IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received November 20, 2020, accepted December 9, 2020, date of publication December 29, 2020, date of current version January 15, 2021.

Digital Object Identifier 10.1109/ACCESS.2020.3047934

A Dynamic Competition and Predation Model for Rumor and Rumor-Refutation

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This work was supported in part by the Humanities and Social Sciences Program of Ministry of Education of China under Grant 19YJC630222, in part by the Major Bidding Program of the National Social Science Foundation of China under Grant 17ZDA286, and in part by the Major Bidding Program of Ministry of Education of China under Grant 17JHQ005.

ABSTRACT Controlling rumors is related to the interplay between the rumor and rumor refutation, which compete to attract the unaware. And rumor refutation preys on the rumors, and try to eliminate rumors. Therefore, there is both a predatory and competitive relationship between the rumors and the rumor refutations. This paper presents a model based on biomathematics theory to describe the interplay between rumors and rumor refutations. The theoretical analysis of the differential equations elucidated three dynamic cases: rumor extinction; rumor refutation extinction; and rumor and rumor refutation coexistence. The subsequent analysis of the equilibrium stability in the three cases revealed both equilibrium stability and model instability. The haze rumor and the official haze refutation data were then crawled from Sina microblog, the rumor propagation process analyzed, an integral method employed for the model parameter estimation, and the proposed model used to estimate and forecast the haze rumor and rumor-refutation evolutions. The findings suggested that rumors and rumor refutations coexist, which was consistent with the theoretical analysis of coexistence. Based on this case and to more deeply analyze the effect of an authority's credibility and the public's cognition, the proposed model was used to suggest several scenarios, and policy suggestions given to assist authorities better manage rumors in emergency events.

INDEX TERMS Rumors, rumor-refutation, competition, predation.

I. INTRODUCTION

Rumors are a common human phenomenon, the transmission of which have exponentially grown since the development of new instant social media platforms such as Facebook, Twitter, and microblogs. People can now receive and publish any information they want at any time anywhere. Further, because a great deal of network communication is anonymous, people are more willing to give their own views regardless of the truth, which has led to an increase in the number of unconfirmed rumors spreading across networks, which in turn can affect the direction of public opinion and government credibility. Rumor propagation in public emergencies in particular can lead to panic and societal instability [1]. To dispel dangerous rumors, governments or the relevant authorities commonly release rumor-refutations to guide and control public opinion. For example, after the loss of the MH370 airliner in 2014, rumors quickly spread through online social media platforms, which contributed to the social risks in a short time, which then prompted the Malaysian government to subsequently release some news to prevent wild rumor propagation. There lease of official rumor-refutations calms the public, eliminates suspicion, and generally quashes the rumors [2]–[4]. However, if there is no authoritative rumor refutation, rumors can propagate rapidly and widely [5].

In 1965, Daley and Kendall argued that rumour spreading has a similarity to the epidemic model, and they constructed a classical mathematical model for rumor spreading [6]. Maki and Thompson [7] then built a variant of the DK model that proposed that a former spreader only stopped spreading rumors when the rumor spreader contacted other spreaders. As this research field has developed, specific rumor propagation features have been identified that have been found to be different from epidemic infection spreads. Based on the classic susceptible, infected and recovered (SIR)

The associate editor coordinating the review of this manuscript and approving it for publication was Hassen Ouakad.

epidemiological model, new rumor propagation models have been developed that include new elements such as a cooling mechanism [8], hesitating mechanism [9], individuals' attitudes [10]. Lu [11] researched an agent-based rumor propagation model, in which the agents interacted with neighbors on a square lattice, with the source spreading rumors to receptors, and if the receptors decided to spread rumors, they became new rumor propagation sources. Amirhosein *et al.* [12] developed a new agent-based rumor propagation model and evaluated it based on real rumor propagation Twitter datasets, finding the that the novel model was better able to represent online social network rumor propagation. Yin *et al.* [13] took a complex network perspective to propose a dynamic model with a double-layer interaction process to explore the impact of network interactions and dynamic evolution.

As most rumor research has been focused on controlling rumor propagation to effectively reduce hazards and losses, the interactions between the rumors and the surrounding environments have attracted research attention, which can be divided into three main areas. The first examines the interplay between rumor propagation and the role of social media. For example, Lee and Choi [14] explored how rational communication was constructed during the 2015 MERS spreading South Korea, and found a mode rating effect between social media informational dependency interactions and false rumor credibility and accuracy-oriented information seeking. Shin *et al.* [15] examined political misinformation diffusion on Twitter, and observed that the rumors resurfaced on partisan news websites that repackaged the old rumors into news, which then gained visibility through influential Twitter users who introduced the rumor into the Twittersphere, and Anjan *et al.* [16] conducted two related studies and demonstrated that denials could be crafted to effectively debunk rumors on social media. The second rumor propagation research area has been focused on the interplay between rumors and authority, for which mean-field equations were derived to describe the rumor propagation and authoritative refutation competition dynamicsl [17]. For example, adynamic 8-state ICSAR model was proposed that considered official rumor refutation and analyzed the rumor propagation mechanism and its ability to assess the effects of official rumor refutation [18], and Xu and Zhang [19] built a predictive model that in corporate four factors including authority,and found that information sharing was related to multimedia cues that indicated source popularity. The third rumor propagation research area has examined the competition between two rumors; for example, Trpevski and Kocarev [20] established an alternate rumor propagation network model in which two rumors, rumor 1 and rumor 2, each of which had different accept probabilities, were shown to propagate between the nodes. To simultaneously consider two rumor propagation types, a competition model was derived from mean-field equations and the system stability of the rumor interactions discussed [21]. A DSIR double-rumor propagation model was developed that considered both old and new rumor propagation, in which it was assumed that the new rumor was launched after the old rumor was propagated [22], and Yan and Jiang [23] proposed an information competition model that described the dynamic changes in the number of information spreaders, compared the competitive positive and negative information, and revealed which was more successful.

However, most rumor propagation models have only considered one piece of information, with only a few models considering the interplay of two pieces of information. After the emergence of a rumor, most previous studies have tended to examine the processes related to the rumor spread rather than examining the interaction of rumors and rumor refutation. As rumor refutation should be given greater attention, this paper surmises that the key to successful rumor refutation is understanding the dynamic interplay between the rumor-refutation and the rumor. Specifically, the authority releases the rumor refutation to clarify, negate and eliminate the rumor and purify the network public opinion. Therefore, the rumor refutation is viewed as a predator and the rumor is the prey, then the relationship between the rumor-refutation and the rumor could be seen to be predatory. After the rumor emergence, the rumors need people to continue the spread throughout the network. Initially, many people believe and forward the rumor and as they are affected by the rumor information, do not always trust the official rumor refutation; therefore, the rumor refutation needs to win the peoples trust to encourage them to spread the refutation in the network. This means that as the rumor and the rumor refutation also compete for the attention and support of network spreaders, the rumor propagation and the rumor refutation have both predatory and competitive relationships.

Biological population interactions have been extensively studied, with competition and predation being two of the most common relationships found between populations. Competition occurs when two distinct populations compete for the same resources. Some classic competition models have also been applied in other fields. For example, the LV model was applied to forecast revenue and analyze the interaction effects between two competing business species [24]. The predator-prey relationship is where a predator kill their prey, which can even lead to the extinction of the prey. Predation models have also been applied to specific problems, for example, Pinheiro studied an optimal harvesting problem associated using a stochastic logistic growth model with a predation term [25]. The mixed competition and predation ecological structures common in nature [26] have been mathematically modeled, such as the intraguild predation(IGP) model, Holt *et al.* [27] developed a general mathematical model describing intraguild predation to describe the interaction between three populations. Therefore, this study borrowed the idea behind the biological population models to establish an interactive predation and competition model to analyze the specific mechanisms in the rumor propagation and rumor refutation populations, after which the model was applied to an actual case to verify its feasibility.

