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Multi-Objective Robust Optimization for the Traffic Sensors Location Problem

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ABSTRACT The main concern of this research is to control traffic flow and monitor highways by installing wireless sensors. Therefore, a new multi-objective model is proposed to find the optimal location of wireless sensors along highways. The sensors are paired and each pair of sensors communicates and interacts with each other to receive information from passing cars. The speed estimation of the vehicles passing through this pair of sensors must have the minimum deviation from the actual speed that is obtained by an accurate measurement. This deviation is called measurement error, which is minimized in the first objective function. In this research when traffic jams happens, some sensors located in proper distance with enough energy, move into the traffic area to reduce the measurement error which is caused by traffic congestion. In fact, for each traffic area a new location problem should be solved to relocate sensors so that the maximum decrease in error rate happens. The second objective maximizes the error reduction resulted from sensors movement. In this paper, movement of sensors is considered based on the amount of solar energy stored in the sensor at that moment. Finally, the third objective function maximizes the benefits resulted from detecting the bottlenecks in highways. Since some parameters of the objective functions such as error rates, error reduction resulted from movement of sensors and benefits are uncertain, this research employs a multi-objective robust optimization approach which results in a traffic control plan which is less sensitive to the realization of uncertain parameters.

INDEX TERMS ε-constraint method, LP metric method, multi objective robust optimization, sensor location problem, wireless sensors network.

I. INTRODUCTION

Transportation system has an important role in economic growth in our societies. It is also one of the main sources of pollution. Therefore managing traffic system becomes crucial. Rapid increase in the number of vehicles leads to traffic congestion which results in delays and more fuel consumption. As capacities of roads are limited, optimal use of available infrastructures is important. To reduce accidents, travel time and traffic congestion, roads need to be under control. Different types of control actions can be used to manage the traffic flow in a highway network.

Pasquale *et al.* [1] divided modelling framework for sustainable freeway traffic control into five categories:

1. Traffic flow models: These models derive from the need to describe the dynamic behavior of real traffic systems through mathematical relations.
2. Emission models: These models consider traffic systems according to harmful substance emission.
3. Consumption models: These models consider traffic systems according to fuel consumption.
4. Dispersion models: These models consider the quantity of pollutants produced by vehicles, wind direction, air temperature and the presence of obstacles as inputs.
5. Safety models: These models evaluate the crash risk on the basis of the layout of the road,

In this paper, traffic flow models are studied to describe the behavior of traffic system.

One of the ways of controlling and monitoring highways is direct police surveillance over highways' length [3], [4], which is an expensive task. Another way is to use sensor technology [5] which is the subject matter of this paper. Gentili and Mirchandani [14] classified all types of sensors used in transportation into four categories: Counting sensors which count cars on network paths, Path-ID sensors which measure number of cars in each path, Image sensors which

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take image of moving flows and Vehicle-ID sensors which identify a specific car on the network.

Sensors require infrastructure and maintenance costs. On the other hand, every sensor needs energy to operate and since the cost and energy are limited [6], [7], we need to achieve the maximum efficiency with the minimum number of sensors. These sensors need to be located at optimal distances to achieve maximum performance with minimum number and cost. If the distance between two sensors is short, the area coverage increases and the error rate reduces but it needs more energy and more budget. Therefore, the optimal number of sensors should be calculated.

Many techniques have been used including over the ground sensors like video image, radars, ultrasonic sensors which have high cost and their accuracy depends on environment's condition, and intrusive sensors like inductive loop detectors which disrupt traffic during installation and repair. Many studies suggested the use of wireless sensor network (WSN) technology for traffic control which is cost effective, easy to install, less maintenance and has potential for large scale deployment.

Wireless sensors are divided into four categories [8]:

1. Fixed: where sensors are located in a fixed locations
2. Moving Sensors: where sensors are located on moving platforms that can move everywhere
3. Combination: means a combination of moving and fixed sensors
4. Robotics: where robots are used to move sensors.

In fact, the exclusive characteristic of WSNs includes the mobility of sensor nodes [2]. Therefore, in this paper, moving wireless sensor networks (WSN) are going to be studied which are more practical in real world situations. Since WSN are small, they can be carried easily by portable devices suchas Robomote [9]. Robomote is a wheel equipped sensor node designed for easy deployment.

Most articles in the field of traffic management are divided into two main groups. The first group includes articles that seek to find optimal traffic light changing time to reduce delays and minimize queues. The second group, which is the main concern of this paper, is about the location of sensors which itself is divided into the following categories:

- Routing: information obtained by sensors helps finding the best routs [10].
- Coverage: sensors are located to achieve maximum coverage in an area [11].
- Infrastructure collapse: which occurs when a road collapses or node failure happens [12].
- Traffic flow control: average speed of passing cars is used in travel time estimation [13].
    1. In this paper, travel time estimation is needed which is obtained from average speed of passing cars and used in traffic flow control. As the speed of a specific car on the network is needed, using vehicle-ID sensors is suggested.

In reality some parameters such as traffic signal systems [15], travel time [16], injury severity [17], traffic condition [18], and so on are uncertain. According to certainty or uncertainty of parameters, traffic articles can be divided into two groups. Some articles dealt with uncertainty through stochastic programming [19] or robust optimization [20]. In this paper a robust optimization approach is employed to deal with uncertainties in parameters such as error rate and the parameter which shows the amount of decrease in error rate as a result of sensor movement. The parameter of error rate is uncertain because of difference between the speed of cars reported by sensors and actual speed of cars.

In this research, we propose a mathematical model to find the optimal location of sensors considering costs and energy consumption to minimize measurement errors. One of the functions of the sensors located along highways is to identify the speed of passing vehicles. The average speed of passing cars can be used in predicting the vehicles' travel time. The sensors must be selected in pairs and each pair of sensors should communicate with each other to receive information from passing cars specially their average speeds. Each of the paired sensors determines the speeds of cars in the distance between these two paired sensors and the average speed of cars can be calculated by these two values. The average speed reported by the pair of sensors is a little different from the actual average speed. This difference is called the speed measurement error. If the distance between the two sensors is short, the error reported by them is lower. Since the numbers of available sensors are limited, finding the optimal configuration of paired sensors is essential.

The amount of measurement error increases when traffic jam happens, because the difference between actual average speed and measured average speed become greater. In this research it is suggested to move some sensors into the traffic areas to reduce the amount of errors. In fact, each traffic area is a new location problem to find the best relocation of sensors. Each sensor gets required energy from solar panels to move. They can move into the traffic area if they have enough energy at the moment of traffic jam.

In sum, the main contributions of this work are listed as follows.

- Considering different time intervals in a day, each of which has its own parameter.
- Considering the possibility of moving sensors.
- Optimizing a multi-objective transportation model.
- Considering enough energy and proper distance of sensors for movement in the model.
- Applying robust optimization to deal with uncertainty.

The remainder of the paper is organized as follows. Section 2 is a review of the work done in the field of wireless sensor network particularly in transportation system. The problem is defined and formulated in section 3 under both deterministic and uncertain parameters. The solution method to simultaneously deal with uncertainty and multi-objective model, i.e. multi-objective robust optimization, is presented in section 4. In section 5 numerical results are presented.

Finally, conclusions and future suggestions are discussed in section 6.

II. LITERATURE REVIEW

Nowadays, with the advancement in wireless technology, wireless sensors can be used to control the transportation system. These sensors do not need wires for communication and therefore they can save money. Wireless sensors can be used in different areas such as monitoring the water supply system to find leakage of water pipes, as well as monitoring oil and gas pipelines. One of the most common uses of this technology is the remote monitoring of the environment such as monitoring and controlling industrial processes, smart homes, farms and traffic areas. In Table 1 some articles related to the wireless sensor network are presented and compared considering their applications, their solving methods, and the uncertainty in the model.

Application of sensors: Sensors have been employed for different applications some of which are listed in the followings.

Viani *et al.* [21] presented the problem of optimizing energy consumption in smart buildings and analyzed the location of wireless sensor network by game theory to help consumers to manage their consumptions. Yoon *et al.* [22] used a wireless sensor network system for detecting and identifying leakage in steamflood and waterflood pipelines in oilfield. Their system aimed to allow continuous monitoring with low cost, short delay, flexible deployment and fine coverage while providing high accuracy in problem detection. Mao *et al.* [23] proposed an approach to monitor carbon dioxide emissions in Wuxi, China. In their paper, the sensors network consists of 100 sensor nodes for monitoring carbon dioxide, and 1096 other nodes for transmitting information received by 100 nodes. They presented a geometric model called the Steiner tree to achieve maximum coverage.

