Received November 28, 2020, accepted December 21, 2020, date of publication December 24, 2020,
date of current version January 5, 2021.

Digital Object Identifier 10.1109/ACCESS.2020.3047203

Scale-Adaptive Growing Neural Network Based on Distortion Error Stability and Its Application in Image Topological Feature Extraction

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This work was supported in part by the Natural Science Foundation of Zhejiang Province China under Grant LY21F030009, in part by the Key Research and Development Program of Zhejiang Province under Grant 2019C04018, in part by the National Natural Science Foundation of China under Grant 62073108, and in part by the application research of Zhejiang Province under Grant 2017C31004.

ABSTRACT Self-organizing neural networks are characterized by topology preservation, dynamic adaptation, clustering, and dimensionality reduction, which prompt their wide application in data mining, knowledge extraction, and image processing. However, existing self-organizing neural networks fail to automatically generate an output space that contains an appropriate number of neurons according to the data input. To address this problem, this paper proposes a growing neural gas (GNG) algorithm with adaptive output network scale, which is called scale-adaptive GNG (SA-GNG) algorithm. The learning process of SA-GNG is divided into two stages: growth and convergence. At the growth stage, distortion error stability is introduced to objectively judge the degree of approximation of the output network to the input space, so that SA-GNG can grow neurons on demand until significant improvement is no longer made for distortion errors. In the convergence stage, neurons are not allowed to be produced, and the similarity between the output network and the input data is improved through continuous learning. SA-GNG promises to autonomously generate an appropriate number of neurons according to the size of the input data, with no need of determining the total number of neurons to be generated in advance, thereby greatly improving its adaptability. As such, the algorithm is especially suitable for the application scenarios where the amount of the data to be input is unknown. The validity and feasibility of the algorithm proposed in this paper are verified by experiments.

INDEX TERMS Self-organizing neural network, feature mapping, topology preservation, image topological feature extraction.

I. INTRODUCTION

The topological structure of images that does not change with parameters such as image size and direction is considered to be an important feature applied to image analysis and recognition [1]. Extracting topological structure from image shape is a process of feature extraction, which means transforming high-dimensional features of pattern space into the low-dimensional features of feature space through mapping or transformation, that is, using fewer new features to describe samples. The self-organizing neural network algorithm is an unsupervised clustering method that can map high-dimensional input data with a low-dimensional output network, realizing dimensionality compression while maintaining topological invariance. Therefore, self-organizing neural networks are widely used in image processing, such as image compression [2], surface reconstruction [3], and topological feature extraction [4].

A self-organizing map (SOM) [5] is a typical self-organizing neural network that represents input space using output neurons with a fixed structure and draws on competitive learning to make the output network successfully approximate the high-dimensional input space. This suggests that the topological relationships input outside can be maintained while complex multi-dimensional relationships be expressed in a low-dimensional space. However, the network structure and number of nodes in a SOM are preset and unchangeable, which, therefore, limits the accuracy and flexibility of the mapping output, impeding the application of SOM to

The associate editor coordinating the review of this manuscript and approving it for publication was Mostafa Rahimi Azghadi.

scenarios where the pattern of input data is unknown. For this reason, many researchers furthered the research based on SOM and proposed a variety of self-organizing growth neural network models, in an attempt to overcome the limitations of SOMs [6], [7]. For growing self-organizing neural networks, there is usually no need to know the distribution of input space beforehand, because they can self-organizingly identify the distribution and topological features of the input data as the output network nodes continuously learn and adjust and the number of nodes grows, and map the features identified to the output space. GNG [8] is also based on the competitive learning mechanism, but its network scale can be increased or decreased in the learning process, so does the number of neurons in GNG when the model performs self-organization. Regarding GNG models, the growth mechanism is originated from the ''growing cell structures'' (GCS) proposed by Bernd Fritzke in 1994 [9], and the topological structure generation mechanism is from the ''competitive hebbian learning'' (CHL) proposed by Martintz and Shulten in 1991 [10]. Li *et al.* [11] proposed a novel self-adjusting feature map (SAM) which can automatically isolate a set of connected neurons. Prasad *et al.* [12] proposed a novel soft-boosted self-constructing neural fuzzy inference network which can reduce the error rate, and improve learning speed. In addition, incremental neural networks are widely applied in many fields, such as recognition, prediction, and knowledge mining [13], [14], and an array of models have been developed [15], [16] [17].

GNG can be used to generate a topological structure equivalent to the input space structure. In the learning process, neurons are inserted at regular intervals. After λ times of iteration, a new neuron is to be inserted between the neuron with the largest error and its nearest neighboring neuron until the number of neurons inserted reaches the preset maximum. When the number of neurons in the GNG network reaches the maximum, no more new neurons are generated, hence resulting in the problem of what's the appropriate number of neurons. When the number of neurons grows to the preset value, a large number of redundant neurons will be generated if the input space is too small; or otherwise, the features that delineate the input space may be lost due to the insufficient number of neurons, thereby affecting the accuracy of feature extraction. As such, the above-mentioned methods are inapplicable to the scenarios where the volume and structure of the input data are uncertain. Therefore, it becomes particularly important to develop a growing self-organizing neural network algorithm that can automatically generate a suitable network scale according to the input space, with the aim of enhancing its adaptability to the input space.