II. MODELLING

A. KEY PROBLEMS DESCRIPTION

Rumor propagation is information that has not been publicly confirmed by the relevant authorities that circulates across social networks [28]. The most common method used to reduce the harm from unfounded rumors has been to issue refutations, with the propagation process involving both the rumor and rumor refutation population co-movements. This section first provides the validity proofs for the classifications of these populations, after which the competitive and predatory relationships between the populations are discussed. The symbols used in this paper are summarized in Table 1.

Notations

t	Time.
$N_1(t)$	Density of the population that spreads the rumors.
$N_2(t)$	Density of the population that spreads the rumor refutation.
R	Density of the unaware population.
a_1	Rate of the connection between rumor spreaders and the unaware population.
a_2	Rate of the connection between rumor refutation spreaders and the unaware population.
b_1	Efficiency at which the rumor population converts the unaware population into the rumor population.
b_2	Efficiency at which the rumor refutation population converts the unaware population into the rumor refutation population.
c_1	Rumor spreader death rate.
c_2	Rumor refutation spreader death rate.
α	Rate at which rumors spreaders encounter the rumor refutation spreaders.
β	Efficiency at which the contacted rumor population is converted into the rumor refutation population.
S	Intensity at which the unaware population enters the system.

1) POPULATION DEFINITION

Similar to biological populations, the rumor and rumor refutation populations can be defined. In this paper, rumors are defined as unconfirmed information that is spread through microblogs that can have an adverse effect on public opinion, and rumor refutation is the clarification and feedback on the rumor information released by official departments through official microblogs and main stream news media that has a positive effect on public opinion. As the rumors and rumor refutations coexist and spread concurrently in the system, the rumor population are people who know of and spread the rumors and the rumor refutation population are the people who know and spread the rumor refutations. Because of the dynamic interactions between the rumor and rumor refuting populations, they may replace each other over time. There is also a population who don't know either the rumor or the rumor refutations (the unaware). The participation of the internet connected population(netizens) has an important influence on the speed and range of the information dissemination, which has an indirect impact on the public opinion in the social network, that is, the faster the information spread and the more people that spread it, the greater the impact of the information on public opinion. As the main players in the rumor and rumor refutation processes are the internet connected population, the influencing factors for rumor suppression are related to their respective behaviors. The spread of government rumor refuting information is affected by the government credibility, and the public can decide whether(or not) to believe the rumor based on their own cognitions, with the rumor and rumor refutation transmissions being related to the network topology. Fig. 1 describes the event, the rumor population, the rumor refutation population, the netizens and the specific interactions.

Rumors and rumor refutations spread together in social networks, which means that the internet connected populations are affected by both. The rumor, rumor refutation and unaware populations, therefore, form a complete ecosystem, the characteristics of which are similar to biological populations. First, these three populations have the same spatial characteristics as the general population, with each being classified by their interest/lack of interest in the rumors or rumor refutations. The number of people in each rumor or rumor-refutation population grows as they become exposed, and the unaware population becomes smaller as they migrate to the rumor or rumor refutation populations, that is, these populations are affected by the initial information release rates, the deletion rates, the immigration rates, and the eviction rates. Second, the rumors and rumor refutation populations each have distinct life cycles, which have a start period, a growth period, a peak period, and a recession period. Finally, these populations have genetic characteristics, that is, the microblogs originate from the same or similar information, and the original content information is spread through multi-layer forwarding.

2) POPULATIONS RELATIONSHIPS

There is both a competitive relationship and a predatory relationship between rumor information and rumor-refuting information in social network. In the period from the appearance of rumors until the death of rumors, rumors and rumors-refuting information exist in the network at the same time. The competition between the rumors and rumor-refuting information is reflected two kinds of information compete on the network potential advocates. Netizens population is shared resource of rumor information and rumor-refuting information. Rumor and rumor-refuting populations expand their influence by striving for ordinary netizens, that is, the two populations compete for the same limited resources. Rumors and rumor-refuting information curb growth of each other through competition. However, due to the authority of government, it will make the public pay less attention on rumor when rumor-refuting information appears.

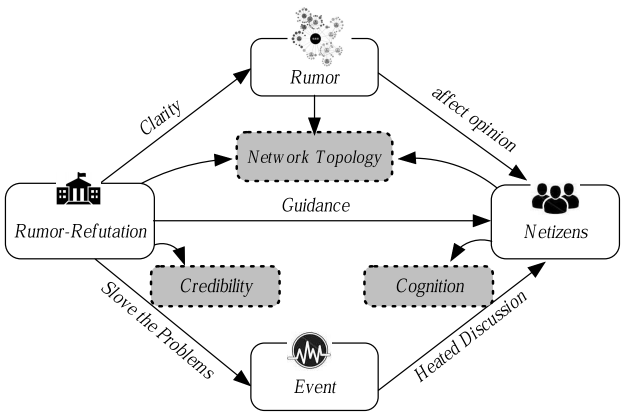

FIGURE 1. Evolution and influencing factors for rumor and rumor refutation.

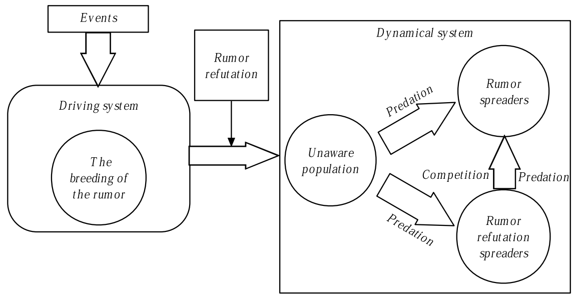

FIGURE 2. Relationship for three populations.

Both competitive and predatory relationships exist in the social network between the rumor and the rumor refutation. In the period from the emergence of the rumor until the death of the rumor, both the rumor and the rumor refutation exist at the same time, which means that the information from both are competing to attract the attention of the unaware population, which also means that they are curbing the growth of the other through this competition. However, because the rumor refutation generally has authority, the public may pay less attention to the rumor when the rumor refutations appear. Rumor refutations and rumors have a predatory relationship, that is, the rumor refutation seeks to kill off the rumors, convert the rumor spreading population, and eliminate the rumors. Therefore, the relationship could be seen to be a mix of a predatory and a competitive relationship, an ecological structure that exists in the wild. The relationships between these populations is shown in Fig. 2.

Most of researches only discuss one aspect of the relationship between rumor and rumor-refutation. For example, a rumor and knowledge information competitive diffusion [29] was are discussed, and in which knowledge diffusion was seen to be the key to controlling the rumor. A rumor propagation model called ILRDS model [30] to describe the rumor dynamics of the debunkers who prey on the rumor spreaders. This paper employed the IGP predation food chain relationship proposed by Holt and Polis [27] to examine the competitive and predatory relationships between the rumor spread and the rumor refutation in the model to more practically depict the interactions between the rumor spreader, rumor refuter and unaware populations.

