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Multi-Period Closed-Loop Supply Chain Network Equilibrium: Perspective of Marketing and Corporate Social Responsibility

CAIQUAN DUAN^{®1}, FENGMIN YAO², GUOYI XIU², YUNHUI ZHANG², AND XIAOLI ZHANG²

¹College of Engineering, Northeast Agricultural University, Harbin 150030, China
²School of Economics and Management, Harbin University of Science and Technology, Harbin 150080, China

Corresponding authors: Fengmin Yao (yao_fengmin@163.com) and Caiquan Duan (duancaiquan@126.com)

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ABSTRACT Marketing and corporate social responsibility (CSR) are crucial factors that affect closed-loop supply chain network (CLSCN) equilibrium. And the environment of enterprises' behaviors often occurs during durations and dynamics. To explore the optimal marketing and CSR strategies in a dynamic multi-period CLSCN system, this paper investigates the dynamic multi-period CLSCN equilibrium problem considering marketing and CSR. A multi-period CLSCN system includes manufacturers, retailers, recyclers, and demand markets. Based on Nash non-cooperative game theory and variational inequality, we design the optimal behavior and equilibrium conditions of members. Then, a new equilibrium model of a multi-period CLSCN is constructed. In this model, marketing is the responsibility of manufacturers and retailers, and CSR is the responsibility of manufacturers. Numerical examples are provided to verify the effectiveness of the model and analyzed to describe how marketing and CSR affect equilibrium results. We find that the retailers are responsible for marketing, and the level of CSR activities of the manufacturers is higher during the early period, which is most beneficial to the multi-period CLSCN system and social welfare. Based on the conclusions of this paper, the management enlightenments from the perspectives of enterprises and government are also proposed.

INDEX TERMS Closed-loop supply chain network, equilibrium model, corporate social responsibility, marketing.

I. INTRODUCTION

Currently, with the growing awareness of circular economy, the closed-loop supply chain (CLSC) introduces remanufacturing and sustainable concepts into the supply chain, which is a hallmark of industrialized civilization. The CLSC adopts technology to reuse end of life (EOL) products to produce remanufactured products. It achieves a closed-loop structure that is described by "products-consumption-recyclingremanufacturing-products" and can prevent the disposal of waste products [1]. Current research on modeling and optimization of CLSC systems in academia can be grouped into two structure categories: simple CLSC (oneto-one, one-to-many, many-to-one) and complex CLSCN (many-to-many) [2]. However, with the development of the economic environment, certain enterprises participate in cooperative and competitive relationships within the CLSC system, and enterprises' behaviors often occur over long durations and dynamics [3]; therefore, the research on dynamic multi-period CLSCN equilibrium problem describes the market environment accurately. Additionally, the behaviors of enterprises are critical factors that influence the multi-period CLSCN system.

In the behaviors of enterprises, scholars and managers pay great attention to marketing and CSR [4], [5]. Enterprises often exhibit marketing behavior to alter the attitudes and behaviors of consumers for their product(s). The type of marketing is different from conventional promotion methods, such as advertising, personnel sales, internet marketing, and business promotion. Current marketing efforts have become

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a way of promoting a combination of various promotional methods [6]. As a critical enterprise's behavior, marketing has positive effects on the supply chain system, including pricing, sales quantity, recovery rate, and profits [7]-[9]. No matter what the structure of the supply chain system (CLSC, CLSCN), different enterprises are responsible for marketing, which will inevitably exhibit different effects on the supply chain system [6]. Moreover, in reality, marketing can promote the purchasing behavior of consumers, and consumers also pay attention to whether relevant enterprises have undertaken CSR activities when purchasing products [4]. Different from marketing, CSR activities usually require relevant enterprises to maximize their financial interests and emphasize their contributions to stakeholders, consumers, the environment, etc. [10]. Since the beginning of this century, based on the hypothesis of irrational economic man, it is discovered that CSR has a positive effect on social welfare, recovery rate, and profits [11]. Additionally, in reality, more enterprises are starting to undertake both marketing and CSR activities; for example, Alibaba's global marketing investment has more than doubled its scale \$4.7 billion in 2019. They also released CSR activities reports at the end of each year, including product innovation, job creation, education, training, environmental protection, etc. However, under the environment of dynamic enterprises' behaviors, the research on the equilibrium problem of dynamic multi-period CLSCN system from the perspective of combining the two behaviors (marketing, CSR) is lacking in existing documents.

Consequently, there are three potentially important issues worth studying. Firstly, when enterprises (manufacturers, retailers) exhibit marketing behavior, how does marketing investment affect the equilibrium results, and which one achieves better results? Secondly, under the environment of static or dynamic CSR activities, how do manufacturers' CSR activities affect the equilibrium results? And, what are the optimal marketing and CSR strategies in the multi-period CLSCN? Thirdly, this paper discusses the results of this research, combined with the development of CLSCN management, and we proposed relevant management inspirations.

The contributions of this paper are in three ways. Firstly, previous studies are mainly research on modeling and optimization of supply chain system (CLSC or static CLSCN) from the perspective of marketing and CSR. In reality, enterprises' behaviors often dynamic, and both behaviors are often existing in a dynamic multi-period CLSCN system. Hence, it is more practical significance to study the dynamic multi-period CLSCN equilibrium from the perspective of marketing and CSR. Secondly, similarly to the research method of literature [12]-[14], based on Nash non-cooperative game theory and variational inequality method, we design a new dynamic multi-period CLSCN equilibrium model. In this model, according to our assumptions, marketing is the responsibility of manufacturers and retailers, and CSR is the responsibility of manufacturers. And the new optimal behavior and equilibrium conditions of members are established. Later, numerical examples are provided to verify

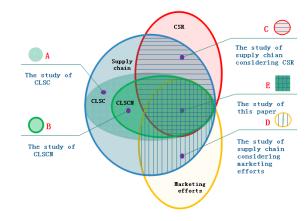


FIGURE 1. Research position of this paper.

the effectiveness of the model and explored the optimal marketing and CSR strategies. Finally, through the equilibrium analysis of the results, the relevant management enlightenments are formulated, which will improve resource utilization and facilitate the sustainable development of multi-period CLSCN management.

This paper is structured in six parts: (1) related studies are summarized and compared, (2) the research problem, assumptions, and annotations are recounted, (3) a new dynamic multi-period CLSCN model is developed by considering marketing and CSR, (4) numerical examples are used to identify the influence of marketing and CSR on equilibrium results, (5) results are discussed, and management inspirations are proposed, and (6) conclusions are provided, including limitations and future work.

II. LITERATURE REVIEW

As explained in the introduction, this paper studies the multiperiod CLSCN equilibrium considering marketing and CSR. Figure 1 describes the research position of this paper through correlation with existing related research.

Figure 1 shows that this paper can be considered to be a cross-question (Part E) of CLSC, CLSCN, supply chain considering marketing, and supply chain considering CSR. Based on the topic of this paper, this literature review will classify and discuss the research status of this topic from three perspectives: 1) Parts A and B, 2) Part C, and 3) Part D. Finally, the research gaps are provided.

A. STUDY OF CLSC AND CLSCN

With the rise of reverse logistics, academia has expanded the scope of supply chain research to closed-loop problems and has achieved many results. The most classic research in this field is a review study by Govindan *et al.* [2], who integrated important literature before 2013, clarified the development context of the CLSC and CLSCN, performed objective evaluations, and noted the future research directions. Since then, scholars have conducted many in-depth studies of the optimization and decisions of CLSC and CLSCN.

In the research field of CLSC (simple structure), scholars have made many achievements and have taken different perspectives to analyze its optimization and decision. In the mainstream research on the impact of variables on the supply chain system, the first type investigates various channels [1], [15], [16], the second type investigates government intervention [17], [18], the third type investigates product diversity [19], the fourth type investigates different rights structures [20], and the fifth type investigates coordination strategies [21], [22]. However, in these studies, the research of optimization and modelling mainly pay attention to the CLSC system. With the change of the supply chain environment, the structure between enterprises in the supply chain is no longer simple; enterprises often exist in a CLSCN system. The equilibrium optimization problem of the complex structure CLSCN has thus become an important topic for research.

In the research field of supply chain network equilibrium (supply chain network, CLSCN), the first study of supply chain network equilibrium was performed by Nagurney et al. [13], who designed a supply chain network equilibrium model using variational inequality. Using the same research method, Hammond and Beullens [23] first combined reverse logistics into a supply chain network, creating a ground-breaking study of the CLSCN equilibrium. After, many scholars used variational inequality to explore the supply chain equilibrium problem [3], [12], [23]-[27]. In the study of CLSCN equilibrium problems, the mainstream studies are static CLSCN equilibrium and dynamic multi-period CLSCN equilibrium. From the static CLSCN equilibrium, the first type is definite demand, and the second type is uncertain conditions (uncertain demand). In terms of definite demand, based on product categories with regard to the differences between new products and remanufactured products, Qiang [12] studied the equilibrium of CLSCN with competitive and remanufacturing design. From the perspective of government behavior, Wenbin et al. [24] designed a new network equilibrium model with government involvement and identified the impact of recovery rate intervention on equilibrium results. In terms of uncertain conditions, Hamidieh and Fazli-Khalaf [28] developed a possibilistic, reliable, and responsive CLSCN design model under uncertain conditions to minimize total network costs. Then, scholars have started to explore the influence of uncertain demand and time-dependent demands on network equilibrium [25], [26]. The study of Kinchan et al. [26] constructed a new CLSCN equilibrium model under time-dependent demands and found that price and transaction quantity of products are strongly affected by time-dependent demands. However, these studies only investigated the one-period CLSCN equilibrium.

In the research field of multi-period CLSCN equilibrium, the early classic studies were performed by Kannan *et al.* [29]. They used batteries as a case to study the equilibrium problem of a multi-period CLSCN. Later, scholars studied the multi-period CLSCN equilibrium problem from more perspectives, including carbon emission constraints [3], [14], government intervention [27]. These scholars optimized their models with the goal of maximizing profits. Conversely, Fazli-Khalaf *et al.* [30] aimed to minimize total costs, constructed a new multi-period CLSCN equilibrium model, and proposed a new effectual robust possibilistic programming (RPP) model.

These scholars studied the equilibrium problem of the CLSCN from different perspectives and thus promoted the sustainable development of CLSCN management. However, modeling marketing and CSR into a multi-period CLSCN model and studying the influence of marketing and CSR on multi-period CLSCN equilibrium problems has not yet been investigated by scholars. In reality, enterprises (such as Apple Group and Alibaba Group) are starting to undertake both marketing and CSR. Therefore, the following two sections systematically discuss the study of supply chain considering marketing and CSR.