Sensor location is a mathematical model which is classified into two categories of binary and probabilistic models. In binary models, the probability of detecting an incident in the coverage range of sensor is one (full coverage) and outside its range is zero. Although full coverage assumption is simpler for modelling, it is not realistic. In real world, detection probabilities can be smaller than one. So it is better to use probabilistic models by knowing the distribution of errors, as what is presented in the paper of Dhillon and Chakrabarty [24]. Full coverage means to cover all parts of an area which is expensive and requires a lot of sensors and makes the problem more complicated. Coverage is classified into two categories: static coverage and dynamic coverage. In the static coverage sensors are placed in fixed locations with the aim of reaching the maximum coverage as done in the paper by Liu *et al.* [25]. In the dynamic coverage sensors are moved to cover different areas at different times and therefore a wider area can be covered [26].

In the literature, mobile sensors are introduced to improve the coverage of an area. In [26], mobile sensors were used to move into uncovered places. The authors employed the game theory approach to determine the appropriate time for moving mobile sensors to achieve maximum level of coverage.

Some papers proposed optimization techniques for dynamic models with mobile sensors, such as Particle Swarm Optimization [27], Ant Colony Optimization [28] and Harmony Search [29].

Location of sensors: Most articles in the literature studied the optimal location of sensors in transportation systems. Gentili and Mirchandani [30] surveyed some articles related to the problem of locating counting sensors and Automatic Vehicle Identification (AVI) readers to estimate travel times on a freeway. Their models were classified into two main approaches: shortest-path based approaches and clustering based approaches. Morrison and Martonosi [31] investigated how minimum number of sensors can be located in the transportation network in order to find the distribution of vehicles in this network. They also studied necessary conditions for the location of sensors in a network to determine the rate of flow everywhere in the network. As it is shown in their article when a set of intersections is monitored, the flow on all roads between intersection M and adjacent intersections A(M) are known, as well as the balancing flows at each centroid in M. They investigated a condition to verify that a proposed set of monitored intersections uniquely determines flow function and the balancing flows. They corrected a slight error in an earlier theorem that addressed this issue by using incident matrix, and presented a stronger necessary condition for this problem that is also sufficient for any unmonitored acyclic subgraph.

Geetla *et al.* [32] employed sensors to detect and prevent accidents and tried to optimize these sensors' locations so that maximum detection capability is achieved and the best route to be serviced after crashes is determined. Although acoustic sensors have more cost but show less error especially in bad weather conditions. The main goal of their paper is to maximize coverage based on budget constraints.

Fei *et al.* [33] examined the uncertainty condition in the transportation system by proposing a two-stage model. In the first stage uncertainty is ignored and only sensor location with the goal of maximum coverage is considered. In the second stage, uncertainty is added to the model and the expected cost of the flow of transportation in uncertain condition is examined. The resulted bi-objective model is solved by a greedy and iterative hybrid algorithm.

Danczyk *et al.* [34] proposed a model considering probabilistic sensors errors and found the optimal configuration of sensors in the highway to minimize measurement errors which measure the difference between the average travel time of direct observation and the average travel time reported by two sensors. In their paper, the failures of sensors along the highway were considered as uncertain parameters and when a sensor failure happens, two other sensors are paired to measure the error.

Fu *et al.* [35] proposed a scenario based two-stage stochastic programming, which considers the uncertainty of the link-paths matrix. In fact, uncertainty exists in different paths

TABLE 1. A review of studies on the wireless sensor network.

Reference	Field of study	Scope of application	Uncertainty	Objective	Move ment	Optimization method
Viani et al. [21]	Energy	Energy consumption in buildings	No	Energy consumption	No	Game theory
Yoon et al.[22]	Pipes monitoring	Gas leakage	No		No	
Mao et al. [23]	Air pollution	Carbon dioxide leakage monitoring	No	Carbon emission measurement	No	Geometric model of Steiner tree
Dhillon and Chakrabarty [24]	Networks	Sensors field on a network	probabilistic	Maximizes coverage	No	
Liu et al. [25]	Networks	Detecting targets on a network	stochastic	Maximize coverage	Yes	Game theory
Liu et al. [26]	Networks	Coverage of a network	Stochastic	Coverage	Yes	Game theory
Gentili and Mirchandani [30]	Transportation	A review article on methods of calculating the travel time	No	Minimize total number of counting sensors Maximize network route flow	No	
Morrison and Martonosi [31]	Transportation	Controlling the transportation flow	No	Traffic flow computing	No	Hybrid genetic algorithm with simulated annealing
Geetla et al. [32]	Transportation	Car crashes	No	Incident detection	Yes	Simulation and neighborhood search
Fei et al. [33]	Transportation	Highway monitoring	Two-stage stochastic	Maximization of OD Flow coverage and minimization of variation of OD matrix	No	Greedy and iterative heuristic algorithm
Danczyk et al. [34]	Transportation	Freeway monitoring	Probabilistic optimization	Minimizing overall monitoring errors	No	Heuristic algorithm
Fu et al. [35]	Transportation	Routing	Two-stage stochastic	Minimizing installation cost and minimized uncovered path	No	Branch and bound method
Zhan and et al. [36]	Transportation	Travel time estimation	No	Traffic simulation	No	Genetic algorithm
Olia et al. [37]	Transportation	Travel time estimation	No	Minimize total number of sensors and travel time error rate	No	Genetic algorithm
Park and Haghani [38]	Transportation	Travel time prediction	Stochastic	Maximizes the reduction of travel time error	Yes	
Salari et al. [39]	Transportation	Sensor failure in traffic network	No	minimize the maximum effect of sensor failure on the link flow	No	Genetic algorithm
Losilla et al. [40]	Transportation	Application of Wireless Sensors in Transportation system	No	Survey		
Gil Jimenez and Fernandez-Getino Garcia [41]	Transportation	Traffic jam	No	Traffic jams avoidance	No	
Chow and Li [42]	Transportation	Traffic jam	Robust optimization formulated as minimax problem with scenarios	Minimizing total delay	No	Two-stage solution algorithm
Current study	Transportation	Highway monitoring	Robust optimization based on Bertsimas and sim approach	Minimizing overall monitoring error Maximizes the reduction of errors Maximizes the benefits of detecting bottlenecks	Yes	ε-Constraint

and the combination of these paths is considered as a scenario. In the first stage, the cost of installing sensors and the expected penalty for uncovered paths are minimized. The second stage attempts to minimize uncovered paths considering a given sensor location and a specific scenario. They used the branch and bound based integer L-shape method to solve the model.

Zhan and *et al.* [36] considered the allocation of additional sensors to improve the coverage of existing deployed sensors considering budget constraint. Their article aims to find the best location of new sensors and the model is solved by a genetic-algorithm.

Olia *et al.* [37]. proposed a multi-objective model and solved it by a genetic algorithm to optimize the number and position of road sensors to estimate travel time. They defined two types of communications; the first type is the relationship between two cars and the second is communication between cars and the road infrastructure. In a case study the authors mentioned that in traffic areas, more numbers of sensors should be used.

Park and Haghani [38] determined the optimal location of sensors in uncertain condition. Because of the uncertainty in travel time error, different scenarios were considered and a stochastic model was presented. The main purpose of their paper was to show where to locate portable sensors on road networks.

Salari *et al.* [39] investigated the effect of node failure over the link flow and aimed to recognize the minimum set of links to be instrumented with counting sensors to reach full flow observability in a traffic network. The authors also studied the location of redundant sensors to maintain the link flow of unobserved links in the event of sensor failure.

Traffic flow control: There are some other studies in literature about traffic flow control in transportation system as follows.

Losilla *et al.* [40] did a survey about application of wireless sensor networks for intelligent transportation systems. According to their paper WSNs can be used to process information, reducing data distribution costs and offering a fast response to critical events.

Gil Jimenez and Fernandez-Getino Garcia [41] compared the use of wireless sensor network with other traffic control methods and the low cost of wireless sensor network is pointed out. They also introduced a variety of sensors used in the traffic area. In their paper, a novel design of a wireless sensor network system for the detection and avoidance of traffic jams is described and analyzed. The systemcan also be used for traffic monitoring and surveillance.