B. MODEL FORMULATION

To control the rumor spread, the relevant authority releases a rumor refutation to dispel the rumor, which not only preys on rumor population, but also preys on the unaware population. And rumor refutation population and the rumor spreading population compete for the unaware population. Therefore, $N_1(t)$ is the density of people spreading the rumor, $N_2(t)$ Is the density of people spreading the rumor refutation, R is the density of all the unaware population, under the assumption that this is constant, that is, the sum of the birth extinction rates of the unaware population is zero in the short term. Each predatory relationship is represented by a linear relationship, which is surmised to be in line with reality because networks are now highly developed. The unaware population are affected by the predators in the information spreading process, that is, the rumor refutation population and the rumor population. As shown in Fig. 3, the rumor and rumor refutation spread rules are as follows:

(1) The rate at which the population spreading the emerging rumors encounters the rumor refutation population is $\alpha > 0$, which is the contact probability. It is assumed that the rumor spreader will become lurker state when the rumor spreader meets the rumor refutation spreader. Due to personal cognitions, logical reasoning abilities and other reasons, the rate at which the rumor spreaders stop spreading the rumors is $c_1 > 0$. Rumor spreaders increase their numbers by capturing the unaware population in netizens; therefore, it is assumed that the rumor spreader population finds the unaware population resource at a rate of $a_1 > 0$, which depends on the connections between the rumor spreaders and the unaware population, and the unaware population becoming rumor spreaders at a rate of $b_1 > 0$. Therefore, the changing speed in the rumor spreader populations differential equation is:

$$\frac{dN_1(t)}{dt} = a_1 b_1 R N_1 - \alpha N_1 N_2 - c_1 N_1$$

(2) Rumor refutation spreaders increase their numbers by capturing people from the unaware population and the rumor spreader population. When a rumor spreader contacts a rumor refutation spreader, the rumor spreader becomes a rumor refutation spreader at the rate of $\beta > 0$. The rumor refutation spreaders find the unaware population resource from netizens at a rate of $a_2 > 0$ which depends on the connection between rumor refutation spreaders and the unaware population; an people from the unaware population become rumor-refutation spreaders at a rate of $b_2 > 0$. Due to boredom, rumor refutation mistrust, or other reasons, the rate of rumor refutation spreaders who stop the rumor refutation spread is $c_2 > 0$. Therefore, based on (1), the changing speed

of the $\frac{dN_2(t)}{dt}$ is proportional to the number of $N_2(t)$, so ;

$$\frac{dN_2(t)}{dt} = a_2b_2RN_2 + \alpha\beta N_1N_2 - c_2N_2$$

(3) The reduced speed of the unaware connected population $\frac{dR}{dt}$ is proportional to the number of existing $N_1(t)$ and $N_2(t)$ from (1) and (2), with the rate of people from the unaware populations entering the systems being $S > 0$, which reflects the intensity of the unaware population entry, so;

$$\frac{dR}{dt} = S - a_1RN_1 - a_2RN_2$$

Based on the previous discussion, by integrating (1)-(3), the following global expected model is formulated to reprensent the spreading process:

$$\begin{cases} \frac{dN_1(t)}{dt} = a_1b_1RN_1 - \alpha N_1N_2 - c_1N_1 \\ \frac{dN_2(t)}{dt} = a_2b_2RN_2 + \alpha\beta N_1N_2 - c_2N_2 \\ \frac{dR}{dt} = S - a_1RN_1 - a_2RN_2 \end{cases} \quad \text{(I)}$$

It is assumed that shared resource R is constant in the process, $\frac{dR}{dt} = S - a_1RN_1 - a_2RN_2 = 0$, that is, $R = \frac{S}{a_1N_1+a_2N_2}$. Therefore, the model is derived as follows,

$$\begin{cases} \frac{dN_1(t)}{dt} = N_1(\frac{a_1b_1S}{a_1N_1 + a_2N_2} - \alpha N_2 - c_1) \\ \frac{dN_2(t)}{dt} = N_2(\frac{a_2b_2S}{a_1N_1 + a_2N_2} + \alpha\beta N_1 - c_2) \end{cases} \quad \text{(II)}$$

where $a_1 > 0$ is the connection rate between the rumor spreaders and the unaware population, $a_2 > 0$ is the connection rate between the rumor refutation spreaders and unaware population, $b_1 > 0$ is the efficiency at which the rumor spreading population converts the unaware connected population resource into the rumor spreader population, $b_2 > 0$ is the efficiency at which the rumor refutation population converts the unaware population resource into the rumor refutation population, $c_1 > 0$ is the death rate at which the rumor spreaders stop spreading rumors, and $c_2 > 0$ is the death rate at which the rumor refutation spreaders stop spreading the rumor refutation, that is, the respective information spreaders quit the population they belong to, $\alpha > 0$ is the rate at which rumor spreaders encounter the rumor-refutation spreaders, $\beta > 0$ is the efficiency at which the rumor population is converted into the rumor refutation population, and $S > 0$ is the intensity at which the unaware population resource enters the system.

C. EQUILIBRIUM ANALYSIS

Initially the rumor propagation system has only the unaware population and the rumor spreaders, with the rumor-refutation spreaders appearing later; however, finally, the system reaches a state of equilibrium, in which there are three possibilities: rumor refutation displacement, rumor displacement, and rumor and rumor refutation coexistence. In the following, the system's steady-state in this period is analyzed. Letting the right side of each of the differential equations be equal to zero in system (I), results in the equation

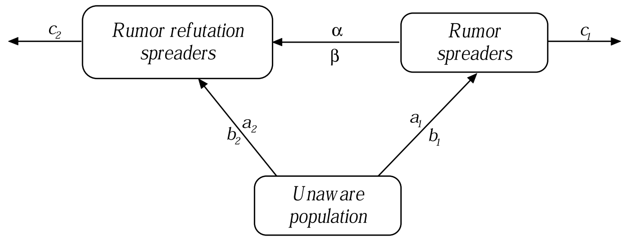

FIGURE 3. Structure of the rumor and rumor-refutation propagation process.

$$a_1b_1RN_1 - \alpha N_1N_2 - c_1N_1 = 0 \quad (1)$$
$$a_2b_2RN_2 + \alpha\beta N_1N_2 - c_2N_2 = 0 \quad (2)$$
$$S - a_1RN_1 - a_2RN_2 = 0 \quad (3)$$

The above equations were solved using MATLAB and the results rewritten; therefore, system (I) has three possible equilibria. (1) The unaware population resource and the rumor refutation population reach equilibrium densities without the rumor spreaders, for which the equilibrium is (0, $\frac{b_2S}{c_2}$, $\frac{c_2}{a_2b_2}$). (2) The unaware and rumor spreader populations are present, but the rumor-refutation population is absent, for which the equilibrium is ($\frac{b_1S}{c_1}$, 0, $\frac{c_1}{a_1b_1}$). (3) The unaware, rumor spreader, and rumor refutation populations are present, there are two possible positive equilibria,

$$(N_1^+, N_2^+, R^+) = (\frac{c_2B + a_2b_2(A - a_1c_2 + a_2c_1\beta)}{B\alpha\beta}, -\frac{c_1B + a_1b_1(A - a_1c_2 + a_2c_1\beta)}{B\alpha}, \frac{-A + a_1c_2 - a_2c_1\beta}{B})$$

$$(N_1^-, N_2^-, R^-) = (\frac{c_2B + a_2b_2(-A - a_1c_2 + a_2c_1\beta)}{B\alpha\beta}, -\frac{c_1B + a_1b_1(-A - a_1c_2 + a_2c_1\beta)}{B\alpha}, \frac{A + a_1c_2 - a_2c_1\beta}{B})$$

where

$$A = \sqrt{a_1^2c_2^2 + 4b_1\alpha Sa_1a_2\beta^2 - 2a_1a_2\beta c_1c_2 - 4b_2\alpha Sa_1a_2\beta + a_2^2\beta^2c_1^2}$$
$$= \sqrt{(a_1c_2 - a_2\beta c_1)^2 - 4\alpha Sa_1a_2\beta(b_2 - \beta b_1)}$$
$$B = 2(a_1a_2b_2 - a_1a_2\beta b_1) = 2a_1a_2(b_2 - \beta b_1)$$

For the given parameters, not all these equilibria exist, and the existence of certain equilibria indicate that the other equilibria are unstable. The three equilibria are respectively discussed in the following section.

Case 1. Equilibrium (1) exists if the rumor-refutation population invades. This equilibrium is stable regardless of perturbations in either the rumor-refutation or the unaware population densities. In this case, if the number in the unaware population is low, the rumor-refutation population cannot subsist on this unaware population resource and must also capture many from the rumor spreader population to survive. If the rumor-refutation population is able to capture sufficient numbers from the rumor spreader population, it will persist. Therefore, when there is a low unaware population resource, a rumor refutation population may be absent from the system if there are no rumor propagators present.

