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# PID Control With Higher Order Derivative Degrees for IPDT Plant Models

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**ABSTRACT** In this paper, we discuss the main features of the generalized higher-order proportional-integrative-derivative control (HO-PID) based on the integral-plus-dead-time (IPDT) plant models. It was developed by extending the traditional PI-control to include  $m$ th-order derivatives and  $n \geq m$ th-order binomial series filters. The HO-PID control provides two additional degrees of freedom, which allow to appropriately modify the speed of the transients and the attenuation of the measurement noise, together with the closed-loop robustness. In this way, it pursues similar goals as an alternative fractional-order PID control. A broad family of the HO-PID controllers with the included low-pass filters is employed to solve a number of new problems. Their integrated suboptimal tuning, based on explicit formulas derived by the multiple real dominant pole (MRDP) method and evaluated by a novel approach that relates the speed of transients to the excessive input and output increments, has been simplified by introducing two integrated tuning procedures (ITPs). The main new finding is that HO-PID control enables faster transients by simultaneously reducing the negative effects of measurement noise and increasing the closed-loop robustness. A brief experimental evaluation using new sensitivity measures fully confirms the excellent HO-PID characteristics and shows that commissioning remains almost as simple as with the filtered PI-control.

**INDEX TERMS** Filtration, higher-order derivative action, multiple real dominant pole method, PID control.

## I. INTRODUCTION

Proportional-integral-derivative (PID) controllers represent the most widely used technology for industrial process control. Their design is typically based on the use of simple delayed plant models [1]. In addition to applications in traditional control areas, current research is dominated by network control and telematic applications [2]. When reviewing the recent development in this area, which can be well represented by the papers appeared at the 3rd IFAC conference on PID control, one cannot overlook the explosion of new fractional-order (FO) solutions. Tepljakov *et al.* [3] identified the possibility of having “two additional tuning knobs that can be used to tune the control law in a way that benefits the control loop” as the reasons for this development, which started from [4]. In other words, the driving force

behind their development is to obtain more degrees of freedom for the designers to satisfy a predefined set of performance criteria [5]. The search for the optimal characteristics of the control loop has introduced numerous optimization approaches to controller design [6], [7]. Tepljakov *et al.* also try to find solutions to questions such as what are the particular advantages of the FO controllers and to what extent can they be preserved by the higher order integer approximations used in their implementation (such as the Crone method [5], [8]).

However, new meaningful degrees of freedom can also be established in the traditional integer-order controllers. It is only necessary to treat more rigorously the filtering unavoidable for the derivative action implementation. Therefore, several recent publications formulate the design problem in a more complex way as a trade-off between the attenuation of load disturbances (IAE), the injection of measurement noise affecting the controller activity (input usage), and robustness

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(usually characterized by the sensitivity peaks  $M_s$  and  $M_T$ ). In the search for the optimal controller parameters, one can again encounter a wide range of different optimization approaches. To evaluate the controller activity, the total variation of the plant input was introduced as a measure of the input usage (controller activity) [9]. Its modified versions focusing on excessive total variation (TV) [10] extend the mathematical-physical interpretation of the redundant changes of input and output. They can be used both for evaluating the influence of the stochastic and deterministic components of the signals at the plant input and output, and for testing the optimality of the closed loop and invariance to parameter uncertainties.

The design of derivative filters is often considered as a tedious problem leading to a trivial solution, which is to eliminate the derivative action and use only the simplest PI control. In textbooks one can still read that either the derivative part is the most difficult to tune [11], the derivative action is not suitable for noisy and time delayed processes [12], or that the PI control is preferred because of its simplicity [13]. The last argument could perhaps be accepted in the time of fluid analog controllers. But in the case of software based controllers, the difference in implementation complexity is not a significant problem and the same conclusions are drawn regarding their application. No doubt, the filtration problem is a major issue not only for PID control [14]–[19]. It affects the control accuracy, energy consumption, heat dissipation, actuator wear, unwanted vibrations, acoustic noise, etc. In this situation, a new integrated filter and controller tuning appeared, which allows to easily deal with filters of arbitrarily chosen order  $n$  and to obtain much better results than the traditional derivative filter design. This allowed to go from the filtered PI and PID control [20], to the less common Proportional-Integrative-Derivative-Accelerative (PIDA) [21]–[25], PIDD<sup>2</sup> [26], [27], or PIDD<sup>2</sup>D<sup>3</sup> [28], [29] controllers with HO derivative actions, up to the introduction of a general and at the same time simple notation of PID<sub>*n*</sub><sup>*m*</sup> control. In such HO-PID control, the use of the derivative actions up to a general degree  $m$ ,  $m = 0, 1, 2, \dots, m \leq n$  proves to be useful both for reducing excessive control effort, increasing transients speed, smoothness, and increasing performance robustness [26], [30]. To compare the FO-PID and HO-PID design, it should be noted that both approaches attempt to expand the applicability of traditional PID. While FO-PID formally simplifies the initial design steps by using non-integer differentiation and integration operators, it encounters problems in the final steps of implementing the proposed solutions through HO approximations. HO-PID is a direct extension of the traditional PID design and synthesizes directly HO control algorithms. New approaches arise in a more rigorous design of necessary filters and the use of new performance and robustness measures and properties, which are much more focused on the actual transient shapes achieved and their correspondence to expected ideal patterns. In order to make these achievements available to a wider public, suitable computer-aided tools are being

developed [31], [32], which may also be helpful in reading this paper.

Based on this preliminary work, this paper extends and tests the basic HO-PID design and its evaluation [33] for the integral plus time delay (IPDT) plant up to the derivative degree  $m = 5$ . It can be considered as an answer to the question [3] “If integer-order approximations are used anyway, why not just use high order integer-order controllers instead of FO-PID approximations?”. Contrary to the expectations [3] that the tuning of all the parameters of an HO controller is more difficult than in the case of an FO-PID, we will show that the complexity of HO-PID tuning remains at the level of a simple filtered PI controller.

Thereby, the paper is structured as follows. Section II presents the performance measures used for dynamics evaluation and optimization. They allow deterministic evaluation of noisy processes without using special statistical methods. Section III brings parameterization of the ideal PID<sup>*m*</sup> controller for the IPDT plant model, the extension to  $n \geq m$  order binomial filters for implementable PID<sub>*n*</sub><sup>*m*</sup> control, and the motivation for treating an HO derivative action. Section IV introduces integrated tuning procedures (ITPs) for simple tuning of HO-PID control based on delay equivalence. It is used for balancing the trade-off between the speed of the control transients and the impact of noise attenuation and uncertainties. In Section V, the characteristics of speed-effort and speed-wobbling in planes are introduced relating the loop retardation (reciprocal to the speed of transients) to the excessive input and output increments. They are used in the analysis of various ITPs characteristics by simulation. Section VI discusses the application of the traditional and the newly proposed closed loop sensitivity functions for robustness analysis. All the main conclusions from simulation are verified by simple real time experiments in Section VII. After the discussion in Section VIII, which relates the obtained experimental results to those of the control loop simulation, the main achievements of the work are summarized in Section IX. Interesting aspects for future development are then discussed in Conclusions.

The novelty of the work lies in the unique combination of several analytical and experimental approaches to optimal control design and performance evaluation for the presented family of HO-PID controllers. It has been compiled with a focus on achieving the fastest possible dynamics while maintaining sufficiently smooth and robust transients. By documenting this under the influence of uncertainties, neglected dynamics, the effects of measurement noise and the actuator constraints, it can be said that the article helps to overcome the traditional reservations about the use of the derivative action in the control of noisy systems.

## II. PERFORMANCE MEASURES AND EVALUATION

Since the settling time used to measure the speed of the control transients at the plant output depends on an ad hoc chosen dead-band around the reference setpoint, the responses can

be evaluated as an integral of the absolute error

$$IAE = \int_0^\infty |e(t)| dt; \quad e = w - y, \quad (1)$$

where  $w$  is the desired reference setpoint,  $y$  is the output, and  $e$  is the control error. Assuming no change in sign of the control error, it can be calculated as the integral of the error  $IAE = IE$ . Performance measures related to the setpoint and disturbance steps are denoted by the subscripts “s” and “d”, respectively. However, since the setpoint responses can also be improved by appropriate feedforward control, the paper focuses on the disturbance responses.

The central concept used in performance evaluation is that of signal monotonicity. As shown in Theorem 1 in [10], an ideal setpoint step response of a single integrator must be monotonic at the output, while at the input it must consist of two monotonic intervals forming a one-pulse (1P) shape. To characterize the output deviations from monotonicity, the excessive total signal variation is used [10]. It is calculated as the total variation (the total sum of absolute increments [9]) exceeding the net output variation  $|y_\infty - y_0|$  (specified by the initial and final values  $y_0$  and  $y_\infty$ ) denoted as

$$TV_0(y_s) = \int_0^\infty \left( \left| \frac{dy}{dt} \right| - \text{sign}(y_\infty - y_0) \frac{dy}{dt} \right) dt \approx \sum_i (|y_{i+1} - y_i|) - |y_\infty - y_0| \quad (2)$$

Since it shows only the contribution of the excessive increments, it provides the best view of the “smoothness” of the output response. The value  $TV_0(y_s) = 0$  corresponds to an ideally smooth (monotonic) output, otherwise  $TV_0(y_s) > 0$ .

An ideal disturbance step response of the integrative first order plants has a one-pulse (1P) shape consisting of two monotonic intervals separated by an extreme point  $y_m \notin (y_0, y_\infty)$ . Therefore, the performance evaluation according to (2) must be performed twice. Deviations of the output  $y(t)$  from an ideal 1P response can then be measured in terms of a modified total variation (TV) denoted as  $TV_1(y_d)$

$$TV_1(y_d) = \sum_i |y_{i+1} - y_i| - |2y_m - y_\infty - y_0| \quad (3)$$

Similarly, deviations of the plant input  $u(t)$  from an ideal 1P step response [10] should be constrained in terms of  $TV_1(u_s)$ , or  $TV_1(u_d)$  measures. In principle, it may be useful for IPDT plant control to consider also an input with a higher number of control pulses, but such a situation already represents an advanced and rarely used option.

Formulating the design problem as a trade-off between the load disturbance attenuation ( $IAE_d$ ), the measurement noise injection affecting the controller activity (input usage  $TV_1(u_d)$ ), and robustness, we look for a minimum value of the “holistic” cost function

$$J_k = IAE_d^k TV_1(u_d) \quad (4)$$

For  $IAE_d > 1$ , the choice  $k > 1$  is a stronger emphasis on the speed of the transients that may dominate over the required controller activity.

### III. IDEAL AND FEASIBLE HO-PID CONTROLLERS FOR THE IPDT PLANT TUNED BY THE MRDP METHOD

Let us approximate the plant responses by an IPDT model

$$S(s) = \frac{Y(s)}{U(s)} = S_0(s)e^{-T_{dp}s}; \quad S_0(s) = \frac{K_{sp}}{s} \quad (5)$$

with the gain  $K_{sp}$  and the dead time  $T_{dp}$ . In the following derivation, the plant model index “p” will be firstly omitted. This corresponds to the model parameters equal to the nominal plant parameters  $K_s$  and  $T_d$ .

