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A New Method for 3-Satisfiability Problem Phase Transition on Structural Entropy

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ABSTRACT The phenomenon of phase transition is an important property of the satisfiability (SAT) problem. It not only determines the difficulty of solving the problem, but also plays an important role in designing fast solving algorithms. The structural entropy is an effective method to measure the complexity of graph structure. It maps the proposition formula to the graph structure, and then gives the structural entropy of it. A new method of 3-SAT phase transition measurement based on structural entropy is proposed. Through numerical experiments on the random 3-SAT instance set, the results show that the satisfiability of the random 3-SAT problem has phase transition with the change of structural entropy. At the same time, it is very difficult to solve the problem in the region near the phase transition point.

INDEX TERMS Phase transition, satisfiability problem, structural entropy.

I. INTRODUCTION

The satisfiability problem is a special kind of constraint satisfaction problem (CSP), which is the core problem in theoretical computer research [1]–[3]. It refers to a given conjunctive normal form (CNF), determining whether there is a set of assignments to make the formula true. Many difficult combinatorial optimization problems can be solved as SAT problems by using polynomial reduction conversion technology [4]. This means that if an effective algorithm can be found to solve the SAT problems, then other problems will be easily solved. Therefore, the research of SAT problem has always been the focus in the field of computer science.

Among the SAT problems, the most studied is the random K-SAT problems. It refers to the SAT problems in which all clauses contain k literals. When $k \geq 3$, the K-SAT problem is the first proven NP complete problem [5]. At present, the research of SAT problem mainly focuses on the aspects of constructing instance generation model, phase transition phenomenon, difficulty of solving the problem, and designing effective algorithms. The phenomenon of phase transition was originally a critical theory in physics, which is the transformation of matter between different physical states. In 1991, Cheeseman and others introduced the theory into the field of computer research, and proved that many NP-complete problems have phase transition phenomena such as hamilton circuit, graph coloring problems, K-SAT problems, traveling salesman problems, etc [6]. Among these problems, the study of the phase transition phenomenon of the random K-SAT problem is still an active research focus in the field of artificial intelligence [7], [8].

In the research of the phase transition in the random K-SAT problem, a critical phenomenon appears between satisfiable and unsatisfiable. In this phenomenon, the clause constraint density α of the CNF formula is a very important structural parameter [9], [10]. α is defined as the ratio of clauses to variables, and α_d is defined as the phase transition point. When $\alpha < \alpha_d$, almost all clauses are satisfying; when $\alpha > \alpha_d$, almost all clauses are unsatisfying. The sudden change clause from satisfiable to unsatisfiable is called the phase transition phenomenon of the random SAT problem. Another property related to the phase transition is the difficulty of solving the problem. Because the solution space of the SAT problem is divided into many smaller solution clusters and these solution clusters are separated from each other, there are a large number of frozen variables in each solution cluster near the

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phase transition point. It is impossible to transform it into a satisfiable assignment in another solution cluster by reversing the assignment of a small number of variables in this solution cluster that satisfies the assignment [11]–[13]. Therefore, it is very difficult to determine the satisfiability of the random SAT problem near the phase transition point, and almost all algorithms for solving the SAT problem cannot exhibit performance well. In the region far from both sides of the phase transition point, most of the instances become easy to solve.However, it is very difficult to find the exact phase transition point. The specific value has not been known so far, and only the upper and lower boundaries of the region where the phase transition point occurs can be estimated. Studies have shown that the phase transition point α_d of random 3-SAT is at least 3.53 [14] and at most 4.506 [15]. When $3.52 < \alpha < 4.506$, the 3-SAT problem is more difficult to solve. Mertens *et al.* [16] and others have used the cavity fields method of statistical physics to obtain an approximate value of $\alpha_d \approx 4.267$. The parameter α_d controls the satisfiability and difficulty of the SAT problem, and most of the algorithms for solving the SAT problem are closely related to α. There are many excellent metaheuristic algorithms [17]–[20] that can be used to solve combinatorial optimization problems. Message propagation algorithm is currently the most effective algorithm for solving 3-SAT problems. Among them, Belief Propagation (BP) is effective for solving the random 3-SAT instances when $\alpha < 3.95$, and the Survey Propagation (SP) for solving the random 3-SAT regional in $\alpha < 4.26$ [21]–[23]. It can be seen that the phenomenon of phase transition is particularly important in the research of random SAT problems. Studying the phase transition phenomena of random SAT problems helps to understand the intractable nature of the problem, so as to propose a faster and more effective solution algorithm.