To solve the aforesaid problem, this paper brings forward the SA-GNG algorithm that consists of two stages, growth and convergence. In the growth stage, distortion error stability is used to objectively judge the degree of approximation of the output space to the input space. With this index, the SA-GNG algorithm can autonomously adapt to the scale of the input space and generate an appropriate number of neurons, with no need of presetting the scale of the output network. Once the network scale grows to a certain degree and meets the judgment index, the algorithm stops generating new network nodes and enters the convergence stage. In the convergence stage, SA-GNG does not create new neurons and instead, uplift the similarity of feature extraction through continuous learning. In these two stages, SA-GNG autonomously generates an appropriate number of neurons according to the volume of the input data, hence unnecessary to pre-set the total number of neurons. As such, SA-GNG can effectively and autonomously extract an output network that has an appropriate number of neurons from different images while maintaining topological features, so it is suitable for application scenarios where the scale and structure of input data are unknown.

The rest of the paper is organized as follows. Section II introduces the related concepts of GNG network. In Section III, the relationship between the stability of the distortion error and the similarity of the input-output network is introduced, and the distortion error stability index is proposed. In Section IV, based on the GNG algorithm, the SA-GNG algorithm is proposed. The algorithm can automatically generate appropriate output network size without pre-setting the network size. Experiments and discussions are given in section V. Finally, the conclusion is made in Section VI.

II. PROBLEM PROPOSED

Growing self-organizing neural networks usually do not need to know the distribution of input space in advance, and self-organizingly identify the distribution and topological features of input data, which are mapped into the output space. Nonetheless, the existing growing self-organizing neural network algorithms lack an effective and stable method to determine the growth scale of the output network, which makes them difficult to autonomously control node growth in a reasonable scale. For example, the GNG algorithm proposed by Fritzke is a typical growing self-organizing neural network algorithm, where the number of nodes in the output network is artificially preset. However, in actual use, it is generally difficult to set an appropriate total number of nodes beforehand due to the uncertainty of input space, so that manual trials have to be made to obtain an appropriate output network scale.

In some cases, there is little or no information about the input distribution or the size of the input data set, and in these cases, it is difficult to determine a priori the number of nodes to use. For example, in the application field of object surface reconstruction, the object surface is first scanned, and then reconstructed on the discrete point set. These sets of data points are complex, volatile and unpredictable. In addition, satellite cloud images are often complex, changeable and unpredictable, too. In these applications, it is difficult to set the appropriate number of neurons of the output network in advance. Since the size and shape of image data are often unknown in advance, it is necessary to have a growing

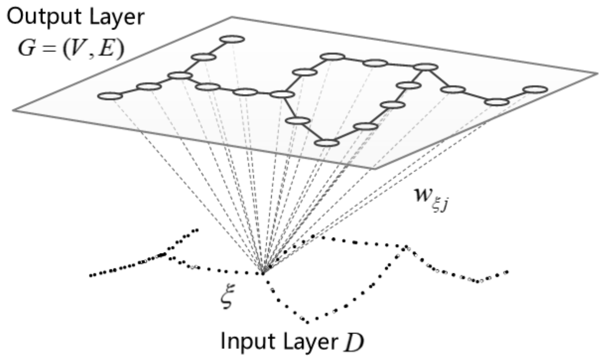
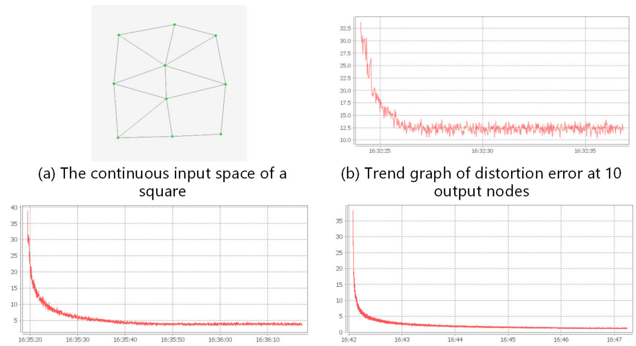


FIGURE 1. An example of self-organizing neural network.



(a) The continuous input space of a square

(b) Trend graph of distortion error at 10 output nodes

(c) Trend of distortion error at 100 output nodes

(d) Trend of distortion error at 1000 output nodes

FIGURE 2. Distortion error trends under different sizes of output network nodes.

self-organizing neural network that can generate the appropriate number of neurons according to the image data.

III. THE CRITERION INDEX BASED ON DISTORTION ERROR STABILITY

A. GROWING NEURAL GAS

GNG network is a kind of self-organizing neural network which can grows incrementally. It is a special kind of SOM network. Through competitive learning, the output layer of GNG network maps the input space D of arbitrary dimension to a topologically ordered output network model $G = (V, E)$ in a self-organizing and self-learning manner, denoted as $\Phi : D \to G$, where Φ represents the mapping algorithm. The mapping algorithm Φ maps the high-dimensional input space to the low-dimensional output network, fulfilling the functions of clustering, dimensionality reduction, and topology preservation. As shown in Fig.1, ξ is the input signal generated randomly in the input space D with the probability $p(\xi) = 1/|D|$, and $w_{\xi i}$ represents the distance between the input signal ξ and the network node v_i. The GNG model is growable, and the neurons in its output network can be increased or decreased with the self-organizing learning process of the model.

