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Responsiveness of Multi-Channel IEC-61850 Substation Communication Network Reliability Performance to Changes in Repair Factors

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ABSTRACT Power distribution centres distribute power to motor control centres in industrial facilities, as well as supply power to medium voltage drives among other loads. In reliability studies, the use of the combinatorial methods to evaluate the reliability of IEC-61850 based SCN in power distribution centres is preferred, owing to the ease of comprehension and applying the methods. The disadvantage of the methods is that repairs are assumed to be perfect and that all system faults are known; whereas it is not the case in practice. Markov model enables the impact of imperfect repairs on the system reliability performance to be investigated using the eigenvalue analysis method based on the concept of linear dynamical systems. The method presented in this paper advances the eigenvalue analysis method and focusses on the incremental responsiveness of the system to imperfect repair factors based on absorbing Markov chain and matrix calculus. The results indicate that even though the system is dependent on repair factors, the system is perfectly inelastic to the repair efficiency factors. However, the diagnostic coverage of the system is the most critical of the two factors, with higher elasticity as the factor approaches 100%. The results also indicate that both sensitivity and elasticity rapidly decrease as the diagnostic coverage of the system decrease. Thus, it is concluded that more emphasis must be put on the system diagnostic coverage because it is embedded in the system design, which can be expensive and practically not achievable once the system is commissioned and is running.

INDEX TERMS Substation communication network (SCN), IEC-61850, sensitivity, elasticity, Markov, matrix calculus.

I. INTRODUCTION

Power distribution centres distribute power to motor control centres in industrial facilities, as well as supply power to medium voltage drives among other loads. In thermal power plants, for instance, the majority of the drives comprise auxiliary boiler loads (viz. pumps, fans and mills). IEC-61850 based Substation Communication Network (SCN) monitors, controls and protects the electrical system; as well as interfaces with external systems such as Boiler Protection System (BPS) for the execution of safety-related trip commands. Hence, high reliability is needed to enable execution of safety-related trips and commands as and when they are issued. In reliability studies, the use of the combinatorial methods to evaluate the reliability of IEC-61850 based SCN is preferred, owing to the ease of applying the methods [1], [2].

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Nevertheless, the disadvantage of the methods is that repairs are assumed to be completed and correctly executed. Also, it is assumed that all system faults are known; whereas it is not the case in practice [2], [3]. In a thermal process plant, the control, monitoring and protection of auxiliary boiler systems is implemented through a dedicated Distributed Control System (DCS) and BPS, respectively. As a Safety-Related System (SRS), BPS is required to meet the requirements of the IEC-61508, which is the standard for SRS. The standard requires that a system diagnostic coverage be considered in the evaluation of SRS reliability performance, where power distribution centre equipment form part of the final element of the SRS to isolate power supply to electrical machines [2], [4].

In [2], The reliability of IEC-61850 based SCN is modelled using Markov, where imperfect repairs and system diagnostic coverage are integrated into the model using Systems Thinking approach [2]. The Markov model approach enables the impact of both repair efficiency and system diagnostic coverage on the system reliability performance and dynamical behaviour to be investigated based on the concept of linear dynamical systems through the observation of the eigenvalues of the system [5]. In cases where the system's state is at the edge of chaos, small changes in system variables could result in the system becoming uncontainable [6], [7]. Hence, the eigenvalue analysis method is used to investigate the dynamics of the system state as it changes.