Because of an invasion by the rumor spreader population, the condition that makes this equilibrium unstable is

$$a_1 b_1 \frac{c_2}{a_2 b_2} - \alpha \frac{b_2 S}{c_2} - c_1 > 0 \tag{4}$$

If inequality (4) is reversed, the rumor spreader population is excluded because there are insufficient unaware population resources to withstand the cumulative mortality imposed by the rumor refutation population and natural death. The rumor refutation capturing ability could also be increased; for example, an increase in the credibility of the official media in real life would attract more of the unaware population resource, reducing the abundance of the unaware population resource(great a_2). At the same time, enhancing the connection between the official media and the unaware populations would improve the exposure to the official information(great a_1). Therefore, given a set of parameters, the rumors spreaders can be excluded by the rumor refutation. In addition, the equilibrium for the rumor-refutation population attracts a greater number of the unaware population resource (large S), making an invasion by the rumor spreaders more difficult, that is, the rumor spreaders number are excluded indirectly by the increased number of rumor refutation.

Case 2. Equilibrium (2) exists if the rumor spreaders successfully invade, with this equilibrium being stable if there are only small changes in rumor spreaders or unaware population densities. When there is an invasion by the rumor-refutation, the condition that makes this equilibrium unstable is

$$a_2 b_2 \frac{c_1}{a_1 b_1} + \alpha\beta \frac{b_1 S}{c_1} - c_2 > 0 \tag{5}$$

If this inequality is reversed, the rumor-refutation population is excluded from the system if there are no available unaware or rumor spreader populations. In fact, if there are few people spreading the rumors and few unaware people, then there is no reason for the rumor refutation population to exist in the system. If the rumor spreader population is a strong competitor and able to attract more of the unaware so that the unaware population falls to a low level (small $\frac{c_1}{a_1 b_1}$), the rumor-refutation may not be strong enough to spread through the media (small $a_2 b_2$) or dissipate the strength of the rumor spreaders (small $\alpha\beta$), which means that the rumor refutation could attract few or none of the unaware or rumor spreaders and be excluded. An increase in the ability of the rumor spreaders to attract more of the unaware population (great a_1) would make a successful invasion by the rumor refutation more difficult, which would also result in an increase in the strength of the rumor spreaders, which in turn would attract more of the unaware population resource; however, if there are many rumors and rumor spreaders, the remaining unaware may be overly attracted, which would significantly reduce the unaware population. Therefore, increasing the rumor attraction rate reduces the overall rumor refutation population supply. For any set of parameters, there is some value in the rumor attraction rate on the unaware population resource, which could lead to an exclusion of the rumor refutation when there are few or no unaware available.

Case 3. The unaware, rumor spreader, and rumor refutation populations are present, with the equilibria densities being positive when the following inequalities hold:

For (N_1^+, N_2^+, R^+), when

$$(\frac{c_2 B + a_2 b_2 (A - a_1 c_2 + a_2 c_1 \beta)}{B\alpha\beta} > 0, \ and \tag{6}$$

$$-\frac{c_1 B + a_1 b_1 (A - a_1 c_2 + a_2 c_1 \beta)}{B\alpha} > 0, \ and \tag{7}$$

$$-\frac{A - a_1 c_2 + a_2 c_1 \beta}{B} > 0 \tag{8}$$

N_1^+, N_2^+, R^+ are positive.

For (N_1^-, N_2^-, R^-), when

$$\frac{c_2 B - a_2 b_2 (A + a_1 c_2 - a_2 c_1 \beta)}{B\alpha\beta} > 0, \ and \tag{9}$$

$$-\frac{c_1 B - a_1 b_1 (A + a_1 c_2 - a_2 c_1 \beta)}{B\alpha} > 0, \ and \tag{10}$$

$$\frac{A + a_1 c_2 - a_2 c_1 \beta}{B} > 0 \tag{11}$$

N_1^-, N_2^-, R^- are positive.

If the quantity $B > 0$, that is $b_2 > b_1\beta$, the rumor spreaders might provide a small benefit to the rumor refutation relative to the unaware population resource. For instance, if the rumors have a low influence, the rumor refutation does not need prey on the rumor population to change the attitude of rumor supporters.

Specifically, from the expression of A, when $b_2 > b_1\beta$, if $a_1 c_2 - a_2 c_1 \beta < 0$, we can get $0 \le A < a_2 c_1 \beta - a_1 c_2$; if $a_1 c_2 - a_2 c_1 \beta > 0$, we can get $0 \le A < a_1 c_2 - a_2 c_1 \beta$. When $a_1 c_2 - a_2 c_1 \beta < 0$, $A - a_1 c_2 + a_2 c_1 \beta > 0$, we can get $N_1^+ > 0, N_2^+ < 0$, so this equilibrium does not exist; and $A + a_1 c_2 - a_2 c_1 \beta < 0$, we can get $N_1^- > 0, N_2^- < 0$, so this equilibrium does not also exist. When $a_1 c_2 - a_2 c_1 \beta > 0$, we can get $A - a_1 c_2 + a_2 c_1 \beta > 0$ and $A + a_1 c_2 - a_2 c_1 \beta > 0$, so N_1^+, N_2^+ and N_1^-, N_2^- both may be positive simultaneously, which depends on the value of the parameters. Obviously, we solve this two inequations (6) and (9), and get $A > a_1 c_2 - a_2 \beta c_1 - \frac{Bc_2}{a_2 b_2}$, and $A < -(a_1 c_2 - a_2 \beta c_1 - \frac{Bc_2}{a_2 b_2})$. If $a_1 c_2 - a_2 \beta c_1 - \frac{Bc_2}{a_2 b_2} > 0$, so there is only one inequation is true in the inequations (6) and (9). A similar analysis can be performed on the inequations (7) and (10). Therefore, there is the unique equilibrium in the positive quadrant.

If the quantity $B < 0$, that is $b_2 < b_1\beta$, because the parameters is positive. When $a_1 c_2 - a_2 c_1 \beta < 0, A > a_2 c_1 \beta - a_1 c_2$, so we can get $A - a_1 c_2 + a_2 c_1 \beta > 0$, and $A + a_1 c_2 - a_2 c_1 \beta > 0$.

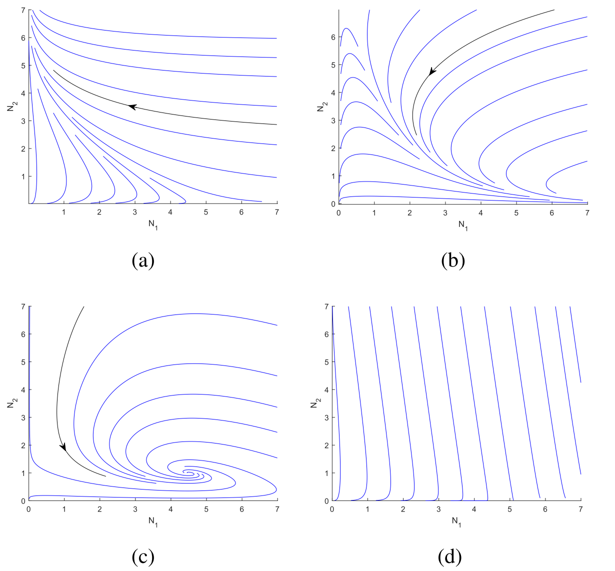

FIGURE 4. The phase portraits (a), (b), (c) in the N_1-N_2 plane corresponding to the three cases, The phase portraits (d) in the N_1-N_2 plane corresponding to instability. (a), a_1=0.5, a_2=1, b_1=0.5, b_2=1, α=0.1, β=0.1, c_1=0.1, c_2=0.1. (b), a_1=1, a_2=0.1, b_1=1, b_2=0.25, α=0.1, β=0.01, c_1=0.1, c_2=0.3. (c), a_1=1, a_2=0.5, b_1=1, b_2=0.5, α=0.1, β=1, c_1=0.1, c_2=0.5. (d), β=10 for (a).