Chow and Li [42] formulated a robust optimization as a minimax problem in an uncertain traffic condition to minimize vehicle delays. In this article traffic demand is uncertain. This model is solved by a two stage algorithm. The first stage tries to minimize the amount of delays, and in the next stage for the desired area, the fundamental diagram is created to show the traffic flow.

Jha *et al.* [43] proposed a model for connectivity restoration for the WSN which consists of two phases: (1) intra-partition, and (2) inter-partition. In the intra-partition phase, the coverage of each partition of a WSN is enhanced by spreading the redundant nodes toward the boundary of the respective partitions they belong. The redundant nodes and the relay nodes situated between partitions are used to make coalition within a cooperative game. The authors proposed a heuristic method to place relay nodes, because finding minimum number of relay nodes is known as an NP-hard problem. In the inter-partition phase a cooperative game theoretic approach has been proposed that keeps the connectivity between different partitions. In this game theory approach residual energy of each node plays important role in making game decisions.

Mehrabipour *et al.* [44] presented a decomposition scheme to find near-optimal solutions for a cell transmission model-based system for an optimal dynamic traffic assignment problem with multiple origin-destination pairs. This technique decomposes the original problem into a set of subproblems. They employed Dantzig-Wolfe technique, which constructs a master problem and a set of subproblems, where each subproblem shows a single origin-destination pair. In their proposed scheme the single origin-destination level solutions are pushed toward the global optimality, that is, the main objective of a dynamic traffic assignment problem with multiple origin-destinations.

Traffic lights control: There are also some works in the literature which study the traffic flow control problem from the traffic lights control perspective. Since it is not the main concern of this paper we just name a few. Liberati [45] proposed a predictive control approach for optimal management of traffic light signals to minimize and as well to balance the queues. Their proposed model is nonlinear and they linearized it by introducing additional auxiliary variables. Han *et al.* [46] investigated the appropriate time for changing traffic lights in order to achieve the lowest total delays in passing cars. They employed the robust optimization approach proposed by Bertsimas and Sim for a signal optimization problem with emission consideration to minimize expected vehicle delays and maximize network throughput. There is a relationship between the aggregate emission rate and the vehicle occupancy on the same link. This relationship is approximated with certain functional forms and the associated uncertainties in approximation errors are handled by using robust optimization techniques.

Bianchin and Pasqualetti [47] studied the problem of optimizing the traffic network overall efficiency by controlling the signalized intersections to optimize vehicle evacuation by designing and controlling the durations of green lights at intersections under congestion conditions.

Liu *et al.* [48] presented a model-based and switching-based control formulation for multi-intersection and multi-phase traffic signal operation. Based on their model, appropriate adaptive dynamic programming methods were used to seek the optimal traffic light policy. They presented a macroscopic traffic flow modeling approach which is useful for development of a model-based and switching-based optimization.

Wu *et al.* [49] proposed a distributed event-triggered strategy for traffic light control in urban traffic networks. In their work, the system is capable to update control signals when triggered by designing the event conditions which are verified by each agent.

In this research the problem of sensor location is considered. Review of the abovementioned literature shows that movement of sensors has less been paid attention in traffic area. While movement of sensors causes less error rate resulted from traffic congestion. Uncertainty is another important parameter which is considered in some papers and in most cases it is dealt with stochastic programming. Some related papers employed robust optimization approach but for a single objective model. In the current research a multi objective model is proposed with uncertain parameters and a multi-objective robust optimization approach is employed to simultaneously deal with multi-objective and uncertainty nature of the problem.

Considering the research gap, the following questions are going to be addressed in this paper.

- Would errors be reduced by moving the sensors to the traffic area?
- Do the sensors move to the traffic area at the time of traffic congestion based on the proposed model?
- What is the suitable mathematical model to locate sensors along highway to minimize reported errors?
- How can uncertain parameters be considered and examined in the proposed multi-objective model?
- Are the results of the robust model less sensitive to uncertainty comparing to the deterministic model

III. PROBLEM DEFINITION AND FORMULATION

The studying problem of this paper is to locate traffic sensors in highways to measure speed of cars and use it to control traffic flows. Sensors are located along highway and are paired to each other to cover highway path (corridor). Each pair reports cars' speeds between two sensors and this report contains error. The main goal of this research is minimizing the total error reported by sensors which is obtained by two methods. The first method is by placing sensors in suitable nodes and connecting them so that they report the minimum error and the second method is to move some sensors into the traffic area to reduce the error rate in traffic congestion time. It is necessary to initially define the error reported by sensors. As it is shown in Fig.1 there are some nodes along the highway which can be selected to locate sensors. In Fig.1 there are six nodes two of which have sensors. When a car traverses a sensor, its speed is reported by that sensor. The average car's speed between two paired sensors is obtained by calculating the average of two speeds reported by these two sensors. This reported speed is a little different from the real speed which is calculated by the speed formula

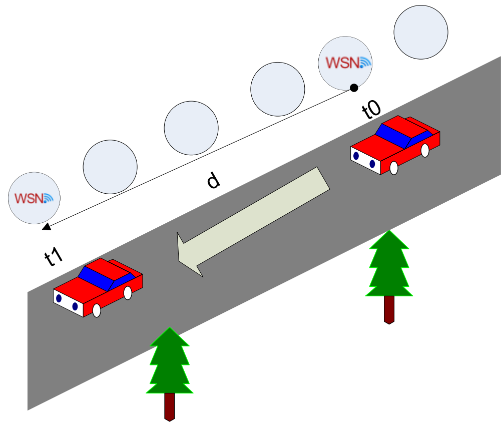

FIGURE 1. Speed of a car reported by two sensors.

which is equal to the distance traveled divided by the traverse time. For example, in Fig.1 a car traverses distance (d) in time interval [t_0, t_1]. Therefore, the real speed is equal to $\frac{d}{t_1-t_0}$. In this research, the difference between the real speed and reported speed by two connected sensors is called "error" which we try to minimize it by connecting sensors in the best way.

Since the traffic behavior of routes in different periods of time can change, it is necessary to consider different time periods in modelling and propose a dynamic model. When traffic congestion occurs in a part of route, the reported error by paired sensors around that area increases. Therefore, this study suggests moving one or more sensors into the traffic area to reduce the error rate. In fact, by movement of some sensors and relocate them into suitable nodes of traffic area, the error decreases. Considering movement of sensors makes a dynamic model by which error resulted from traffic in each time period, decreases.

In Fig.2 sensors are connected according to sequence 1-2-3-6-8, respectively. As it is shown in Fig. 2 there are eight nodes for allocating sensors five of which are chosen to have sensors. The nodes are selected to have sensors and the sensors are paired so that the total reported error is minimized.

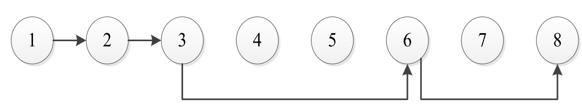

FIGURE 2. Connection of nodes before traffic congestion.

Then traffic congestion happens in nodes 4 and 5. Therefore, the error reported by the pair of sensors located in nodes 3 and 6 increases and it is necessary to move some sensors to the traffic area.

When traffic congestion occurs we face with a new location problem and must decide which sensors should move to the traffic area to reduce error rate, and which node in traffic area should be selected to yield maximum reduction in error rate which is added as a result of traffic congestion happening.

It should be noted that the energy of each sensor is stored by solar panels. If a sensor does not have enough energy to move into the traffic area at the moment of traffic congestion, other sensors located in proper distances with enough energy will be selected.

By moving sensors to the traffic area, the amount of error will reduce owing to the new location and new configuration of sensors.

In this research, three objective functions are considered as follows:

1. The first objective function seeks to minimize the overall error reported by paired sensor
2. The second objective function attempts to maximize the reduction of errors by moving sensors into the traffic congestion are
3. The third objective function seeks to maximize the total benefit resulted from detecting bottlenecks

The optimal decision variables of the first objective function give a configuration which is not changeable during the traffic period. While traffic congestion happens the second objective function is suggested to reduce the error rate by moving some sensors into the traffic congestion area. The proposed model not only seeks to find the optimal configuration of sensors for minimizing the error rate but also to find bottlenecks by this configuration at the shortest time.

To solve the proposed multi-objective model, the present study employs the ε-constraint method. And as the errors reported by the sensors are uncertain values, this research proposes a robust optimization approach to deal with this uncertainty.

At first the proposed model is formulated with certain parameters, then uncertainty is considered in parameters and finally a robust optimization approach is presented to deal with uncertainty.