At present, the research on SAT problem is slower and more complexing. Therefore, this paper proposes a new method to measure the phase transition phenomenon of random 3-SAT problems. In the SAT problem, each CNF formula can be represented by a factor graph. For instances where the factor graph is a tree structure, the information propagation algorithm can effectively converge; but for instances where the factor graph contains multiple loops, the information propagation algorithm often shows un-convergence [24]. It can be seen that the factor graph structure affects the performance of the algorithm. Structural entropy can measure the structural information embedded in the graph, reflecting the dynamic complexity of the factor graph. Therefore, it is proposed to use structural entropy to measure the phase transition phenomenon and the degree of difficulty in random 3-SAT problems. The random 3-SAT problem is mapped into a factor graph, and the factor graph is divided into independent nodes by the minimum cut method. Calculating the structural entropy of the factor graph through the encoding tree. Finally, the phase transition of the 3-SAT problem is analyzed by the structural entropy of the factor graph. The relationship between the structural entropy of the formula factor graph and the satisfiability and difficulty of the 3-SAT problem was observed experimentally. The experimental results show that the satisfiability of the 3-SAT problem undergoes a phase transition with the change of structural entropy, and near the phase transition point, the problem is very difficult to solve.

In real life, there are many important and difficult to solve verification and combinatorial optimization problems that can be transformed into SAT problems to solve, such as the traveling salesman problem and the 0-1 knapsack problem. The phase transition phenomenon of the 3-SAT problem is measured by structural entropy, and the phase transition measurement method is transformed from the property of the formula to the structural property of the factor graph. The original problem can be directly calculated without converting the original problem into a CNF formula with a specific structure. With the graph model, we can know the satisfiability of the SAT problem after conversion. This method has simple judgment conditions and a wide range of applications. It provides an experimental basis for the next step to explore the features of difficult examples, and also provides new ideas and directions for designing fast and effective solving algorithms. Moreover, the method can also be extended to other research directions, such as Pythagorean fuzzy uncertain environment [25] and other issues.

II. BASIC KNOWLEDGE

A. FACTOR GRAPH

Set $F = \{C_1, C_2, \cdots, C_m\}$ as a CNF formula which contains n boolean variables $\{x_i \in \{0, 1\}\}$ (where 0 is false, 1 is true with $i \in \{1, 2, \cdots, n\}$). The variable node set is $x = \{x_1, x_2, \cdots, x_n\}$, the clause node set is $C = \{C_1, C_2, \cdots, C_m\}$,and the edges in graph G are divided into two categories: solid line and dotted line [26].

Every SAT problem can be represented by a factor graph $G = (C \cup X, E)$, which is composed of function nodes, variable nodes and connected edges. Every clause has a corresponding function node (represented by a square in the figure), and every variable has a corresponding variable node (represented by a circle in the figure). When a positive literal x_j appears in the clause C_i, the function node C_i and the variable node x_j are connected by a solid line; when a negative literal $\neg x_j$ appears in the clause C_i, the clause node C_i and the argument node x_j Connect with dotted lines. Among them, the weight of the edge is represented by $w(i, j)$. The value of the weight of the edge in this paper is as follows:

$$w(i,j) = \begin{cases} 1-\delta & x_i \in C_j \\ \delta & \neg x_i \in C_j \end{cases} \left(0 < \delta \leq 2^{-3}\right) \quad (1)$$

The factor graph corresponding to the formula $F = (x_1 \vee x_2 \vee \overline{x_3}) \wedge (x_2 \vee \overline{x_3} \vee x_4) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_4})$ is shown in Figure 1.