According to the expression of $N_1^+, N_2^+, R^+, N_1^-, N_2^-, R^-$, N_1, N_2, R cannot be all positive. When $a_1c_2 - a_2c_1\beta > 0$, $A > a_1c_2 - a_2c_1\beta$, so we can get $A - a_1c_2 + a_2c_1\beta > 0$, and $A + a_1c_2 - a_2c_1\beta > 0$. In this situation, only N_1^-, N_2^- may be positive, which also depends on the value of the parameters.

If the quantity $B = 0$, that is $b_2 = b_1\beta$, after slove the equations (1)-(3), we have

$$(N_1, N_2) = (\frac{a_2b_1S\alpha\beta^2 + c_2(a_2c_1\beta - a_1c_2)}{\alpha\beta(a_2c_1\beta - a_1c_2)}, \frac{c_1(a_2c_1\beta - a_1c_2) + a_1b_1S\alpha\beta}{\alpha(a_1c_2 - a_2c_1\beta)})$$

Finally, if $b_2 = b_1\beta$ and $a_1c_2 - a_2c_1\beta$=0, there is no solution to (1)-(3).

Let (N_1^*, N_2^*) be an equilibrium with $N_1^* > 0$ and $N_2^* > 0$. Linearizing the right-hand side of (II) about (N_1^*, N_2^*) yields the equation can be derived, as shown at the bottom of next page

The trace of J is negative. the determinant of J is given by

$$detJ = \alpha N_1^* N_2^* (\frac{a_1a_2S(b_1\beta - b_2)}{(a_1N_1^* + a_2N_2^*)^2} + \alpha\beta)$$

From the above equation, if $b_2 = b_1\beta$, because $\alpha > 0, \beta > 0$, and $N_1^* > 0, N_2^* > 0$, $\det J = \alpha^2\beta N_1^* N_2^* \neq 0$. Alternatively, if $b_2 \neq b_1\beta$, the characteristic equation is $\lambda^2 + d_1\lambda + d_2 = 0$, where

$$d_1 = \frac{a_1^2b_1SN_1^*}{(a_1N_1^* + a_2N_2^*)^2} + \frac{a_2^2b_2SN_2^*}{(a_1N_1^* + a_2N_2^*)^2}$$
$$d_2 = \frac{a_1a_2\alpha SN_1^*N_2^*(b_1\beta - b_2)}{(a_1N_1^* + a_2N_2^*)^2} + \alpha^2\beta N_1^* N_2^*$$

The equilibrium is locally stable if the real parts of the characteristic equation roots are negative. The Routh-Hurwitz criteria for this to hold are given by the inequalities $d_1 > 0$ and $d_2 > 0$. By inspection, d_1 is positive, d_2 is definitely positive when $\beta b_1 - b_2 > 0$, and d_2 may be positive when $\beta b_1 - b_2 < 0$, and system (II) exists and $d_2 < 0$, then the equilibrium is locally unstable.

If equilibrium (3) exists and $d_2 > 0$, then either the rumor or the rumor-refutation can invade and they are at equilibrium. However, if the full system has large fluctuations, this does not guarantee persistence in either the rumor or the rumor-refutation. Although the equilibrium is globally stable for many parameter choices, many parameter combinations can result in a highly unstable system [27]. If $\beta b_1 - b_2 < 0$, but $d_2 > 0$, the Routh-Hurwitz criteria are satisfied and the equilibrium is stable; conversely, $\beta b_1 - b_2 < 0$, but $d_2 < 0$, then the equilibrium is unstable. Fig. 4 gives examples of the domains of stability and instability for the proposed model in which three cases are shown: when $\beta b_1 - b_2 < 0$ in (a) and (b); when $\beta b_1 - b_2 > 0$ in (c); and (d), which shows instability $\beta b_1 - b_2 > 0$.

III. EMPIRICAL ANALYSIS AND RESULTS

In this section, a case is given to describe the competition and predation behaviors between rumor propagation and rumor refutation, each of which is represented by the parameter values and dynamic model analysis. The model's parameters were estimated using the integral method, which can be easily adapted to a variety of mathematical models to describe interactions [31].

A. DESCRIPTION OF THE DATA

A microblog "haze" rumor and the associated rumor refutation were chosen for this case. On January 5, 2017 at 9:43, a microblog on "haze" was published online by the famous director Lu Chuan, the main content of which was that "the inhalation of haze into the lungs will never be discharged, and will cause an erosion of the body and lungs, which will become apparent in the onset of disease in 10 to 20 years". Even though this microblog haze rumor included few facts or evidence, because it was related to the people's health, many people forwarded this information to others. To dispel these rumors, on January 6, 2017 at 14:21, the Beijing Concorde Hospital released a microblog. From data extracted from the Sina microblog, this case study examined five days of this event from January 5, 2017 to January 10, 2017. The "Luchuan Haze rumor incident" began on January 5 and quickly entered a hot debate stage, which began to taper off on January 10 when no comments/articles were forwarded, after which the public comment died.

Python was used to crawl the network Sina microblog data to study the rumor and official information propagation characteristics related to this event. Because of the wide data text variety and volume in the microblogs, only the forwarding quantities from two microblogs were examined; the total quantities can be found on the microblog page.

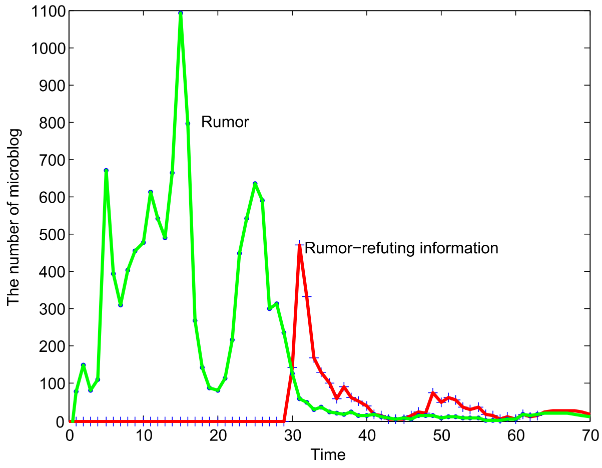

FIGURE 5. **The number of two groups in case study.**

To extract the forwarding time information and study its characteristics in different time periods, a Python code was written to simulate user behavior and crawl the data, which allowed for the collection of the forwarding content, which included the user nicknames, user ids, and user forwarding times; however, because some users had set up directional forwarding, some data were not visible. In total, 12,178 rumor forwarding data and 2,284 official refutation forwarding data were examined. After removing all post January 10 data, there were 12,033 rumor forwarding data entries and 2,227 official refutation data entries. The data were sorted into each hour from January 5 to January 7, and because there were fewer entries on January 9 and 10, only two representative time periods were selected; from 9 a.m. to 10 a.m. and from 10 p.m. to 11 p.m.. Finally, taking 9 a.m. as time 0 and taking the time as the horizontal coordinate and the number of forwards in unit hours as the vertical coordinate, a figure was drawn to reflect the changes over time in the forwarding rate for the haze rumors and the refutation. The data are shown in Table 1; number 1 shows the number of rumor forwards and number 2 shows the number of rumor refutation forwards. The trend in the two groups is intuitively shown in Fig.5.