The technique presented in this paper advances the eigenvalue analysis method, it uses sensitivity analysis to evaluate the responsiveness of the system reliability performance to incremental changes in repair efficiency and system diagnostic coverage [1], [2], [5], being cognizant that any factors of interest can be modelled based on the desired level of system abstraction. Hence, the sensitivity analysis approach enables the determination of the impact of imperfect repairs between any two system states; which in turn enables system optimisation at subsystem level compared to the eigenvalue method that only determines high-level behavioural dynamics based on the layout formation of the eigenvalues and the spectral gap of the second largest eigenvalue(s) [8]. The disadvantage of only using the eigenvalue method is that mean state transitions of the system cannot be determined since the focus is asymptotic and not transient. Thus, sensitivity analysis complements the eigenvalue analysis methods [9]–[11]. The contributions of the paper are the following:

a) The determination of transient system dynamics in IEC-61850 based SCN based on the concept of absorbing Markov chain and matrix calculus.
b) Analysis of the responsiveness of system reliability performance to repair factors based on system mean state transitions.

The layout of the paper is as follows: The context of the sensitivity and elasticity analysis of the system reliability based on mean system state transitions to the repair efficiency and diagnostic coverage factors are presented in section II. Section III presents an overview of a power distribution centre under consideration in this paper and the associated SCN architectures. Also, a review of IEC-61850 based SCN reliability, availability and evaluation methods are presented in section IV. This section also discusses advanced reliability and availability studies of repairable multi-channel IEC-61850 based SCN and its dynamical behaviour. Section V presents the preliminaries of matrix calculus and the notation used in sections VI and VII. The derivation of sensitivity and elasticity of the responsiveness of system reliability performance based on absorbing Markov Chain is presented in section VI. Section VII presents the sensitivity and elasticity of the 'one-out-of-two' scheme to imperfect repair factors based on matrix calculus. The results and discussions of the case studies are presented in section VIII. Section IX highlights the findings of the research, and thus concludes the paper.

II. SENSITIVITY AND ELASTICITY STUDIES

The results of the eigenvalue analysis method indicate that the response of their magnitudes to changes in repair factors (viz. repair efficiency and system diagnostic coverage) is not linear [5], [6], of which the finding necessitate a new method that can be used to determine the impact in quantity or percentage to enable performance optimisation of the system. Sensitivity studies are used in science and other fields, as well as in engineering analysis to investigate the responsiveness of a system to some dependent variables [8], [12], [13]. In this paper, the approach is used to determine the responsiveness of the system reliability performance based on the system mean state transitions to repair factors.

The method presented in this paper is based on the concept of absorbing Markov chain to determine the fundamental matrix while matrix calculus techniques are employed to determine the responsiveness of the system [3], [14], [15]. The advantage of using the fundamental matrix is that system variables of choice can be investigated based on the level of system abstraction to determine whether incremental changes of the individual repair factors of the respective subsystems is beneficial, given a specific system performance level [15]–[18]. Hence, system optimisation can be achieved at the subsystem level as desired. The next section presents an overview of a power distribution centre and the basis for the study.

III. OVERVIEW OF AN INDUSTRIAL POWER DISTRIBUTION CENTRE

The reliability of electrical protection system in a power distribution centre of a process plant is equally important if not more than the need of maintaining the continuous supply power to the various parts of the plant to ensure process continuity. Hence, repairable multi-channel IEC-61850 based SCN schemes with a minimum of 'one-out-of-two' voting capability are employed to ensure that a single point of failure in the protection scheme does not result in the scheme being inoperable [19], [20]; this configuration enables one channel to be isolated for repairs while the other channel is in service [21], [22]. Two independent channels (viz. cascade and star configurations) based IEEE Power System Relaying Committee (PSRC) SCN architectures are considered [23].

A. OUTLINE OF A POWER DISTRIBUTION CENTRE AND COMMUNICATION ARCHITECTURE

In a power distribution centre, a single incoming supply to the switchboard supplies power to multiple circuits that in turn supply various loads. Most of the power distribution centre loads in a thermal plant are auxiliary boiler drives [2], [24], [25]. The reliability and availability of the draught system protection circuits are critical to ensuring the safety of the plant and personnel if the process becomes unstable and needs to be stopped with urgency [24], [26]. The protection of the draught system is per IEC-61508 that address the requirements of safety-related systems, particularly IEC-61511 in process plants; which requires the consideration of system diagnostic coverage in determining the reliability of the protection system. Figure 1 depicts a typical IEC-61850 SCN that interfaces to a BPS through