B. THE CONCEPT OF DISTORTION ERROR

A self-organizing neural network is a typical competitive neural network, whose output layer maps high-dimensional inputs to low-dimensional outputs in a self-organizing and self-learning manner. The output network should maintain a similar relationship with the original data, with the goal of generating a set of neurons V so that each input signal ξ can find a neuron v closest to it in V to represent ξ. The Euclidean distance between the nearest neuron v and the input signal ξ is the distortion error of neurons, denoted as $E(\xi, v)$.

$$E(\xi, v) = \| \xi - w_v \| \tag{1}$$

where w_v is the reference vectors of neuron v, representing the position of the neuron v in the input space. The learning of GNG network is an iterative process. Every λ^{th} iteration a new node is inserted between the node with the largest error and the adjacent node with the largest error. Therefore, the GNG network distortion error $E(G)$ for one iteration period is then defined as

$$E(G) = \frac{1}{\lambda} \sum_{\lambda} \| \xi - w_v \| \tag{2}$$

The self-organizing neural network optimizes and adjusts the neurons in the output network by a gradient descent method, so that $E(\xi, v)$ converges to an approximate optimal solution, which, in turn, makes $E(G)$ gradually decrease and tend to stabilize and the mapping of the output space to the input space more and more stable. Figure2 is the tendency chart of the output network distortion error caused by output spaces of different scales representing the same input space. As illustrated in the Figure2, the output network distortion error is decreasing and tends to stabilize as the output space scale increases, suggesting a gradually stable mapping effect of the output space scale to the input space. The stability is mainly reflected in the decreasing fluctuation amplitude of $E(G)$, in other words, the variance of $E(G)$ in some segments is getting smaller, which implies an increasing and stabilizing similarity between the output space and the input space. The curve in Figure2 represents the learning curve of the output network distortion error $E(G)$, showing that both $E(G)$ and oscillation amplitude tend to decrease and stabilize. By drawing on the feature of $E(G)$ tending to stabilize, this paper samples a certain number of nearest error values to analyze the variance of $E(G)$ which is considered as a stability index of output network distortion errors. The smaller the value of the variance, the more stable the distortion errors, the higher the similarity of output networks.

C. THE CALCULATION OF DISTORTION ERROR STABILITY

During the execution of the self-organizing neural network algorithm, $E(G)$ is continuously output as a data stream. Thus, the distortion error $E(G)$ at time t is defined as $E_G(t)$.

$$E_G(t) = \frac{1}{\lambda} \sum_{\lambda,t} \| \xi - w_v \| \tag{3}$$

The most recent sequence of data stream $E_G(t)$ reflects the degree of the similarity between the current output network and the input space. With the execution of the algorithm, the value of $E_G(t)$ decreases gradually and tends to be stable. At this time, even if the self-organizing neural network continues to learn and the network scale continues to grow, $E_G(t)$ will not change significantly and tends to be stable, which reflects the node scale is a suitable value, and can effectively express the distribution and topological structure of the input space. Then a linked list L with a fixed capacity of T is used to store a certain amount of the distortion errors $E_G(t)$. By analyzing the variance of these error data, the stability of distortion error is analyzed.

$$D(L) = \frac{1}{T-1}\sum_{k=1}^{T}[\overline{L} - E_G(k)]^2 \tag{4}$$

where $\overline{L}$ is the mean value of T distortion errors. The variance $D(L)$ can be used to reflect the stability of the distortion error of the output network, so it can be used as the judgment index of the stability of the similarity between the input space and output networks. When the $D(L)$ is less than the threshold $T_{D(L)}$, the node size of the output network grows to an appropriate value, and the characteristics of the input space can be fully expressed at this time, so as to stop growing new nodes.

D. TOPOLOGY PRESERVATION MEASUREMENT BASED ON MAXIMUM ENTROPY PRINCIPLE

Maximum entropy principle(MEP)has been applied in most information systems [18], [19]. Entropy, by definition, describes the uncertainty of a random variable. The maximum value of entropy means the maximum uncertainty, that is, the random variable is the most random, and it is the most difficult to predict its behavior accurately. In this sense, the essence of the maximum entropy principle is that we should choose the probability distribution with the largest entropy value on the premise of knowing part of the knowledge. From this point of view, the optimal solution of the self-organizing growth neural network learning is that for the random input signal ξ, the probability of mapping to each node c_i in the output space $V = \{c_1, c_2, \cdots, c_n\}$ is equal.

$$P(s(\xi) = c_i) = \frac{1}{|V|}, c_i \in V \tag{5}$$

where $s(\xi) = c_i$ means that the winning node of signal ξ is c_i; The formula 5 means that each node c_i in the output network has the same chance to be winner for a randomly generated input signal ξ, and the value of $c_i \in V$ can be assigned to the random variable X, that is, the information entropy $H(X)$ is maximum.

$$H(X) = -\sum_{i=1}^{n} P(c_i)log_n(P(c_i)), c_i \in V \tag{6}$$

where $P(c_i)$ is the probability that the node c_i wins; n is the number of nodes in the output network V. The value of information entropy $H(X)$ reflects the degree of topology preservation between the output network and the input data. The closer the value of information entropy $H(X)$ is to 1, the closer the distribution of the output network is to the optimum, and the higher the degree of topology preservation between the output network and the input space is. Therefore, based on the theory of MEP, this paper uses the value of the information entropy to evaluate the quality of topological feature extraction.