B. STRUCTURAL ENTROPY

Shannon information theory is the theoretical basis of data analysis and information processing. However, Shannon entropy can only measure the amount of unstructured probability distribution information, and cannot measure the information embedded in the structure. Li and Pan [27] proposed a method of measuring system information with structural entropy, which solved the information measurement problem proposed by Brooks [28] and Shannon [29]. Structural entropy is a natural extension of shannon entropy, which is a dynamic measurement method. Shannon entropy can only be used to measure the single information between points. Structural entropy makes up for the defect that shannon entropy cannot accurately reflect the complexity of the network structure. It can be used to measure the dynamical complexity of networks and the complexity of internal structure. Clause nodes and variable nodes in a factor graph can be directly connected through edges, while clause nodes can only be indirectly connected through other nodes. Similarly, variable nodes can only be connected indirectly. The special relationship between the nodes makes the factor graph produce a multi-level complex structure. Therefore, this paper introduces structural entropy to measure factor graphs. For a given undirected connected graph $G = (V, E)$, V is the set of all nodes in G, and E is the set of all edges in G. T is an encoding tree of G, ε is the node in the encoding tree, ε^- is the parent node of ε, and the structural entropy is defined as:

$$H^T(G, \varepsilon) = -\frac{g_\varepsilon}{2m} log_2 \frac{V_\varepsilon}{V_{\varepsilon^-}} \tag{2}$$

$$H^T(G) = \sum_{\varepsilon \in T, \varepsilon \neq \lambda} H^T(G, \varepsilon) \tag{3}$$

Among them, $m = |E|$; g_ε is the number of edges sent from nodes in T_ε to nodes outside T_ε (for a weighted graph G, g_ε is the sum of the weights of the edges sent from nodes in T_ε to nodes outside T_ε); V_β is the volume of the set T_β is the sum of the degrees of all nodes in T_β. Research [22] has shown that minimizing the entropy of the k-dimensional structure is the principle of detecting the true structure in the natural network. The k-dimensional structural entropy of G is defined as follows:

$$H^K(G) = min_{T:h(T) \leq K} \left\{ H^T(G) \right\} \tag{4}$$

According to the definition of k-dimensional structural entropy, we can find an optimal encoding tree $T^*(h(T^*) \leq K)$ such that $H^{K+1}(G) \leq H^{T*}(G)$. Therefore, the k-dimensional structural entropy of G can also be defined as: $H^K(G) = H^{T*}(G)$.

III. STRUCTURAL ENTROPY MEASURES THE PHASE TRANSITION OF 3-SAT PROBLEM

According to the definition of structural entropy, in order to obtain the k-dimensional structural entropy of the factor graph, the main task is to find an optimal segmentation method to obtain the encoding tree T^*, which minimizes the value of the k-dimensional structural entropy. In recent years, typical segmentation methods based on graph cut theory include minimum cut, standard graph cut, minimum and maximum graph cut, etc. The minimum cut problem has been studied and applied by mathematicians from the past to the present. Sixty years ago, the minimum cut and maximum flow algorithm based on graph theory was first proposed, until now a complete set of theory and application system has been formed. More and more fields can get solutions from the minimum cut and maximum flow model. The minimum cut algorithm is very suitable for solving combinatorial optimization problems. At present, there are many researches on community discovery using the minimum cut algorithm in the academic field [30]. The minimum cut is defined as: among all the cuts in the graph, the cut with the smallest sum of edge weight is the minimum cuts. Therefore, the cyclic use of the minimum cut for the factor graph can reduce the structural information loss of the structural entropy in the calculation process, obtain the desired encoding tree T^*, and obtain the smallest k-dimensional structural entropy.

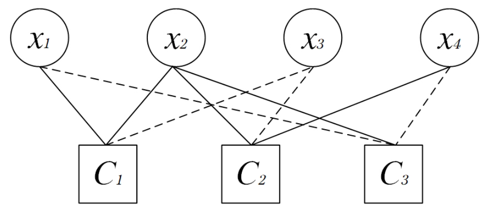

FIGURE 1. Factor graph.