#### A. IDEAL HO-PID CONTROL

*Definition 1 (PID<sup>m</sup> Controller):* A generalization of the PID control with possible derivative terms up to an integer degree “m” will be proposed as

$$PID^m(s) = K_c \left[ \frac{1 + T_i s}{T_i s} + T_{D1}s + \dots + T_{Dm}s^m \right] \quad (6)$$

Although for  $m > 0$  such a controller may not be implemented without a filter, its concept still may be useful in exploring more complex systems. For the nominal plant (5), the disturbance-to-output transfer function corresponding to PID<sup>m</sup> control is

$$F_{dy}(s) = \frac{Y(s)}{D_i(s)} = \frac{K_s T_i s}{T_i s^2 e^{T_d s} + K_c K_s [1 + s T_i (1 + T_{D1}s + \dots + T_{Dm}s^m)]} \quad (7)$$

To get dimensionless parameters [34], let us denote

$$p = s T_d; \quad \kappa = K_c K_s T_d \quad \tau_i = \frac{T_i}{T_d}; \quad \tau_j = \frac{T_{Dj}}{T_d}; \quad \kappa_m = \kappa \tau_j; \quad j = 1, 2, \dots, m \quad (8)$$

*Theorem 1 (MRDP-Optimal PID<sup>m</sup> Parameters):* Optimal controller parameters determined by the MRDP method guarantee the  $m + 3$ -tuple real dominant poles  $s_o$  and  $p_o$

$$p_o = s_o T_d = \sqrt{m + 2} - (m + 2) \quad (9)$$

For  $m \in [0, 5]$  the corresponding “optimal” values  $\kappa_o, \tau_{io}, \tau_{jo}; j = 1, 2, \dots, m$  are summarized in Tab. 1.

The  $m + 2$  controller parameters of the normed quasi-polynomial

$$P(p) = \tau_i p^2 e^p + \kappa [1 + p \tau_i (1 + \tau_{D1} p + \dots + \tau_{Dm} p^m)] = \kappa + \tau_i [p^2 e^p + \kappa p + \kappa_1 p^2 + \dots + \kappa_m p^{m+1}] \quad (10)$$

are determined to get an  $m + 3$ -tuple real dominant pole  $p_o$ . It is,  $p_o$  and the normed controller parameters have to fulfill the following system of equations

$$\left[ P(p); \frac{dP(p)}{dp}; \dots; \frac{d^{m+2}P(p)}{dp^{m+2}} \right]_{p=p_o} = \mathbf{0} \quad (11)$$

According to its definition, from the roots of the last equation

$$\frac{d^{m+2}P(p)}{dp^{m+2}} = (p^2 + 2(m + 2)p + (m + 2)(m + 1)) \tau_i e^p = 0 \quad (12)$$

**TABLE 1.** Optimal PID<sup>m</sup> parameters corresponding to (9), m ∈ [0, 5].

	m = 0	m = 1	m = 2	m = 3	m = 4	m = 5
κ <sub>o</sub>	0.4612	0.78361	1.08268	1.37114	1.65330	1.93117
τ <sub>io</sub>	5.8284	3.73205	3.00000	2.61803	2.37980	2.21527
τ <sub>1o</sub>	0	0.26289	0.37500	0.43673	0.47525	0.50120
τ <sub>2o</sub>	0	0	0.04167	0.07492	0.09972	0.11843
τ <sub>3o</sub>	0	0	0	0.00474	0.01020	0.01526
τ <sub>4o</sub>	0	0	0	0	0.00042	0.00105
τ <sub>5o</sub>	0	0	0	0	0	0.00003

**TABLE 2.** Optimal normed IAE<sub>d</sub> = IAE<sub>d</sub> / (K<sub>s</sub>T<sub>f</sub><sup>2</sup>) values corresponding to unit input disturbance step responses of PID<sup>m</sup> from Tab. 1.

-	m = 0	m = 1	m = 2	m = 3	m = 4	m = 5
IAE <sub>d</sub>	12.639	4.763	2.771	1.909	1.439	1.14711

the dominant pole (9) is chosen as the pole closest to the origin. After substituting p = p<sub>o</sub> into dP(p)/dp = P<sub>1</sub>(p), . . . , dP<sup>m+1</sup>/dp<sup>m+1</sup> = P<sub>m+1</sub>(p)

$$\begin{aligned}
 &P_1(p) \\
 &= \tau_i \left[ (p^2 + 2p)e^p + \kappa + 2\kappa_1 p + \dots + (m + 1)\kappa_m p^m \right] \\
 &\vdots \\
 &P_{m+1}(p) \\
 &= \tau_i \left[ (p^2 + 2(m+1)p + m(m+1))e^p + (m+1)! \kappa_m \right] \quad (13)
 \end{aligned}$$

we get from (11) a triangular system of equations (13) regarding the parameters κ, κ<sub>1</sub>, . . . , κ<sub>m</sub> which can be solved from the end. The solution finishes with deriving τ<sub>io</sub> from P(p<sub>o</sub>) = 0 and with expressing τ<sub>jo</sub> = κ<sub>jo</sub>/κ<sub>o</sub>, j = 1, . . . , m. Results of the solutions for particular m ∈ [0, 5] are in Tab. 1.

*Remark 1 (Optimality of the MRDP Tuning):* When changing the parameters of a given controller from the optimum corresponding to (11), some modes of transients accelerate, while others slow down. Since the speed of transients is dominated by the slow ones, in terms of the control velocity, the mutual balance corresponds to the optimum with equally fast (slow) dominant poles. This corresponds to a multiple real dominant pole of the characteristic closed loop polynomial (see, for example [10]).

**B. IDEAL IAE VALUES**

Reasons for introducing higher order derivative actions will be demonstrated by IAE<sub>do</sub> = -IE<sub>do</sub> values

$$IAE_{do} = - \lim_{s \rightarrow 0} F_{dy}(s)/s = T_i/K_c \quad (14)$$

corresponding to unit input disturbance step responses and normed controller tuning (8) from Tab. (1). Obviously, by increasing m from m = 0 to m = 5 the IAE<sub>d</sub> values decrease more than 10 times.

When wishing to examine the most important question, how far such improvements are reachable in practice, we have to replace the ideal PID<sup>m</sup> controllers with a filtered PID<sup>m</sup><sub>n</sub>, m ≤ n control. In traditional PID design, the filter delay is mostly added to the plant. Analogically, in this paper,

the total loop delay will be expressed in terms of the plant dead time T<sub>dp</sub> increased by an equivalent amount spent on filtration.

**C. FEASIBLE HO-PID CONTROLLER**

One of the basic advantages of the new concept against the FO-PID control is that the PID<sup>m</sup> controller may be simply implemented in combination with an n<sup>th</sup> order series binomial filter

$$Q_n(s) = 1/(T_f s + 1)^n; \quad n = 1, 2, \dots; n \geq m \quad (15)$$

*Definition 2 (PID<sup>m</sup><sub>n</sub> Controller):* A combination of the PID<sup>m</sup> controller with a binomial low-pass filter Q<sub>n</sub> of an appropriate order n will be denoted as PID<sup>m</sup><sub>n</sub> controller

$$PID_n^m(s) = PID^m(s)Q_n(s); \quad m \leq n \quad (16)$$

*Remark 2 (Motivation to Use Higher Order Filters):* The transfer function PID<sup>m</sup><sub>n</sub>(s) becomes proper for n = m. Furthermore, the measurement noise may be significantly better attenuated for strictly proper PID<sup>m</sup><sub>n</sub>(s) with n > m. Although already for n = 1 there are no clear guidelines for tuning the filter parameters [11]–[13], this paper will show relatively simple tuning possibility also for higher orders m and n ≥ m.

**IV. TUNING SCENARIOS**

One possible solution to get a simple analytical Q<sub>n</sub>(s) design is to approximate the effect of its n time constants T<sub>f</sub> by an equivalent dead time T<sub>e</sub>. In [33], its value has been derived by a delay equivalence based on keeping an equal position of the dominant closed loop poles corresponding to:

- 1) PID<sup>m</sup> control of an IPDT plant S(s) yielding the pole s<sub>o</sub> (9) with some T<sub>dp</sub>.
- 2) PID<sup>m</sup><sub>n</sub> control of the delay free integral plant S<sub>0</sub>(s) (i.e. T<sub>dp</sub> = 0) with the filter (15) time constant T<sub>f</sub> yielding the characteristic polynomial

$$\begin{aligned}
 P(s) &= T_i s^2 (1 + T_f s)^n + K_c K_s \\
 &\times [1 + T_i s (1 + T_{D1} s + \dots + T_{Dm} s^m)] \quad (17)
 \end{aligned}$$

with the MRDP s<sub>n</sub>.

- 3) T<sub>dp</sub> = T<sub>e</sub> in 1. yielding for T<sub>fe</sub> = T<sub>f</sub> in 2. the equality s<sub>o</sub> = s<sub>n</sub> may then be denoted as the equivalent time delay T<sub>e</sub> (equivalent time constant T<sub>fe</sub>).

*Lemma 1 (Dominant Poles for PID<sup>m</sup><sub>n</sub> With S<sub>0</sub>(s)):* For (17) the MRDP method yields the dominant pole

$$s_n = \frac{\sqrt{(m+2)(n+1)(n-m)} - (m+2)(n+1)}{(n+1)(n+2)T_f} \quad (18)$$

The proof is again based on introducing normed parameters p = sT<sub>f</sub>, τ<sub>i</sub> = T<sub>i</sub>/T<sub>f</sub>, τ<sub>j</sub> = T<sub>Dj</sub>/T<sub>f</sub><sup>j</sup>, κ = K<sub>c</sub>K<sub>s</sub>T<sub>f</sub>, κ<sub>j</sub> = τ<sub>j</sub>κ and looking for the m + 3-tuple pole p<sub>n</sub> = s<sub>n</sub>T<sub>f</sub> of the characteristic polynomial

$$P(p) = \kappa + \tau_i \left[ p^2 (1 + p)^n + \kappa p + \kappa_1 p^2 + \dots + \kappa_m p^{m+1} \right] \quad (19)$$

The solution has to fulfill

$$\begin{aligned} & \frac{d^{m+2}P(s)}{dp^{m+2}} \\ &= n(n-1)\dots(n-m+1)(p+1)^{n-m-2} \\ & \cdot \left[ (n+1)(n+2)p^2 + 2(m+2)(n+1)p + (m+2)(m+1) \right] \\ &= 0 \end{aligned} \quad (20)$$

The dominant pole  $p_n = s_n T_f$  corresponds to the root of (20) closer to the imaginary axis, which then yields (18).

*Lemma 2 (Equivalence of Time Delays):* A time constant  $T_f = T_{fe}$  of the filter  $Q_n(s)$  will be considered to be equivalent to a dead time  $T_e$ , when the corresponding dominant pole (18) equals to the pole  $s_o = p_o/T_d$  (9) calculated for  $T_d = T_e$ , i.e.

$$s_o = s_n \quad (21)$$

Solving (21) for  $T_f$  with  $T_d = T_e$  and  $n \geq m \geq 0$  yields

$$T_{fe} = \frac{\sqrt{(m+2)(n+1)(n-m)} - (m+2)(n+1)}{(n+1)(n+2)[\sqrt{m+2} - (m+2)]} T_e \quad (22)$$

*Remark 3 (Loop Properties Imposed by the Delay Equivalence):* Similarly, as in traditional PID control, the key problem of the introduced  $PID_n^m$  control is that the ideal  $PID^m$  control must be augmented by a low-pass implementation filter, which requires to deal with mixed types of delays. In such situations, a consistent “optimal” approach becomes unenforceable. Therefore, we will propose simplified integrated tuning procedures which, however, need to check the corresponding loop performance in all possible limit applications.