B. SUPPLY CHAIN CONSIDER MARKETING

Currently, supply chain management has gradually transformed from the traditional type that is dominated by core enterprises to the new type of supply chain that is dominated by markets. Enterprises have gradually begun to consider the issue of marketing [6]. In academia, the problem of marketing has been studied as early as 2002 [31]. Later, scholars have considered marketing into the supply chain system [32], [33]. Their research laid the foundation for the modeling and optimization of supply chain under marketing. In the last five years, in the study of the supply chain (forward, CLSC, CLSCN) considering marketing, scholars have focused their attention on the best marketing strategies, e-commerce channel, pricing decisions, coordination mechanism, and network equilibrium problem.

In the research field of the forward supply chain considering marketing, under the assumption that the retailer exhibits certain marketing, Pal et al. [7] and Ranjan et al. [8] identified the optimal marketing' input and pricing decisions. In the research field of the CLSC considering marketing, Ma et al. [5] studied the impact of marketing on the pricing, recovery rate, and profits of different supply chain structures, constructed four reverse channels CLSC models, and they finally identified the law of influence. Under the assumption that the retailer exhibits certain marketing, Zerang et al. [9] studied a supply chain's decision pricing. To determine whose marketing is better, the impact of manufacturer and retailer are respectively responsible for marketing on the performance of dual-channel CLSC was studied by Taleizadeh et al. [6], who finally revealed that retailer's marketing achieves better results. However, these scholars investigated only based on the simple structure supply chain. In the research field of the CLSCN considering marketing, the impact of marketing investment for multi-period CLSCN equilibrium under stochastic demand was studied by Zhang et al. [34], who used variational inequality to build a multi-period CLSCN equilibrium model and conducted a sensitivity analysis by numerical examples. The above literature studied the impact of marketing on the supply chain

 TABLE 1. Classification and comparison of the above literature.

literature	CSR	Marketing	Simple structure		Network structure				
			Forward	Closed-loop	Forward		Closed-loop		variational
					Static	dynamic	Static	dynamic	inequality
[1], [6], [16], [17], [18], [19], [20], [21], [22]				0					
[23], [24], [25], [28]							0		0
[3], [12], [14], [26], [34]								0	0
[29], [30]								0	
[27]		0					0		0
[7], [8], [32], [33]		0	0						
[5], [9]		0		0					
[4], [37]	0			0					
[10], [36], [38], [39]	0		0						
[40]	0				0				0
[41], [42]	0					0			0
[43]	0							0	
[44]	0					0			
This paper	0	0						0	0

based on the hypothesis of rational economic man. However, with increasing awareness of CSR, enterprises have gradually begun to engage in CSR activities. Next, we analyze and summarize the literature of the supply chain that considers CSR activities.

C. SUPPLY CHAIN CONSIDER CSR

Regarding supply chains with CSR, pioneering research was first proposed by Carter and Jennings [35], who investigated CSR's value in supply chain decisions through case studies. Then, scholars have conducted more research to identify the influence of CSR on CLSC and CLSCN equilibrium.

In the research field of CLSC considering CSR, most researchers investigate the impact of CSR on the supply chain system based on the hypothesis of irrational economic man to study. The most classic research was provided by Panda et al. [4], who showed that manufacturer' CSR activities increase social welfare and the rate of recovery. Inspired by the study of [4], [36], Jokar and Hosseini-Motlagh focused their research on the different participants' CSR effects, and they eventually found that the manufacturer performs CSR better. Then, under manufacturer's CSR activities, Shu et al. [37], Yong et al. [38], and Li [39] respectively showed that government intervention (carbon emission constraints, credit support, and government subsidies) could help promote the enterprises' CSR activities and improve social welfare. Particularly, Stekelorum [11] reviewed 63 documents to describe the impact of SME CSR behavior on the supply chain. However, the above literature studied the impact of CSR on simple structure supply chains.

In the research of many-to-many structure supply chain (supply chain network, CLSCN) considering CSR, few studies of the network equilibrium problem considering CSR. Early on, the coordination mechanism of CSR was proposed by Hsueh and Chang [40] to determine an optimal supply chain network system. Cruz and Wakolbinger [41] studied CSR's effects for transaction costs, emissions, and risks on the supply chain network, and also identified the optimal CSR level. After, based on a multi-period supply chain network [42] or multi-period CLSCN system [43], the equilibrium problem has been studied. However, they have investigated equilibrium problems under the definite demand. From the uncertain perspectives, a socially responsible supplier selection model was constructed by Fazli-Khalaf and Nemati [44]. These studies have played an important role in the study of network equilibrium problem considering CSR.

Summarily, according to the above literature, research gaps still exist in this field. These research gaps are described as follows, and a more detailed classification and comparison of the above literature is shown in Table 1. Firstly, to study CLSCN equilibrium, most of the literature designed the equilibrium model and used the variational inequality method, which is similar to the modeling method of this paper. However, in the current market environment, marketing and CSR are particularly important. Academia rarely considers both marketing and CSR into the dynamic multi-period CLSCN system. Secondly, to investigate the impact of marketing on the supply chain, scholars are primarily based on simple structure supply chain or only one period supply chain network. Under the multi-period CLSCN structure, scholars studied the impact of recyclers responsible for marketing on equilibrium results. However, when both manufacturers and retailers exhibit marketing, research on the optimal marketing leader is not sufficient. Thirdly, in the process of studying the impact of CSR on the supply chain, most of the literature considers the manufacturer engaging in CSR activities and the manufacturers satisfy the hypothesis of irrational economic man, which is consistent with the research hypothesis of this paper. However, starting from the network structure of a multi-period CLSCN system, research on the problem of equilibrium optimization considering the static or dynamic level of CSR activities is lacking. Therefore, this paper begins from these research gaps and supplements the literature on CLSCN equilibrium. We study the equilibrium problem of the dynamic multi-period CLSCN from the perspective of marketing and CSR.

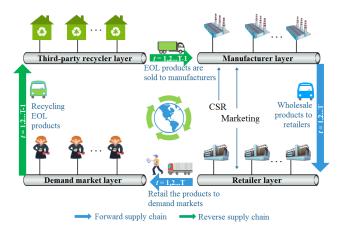


FIGURE 2. Multi-period CLSCN structure diagram of this paper.

III. PROBLEM DESCRIPTION, ASSUMPTIONS, AND SYMBOLS

The multi-period CLSCN system in this paper includes a manufacturer layer, a retailer layer, a third-party recycler layer (recycler layer), and a demand market layer. The manufacturer layer includes multiple manufacturers, the retailer layer includes multiple retailers, the recycler layer includes multiple recyclers, and the demand market layer includes multiple demand markets. Figure 2 shows a complete description of the multi-period CLSCN structure in this paper. The symbols and their definitions in this paper are shown in Tables 2–4. The behavior of each layer participant and assumptions are as follows:

- The manufacturer layer produces new products using original materials and remanufactured products using EOL products. There is no difference between these two types of products [14], [24], and this paper refers to both new product and remanufactured product as "product."
- 2) Products are sold to the demand market layer through the retailer layer. The retailer layer wholesales products from the manufacturer layer based on the demand for products and then sells them to the demand market layer.
- The manufacturer layer has CSR activities [4], [36], [37]. Under CSR activities, manufacturers focus on their total profits (profits), other layer's profits, and entire multi-period CLSCN's profits [4]. To promote product demand, both the manufacturer and retailer layers can conduct marketing (apportionment mechanism) [6].
- 4) The recycler layer is responsible for the recycling of EOL products from the demand market layer at the end of each period and then sell them to the manufacturer layer. In the final period, EOL products do not have remanufacturing value; therefore, EOL products are not recycled in the final period [14].
- 5) To model real enterprises more accurately, we assume that manufacturing enterprises (manufacturers) and recycling enterprises (recyclers) exhibit inventory

TABLE 2. Basic symbols and definitions.

Symbol	Definition
m	A manufacturer of manufacturer layer, $m = 1, 2,, M$
r	A recycler of recycler layer, $r = 1, 2,, R$
S	A retailer of retailer layer, $s = 1, 2,, S$
d	A demand market of demand market layer, $d = 1, 2,, D$
t	A period of multi-period, $t = 1, 2,, T$
b	Sensitivity coefficient of the price
е	Marketing cost coefficient
u_m^i	Conversion rate. <i>i</i> =0: raw material into product. <i>i</i> = <i>h</i> : EOL product into product.
e_m	Marketing apportionment factor of m
e_s	Marketing apportionment factor of s, $e_s = 1 - e_m$
$h_m(t)$	Level of CSR activities of <i>m</i> in <i>t</i>

TABLE 3. Transaction price, transaction volume, and inventory.

Symbol	Definition
$q_{ms}(t)$	In period t, the number of products sold by m to s, $\sum_{m=1}^{M} \sum_{s}^{S} q_{ms}(t) = Q^{MS}(t) \text{, all } q_{ms}(t) = Q^{MST} \in R_{+}^{MST}$
$q_{rd}(t)$	In period <i>t</i> , the number of EOL products recycled by <i>r</i> from $d, \sum_{r=1}^{R} \sum_{d}^{D} q_{rd}(t) = Q^{RD}(t)$, all $q_{rd}(t) = Q^{RDT} \in R_{+}^{RDT}$
$q_m^h(t)$	In period <i>t</i> , the number of EOL products used by <i>m</i> for remanufacturing, $\sum_{m}^{M} q_{m}^{h}(t) = Q_{H}^{M}(t), \text{ all } q_{m}^{h}(t) = Q_{H}^{M(T-1)} \in R_{+}^{M(T-1)}$
$q_{mr}(t)$	In period t, the number of EOL products sold by r to m, all $q_{mr}(t) = Q^{MR(T-1)} \in R_+^{MR(T-1)}$
$q_{sd}(t)$	In period <i>t</i> , the number of products sold by <i>s</i> to <i>d</i> , all $q_{sd}(t) = Q^{SDT} \in R^{SDT}_+$
$q_m^i(t)$	The inventory of <i>m</i> in <i>t</i> . all $q_m^i(t) = Q_I^{MT} \in R_+^{MT}$
$q_r^i(t)$	The inventory of <i>r</i> in <i>t</i> . all $q_r^i(t) = Q_I^{R(T-1)} \in R_+^{M(T-1)}$
$p_{ms}(t)$	Product's sale price between m and s in t
$p_{mr}(t)$	EOL product's price between r and m in t
$p_{sd}(t)$	Product's retail price between s and d in t
$p_{rd}(t)$	EOL product's recycling price between r and d in t

transfer behavior with multi-period characteristics. Thus, manufacturers and recyclers conduct inventory management of products (EOL products) in period t, and sell products (EOL products) in period t + 1, thereby ensuring the continuity of products circulation in multi-period CLSCN [14].

IV. MODEL ESTABLISHMENT

This section establishes a multi-period CLSCN equilibrium model considering marketing and CSR. Based on Nash noncooperative game theory, we use the variational inequality method to design a new network equilibrium model. Variational inequality is an important method to study the problem of supply chain network equilibrium.