IV. SCALE-ADAPTIVE GROWING NEURAL NETWORK ALGORITHM

The traditional GNG algorithm generates new neurons with a fixed period of λ and inserts them into the GNG network until the number of neurons reaches a preset number. However, it is difficult to know the size of the input space in advance, and it is also difficult to give a suitable total number of neurons in advance. In this paper, the growth mode of traditional GNG algorithm is improved, which is called SA-GNG algorithm. The learning process of SA-GNG algorithm is divided into two stages: growth stage and convergence stage. In the growth stage, the distortion error stability is introduced to adaptively adjust the size of the network growth until the distortion error is not significantly improved. In the convergence stage, the SA-GNG algorithm does not create any new neurons, and the similarity of feature extraction is improved through continuous learning.

The proposed SA-GNG algorithm can be summarized as the following steps:

(Step 1) Initialize the output network $G = (V, E)$. Randomly generating two nodes $V = \{v_1, v_2\}$ according to the probability $p(\xi) = 1/|D|$, where $|D|$ is the size of the input data set, and setting the adjacent connection set E as an empty set $E = 0$;

(Step 2) A new input signal ξ is randomly generated based on the $p(\xi)$. Determine the winning node s_1 and the second-nearest winner $s_2(s_1, s_2 \in V)$ by:

$$s_1 = argmin_{v\in V} \parallel \xi - w_v \parallel \tag{7}$$

$$s_2 = argmin_{v\in V\ \{s_1\}} \parallel \xi - w_v \parallel \tag{8}$$

(Step 3) Make adjustments to s_1 and s_2.

- If the edge between s_1 and s_2 does not exist, then create an edge.

$$E = E \cup \{(s_1, s_2)\} \tag{9}$$

- Set the age of the new edge $age_{(s_1,s_2)} = 0$.
- Adjust the local accumulated error of the winning node s_1.

$$E_{s_1} = E_{s_1} + \parallel \xi - w_{s_1} \parallel^2 \tag{10}$$

- Adapt the reference vectors of the winner and its direct topological neighbors by fractions ε_b and ε_n, respectively.

$$w_{s_1} = w_{s_1} + \varepsilon_b(\xi - w_{s_1}) \tag{11}$$

$$w_i = w_i + \varepsilon_b(\xi - w_i) \quad (\forall i \in N_{s_1}) \tag{12}$$

- Adjust *age* of all edges connected to node s_1

$$age_{(s_1,i)} = age_{(s_1,i)} + 1 \quad (\forall i \in N_{s_1}) \tag{13}$$

(Step 4) Removes all edges where its *age* is greater than the threshold value a_{Max}, and also deletes nodes with no connected edges.

(Step 5) To determine whether to stop growing new neurons.

- Obtain the distortion error $E_G(t)$ at time t

$$E_G(t) = \frac{1}{\lambda} \sum_{\lambda,t} \parallel \xi - w_v \parallel \tag{14}$$

- Store $E_G(t)$ in the linked list L, if the number of elements in L is less than the value T, go to Step 2;
- Calculate the variance $D(L)$ of the elements in the linked list L, and clear the list.
- If the value of $D(L)$ is less than the threshold $T_{D(L)}$, the SA-GNG algorithm stops generating new neurons, and enters the convergence stage, and goes to Step 2.

(Step 6) Generate new node.

- If the input signal ξ is generated an integer multiple of λ, the node p with the largest local cumulative error is found.

$$p = argmax_{p \in V} E_p \tag{15}$$

- Find the neighbor node q of p, which has the largest local accumulated error among all neighbor nodes of p, and insert a new node r between p and Q.

$$V = V \cup \{r\} \tag{16}$$
$$w_r = (w_p + w_q)/2 \tag{17}$$

- Insert edges between r and p and between r and q, and delete the original edge between q and p.

$$E = E \cup \{(r,p), (r,q)\} \tag{18}$$
$$E = E \setminus \{(p,q)\} \tag{19}$$

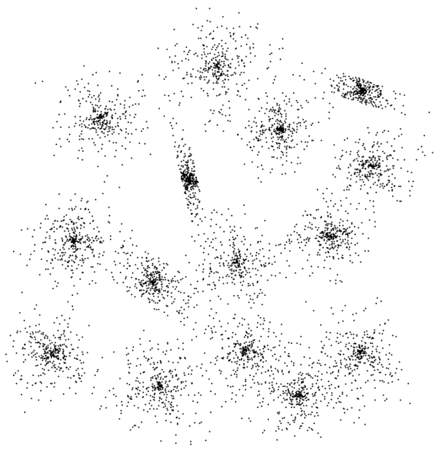
- Adjust the local accumulated error of p and q and set the local accumulated error of node r

$$E_p = (1-\alpha) * E_p \tag{20}$$
$$E_q = (1-\alpha) * E_q \tag{21}$$
$$E_r = (E_p + E_q)/2 \tag{22}$$