This paper focuses on observing the phase transition phenomenon and the degree of difficulty of the random 3-SAT problem. Generate random 3-SAT instances by using the $G(n, 3, m)$ model to construct factor graphs of the instances. After a large number of experiments, it is shown that when the problem scale is small, the change of δ value does not have much influence on the result of structural entropy. As the problem scale increases, the larger the value of δ, the smaller the value of structural entropy. Therefore, this paper sets δ to 0.125. According to formula (1), when the clause i contains a positive literal j, $w(i, j) = 0.875$; when the clause i contains a negative literal j, $w(i, j) = 0.125$. After determining the edge weights, the factor graph is converted to an undirected graph.

This section proposes an algorithm to obtain the structural entropy value of the random 3-SAT instance. As shown in algorithm 1, Generating a random 3-SAT instance, and using a factor graph to represent the instance, then converting the factor graph into an undirected graph G, and further constructing the encoding tree of graph G to obtain the structural entropy value $H_K(G)$.

Convert the factor graph shown in Figure 1 into an undirected connected graph according to the algorithm1, the undirected graph is tree segmented by the minimum cut, and the structural entropy of the random 3-SAT problem is solved by the undirected graph and the encoding tree. The specific implementation process is as follows. The process of converting the factor graph corresponding to formula $F = (x_1 \vee x_2 \vee \overline{x_3}) \wedge (x_2 \vee \overline{x_3} \vee x_4) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_4})$ into a weighted undirected graph is shown in Figure 2.

Algorithm 1 Random 3-Sat Instance Solving Structural Entropy Algorithm

Input: random 3-sat instance
Output: structural entropy

1: $X \leftarrow \{x_1, x_2, \cdots x_n\}$
2: $C \leftarrow \{c_1, c_2, \cdots c_m\}$
3: $E.SolidLine \leftarrow 1-\delta$
4: $E.DottedLine \leftarrow \delta$
5: $G \leftarrow (C \cup X, E)$
6: encoding tree $\leftarrow \varnothing$
7: **if** G connect **then**
8: divide G into subgaphs with minimum cut
9: **if** all subgraphs are single node **then**
10: go to step 14
11: **else**
12: $G \leftarrow subgraph$
13: go to step 8
14: construct the encoding tree
15: $H^T(G,\varepsilon) = -\frac{g_\varepsilon}{2m} log_2 \frac{V_\varepsilon}{V_{\varepsilon^-}}$
16: $H^T(G) = \sum_{\varepsilon \in T, \varepsilon \neq \lambda} H^T(G,\varepsilon)$
17: **return** $H^T(G)$

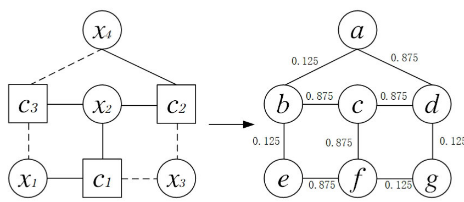

FIGURE 2. The process of factor graph conversion to undirected graph.

The graphs in this paper are all undirected connected graphs. According to the minimum cut algorithm to segment the weighted undirected graph in Figure 2. Dividing the undirected graph into two parts T1 and T2, and then dividing the T1 and T2 parts respectively until all the divided parts contain only a single node. The specific segmentation process is shown in Figure 3 (for the picture to be concise and clear, please refer to the edge weights in Figure 2).

In order to calculate the k-dimensional structural entropy of the factor graph, it is necessary to generate the corresponding decomposition tree T according to the segmentation process of the undirected graph as the encoding tree of G. The encoding tree has the following attributes [27]:

(1) For the root node denoted λ, defifining the set $T_\lambda = V$.

(2) For every $\varepsilon \in T$, the immediate successors of ε are ε^j for j from 1 to a natural number N ordered from left to right as j increases. Therefore, ε^i is to the left of ε^j written as $\varepsilon^i < L\varepsilon^j$, if and only if $i < j$.