A. DETERMINISTIC MODEL

Indices

i, j, k: Sensor nodes

t: Time periods

i': Start node

j': End node

Parameters

E_{ijt}: The error reported by two sensors located in nodes i and j at time t

a_{ijt}: The amount of decrease in total error by moving the sensor from node i to node j at time t

b_{ijt}: The benefit of detecting the bottleneck by two sensors located in nodes i and j at time t

R: maximum number of sensors allotted to model

l_{ij}: The distance between node i and node j

L: The maximum distance that a sensor can move

T: Total number of time periods in a day

N: Total number of nodes

Variables

x_{ijt}: A binary variable which takes 1 when two sensors in nodes i and j are connected at time t

y_{it}: A binary variable which takes 1 when a sensor is located in node i at time t

z_{ijt}: A binary variable which takes 1 when the sensor moves from node i to node j at time t

Mathematical model

$$\text{Min} \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} E_{ijt} \times x_{ijt} \tag{1}$$

$$\text{Max} \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} a_{ijt} \times z_{ijt} \tag{2}$$

$$\text{Max} \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} b_{ijt} \times x_{ijt} \tag{3}$$

$$\text{Subject to} \sum_{j=i+1}^{j'} x_{ijt} - y_{it} = 0 \quad \forall i, t \tag{4}$$

$$\sum_{i=i'}^{j} x_{ijt} = \sum_{k=j}^{j'} x_{jkt} \quad \forall j, t \tag{5}$$

$$\sum_{i=i'}^{N} x_{ij't} = 1 \quad \forall t \tag{6}$$

$$\sum_{j=1}^{j'} x_{i'jt} = 1 \quad \forall t \tag{7}$$

$$\sum_{j=1}^{N} z_{ijt} \le y_{it-1} \quad \forall i, t \tag{8}$$

$$\sum_{i=1}^{N} z_{ijt} \le y_{jt} \quad \forall j, t \tag{9}$$

$$\sum_{j=1}^{N} z_{ijt} \times y_{it} = 0 \quad \forall i, t \tag{10}$$

$$\sum_{i=1}^{N} z_{ijt} \le 1 \quad \forall j, t \tag{11}$$

$$\sum_{j=1}^{N} z_{ijt} \le 1 \quad \forall i, t \tag{12}$$

$$\sum_{j=1}^{N} y_{jt} \le R \quad \forall t \tag{13}$$

$$x_{ijt} = 0 \quad \forall i \ge j, \forall t \tag{14}$$

$$z_{ijt} = 0 \quad \forall i = j, \quad \forall t \tag{15}$$

$$z_{ijt} \le LF_{ij} \times EF_{ijt} \quad \forall i, j, t \tag{16}$$

$$z_{ijt} \in \{0, 1\} \quad \forall i, j, t \tag{17}$$

$$x_{ijt} \in \{0, 1\} \quad \forall i, j, t \tag{18}$$

$$y_{it} \in \{0, 1\} \quad \forall i, t \tag{19}$$

The first objective function defined in (1) seeks to minimize the overall error with a given configuration of sensors. The second objective function defined in (2) maximizes the overall reduction in total error by moving sensors into the traffic area. The third objective function defined in (3) maximizes the total benefit resulted from bottlenecks detection, based on allocation of sensors [50]. In the third objective function a parameter (b_{ij}) is defined as a benefit of detecting bottlenecks. This benefit takes greater values if bottlenecks are detected in closer nodes (see denominator of Eq. (20)).

$$b_{ij} = \max(0, \frac{\sum_{t=0}^{T}(v_j^t - v_i^t)}{s_j - s_i}) \tag{20}$$

In (20) v_j^t shows the average speed of cars reported by sensor in node j at time t, and $s_j - s_i$ is the distance between node i and node j. Relation (20) shows when the difference between reported speeds by two sensors in nodes i and j is greater, there is a bottleneck between them and it is better to locate two sensors in nodes i and j to reduce total error.

Constraint (4) ensures that at time t, sensors located in nodes i and j can be connected as long as a sensor is allocated to i. Constraint (5) declares that at time t, if a link is available between node j and one of the previous nodes, another link will be available between node j and one of the next nodes. Constraint (6) guarantees that one link starts at the corridor's start node (i') while constraint (7) guarantees that one link terminates at the corridor's end node (j'). According to (8) a sensor can move from node i to node j at time t (i.e. $z_{ijt} = 1$), if and only if a sensor be available in node i at time (t-1). Constraint (9) states if a sensor moves to j from one of the other nodes, node j has a sensor at time t. Constraint (10) ensures if a sensor moves from i to j at time t (i.e. $z_{ijt} = 1$) there won't be any sensor at node i. Constraint (11) states that in each period only one sensor can move to node j and (12) declares that the sensor located in node i can only move to one of the other nodes in each period. Constraint (13) limits the number of sensors to R. Constraint (14) allows for only links traveling in downstream direction to receive coverage by setting all other possible links to a value of zero. Constraint (15) prevents a movement from a node to itself. Constraint (16) forces two conditions of sufficient energy and proper distance on movement. In this constraint EF_{ijt} and LF_{ij} are two parameters defined as follows.

EF_{ijt} is a parameter which takes value of 1 when the needed energy to move a sensor is enough as it is shown in (21).

$$EF_{ijt} = 1 \quad \text{if } U_{ijt} \le F_{it} \text{ and } EF_{ijt} = 0 \text{ otherwise} \tag{21}$$

where, F_{it} represents the amount of energy stored in node i at the beginning of the period t and U_{ijt} represents the energy needed to move the sensor from node i to node j at time t. The energy is stored by solar panels and used to move sensors to the traffic congestion areas. When a sensor is selected to move, it must have enough energy needed for movement. This condition is shown in Eq. (22).

$$U_{ijt} \le F_{it} \tag{22}$$

Furthermore, LF_{ij} is another parameter, which takes value of 1 when the distance between two nodes (l_{ij}) is not greater than the maximum distance that a sensor can move (i.e. L) and this condition is shown in (23).

$$l_{ij} \le L \tag{23}$$

By defining the above mentioned parameters, the constraint (16) is defined, which denotes for moving sensors into the traffic congestion areas, enough energy and proper distance are necessary. Constraints (17)-(19) are structural constraints which declare z_{ijt}, x_{ijt}, y_{it} are binary variables.

IV. SOLUTION METHODS

A. ROBUST OPTIMIZATION APPROACH

As the values of parameters used in objective functions of this research are uncertain, two approaches can be considered to deal with this uncertainty. The first approach is the probabilistic approach that considers the mean of parameters and the second approach is the robust optimization approach which regards the worst value of parameters. In this research a robust optimization approach is employed to deal with uncertainty.

Soyster [51] presented the initial idea of robust optimization, considering the worst possible value of uncertain parameters. His proposed approach was too conservative. Later, Bertsimas and Sim [52] proposed a less conservative robust approach in which violations of constraints are considered in the objective function. The robust approach proposed by Bertsimas and Sim [52] allows us to determine the degree of conservatism by defining a protection parameter denoted by Γ. Keeping the model linear is another advantage of this approach. Therefore, the Bertsimas and Sim [52] robust approach is used in this research to deal with uncertainties of the objective functions' parameters, i.e. $\hat{E_{ijt}}$, $\hat{a_{ijt}}$ and $\hat{b_{ijt}}$, where

$$\tilde{E_{ijt}} \in [\hat{E_{ijt}} - E'_{ijt}, \hat{E_{ijt}} + E'_{ijt}]$$
$$\tilde{a_{ijt}} \in [\hat{a_{ijt}} - a'_{ijt}, \hat{a_{ijt}} + a'_{ijt}]$$
$$\tilde{b_{ijt}} \in [\hat{b_{ijt}} - b'_{ijt}, \hat{b_{ijt}} + b'_{ijt}]$$

Defining normalized perturbation vector ξ, results in $\tilde{E_{ijt}} = \hat{E_{ijt}} + \xi \times E'_{ijt}$, $\tilde{a_{ijt}} = \hat{a_{ijt}} + \xi \times a'_{ijt}$ and $\tilde{b_{ijt}} = \hat{b_{ijt}} + \xi \times b'_{ijt}$ where $\xi \in [-1, 1]$ which in this research is a box-polyhedral (budgeted) uncertainty set. Accordingly, the robust counterpart of the proposed model is formulated as follows.