- Adjust the local accumulated error of all nodes.

$$E_v = (1-\beta) * E_v \quad (\forall v \in V) \tag{23}$$

SA-GNG can autonomously adjust the growth size of the output network to generate a topological structure similar to the input space. While the SA-GNG algorithm is executing, the distortion error stability $D(L)$ between the output network and the input data is evaluated in real time, and if $D(L)$ reaches the threshold $T_{D(L)}$, the SA-GNG stops adding new neurons and enters the convergence state. Step 5 is the main content of the improvement of the traditional GNG algorithm by SA-GNG. Through these steps, the SA-GNG algorithm can generate an appropriate number of output neurons without setting the number of output neurons in advance, and the generated topological network maintains a high similarity to the input data.



(a) First discrete input data set, 5000 data points

(b) Second discrete input data set, 100,000 data point

FIGURE 3. Two discrete data sets.

V. EXPERIMENTS AND DISCUSSION

To verify the effectiveness and feasibility of the algorithm proposed, this study performs an experimental analysis of discrete input space and continuous input space, and the analysis results are applied to the extraction of image topological features.

A. EXPERIMENTAL ANALYSIS IN DISCRETE INPUT SPACE AND CONTINUOUS INPUT SPACE

1) DISCRETE INPUT SPACE

Two sets of typical discrete input data were used to verify the SA-GNG algorithm, as shown in Figure3. Figure3(a) is a discrete data set consisting of 5000 data points in a relatively scattered distribution. Figure3(b) is a discrete data set consisting of 100,000 data points; the amount of data is very large, and the distribution is wave-shaped. This experiment doesn't pre-set the number of output network nodes, and instead, independently evaluate and control node growth by the SA-GNG algorithm, with the aim of analyzing the algorithm's adaptability to input data as well as its capabilities in clustering, dimensionality reduction, and topology learning. SA-GNG algorithm parameters are set as follows: $\lambda = 600$; $\epsilon_b = 0.1$; $\epsilon_n = 0.001$; $\alpha = 0.5$; $\beta = 0.0005$; $a_{max} = 88$; $T = 50$; and $T_{D(L)} = 0.05$, where $T = 50$ indicates that the capacity of the linked list is 50, $T_{D(L)} = 0.05$ indicates that the threshold of the distortion error variance is 0.05, and when the $D(L)$ is less than the threshold $T_{D(L)}$, the generation of a new node is stopped.

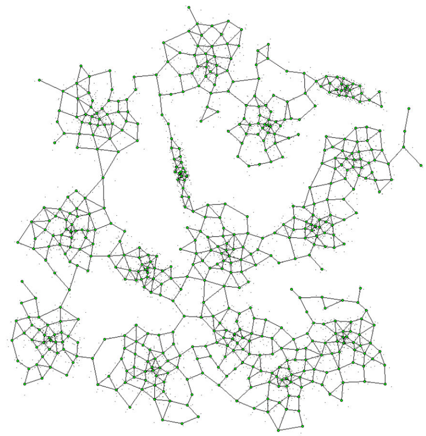
Figure4 presents the learning results of the SA-GNG algorithm in two discrete input spaces with different scales and distribution features, whose output network scales were 589 and 93 data points. For the data set shown in Figure3(a), the SA-GNG algorithm autonomously generated 589 neurons, reflecting the distribution and topological features of the input discrete data. Although the data set in Figure3(b) has a large data scale, reaching 100,000 data points, their distribution is extremely concentrated, so that only 93 neurons were generated to describe the distribution and topological features of the data set. Figure5 is the trend diagram of output network distortion error caused by the learning of the improved GNG algorithm in a discrete data set. As illustrated, as the learning progresses, the distortion error gradually tends to stabilize with no new output network notes generated when its stability meets the threshold of the similarity judgment index. It can be seen that SA-GNG can self-organizingly identify the distribution and topological features of the data in the input space with the continuous learning of the network and map the features identified to the output space. The data-intensive areas in the input space have more network nodes, which contribute to the accurate reflection of the input space's pattern clustering and topological features. From the point of view of information entropy, the entropy values of the generated networks are 0.966 and 0.994 respectively. They are very close to 1, which shows that the generated network has a very good topological preservation characteristics, and can extract the topological features of the input data very well. Meanwhile, the algorithm successfully converges the output network to a suitable scale through the distortion error stability judgment strategy. According to the learning results, the distortion error stability judgment index is suitable for both densely- and sparsely-distributed input spaces, suggesting the wide adaptability of SA-GNG.

