It should, however, be noted that the spectral null PCs involved in this work will be limited to cases where M is a prime number, on the basis of the expression in (3).

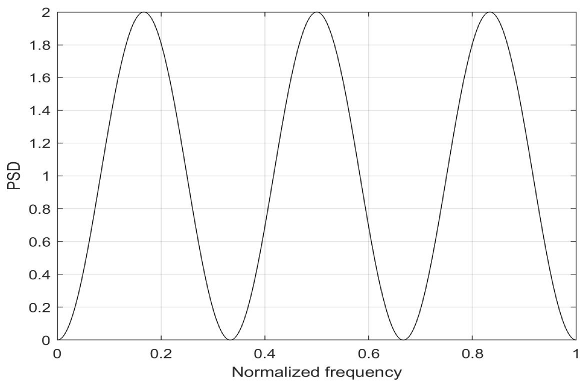

FIGURE 1. **The spectrum of a binary block codebook.**

III. PERMUTATION CODING WITH INJECTIONS

PC is not a new study and its usage for power line communications (PLC) was proposed by Vinck and Häring [3]. Afterwards, a number of researches done in this regard have been published [1], [4]–[9], [23]. PC is made up of codewords C_τ with non-repetitive symbols from the set $Q = \{0, 1, \ldots, q-1\}$ [4], [23], [24], where q is the symbol size. Usually in the case of PC, $q = M$. The cardinality $|\mathcal{C}|$ of a PC is known as the total number of possible codewords in the codebook $\mathcal{C}$, and it is bounded by

$$|\mathcal{C}| \leq \frac{M!}{(H_{\min}-1)!}, \tag{7}$$

where $H_{\min}$ is the minimum Hamming distance, which is the least possible distance between any two codewords. As such, we simply denote a permutation codebook by $\mathcal{C}(M, H_{\min})$.

1) MAPPING INFORMATION BITS TO PERMUTATION SYMBOLS

When PC is used for transmission, information bits are grouped into n bits per group and converted (i.e., mapped) to the corresponding permutation codeword of length M. An example is

$$\left\{\begin{array}{cccc} 00 & 01 & 10 & 11 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 201 & 210 & 102 & 120 \end{array}\right\}. \tag{8}$$

Here, the upper part of (8) are the grouped information bits and the lower part are the permutation codewords. With this, $n = 2$ and $M = 3$. Hence, the rate is given by

$$r = n/M = 2/3. \tag{9}$$

2) MAPPING PERMUTATION SYMBOLS TO CHANNEL SYMBOLS

Given a codebook with M symbols, each symbol can be mapped to a channel symbol using

$$\left\{\begin{array}{ccccccc} 0 & 1 & \ldots\ \frac{M}{2} & \ldots & M-2 & M-1 \\ \downarrow & \downarrow & \downarrow\ \downarrow & \downarrow & \downarrow & \downarrow \\ -\frac{M}{2} & -(\frac{M}{2}-1) & \ldots\ 0 & \ldots & \frac{M}{2}-1 & \frac{M}{2} \end{array}\right\} \tag{10}$$

for odd M, or, as shown at the bottom of the next page for even M.

3) INJECTION CODING

The technique of injection coding got motivated from some deficiencies exhibited by PC. For instance, its cardinality is upper bound by (7). Other PCs parameters with deficiencies were identified in [1], [5]. Injection codes thus offer alternatives for applications with such constraining parameters in PCs.

Given that $C_\tau = \{c_0, c_1, \ldots, c_{M-1}\}$, $C'_\tau \subset C_\tau$ and $Q' \subset Q$, with the length of Q' being M', then an M'-arrangement of C_τ is an injection of C'_τ into C_τ, provided that $M \geq M'$. This implies that a PC has all its codewords from the permutation of all the symbols in Q, while for an injection code, all the codewords are derived by permuting a subset of symbols from Q. For non-binary codewords, $c_i \in \{0, 1, 2, \ldots, M-1\}$. This is best understood using an example.