Variational inequality was first proposed by Lions and Stampacchia *et al.* [45]. Later, with the continuous

TABLE 4. Functions and other variables.

Symbol	Definition
$C_{ms}^m(q_{ms}(t))$	Manufacturer's product transaction cost in t
$C_m^m(q_m(t))$	Manufacturer's product production cost (using raw material) in t
$C_x^i(q_x^i(t))$	In period <i>t</i> , <i>x</i> 's inventory cost, $x = m, r$
$q_d(t)$	In period t , demand function of d for the products
$C_{mr}^m(q_{mr}(t))$	Manufacturer's EOL product transaction cost in t
$C^h_m(u^h_m,q^h_m(t))$	Manufacturer's product production cost (using EOL product) in t
$f_{ms}(e,e_m,q_{ms}(t))$	Manufacturer's marketing cost in t
$f_m^{CSR}(t)$	Manufacturer's CSR cost in t
$(Q^{MS}(t))^2(h_m(t)/2b)$	Consumer surplus (CS) in t [4]
$C^s_{sd}(q_{sd}(t))$	Retailer's product transaction cost in t (with demand market)
$C_{ms}^s(q_{ms}(t))$	Retailer's product transaction cost in t (with manufacturer)
$f_{sm}(e,e_s,q_{ms}(t))$	Retailer's marketing cost in t
$C_{mr}^r(q_{mr}(t))$	Recycler's EOL product transaction cost in t (with manufacturer)
$C_{rd}^r(q_{rd}(t))$	Recycler's EOL product transaction cost in <i>t</i> (with demand market)
$V_d(t)$	Demand market's negative utility in <i>t</i>
π_m^p	Manufacturer <i>m</i> 's pure profits
π_m	Manufacturer <i>m</i> 's profits (total profits)
π_i	The profits of i , $i = s, r$
$\sum \pi$	The profits of entire multi-period CLSCN system

improvement of the variational inequality, it has been applied to traffic, transportation, economic problems, etc. With regard to the equilibrium of the supply chain network, Nagurney et al. [13] introduced equilibrium theory and proposed the well-known model of spatial price equilibrium. They used Nash non-cooperative game theory, spatial price equilibrium model, and variational inequality to create a new supply chain network equilibrium model. In their model, the optimal behavior of members (manufacturers, retailers, consumers) have been described by equilibrium price, equilibrium transaction quantity, and optimal profits. After Nagurney et al. [13], many scholars used variational inequality to solve the problem of supply chain equilibrium [3], [12], [14], [23]–[27], highlighting that the supply chain network equilibrium problem can be transformed into the corresponding variational inequality problem.

Inspired by the research of [12]-[14], [24], we design an equilibrium model of multi-period CLSCN considering marketing and CSR. First, the marketing has been added the objective function of manufacturer *m* and retailer *s*. Second, under the hypothesis of irrational economic man among manufacturers [4], the CSR has been added to the manufacturer *m*'s objective function. Third, we have considered products retail and EOL products recycling using a spatial price equilibrium model that includes forward and reverse logistics. Fourth, using variational inequality, a multi-period CLSCN equilibrium problem that considers marketing and CSR have been transformed into a variational inequality problem. Finally, by solving the variational inequality model of this section, the equilibrium result of multi-period CLSCN can be obtained, which can explore the optimal marketing and CSR strategies.

A. OPTIMAL BEHAVIOR AND EQUILIBRIUM CONDITIONS OF MANUFACTURER LAYER

All manufacturers in the manufacturer layer are responsible for producing the products using raw materials across multiple periods (t = 1, 2, ..., T). Meanwhile, they are also responsible for converting EOL products remanufacturing into products across multiple periods ($t = 2, 3 \cdots, T$) [14]. When not considering CSR activities, the costs of manufactures include transaction, production, inventory, materials, and marketing. The revenues of manufacturers are defined as the complete wholesale of products to retailers. The pure profits maximization of *m* without considering CSR activities can be described as follows:

$$\max \pi_m^p = \sum_{t=1}^T \sum_{s=1}^S p_{ms}(t) q_{ms}(t) - \sum_{t=1}^{T-1} \sum_{r=1}^R q_{mr}(t) p_{mr}(t) - \sum_{t=1}^T \sum_{s=1}^S C_{ms}^m(q_{ms}(t)) - \sum_{t=1}^{T-1} \sum_{r=1}^R C_{mr}^m(q_{mr}(t))$$

$$-\sum_{t=2}^{T} C_m^h(u_m^h, q_m^h(t)) - \sum_{t=1}^{T} C_m^m(q_m(t)) \\ -\sum_{t=1}^{T} C_m^i(q_m^i(t)) - \sum_{t=1}^{T} \sum_{s=1}^{S} f_{ms}(e, e_m, q_{ms}(t))$$
(1)

Equation (1) shows that when manufacturer m does not have CSR activities, and the manufacturer m satisfies the hypothesis of rational economic man, only considering the maximization of pure profits without considering social welfare. Manufacturer m's pure profits maximization is described by Equation (1), whose terms are from left to right: revenue from the wholesale of products to the retailers, minus the purchase cost of EOL products from the recyclers, transaction costs with the retailers, transaction costs with the recycles, the manufacturing cost of products using raw materials, the manufacturing cost of products using EOL products, inventory costs, and marketing costs.

However, when CSR activities are considered, the manufacturer m aims to maximize social welfare under the hypothesis of irrational economic man. Based on economic assumptions, social welfare is equal to producer surplus (manufacturer's profits) plus consumer surplus (*CS*), where *CS* is the difference between the product's highest price and its real price (i.e., willingness to pay of consumers) [4]. Therefore, the total profits (profits) maximization of m considering CSR activities can be described as follows:

$$\max \pi_{m} = \max \pi_{m}^{p} - \sum_{t=1}^{T} f_{m}^{CSR}(t) + \sum_{t=1}^{T} \frac{h_{m}(t)}{2b} (Q^{MS}(t))^{2}$$

$$= \sum_{t=1}^{T} \sum_{s=1}^{S} p_{ms}(t)q_{ms}(t) - \sum_{t=1}^{T} \sum_{s=1}^{S} C_{ms}^{m}(q_{ms}(t)) \quad (2)$$

$$- \sum_{t=1}^{T} C_{m}^{m}(q_{m}(t)) - \sum_{t=1}^{T-1} \sum_{r=1}^{R} [q_{mr}(t)p_{mr}(t)$$

$$+ C_{mr}^{m}(q_{mr}(t))]$$

$$- \sum_{t=1}^{T} C_{m}^{i}(q_{m}^{i}(t)) - \sum_{t=2}^{T} C_{m}^{h}(u_{m}^{h}, q_{m}^{h}(t))$$

$$- \sum_{t=1}^{T} \sum_{s=1}^{S} f_{ms}(e, e_{m}, q_{ms}(t))$$

$$- \sum_{t=1}^{T} f_{m}^{CSR}(t) + \sum_{t=1}^{T} \frac{h_{m}(t)}{2b} (Q^{MS}(t))^{2}$$
s.t. $q_{m}^{i}(t-1) + u_{m}q_{m}(t) + u_{r}q_{m}^{h}(t) = q_{m}^{i}(t) + \sum_{s=1}^{S} q_{ms}(t)$

$$u_{m}(t-1) + u_{m}q_{m}(t) + u_{r}q_{m}^{*}(t) = q_{m}^{*}(t) + \sum_{s=1}^{r} q_{ms}(t)$$
(3)

$$q_m^h(t) \le \sum_{r=1}^R q_{mr}(t-1)$$
 (4)

Here,
$$f_m^{CSR}(t) = f_m^{CSR}(h_m(t), u_m q_m(t), u_m^h q_m^h(t))$$

Equation (2) shows that when manufacturer m has CSR activities, the manufacturer m must pay additional CSR costs. Therefore, the costs of manufacturer m must include CSR activities costs. Consumer surplus CS is created due to the CSR activities of the manufacturer m. Similar to the study of Panda *et al.* [4], the total profits maximization function of manufacturer m must subtract CSR activities costs and add consumer surplus CS based on Equation (1).

Constraints (3) and (4) represent *m*'s inventory constraint and EOL products remanufacturing constraint in period *t*, respectively. Let $\lambda_m(t)$ and $\mu_m(t)$ be the Lagrange multipliers of constraints (3) and (4), $\sum_{t=1}^{T} \sum_{m=1}^{M} \lambda_m(t) = \Lambda^{MT} \in \mathbb{R}^{MT}_+$, $\sum_{t=1}^{T} \sum_{m=1}^{M} \mu_m(t) = M^{MT} \in \mathbb{R}^{MT}_+$. According to the literature of [13], [14], when manufacturers achieve equilibrium, they should satisfy Property 1.

Property 1: The equilibrium conditions of all manufacturers at the manufacturer layer ensure that $(Q^{MST}, Q^{MT}, Q^{MR(T-1)}, Q_H^{M(T-1)}, Q_I^{MT}, \Lambda^{MT}, M^{MT}) \in \Omega^M$ satisfies:

$$\begin{split} \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{s=1}^{S} \left[\frac{\partial C_{ms}^{m}(q_{ms}^{*}(t))}{\partial q_{ms}(t)} - \frac{h_{m}(t)Q^{MS*}(t)}{b_{m}} \right] \\ + \lambda_{m}^{*}(t) - p_{ms}^{*}(t) \\ + \frac{\partial f_{ms}(e, e_{m}, q_{ms}^{*}(t))}{\partial q_{ms}(t)} \right] \\ \times [q_{ms}(t) - q_{ms}^{*}(t)] \\ + \sum_{t=1}^{T} \sum_{m=1}^{M} \left[\frac{\partial C_{m}^{m}(q_{m}^{*}(t))}{\partial q_{m}(t)} + \frac{\partial f_{m}^{CSR*}(t)}{\partial q_{m}(t)} - \lambda_{m}^{*}(t) \right] \\ \times [q_{m}(t) - q_{m}^{*}(t)] + \sum_{t=1}^{T} \sum_{m=1}^{M} \left[\frac{\partial C_{m}^{i}(q_{m}^{i*}(t))}{\partial q_{m}(t)} \right] \\ + \lambda_{m}^{*}(t) - \lambda_{m}^{*}(t+1)] \times [q_{m}^{i}(t) - q_{m}^{i*}(t)] \\ + \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{r=1}^{R} [p_{mr}^{*}(t) + \frac{\partial C_{mr}^{m}(q_{mr}^{*}(t))}{\partial q_{mr}(t)} \\ - \mu_{m}^{*}(t)] \times [q_{mr}(t) - q_{mr}^{*}(t)] \\ + \sum_{t=2}^{T} \sum_{m=1}^{M} \left[\frac{\partial C_{m}^{r}(u_{m}^{h}, q_{m}^{h*}(t)}{\partial q_{m}^{h}(t)} + \frac{\partial f_{m}^{CSR*}(t)}{\partial q_{m}^{h}(t)} \right] \\ \times [q_{m}^{h}(t) - q_{m}^{h*}(t)] \\ + \sum_{t=2}^{T} \sum_{m=1}^{M} \left[\frac{\partial C_{m}^{r}(u_{m}^{h}, q_{m}^{h*}(t)}{\partial q_{m}^{h}(t)} + \frac{\partial f_{m}^{CSR*}(t)}{\partial q_{m}^{h}(t)} \right] \\ \times [q_{m}^{h}(t) - q_{m}^{h*}(t)] \\ + \sum_{t=1}^{T} \sum_{m=1}^{M} \left[\frac{q_{m}^{i}(t-1) + q_{m}(t) + u_{r}q_{m}^{h}(t)}{-q_{m}^{i}(t) - \sum_{s=1}^{S} q_{ms}(t)} \right] \\ \times [\lambda_{m}(t) - \lambda_{m}^{*}(t)] + \sum_{t=1}^{T} \sum_{m=1}^{M} \left[\sum_{r=1}^{R} q_{mr}(t) - q_{m}^{h}(t+1) \right] \\ \times [\mu_{m}(t) - \mu_{m}^{*}(t)] \ge 0 \end{split}$$