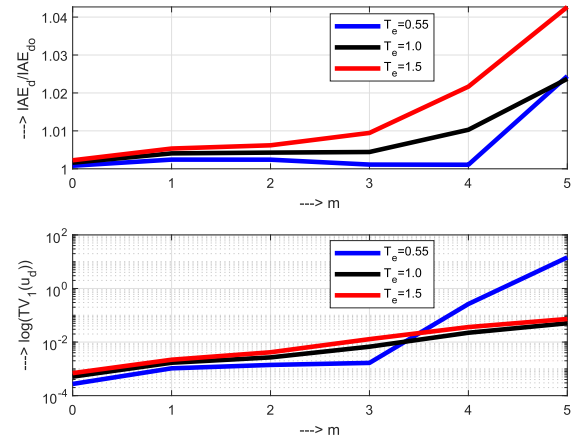
*Remark 4 (Limiting the Filter Orders Used):* By the number of tuning degrees of freedom given by the parameters  $m, n$  and  $T_f$ , we have created a huge family of  $PID_n^m$  controllers [33] which, of course, cannot be explored in all details in a single article. Because the key new aspect of  $PID_n^m$  control relates to the higher order derivative actions, we have constrained ourselves by evaluating only the cases with  $n = m+2$ , i.e. with the  $PID_{m+2}^m$  control. Namely, as it was derived in [35], the noise attenuation does not increase significantly for  $n > m+2$ . Together with the delay equivalence (22), this assumption reduces the number of adjustable parameters to only two:  $m$  and  $T_e$ .

*Definition 3 (Integrated Tuning Procedure ITP):* In order to consider both the IPDT plant (5) and the unavoidable binomial implementation filter  $Q_n(s)$  (15) a sequence of steps for the  $PID_n^m$  controller tuning will be proposed consisting of:

- 1) After identifying the plant model parameters  $K_{sp}$  and  $T_{dp}$ , choose an appropriate value  $T_e > 0$  corresponding to a required degree of filtration;
- 2) Specify a derivative degree  $m$  and calculate the controller (9) corresponding to the total loop delay  $T_d$ ,

$$T_d = T_{dp} + T_e \quad (23)$$

- 3) Choose a filter order  $n \geq m$  and by a delay equivalence (22) specify the filter time constant  $T_{fe} = f(m, n, T_e)$ ;



**FIGURE 1.** Verification of the ITP with  $T_e = const$  by comparing the analytical  $IAE_{d0}$  values with values  $IAE_d$  (1) from loop simulation,  $T_e = \{0.55, 1.0, 1.5\}T_{dp}$ ,  $T_{dp} = 1$ ,  $\delta = 0$ ,  $T_s = 0.001T_{dp}$ ,  $m \in [0, 5]$ .

- 4) Check, if the calculated value  $T_{fe}$  fulfills the requirement  $T_{fe} \gg T_s$ , with  $T_s$  representing the sampling period used for the quasi-continuous control implementation. If not, you should either decrease  $T_s$ , or  $n$ , which must fulfill the condition  $n \geq m$ . In the worst, case you may still use the non filtered PI control.
- 5) By experimentally evaluating properties of the noisy loop for different  $m$  and  $n$ , choose the controller parameters fitting optimally the required loop performance.

It will be denoted as an integrated tuning procedure (ITP). Since this approach deals with numerous free parameters, it is useful to explore and illustrate its properties by analyzing several possible scenarios.

### A. ITP-Te

With respect to the fact that several dead time elements of a control loop may be replaced by their sum without changing the corresponding optimal controller tuning, first ITP based on the delay equivalence (22) may be formulated as follows:

*Theorem 2 (Nearly Homogeneous Shape Properties Under ITP-Te With  $T_e = const$ ):* ITP considering some

$$T_e = const > 0 \quad (24)$$

guarantees in a broad range of the parameters  $m$  and  $n = m+2$  nearly ideal shapes of the transients at the plant input and output with the speed of transients increasing with  $m$  and decreasing with  $T_e$ .

A simple verification of the ITP-Te with  $T_e = const$  may be based on comparing the expected  $IAE_{d0}$  values (14) corresponding to  $T_d$  (23) for the chosen  $m$  and  $T_e$  with  $IAE_d$  values (1) achieved by loop simulation. The simulated results (Figure 1) enable two conclusions:

- In a broad range of considered parameters  $m, n = m+2$  and  $T_e$  the values achieved by simulation correspond with a high precision to those derived analytically (the differences are below 1%).

**TABLE 3.** Weights of the IAE values in (26) corresponding to the unit input disturbance step responses for PID<sup>m</sup> parameters from Tab. 1.

-	$m = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$
$\gamma_d$	0.2813	0.4582	0.6007	0.7236	0.8335	0.9337

- In such situations also  $TV_1(u_d) \rightarrow 0$ , which corresponds to no error sign change considered ideally in the derivation of (1) by means of  $IAE = |E|$ .

Differences that occur at greater values  $m$  are of two kinds:

- In the range with  $T_e \leq 0.6T_{dp}$ , numerical instability occurs due to simulation (Figure 2, transients above) of the loop with a significant dead time. In real time control, when confined only to a controller simulation (not the entire loop), instability was not present at the tested values of  $T_e$  (see Section VIII-C).
- For larger values of  $T_e$ , the relatively small shape deviations (output undershooting, Figure 2, transients below) are related to the approximate  $T_{fe}$  calculation according to (22). If necessary, these slight shape deviations could be eliminated by  $T_{fe}$  corrections.

In exploring the impact of all three degrees of freedom (two for  $n = m + 2$ ), also an alternative ITP may be chosen.

**B. ITP-IAEd**

By using (14) it is possible to propose ITP for specifying the controller parameters guaranteeing some (realistically chosen) required value  $IAE_{dw} = const$

$$IAE_{do} = IAE_{dw} > IAE_{do,0} \tag{25}$$

Thereby, for  $T_d$  specified by (23),  $IAE_{do,0} = 12.639K_sT_d^2$  corresponds to some  $T_e > 0$  and  $m = 0$  in Tab. 2.

*Theorem 3 (Improving Noise Attenuation Under ITP-IAEd):* By increasing  $m$ , the ITP-IAEd carried out with the requirement (25) may to some degree contribute to improved noise attenuation.

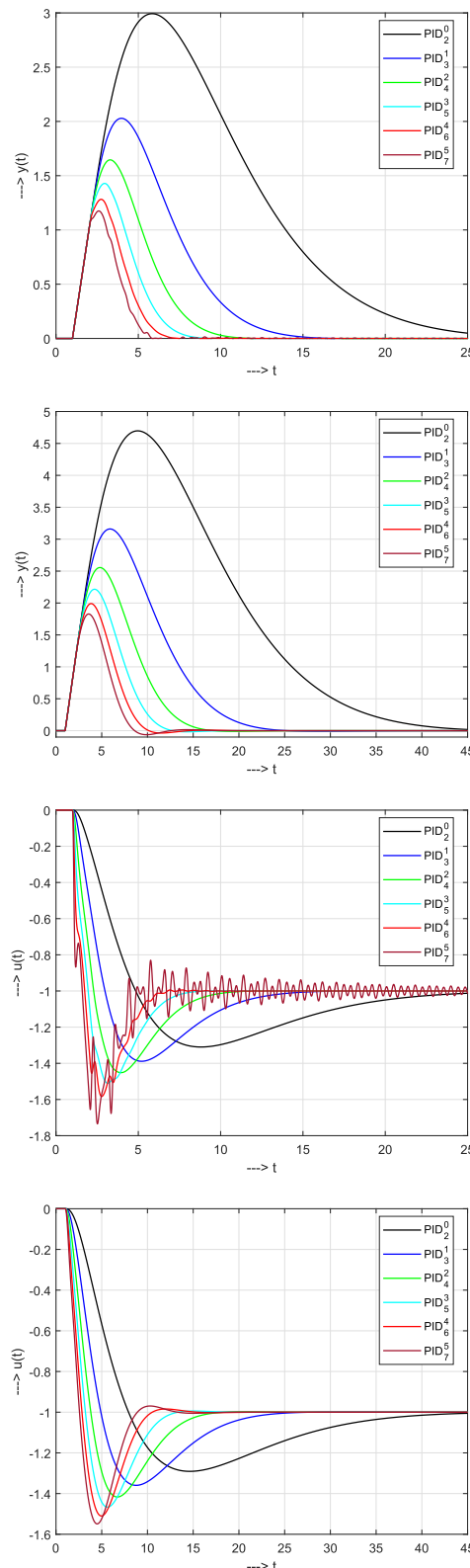
First, it will be shown that for ITP-IAEd based on  $IAE_d = const$ , an improved noise attenuation is achieved by an increase of the highest derivative degree  $m$  of an ideal PID<sup>m</sup>.

Second, due to the imperfections of PID<sub>n</sub><sup>m</sup> tuning, which are increasing with rising  $m$ , it will always be possible to find an optimal  $m$ .

By substituting for  $T_i = T_{dp}\tau_{io}$  and  $K_c = K_o/(K_{sp}T_{dp})$  into (14) with  $\tau_{io}$  and  $K_o$  taken from Tab. 1 it is possible to derive values  $\gamma_d = \sqrt{K_o/\tau_{io}}$  in (26) guaranteeing for chosen PID<sup>m</sup> the required  $IAE_{dw}$  (see Tab. 3). Then, by considering steps 2-5 from Definition 3 with (23) and

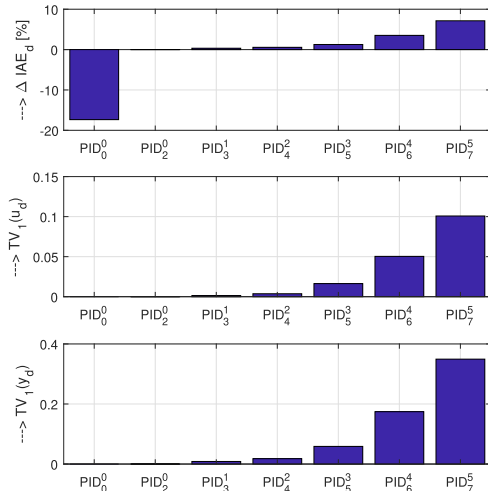
$$T_e = \sqrt{\frac{IAE_{dw}K_o}{\tau_{io}K_{sp}}} - T_{dp} = \gamma_d \sqrt{\frac{IAE_{dw}}{K_{sp}}} - T_{dp} \tag{26}$$

we get new modified tuning formulas. Thereby, the values in Tab. 3 show that for a given  $T_{dp}$  and  $K_{sp}$  and some  $IAE_{dw}$  satisfying (25), it is possible to spend on filtration equivalent dead time  $T_e$  which is increasing by increasing  $m$ . Thus, the increased derivative degrees  $m$  indicate the potential to contribute to stronger noise suppression. However, due



**FIGURE 2.** Verification of the ITP-Te by simulation: plant outputs and inputs for  $T_e = 0.55$  (above) and for  $T_e = 1.5$  (below),  $T_{dp} = 1$ ,  $\delta = 0$ ,  $T_s = 0.001$ ,  $m \in [0, 5]$ .

to the imperfections of the ITP-IAEd (based on the delay equivalence (22) and the MRDP based controller tuning (9)), an increasing  $m$  contributes also to some shape related



**FIGURE 3.** Performance for the disturbance steps with  $\delta = 0$  and tuning (25) with  $IAE_{dw}/IAE_{d,0} = 1.21$ ;  $\Delta IAE_d = IAE_d - IAE_{dw}$ .

deviations at the input and output which makes the final judgment more complicated.