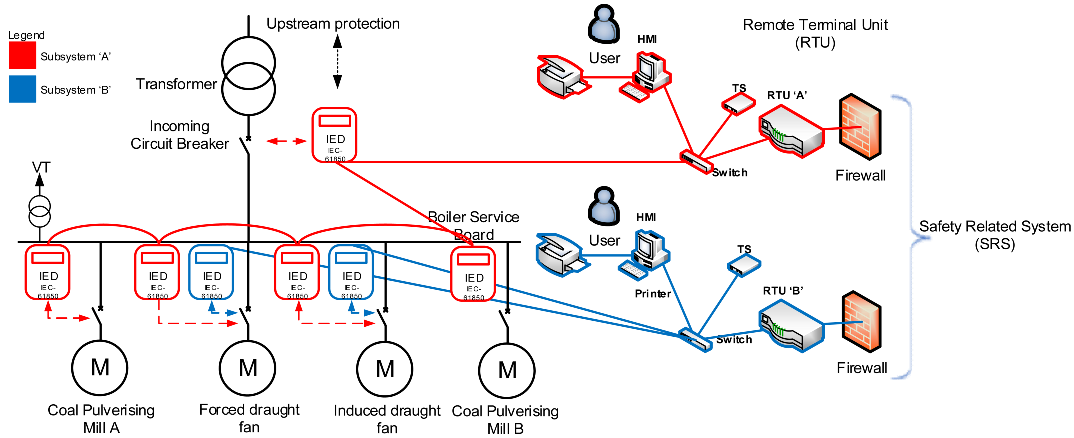

FIGURE 1. Typical IEC-61850 based thermal industrial power distribution centre [2], [6], [23], [24].

Remote Terminal Units (RTU) to a boiler protection system [2], [23], [24].

The study presented in this paper assumes the following in order to obtain the realistic performance level of the system, as well as simplify the modelling effort needed:

a) The most distant IEDs is considered as the worst case is for the reliability and availability performance evaluation [20], [27].
b) Hardwiring method of the tripping output signals from the individual IEDs to the associated circuit breakers is used [1].
c) The reliability of the communication network links is reasonably very high and assumed to have no adverse effect on the calculations presented here [27], [28].
d) The individual channels are entirely independent [3].

B. RELIABILITY DATA OF SUBSTATION DEVICES

The Mean Time To Repair (MTTR) of each subsystem is assumed to be 12 hours [2], [6], while the reliability data of the substation communication devices applied in this case study is obtained from [20], [27]. Hence, the failure rates λ_A and λ_B of the subsystems A and B are presented in Table 1, respectively [3], [20], [27].

TABLE 1. Subsystem failure rates.

Subsystem	A	B
Failure rate (λ)	0.13999	0.11333

IV. A REVIEW OF IEC-61850 SUBSTATION COMMUNICATION NETWORKS

This section reviews the basic, as well as the advanced reliability methods of IEC-61850 based SCN.

A. RELIABILITY, AVAILABILITY AND EVALUATION METHODS

In [1], [2], [29], a comprehensive model review of IEC-61850 SCN is presented and discussed. The papers [1], [2], [29] discuss the reliability, availability and data traffic flow performance of IEC-61850 based SCN. The reliability and availability performance is evaluated using the RBD while Optimised Network Engineering Tools (OPNET) software is used in [27], [30], [39]–[43], [31]–[38]. In the studies presented in [27], [30], [39]–[43], [31]–[38], system diagnostic coverage is not considered in the evaluation of the system reliability performance [2], [6]. This paper uses the eigenvalue analysis method to demonstrates that system reliability performance is dependent on both the diagnostic coverage and repair efficiency [6]. Also, the paper demonstrates that sensitivity and elasticity analysis can be employed to determine the incremental effect of both the diagnostic coverage and repair efficiency on the system to advance the level of detail provided by the eigenvalue analysis method.