 $\begin{array}{l} \forall (Q^{MST}, Q^{MT}, Q^{MR(T-1)}, Q_H^{M(T-1)}, Q_I^{MT}, \Lambda^{MT}, M^{MT}) & \in \\ \Omega^M, \text{ where, } \Omega^M = R_+^{MST+4MT+MR(T-1)+M(T-1)}. \end{array}$

Property 2: Based on Property 1, the equilibrium result of manufacturer *m* is:

$$u_r[-\frac{\partial C_{ms}^m(q_{ms}^*(t+1))}{\partial q_{ms}(t+1)} + \frac{h_m(t+1)Q^{MS*}(t+1)}{b_m}$$

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$$\begin{split} +p_{ms}^{*}(t+1) &- \frac{\partial f_{ms}(e,e_{m},q_{ms}^{*}(t+1))}{\partial q_{ms}(t+1)}] \\ &- \frac{\partial C_{m}^{r}(u_{m}^{h},q_{m}^{h*}(t+1))}{\partial q_{m}^{h}(t+1)} - \frac{\partial f_{m}^{CSR*}(t+1)}{\partial q_{m}^{h}(t+1)} \\ &= p_{mr}^{*}(t) + \frac{\partial C_{mr}^{m}(q_{mr}^{*}(t))}{\partial q_{mr}(t)}; \frac{\partial C_{ms}^{m}(q_{ms}^{*}(t))}{\partial q_{ms}(t)} \\ &+ \frac{h_{m}(t)Q^{MS*}(t)}{b_{m}} + p_{ms}^{*}(t) = \frac{\partial f_{ms}(e,e_{m},q_{ms}^{*}(t))}{\partial q_{ms}(t)} \\ &+ \frac{\partial C_{m}^{m}(q_{m}^{*}(t))}{\partial q_{m}(t)} + \frac{\partial f_{m}^{CSR*}(t)}{\partial q_{m}(t)}, \end{split}$$

Based on Property 2, in period t, the product's wholesale price of manufacturer m is positively related to the marginal marketing cost. The EOL product's purchasing price of manufacturer m is negatively related to marketing. And the level of m's CSR activities is negatively related to the wholesale price and is positively related to social welfare.

B. OPTIMAL BEHAVIOR AND EQUILIBRIUM CONDITIONS OF RETAILER LAYER

All retailers in the retailer layer are responsible for product sales across multiple periods (t = 1, 2, ..., T). Retailers act as bridges between manufacturers and demand markets. If retailers perform marketing, the costs of retailers include transaction, wholesale, and marketing. The revenues of retailers include the sales of retail products to markets. Therefore, the profits maximization of *s* can be described as follows:

$$\max \pi_{s} = \sum_{t=1}^{T} \sum_{d=1}^{D} p_{sd}(t) q_{sd}(t) - \sum_{t=1}^{T} \sum_{D=1}^{D} C_{sd}^{s}(q_{sd}(t))$$
$$- \sum_{t=1}^{T} \sum_{m=1}^{M} p_{ms} q_{ms} - \sum_{t=1}^{T} \sum_{m=1}^{M} C_{ms}^{s}(q_{ms}(t))$$
$$- \sum_{t=1}^{T} \sum_{m=1}^{M} f_{sm}(e, e_{s}, q_{ms}^{*}(t))$$
(6)

s.t.
$$\sum_{d=1}^{D} q_{sd}(t) \le \sum_{m=1}^{M} q_{ms}(t)$$
 (7)

Constraint (7) represents *s*'s sales constraint. Let $\theta_s(t)$ be the Lagrange multiplier of constraint (7), $\sum_{t=1}^{T} \sum_{s=1}^{S} \theta(t) = \Theta^{ST} \in \mathbb{R}^{ST}_+$. According to the literature of [13], [14], when retailers achieve equilibrium, they should satisfy Property 3.

Property 3: The equilibrium conditions of all retailers at the retailer layer ensure that $\forall (Q^{MST}, Q^{SDT}, \Theta^{ST}) \in \Omega^S$, satisfies:

$$\sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{m=1}^{M} \begin{bmatrix} \frac{\partial C_{ms}^{s}(q_{ms}^{*}(t))}{\partial q_{ms}(t)} + p_{ms}^{*} - \theta_{s}^{*}(t) \\ + \frac{\partial f_{sm}(e, e_{s}, q_{ms}^{*}(t))}{\partial q_{ms}(t)} \end{bmatrix}$$
$$\times [q_{ms}(t) - q_{ms}^{*}(t)]$$
$$+ \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{d=1}^{D} [\frac{\partial C_{sd}^{s}(q_{sd}^{*}(t))}{\partial q_{sd}(t)} - p_{sd}^{*}(t) + \theta_{s}^{*}(t)]$$

$$\times [q_{sd}(t) - q_{sd}^{*}(t)] + \sum_{t=1}^{T} \sum_{s=1}^{S} [\sum_{m=1}^{M} q_{ms}(t) - \sum_{d=1}^{D} q_{sd}(t)] \times [\theta_{s}(t) - \theta_{s}^{*}(t)] \ge 0$$
(8)

 $\forall (Q^{MST}, Q^{SDT}, \Theta^{ST}) \in \Omega^S$, where $\Omega^S = R_+^{MST+SDT+ST}$.

Property 4: Based on Property 3, the equilibrium result of retailer *s* is:

$$\frac{\partial C_{ms}^{s}(q_{ms}^{*}(t))}{\partial q_{ms}(t)} + p_{ms}^{*} + \frac{\partial f_{sm}(e, e_s, q_{ms}^{*}(t))}{\partial q_{ms}(t)}$$
$$= p_{sd}^{*}(t) - \frac{\partial C_{sd}^{s}(q_{sd}^{*}(t))}{\partial q_{sd}(t)}$$

Based on Property 4, in period t, the retail price of s is positively related to the marginal transaction costs. The retail price of s is positively related to the wholesale price. And the retail price of s is positively related to marketing costs.

C. OPTIMAL BEHAVIOR AND EQUILIBRIUM CONDITIONS OF THIRD-PARTY RECYCLER LAYER

All recyclers in the recycler layer are responsible for the EOL products' recycling across multiple periods (t = 1, 2, ..., T - 1). Because recyclers pursue profit maximization, the final period of recycling is performed by the government or social organizations [14], [34]. The costs of recyclers include transaction and EOL products' recycling. The revenues of recyclers include selling EOL products to manufacturers. Therefore, the profits maximization of recycler *r* can be described as follows:

$$\max \pi_r = \sum_{t=1}^{T-1} \sum_{m=1}^{M} p_{mr}(t) q_{mr}(t) - \sum_{t=1}^{T-1} \sum_{m=1}^{M} C_{mr}^r(q_{mr}(t)) - \sum_{t=1}^{T-1} \sum_{d=1}^{D} p_{rd}(t) q_{rd}(t) - \sum_{t=1}^{T-1} \sum_{d=1}^{D} C_{rd}^r(q_{rd}(t)) - \sum_{t=1}^{T-1} C_r^i(q_r^i(t))$$
(9)

s.t.
$$\sum_{m=1}^{M} q_{mr}(t) \le \sum_{d=1}^{D} q_{rd}(t)$$
 (10)

$$q_r^i(t-1) + \sum_{d=1}^{D} q_{rd}(t) = q_r^i(t) + \sum_{m=1}^{m} q_{mr}(t)$$
(11)

Constraints (10) and (11) represent *r*'s EOL products recycling constraint and inventory constraints in period *t*, respectively. Let $\chi_r(t)$ and $\sigma_r(t)$ be the Lagrange multipliers of constraints (10) and (11), $\sum_{t=1}^{T-1} \sum_{r=1}^{R} \chi_r(t) = X^{R(T-1)} \in R^{R(T-1)}_+$, $\sum_{t=1}^{T-1} \sum_{r=1}^{R} \sigma_r(t) = P^{R(T-1)} \in R^{R(T-1)}_+$. According to the literature of [13], [14], when recyclers achieve equilibrium, they should satisfy Property 5.