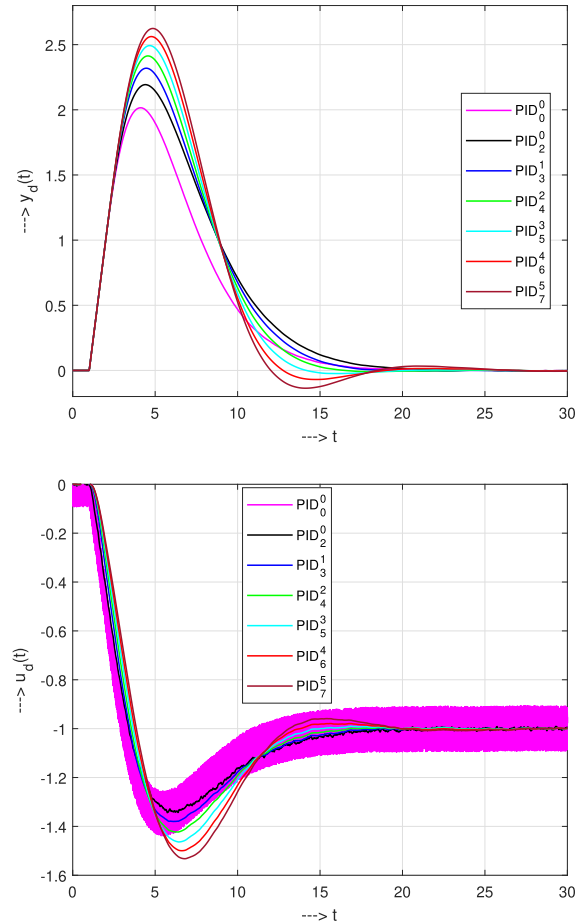
Thus, by simulation of a *noise-free loop*, it can be shown first that due to ITP-IAEd tuning imperfections, with increasing  $m$  the  $IAE_{do}$  values slightly increase over the chosen value  $IAE_{dw}$  (Figure 3). Thereby, also the  $TV_1(u_d)$  and  $TV_1(y_d)$  values increase (even for no external noise  $\delta = 0$ ), which signals deviations from ideal IP shapes at the input and output. For example, with

$$T_{dp} = 1; \quad T_e = 0.1; \quad T_d = T_{dp} + T_e = 1.1. \quad (27)$$

For  $IAE_{dw} = 1.21IAE_{do,0}$  chosen as the reference value for all the filtered alternatives, the non-filtered  $PID_0^0$  controller with  $T_e = 0$  may ideally have  $IAE_{d,0}$  lower by nearly 20%. Therefore, in applications with a negligible noise level, the simplest  $PID_0^0$  still represent an optimal option for practical use. In such situations, the above mentioned shape imperfections of the proposed ITPs may be visible, when they lead to increasing IAE and TV values (Figure 4) and to a moderate undershooting of the disturbance responses at the plant output.

Whereas in loops with *noise amplitudes*  $|\delta| \leq 0.2^1$  (Figure 3) the IAE values do not change significantly and different  $PID_{m+2}^m$  controllers tuned according to (26) for (27) still show similar shapes of transients with some moderate undershooting at the output, the shape related performance measures change in a wide range. For that reason (Figure 5 above), the logarithmic scale is used. They show that with respect to the minimal noise impact at the plant input it makes sense to work with  $m = 5$ , whereas the lowest noise impact at the plant output is obtained with  $m = 2$ .

On the other hand, for the disturbance step responses with a much lower noise level  $|\delta| \leq 0.02$  (Figure 5 below), the minimal noise impact at the plant input corresponds to  $m = 3$ , while the lowest noise impact at the plant output is achieved with  $m = 1$ .



**FIGURE 4.** Unit disturbance step responses at the input and output of the plant (5) with different controllers tuned according to (26),  $|\delta| \leq 0.2$ ,  $T_s = 0.001$ , for  $PID_0^0$  with  $T_e = 0$ , else with  $T_e = 0.1$ ,  $T_{dp} = 1$ ,  $K_{sp} = 1$  yielding  $IAE_{dw}/IAE_{d,0} = 1.21$ .

*Remark 5 (Optimal Controller Tuning for Noisy Loops):* Thus, for noisy loops there exists no “globally optimal” controller tuning. Its optimization has always to consider the existing noise level and the requirements of practice.

*Remark 6 (Experimental Specification of the Optimal Tuning):* Hence, due to the approximative character of the ITP-IAEd (25) derived for  $IAE_d = const$ , the simulation experiments show that for the  $PID_{m+2}^m$  control it yields just  $IAE_d \approx const$  performance with slight deviations from ideal shapes at the plant input and output. These imperfections are, however, negligible with respect to the noise impact. Thereby, without further specification of experimental conditions, it cannot be clearly concluded beforehand, which derivative order  $m$  (with the corresponding filter order  $n = m + 2$ ) is the optimal one. Furthermore, despite the perceived ideas, under  $IAE_d \approx const$  the use of controllers with HO derivatives can be paid just in control of noisy time delayed processes.

## V. SPEED-EFFORT (SE) AND SPEED-WOBBLING (SW) CHARACTERISTICS

*Definition 4 (Speed-Effort and Speed-Wobbling Characteristics):* In order to demonstrate the trade-off between load

<sup>1</sup>generated in Matlab/Simulink by the block “Uniform Random Number”

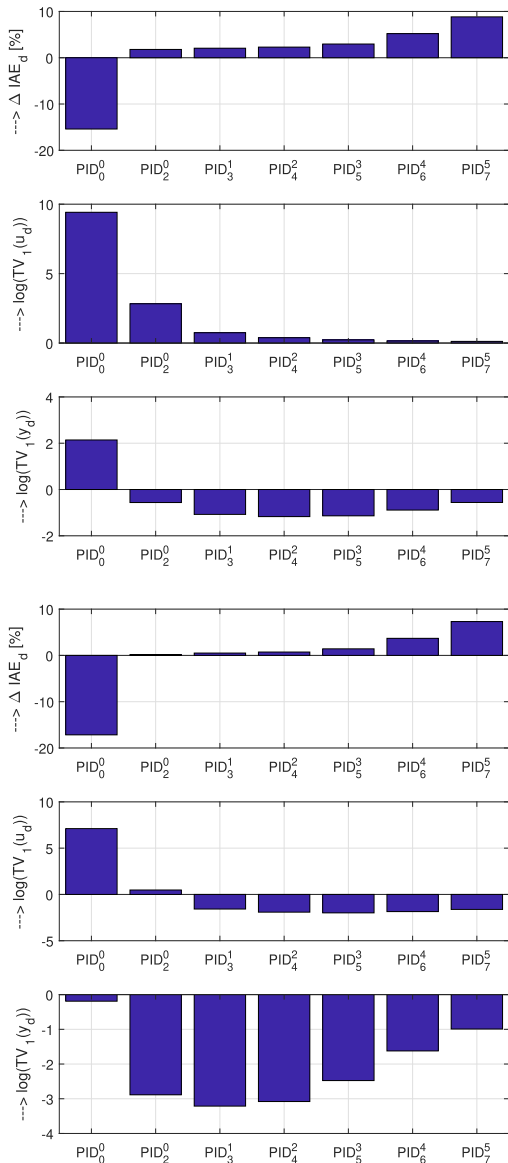


FIGURE 5. Performance measures for the disturbance responses and a measurement noise with  $|\delta| \leq 0.2$  (above) and  $|\delta| \leq 0.02$  (below).

disturbance attenuation and the measurement noise injection under the cost function (4), the input and output characteristics in  $\xi, \eta$  planes will be used<sup>2</sup> with the coordinates

$$\begin{aligned} \xi &= TV_1(u_d), & \eta &= IAE_d^k, \text{ or} \\ \xi &= TV_1(y_d), & \eta &= IAE_d^k \end{aligned} \quad (28)$$

Here they will be denoted as speed-effort and speed-wobbling characteristics.

**A. IDEAL SPEED-EFFORT CHARACTERISTICS - NO EXTERNAL NOISE**

Ideally, in loops without an external noise, the shape related deviations should converge to zero. Their positive values

<sup>2</sup>introduced for  $k = 1$  in [35]

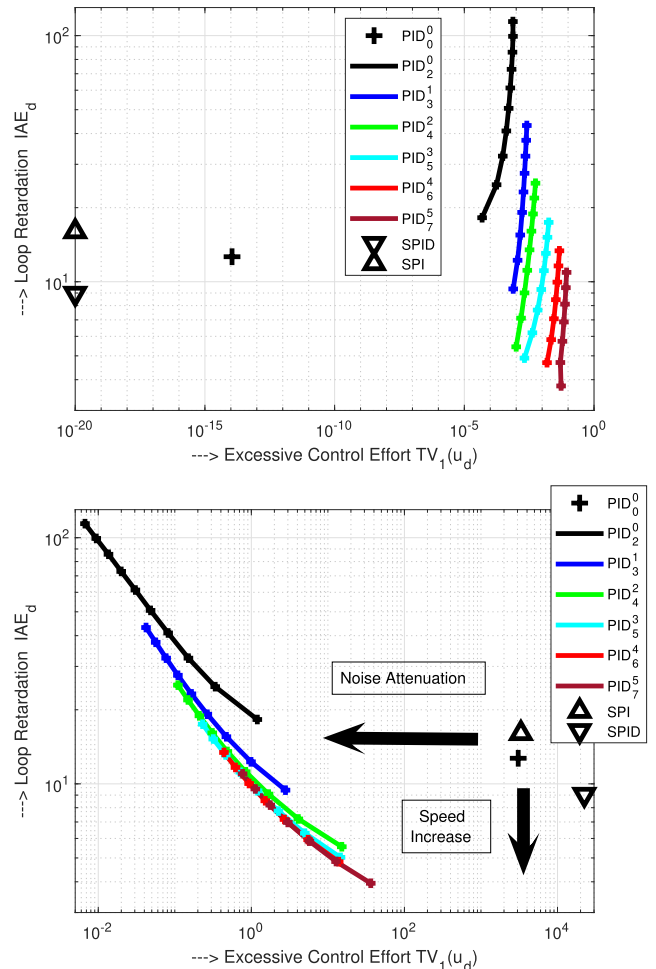


FIGURE 6. SE-characteristics:  $IAE_d$  versus shape related deviations at the input for the plant (5) and different controllers, noise with the amplitudes  $\delta = 0$  (above) and  $|\delta| \leq 0.1$  (below) generated in Matlab/Simulink by the block “Uniform Random Number”,  $T_s = 0.001$ , for  $m = 0$ ,  $T_e/T_{dp} \in [0.2, 2]$ ; for  $m = 1, 2$   $T_e/T_{dp} \in [0.4, 2]$ ; for  $m = 3$ ,  $T_e/T_{dp} \in [0.6, 2]$ ; for  $m = 4, 5$ ,  $T_e/T_{dp} \in [0.8, 2]$ ;  $\Delta T_e = 0.2T_{dp}$ ,  $T_{dp} = 1$ ; SPI=SIMC-PI, SPID=iSIMC-PID according to [13].

may result from the final precision of numerical calculations (resulting in an “internal noise”) and from non-perfect shapes of transients caused dominantly by the simplified controller tuning. By increasing  $T_e$  and using the ITP with  $IAE_d = const$  for a negligible measurement noise, the speed-effort characteristics in Figure 6 above, should ideally move vertically. Due to the above mentioned ITP imperfections, the corresponding characteristics are slightly bent: all the considered filtered controllers (including the filtered PI control denoted as  $PID_2^0$ ) exhibit certain levels of an excessive input effort, which is higher than for the simplest non-filtered PI control ( $PID_0^0$ ).

**B. SIMC AND iSIMC TUNING RULES FOR TRADITIONAL PID CONTROL**

As a reference example for comparison with the traditional PID control, the SIMC tuning rules have been chosen with respect to their ambition to be used as the best simple PID tuning rules in the world [36].