Example 1: Let us consider a permutation codebook [23]

$$\left\{\begin{array}{l} 012543, 102453, 512043, 152403, 042513, 402153, \\ 542013, 452103, 013542, 103452, 513042, 153402, \\ 043512, 403152, 543012, 453102 \end{array}\right\} \tag{12}$$

where $M = 6$, $H_{\min} = 2$ and $|\mathcal{C}| = 16$. We can derive a new codebook from (12) by removing the first symbol (i.e., column $i = 0$) from each codeword. This process is as illustrated in Fig. 2.

Here, we denote the number of columns removed by δ, and h denotes the index of the column removed. The resulting

$$\left\{\begin{array}{lcccccc} & c_0 & c_1 & c_2 & c_3 & c_4 & c_5 \\ C_1: & 0 & 0 & 0 & 1 & 1 & 1 \\ C_2: & 0 & 0 & 1 & 1 & 1 & 0 \\ C_3: & 0 & 1 & 0 & 1 & 0 & 1 \\ C_4: & 0 & 1 & 1 & 1 & 0 & 0 \\ C_5: & 1 & 0 & 0 & 0 & 1 & 1 \\ C_6: & 1 & 0 & 1 & 0 & 1 & 0 \\ C_7: & 1 & 1 & 0 & 0 & 0 & 1 \\ C_8: & 1 & 1 & 1 & 0 & 0 & 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{lcccccc} & x_0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ X_1: & -1 & -1 & -1 & +1 & +1 & +1 \\ X_2: & -1 & -1 & +1 & +1 & +1 & -1 \\ X_3: & -1 & +1 & -1 & +1 & -1 & +1 \\ X_4: & -1 & +1 & +1 & +1 & -1 & -1 \\ X_5: & +1 & -1 & -1 & -1 & +1 & +1 \\ X_6: & +1 & -1 & +1 & -1 & +1 & -1 \\ X_7: & +1 & +1 & -1 & -1 & -1 & +1 \\ X_8: & +1 & +1 & +1 & -1 & -1 & -1 \end{array}\right\} \tag{5}$$

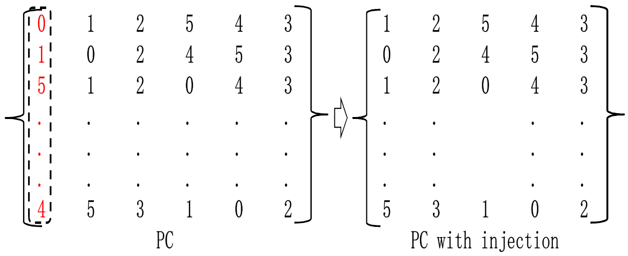

FIGURE 2. **Illustrating PC to injection code.**

entire injection codebook is given by

$$\left\{\begin{array}{l} 12543, 02453, 12043, 52403, 42513, 02153 \\ 42013, 52103, 13542, 03452, 13042, 53402, \\ 43512, 03152, 43012, 53102 \end{array}\right\} \quad (13)$$

Observing the new set of codewords produced from (12), we can see that each codeword is a subset of Q. These two types of codebooks have the same cardinality but different codeword lengths M and M', where notations without the superscript ($'$) stand for ordinary PC, and those with ($'$) are for injection codebook. The injected code has a lower minimum Hamming distance $H'_{\min}$, but with the advantage of achieving increased transmission rate r', since n bits are mapped onto M' symbols.

It is worth noting that, regardless of which column is removed from the PC, the resulting injection codebooks have the same $H'_{\min}$ in either case. This may however have an impact on the frequency spectrum, as will be discussed later in Section IV.

For simplicity, let us denote an injection codebook by $\mathcal{C}'(M', H'_{\min})$, similar to the notation of an ordinary PC. As such, the cardinalities of the ordinary PC and injection code can be denoted as $|\mathcal{C}|$ and $|\mathcal{C}'|$, respectively. Hence, a PC and an injection coding with the same cardinality $|\mathcal{C}| = |\mathcal{C}'|$, are related by the expression:

$$\left.\begin{array}{l} H'_{\min} < H_{\min}, \;\; M' < M, \\ Q' \subset Q \;\text{ and} \\ (r' = n/M') > (r = n/M) \end{array}\right|, \;\text{ if } |\mathcal{C}| = |\mathcal{C}'|. \quad (14)$$

Example 2: Provided that it is desired to generate a codebook $\mathcal{C}(6, 5)$, the maximum possible value of $|\mathcal{C}|$ is 30 for a PC, by using the expression in (7). In the case of injection coding, it may be worthwhile to increase the symbol size to 7, construct a PC of $H_{\min} = 6$ and remove one column, thereby yielding an injection codebook $\mathcal{C}'(6, 5)$ with $|\mathcal{C}'| = 42$ and $q' = 7$. This implies that $H'_{\min} = H_{\min}$, $M = M'$, but $|\mathcal{C}'| > |\mathcal{C}|$ and $q' > q$.