$$\text{Min} \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} \hat{E_{ijt}} \times x_{ijt} + \omega_1 \times \Gamma_1 + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} P_{ijt}^1 \tag{24}$$

$$\text{Max} \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} \hat{a_{ijt}} \times z_{ijt} - \omega_2 \times \Gamma_2 - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} P_{ijt}^2 \tag{25}$$

$$\text{Max} \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} \hat{b_{ijt}} \times x_{ijt} - \omega_3 \times \Gamma_3 - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} P_{ijt}^3 \tag{26}$$

$$-W_{ijt}^{1} \leq z_{ijt} \leq W_{ijt}^{1} \quad \forall i, j, t \tag{27}$$

$$-W_{ijt}^{2} \leq x_{ijt} \leq W_{ijt}^{2} \quad \forall i, j, t \tag{28}$$

$$\omega_1 + P_{ijt}^{1} \geq E'_{ijt} \times W_{ijt}^{2} \tag{29}$$

$$\omega_2 + P_{ijt}^{2} \geq a'_{ijt} \times W_{ijt}^{1} \tag{30}$$

$$\omega_3 + P_{ijt}^{3} \geq b'_{ijt} \times W_{ijt}^{2} \tag{31}$$

$$\omega_1, \omega_2, \omega_3, P_{ijt}^{1}, P_{ijt}^{2}, P_{ijt}^{3}, W_{ijt}^{1}, W_{ijt}^{2} \geq 0 \tag{32}$$

Other constraints of the deterministic model, i.e. constraints (4) to (17), hold for this robust model.

In the constraint (29), E'_{ijt} denotes the amount of deviation from the nominal value of $\hat{E}_{ijt}$, and in the constraints (30) and (31), a'_{ijt} and $b'_{ijt}$are deviations from the nominal values of $\hat{a}_{ijt}$ and $\hat{b}_{ijt}$, respectively. As it was mentioned, Γ_k is a parameter that controls the level of conservatism of objective function k. If Γ_k takes 0, the values of the parameters are nominal values and the robust model is the same as the deterministic model. By increasing the amount of Γ_k the model becomes more conservative and results will be close to the Soyster approach's results. Finally, ω_k and p_{ijt}^{k} are positive variables of the dual problem of the protection function of objective function k. Other variables and parameters are the same as the deterministic model.

B. ε-CONSTRAINT METHOD

In this research the ε-constraint method is used to solve the three objective functions simultaneously [54]. Since in this method one of the objective functions must be considered as the main objective function, in this research the first objective function which minimizes overall error is regarded as the main objective function and two other objective functions are added to the model as constraints according to (33).

$$\begin{aligned}
\text{Min: } & \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} E_{ijt} \times x_{ijt} \\
\text{St: } & \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} a_{ijt} \times z_{ijt} \geq \varepsilon_2 \\
& \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} b_{ijt} \times x_{ijt} \geq \varepsilon_3 \\
& \text{Other constraints hold}
\end{aligned} \tag{33}$$

Initially, a payoff table must be constructed as follows. At first, the individual optimal value of each objective function must be calculated and written in the main diagonal of the payoff table. As the proposed model of this paper includes three objective functions, the payoff table has three rows and columns. Next, to calculate off-diagonal values of each row, the diagonal value which is the optimal value of the corresponding objective function is considered as a constraint and accordingly the values of other functions are calculated. The jth column of the payoff table includes the obtained values for the objective function fj among which the minimum and maximum values indicate the range of the objective function f_j for the ε-constraint method.

The best value of each column, which corresponds to objective function k, is called Utopia point (f_k^u) and the worst value is called Nadir point (f_k^n) and the difference between these two values represents the range. Then, the value of ε_k for objective function k is calculated by the Equation (34).

$$\varepsilon_k = f_k^n + \frac{(f_k^u - f_k^n)}{6} \times n_k \quad k = 2, 3 \tag{34}$$

The calculated values of ε_2 and ε_3 are inserted into model (33). For different values of ε_2 and ε_3, model (33) is solved and the Pareto solutions are obtained in which dominated solutions are eliminated.

C. MULTI OBJECTIVE ROBUST OPTIMIZATION

In order to calculate the multi-objective robust solutions, multi-objective robust model (35) along with other constraints of the proposed robust model i.e. (27)-(32) and also constraints (4)-(17), are solved.

$$\begin{aligned}
\text{Min } & \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} \hat{E_{ijt}} \times x_{ijt} + \omega_1 \times \Gamma_1 + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} P_{ijt}^{1} \\
\text{St: } & \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} \hat{a_{ijt}} \times z_{ijt} - \omega_2 \times \Gamma_2 - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} P_{ijt}^{2} \geq \varepsilon_2 \\
& \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} \hat{b_{ijt}} \times x_{ijt} - \omega_3 \times \Gamma_3 - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} P_{ijt}^{3} \geq \varepsilon_3 \\
& \text{Other constraints hold}
\end{aligned} \tag{35}$$

V. NUMERICAL RESULT

In this section, the performance of the proposed deterministic and robust models for traffic sensors location is investigated and all the optimal solutions are obtained by COUENNE solver of GAMS software. For this purpose, a highway with 10 nodes is considered for which 5 sensors are available to be allocated to suitable nodes. It is assumed that there are three traffic periods and the traffic congestion happens in second period, in nodes 4, 5 and 6.

A. RESULTS OF THE DETERMINISTIC MODEL

To solve the proposed multi-objective deterministic model, the ε-constraint method, which was fully described in the previous section, is applied. Initially, the payoff table is calculated and presented in Table 2.

TABLE 2. Payoff table.

First objective function	Second objective function	Third objective function
45	0	60
85	51	66
125	17	73

Then, for each range the values of ε_1, ε_2 and ε_3 are calculated by Equations (34) and presented in Table 3.

TABLE 3. ε **values for deterministic model.**

N	ε_1	ε_2	ε_3
0	125	0	60
1	111.67	8.5	62.17
2	98.33	17	64.33
3	85	25.5	66.5
4	71.67	34	68.67
5	58.33	42.5	70.83
6	45	51	73

Afterwards, for different values of ε_2 and ε_3, model (33) is solved and the results are presented in Table 4. For example, in the first row of Table 4 for $\varepsilon_2 = 0$ and $\varepsilon_3 = 60$, the values of the first, second, and third objective functions are 45, 0 and 60, respectively.

In Fig. 3, all the solutions of the deterministic model are drawn by MATLAB software.

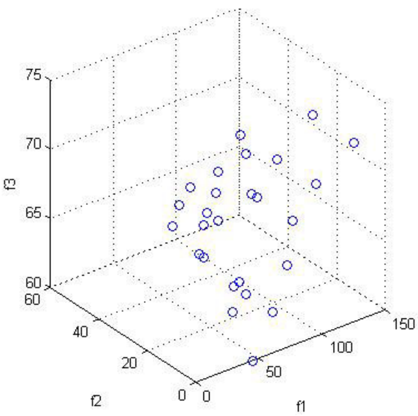

FIGURE 3. **Solutions of the deterministic model.**

Among these solutions, there are a number of dominated solutions that must be deleted in order to have only non-dominated solutions which are shown in Table 5.

In order to help decision makers to choose a suitable solution, LP metric method is applied to nondominated results [54]. For this purpose, all the objective functions are converted into the minimization form by multiplying the second and third objectives' results by (-1). The objective functions are normalized through relation (36), where, $f_k^{\min}$ and $f_k^{\max}$ are the minimum and maximum values for objective function f_k.

$$\frac{f_k - f_k^{\min}}{f_k^{\max} - f_k^{\min}} \tag{36}$$

Table 6 shows the results of Table 5 after normalization. In the last column of Table 6, the values obtained by using the LP metric method for $p = 2$ are shown which are achieved through relation (37).

$$\sqrt{f_1^2 + f_2^2 + f_3^2} \tag{37}$$

The results show that solution 12 is the best solution with the lowest L_2 norm value equal to 0.639. For this solution

TABLE 4. ε**-constraints results for deterministic model.**

	First objective function	Second objective function	Third objective function
1	45	0	60
2	61	11	63
3	62	17	61
4	68	32	63
5	68	34	63
6	76	49	63
7	85	51	64
8	61	0	63
9	61	16	63
10	67	17	63
11	68	34	63
12	68	34	63
13	76	49	63
14	85	51	64
15	72	0	66
16	72	28	66
17	72	33	66
18	72	34	65
19	72	34	65
20	94	51	65
21	94	51	65
22	Infeasible		
23	77	17	69
24	77	31	69
25	77	31	69
26	81	34	69
27	Infeasible		
28	Infeasible		
29	77	0	69
30	77	15	69
31	77	17	69
32	77	31	69
33	105	34	69
34	Infeasible		
35	Infeasible		
36	95	0	71
37	95	16	71
38	95	31	71
39	95	31	71
40	Infeasible		
41	Infeasible		
42	Infeasible		
43	125	0	73
44	125	17	73
45	125	17	73
46	Infeasible		
47	Infeasible		
48	Infeasible		
49	Infeasible		

and for the first period (before traffic congestion) sensors are connected according to Figure 4, that is $x_{1-2} = x_{2-3} = x_{3-8} = x_{8-10} = 1$.