The SIMC PI controller [9], [13] denoted as SPI

$$SPI : C(s) = K_c \frac{1 + sT_i}{sT_i}; \quad K_c = \frac{0.5}{K_s}; \quad T_i = 8T_d \quad (29)$$

and the series iSIMC PID controller [13]

$$SPID : C(s) = K_c \frac{(1 + sT_i)(1 + sT_D)}{sT_i(sT_D/N + 1)}$$

$$K_c = \frac{0.5}{K_s T_d}; \quad T_i = 6T_d; \quad T_D = 0.33T_d; \quad N = 5 \quad (30)$$

have been included in the comparative framework. The recently published improved set of tuning rules denoted as iSIMC (improved SIMC) introduces the PID controller (30) as a possibility to reduce  $IAE_d$  of IPDT systems by 40%. As obvious from Figure 6 above, without an external noise, both SPI and SPID yield extremely smooth transients. However, their authors warn that SPID can only be used for noise-free signals.

**C. SPEED-EFFORT CHARACTERISTICS WITH EXTERNAL MEASUREMENT NOISE**

When working with statistical variables, different probability distributions are typically used. Since in practice they require relatively long measurement intractable under frequently changing working conditions, the presented approach does not consider their identification.

*Remark 7 (Sampling Period Impact):* In the numerical simulation of continuous-time processes, the sampling period  $T_s$  has to be chosen relatively short with respect to the shortest time constants considered. For higher-order filters tuned according to Section IV, it approximately holds  $nT_f \approx T_e$ . In order to keep the ratio  $T_f/T_s \gg 1$  also for higher  $n$ , the sampling time should fulfill the requirement

$$T_s \ll T_e/n \quad (31)$$

In Matlab/Simulink simulation limited for  $k = 1$  to the SE ( $IAE - TV_1(u)$ ) characteristics, the measurement noise has been generated by the block "Uniform Random Number" with the sampling period  $T_s = 0.001T_{dp}$  and with several amplitudes  $|\delta|$ .

*Remark 8 (Specifics of Noisy Measurement Evaluation):* In a  $PID_{m+2}^m$  control evaluation according to the chosen performance measures (1)-(3) characterizing speed of the transients and the input usage (controller activity), it is to remember that in a noisy loop both performance measures permanently increase and thus they depend on the length of the measurement window. Thus, it is possible to compare just results corresponding to the same  $t_{max}$ , in our case  $t_{max} = 100$ .

Under external measurement noise, the excessive input increments due to the ITP and simulation imperfection from Figure 6 above show to be negligible in comparison with those appearing due to the external (measurement) noise in Figure 6 below. Due to such a noise induced error,

the  $IAE_d$  values are slightly increased (vertical shifts). On the other side, much stronger horizontal shifts reflect significant changes in performance measures corresponding to excessive input increments.

By a filtration corresponding to  $IAE_d \approx const$ , the excessive plant input effort of the nonfiltered  $PID_0^0$ , SPI, or SPID control, evaluated in terms of  $TV_1(u_d)$ , may be decreased nearly  $10^4$  times. Hence, the new degrees of freedom may be used not just for noise attenuation (horizontal shift), but also for accelerating transient responses (vertical shift of the working point), or for both of them. Particular properties depend both on  $m$  and  $T_e$  which may seem to lead to an ambiguous tuning. However, by analyzing the speed/wobbling ( $IAE - TV_1(y)$ ) characteristics, it may be recommended to work with values  $m$  [33] as low as possible. This problem will also be examined from the robustness point of view.

For  $PID_2^0$ , noise attenuation may be achieved just on the cost of slower dynamics, since the transients are always slower than for the non-filtered PI control.

**VI. ROBUSTNESS ISSUES**

Next, we will analyze how a possible plant - model mismatch influences the closed loop stability and performance.

**A. SIMPLE ROBUSTNESS TESTS AND NEW PERFORMANCE SENSITIVITY MEASURES**

The robustness analysis based on sensitivity functions [12]

$$M_s = \max \left\{ \left| \frac{1}{1 + L(j\omega)} \right| \right\}; \quad M_t = \max \left\{ \left| \frac{L(j\omega)}{1 + L(j\omega)} \right| \right\}$$

$$\omega \geq 0; \quad L(s) = PID_n^m S(s) \quad (32)$$

defined for the nominal plant (5) and the  $PID_{m+2}^m$  controller is primarily related to the loop stability.

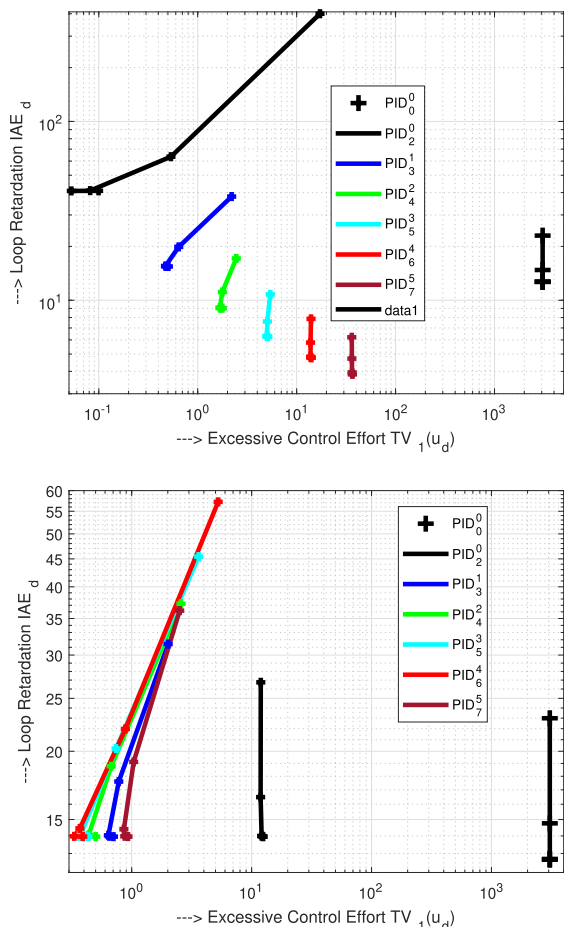
In order to test the closed loop robustness corresponding to particular ITPs from the previous section,  $PID_{m+2}^m$  controllers derived for the integrating plant (5) will be applied to the plant

$$S(s) = \frac{Y(s)}{U(s)} = \frac{e^{-s}}{s + a}; \quad a \in [-0.2, 0.2] \quad (33)$$

Its internal feedback quantified by the pole  $s = -a$  transforms the IPDT plant into the first-order-time-delayed (FOTD) system.

By a simple counter-example in [37], it has been shown that from the performance point of view the robust control design based on the traditional sensitivity functions (32) does not necessarily lead to the intuitively expected results. This gives us the motivation to look for alternative more eloquent measures.

By changing  $a$  in (33), the performance measures corresponding to some chosen  $PID_{m+2}^m$  controllers trace out trajectories in the SE ( $IAE_d - TV_1(u_d)$ , Figure 7) plane. Under particular controller type, longer trajectories correspond to more significant performance changes and thus also to higher sensitivity (lower closed loop robustness). When denoting the



**FIGURE 7.** Robustness characteristics expressing  $IAE_d$  changes due to uncertainty of  $a$  versus the shape related deviations at the input of the plant (33),  $\Delta a = 0.1$  and different controllers tuned with  $T_e = 0.8$  (above) and  $IAE_{dw} = 13.93$  (25) (below), noise with the amplitudes  $|\delta| \leq 0.1$  generated in Matlab/Simulink by the block “Uniform Random Number”,  $T_s = 0.001$ .

particular uncertain parameter values as

$$a_i = a_{min} + (i - 1)\Delta a; \quad i = 1, 2, \dots, N$$

$$\Delta a = (a_{max} - a_{min}) / (N - 1) \quad (34)$$

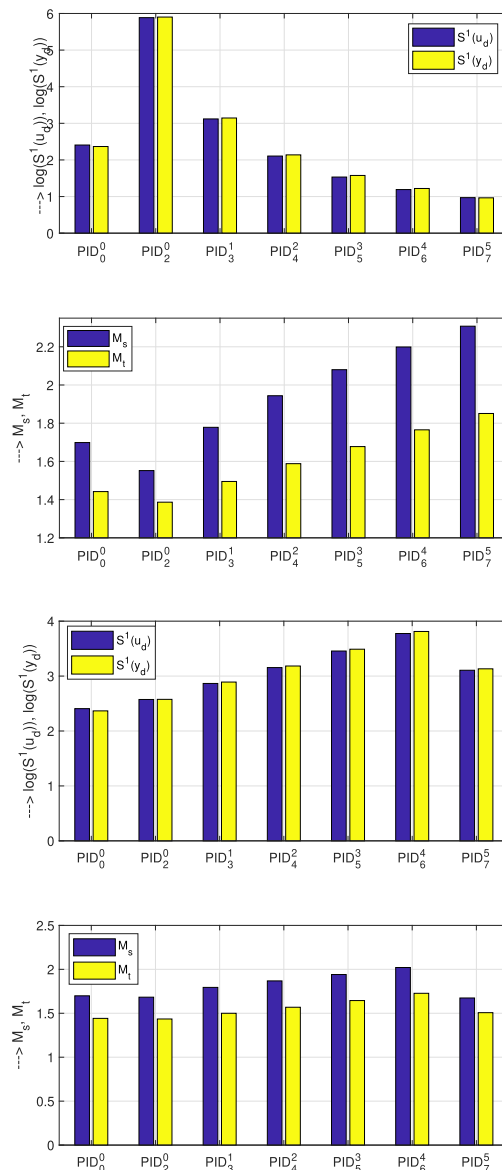
according to (28) with  $\xi = TV_1(u_d)$ , the corresponding sensitivity measure for the disturbance responses at the plant input may be introduced as

$$S^k(u_d) = \sum_{i=1}^{N-1} \sqrt{(\xi_i - \xi_{i+1})^2 + (\eta_i - \eta_{i+1})^2} \quad (35)$$

Similarly, for  $\xi = TV_1(y_d)$  the sensitivity measure of the disturbance responses at the plant output may be defined as

$$S^k(y_d) = \sum_{i=1}^{N-1} \sqrt{(\xi_i - \xi_{i+1})^2 + (\eta_i - \eta_{i+1})^2} \quad (36)$$

*Remark 9 (Contradictions of the Traditional Sensitivity Measures):* For the ITP-Te (24) with  $T_e = const$  and  $k = 1$ , the lowest excessive changes at the controller output (i.e. those with the shortest trajectories in Figure 7 above), are obtained with  $PID_7^5$  control. The traditional sensitivity measures (32) take the optimal values for the  $PID_0^0$  control



**FIGURE 8.** Above two bar-graphs: new sensitivities (35), (36) and the traditional sensitivity functions for the plant (33) with uncertain  $a$ ,  $\Delta a = 0.1$  and  $PID_{m+2}^m$  controllers tuned according to (24),  $T_s = 0.001$ ;  $T_e = 0.8$  and (below two bar-graphs) for  $IAE_{dw} = 13.93$  (25),  $|\delta| \leq 0.1$ ,  $T_s = 0.001$ ,  $T_{e0} = 0.05$  (right).

(Figure 8 above) for which the real loop behavior (reflected better by the new measures) shows the highest sensitivity. Furthermore, by increasing  $m$ , when the traditional sensitivity peaks already exceed the textbook recommendations  $M_s \in [1.2, 2]$ , the new  $S^1(u_d)$  sensitivity values (35)-(36) decrease to their optima. This again confirms the inconsistency and counter-productivity of the traditional sensitivity measures. Here, it is obviously incorrect to conclude that “higher values of  $M_s$  and  $M_t$  lead to oscillatory responses to step disturbances and higher overshoots for step references” [12].