From the scenario in *Example 2*, the PC can map only $n = 4$ bits onto M symbols, while the injected PC is able to map $n = 5$ bits onto M' symbols.

Henceforth, if an injection codebook is derived from a PC, we shall refer to the resulting codebook as the target codebook, while the PC will be called the source codebook.

IV. INJECTION CODING WITH SPECTRAL SHAPING CAPABILITY

We shall give a brief overview of spectral null PCs before proceeding to describe injection coding with spectral shaping.

1) PERMUTATION CODING WITH SPECTRAL NULLS

Literature has reported various ways by which spectral null PCs can be constructed [10], [19], [20]. According to (3), the condition for a spectral null codebook is that M is made to be a multiple of G, such that $M = Gg$, where g is an integer. Our focus is on the case where M is even and $g = 2$. Hence, if PC symbols $(0, 1, \ldots, M)$ are mapped based on (11), as shown at the bottom of the page, each channel symbol will always have its complement in the arrangement. This implies that the summation of each codeword, after being mapped to the PAM symbol will yield zero. Let us consider the following spectral null PC mapping as an example, where $M = 4$, $g = 2$ and $G = 2$:

$$\left\{\begin{array}{lcccc} & c_0 & c_1 & c_2 & c_3 \\ C_1 : & 0 & 1 & 2 & 3 \\ C_2 : & 1 & 0 & 2 & 3 \\ C_3 : & 1 & 3 & 2 & 0 \\ C_4 : & 0 & 2 & 3 & 1 \end{array}\right\} \Rightarrow \left\{\begin{array}{lcccc} & x_0 & x_1 & x_2 & x_3 \\ X_1 : & -1.5 & -0.5 & +1.5 & +0.5 \\ X_2 : & -0.5 & -1.5 & +0.5 & +1.5 \\ X_3 : & -0.5 & +1.5 & +0.5 & -1.5 \\ X_4 : & -1.5 & +0.5 & +1.5 & -0.5 \end{array}\right\} \quad (15)$$

According to (15), the mapping is done by deriving the first $M/2$ columns (i.e., first half) of the channel symbols from (11) and the remaining $M/2$ columns (i.e., second half) are constructed such that, for every column in the first half, there exists an exact complement in the second half. That is, column $i = 0$ is the complement of column $i = 2$, and column $i = 1$ is the complement of column $i = 3$. With this, the conditions in (4) are fulfilled.

2) INJECTION CODING WITH SPECTRAL SHAPING

In Section III, we have demonstrated that injection codebooks can be derived from permutation codebooks. We have as well considered the characteristics of spectral null PCs in the above sub-section. The challenge is therefore to determine whether an injection codebook derived from such PC will exhibit spectral shaping capability or not.

Let us consider the case in *Example 1*. If each permutation codeword in (12) is compared with the corresponding injection codeword in (13), we will discover that one element

$$\left\{\begin{array}{cccccccccc} 0 & 1 & \ldots & \frac{M}{2}-1 & \frac{M}{2} & \ldots & M-2 & M-1 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\;\downarrow & & \downarrow & \downarrow \\ -\frac{M-1}{2} & -(\frac{M-1}{2}-1) & \ldots & -\frac{1}{2} & \frac{1}{2} & \ldots & \frac{M-1}{2}-1 & \frac{M-1}{2} \end{array}\right\} \quad (11)$$