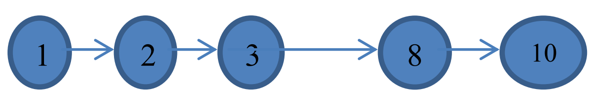

FIGURE 4. **Sensors arrangement before traffic congestion period (period 1) resulted from the proposed deterministic model.**

In the second period when the traffic congestion in nodes 4, 5 and 6 occurs, the sensors start to move according to Fig. 5.

TABLE 5. Non-dominated solutions for deterministic model.

	First objective function	Second objective function	Third objective function
1	45	0	60
2	62	17	61
3	61	16	63
4	67	17	63
5	68	34	63
6	76	49	63
7	85	51	64
8	72	33	66
9	72	34	65
10	94	51	65
11	81	34	67
12	77	31	69
13	105	34	69
14	95	31	71
15	125	17	73

TABLE 6. Results of using LP metric method for deterministic model.

	Normalized first objective function	Normalized second objective function	Normalized third objective function	LP metric results
1	0	1	1	1.414
2	0.2125	0.667	0.923	1.158
3	0.2	0.6862	0.769	1.05
4	0.275	0.667	0.769	1.054
5	0.2875	0.333	0.769	0.886
6	0.3875	0.039	0.769	0.862
7	0.5	0	0.692	0.853
8	0.3375	0.353	0.538	0.727
9	0.3375	0.333	0.615	0.777
10	0.6125	0	0.615	0.868
11	0.45	0.333	0.461	0.726
12	0.4	0.392	0.308	0.639
13	0.75	0.333	0.308	0.876
14	0.625	0.392	0.154	0.754
15	1	0.667	0	1.202

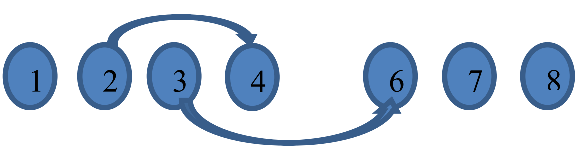

FIGURE 5. Movement of sensors in traffic congestion time (period 2) resulted from the proposed deterministic model.

The sensor located in node 2 moves to node 4 ($z_{2-4} = 1$), and the sensor in node 3 moves to node 6 ($z_{3-6} = 1$).

Therefore, in the traffic congestion period, sensors are connected according to sequence 1-4-6-8-10 respectively.

B. RESULTS OF THE ROBUST MODEL

To solve the multi-objective robust model, first the robustification method proposed by Bertsimas and Sim [52] is employed and model (35) is obtained. Then the ε-constraints method is applied to solve the resulted multi-objective robust model. By setting the Γ value equal to zero ($\Gamma = 0$), the results are the same as the results of the deterministic model. This verifies the proposed multi-objective robust model. By increasing Γ, conservatism level increases as well. The resulted non-dominated solutions are presented in Table 7.

TABLE 7. Non-dominated solutions for robust model.

	First objective function	Second objective function	Third objective function
1	60	13.013	44.04
2	79	17.5	44
3	80	22.031	44.063
4	80	31	44
5	91	40.02	41.4
6	91	40	44

To decide about a suitable robust solution for sensor location problem, LP metric method is applied and results are presented in Table 8.

TABLE 8. Results of using LP metric method for robust model.

	First objective function	Second objective function	Third objective function	LP metric results
1	0	1	0.008	1
2	0.613	0.834	0.02	1.035
3	0.645	0.666	0	0.927
4	0.645	0.334	0.02	0.727
5	1	0	1	1.414
6	1	0.0007	0.02	1.0002

As it is shown, solution 4 is the best solution with the lowest L_2 norm value (0.727). For this solution the first, second and third objective functions have 80, 31 and 44 values, respectively. For this solution at the first period (before traffic congestion), sensors are connected according to sequence 1-2-3-8-10, respectively. As traffic congestion happens at the second period some sensors move into the traffic congestion area according to Fig. 6.

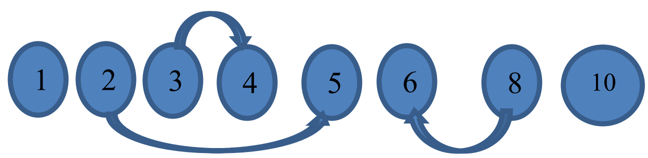

FIGURE 6. Movement of sensors in traffic congestion period resulted from the proposed robust model.

The sensor in node 2 moves to node 5, sensor in node 3 moves to node 4, and sensor in node 8 moves to node 6. Therefore, the sensors are connected according to sequence 1-4-5-6-10.

Table 9 summarizes the results and compares both deterministic and robust models with and without movement. In general before traffic the measurement error is lower than after traffic and movement is applicable after traffic. To test the effect of movement, first the single objective model with minimizing measurement error as objective is run once without considering movement (row 2) and once with considering

TABLE 9. Comparing models with and without movement.

Model	Measurement error (f_1)	Error reduction (f_2)	Benefits (f_3)	Running time (In seconds)
Single objective Deterministic model (without movement)	Before Traffic= 13 After Traffic= 96	-	-	0.5
Single objective Deterministic model (with movement)	45	51	73	17
The proposed multi-objective deterministic model (with movement)	77	31	69	20
Robust single objective model (without movement)	Before Traffic=16 After Traffic=131	-	-	1
The proposed multi-objective robust model (with movement)	91	40	44	101

movement (row 3). As can be seen, movement of sensors after traffic decreases measurement error. This result is also true for multi-objective model with movement (row 4). However, for multi-objective model the measurement error is more than the single objective model (see row 4) due to making a compromise amongst objectives. In both cases, i.e. models with and without movements, the objective functions of the robust model is not better than those of the deterministic model due to protection functions considered in the objective functions of the robust model (compare rows 4 and 6).

Considering movement of sensors, to compare the effect of having multiple objectives simultaneously, the single objective model with minimization of error is run and the resulted solution is put into second and third objective functions. As shown in rows 3 and 4, in the multi-objective model the objectives become worse due to making a compromise amongst objectives.

C. VALIDATION OF THE PROPOSED ROBUST MODEL

In this paper, Monte Carlo simulation method is used to generate random realization of uncertain parameters and test validation of the proposed robust model. Since the probability distribution of parameters is unknown, 1000 random numbers with two normal and uniform distributions are generated for the uncertain parameters.

For example for parameter $\tilde{E_{ijt}} \in [\hat{E_{ijt}} - E'_{ijt}, \hat{E_{ijt}} + E'_{ijt}]$, first it is supposed that $\tilde{E_{ijt}}$ is uniformly distributed in the considered interval. Therefore, considering lower and upper bounds of interval, uniform random numbers are generated. In case of uniform distribution the expected value (EV) and variance (Var) are equal to $EV(\tilde{E_{ijt}}) = \hat{E_{ijt}}$ and $Var(\tilde{E_{ijt}}) = \frac{(2\times E'_{ijt})^2}{12} = \frac{(E'_{ijt})^2}{3}$, respectively. Then it is supposed that $\tilde{E_{ijt}}$ is normally distributed. Therefore, random numbers from normal distribution are generated with the same mean $EV(\tilde{E_{ijt}})$ and variance $Var(\tilde{E_{ijt}})$. The same processes are done for uncertain parameters $\tilde{a_{ijt}}$ and $\tilde{b_{ijt}}$.

Considering the decision variables values specified by the LP metric method for both multi-objective deterministic and robust models (shown in Table 10), and generated random values for $\tilde{E_{ijt}}$, $\tilde{a_{ijt}}$ and $\tilde{b_{ijt}}$, the objective functions of both deterministic and robust models are calculated.