For the ITP-IAEd and increasing  $m$ , the trajectories in the SE plane stretch (Figure 7 below). The loop robustness decreases and the noise attenuation significantly increases up

to  $m = 4$ . In this case (Figure 8 below), both the traditional and the new sensitivities yield the same trends.

In order to be able to cover both sets of experiments, in the context of robust performance evaluation we will preferably use the newly introduced sensitivity measures (35)-(36).

*Remark 10 (Application to Stable/Unstable FOTD Plants):* Under both ITPs, the positive and negative perturbations of  $a$  have a strongly asymmetric impact on the loop performance (Figure 7). Since the performance changes for  $a > 0$  are negligible, we may ask, if it is eventually possible to use the controller derived for the IPDT plant with  $a = 0$  as a nearly-optimal one at least for stable FOTD plants ( $a > 0$ ), i.e. without identifying the third model parameter  $a$ .

### B. ANTI-WINDUP CONTROLLER IMPLEMENTATION

In practice, we have to take into account that every process input signal is limited (e.g. opening of a valve, motor speed, power of the heater, etc.). When speeding up processes by HO-PID control, due to the limitations, the windup phenomenon (exaggerated process output overshoots) can occur [38]–[41]. Significant output overshooting or undershooting (as in Figure 9) considerably prolong the transients and increase the excessive control effort. Although the impact of redundant integration is worst in the case of filtered PI control, the implementation of the HO-PID controller, without the appropriate anti-windup solution, is, therefore, quite limited in practice.

One of the most successful and simple anti-windup solutions is the Conditioning technique method [42]. In principle, the method is based on dividing the proper controller transfer function into two parts: the direct part and the feedback part, where the direct part is the controller’s gain at high frequencies.

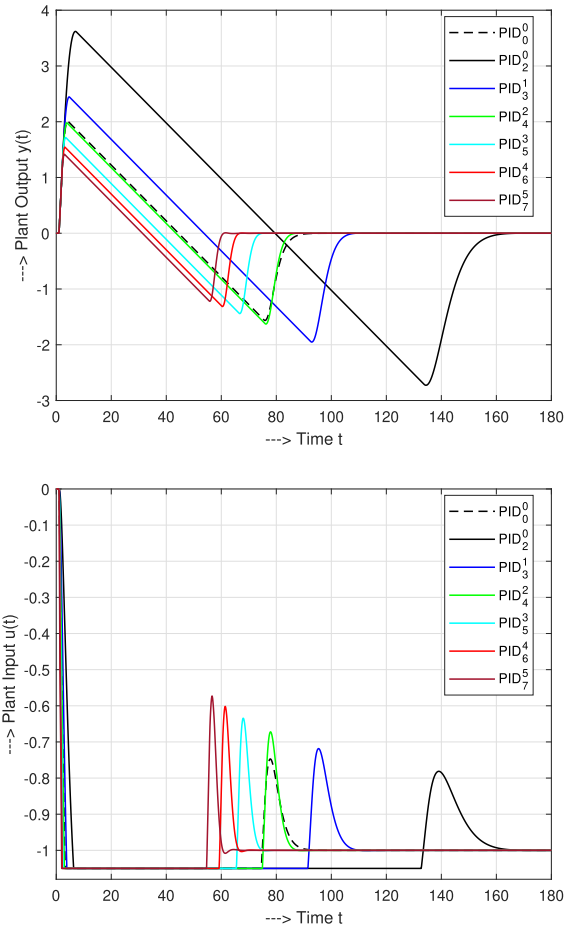
For constrained PI control, the anti-windup controller implementation in Figure 11 is close to the series “automatic reset” controllers [12], [39].  $K_{CD} = K_c$  and the I action is implemented using positive feedback from the limited output of the controller. The time constant of the delay  $1/(1 + T_i s)$  of the used feedback gives directly the integration constant  $T_i$ . Any filter is included beyond the saturation.

Let us illustrate the method on the  $PID_2^2$  controller with the second order derivative and the same order filter:

$$U(s) = G_C(s)E(s) = \frac{(K_0 + K_1 s + K_2 s^2 + K_3 s^3)}{(s(1 + sT_f)^2)} E(s) \quad (37)$$

where  $G_C(s) = PID_2^2(s)$  is the controller transfer function,  $K_0 = K_c/T_i$ ,  $K_1 = K_c$ ,  $K_2 = K_c T_{D1}$  and  $K_3 = K_c T_{D2}$  are the integral, proportional, derivative and accelerative gains of the controller, respectively,  $T_f$  is the filter time constant and  $E(s)$  is control error signal. By a simple block manipulation, the equivalent controller can also be realized in the following form:

$$U(s) = K_{CD}E(s) + G_{CFB}U_r(s) \quad (38)$$



**FIGURE 9.** Unit disturbance step responses at the input and output of the plant (5) with  $PID_n^m$  controllers implemented according to Figure 10 ( $G_c = PID_n^m$ ) followed by saturation to an admissible interval  $u \in [-1.05, 0.05]$ ,  $T_e = 0.9$ ,  $T_{dp} = 1$ ,  $K_{sp} = 1$ .

where  $K_{CD}$  is the controller gain at high frequencies (obtained by calculating  $\lim_{s \rightarrow \infty} G_C(s)$ ):

$$K_{CD} = \frac{K_3}{T_f^2} \quad (39)$$

and

$$G_{CFB} = \frac{K_0 + \left(K_1 - \frac{K_3}{T_f^2}\right)s + \left(K_2 - \frac{2K_3}{T_f}\right)s^2}{K_0 + K_1 s + K_2 s^2 + K_3 s^3} \quad (40)$$

The corresponding block manipulation is also illustrated in Figure 10 below. The Conditioning technique slightly modifies the expression (38) in such a way that the signal, fed to the feedback part of the controller, should be the limited (restricted) control signal  $U_r$  instead of the unlimited signal  $U$ :

$$U(s) = K_{CD}E(s) + G_{CFB}U_r(s) \quad (41)$$

which is illustrated in Figure 11.

In a similar manner, the higher-order controllers can be obtained as well. For example, the fourth-order controller  $PID_3^3$  with  $K_0 = K_c/T_i$ ,  $K_1 = K_c$ ,  $K_2 = K_c T_{D1}$   $K_3 = K_c T_{D2}$

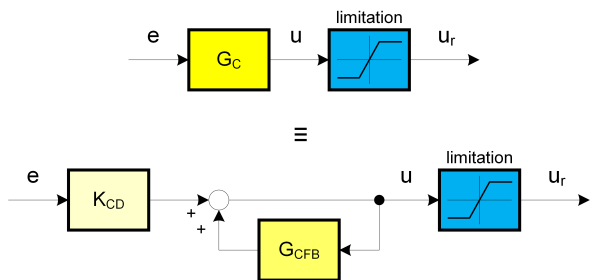


FIGURE 10. The controller realizations without anti-windup protection. The upper and the lower controller realizations are equivalent.

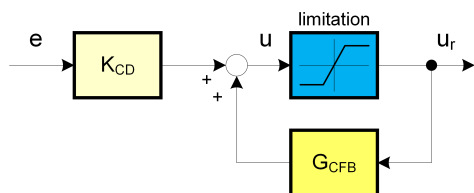


FIGURE 11. The controller with the Conditioning technique anti-windup protection (self-conditioning realization).

and  $K_4 = K_c T_{D3}$ :

$$U(s) = G_C(s)E(s) = \frac{(K_0 + K_1s + K_2s^2 + K_3s^3 + K_4s^4)}{(s(1 + sT_f)^3)} E(s) \quad (42)$$

can be realized as:

$$K_{CD} = \frac{K_4}{T_f^3} \quad (43)$$

and

$$G_{CFB} = \frac{K_0 + \left(K_1 - \frac{K_4}{T_f^3}\right)s + \left(K_2 - \frac{3K_4}{T_f^2}\right)s^2 + \left(K_3 - \frac{3K_4}{T_f}\right)s^3}{K_0 + K_1s + K_2s^2 + K_3s^3 + K_4s^4} \quad (44)$$

If the controller transfer function  $G_C(s) = \text{PID}_n^m$ ,  $n > m$  is strictly proper (the denominator's degree is higher than the numerator's degree):

$$G_C(s) = \frac{K_0 + K_1s + \dots + K_{m+1}s^{m+1}}{s(1 + sT_f)^m(1 + sT_f)^k} \quad (45)$$

where  $k = n - m > 0$ , the  $G_C(s)$  can be divided into two terms:

$$G_C(s) = G_{C0}(s)G_{FR}(s), \quad (46)$$

the term  $G_{C0}(s)$  with the relative degree equal to zero and the remaining filter term  $G_{FR}(s)$ :

$$G_{C0}(s) = \frac{K_0 + K_1s + \dots + K_{m+1}s^{m+1}}{s(1 + sT_f)^m} \quad (47)$$

$$G_{FR}(s) = \frac{1}{(1 + sT_f)^{n-m}}$$

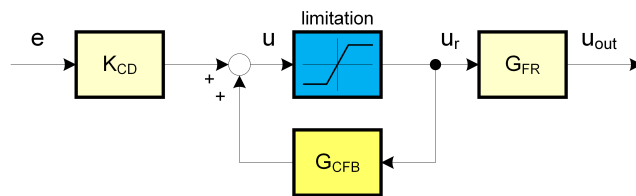


FIGURE 12. The realization of the controller with anti-windup protection for strictly proper controllers.

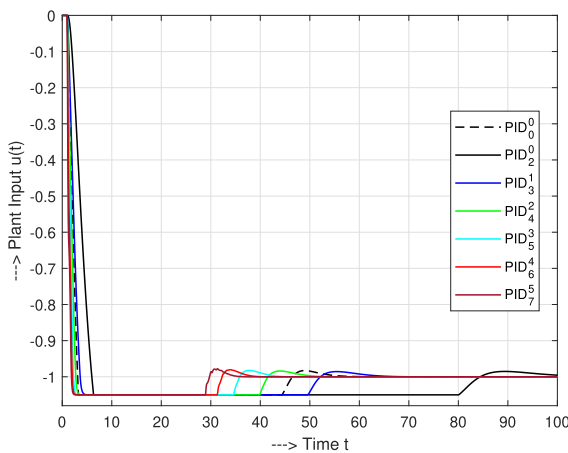
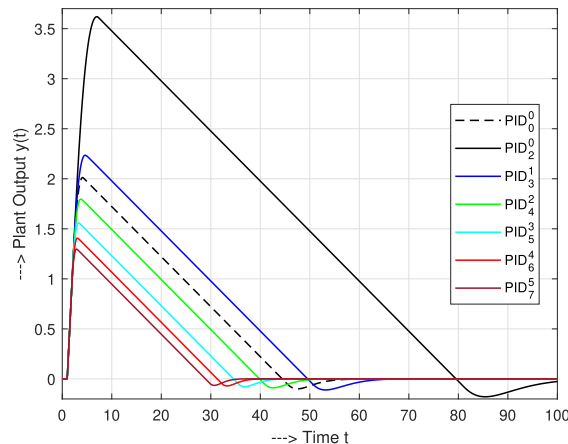


FIGURE 13. Unit disturbance step responses at the input and output of the plant (5) with anti-windup  $\text{PID}_n^m$  controller implementation according to Figure 12 ( $G_C = \text{PID}_n^m$ ) followed by saturation to an admissible interval  $u \in [-1.05, 0.05]$ ,  $T_e = 0.9$ ,  $T_{dp} = 1$ ,  $K_{sp} = 1$ .