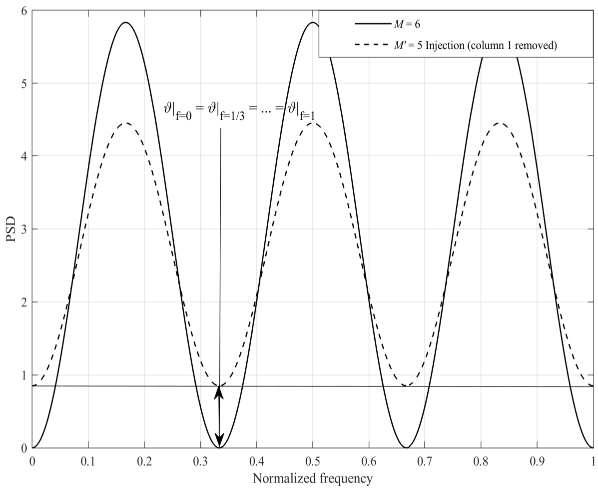

FIGURE 3. **The spectrums of $M = 6$ PC and $M' = 5$ injection codebooks (12), (13).**

in permutation codeword 1 ($C_1 = 012543$), for instance, is not present in the corresponding injection codeword 1 ($C'_1 = 12543$). In addition, by examining the elements of the permutation codebook in (12) as a whole, we discover that every individual element represented therein is also represented in the target injection codebook in (13) as a whole. As such, if the spectrums of the two codebooks are plotted, they are expected to be relatively similar. However, since $C'_1 \neq C_1$ and $M' < M$, this therefore breaks the conditions described in (3) and (4). We should therefore expect a slight variation in their average frequencies. This is as validated in Fig. 3.

According to Fig. 3, the spectrum curves are presented in a normalized frequency format, ranging from 0 to 1. For any spectral null codebook, the amplitude of the spectrum is approximately zero at certain frequency intervals. That is why the spectrum curve for the $M = 6$ PC is touching the frequency axis (i.e., zero amplitude) at k sub-multiples of $1/G$, where $G = 3$ and $k = 0, 1, \ldots, G$. The amplitudes are non-zero at any other frequency other than k/G. A similar pattern is expected for all spectral null codebooks in this work.

Since the spectrums of the source PC and the target injection codebook in Fig. 3 exhibit low energy contents at frequency sub-multiples of $1/G = 1/3$, they are both said to have spectral shaping capability. Furthermore, the PC is also considered a spectral null codebook, since its energy is completely zero at sub-multiples of $1/G = 1/3$, while the injection codebook is not. Henceforth, we shall refer to the gap between a spectrum's lowest energy point (i.e. notch) and the zero mark as the spectrum variation ϑ.

Proposition 1: Given a permutation code with spectral nulls at frequency sub-multiples of $1/G$, that is $f = k/G$, it is possible to create injection codes that have notches at the same frequency sub-multiples. If $\delta = 1$, the notches for such injection codes exhibit a spectrum variation ϑ above zero energy frequency, which is given by

$$\vartheta|_{f=k/G} = \frac{1}{L(M-\delta)} \sum_{\tau=1}^{L} \left|s'_\tau\right|^2, \tag{16}$$

where

$$s'_\tau = \sum_{i=h}^{(h+\delta)-1} x_i e^{-j2\pi fT(i+1)}, \tag{17}$$

δ is the number of columns removed from the source PC, h is the index of the column removed, L is the number of sequences per codebook, and $k = 0, 1, \ldots, G$.

Proof: In comparison with (1), we observe that (16) represents the spectrum of δ number of columns removed from the source PC. As stated in Section IV-4, each column has its exact complement in the entire codebook after being mapped to channel symbols. This implies that, at $f = k/G$, the spectral null source PC will have $s_\tau = 0$. However, if $\delta = 1$ column is removed, the remaining columns will sum up to zero, according to (1), leaving the exact complement of the column removed. In other words, the spectrum of the resulting target injection codebook, if evaluated at $f = k/G$, is the same as the spectrum of the column removed, which is the variation ϑ above zero in the spectrum. This thus proves (16).

Proposition 2: For $\delta = 1$, the spectrum variation ϑ of a target injection code has a maximum and minimum possible value bounded by

$$\frac{M-1}{4} \geq \vartheta|_{f=k/G} \geq \frac{1}{4(M-1)}, \tag{18}$$

if it is evaluated at frequency positions $1/G$, where $k = 0, 1, \ldots, G$.

Proof: According to (11), channel symbols with the absolute minimum and maximum energy will be $-1/2$ or $1/2$ and $-(M-1)/2$ or $(M-1)/2$, respectively.