A comprehensive review of the reliability and availability evaluation methods of IEC-61850 based SCN is presented in [1], [2]. Markov process can model repairable multi-state systems because the dependencies of the states are naturally included in the model [44]. Even though the stochastic property of the Markov process is considered to be a drawback [3], [45], the advantages of the process concerning the comprehension and the model simplification out-weighs the disadvantages presented by the process [2], [46], [47]. Hence, the Markov is chosen to model the reliability and availability of IEC-61850 based SCN in this paper.

B. ADVANCED RELIABILITY AND AVAILABILITY PERFORMANCE STUDIES

The need for IEC-61850 based SCN to execute SRS mission-critical functions is comprehensively discussed in [48], [49]. In [48], a question is asked, "can we use IEC-61850 for safety-related functions?". Perhaps a more precise question to ask would be how dependable is IEC-61850 to be used for IEC-61508 based safety-related applications? Detailed experimental studies presented in [49] confirmed that IEC-61850 satisfy all the required qualitative attributes of dependability stipulated in IEC-61784-3 (viz. safety and integrity), and therefore the requirement of IEC-61508. In [1], [2], it was demonstrated that the reliability and availability studies of SCN for mission-related systems requires consideration of system diagnostic coverage according to the requirements of IEC-61508. The work presented in [2] integrated the diagnostic coverage and repair efficiency factors of the system into the Markov reliability model using Systems Thinking approach.

In [6], Markov and symbolic dynamics based on the concept of linear dynamical systems are used to investigate the impact of both the repair and system diagnostic coverage factors on the dynamical behaviour of the system. The study outcome indicates that the eigenvalues of a transition probability matrix can be used to determine the system periodicity, as well as the nature of the state transitions. The outcome also indicates that the spectral gap between the eigenvalue of magnitude one and the second largest eigenvalue determines the rate at which the system converges to the absorbing state.

Additional to the outcome already listed above, the research work presented in [6] indicates the effectiveness of both the repair efficiency and diagnostic coverage factors on the spectral gap of the system as they vary from 0% to 100%; which indicates the system reliability response based on the mean system state transitions before complete system failure.

In the case study of [6], it is noticeable that increasing repair efficiency levels much closer to 100% has a small impact on the magnitudes of the eigenvalue. This result becomes more pronounced when the system has a higher diagnostic coverage factor closer to 100%, which suggests that the factors can be optimised. Although the eigenvalue analysis method can reveal the increase or decline of system mean state transitions when one of the system parameters is adjusted, the analysis does not determine the number of state transitions. On the contrary, the eigenvalue analysis method can determine the dynamical state of the system. Further, the change in layout formation of the eigenvalues on the complex plane can be used to determine any stability, improvement or worsening system dynamical behaviour.

C. EIGENVALUE ANALYSIS METHOD AND SYSTEM DYNAMICS

An overview of system diagnostic coverage levels based on ISO 13849-1 is presented [2], [6] and forms the basis of the case studies presented here. The levels of system diagnostic coverage are presented in Table 2 according to their denotation for ease of reference [50]–[52].

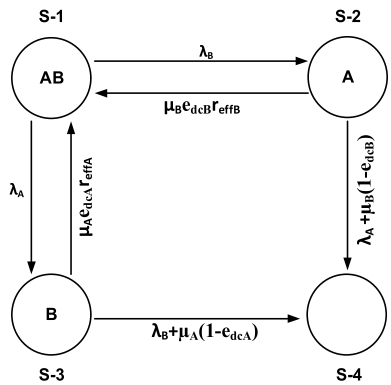

FIGURE 2. State transition probability diagram of 'one-out-of-two' SCN scheme [2], [6].

TABLE 2. Denotation of diagnostic coverage levels and ranges [2], [6], [50]–[52].