The actual realization of the controller with anti-windup protection is then the one depicted in Figure 12, where  $K_{CD}$  and  $G_{CFB}$  are calculated from  $G_{C0}(s)$ .

The transients corresponding to the anti-windup implementation of the  $\text{PID}_n^m$  control in Figure 13 show a significant reduction in transients settling time to about 60% compared to Figure 9 as well as a significant reduction in the undershooting of the output variable. Excessive control efforts will also be significantly reduced.

### VII. EXPERIMENTAL EVALUATION

To illustrate the above design and to compare the carried out simulation analysis with real time experiments, the  $\text{PID}_n^m$



FIGURE 14. Arduino-Due based thermo-opto-mechanical laboratory system TOM1A.

control has been applied to the thermal channel of the thermo-opto-mechanical laboratory TOM1A system [43] (Figure 14). It consists of a heat source (bulb 5W/12V), temperature sensor (Pt1000), and a cooling fan which together yield a strongly nonlinear system with a considerable measurement noise. Typical thermal plants are with several modes of heat transfer (at least two - the fast and slow) [44], which require a special pole-assignment controller. Because the situation with two modes has not been addressed yet by the analysis considering measurement noise attenuation and robustness, the question arises as to whether the designed  $PID_n^m$  controllers are sufficiently robust even in such a situation. Since in the majority of situations the plant has a monotonic step response [45] which may well be approximated by the FOTD model, with respect to Remark 10 in Section VI-A, also PI control and its generalizations may be expected to yield satisfactory results.

*Remark 11 (A Typical Problem of Traditional Optimization):* For systems with two modes, the PI controller tuning derived by the non-convex optimization [46] illustrates the typical handicaps of traditional optimization approaches:

- 1) They may be applied just to plants characterized by a fixed set of identified parameters considered in the carried out optimization (In [46] the authors noticed a similarity between the controller tunings for the plants with one (fast) and two (fast and slow) modes, but did not notice the tuning similarities for the one-mode static and integral models.)
- 2) Neither they allow modification of the resulting performance - in the given example [46] one could require avoidance of overshooting and achievement of possibly fast transients with acceptable noise impact and a minimal number of monotonic intervals.

Hence, when wishing to use the controller tuning based on a traditional optimization method, it is necessary to repeat it for each new set of identified model parameters and performance requirements.

**A. SIMPLIFIED PLANT IDENTIFICATION**

With respect to Remark 10, we may also ask if the  $PID_n^m$  controller design based on the IPDT model (which does not require identification of the internal plant feedback parameter  $a$ ) is able to yield nearly optimal responses also for thermal plants having a significantly different stable dynamics.

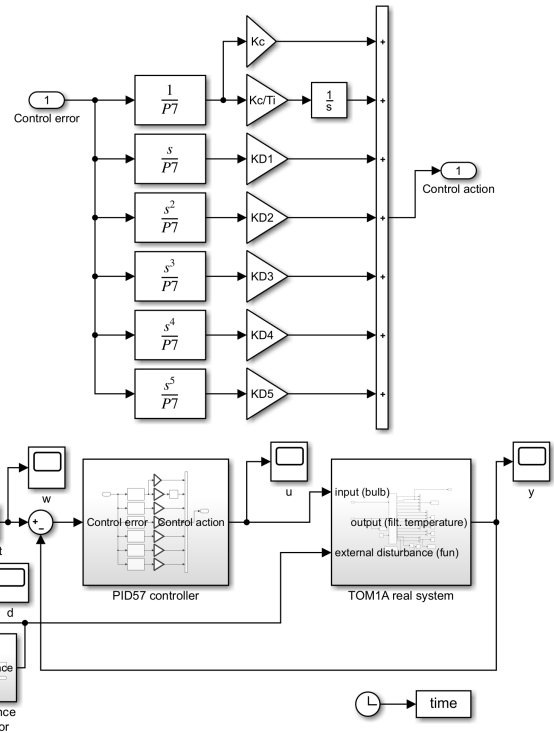


FIGURE 15. Matlab/Simulink control scheme for real time experiment (below) with  $PID_7^5$  controller sub-block (above);  $KDi = K_c T_{Di}; i \in [1, 5]$ ,  $P7$  corresponds to  $(1 + T_f s)^7$ .

Thereby, one of the main advantages of the IPDT based approximations is that to get the plant model it is enough to identify just the introductory part of its step response and to replace a possibly long lasting complete step response measurement with a significantly shorter response to a rectangular pulse.

According to [43], the model parameters (5)

$$K_{sp} = 0.01; \quad T_{dp} = 5.5s \quad (48)$$

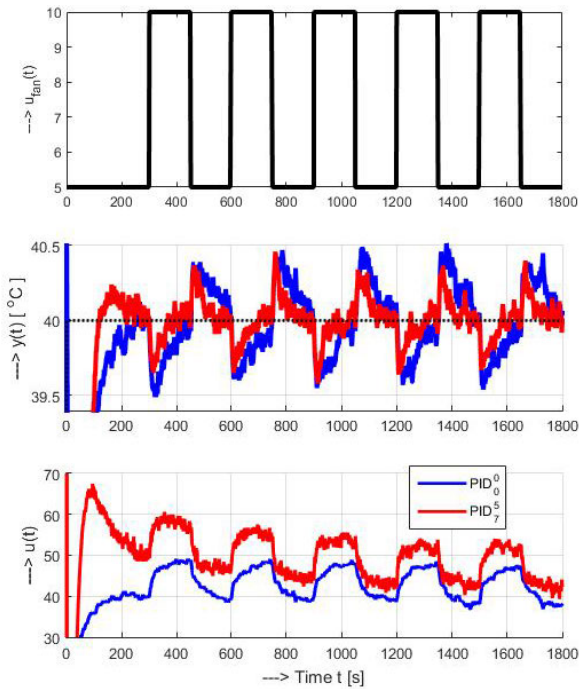
have been determined to approximate the steepest initial segment of the step response curve.

**B. EXPERIMENT ORGANIZATION - PERIODIC COOLING SIGNAL**

The control experiments based on the control scheme in Figure 15 have been organized in such a way that the output temperature was first set by the  $PID_7^5$  controller to the reference setpoint level of  $40^\circ C$  (Figure 16) with the cooling fan input equal to  $u_{fan,2} = 5$ . Then the fan has been periodically excited (with the period 300s) by changing its input between levels  $u_{fan,1} = 5$  and  $u_{fan,2} = 10$ . The resulting performance measures correspond to average values calculated of 10 measured disturbance step responses.

Next, the experiment continued with the decreased derivative action degree of the  $PID_6^4$  controller. As the last in the sequence, the  $PID_0^0$  and  $PID_0^0$  controllers have been applied.

In the evaluation, it is to remember that when starting with a “cold” system under  $PID_7^5$  control, the slow heat transfer



**FIGURE 16.** Overall transients for  $PID_7^5$ ,  $T_e = T_d$  and  $PID_0^5$  consisting of the setpoint step response to  $w = 40$  [°C] within  $t \in [0, 300]$ s and 5 cycles of the periodic disturbance fan signal  $u_{fan,1} = 5$  and  $u_{fan,2} = 10$  with the period 300s,  $T_s = 0.03$ s.

mode by convection leads to a slow decline of the control signal (Figure 16 below), whereas the output remains in the vicinity of the reference value (analogy of the “zero dynamics” known from nonlinear feedback linearization [47]).

**C. EQUIVALENT DELAY CHOICE**

Above described sequences including all the considered controllers have been carried out for 3 different values

$$T_e = \{0.5, 1, 1.5\}T_{dp}, \quad T_s = 0.03s \quad (49)$$

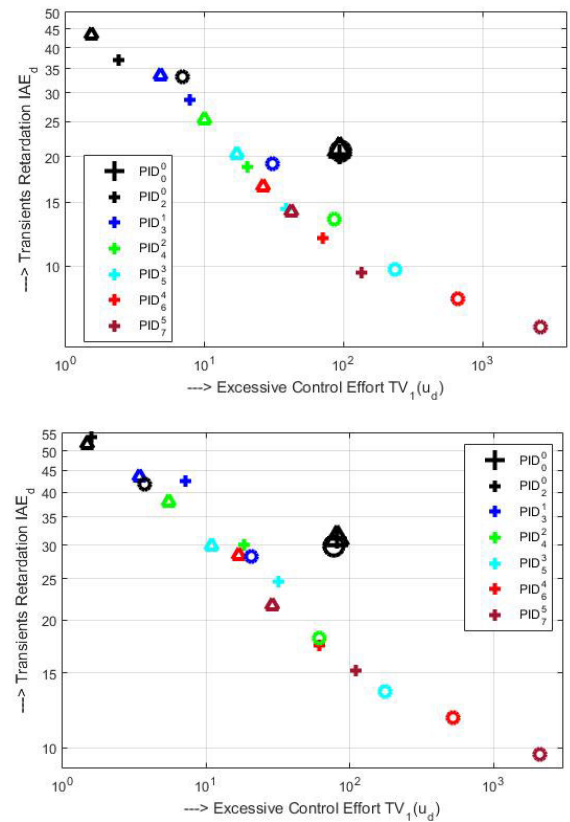
Results of these experiments are displayed in the SE plane  $IAE_d - TV_1(u_d)$  (Figure 17) for three values of the dead time estimate

$$T_{dp} = \{4.5, 5.5, 6.5\}s \quad (50)$$

(for roughly  $\pm 18\%$  changes of the identified  $T_{dp}$  value (48), Figure 17). In all three cases, at the output, we may see results in line with the expected speed-effort characteristic in Figure 6 below. They indicate that new HO-PID controllers are not more sensitive to determination accuracy of  $T_{dp}$  than the traditional non-filtered PI. Thereby, the possibility to push  $PID_{m+2}^m$  working points to the left, or below the  $PID_0^0$  working point does not significantly depend on the accuracy of the  $T_{dp}$  identification.

**D. MODIFIED ROBUSTNESS TEST**

More detailed robustness evaluation with calculating  $S^k(u_d)$  according to (35) for  $k = \{1, 5\}$ ,  $i \in [1, 5]$  and  $T_e$  (49)



**FIGURE 17.** Performance evaluation of the disturbance responses for  $T_{dp} = 4.5s$  (above)  $T_{dp} = 5.5s$  (middle) and  $T_{dp} = 6.5s$  (below):  $T_d = T_{dp} + T_e$ ,  $T_e = 0.5T_{dp}$  (o),  $T_e = T_{dp}$  (+),  $T_e = 1.5T_{dp}$  ( $\Delta$ ).

focusing on  $T_{dp}$  impact has been based on real time experiments repeated for

$$T_{dp} = \{4.5, 5, 5.5, 6, 6.5\}s \quad (51)$$

In Figure 18 above, mean values of  $IAE_d$  and  $TV_i(u_d)$  from 5 upwards and 5 downwards steps measurements over the interval of  $T_{dp}$  values confirm expectations that the low-pass filtration reduces the excessive control effort on costs of increased loop retardation which may be eliminated by increased derivative action.