Property 5: The equilibrium conditions of all recyclers at the recycler layer ensure that $(Q^{MR(T-1)}, Q^{RD(T-1)}, Q_I^{R(T-1)})$,

 $X^{R(T-1)}, P^{R(T-1)}) \in \Omega^R$ satisfies:

$$\sum_{t=1}^{T-1} \sum_{r=1}^{R} \sum_{d=1}^{D} \left[\frac{\partial C_{rd}^{r}(q_{rd}^{*}(t))}{\partial q_{rd}(t)} + p_{rd}^{*}(t) - \chi_{r}^{*}(t) - \sigma_{r}^{*}(t) \right] \\ \times \left[q_{rd}(t) - q_{rd}^{*}(t) \right] + \sum_{t=1}^{T-1} \sum_{r=1}^{R} \sum_{m=1}^{M} \left[\frac{\partial C_{mr}^{r}(q_{mr}^{*}(t))}{\partial q_{mr}(t)} - p_{mr}^{*}(t) \right] \\ \times \left[\frac{\partial C_{mr}^{r}(q_{mr}^{*}(t))}{\partial q_{mr}(t)} - p_{mr}^{*}(t) \right] \\ \times \left[q_{mr}(t) - q_{mr}^{*}(t) \right] + \sum_{t=1}^{T-1} \sum_{r=1}^{R} \left[\frac{\partial C_{r}^{i}(q_{r}^{i*}(t))}{\partial q_{r}^{i}(t)} + \sigma_{r}^{*}(t) - \sigma_{r}^{*}(t+1) \right] \times \left[q_{r}^{i}(t) - q_{r}^{i*}(t) \right] \\ + \sum_{t=1}^{T-1} \sum_{r=1}^{R} \left[q_{r}^{i}(t-1) + \sum_{d=1}^{D} q_{rd}(t) - q_{r}^{i}(t) \right] \\ - \sum_{m=1}^{M} q_{mr}(t) \right] \times \left[\sigma_{r}(t) - \sigma_{r}^{*}(t) \right] + \sum_{t=1}^{T-1} \sum_{r=1}^{R} \left[\sum_{d=1}^{D} q_{rd}^{*}(t) - q_{rd}^{i}(t) \right] \\ - \sum_{m=1}^{M} q_{mr}^{*}(t) \right] \times \left[\chi_{r}(t) - \chi_{r}^{*}(t) \right] \ge 0$$

$$(12)$$

 $\forall (Q^{MR(T-1)}, Q^{RD(T-1)}, Q_I^{R(T-1)}, X^{R(T-1)}, P^{R(T-1)}) \in \Omega^R,$ where $\Omega^R = R_+^{(T-1)(MR+RD+3R)}.$

Property 6: Based on Property 5, the equilibrium result of recycler *r* is:

$$\frac{\partial C_{rd}^r(q_{rd}^*(t))}{\partial q_{rd}(t)} + p_{rd}^*(t) = p_{mr}^*(t) - \frac{\partial C_{mr}^r(q_{mr}^*(t))}{\partial q_{mr}(t)}$$

Based on Property 6, in period t(t = 1, 2, ..., T - 1), the EOL product's sell price of recycler r is positively related to the recovery price. And the EOL product's recovery price of recycler r is positively related to the costs of marginal transaction.

D. OPTIMAL BEHAVIOR AND EQUILIBRIUM CONDITIONS OF DEMAND MARKET LAYER

The demand market layer describes consumer demand for products. The behavior of demand markets in the CLSCN is responsible for product purchases from retailers across multiple periods (t = 1, 2, ..., T). The demand markets must pay the corresponding product price when buying products; thus, the price and demand of products should satisfy Equation (13) [13]:

$$q_d(t) \begin{cases} = \sum_{\substack{s=1\\s}}^{S} q_{sd}^*(t), p_{sd}^*(t) > 0 \\ \ge \sum_{s=1}^{S} q_{sd}^*(t), p_{sd}^*(t) = 0 \end{cases}$$
(13)

For the multi-period reverse supply chain network, a certain compensation must be given to demand markets by recyclers if the recyclers recycle the EOL products from demand markets. This type of compensation primarily described by the price [23]:

$$V_d^*(t) = \begin{cases} = p_{rd}^*(t), & q_{rd}^*(t) > 0\\ \ge p_{rd}^*(t), & q_{rd}^*(t) = 0 \end{cases}$$
(14)

s. t.
$$\sum_{r=1}^{R} q_{rd}^*(t) \le \sum_{s=1}^{S} q_{sd}^*(t)$$
 (15)

Constraint (15) represents demand market *d*'s EOL products that could be recycled in period *t*, (*t* = 1, 2, ..., *T* - 1). Let $\gamma_d(t)$ be the Lagrange multiplier of the constraint (15), $\sum_{t=1}^{T-1} \sum_{d=1}^{D} \eta_d(t) = I^{D(T-1)} \in R^{D(T-1)}_+$, which is based on the requirements of demand markets equilibrium. When the formulas (13)-(15) are all satisfied, the demand markets achieve equilibrium and should satisfy Property 7:

Property 7: The equilibrium conditions of all demand markets at the demand market layer ensure that $(Q^{RD(T-1)}, P^{SDT}, I^{D(T-1)}) \in \Omega^R$ satisfies:

$$\sum_{t=1}^{T-1} \sum_{r=1}^{R} \sum_{d=1}^{D} \left[V_d^*(Q^{RD*}(t)) - p_{rd}^*(t) + \eta_d^*(t) \right] \\ \times \left[q_{rd}(t) - q_{rd}^*(t) \right] + \sum_{t=1}^{T} \sum_{d=1}^{D} \left[\sum_{s=1}^{S} q_{sd}^*(t) - q_d(t) \right] \\ \times \left[p_{sd}(t) - p_{sd}^*(t) \right] + \sum_{t=1}^{T-1} \sum_{d=1}^{D} \left[\sum_{s=1}^{S} q_{sd}^*(t) - q_d(t) \right] \\ - \sum_{r=1}^{R} q_{rd}^*(t) \right] \times \left[\eta_d(t) - \eta_d^*(t) \right] \ge 0$$
(16)

 $\forall (Q^{RD(T-1)}, P^{SDT}, I^{D(T-1)}) \in \Omega^{D}, \text{ where } \Omega^{D} = R^{RD(T-1)+SDT+D(T-1)}_{+}$

Property 8: Based on Property 7, the equilibrium result of demand market d is $p_{rd}^*(t) = V_d^*(Q^{RD*}(t)) - \eta_d^*(t)$. Therefore, in period t, the EOL product's recovery price from demand market d is positively related to the negative effect of demand market d.

E. MULTI-PERIOD CLSCN EQUILIBRIUM MODEL

The relationship of participants at each level is described by Nash non-cooperative game theory in the network system. When the manufacturer, retailer, recycler, and demand market layers all achieve equilibrium conditions simultaneously, the entire network system could reach equilibrium. Therefore, when Properties 1, 3, 5, and 7 are satisfied simultaneously, the multi-period CLSCN of this paper achieves equilibrium.

Property 9: The equilibrium of the multi-period CLSCN ensures that $\forall (Q^{MST}, Q^{MR(T-1)}, Q^{M(T-1)}, Q^{MT}, W^{MT}, W^{R(T-1)}, Q^{RD(T-1)}, Q^{SDT}, P^{SDT}, \Lambda^{MT}, M^{MT}, \Theta^{ST}, X^{R(T-1)}, T^{R(T-1)}, I^{D(T-1)}) \in \Omega$ satisfies, (17), as shown at the bottom of the next page, *Property 10:* Based on the Property 9, $p_{ms}^*(t)$, $p_{mr}^*(t)$, $p_{rd}^*(t)$ are endogenous variables. Therefore, according to the variable inequalities (5), (8), (12), and (16), we can achieve endogenous variables:

$$p_{mr}^{*}(t) = \frac{\partial C_{mr}^{r}(q_{mr}^{*}(t))}{\partial q_{mr}(t)} + \chi_{r}^{*}(t) + \sigma_{r}^{*}(t), p_{rd}^{*}(t)$$
$$= V_{d}^{*}(Q^{RD*}(t)) - \eta_{d}^{*}(t)$$
$$p_{ms}^{*}(t) = \frac{\partial C_{ms}^{m}(q_{ms}^{*}(t))}{\partial q_{ms}(t)} - \frac{h_{m}(t)Q^{MS*}(t)}{b_{m}}$$
$$+ \frac{\partial f_{ms}(e, e_{m}, q_{ms}^{*}(t))}{\partial q_{ms}(t)} + \lambda_{m}^{*}(t)$$

V. NUMERICAL EXAMPLES

In this section, numerical examples are given to verify the effectiveness of the model and analyzed to describe how relevant parameters $h_m(t)$, e_r and e_m affect equilibrium results. To solve the multi-period CLSCN model, the modified project contraction algorithm is conducted to solve the Equation (17) [13], [14], [24]. This algorithm has simple steps

and can solve all variables and Lagrange multipliers. We get the equilibrium result by programming with MATLAB. The iterating step is 0.01, and the convergence criterion between two steps is lower than or equal to 10^{-6} . We assume that two manufacturers (M = 2), two retailers (S = 2), two recyclers (R = 2), and two demand markets (D = 2) existing in a three-period (T = 3) CLSCN system (Figure 3), and $u_m = 0.95$, $u_m^h = 0.75$, b = 10. Table 5 shows the associated cost function [13], [14], [24].

To more accurately determine the impact of marketing and CSR for multi-period CLSCN equilibrium, this section primarily investigates the following issues:

 When the level of manufacturers' CSR activities is static (i.e., the level of CSR activities does not change within each period), and both manufacturers and retailers can conduct marketing efforts (marking apportionment), how do marketing and CSR affect equilibrium