Denotation	Range
None	$e_{dc} < 60\%$
Low	$60\% \leq e_{dc} < 90\%$
Medium	$90\% \leq e_{dc} < 99\%$
High	$99\% \leq e_{dc}$

In order to demonstrate the eigenvalue analysis method presented in [6], the 'one-out-of-two' system of Figure 1 presented in section III is considered. Figure 2 depicts the Markov state transition probability diagram of the SCN presented in Figure 1, where the state transition probability matrix is given in (1), as shown at the bottom of the page.

Three cases of individual system diagnostic capabilities are presented in Table 3, where the diagnostic coverage factors have been arbitrarily chosen to illustrate the eigenvalue analysis method [6].

The magnitude responses of the eigenvalues of the transition probability matrix are depicted in Figure 3 as the repair efficiency is varied from 5% to 100% for the case studies presented in Table 3. It is noticeable in Figure 3(C-1), Figure 3(C-2) and Figure 3(C-3) that the magnitude of eigenvalues eigV1 and eigV2 increase with increasing repair efficiency. However, it is also noticeable that the spectral gap decreases with higher system diagnostic coverage for a given

$$P = \begin{bmatrix} 1-\lambda_A-\lambda_B & \lambda_A & \lambda_B & 0 \\ \mu_A e_{dcA} r_{effA} & 1-\mu_A e_{dcA} r_{effA}-(\lambda_B+\mu_A(1-e_{dcA})) & 0 & \lambda_B+\mu_A(1-e_{dcA}) \\ \mu_B e_{dcB} r_{effB} & 0 & 1-\mu_B e_{dcB} r_{effB}-(\lambda_A+\mu_B(1-e_{dcB})) & \lambda_A+\mu_B(1-e_{dcB}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

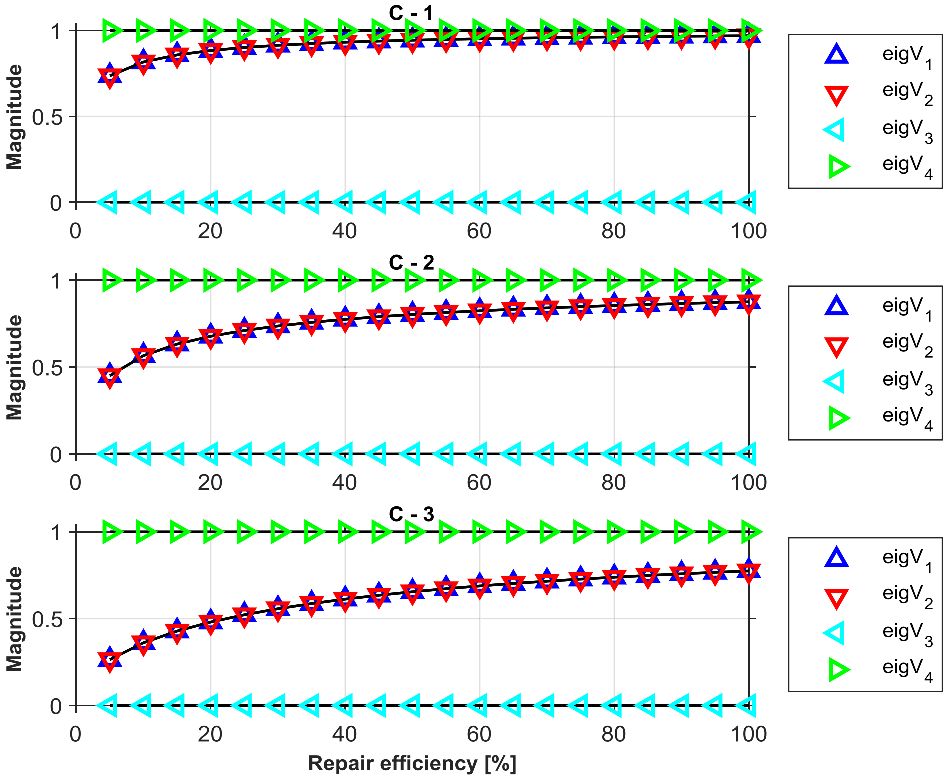

FIGURE 3. Case studies of 'one-out-of-two' scheme based on eigenvalue analysis.