2) CONTINUOUS INPUT SPACE

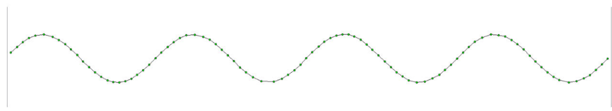
For continuous input spaces, the SA-GNG algorithm also shows self-adaption. Figure6(a) presents the learning result of the algorithm on a complex continuous input space, and there were 472 neurons autonomously generated. Figure6(b) presents the distortion error tendency chart. As illustrated, the distortion error quickly converges and then keeps stable. When the index $D(L)$ is smaller than the threshold, new network nodes stop growing. It's information entropy $H(X)$ is 0.997. Thus it can be seen that the SA-GNG algorithm performs noticeably well in digging out the spatial distribution and topology information in continuous input spaces, thereby effectively suitable for continuous input spaces.

B. APPLICATION IN IMAGE TOPOLOGICAL FEATURE EXTRACTION

Nowadays, as a result of the development of information technology, a large amount of image data is collected and stored. However, the data is usually unlabeled, so that algorithms are needed to dig out the hidden patterns and internal relationships among the data. GNG is a powerful tool for analyzing raw image data. With GNG, structured and organized feature information of data can be extracted, and object recognition and knowledge extraction realized. Therefore, the network has been widely used [20], [21]. Despite that, it is difficult to determine the output scale of GNG in advance due to the unknownness, diversity, and differences of input images. Cases in point are hand gesture and handwritten digit images, for which it is difficult to preset the scale of the output network. Nonetheless, this problem can be effectively solved by the SA-GNG algorithm proposed. Figure7,8,9,10 shows the learning result of the SA-GNG algorithm on four pictures retrieved from the network. The parameters of the

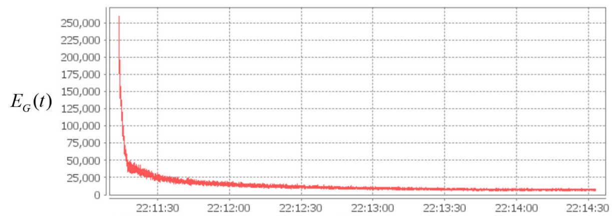


(a) 589 output neurons were generated autonomously

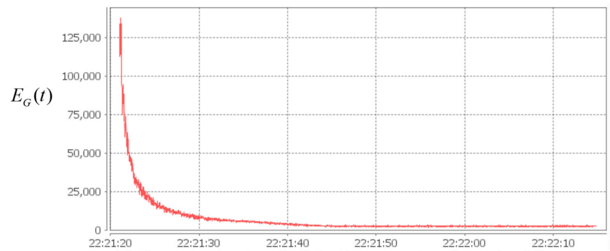


(b) Autonomously generate 93 output neurons

FIGURE 4. The learning results of the SA-GNG algorithm for the two discrete input spaces, 589 and 93 nodes are generated automatically, respectively, and the corresponding information entropies $H(X)$ are 0.966 and 0.994.

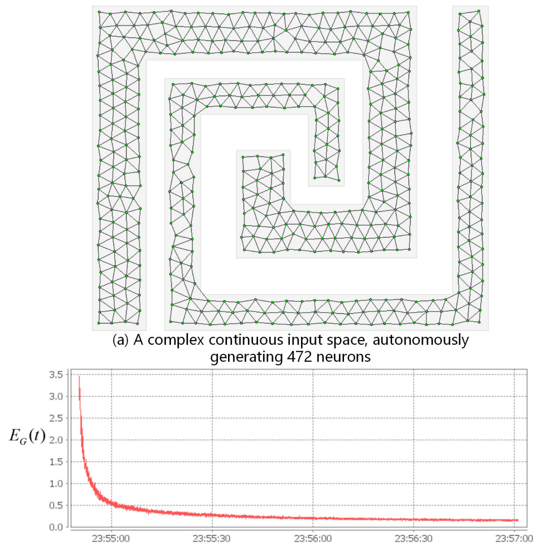


(a) The output network distortion error trend graph of the first discrete data set



(a) The output network distortion error trend graph of the second discrete data set

FIGURE 5. Distortion error trend diagram of learning process in discrete data sets based on the SA-GNG algorithm.



(a) A complex continuous input space, autonomously generating 472 neurons

(b) Output network distortion error trend

FIGURE 6. SA-GNG networks are generated autonomously in continuous input space, and the information entropy $H(X)$ is 0.997.

FIGURE 7. Topological network of 264 nodes is automatically generated to extract the digital features of the license plate, and the information entropy $H(X)$ is 0.998.

FIGURE 8. Topological features are extracted from the logo BIC, and 535 nodes are generated automatically, and the information entropy $H(X)$ is 0.999.

SA-GNG algorithm were set the same as those in the previous section and the value of $T_{D(L)}$ is set to 0.04. As illustrated, the SA-GNG algorithm successfully extracted the overall topological structure features of the objects in the image, and autonomously generated 264, 535, 406, and 543 network nodes in absence of knowing the scale of the output network, respectively. The information entropies of the generated networks are very close to 1, which shows that the SA-GNG algorithm can extract the topological features of the input image well without setting the number of network nodes in advance. According to the topology network constructed, a small number of output network nodes were used to express a large amount of original information in the image, satisfactorily describing the structure information of the objects and performing noticeable well in dimensionality reduction, clustering, and topology preservation.