(3) For every $\varepsilon \in T$, there is a subset $T_\varepsilon \subset V$ that is associated with ε. For ε and λ, we use $\varepsilon \subset \beta$ to denote that ε is an initial segment of λ. For every node $\varepsilon \neq \lambda$, we use ε^- to denote the longest initial segment of ε, or the longest β such that $\beta \subset \varepsilon$.

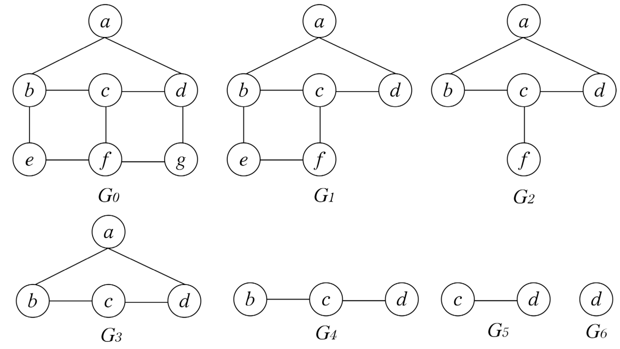

FIGURE 3. Undirected graph segmentation process.

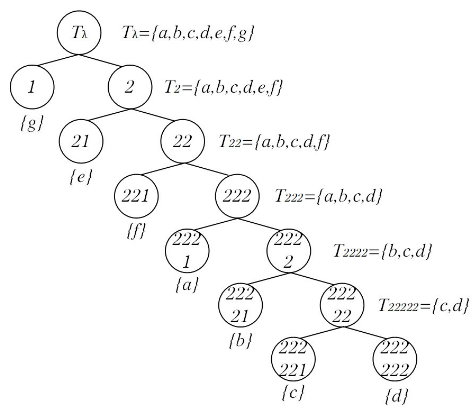

FIGURE 4. Encoding tree for undirected graph.

(4) For every i, $\{T_\varepsilon | h(\varepsilon) = i\}$ is a partition of V, where $h(\varepsilon)$ is the height of ε (note that the height of the root node λ is 0, and for every node $\varepsilon = \lambda$, $h(\varepsilon) = h(\varepsilon^-) + 1$.

(5) For every ε, T_ε is the union of T_β for all $\beta' s$ such that $\beta^- = \varepsilon$; thus, $T_\varepsilon = \cup_{\beta^- = \varepsilon} T^\beta$.

(6) For every leaf node ε of T, T_ε is a singleton; thus, T_ε contains a single node of V.

Combining the segmentation process of the undirected graph in Figure 3 and the nature of the encoding tree, the encoding tree corresponding to the undirected graph in Figure 2 is shown in Figure 4.

After the encoding tree is constructed, the entropy value $H^T(G,\varepsilon)$ of each node in the segmentation tree can be obtained by formula (2). According to formula (3), the sum of $H^T(G,\varepsilon)$ of all nodes is $H^T(G)$. From formula (4), it can be seen that the structural entropy is ultimately to find an optimal segmentation method to minimize the value. In the above, the minimum cut has been selected as a tool for segmenting undirected graphs to obtain the best encoding tree T^*. Therefore, the structural entropy of the weighted undirected graph in Figure 2 can be obtained as $H = 2.335$.

IV. EXPERIMENTAL ANALYSIS

The hardware environment for the experiment is Intel Core i7 2 600k + GTX 1 080 8 GHg + 16 GB RAM, and the software environment is Windows 10 ×64 + ubuntu18.04 + xshell + Python3.7 + Matlab. In the following numerical experiments,