B. PARAMETER ESTIMATION

Because of the complexity of finding the exact parameters to fit the model, a parameter $a_1 = a_2$ was assumed, which was considered reasonable, because the rumor and rumor refutation rates to attract the unaware population were the same as in an open network space. First, the integral algorithms [31] were applied to the dataset, from which the parameters of

TABLE 1. **The number of microbolgs.**

Hour	1	2	3	4	5	6	7	8
number1	78	150	83	112	671	395	310	403
Hour	9	10	11	12	13	14	15	16
number1	455	479	614	542	489	663	1093	797
Hour	17	18	19	20	21	22	23	24
number1	270	142	87	83	114	218	449	541
Hour	25	26	27	28	29	30	31	32
number1	636	589	300	312	238	127	58	50
number2						144	471	333
Hour	33	34	35	36	37	38	39	40
number1	29	38	23	22	19	24	16	14
number2	169	129	100	59	91	64	52	41
Hour	41	42	43	44	45	46	47	48
number1	18	11	7	4	9	4	15	15
number2	17	14	5	4	4	14	24	21
Hour	49	50	51	52	53	54	55	56
number1	14	7	11	11	8	8	7	3
number2	75	50	63	56	37	32	36	18
Hour	57	58	59	60				
number1	1	2	5	6				
number2	15	5	11	6				

(I) were determined. The detailed calculation process was as follows.

The integral method was then applied to estimate the coefficients for the first differential equation in system (II). As the coefficients in the second equation were able to be obtained in a similar manner, the first equation for species N_1 was given

$$\frac{dN_1(t)}{dt} = N_1\left(\frac{b_1 S}{N_1 + N_2} - \alpha N_2 - c_1\right),$$

where N_1, N_2 were functions of t. Integrating both sides of above equation with respect to t over the interval $[t_0, t_n]$ yields

$$\int_{t_0}^{t_n} \frac{dN_1(t)}{dt} dt = \int_{t_0}^{t_n} N_1\left(\frac{b_1 S}{N_1 + N_2} - \alpha N_2 - c_1\right) dt,$$

The time interval $[t_0, t_n]$ was divided into n sub-intervals. For each of these intervals, it follows that

$$N_1 \Big|_{t_j}^{t_{j+1}} = \int_{t_j}^{t_{j+1}} N_1\left(\frac{b_1 S}{N_1 + N_2} - \alpha N_2 - c_1\right) dt, \quad j = 0, 1, 2, \ldots, n-1$$

so that

$$N_1(t_{j+1}) - N_1(t_j) = b_1 S \int_{t_j}^{t_{j+1}} \frac{N_1}{N_1 + N_2} dt - \alpha \int_{t_j}^{t_{j+1}} N_1 N_2 dt - c_1 \int_{t_j}^{t_{j+1}} N_1 dt$$

$$J = \begin{pmatrix} -\dfrac{a_1^2 b_1 S N_1^*}{(a_1 N_1^* + a_2 N_2^*)^2} & -\dfrac{a_1 a_2 b_1 S N_1^*}{(a_1 N_1^* + a_2 N_2^*)^2} - \alpha N_1^* \\ -\dfrac{a_1 a_2 b_2 S N_2^*}{(a_1 N_1^* + a_2 N_2^*)^2} + \alpha \beta N_2^* & -\dfrac{a_2^2 b_2 S N_2^*}{(a_1 N_1^* + a_2 N_2^*)^2} \end{pmatrix}$$

Any numerical method can be used to estimate each of the integrals on the right-hand side of above equation. Remembering that the intervals $[t_j, t_{j+1}]$ are of unit length and using the Trapezium rule, it follows that

$$b_1S\int_{t_j}^{t_{j+1}}\frac{N_1}{N_1+N_2}dt \approx \frac{t_{j+1}-t_j}{2}[b_1S(\frac{N_1(t_{j+1})}{N_1(t_{j+1})+N_2(t_{j+1})} + \frac{N_1(t_j)}{N_1(t_j)+N_2(t_j)})]$$

$$\alpha\int_{t_j}^{t_{j+1}}N_1N_2dt \approx \frac{t_{j+1}-t_j}{2}[\alpha(N_1(t_{j+1})N_2(t_{j+1}) + N_1(t_j)N_2(t_j)]$$

$$c_1\int_{t_j}^{t_{j+1}}N_1dt \approx \frac{t_{j+1}-t_j}{2}[c_1(N_1(t_{j+1})+N_2(t_j))]$$

The statistical data for $x(t_j)$, $y(t_j)$ and $z(t_j)$ are shown in table 1, the set of linear equations above were then represented in matrix notation as

$$\begin{pmatrix} d_{1,0} \\ d_{2,1} \\ \vdots \\ \vdots \\ \vdots \\ d_{n,n-1} \end{pmatrix} = \begin{pmatrix} x_{1,0} & y_{1,0} & z_{1,0} \\ x_{2,1} & y_{2,1} & z_{2,1} \\ \vdots & \vdots & \vdots \\ x_{n,n-1} & y_{n,n-1} & z_{n,n-1} \end{pmatrix} \begin{pmatrix} b_1S \\ \alpha \\ c_1 \end{pmatrix}$$

$$or\ \ d = AX$$

with $d_{j+1,j} = N_1t_{j+1} - N_1(t_j), j = 0, 1, 1, \ldots, n-1$. The matrix A contains

$$x_{j+1,j} = \frac{t_{j+1}-t_j}{2}(\frac{N_1(t_{j+1})}{N_1(t_{j+1})+N_2(t_{j+1})} + \frac{N_1(t_j)}{N_1(t_j)+N_2(t_j)})$$

$$y_{j+1,j} = \frac{t_{j+1}-t_j}{2}(N_1(t_{j+1})N_2(t_{j+1}) + N_1(t_j)N_2(t_j)$$

$$z_{j+1,j} = \frac{t_{j+1}-t_j}{2}(N_1(t_{j+1}) + N_2(t_j))$$

The matrix X contains the unknown parameters to be determined. To solve the unknown parameters X in $d = AX$, the transpose of A was considered, namely A', so that

$$A'd = A'AX$$

The estimation of X was then given by

$$X = (A'A)^{-1}A'd$$

As the values $x_{j+1,j}$,$y_{j+1,j}$, and $z_{j+1,j}$ can be derived from Table 1, Matlab software was employed to solve X.

Similarly, for the second equation in system (2),

$$\frac{dN_2(t)}{dt} = N_2(\frac{b_2S}{N_1+N_2} + \alpha\beta N_1 - c_2)$$

the parameter estimation for b_2S, $\alpha\beta$, c_2 was determined.

With all the above methods being made, the parameter b_1S, α, c_1, b_2S, $\alpha\beta$, c_2 were determined. Substituting the estimated values into Equation (2) yielded the system for Equation (9), which describes the dynamics of the rumor refutation system:

$$\begin{cases} \frac{dN_1(t)}{dt} = N_1(\frac{6.9405}{N_1+N_2} - 0.0025N_2 - 0.0115) \\ \frac{dN_2(t)}{dt} = N_2(\frac{4.2226}{N_1+N_2} + 0.0227N_1 - 1.1861) \end{cases} \tag{12}$$

In Equation (12) the estimated parameters provide a useful insight to the type of interactions and the churn effects between the rumor and the rumor refutation. All parameters indicate that the competition and predation interactions describe these two populations, which was the expected result as the rumor and the rumor refutation are competing for the unaware population, and the rumor refutation is also preying on the rumor spreader population.

b_1S=6.9405, b_2S=4.2226, $b_1/b_2 = 1.6437$ indicate that the efficiency at which the rumor spreader population converted the unaware population into the rumor population was higher than the efficiency at which the rumor refuting population converted the unaware population into the rumor refutation population, that is the rumors were strong enough to convince the unaware population to believe the rumors rather than the rumor refutation. The case data indicated that the haze rumor incident began on January 5, quickly entered a hot debate stage, and reached a peak of 1093 after 15 hours. The Beijing Concorde Hospital released a microblog to dispel the rumors after 30 hours, after which the rumor refutation forwarding rose sharply, and reached a peak of 471, which was far less than the rumor peak. As shown in Equation (9), the rate $\alpha = 0.0025$ at which the rumor spreaders encountered the rumor refutation spreaders was very low, that is, the rumor-refutation capture rate of the rumor spreader population was low.