TABLE 10. Optimal values of decision variables for robust and deterministic models.

	$X_{ijt} = 1$	$Z_{ijt} = 1$
Deterministic model	$X_{1\text{-}2\text{-}1}, X_{2\text{-}3\text{-}1}, X_{3\text{-}8\text{-}1}, X_{8\text{-}10\text{-}1}$	$Z_{2\text{-}4\text{-}2}, Z_{3\text{-}6\text{-}2}$
Robust model	$X_{1\text{-}2\text{-}1}, X_{2\text{-}3\text{-}1}, X_{3\text{-}8\text{-}1}, X_{8\text{-}10\text{-}1}$	$Z_{2\text{-}5\text{-}2}, Z_{3\text{-}4\text{-}2}, Z_{8\text{-}6\text{-}2}$

($Z_{ijt} = 1$, means sensor at node i moves to node j at time t, and $X_{ijt} = 1$declares two sensors at nodes i and j at time t are connected)

After generating a thousand values for uncertain parameters of the objective functions in both robust and deterministic models by two uniform and normal distributions, the mean and standard deviation of these objective functions are calculated and shown in Table 11 and Table 12.

TABLE 11. Mean of objective functions in both robust and deterministic models.

Model	Distribution function	First objective function	second objective function	third objective function
Deterministic	Uniform	77.1	30.9	69.22
	Normal	77.27	31.1	69.1
Robust	Uniform	76.1	46.85	62.86
	Normal	76.2	47.17	62.78

TABLE 12. Standard deviation of objective functions in both robust and deterministic models.

Model	Distribution function	First objective function	second objective function	third objective function	Summation of standard deviations
Deterministic	Uniform	3.5	4.55	5.69	13.74
	Normal	8.5	14.36	14.38	37.24
Robust	Uniform	3.8	4.54	4.52	12.86
	Normal	8.89	13.78	8.17	30.84

As shown in Table 12, the summation of the standard deviations in the robust model is less than the summation of standard deviation in the deterministic model for both normal and uniform distributions. These results validate the proposed robust model which has less standard deviation for any realization of uncertain parameters.

VI. CONCLUSION

In this paper, a model was proposed to find the optimal location of sensors along highways to minimize the total error reported by them and also to find the best way of connecting sensors to find bottlenecks. As the error rate increases when traffic congestion happens, this research suggests movement of sensors with enough energy and locating in a proper distance from traffic congestion areas to reduce the total errors. Since the parameters of the objective functions of the proposed model are not known with certainty, a robust approach was used to deal with this uncertainty. The robust approach proposed by Bertsimas and Sim [52] was used in this paper because the level of conservatism can be adjusted by this approach. To test the validity of the proposed robust model, the Mont Carlo simulation was used. The results showed the total deviations in the robust model are less than the total deviations in the deterministic model and validity of the robust model is proved.

For the future study, the idea of sensor movement can be expanded to the case that one of the sensors is broken, and another sensor can move to replace with the broken sensor so that the error caused by the broken sensor is reduced. In some special situations such as earthquake, flood, accidents, road icing or road damage, the effect of movement of sensors on traffic flow can be investigated too. Furthermore, in this study, the coverage of the area where the sensor is located is assumed to be complete, i.e. 100% which is not compatible with many real world situations. In fact, the coverage follows a probability distribution which can also be considered in the model. Finally, in this research, we used an uncertainty set, i.e. box-polyhedral set, for uncertain parameters. In some cases, one can consider some scenarios for uncertain parameters and apply a scenario-based robust optimization approach for sensor location problem.

REFERENCES

[1] C. Pasquale, S. Sacone, S. Siri, and A. Ferrara, "Traffic control for freeway networks with sustainability-related objectives: Review and future challenges," *Annu. Rev. Control*, vol. 48, pp. 312–324, 2019.

[2] K. Nellore and G. Hancke, "A survey on urban traffic management system using wireless sensor networks," *Sensors*, vol. 16, no. 2, p. 157, Jan. 2016.

[3] B. Moghadas poor, F. Sabouhi, A. Bozorgi-Amiri, and M. S. Jbalameli, "Location-allocation of traffic police patrols in the suburban network," *Iranian J. Oper. Res.*, vol. 10, no. 1, pp. 103–115, Jul. 2019.

[4] H. Al-Harthei, O. B. Ali, and A. Garib, "Real-time traffic patrol allocation for Abu Dhabi emirate (UAE)," *J. Traffic Logistics Eng.*, vol. 1, no. 1, pp. 64–68, 2013.

[5] J. Guerreo-Ibanez, S. Zeadally, and J. Contreras-Castillo, "Sensor technologies for intelligent transportation systems," *Sensors*, vol. 18, no. 4, pp. 1–24, Apr. 2018.

[6] M. Bernas, B. Placzek, W. Korski, P. Loska, and J. Smyla, "A survey and comparison of low-cost sensing technologies for road traffic monitoring," *Sensors*, vol. 18, no. 10, pp. 1–24, Sep. 2018.

[7] W.-H. Liao, S.-C. Kuai, and M.-S. Lin, "An energy-efficient sensor deployment scheme for wireless sensor networks using ant colony optimization algorithm," *Wireless Pers. Commun.*, vol. 82, no. 4, pp. 2135–2153, Feb. 2015.

[8] S. M. Mohamed, H. S. Hamza, and I. A. Saroit, "Coverage in mobile wireless sensor networks (M-WSN): A survey," *Comput. Commun.*, vol. 110, pp. 133–150, Sep. 2017.

[9] K. Dantu, M. Rahimi, H. Shah, S. Babel, A. Dhariwal, and G. S. Sukhatme, "Robomote: Enabling mobility in sensor networks," in *Proc. 4th Int. Symp. Inf. Process. Sensor Netw. (IPSN)*, Boise, ID, USA, 2005, pp. 404–409.

[10] I. Banerjee, P. Chanak, B. K. Sikdar, and H. Rahaman, "EERIH: Energy efficient routing via information highway in sensor network," in *Proc. Int. Conf. Emerg. Trends Electr. Comput. Technol.*, Nagercoil, India, Mar. 2011, pp. 1057–1062.

[11] K. Tarnaris, I. Preka, D. Kandris, and A. Alexandridis, "Coverage and K-coverage optimization in wireless sensor networks using computational intelligence methods: A comparative Study," *Electronics*, vol. 9, no. 4, pp. 1–24, Apr. 2020.

[12] V. Chandankede, "Monitoring and maintenance of highway bridges using wireless sensor networks," *Int. Res. J. Power Energy Eng.*, vol. 3, no. 2, pp. 87–91, Nov. 2017.

[13] L. Po, F. Rollo, C. Bachechi, and A. Corni, "From sensors data to urban traffic flow analysis," in *Proc. IEEE Int. Smart Cities Conf. (ISC)*, Casablanca, Morocco, Oct. 2019, pp. 478–485.

[14] M. Gentili and P. B. Mirchandani, "Locating sensors on traffic networks: Models, challenges and research opportunities," *Transp. Res. C, Emerg. Technol.*, vol. 24, pp. 227–255, Oct. 2012.

[15] S. V. Ukkusuri, G. Ramadurai, and G. Patil, "A robust transportation signal control problem accounting for traffic dynamics," *Comput. Oper. Res.*, vol. 37, no. 5, pp. 869–879, May 2010.

[16] J. Lee and H. J. Miller, "Robust accessibility: Measuring accessibility based on travelers' heterogeneous strategies for managing travel time uncertainty," *J. Transp. Geography*, vol. 86, Jun. 2020, Art. no. 102747.

[17] Z. H. Khattak and M. D. Fontaine, "A Bayesian modeling framework for crash severity effects of active traffic management systems," *Accident Anal. Prevention*, vol. 145, Sep. 2020, Art. no. 105544.

[18] J. Guo and B. M. Williams, "Real-time short-term traffic speed level forecasting and uncertainty quantification using layered Kalman filters," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2175, no. 1, pp. 28–37, Jan. 2010.

[19] Y. Tong, L. Zhao, L. Li, and Y. Zhang, "Stochastic programming model for oversaturated intersection signal timing," *Transp. Res. C, Emerg. Technol.*, vol. 58, pp. 474–486, Sep. 2015.

[20] N. Zhu, C. Fu, and S. Ma, "Data-driven distributionally robust optimization approach for reliable travel-time-information-gain-oriented traffic sensor location model," *Transp. Res. B, Methodol.*, vol. 113, pp. 91–120, Jul. 2018.