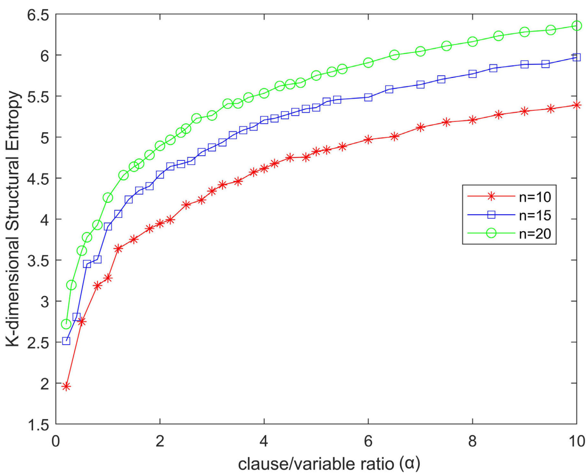

FIGURE 5. The change of K-dimensional structural entropy with α.

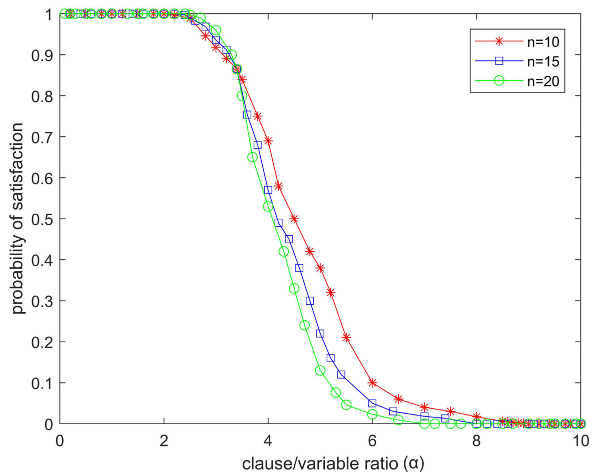

FIGURE 6. The satisfiable probability changes with α.

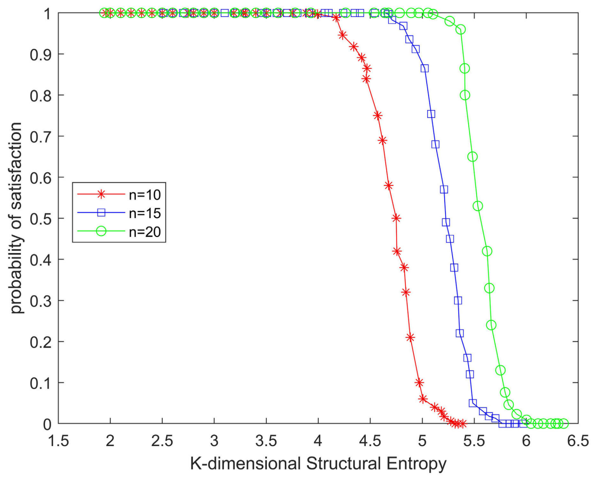

FIGURE 7. The satisfiable probability changes with k-dimensional structural entropy.

every data point is composed of the mean value of 100 random instances generated by the $G(n, 3, m)$ model, where n is the number of variables and m is the number of clauses. Since the clauses increases exponentially with the increase of the variables, the larger the variables is, the more complex the structure of the corresponding factor graph. Therefore, this paper set $n = 10$, $n = 15$ and $n = 20$ to research the phase transition phenomenon of the random 3-SAT problem.

When the value of n is fixed, as α increases, the structural change of the factor graph will directly affect the value of structural entropy. First, observing the relationship between α and structural entropy under different problem scales, as shown in the Figure 5. It can be seen that when the value of n is fixed, as α increases, the slope of the curve gradually decreases, and the increase of entropy value becomes more and more slowly; when the value of α is fixed, the entropy value increases with the increase of n. This is because in the simple factor graph structure, any changes, such as the increase of loops and the change of graph scale, will have a greater impact on the factor graph structure. When the scale of the factor graph increases to a certain extent, the increase of the loop and the size of the factor graph within a certain range will significantly reduce the impact on the dynamic complexity of the graph, the state of the factor graph is more stable. Besides, an interesting phenomenon is that when $\alpha \geq 2$, the value of the distance between any two curves seems to be fixed within a certain range. For example, the distance between the two curves of $n = 10$ and $n = 15$ is 0.56 ± 0.07. This is because when the scale of the factor graph is increased to a certain degree under different scales, the change in the distribution of variables and clauses has a very limited impact on the overall structure of the factor graph. And each group selected a large number of instances for experiments, the structure entropy of the instances changed relatively stable, under the same control parameters, the difference value between the structure entropy of different scale examples was very close. The difference can reflect the stable difference of the structural complexity of the instances of different scales. As the size of the instance increases, the change in structural complexity will become smaller and smaller, so the difference value in structural entropy will also become smaller.