In the case of injection code, where $\delta = 1$, if the column removed from the source PC contains a mixture of only $-1/2$ and $1/2$, (16) can be evaluated at any sub-multiples of $1/G$ as

$$\frac{L(1/2)^2}{L(M-1)} = \frac{(1/2)^2}{(M-1)} = \frac{1}{4(M-1)}. \tag{19}$$

This simply implies that any channel symbol combination in the removed column can not yield any notch value (i.e., ϑ evaluated at any sub-multiple of $1/G$) that is less than (19).

Also, if the column removed from the source PC contains a mixture of only $-(M-1)/2$ and $(M-1)/2$, (16) can be evaluated at any sub-multiples of $1/G$ as

$$\frac{L((M-1)/2)^2}{L(M-1)} = \frac{((M-1)/2)^2}{(M-1)} = \frac{(M-1)^2}{4(M-1)} = \frac{(M-1)}{4}. \tag{20}$$

This therefore implies that any channel symbol combination in the removed column can not yield any notch value that is greater than (20). Both (19) and (20) thus proves (18).

Depending on the desired design, it is possible to remove more than one columns from the source code in order to achieve the needed target codebook. For instance, an $M = 12$ source code has indices $i = 0, 1, 2, \ldots, 11$. Hence, if $M' = 9$ target codebook is required by starting to delete from the tenth column onwards, then $\delta = 3$ while $h = [9, 10, 11]$.

As M increases, with δ kept fixed, the variation ϑ reduces, thereby causing both the source PC and the target injection codebooks to have relatively overlapping notches on the frequency axis, according to (16). Also, as δ increases while keeping M fixed, the variation ϑ increases.

It should, however, be noted that (18) only holds for cases where $\delta = 1$. As such, we have focused the evaluations done in this work to such cases and present limited results on cases where $\delta > 1$. In the following section, we shall evaluate M' spectral shaping injection codebooks from the corresponding spectral null PC codebooks. To avoid repetition of knowledge, the reader is hereby referred to [10], [11], [20] for details on spectral null PC constructions.

It is worth noting that the proposed scheme is also applicable to QAM channel symbols since its constellations are symmetrical and can be built from two orthogonal PAM components.

For simplicity, we shall henceforth denote a spectral null PC by $\mathcal{C}_{[M,G]}$ and the corresponding injection codebook by $\mathcal{C}'_{[M'G,\delta,h]}$, where M, M', G, δ and h are the same as previously defined in the previous sections. For example, in the case of *Example 1*, $M = 6$ and $G = 3$. Hence, the spectral null PC is $\mathcal{C}_{[6,3]}$. The corresponding injection codebook derived from such PC, by removing column $h = 0$, is also denoted by $\mathcal{C}'_{[5,3,1,0]}$

V. EVALUATION AND PERFORMANCE RESULTS

In this section, we shall construct some target spectral shaping injection codebooks from their corresponding spectral null source PC counterparts, after which their spectrum curves are each drawn, using a normalized frequency range, for comparison purposes.

We first consider a $\mathcal{C}_{[4,2]}$ PC, which is an $M = 4$ PC with spectral nulls at sub-multiple of frequency $1/G$, where $G = 2$ [10]. According to (18), we have

$$\frac{3}{4} \geq \vartheta|_{f=k/2} \geq \frac{1}{12}. \tag{21}$$

From this PC, we constructed $\mathcal{C}'_{[3,2,1,0]}$ injection codebook by removing column index $h = 0$ from the source PC. Using (16), ϑ at frequency sub-multiples of $1/2$ is calculated as

$$\vartheta|_{f=0} = \vartheta|_{f=1/2} = \vartheta|_{f=1} = 0.4167, \tag{22}$$

which falls within the range calculated in (21).

Fig. 4 shows the resulting spectrums of these 2 codebooks. As predicted, the $\mathcal{C}_{[4,2]}$ and $\mathcal{C}'_{[3,2,1,0]}$ codebooks exhibit spectral shaping characteristics, having their notches at $f = 0, 1/2$ and 1. The notches of the target code's spectrum also have the same values as calculated in (22).