TABLE 3. Eigenvalue analysis case studies.

Case study	C-1	C-2	C-3
Subsystem 'A' diagnostic coverage (e_{dcA})	90%	90%	60%
Subsystem 'B' diagnostic coverage (e_{dcB})	99%	60%	60%

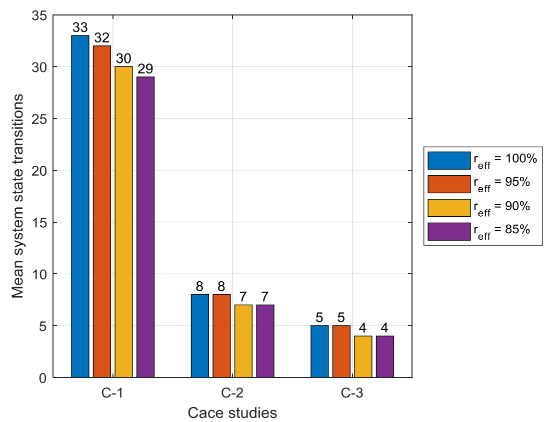

FIGURE 4. Case studies of 'one-out-of-two' scheme based on mean system state transitions.

level of repair efficiency. Another observation of interest in Figure 3 is that the effect of repair efficiency decreases as it approaches 100%, particularly so for systems with high diagnostic coverage as depicted in Figure 3(C-3); which demonstrate a nonlinear response of the system reliability based on the eigenvalue magnitudes as they impact the mean system state transitions. On the other hand, the asymptotic behavioural dynamics of the system is clearly illustrated by the eigenvalue magnitudes.

Nevertheless, the disadvantage of the eigenvalue method in providing a comprehensive system analysis is for the fact that it is not able to provide transient system performance indicators based on the mean system state transitions for optimisation purposes. A summary of the mean system states transitions of the case studies of the system in Table 2 and Figure 3 is depicted in Figure 4, where repair efficiency 85% to 100% is presented to enable comparison.

A comprehensive description of the eigenvalue analysis method is presented in work presented in [5], [6]. The remainder of this paper presents a complementary analysis method that is based on sensitivity and elasticity analysis of the system to imperfect repairs in order to enable optimisation of the system performance. The next section presents the concepts and methods of matrix calculus and notation.

V. PRELIMINARIES OF MATRIX CALCULUS AND NOTATION

Matrix calculus methods enable logical differentiation of various forms of valued functions (viz. scalar, vector and matrices). Vector arrangement is used in this paper because it is conceptually simple to comprehend and logical to apply compared to many other conventions of matrix calculus methods [15], [53]–[55]. Also, the implementation of the vector arrangement concept in software packages is achievable without losing the original mathematical formation of the problem. This aspect eases the level of the required effort to present, interpret and analyse the results. In this paper, the notation used to represent scalars is non-bold non-capital letters, whereas vectors are represented by bold non-capital letters. Bold capital letters are used to represent matrices.