[21] F. Viani, E. Giarola, F. Robol, G. Oliveri, and A. Massa, "Distributed monitoring for energy consumption optimization in smart buildings," in *Proc. IEEE Conf. Antenna Meas. Appl. (CAMA)*, Antibes Juan-les-Pins, France, Nov. 2014, pp. 1–3.

[22] S. Yoon, W. Ye, J. Heidemann, B. Littlefield, and C. Shahabi, "SWATS: Wireless sensor networks for steamflood and waterflood pipeline monitoring," *IEEE Netw.*, vol. 25, no. 1, pp. 50–56, Jan. 2011.

[23] X. Mao, X. Miao, Y. He, X.-Y. Li, and Y. Liu, "CitySee: Urban CO_2 monitoring with sensors," in *Proc. IEEE INFOCOM*, Orlando, FL, USA, Mar. 2012, pp. 1611–1619.

[24] S. S. Dhillon and K. Chakrabarty, "Sensor placement for effective coverage and surveillance in distributed sensor networks," in *Proc. IEEE Wireless Commun. Netw. (WCNC)*, New Orleans, LA, USA, Mar. 2003, pp. 1609–1614.

[25] B. Liu, P. Brass, and O. Dousse, "Mobility improves coverage of sensor networks," in *Proc. Mobihoc*, vol. 5, May 2005, pp. 300–308.

[26] B. Liu, O. Dousse, P. Nain, and D. Towsley, "Dynamic coverage of mobile sensor networks," *IEEE Trans. Parallel Distrib. Syst.*, vol. 24, no. 2, pp. 301–311, Feb. 2013.

[27] X. Wang, S. Wang, and J.-J. Ma, "An improved co-evolutionary particle swarm optimization for wireless sensor networks with dynamic deployment," *Sensors*, vol. 7, no. 3, pp. 354–370, Mar. 2007.

[28] C. Ozturk, D. Karaboga, and B. Gorkemli, "Probabilistic dynamic deployment of wireless sensor networks by artificial bee colony algorithm," *Sensors*, vol. 11, no. 6, pp. 6056–6065, Jun. 2011.

[29] R. Falcon, X. Li, A. Nayak, and I. Stojmenovic, "A harmony-seeking firefly swarm to the periodic replacement of damaged sensors by a team of mobile robots," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Ottawa, ON, Canada, Jun. 2012, pp. 4914–4918.
[30] M. Gentili and P. B. Mirchandani, "Review of optimal sensor location models for travel time estimation," *Transp. Res. C, Emerg. Technol.*, vol. 90, pp. 74–96, May 2018.
[31] D. R. Morrison and S. E. Martonosi, "Characteristics of optimal solutions to the sensor location problem," *Ann. Oper. Res.*, vol. 226, no. 1, pp. 463–478, Jun. 2014.
[32] T. Geetla, R. Batta, A. Blatt, M. Flanigan, and K. Majka, "Optimal placement of omnidirectional sensors in a transportation network for effective emergency response and crash characterization," *Transp. Res. C, Emerg. Technol.*, vol. 45, pp. 64–82, Aug. 2014.
[33] X. Fei, H. S. Mahmassani, and P. Murray-Tuite, "Vehicular network sensor placement optimization under uncertainty," *Transp. Res. C, Emerg. Technol.*, vol. 29, pp. 14–31, Apr. 2013.
[34] A. Danczyk, X. Di, and H. X. Liu, "A probabilistic optimization model for allocating freeway sensors," *Transp. Res. C, Emerg. Technol.*, vol. 67, pp. 378–398, Jun. 2016.
[35] C. Fu, N. Zhu, and S. Ma, "A stochastic program approach for path reconstruction oriented sensor location model," *Transp. Res. B, Methodol.*, vol. 102, pp. 210–237, Aug. 2017.
[36] F. Zhan, X. Wan, Y. Cheng, and B. Ran, "Methods for multi-type sensor allocations along a freeway corridor," *IEEE Intell. Transp. Syst. Mag.*, vol. 10, no. 2, pp. 134–149, Apr. 2018.
[37] A. Olia, H. Abdelgawad, B. Abdulhai, and S. N. Razavi, "Optimizing the number and locations of freeway roadside equipment units for travel time estimation in a connected vehicle environment," *J. Intell. Transp. Syst.*, vol. 21, no. 4, pp. 296–309, Jul. 2017.
[38] H. Park and A. Haghani, "Optimal number and location of Bluetooth sensors considering stochastic travel time prediction," *Transp. Res. C, Emerg. Technol.*, vol. 55, pp. 203–216, Jun. 2015.
[39] M. Salari, L. Kattan, W. H. K. Lam, H. P. Lo, and M. A. Esfeh, "Optimization of traffic sensor location for complete link flow observability in traffic network considering sensor failure," *Transp. Res. B, Methodol.*, vol. 121, pp. 216–251, Mar. 2019.
[40] F. Losilla, A.-J. Garcia-Sanchez, F. Garcia-Sanchez, J. Garcia-Haro, and Z. J. Haas, "A comprehensive approach to WSN-based ITS applications: A survey," *Sensors*, vol. 11, no. 11, pp. 10220–10265, Oct. 2011.
[41] V. P. Gil Jiménez and M. J. Fernández-Getino García, "Simple design of wireless sensor networks for traffic jams avoidance," *J. Sensors*, vol. 2015, pp. 1–7, Jan. 2015.
[42] A. H. F. Chow and Y. Li, "Robust optimization of dynamic motorway traffic via ramp metering," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 3, pp. 1374–1380, Jun. 2014.
[43] V. Jha, N. Prakash, and A. K. Mohapatra, "Energy efficient model for recovery from multiple nodes failure in wireless sensor networks," *Wireless Pers. Commun.*, vol. 108, no. 3, pp. 1459–1479, May 2019.
[44] M. Mehrabipour, L. Hajibabai, and A. Hajbabaie, "A decomposition scheme for parallelization of system optimal dynamic traffic assignment on urban networks with multiple origins and destinations," *Comput.-Aided Civil Infrastruct. Eng.*, vol. 34, no. 10, pp. 915–931, Oct. 2019.
[45] F. Liberati, "Model predictive control of a road junction," *Smart Cities*, vol. 3, no. 3, pp. 806–817, Aug. 2020.
[46] K. Han, H. Liu, V. V. Gayah, T. L. Friesz, and T. Yao, "A robust optimization approach for dynamic traffic signal control with emission considerations," *Transp. Res. C, Emerg. Technol.*, vol. 70, pp. 3–26, Sep. 2016.
[47] G. Bianchin and F. Pasqualetti, "Gramian-based optimization for the analysis and control of traffic networks," *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 7, pp. 3013–3024, Jul. 2019.
[48] D. Liu, W. Yu, S. Baldi, J. Cao, and W. Huang, "A switching-based adaptive dynamic programming method to optimal traffic signaling," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 50, no. 11, pp. 4160–4170, Nov. 2020.
[49] N. Wu, D. Li, Y. Xi, and B. de Schutter, "Distributed event-triggered model predictive control for urban traffic lights," *IEEE Trans. Intell. Transp. Syst.*, early access, Mar. 23, 2020, doi: 10.1109/TITS.2020.2981381.
[50] H. X. Liu and A. Danczyk, "Optimal sensor locations for freeway bottleneck identification," *Comput.-Aided Civil Infrastruct. Eng.*, vol. 24, no. 8, pp. 535–550, Nov. 2009.
[51] A. L. Soyster, "Inexact linear programming with generalized resource sets," *Eur. J. Oper. Res.*, vol. 3, no. 4, pp. 316–321, Jul. 1979.
[52] D. Bertsimas and M. Sim, "The price of robustness," *Oper. Res.*, vol. 52, no. 1, pp. 35–53, Feb. 2004.
[53] J. Aghaei, N. Amjady, and H. A. Shayanfar, "Multi-objective electricity market clearing considering dynamic security by lexicographic optimization and augmented epsilon constraint method," *Appl. Soft Comput.*, vol. 11, no. 4, pp. 3846–3858, Jun. 2011.
[54] S. J. Sadjadi, A. A. Tofigh, and R. Soltani, "A new nonlinear multi-objective redundancy allocation model with the choice of redundancy strategy solved by compromise programming approach," *Int. J. Eng.*, vol. 27, no. 4, pp. 545–552, Apr. 2014.

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