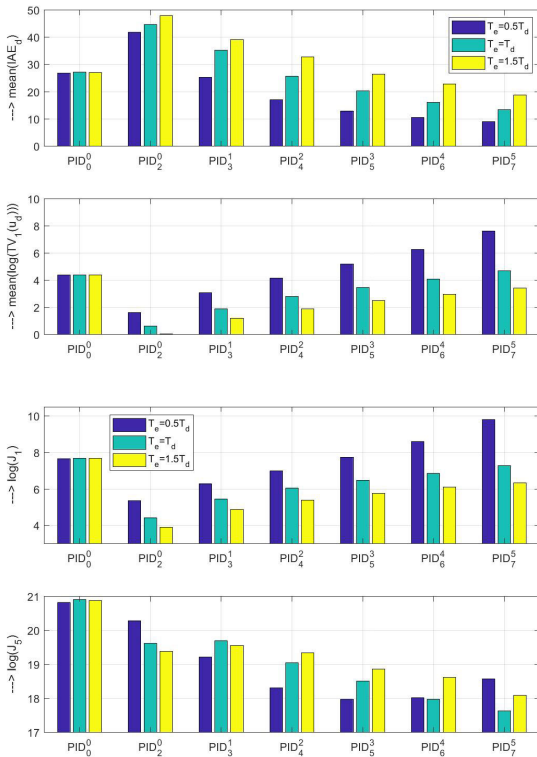


FIGURE 18. Mean performance measures for the real time disturbance responses evaluated for  $T_{dp}$  (51) with three different values of  $T_e$  (49);  $T_s = 0.03s$  (above) and the corresponding mean values of the “holistic” cost function (4) calculated for  $k = 1$  and  $k = 5$  (below).

The optimal balance between loop retardation (speed of transients) and excessive control effort can be analyzed by introducing a cost function (4) combining both main performance attributes with different weighting coefficients. For all three  $T_e$  values (49), with  $k = 1$  (Figure 18 below) the optimal transients correspond to  $PID_2^0$ . The nonfiltered PI (i.e.  $PID_0^0$ ) yields worse performance than the majority of the filtered alternatives.

For  $k = 5$  (emphasizing fast transients) and  $T_e = 0.5T_{dp}$  the optimum moves to  $PID_3^3$  and for the remaining two  $T_e$  values to  $PID_7^5$ . PI control lags behind even the most filtered solutions. In this way, by stressing the speed of transients, the optimum would obviously move to higher  $m$  and lower  $T_e$ .

The sensitivity of particular controllers evaluated according to (35) may be illustrated in Figure 19. The traditional PI obviously lags behind numerous filtered solutions. For  $k = 1$ , significant differences between the controlled system and its approximation are manifested by higher sensitivity at  $T_e = 0.5T_{dp}$ . On the other hand, for  $k = 5$  higher sensitivities correspond to  $T_e = 1.5T_{dp}$ .

VIII. DISCUSSION

By offering a broad spectrum of achievable performance variations, results of real time experiments clearly demonstrate the benefits of the proposed PID generalizations. Similarly, they also confirmed the limited acceptability of the traditional

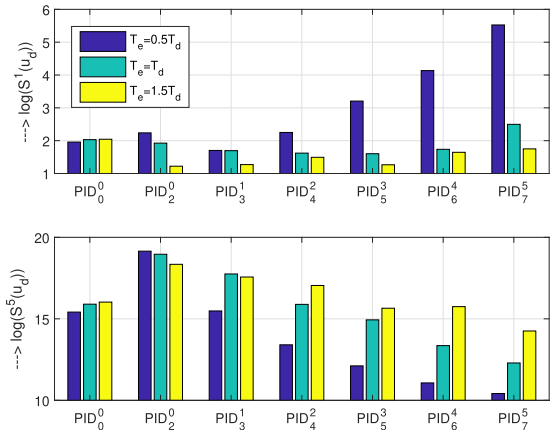


FIGURE 19. Evaluation of the sensitivity (35) with respect to  $T_{dp}$  (51) for three different values of  $T_e$  (49) with  $k = 1$  (above) and  $k = 5$  (below);  $T_s = 0.03s$ .

robustness measures and the necessity to use more concise robustness indicators.

A. PERFORMANCE BENEFITS

In the SE plane (Figure 17), limits of the achievable performance may be roughly approximated by a falling, just slightly bend, curve shifted to the left and below of the reference  $PID_0^0$  controller working point. Again, the results of the simulation analysis from Subsection V-C have been fully confirmed. By moving the working point of the non-filtered PI to this limit curve horizontally (by keeping  $IAE_d = const$ ), the excessive control effort may be reduced slightly less than 10 times. By the vertical movement, in the nominal case, the loop retardation evaluated in terms of  $IAE_d$  may be reduced to nearly a half when compared to  $PID_0^0$ . A more radical  $IAE_d$  decrease to approximately 1/3 of the reference  $PID_0^0$  level may be achieved at the cost of significantly increased control effort. As the key factor of these performance improvements, we may denote the sampling period available.

B. AS SIMPLE AS PI CONTROL

Tuning of the seemingly complex multi-parameter  $PID_n^m$  (or in our case  $PID_{m+2}^m$ ) controller is still based on the two-parameter IPDT plant model (48). From this point of view, it fully preserves the simplicity of  $PID_0^0$  tuning. Similarly to looking for the “optimal” closed loop poles location in the pole-assignment controller tuning, the free parameters  $m$  and  $T_e$  may be determined by a “trial and error” approach.

C. IMPLEMENTATION PROBLEMS

Concerning the implementation complexity, higher order filters require some additional computational power. However, similar challenges solved with FO-PID controllers [3], [48]<sup>3</sup> do not offer simpler solutions. The numerical problems which

<sup>3</sup>Implementation of FO-PID controllers is generally based on approximations by higher order IO transfer function [5], [8], [49], but together with the fractional-orders choice it further requires specification of several additional filter parameters

may degrade the simulation of loops with long dead time for short  $T_e$  [33], [50] disappeared in real time control, where the solvers restricted themselves to the (much simpler) controller simulation. In the carried out thermal plant control, the limitation (31) due to the sampling period value was not active. For example, for  $T_{dp} = 5\text{s}$ ,  $T_e = 0.5T_{dp}$  and  $n = 7$  calculated according to (22), the time constant  $T_{fe} = 0.3643\text{s}$  obviously fulfills the filtration condition  $T_{fe} \gg T_s = 0.03\text{s}$ .

#### D. LIMITATIONS OF THE CARRIED OUT ANALYSIS

Note that the possibilities of accelerating the system dynamics by the reduction of  $T_e$  are always limited, since at low  $T_e$  values the amplitudes of the noise-excited control tend to overlap the entire admissible control range and, due to the control limitations, they degrade performance and endanger the stability of the transient responses. The increasingly noise-excited process output contributes to increased  $IAE_d$  values. Hence, the corresponding  $IAE_d$  may start to increase, even though the transient responses seem to be faster.

#### E. MAGIC OF INTEGRAL MODELS

Once a chosen controller based on the IPDT model guarantees acceptable results for the particular (stable) plant, its tuning is supposed to be close to the tuning achievable by the more complex FOTD models. In  $PID_n^m$  control this convergence improves with increasing  $m$  [26]. Starting from the controller tuning by Ziegler and Nichols [51], similar properties applicable to simplification of the controller tuning may also be found in other model-based control structures [50], [52], even when they use the adjective model-free. The magic of integral models denoted as “ultra-local” ones, in contrast to the usual “local” linear models, is used both in the intelligent PID control [53] and in the advanced disturbance rejection control [54]. In the difference to the feedback linearization [47] which transforms a nonlinear plant to a chain of integrators with the plant nonlinearity transformed to its input, in the considered approach this nonlinearity is locally “frozen” to a constant signal. Thus, since it is possible to design fully reliable controllers by using few appropriately chosen integral plant models, a question arises, when we actually need all different modeling and the subsequent optimization approaches as treated, for example, in [3], [46], [55]–[57].

#### IX. SUMMARY

- 1) HO-PID control was introduced as a generic term for all integer-order  $PID_n^m$  controllers with  $m$ th-order derivative action and  $n$ th-order low-pass binomial filters ( $n \geq m$ ).
- 2) In order to manage and demonstrate a wide range of possible performance nuances, two different integrated tuning procedures (ITPs) for tuning  $PID_n^m$  controllers were proposed and investigated based on explicit formulas derived by the MRDP method and delay equivalence.

- 3) In this work, a three-step evaluation was performed, which first considers a noise-free setpoint control to evaluate inadequacies of the considered ITPs due to the applied delay equivalence.
- 4) In the second step, the measurement noise attenuation evaluation was performed based on numerical simulations of the nominal system in Matlab/Simulink.
- 5) In the third step, the robustness aspects of the two ITPs were tested using traditional and new sensitivities through simulations of a FOTD system under derived  $PID_n^m$  control in Matlab/Simulink.
- 6) In all the aspects considered, the results of the simulation analysis were confirmed by real-time experiments on the control of the thermal system.

#### X. CONCLUSION

In this article, the most important aspects related to the design, optimal setting and deployment of HO-PID controllers have been briefly analyzed. Due to the scope and the relatively large number of family members of the considered controllers, the presentation could not focus on a more detailed analysis of individual aspects. Rather, an attempt was made to provide a comprehensive picture of the various perspectives of the possible applications. A more detailed elaboration of individual problems as well as applications of the HO-PID controller remains a topic for future research. As the main field of application of HO-PID control, various tasks can be expected where the focus is on the speed of transients. At the same time, it can be used to achieve the highest possible smoothing of transients at the input or output of the system. Such controllers also bring an increase in robustness to uncertainties and neglected loop dynamics. They also make it possible to use highly simplified methods of system identification. Some aspects of real-time applications are discussed in Section VIII.

The inadequately managed filtering function in traditional PID control contributed to its unflattering image in terms of proper noise attenuation, time-delayed loop dynamics, and ease of commissioning. Through simulations and real-time control, it was shown that traditional PID control covers only a fraction of the achievable performance spectrum. To explain and overcome its limitations, we introduced a whole family of controllers with higher order derivative actions and developed the ability to easily adjust their parameters, including the inevitable filters. In contrast to the poor PID image, when controlling systems with long delays, measurement noise and uncertainties, the  $PID_n^m$  control allows to increase the robustness of the loop and improve the settling speeds in the desired way. The resulting form of HO-PID controllers, together with the motivation for their introduction, point to analogies with the context of fractional-order PID control (FO). Compared to the FO-PID control, the proposed HO-PID solutions are directly applicable, do not require further approximations and do not require a complex mathematical apparatus. In this paper, they have been studied up to the derivative degree  $m = 5$  and  $n = m + 2 > m$ , but the experimental results



suggest that it might be interesting to further investigate systems with higher  $n$  and  $m$  as well. However, before approaching such a task, stability regions of the HO-PID should be analyzed and numerical problems arising in the simulation should be explored.

On the one hand, it has been shown that tuning this family of controllers is almost as simple as tuning the filtered PI control. It can be based on estimates of two IPDT model parameters  $T_{dp}$  and  $K_{sp}$ . After choosing the derivative degree  $m$ , the filtration intensity corresponding to the filter order  $n = m+2$  was set by its filter time constant  $T_f$  calculated from the equivalent time delay  $T_e$  (22). However, the wide variety of practical requirements cannot be satisfied by a single universal optimal tuning, nor by a large number of methods for PID optimization. The real flexibility that allows “fine tuning” can only be achieved by combining the analytically tuned controllers with their “trial and error” commissioning. The new ITPs for  $PID_n^m$  tuning provide this capability.

The extended capabilities of HO -PID control have also raised new requirements for criteria for its performance and robustness evaluation. With respect to performance robustness, traditional sensitivity functions agree only with a narrow range of practical experience. In some aspects of performance robustness, they can give completely counterproductive results. In this context, much depends on the priorities in the performance evaluation (choice of the parameter  $k$  in the cost function (4)). New sensitivity measures have been proposed in order to cope with all the situations considered.

In future research, it would be appropriate to reduce the ITP imperfections shown in Figs 3-5. It is also possible to achieve further improvement of the performance in HO -PID controllers with anti-windup implementation by generalizing the techniques presented in [58].

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