This paper have done a comparative experiment on the variation of the satisfiable probability curve with different control parameters, as shown in Figure 6 and Figure 7. Defining the transition position of the random 3-SAT problem that satisfies the probability as the control parameter when the probability is 0.5, and the transition width is the difference of the parameter value corresponding to the satisfiability probability from 1 to 0. As shown in Figure 6, with the increase of the ratio α, the satisfiable probability curve of the random 3-SAT problem transitions from probability 1 to 0. As n increases, the transition width of the probability curve becomes narrower, and the transition point moves from right to left. Different curves have a common intersection point, and they rotate around this intersection point. The usual assumption is that in the limit state, the curve passes through this point and is close to the step function, the ratio is the critical phase transition value of random 3-SAT. For different control parameters, a novel phase transition

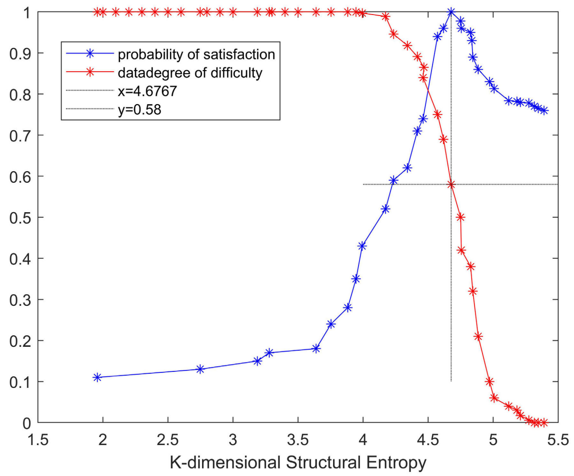

FIGURE 8. Satisfying probability and difficulty change with k-dimensional structural entropy (n=10).

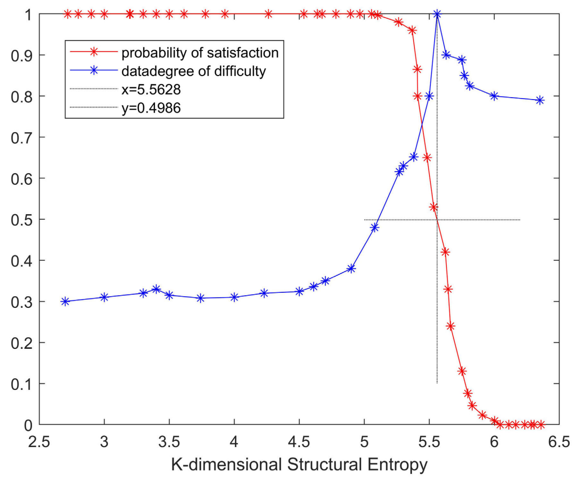

FIGURE 10. Satisfying probability and difficulty change with k-dimensional structural entropy (n=20).

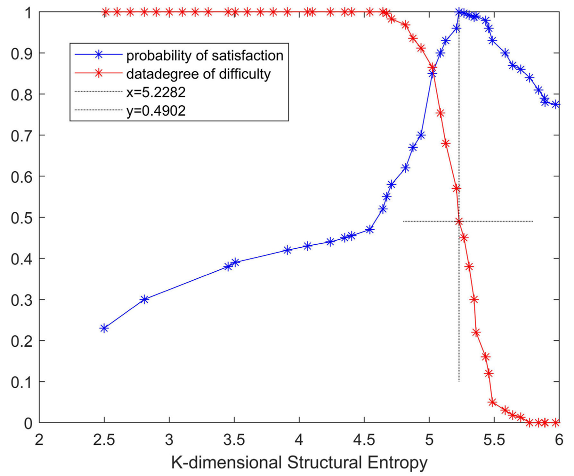

FIGURE 9. Satisfying probability and difficulty change with k-dimensional structural entropy (n=15).