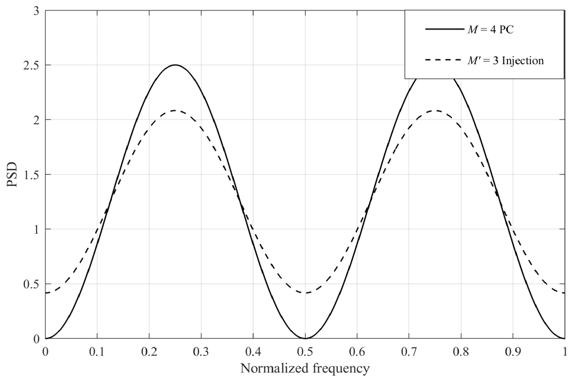

FIGURE 4. The spectrums of an $M = 4$, $G = 2$ spectral null PC and target injection codebooks.

Table 1 shows $M = 6$ PC with spectral nulls at sub-multiple of frequency $1/G$, where $G = 3$ [10]. According to (18), we have

$$\frac{5}{4} \geq \vartheta|_{f=k/3} \geq \frac{1}{20}. \tag{23}$$

The corresponding target injection codebook was constructed by removing column index $i = 0$ for $M' = 5$, from the source codebook. Using (16), ϑ is given by

$$\vartheta|_{f=0} = \vartheta|_{f=1/3} = \cdots = \vartheta|_{f=1} = 0.45, \tag{24}$$

which falls within the range calculated in (23).

Fig. 5 shows the resulting spectrums of the codebooks in Table 1. Here, codebook $M = 6$, $M' = 5$ is seen to exhibit spectral shaping capabilities. We also observe that the spectrum variation obeys the expressions in *Proposition 1* and *Proposition 2*.

For $M = 8$, a sample $\mathcal{C}_{[8,4]}$ source PC was considered, from which a corresponding $\mathcal{C}'_{[7,4,1,0]}$ target injection codebook was derived. Its spectrum variation is given by

$$\vartheta|_{f=0} = \vartheta|_{f=1/4} = \cdots = \vartheta|_{f=1} = 1.321, \tag{25}$$

This also falls within the lower and upper bounds calculated from (18) as

$$\frac{7}{4} \geq \vartheta|_{f=k/4} \geq \frac{1}{28}. \tag{26}$$

To demonstrate what effect the choice of the column selected for deletion has, the last column (i.e., $h = 7$) was removed to get another $\mathcal{C}'_{[7,4,1,7]}$ target codebook whose ϑ is calculated to be

$$\vartheta|_{f=0} = \vartheta|_{f=1/4} = \cdots = \vartheta|_{f=1} = 0.0357. \tag{27}$$

The variation ϑ from (27) is exactly the same value as the lower bound computed in (26). This is due to the fact that last column of the source PC has only a mixture of $-1/2$ and $+1/2$ symbols.

With a view to analysing cases of $\delta > 1$, we also derived $\mathcal{C}'_{[6,4,2,h]}$ codebook (where $h = [0, 7]$ columns are removed), $\mathcal{C}'_{[5,4,3,h]}$ codebook (where $h = [0, 1, 7]$

TABLE 1. $M = 6$, $G = 3$ **Source PC and Corresponding Target Injection Codebooks With Spectral Shaping.**

$M = 6$	$M' = 5$
012543	12543
013542	13542
042513	42513
043512	43512
512043	12043
513042	13042
542013	42013
543012	43012
102453	02453
103452	03452
402153	02153
403152	03152
152403	52403
153402	53402
452103	52103
453102	53102
210345	10345
310245	10245
240315	40315
340215	40215
215340	15340
315240	15240
245310	45310
345210	45210
201354	01354
301254	01254
204351	04351
304251	04251
251304	51304
351204	51204
254301	54301
354201	54201