$$\begin{split} \sum_{i=1}^{T} \sum_{m=1}^{M} \sum_{s=1}^{S} \begin{bmatrix} \frac{\partial C_{ms}^{m}(q_{ms}^{*}(t))}{\partial q_{ms}(t)} + \frac{\partial C_{ms}^{*}(q_{ms}^{*}(t))}{\partial q_{ms}(t)} + \frac{\partial f_{ms}(e, e_{m}, q_{ms}^{*}(t))}{\partial q_{ms}(t)} + \frac{\partial f_{ms}(e, e_{m}, q_{ms}^{*}(t))}{\partial q_{ms}(t)} + \lambda_{m}^{*}(t) - \theta_{s}^{*}(t) \end{bmatrix} \\ & + \sum_{i=2}^{T} \sum_{m=1}^{M} \begin{bmatrix} \frac{\partial C_{ms}^{*}(q_{m}^{*}q_{ms}^{*}(t))}{\partial q_{ms}^{*}(t)} + \frac{\partial f_{ms}^{*}(e, e_{m}, q_{ms}^{*}(t))}{\partial q_{ms}(t)} + \lambda_{m}^{*}(t) - \theta_{s}^{*}(t) \end{bmatrix} \\ & + \sum_{i=2}^{T} \sum_{m=1}^{M} \begin{bmatrix} \frac{\partial C_{ms}^{*}(q_{ms}^{*}q_{ms}^{*}(t))}{\partial q_{m}(t)} + \frac{\partial f_{ms}^{*}(e, e_{m}, q_{ms}^{*}(t))}{\partial q_{ms}(t)} + \lambda_{m}^{*}(t) - q_{ms}^{*}(t) \end{bmatrix} \\ & \times \left[q_{ms}(t) - q_{ms}^{*}(t) \right] \\ & + \sum_{i=1}^{T} \sum_{m=1}^{M} \begin{bmatrix} \frac{\partial C_{ms}^{*}(q_{ms}^{*}q_{ms}^{*}(t))}{\partial q_{m}(t)} + \frac{\partial f_{ms}^{*}(s)}{\partial q_{m}(t)} - \lambda_{m}^{*}(t) \right] \\ & \times \left[q_{m}(t) - q_{ms}^{*}(t) \right] \\ & + \sum_{i=1}^{T} \sum_{m=1}^{M} \begin{bmatrix} \frac{\partial C_{ms}^{*}(q_{ms}^{*}(t))}{\partial q_{m}(t)} + \frac{\partial f_{ms}^{*}(s)}{\partial q_{m}(t)} - \lambda_{m}^{*}(t) \right] \\ & \times \left[q_{m}(t) - q_{ms}^{*}(t) \right] \\ & + \sum_{i=1}^{T} \sum_{m=1}^{M} \begin{bmatrix} \frac{\partial C_{ms}^{*}(q_{ms}^{*}(t))}{\partial q_{m}(t)} + \frac{\partial f_{ms}^{*}(s)}{\partial q_{m}(t)} - \lambda_{m}^{*}(t) \right] \\ & \times \left[q_{mi}(t) - q_{mi}^{*}(t) \right] \\ & + \sum_{i=1}^{T} \sum_{m=1}^{T} \begin{bmatrix} \frac{\partial C_{ms}^{*}(q_{ms}^{*}(t))}{\partial q_{m}(t)} + \lambda_{m}^{*}(t) - \lambda_{m}^{*}(t) + \left[q_{mi}^{*}(t) - q_{mi}^{*}(t) \right] \\ & + \sum_{i=1}^{T} \sum_{m=1}^{T} \begin{bmatrix} \frac{\partial C_{ms}^{*}(q_{ms}^{*}(t))}{\partial q_{m}(t)} + \lambda_{m}^{*}(t) - \chi_{m}^{*}(t) + \eta_{m}^{*}(t) \right] \\ & + \sum_{i=1}^{T} \sum_{m=1}^{T} \sum_{m=1}^{T} \begin{bmatrix} \frac{\partial C_{ms}^{*}(q_{ms}^{*}(t)}{\partial q_{m}(t)} + \lambda_{m}^{*}(t) - \chi_{m}^{*}(t) + \eta_{m}^{*}(t) + \eta_{m}^{*}(t) \right] \\ & + \sum_{i=1}^{T} \sum_{m=1}^{T} \begin{bmatrix} \frac{\partial C_{ms}^{*}(q_{ms}^{*}(t)}{\partial q_{m}(t)} + \lambda_{m}^{*}(t) - \eta_{m}^{*}(t) + \eta_{m}^{*}(t) - \eta_{m}^{*}(t) \right] \\ & + \sum_{i=1}^{T} \sum_{m=1}^{T} \begin{bmatrix} \frac{\partial C_{ms}^{*}(q_{ms}^{*}(t)}{\partial q_{ms}(t)} + \lambda_{m}^{*}(t) - \eta_{m}^{*}(t) + \sum_{i=1}^{T} \sum_{m=1}^{T} \begin{bmatrix} \frac{\partial C_{ms}^{*}(q_{ms}^{*}(t)}{\partial q_{ms}(t)} + \eta_{m}^{*}(t) - \eta_{m}^{*}(t) \right] \\ & + \sum_{i=1}^{T} \sum_{m=1}^{T} \sum_{m=1}^{T} \begin{bmatrix} \frac{\partial C_{ms}^{*}(q_{ms}^{*}(t)}{\partial q_{ms}^{*}(t)} + \lambda_{m}^{*}(t) - \eta_{m}^{*}(t) - \eta_{m}$$

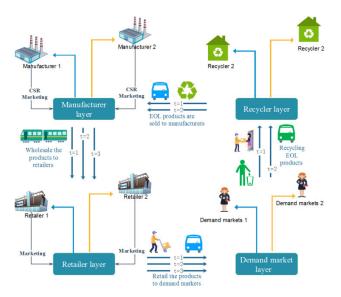


FIGURE 3. Three-period CLSCN structure.

TABLE 5. Cost functions in the CLSCN.

Function settings

$$\begin{split} C_{ms}^{m}(q_{ms}(t)) &= 0.15q_{ms}(t)^{2} + 0.15q_{ms}(t) + 0.1 \\ C_{m}^{m}(q_{m}(t)) &= (0.15 + 0.25t)(u_{m}q_{m}(t))^{2} + u_{m}q_{m}(t) + 0.1 \\ C_{m}^{i}(q_{m}^{i}(t)) &= (0.15 + 0.1t)q_{m}^{i}(t) \\ C_{r}^{i}(q_{r}^{i}(t)) &= (0.15 + 0.1t)q_{r}^{i}(t) \\ C_{mr}^{m}(q_{mr}(t)) &= 0.15q_{mr}(t)^{2} + 0.15q_{mr}(t) + 0.1 \\ C_{m}^{m}(u_{m}^{h}, q_{m}^{h}(t)) &= (1 - 0.25t)(u_{m}^{h}q_{m}^{h}(t))^{2} + u_{m}^{h}q_{m}^{h}(t) + 0.1 \\ f_{ms}(e, e_{m}, q_{ms}(t)) &= 0.15e^{2}e_{m}q_{ms}(t) \\ f_{ms}^{CSR}(h_{m}(t), u_{m}q_{m}(t), u_{m}^{h}q_{m}^{h}(t)) &= 0.25h_{m}(t)(u_{m}q_{m}(t) + u_{m}^{h}q_{m}^{h}(t)) \\ C_{sd}^{s}(q_{sd}(t)) &= 0.25q_{sd}(t)^{2} + 0.15q_{sd}(t) + 0.1 \\ f_{sm}(e, e_{s}, q_{ms}(t)) &= 0.15e^{2}e_{s}q_{ms}(t) \\ f_{sm}^{c}(e, e_{s}, q_{ms}(t)) &= 0.15e^{2}e_{s}q_{ms}(t) \\ C_{mr}^{r}(q_{mr}(t)) &= 0.25q_{rd}(t)^{2} + 0.15q_{rd}(t) + 0.1 \\ C_{rd}^{r}(q_{rd}(t)) &= 0.25q_{rd}(t)^{2} + 0.15q_{rd}(t) + 0.1 \\ q_{d}(1) &= (100 + 10t) - (10 - t)p_{si}(t) - 5p_{s(3-i)}(t) + be, i = 1,2 \\ V_{d}(t) &= (0.25 - 0.1t)Q^{RD}(t) \end{split}$$

results? Meanwhile, what are the optimal CSR and marketing strategies?

2) When the level of manufacturers' CSR activities is dynamic (i.e., the level of CSR activities changes within each period), and both manufacturers and retailers can conduct marketing (marking apportionment), how do marketing and CSR affect multi-period CLSCN equilibrium results? Meanwhile, what are the optimal CSR and marketing strategies?

The detailed results and analysis of the above issues are shown in the section of Numerical Example 1 and Numerical Example 2.

A. NUMERICAL EXAMPLE 1

To explore the issue 1, the equilibrium analysis is conducted by changing the value of parameters $h_m(t)$, e_r , and e_m , $(e_m = 1 - e_r)$. Table 6 illustrates the effects of $h_m(t)$ and e_r on transaction quantity and transaction price. Figures 4–6 illustrate the effects of $h_m(t)$ and e_r on profits of the multiperiod CLSCN system.

Conclusion 1: Under the static level of CSR activities, when the value of retailers' marketing apportionment factor e_r is larger, it is more beneficial to the multi-period CLSCN system, which marketing should be assigned to the retailers instead of the manufacturers.

Conclusion 2: Under the static level of CSR activities, the level of manufacturers' CSR activities is positively correlated with transaction quantity, the profits of enterprise and the entire network system, and social welfare. However, the level of manufacturers' CSR activities is negatively correlated with the pure profits of manufacturers, wholesale price, and retail price.

Conclusion 3: To promote social welfare and the sustainable development of multi-period CLSCN, the optimal strategy of CSR and marketing is that the manufacturers have a relatively high level of CSR activities under the retailers' marketing.

As shown in Table 6, in terms of transaction quantity and price, when the level of manufacturers' CSR activities remain unchanged, the value of retailers' marketing apportionment factor $e_r(e_r \in [0, 1])$ increases, the transaction price (wholesale price, retail price) are lower, and transaction quantity is greater. This phenomenon occurs because manufacturers have fewer or do not need to pay the costs of marketing when the retailer's marketing apportionment factor e_r is larger $(e_r \rightarrow 1)$; thus, the wholesale price is lower. Meanwhile, when retailers are responsible for marketing or marketing apportionment factor e_r is larger, retailers should pay an individual marketing cost, can have a larger retail price space, and will exhibit better behaviors to stimulate consumption to expand market demand. Additionally, whether the marketing behaviors are led by manufacturers or retailers, the manufacturers' CSR activities increase the transaction quantity and decrease the wholesale and retail price. This result is similar to the conclusion of Panda et.al [4], that is, the manufacturers' CSR activities reduce the wholesale price, thereby prompting retailers to reduce the retail price, which describes the effect of CSR activities in increasing social welfare.

As shown in Figures 4–6, in terms of profits, when the level of manufacturers' CSR activities remains unchanged, the value of retailers' marketing apportionment factor $e_r(e_r \in [0, 1])$ increases, all members and the entire network's profits are higher. The reason for this phenomenon is that when retailers are responsible for marketing, retailers directly face contact consumers, the effect of marketing is clear, and the number of transactions increases. Regardless of whether marketing is led by manufacturers or retailers, the pure profits of manufacturers decrease, and the profits of manufacturers, recyclers, retailers and the entire multi-period

TABLE 6. Impact of static.