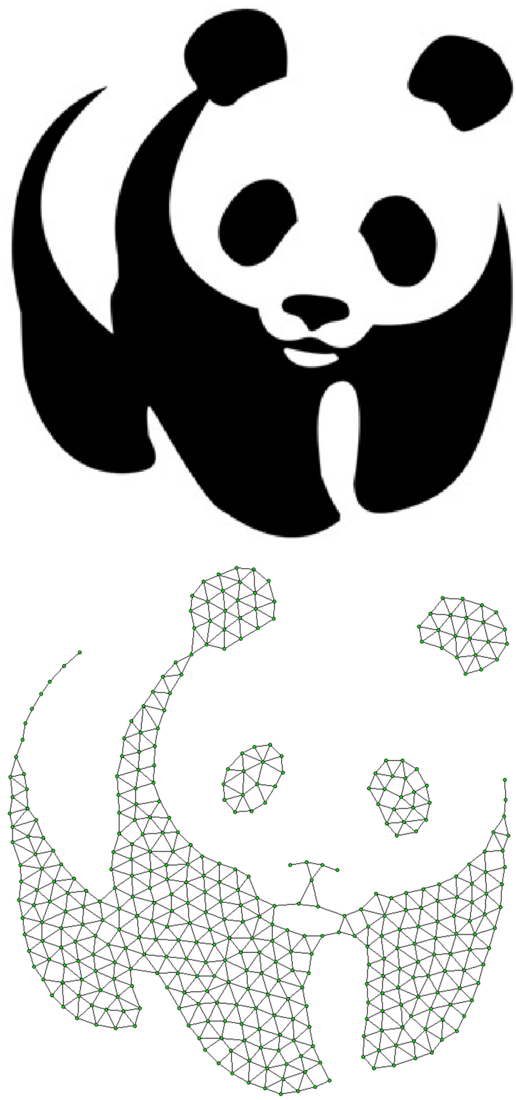


FIGURE 9. Topological feature extraction of input image based on the SA-GNG algorithm. A topology network with 406 nodes is automatically generated to represent the original input image, and the information entropy $H(X)$ is 0.999.

C. COMPARATIVE ANALYSIS WITH TRADITIONAL GNG ALGORITHM

Traditional GNG algorithms lack the ability of self-adaptation to the output network, so that the number of

FIGURE 10. Topological feature extraction of input image based on the SA-GNG algorithm. The topology network of 543 nodes is generated automatically, and the information entropy $H(X)$ is 0.998.

output network nodes has to be artificially preset. However, in general cases, the volume of the input data is unknown in advance, leading to the difficulty in presetting an appropriate number for network nodes. Hence, in order to work out the appropriate scale of the output network, repeated trials are generally needed. In this paper, we preset 100, 500 and 1000 nodes to perform clustering and topology learning for the two discrete input spaces and one continuous input space shown in the above section and compared the learning results with those obtained by SA-GNG. Figure11 presents the learning results obtained by traditional GNG algorithms. As illustrated, it is not easy to determine an appropriate output network scale in advance. For example, although the volume of discrete data set 2 is very large, the output network volume is found excessive when the numbers of nodes are set to 500 and 1000, as shown in Fig.11(d-f). For data set 1, the scale of the output network containing 100 nodes appears to be insufficient, while that containing 1000 nodes appears to be excessive, as shown in Figure11(a-c). As illustrated in Figure11(a), when the scale of the output network is 100 nodes, the distribution and topological features of the input space was not well extracted due to the insufficient scale of the output network. As illustrated in Figure11(c), when the scale of the output network is 1000 nodes, the output network appeared to be partially fragmentized, failing to completely express the distribution and topological features