As the rumor refutation did not enter the system until 30 hours after the incident, the rumors entered a dissolution stage, with the number of rumor forwards dropping to 127, which indicated that the probability of the rumor and the rumor refutation meeting was very small; $\alpha = 0.0025$, $\alpha\beta = 0.0227$, $\beta = 0.0227/0.0025 = 9.08$, Although, the efficiency at which the rumor population converted into the rumor refutation population was high, there was still a limit to the increase in the rumor refutation capture of the rumor population because of the low encounter rate. In addition, the low, but steady rumor death rate (0.0115) implied that people were maintaining an interest. Even though this microblog haze rumor had few facts or evidence, because it was related to health, many people forwarded the information to others. Therefore, the rumors did not die quickly regardless of the lack of facts. The rumor-refutation death rate (1.1861) was higher than the rumor death rate. By the time the rumor-refutation was released, the rumors had been spreading for 30 hours and had entered the public consciousness, which meant that the rumor refutation was less attractive to the public. When an authority's credibility is low, the rumor refutation does not gain public trust and the public refuses to spread it, which means that the rumor refutation

TABLE 2. Equilibrium points of the system.

	N_1(Rumors)	N_2(Rumor-refuting information)
1	0	3.56
2	603.52	0
3	58	-90.37
4	49.93	30.09

disappears very quickly. This theoretical analysis is consistent with reality.

To calculate the equilibrium state of the system for the equations in (9), they were simultaneously set as equal to zero, that is:

$$\begin{cases} \frac{dN_1(t)}{dt} = N_1(\frac{6.9405}{N_1+N_2} - 0.0025N_2 - 0.0115) = 0 \\ \frac{dN_2(t)}{dt} = N_2(\frac{4.2226}{N_1+N_2} + 0.0227N_1 - 1.1861) = 0 \end{cases} \quad (13)$$

The solution to (13) has with 4 equilibrium points, as shown in Table 2, two of which lead to rumor extinction or rumor refutation extinction, with the remaining two indicating a coexistence of the two populations.

However, not all are stable equilibrium points. After dropping the non-valid equilibrium points (not belonging to the positive quadrant), an eigenvalue analysis was performed to examine the stability of the rest of the points. The eigenvalues and eigenvectors for each valid equilibrium point were derived by calculating the corresponding Jacobian matrices in Equation (2). The analysis found only one stable equilibrium point, that is, the fourth critical point in Table 2, which indicated that none of the competing populations became extinct at the equilibrium. The remainder were found to be unstable as the Jacobian matrices' eigenvalues had different signs; therefore, in the derived general solutions, one of the variables dominated and caused the system to become unbounded and unstable [32].

A substitution of the estimated values and the fourth equilibrium point into the Jacobian matrix yielded the matrix

$$J = \begin{pmatrix} -0.0555 & -0.1803 \\ 0.6626 & -0.0204 \end{pmatrix}$$

As the characteristic value for the above matrix was $\lambda_1 = -0.0380+0.3452i$, $\lambda_2 = -0.0380-0.3452i$, there were real negative parts. According to hurwitz stability criterion, the equilibrium was locally asymptotically stable. Fig. 6 shows the phase portraits from the case study, which illustrates that the rumor and rumor refutation populations finally coexisted in the system.

IV. DISCUSSION

Based on the case study results, two case are given to examine the effect of the authority's credibility and public cognition. By varying parameters p_1 and p_2, simulations were conducted in MATLAB using the Runge-Kutta method to examine the changes in the rumor spreader population proportion over time. From the parameter analyses in the different scenarios, a better understanding of rumor propagation was obtained.

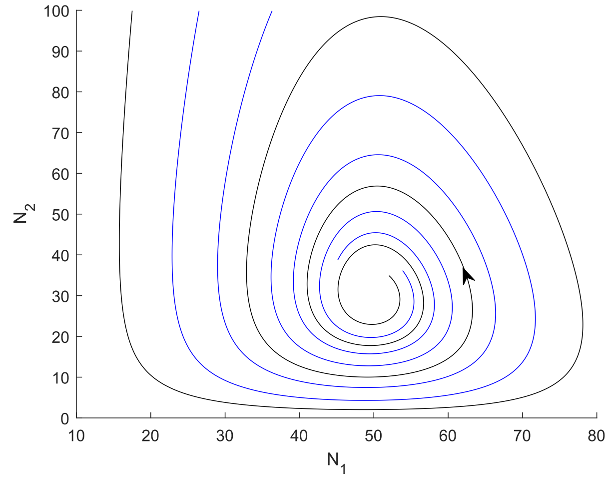

FIGURE 6. The phase portraits of case study for the $N_1 - N_2$ plane.

Then, combined with the previous theoretical analysis, some suggestions were developed for the management and control of rumors.

A. PARAMETER ANALYSIS

The effects of parameters b_1 and b_2 on rumor propagation were examined, where $b_1 > 0$ is the efficiency at which the rumor spreaders convert the unaware population into rumor spreaders, and $b_2 > 0$ is the efficiency at which the rumor refutation spreaders convert the unaware population into rumor refutation spreaders. The previous analysis revealed that parameter b_1 is mainly affected by the public's cognition, and that if this cognition was strong, this parameter was small. Parameter b_2, however, was shown to be mainly affected by the authority's credibility, that is, the higher the credibility, the larger parameter b_2. Two simulations were conducted; $a_1 = a_2 = 1$, and $S = 50$, with the other parameters being the same as those fitted for the above case. Specifically, the initial parameter values for dissemination model were given as follows: $N_1(0) = 0.0001$, $N_2(0) = 0.0001$, $a_1 = a_2 = 1$, and $S = 50$, $\alpha = 0.0025$, $\beta = 9.08$, $c_1 = 0.0115$, $c_2 = 1.1861$, and $b_2 = 0.084$ when b_1 take different values, and $b_1 = 0.139$ when b_2 take different values. Simulations were conducted using the Runge-Kutta method in MATLAB.

1) INFLUENCE OF PUBLIC COGNITION ON RUMOR DISSEMINATION

To analyze the affect of b_1 on the rumor propagation in the system, Matlab was used to solve system (II) and determine the rumor propagation trends. Fig. 7 illustrates the changes in the rumor spreader density over time for different values of b_1. From a macroscopic perspective, it was found that as parameter b_1 increased, the number of spreaders increased, because a larger b_1 indicated a weaker public cognition. Of the three curves, the green curve indicates a scenario for a strong public cognition and the red curve indicates a scenario for a weak public cognition. It can be seen that the higher the parameter b_1, the larger the peak spreader value, and the

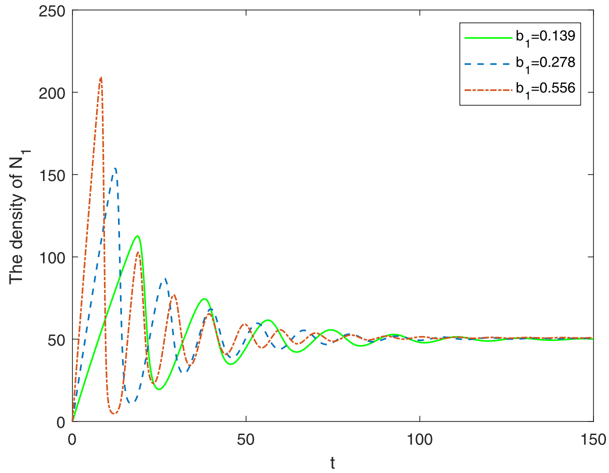

FIGURE 7. Density of rumor spreaders vary with the parameter b_1.