	$(e_r, e_m) \setminus h_m(t)$	0.1	0.5	1.0	1.5	2.0	2.5	3.0
	$e_r = 0$	7.3840	7.1604	6.8192	6.3940	5.8858	5.2733	4.5815
		8.6135	8.3790	8.0175	7.5700	7.0401	6.3903	5.6999
	$e_m = 1$	9.8920	9.6497	9.2828	8.8316	8.2981	7.6567	6.9645
	$e_r = 0.3$	7.3480	7.1217	6.7802	6.3550	5.8468	5.2342	4.5599
	$e_m = 0.7$	8.5746	8.3374	7.9757	7.5282	7.0004	6.3506	5.6811
$p_{sd}^{*}(t)$		9.8545	9.6103	9.2432	8.7919	8.2584	7.6170	7.0069
$P_{sd}(t)$	$e_r = 0.7$	7.2745	7.0427	6.7007	6.2754	5.7672	5.1545	4.5159
		8.4952	8.2525	7.8904	7.4428	6.9192	6.2694	5.6426
	$e_m = 0.3$	9.7795	9.5298	9.1623	8.7109	8.1774	7.5358	7.0933
	$e_r = 1$	7.2340	6.9991	6.6569	6.2315	5.7233	5.1105	4.4916
		8.4515	8.2057	7.8434	7.3957	6.8745	6.2247	5.6214
	$e_m = 0$	9.7390	9.4855	9.1177	8.6663	8.1327	7.4911	6.8710
	$e_r = 0$	16.6240	19.7544	24.5312	30.4840	37.5988	46.1738	55.8590
		18.0245	21.0730	25.7725	31.5900	38.4787	46.9261	55.9013
	$e_m = 1$	21.2960	24.2036	28.6064	34.0208	40.4228	48.1196	56.4260
	$e_r = 0.3$	17.1280	20.2962	25.0772	31.0300	38.1448	46.7212	56.1614
		18.5302	21.6138	26.3159	32.1334	38.9948	47.4422	56.1457
* (1)	$e_m = 0.7$	21.7460	24.6764	29.0816	34.4972	40.8992	48.5960	55.9172
$q_{sd}^{*}(t)$	$e_r = 0.7$	18.1570	21.4022	26.1902	32.1444	39.2592	47.8370	56.7774
	•	19.5624	22.7175	27.4248	33.2436	40.0504	48.4978	56.6462
	$e_m = 0.3$	22.6460	25.6424	30.0524	35.4692	41.8712	49.5704	54.8804
	$e_r = 1$	18.7240	22.0126	26.8034	32.7590	39.8738	48.4530	56.1176
		20.1305	23.3259	28.0358	33.8559	40.6315	49.0789	56.9218
	$e_m = 0$	23.1320	26.1740	30.5876	36.0044	42.4076	50.1068	57.5480
	$e_r = 0$	5.9840	5.7604	5.4192	4.9940	4.4858	3.8733	3.1815
		7.2135	6.9790	6.6175	6.1700	5.6401	4.9903	4.2999
	$e_m = 1$	8.4920	8.2497	7.8828	7.4316	6.8981	6.2567	5.5645
	$e_r = 0.3$	5.9480	5.7217	5.3802	4.9550	4.4468	3.8342	3.1599
		7.1746	6.9374	6.5757	6.1282	5.6004	4.9506	4.2811
*	$e_m = 0.7$	8.4545	8.2103	7.8432	7.3919	6.8584	6.2170	5.6069
$p_{ms}^*(t)$ -	$e_r = 0.7$	5.8745	5.6427	5.3007	4.8754	4.3672	3.7545	3.1159
		7.0952	6.8525	6.4904	6.0428	5.5192	4.8694	4.2426
	$e_m = 0.3$	8.3795	8.1298	7.7623	7.3109	6.7774	6.1358	5.6933
	$e_r = 1$	5.8340	5.5991	5.2569	4.8315	4.3233	3.7105	3.0916
	·	7.0515	6.8057	6.4434	5.9957	5.4745	4.8247	4.2214
	$e_m = 0$	8.3390	8.0855	7.7177	7.2663	6.7327	6.0911	5.4710

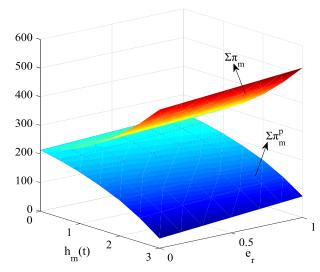


FIGURE 4. The profits of manufacturer layer of e_r and static $h_m(t)$.

CLSCN increases when the level of manufacturers' CSR activities increases. Although the total profits of manufacturers are positively correlated with the level of their CSR activities, the pure profits of manufacturers decrease as the

level of manufacturers' CSR activities increases. Therefore, manufacturers should not over-invest in their own CSR activities level because they will not be able to obtain pure profits to maintain business operations. These results indicate that manufacturers have increased social welfare by reducing their pure profits. Additionally, when the retailers' marketing apportionment factor e_r is larger ($e_r \rightarrow 1$), and manufacturers have a higher level of CSR activities, the profits of recyclers greater. This phenomenon will help recyclers increase their recycling power, thereby avoiding environmental pollution from EOL products, improving resource utilization, and facilitating the sustainable development of the multi-period CLSCN system.

B. NUMERICAL EXAMPLE 2

To explore the issue 2, the equilibrium analysis is also conducted by changing the value of parameters $h_m(t)$, e_r , and e_m , $(e_m = 1 - e_r)$. As the surrounding environment changes in enterprises, the behavior of manufacturers also changes; therefore, the level of manufacturers' CSR activities should also change during multi-period. Additionally, considering the situation as dynamic makes modeling more accurate.

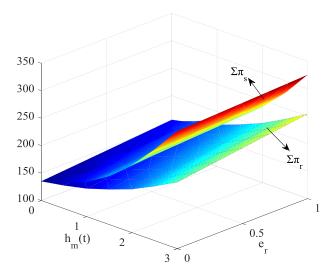


FIGURE 5. The profits of retailer or recycler layer of e_r and static $h_m(t)$.

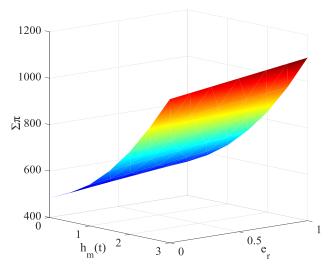


FIGURE 6. The profits of multi-period CLSCN of e_r and static $h_m(t)$.

TABLE 7. Five dynamic CSR levels situation.

Symbol	Investment of CSR-Activities
h _{csr1}	$h_m(1) = 1.25, h_m(2) = 0.5, h_m(3) = 1.25$
h _{csr2}	$h_m(1) = 0, h_m(2) = 3.0, h_m(3) = 0$
h _{csr3}	$h_m(1) = 1.50, h_m(2) = 0, h_m(3) = 1.50$
h _{csr4}	$h_m(1) = 0.5, h_m(2) = 1.0, h_m(3) = 1.5$
h _{csr5}	$h_m(1) = 1.5, h_m(2) = 1.0, h_m(3) = 0.5$

The dynamics considered in this paper are divided into five situations. Table 7 and Figure 7 show the dynamic level of the manufacturers' CSR activities. Table 8 illustrates the effects of $h_m(t)$ and e_r on transaction quantity and transaction price. Figure 8 illustrates the effects of $h_m(t)$ and e_r on the profits of the multi-period CLSCN system.

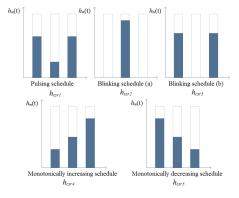


FIGURE 7. Five dynamic levels of CSR activities.

Conclusion 4: Compared to the static level of CSR activities, when the dynamic level of CSR activities, the influence of the marketing and CSR on the equilibrium results shows a consistent trend.

Conclusion 5: The level of CSR activities in the previous periods has a more significant impact on the equilibrium result.

Conclusion 6: For the optimal marketing and CSR strategy, the retailers are responsible for marketing, and the manufacturers have a higher level of CSR activities in the early periods.

In terms of product transaction quantity and price, as shown in Table 8, under the same level of manufacturers' CSR activities, when the value of retailers' marketing apportionment factor e_r is higher, retail and wholesale prices are lower, and transaction quantity is more heightened. The above performance is most obvious when $e_r = 1$. These results are consistent with the static situation of CSR activities, which the retailers' marketing is more conducive to reducing product prices and increasing product transaction quantity. These results can also explain from another perspective: retailers' marketing promotes increasing social welfare. Under the same marketing behaviors, when the level of manufacturers' CSR activities is h_{csr5} , the average price of products is the lowest, and the total transaction quantity is the highest. Conversely, when the level of manufacturers' CSR activities is h_{csr2} , the average price of products is the highest, and the total transaction quantity is the lowest. By comparing the level of manufacturers' CSR activities in h_{csr1} , h_{csr3} , and h_{csr4} , the average price of products is higher, and the total transaction quantity is lower when the level of manufacturers' CSR activities is h_{csr4} . Also, by comparing the level of manufacturers' CSR activities in h_{csr1} and h_{csr3} , we have found that when the level of manufacturers' CSR activities is h_{csr3} , the average price is lower, and the total transaction quantity is higher.

In terms of profits, as shown in Figure 8, the level of manufacturers' CSR activities is in h_{csr1} , h_{csr2} , h_{csr3} , h_{csr4} , and h_{csr5} . When retailers' marketing apportionment factor e_r is larger ($e_r \rightarrow 1$), the profits of manufacturers, retailers, and recyclers are higher, as does the multi-period CLSCN's profits. Notably, the profits of the entire multi-period CLSCN

				$h_m(t)$		
		h _{csr1}	h_{csr2}	h _{csr3}	h _{csr4}	h _{csr5}
	$e_{r} = 0$	6.5641	7.4287	6.4387	7.2163	6.4790
		8.3067	6.7702	8.7207	7.8385	7.7938
	$e_m = 1$	9.1023	9.7353	8.8742	8.9218	9.7103
	$e_r = 0.3$	6.5455	7.2124	6.3997	7.1783	6.4400
	'	8.2667	6.4768	8.6810	7.7967	7.7520
$p_{sd}^{*}(t)$	$e_m = 0.7$	9.0627	9.6992	8.8340	8.8822	9.6716
$P_{sd}(t)$	$e_r = 0.7$	6.5076	7.1389	6.3201	7.1006	6.3604
		8.1849	5.8778	8.6000	7.7113	7.6666
	$e_m = 0.3$	8.9817	9.6256	8.7521	8.8012	9.5925
	$e_{r} = 1$	6.4867	7.2787	6.2762	7.0578	6.3166
	'	8.1332	5.5477	8.5553	7.6643	7.6196
	$e_m = 0$	8.9371	9.5850	8.7069	8.7566	9.5489
	$e_{r} = 0$	28.1026	15.9982	29.8582	18.9718	29.2940
	'	22.0129	41.9874	16.6309	28.0995	28.6806
	$e_m = 1$	30.7724	23.1764	33.5096	32.9384	23.4764
	$e_r = 0.3$ $e_m = 0.7$	28.3630	19.0264	30.4042	19.5038	29.8400
		22.5329	45.8016	17.1470	28.6429	29.2240
$q_{sd}^{*}(t)$		31.2476	23.6096	33.9920	33.4136	23.9408
$q_{sd}(t)$	$e_r = 0.7$	28.8936	20.0554	31.5186	20.5916	30.9544
	<i>e_m</i> = 0.3	23.5963	53.5886	18.2000	29.7531	30.3342
		32.2196	24.4928	34.9748	34.3856	24.8900
	$e_r = 1$ $e_m = 0$	29.1862	18.0982	32.1332	21.1908	31.5676
		24.2684	57.8799	18.7811	30.3641	30.9452
		32.7548	24.9800	35.5172	34.9208	25.4132
	$e_{r} = 0$	5.1641	6.0287	5.0387	5.8163	5.0790
		6.9067	5.3702	7.3207	6.4385	6.3938
	$e_m = 1$	7.7023	8.3353	7.4742	7.5218	8.3103
	$e_r = 0.3$ $e_m = 0.7$ $e_r = 0.7$	5.1455	5.8124	4.9997	5.7783	5.0400
		6.8667	5.0768	7.2810	6.3967	6.3520
$p_{ms}^{*}(t)$		7.6627	8.2992	7.4340	7.4822	8.2716
$P_{ms}(t)$		5.1076	5.7389	4.9201	5.7006	4.9604
	$e_m = 0.3$	6.7849	4.4778	7.2000	6.3113	6.2666
		7.5817	8.2256	7.3521	7.4012	8.1925
	$e_r = 1$ $e_m = 0$	5.0867	5.8787	4.8762	5.6578	4.9166
		6.7332	4.1477	7.1553	6.2643	6.2196
		7.5371	8.1850	7.3069	7.3566	8.1489