mode appears in random 3-SAT. As shown in Figure 7, with the increase of structural entropy, the satisfiable probability curve still has a sharp transition from satisfiable to unsatisfiable. As n increases, the transition width of the probability curve becomes narrower, and more problem instances can be satisfied. The difference with α is that with the increase of n, the transition point moves from left to right, but there is no common intersection point between the curves, and the precise position of the transition cannot be determined intuitively. This is similar to the exact phase transition point of the α metric random 3-SAT problem. It is still an interesting open problem to determine the transition position of the structural entropy metric random 3-SAT.

In order to facilitate the observation of the relationship between the satisfiable probability of the random 3-SAT problem and the difficulty of its solution, the following normalization preprocessing is performed on the experimental data. When $n = 10$, the maximum solution time of zchaff is 163 seconds, so the ratio of the time to solve each instance to the maximum solution time is taken as the average difficulty of solving the problem. Similarly, when $n = 15$, the maximum solution time of zchaff is 209 seconds; when $n = 20$, the maximum solution time of zchaff is 255 seconds. Figures 8-10 respectively depict the satisfiable probability and the solving difficulty curve with structural entropy for $n = 10$, $n = 15$, $n = 20$. In these figures, we can see that with the increase of n, the random 3-SAT problem becomes more difficult to solve, and the curve measuring difficulty with H also shows the same easy-hard-easy pattern as the curve measuring difficulty with α, and the transition position of the peak of the difficulty curve of solving the difficulty moves from left to right. All three groups of experimental show that the problem is the most difficult to solve near the transition point.When the probability of the random 3-SAT problem is 1, the difficulty of solving linearly increases. When the probability of the random 3-SAT problem is within the interval of [0.5,1), the difficulty of solving increases exponentially. Experiments show that the relationship between the satisfiability of the structural entropy measurement formula and the difficulty of solving is consistent with the problem of α measurement. When $n = 10$, the transition point of the phase transition curve of the random 3-SAT problem is 4.68. When the value of the structural entropy is lower than the critical value, the random 3-SAT has a high probability of being satisfied, and when the entropy exceeds the critical value, it is almost always unsatisfiable. In addition, it is the most difficult to solve near the transition point. Similarly, when $n = 15$, the transition point of the random 3-SAT problem is 5.23; when $n = 20$, the transition point of the random 3-SAT problem is 5.56.

V. CONCLUSION

Experiments show that the satisfiability of a random 3-SAT instance undergoes a phase transition with the change of structural entropy. When the structural entropy H crosses the critical point H_d, the instance suddenly changes from satisfiable to unsatisfiable. And near the critical point H_d, the problem is more difficult to solve. Structural entropy is a parameter that measures the structure information of the graph, and represents the complexity of the factor graph. The more complex the structure of the factor graph is, the larger the value of the structural entropy. Therefore, when n is fixed, the value of structural entropy increases with the increase of α, and when a certain critical point is reached, the value of structural entropy increases relatively slowly. The research in this paper shows that in addition to the ratio of clauses to variables, the random SAT problem also has other control parameters that make it appear phase transition, and there is a close relationship between the structural entropy and the ratio of clauses to variables. Analyzing the phase transition phenomenon of SAT problems from different perspectives can enable us to better understand the essence of difficult problems, and providing an experimental basis for the next step to explore the features of difficult cases. In the next step, we intend to embed the structure information of the proposition formula into the message propagation algorithm to improve the performance of the algorithm, and hope to obtain some specific research results.

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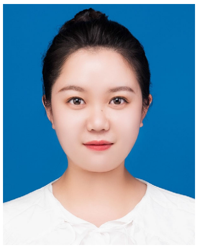

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