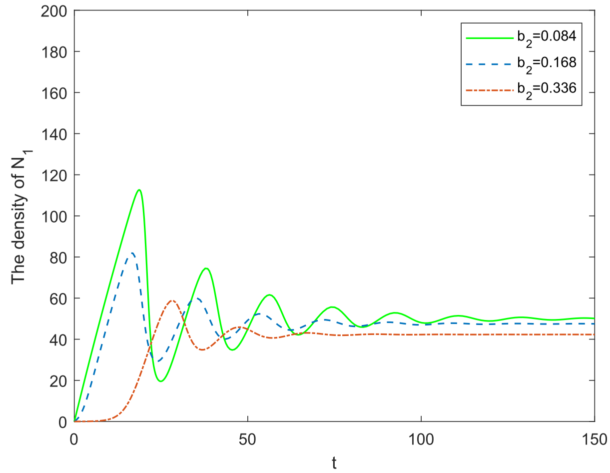

FIGURE 8. Density of rumor refutation spreaders vary with the parameter b_2.

earlier it reaches a peak value. Clearly, the larger the value of b_1 when the other parameters are fixed, the wider the rumor influence, and the higher the peak value, the faster the rumor spread, and the earlier it reaches a peak.

The analysis of Fig. 7 indicated that public cognition plays an important role in rumor propagation. Because cognition is related to the acquisition or application of knowledge, in real life, if people come into contact with rumors, they identify whether the rumor is true or not using their knowledge and experience. If people have enough reasoning and discrimination, they see the flaws in the rumor or do not fully believe the rumor. However, if people do not have a strong intellectual background, they are more likely to follow the crowd and believe the rumors. For example, after the nuclear disaster in Japan in March 2011, there were rumors that iodized salt could protect people against radiation, resulting in many Chinese residents rushing to buy iodized salt. Experts quickly quelled these rumors by providing evidence that that iodized salt did not protect against radiation, thereby illustrating that people with higher scientific and cultural literacy are able to think independently and critically, identify the truth, and quash obviously false rumors.

2) INFLUENCE OF GOVERNMENT CREDIBILITY ON RUMOR DISSEMINATION

A change in government credibility may affect rumor propagation. To study the influence of this factor, different scenarios were established. With all other parameters remaining unchanged, b_2 was changed: b_2=0.084; b_2=0.168; b_2=0.336; and the differences in the rumor propagation under the different values for b_2 examined. Fig. 8 illustrates the changes in the rumor spreader densities over time with different values for b_2. As can be seen, with an increase in b_2, the spreader proportion peak and the spread duration both decreased. When the value of the influencing factor b_2 was 0.336, the rumor spreader peak was low, which indicated that the government credibility was high and the influence on the rumor propagation was very great, and when b_2 was greater than a certain level, there was no rumor spread.

Clearly, the smaller the value of b_2, the broader the rumor's influence, and the longer the rumor duration. In real life, after rumors start, the government takes measures to deal with them, one of the more common of which is to release official refutations. Generally, the public prefers to trust official information because of government credibility and authority. However, if the government loses credibility, the public may lose trust in official rumor refutation and choose to believe unofficial information such as rumors, which is known as the Tacitus trap, that is, when a government or an organization loses credibility, it is considered to be lying whether it is telling the truth or not. Therefore, strengthening government credibility and increasing the public's trust in the government increase the governments ability to refute unfounded rumors.

B. MANAGERIAL IMPLICATIONS

Keep rumor refutation attractive. The proposed mixed dynamic model theoretical analysis was able to accurately describe the relationships between rumors and rumor refutations and reveal the competition and prey behaviors between the rumor and rumor refutations in cyberspace. The model suggested that a rumor and rumor refutation coexistence occurs when the rumor is more attractive than the rumor refutation and the ability to convert the unaware populations into a rumor population is stronger. Therefore, if an authority wants to dispel rumors, they need to make sure that the rumor-refutation is more attractive and is spread widely, which relies on knowing what the public wants to know. If the information is not information public wants to know, they will ignore the official information. Second, authorities need to maintain high credibility. If the government has the higher authority, it is the leader in the social system and the information the government publishes will generally attract the public.

Propose countermeasures from individual perspectives. In the theoretical analysis of extinction of rumor population, the results show that rumor refutation is more likely to be successful in dispelling rumors if there is a large unaware population. As rumor refutations are released to quash rumors, they are generally released a little later than the rumors, which means that the initial rumor has a spread advantage at the beginning. As the rumors are believed and spread by some of the previously unaware population, it increases the refutation's competitive pressure when the rumor-refutation and the rumor are competing for the population resources that have not yet been converted. If a rumor is widespread, a large number of the unaware population will be attracted to it; therefore, the government cannot let rumors spread widely to attract the majority of the unaware population as this would decrease the effect of the refutation. If there are few unaware population resources, nobody would be available to propagate the rumor refutation and the rumor refutation, which would find it difficult to survive without preying on the rumor spreader population. Therefore, it is necessary to think about the possible countermeasures that could be used to to control rumor propagation. Obviously, improving the public's scientific knowledge and the abilities to think independently and critically and identify rumors would be one of the best methods to reduce rumor propagation.

Establish an early warning rumor mechanism. The case analysis revealed that rumors and rumor-refutations coexist, the specific features for which are; (1) the unaware population conversion efficiency into rumor spreaders is high; (2) the probability is low that the rumor spreader encounters the rumor refutation; and (3) the rumor refutation death rate is higher than the rumor death rate. The case revealed that the haze problem was a serious issue for the public; therefore, the haze rumor quickly attracted the public's attention. Therefore, authorities should strengthen their monitoring of the online network so that prompt measures can be taken if rumors emerge that are closely related to public livelihoods. Second, the rumor-refutation entered the system too late. If the rumor-refutation lags significantly behind the rumor, it becomes more difficult to control the rumor. Therefore, the earlier the release of the rumor refutation after the rumor begins to circulate, the easier it is for the rumor to be quashed. Overall, when there is an emergency, the government needs to establish an early warning rumor mechanism, supervise public opinion, and release official information before the rumors arise.

V. CONCLUSION

Rumor propagation is a widespread social phenomenon, which is marked by periods of uncertainty and anxiety and can cause damage to individuals and society. Therefore, government rumor refutations that either refute the rumors or confirm the facts provide valuable emergency notifications to dispel and discharge rumors. The success of refutation depends on the interplay between the rumor refutation and rumors, that is, the rumor refutation and the rumor compete for the attention of the unaware population and also are locked in predator-prey interactions.

This paper presented a model that examined the interplay between rumor and rumor refutation propagation, which was found to provide excellent explanations for the rumor and rumor-refutation mixed competition and predation behaviors, better reflected the actual circumstances, and was easy to understand. Possible equilibrium points were derived for three cases; rumor-refutation displacement, rumor displacement, and rumor-refutation and rumor coexistence. The equilibrium stability and instability points were discussed, some general principles of coexistence elucidated for the rumor and rumor refutation propagation system, and a realistic picture given on the complications in the conditions and results.

A case was given to better illuminate the rumor and rumor-refutation competition and predation behaviors. Network data from the Sina microblog were extracted, and the model parameters estimated using the integral method. The empirical analysis revealed that at equilibrium, both the rumor and the rumor-refutation coexist and that the system is locally asymptotically stable at the end. It was also shown that rumors and rumor refutations compete for the available unaware populations and that rumor refutations prey on the rumor spreading population. From the values for the parameters b_1S and b_2S, rumors can make more populations believe the rumors and dismiss the rumor refutation, and from the values for parameters α and β, it was found that rumor refutations may have limited encounters with the rumor spreader population. As the theoretical analysis was consistent with reality, management suggestions were proposed based on the theoretical simulation analyses.

The rumors and rumor refutation propagation interplay model developed in this study contributes to rumor management by offering a more comprehensive analysis. Although the model did not consider any microscopic factors, which would be desirable for a more realistic description, it was shown to be appropriate for a macro structure analysis of the competitive and predator dynamics existing in rumor and rumor refutation information systems. Further, as the parameter estimations for differential equations are difficult when the differential equations are complex, it was assumed that $a_1 = a_2$ in this paper. Future work directions include the development of suitable methodologies based on the other algorithmic approaches to determine more accurate parameter estimates.

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