TABLE 8. Impact of dynamic $h_m(t)$ and e_r on equilibrium transaction quantity and price.

and social welfare are the highest of $e_r = 1$, which is the same as the static level of manufacturers' CSR activities. Next, we compare the impact of the five levels (h_{csr1} , h_{csr2} , h_{csr3} , h_{csr4}, h_{csr5}) of manufacturers' CSR activities on the profits of members and entire multi-period CLSCN under the same marketing. When the level of manufacturers' CSR activities is h_{csr5} , the profits of members and entire multi-period CLSCN are the highest, the pure profits of manufacturers are the lowest. Conversely, when the level of manufacturers' CSR activities is h_{csr2} , the pure profits of manufacturers are the highest, the profits of other members and entire multi-period CLSCN are the lowest. Comparing the level of manufacturers' CSR activities in h_{csr1} and h_{csr4} , manufacturers' pure profits are higher, and members' profits and the entire network system's total profits are lower when the level of manufacturers' CSR activities is h_{csr4} . And comparing the level of CSR activities in h_{csr1} and h_{csr3} , the pure profits of manufacturers is higher, and the total profits of enterprises and entire network system are higher when the level of manufacturers' CSR activities is h_{csr3} . Finally, by comparing the level of manufacturers'

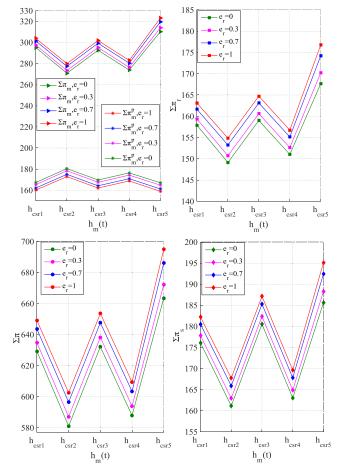


FIGURE 8. Profits of multi-period CLSCN of e_r and dynamic $h_m(t)$.

CSR activities in h_{csr1} , h_{csr3} and h_{csr4} , we find that the h_{csr3} exhibits a higher level of manufacturers' CSR activities in the first period. Therefore, the profits of enterprises and entire multi-period CLSCN are greater in the higher level of manufacturer's CSR in the first or early periods.

VI. DISCUSSION AND MANAGERIAL INSPIRATION

A. DISCUSSION

Through comprehensive analysis of the results of Numerical Examples, this paper shows that manufacturers and retailers responsible for marketing improves the economic performance of multi-period CLSCN, which agrees with the conclusions of Taleizadeh et al. [6] and Zhang et al. [34]. Additionally, based on conclusions 1 and 4, we have found that retailers' marketing has better results. Different from the literature of [6], [34], we also consider the manufacturers' CSR activities. We found that whether manufacturers or retailers lead marketing, manufacturers' CSR activities benefit the profits of retailers and recyclers, which shows that the manufacturers' CSR activities care about the interests of their stakeholders [4]. At this point, manufacturers sacrifice their pure profits to increase the economic performance of their stakeholders. As manufacturers' CSR activities increases, social welfare and the profits of entire multi-period

CLSCN improve. This conclusion agrees with the study of Shu *et al.* [37]; however, they did not investigate the impact of the dynamic level of manufacturers' CSR activities on a multi-period CLSCN. Our research expands the study of [37] and analyses the level of manufacturers' CSR activities in dynamic situations. As analyzed in numerical example 2, under the five dynamic levels of CSR activities, the better levels of CSR activities are h_{csr3} and h_{csr5} , the worst level of CSR activities is the h_{csr2} . Additionally, this study shows that the level of manufacturers' CSR activities strongly affects equilibrium quantity, equilibrium price, profits, and social welfare in the first period. Therefore, the optimal marketing and CSR strategies in the multi-period CLSCN is that retailers are responsible for marketing, and manufacturers have a higher level of CSR activities in the first or early period.

B. MANAGERIAL INSPIRATION

1) MANUFACTURERS/RETAILERS/RECYCLERS

First, the equilibrium analysis of this paper shows that manufacturers' CSR activities have a positive effect on a multi-period CLSCN system. When manufacturers increase their profits, they should also be responsible for the profits of other members and the entire system. To improve the level of CSR activities, manufacturers should consider environmental protection, anti-corruption, integrity, charity, etc. Particularly, manufacturers should consider the nature of their products' technical ability (e.g., scientific and technological means) to increase recovery rate, improve resources' sustainability, and protect the environment. Technological innovation can reduce the price of products, which can increase consumer welfare. Moreover, manufacturers should consider consumers' loyalty for products and adopt some specific strategies to improve consumers' product preference. The American economists Reikelder and Sass have conducted long-term observations and analyses of some industries, and they found that the loyalty product of consumers promotes the value of the enterprise. If consumer loyalty for products increases by 5%, the profits of the enterprise could increase by 25%-85%, and the costs of services for an old consumer may decrease steadily; therefore, manufacturers should strive to improve consumers' product loyalty while enhancing consumers' product preferences. Specific strategies include establishing a customer loyalty database and using big data to develop consumer loyalty strategies. Concurrently, manufacturers should understand the preferences and buying habits of consumers and then formulae dynamic management decisions, which can improve service quality and social welfare.

Second, in terms of retailers, retailers' marketing is more beneficial than manufacturers' marketing. Retailers should increase marketing investment, which is more conducive to the entire network system. However, retailers' marketing can increase other members' economic benefits, but retailers need to pay some marketing costs. This result can lead retailers to reduce marketing behavior. Therefore, to promote retailers' willingness to engage in marketing, manufacturers should Third, in terms of recyclers, retailers act as bridges between manufacturers and demand marketing in a reverse supply chain system. As shown in Property 8, when the adverse effects of recycling EOL products to demand markets are greater, the recycling price is greater. Therefore, recyclers should reduce the adverse effects on consumers through certain methods, such as door-to-door recycling, fast recycling, transportation subsidies, etc.

2) GOVERNMENT

The government is also important for the multi-period CLSCN system [27]. The formulation of government policies and regulations plays a decisive role, and therefore, recommendations to governments include the following:

First, the government has a responsibility to urge manufacturers to conduct CSR activities. This study shows that the manufacturers' CSR activities can reduce the pure profits of manufacturers; thus, the government should develop regulations, including certain supervision policies and compulsory measures, that increase the level of manufacturers' CSR activities. Additionally, our work shows that the manufacturers' CSR activities contribute to the profitability of the entire multi-period CLSCN system and help increase social welfare. Therefore, the government's policies can improve the sustainable development of the multi-period CLSCN system.

Second, increasing the recycling price will help increase the recycling rate. These increments often require subsidizing recyclers, which can enhance the recovery rate and reduce the environmental hazards of EOL product disposal. The government should work with manufacturers to use technology to understand and predict the demand markets' preference for products. After knowing the demand markets' preference for products, dynamic government intervention strategies, including dynamic subsidy policies, dynamic minimum recovery requirements, and dynamic patent regulations, should be developed. Only through dynamic intervention can the government's intervention utility be maximized.

Finally, regarding supervision policies, the government should also provide subsidies [18]. Our work shows that CSR activities will reduce manufacturers' pure profits. Manufacturers may thus decrease their CSR activities based on economics. This paper suggests that the government should adopt staged subsidy policies to prevent a decrease in manufacturers' CSR activities. For example, different CSR activities should be considered, and a step-by-step subsidy policy should be adopted. This policy could include different enterprise scales, industries, consumer groups, etc. and could increase the efficiency of government subsidies while maximizing social welfare.

VII. CONCLUSION

This paper studied the multi-period CLSCN equilibrium considering marketing and CSR. The equilibrium model is

established by using variational inequality. The influences of marketing and CSR on equilibrium results are described by numerical examples. The primary conclusions of this study are as follows: (1) regardless of whether manufacturers' CSR activities are dynamic or static, retailers' being responsible for marketing is better than manufacturers'; (2) increasing the level of manufacturers' CSR activities can help improve social welfare and the profits of the entire multi-period CLSCN system when the level of marketing remains unchanged; (3) when the level of manufacturers' CSR activities are dynamic, the best level of manufacturers' CSR activities are the flashing schedule b and the pulse schedule, and the worst level of manufacturers' CSR activities is the flashing schedule a regardless of whether retailers or manufacturers manage marketing. The level of CSR activities has the most significant impact on equilibrium results in the first period; and (4) the optimal strategy of marketing and CSR in the multi-period CLSCN system is that retailers are responsible for marketing, and the manufacturers have a higher level of CSR activities in the earlier periods.

These conclusions are appropriate for multi-period CLSCN with the assumptions outlined in this paper. These conclusions are also suitable for a multi-period CLSC under the same assumptions of this paper. Moreover, the conclusion of manufacturers' CSR static level is also suitable for a supply chain or static CLSCN equilibrium problem. Moreover, under the same assumptions, these conclusions are also suitable for the CLSCN equilibrium when other members are responsible for recycling EOL products.

Future research will consider studying the structure of a dual-channel multi-period CLSCN with CSR and marketing. Additionally, this study only conducted a sensitivity analysis from the method of numerical examples and did not include case studies; therefore, we plan to consider case studies. This paper also only considers manufacturers' CSR activities; thus, we hope to study the multi-period CLSCN equilibrium considering other members' CSR activities.

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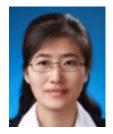




CAIQUAN DUAN received the Ph.D. degree in management science and engineering from the Harbin University of Science and Technology, Harbin, China. She is currently a Lecturer with the Department of College of Engineering, Northeast Agricultural University. Her research interests include supply chain system optimization and modeling, optimization approach, and closed-supply chain management.

FENGMIN YAO received the Ph.D. degree in management science and engineering from the Harbin University of Science and Technology, Harbin, China. He is currently a Professor and the Ph.D. Director of the School of Economics and Management, Harbin University of Science and Technology. His research interests include supply chain system optimization and modeling, optimization approach, and supply chain management.

GUOYI XIU received the Ph.D. degree in management science and engineering from Harbin Engineering University, Harbin, China. He is currently a Professor and the Ph.D. Director of the School of Economics and Management, Harbin University of Science and Technology, China. His research interests include management innovation and engineering, virtual enterprise system management, and supply chain management.







XIAOLI ZHANG received the M.Sc. degree in economics from the Harbin University of Commerce, Harbin, China. She is currently pursuing the Ph.D. degree with the School of Economics and Management, Harbin University of Science and Technology, Harbin. She is also a Lecturer with the Harbin Institute of Finance, Harbin. Her current research interests include supply chain system optimization and modeling.

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