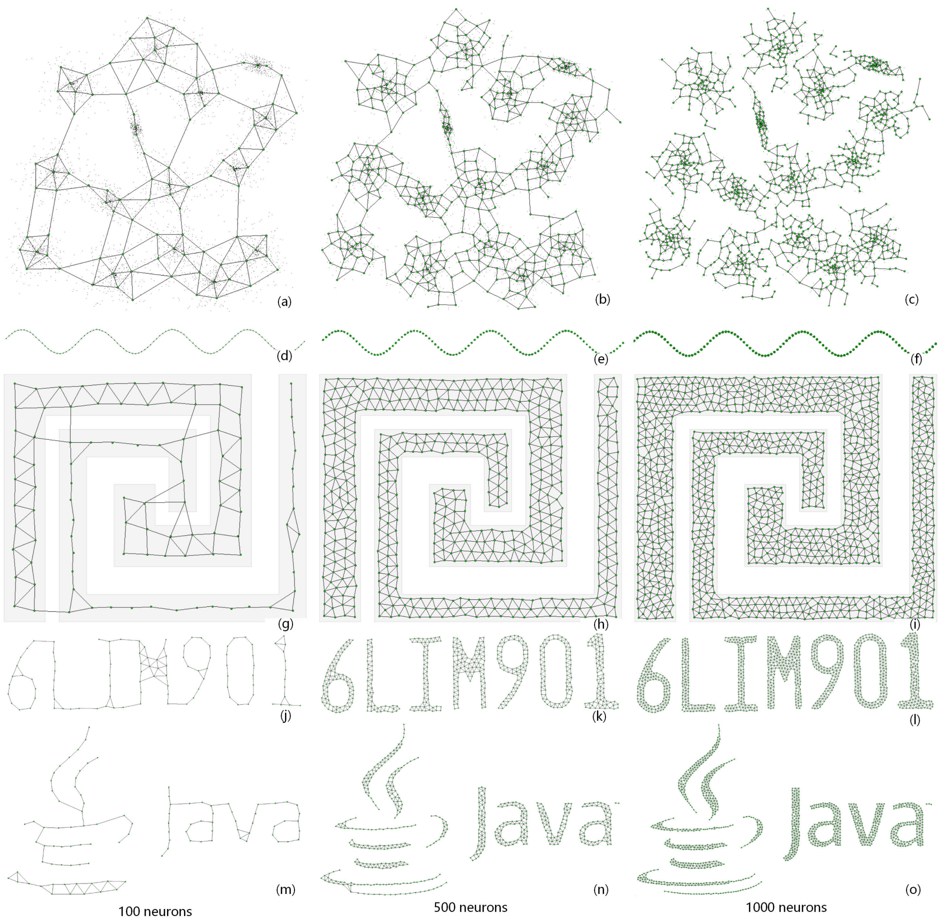


FIGURE 11. The learning results of traditional GNG network algorithm. 100, 500 and 1000 output neurons are preset, respectively.

TABLE 1. Comparative Analysis between SA-GNG and Traditional GNG Algorithm.

Input	GNG			SA-GNG		
	Neurons	$D(L)$	$H(X)$	Neurons	$T_{D(L)}$	$H(X)$
5000 data points	100	0.483	0.950	589	0.05	0.966
	500	0.051	0.966			
	1000	0.012	0.966			
100,000 data points	100	0.041	0.999	93	0.05	0.994
	500	0.003	0.995			
	1000	0.001	0.992			
continuous input space	100	0.202	0.963	472	0.05	0.997
	500	0.048	0.995			
	1000	0.024	0.998			
license plate	100	0.092	0.923	264	0.04	0.998
	500	0.025	0.999			
	1000	0.010	0.999			
java logo	100	0.184	0.912	543	0.04	0.998
	500	0.042	0.998			
	1000	0.009	0.998			

of the input data. This same goes for the continuous input space and the images, as shown in Figure11(g-o). Table 1 shows the comparative analysis between the SA-GNG and GNG. From the value of information entropy, it can be seen that both GNG and SA-GNG can generate networks with similar topological structure from the input data due to the use of competitive hebbian learning rule. As the number of default nodes increases from 100 to 1000, the variance of the distortion error of the GNG network gradually decreases. Because the GNG network algorithm can not automatically generate the appropriate scale of the topology network according to the input data, it often needs repeated experiments to get the output network with the appropriate scale. For example, for the discrete data set with 100,000 points, the output network with 100 nodes is better than that with 500 and 1000 nodes, and the value of $D(L)$ is 0.041, which is close to 0.05. When the number of nodes of the output network is 500 and 1000, it can be seen from the figure11(e,f) that the scale of the network is excessive, and the values of $D(L)$ are 0.003 and 0.001, respectively, much less than 0.05. For the two input data of continuous input space and java logo, the output network of 500 nodes is relatively suitable, and the values of $D(L)$ are 0.048 and 0.042 respectively, which are not much different from 0.05. The threshold value of $T_{D(L)}$ is typically set to 0.05, which is an empirical value.

In a word, comparing the learning results of the traditional GNG algorithm with the SA-GNG algorithm, it can be seen that the SA-GNG algorithm can autonomously generate an appropriate output network size without pre-setting. This not only effectively improves the autonomy and adaptability of the self-organizing growth neural network algorithm, can accurately extract the clustering characteristics and topological characteristics of the input space, but also overcomes the difficulty of presetting the appropriate size of the output network.

VI. CONCLUSION

Self-organizing growing neural networks online incrementally learn input data in an unsupervised manner and represent the clustering and topology of input space in the absence of prior knowledge. However, existing growing self-organizing neural network algorithms cannot effectively evaluate and keep node growth on a reasonable scale, thereby failing to autonomously generate an appropriate total number of nodes. To address this problem, this paper proposes the SA-GNG and introduces a judgment index that can evaluate node growth in the output network. This index allows SA-GNG to adapt to the input space autonomously, with no need of knowing the distribution of the input space in advance nor presetting the scale of the output network in advance. When the network scale increases to a certain extent and meets the threshold condition of the judgment index, the distortion error between the output network and the input data becomes stable at this time, and the algorithm stops growing new network nodes. Experimental analysis was made in a variety of discrete input spaces and continuous input spaces, and the results verified the effectiveness and feasibility of the judgment method based on output network distortion error stability. Therefore, this improvement addresses the problem that GNG cannot work out the appropriate number of output network nodes beforehand, thereby enhancing the autonomy and adaptability of growing self-organizing neural network algorithms.

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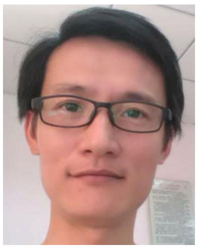

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