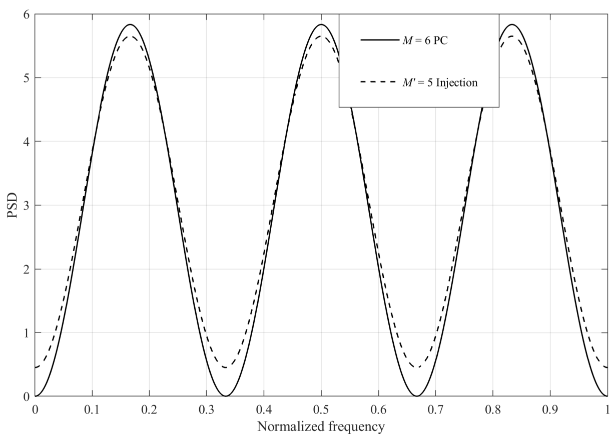

FIGURE 5. **The spectrums of an** $M = 6$, $G = 3$ **spectral null PC and target injection codebooks.**

columns are removed) and $\mathcal{C}'_{[5,4,4,h]}$ codebook (where $h = [0, 1, 6, 7]$ columns are removed) from the $\mathcal{C}_{[8,4]}$ source PC.

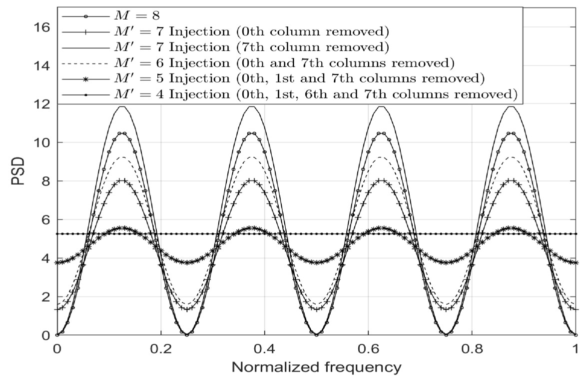

FIGURE 6. **The spectrums of an** $M = 8$, $G = 4$ **spectral null PC and target injection codebooks.**

Fig. 6 shows the spectrums of these codebooks. All the curves exhibit notches at the expected rational sub-multiples of $1/4$. For $\mathcal{C}'_{[7,4,1,7]}$, the spectrum is seen to overlap with that of the source PC's spectrum at frequencies 0, $1/4$, $1/2$, $3/4$ and 1. This is due to the fact that ϑ values are extremely low. For $\delta > 1$, we observe that the spectral shaping capability only gets severely affected at $\delta = 4$. Hence, for $M = 8$ PC, we can infer that, from $\delta = 1$ to $\delta = 2$, high rates can be attained while still achieving spectral shaping. It should however be noted that the obtained results are specific for cases of $\delta > 1$, where columns $h = [0, 7]$, $h = [0, 1, 7]$ and $h = [0, 1, 6, 7]$. Any other columns combination selected for removal will result into spectrums with notches that are completely different from those in Fig. 6.

Based on all the results from Fig. 4 to Fig. 6, it is clear that a target injection codebook derived from a source spectral null PC will also exhibit spectral shaping capabilities. The expression in (16) also perfectly helps to predict the notch values of the target codebooks. As compared to other possible column injections, any column injection that yields the smallest value of ϑ, as computed from (16), will produce an injection codebook with the best spectrum shaping. This thus validates our statement in *Proposition 1*. We also observe that the notches of the target injection codebooks fall within the lower and upper bounds presented in *Proposition 2*. Therefore, in code construction for injection codebooks, (16) becomes very useful in arriving at the codebook with the best spectrum attribute, and (18) helps to predetermine the worst case scenarios in such codebooks.

VI. CONCLUSION

We have reported a special form of non-binary coding scheme, which is able to exhibit spectral shaping at certain frequency sub-multiples. This evolved from the injection of selected symbols from the conventional spectral null PCs. It was established that the effect of average frequency variation can be drastically minimized, if the column injected is strategically selected with the help of the expression derived in *Proposition 1*. The injected PC is able to attain a higher symbol rate. The proposed spectral shaping injection coding scheme is a good candidate for matching specific transmission, multiplexing, storage system design or digital recording requirements. Also, when used in PAM systems, it may be employed in applications such as LAN systems, and in fiber channels. Since most of the analyses presented in this work is emphasised on cases where $\delta = 1$, an extended work will extensively consider cases where $\delta > 1$.

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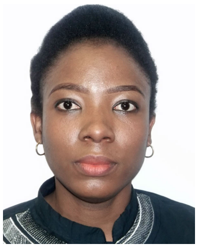

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