A. DERIVATIVES

The derivative of a scalar function y with respect to a scalar function x is given by $\frac{dy}{dx}$. If the function $\boldsymbol{y}$ is a vector and x a scalar, the derivative of the vector function $\boldsymbol{y}$ with respect to a scalar function x is given by a n x 1 vector (2), where n is the length $\boldsymbol{y}$ and $\boldsymbol{b}^{\mathrm{T}}$ is a transpose of $\boldsymbol{y}$.

$$\frac{d\boldsymbol{y}}{dx} = \begin{pmatrix} \frac{dy_1}{dx} & \cdots & \frac{dy_n}{dx} \end{pmatrix}^{\mathrm{T}} \tag{2}$$

The derivative of a scalar function y with respect to a vector function $\boldsymbol{x}$ is given by a 1 x m gradient vector (3), where m is the length of $\boldsymbol{x}$.

$$\frac{dy}{d\boldsymbol{x}^{\mathrm{T}}} = \begin{pmatrix} \frac{dy}{d\boldsymbol{x}_1} & \cdots & \frac{dy}{d\boldsymbol{x}_m} \end{pmatrix} \tag{3}$$

The difference between (2) and (3) is that (2) is a column vector while (3) is a row vector. This formation of vector representation is maintained throughout the paper. Given the results of (2) and (3), if both $\boldsymbol{x}$ and $\boldsymbol{y}$ are vector functions, then the derivative of $\boldsymbol{y}$ with respect to $\boldsymbol{x}$ is a n x m Jacobian matrix given by (4) if $\boldsymbol{y}$ is a n x 1 vector and $\boldsymbol{x}$ is a m x 1 vector [15], [53]–[55].

$$\frac{d\boldsymbol{y}}{d\boldsymbol{x}^{\mathrm{T}}} = \left(\frac{d\boldsymbol{y}_i}{d\boldsymbol{x}_j}\right) \tag{4}$$

Matrix derivatives are computed by first transforming the matrix into a vector formation using the vector operator and then applying the principles of vector differentiation to the vector functions. In order to maintain consistent notation, the vector operator is written as ′vec′ operator from this point onward. The ′vec′ operator stacks the columns of a n x m matrix to a nm x 1 vector, such that, if $\boldsymbol{X}$ is a n x m matrix and $\boldsymbol{Y}$ is a p x q matrix, the derivative of $\boldsymbol{Y}$ with respect to $\boldsymbol{X}$ is a matrix nm x pq given by (5) [15], [17], [18], [53].

$$\frac{d\mathrm{vec}\boldsymbol{Y}}{d\mathrm{vec}\boldsymbol{X}^{\mathrm{T}}} = \left(\frac{d\mathrm{vec}\boldsymbol{Y}_i}{d\mathrm{vec}\boldsymbol{X}_j}\right) \tag{5}$$

Thus, by chain rule, if $\boldsymbol{Y}$ is a function of $\boldsymbol{X}$, and $\boldsymbol{X}$ is a function $\boldsymbol{Z}$; then (6) holds.

$$\frac{d\mathrm{vec}\boldsymbol{Y}}{d\mathrm{vec}\boldsymbol{Z}^{\mathrm{T}}} = \frac{d\mathrm{vec}\boldsymbol{Y}}{d\mathrm{vec}\boldsymbol{X}^{\mathrm{T}}}\frac{d\mathrm{vec}\boldsymbol{X}}{d\mathrm{vec}\boldsymbol{Z}^{\mathrm{T}}} \tag{6}$$

B. THE KRONECKER PRODUCT AND ROTH'S THEOREM

The Kronecker product is given by (7). The product is also referred to as a tensor or direct product [15], [55].

$$\boldsymbol{A}\otimes\boldsymbol{B} = \left(a_{ij}\boldsymbol{B}\right) \tag{7}$$

The Kronecker product is related to the vec operator by Roth's theorem, such that if (7) holds, then (8) defines Roth's relation on block matrices [53]–[55].

$$\boldsymbol{D} = \boldsymbol{ABC} \tag{8}$$

$$\mathrm{vec}\,\boldsymbol{D} = \left(\boldsymbol{C}^{\mathrm{T}}\otimes\boldsymbol{A}\right)\mathrm{vec}\,\boldsymbol{B} \tag{9}$$

The next section employs matrix calculus concepts and principles to determine the sensitivity of the fundamental matrix to repair factors of choice. The sensitivity of the matrix represents the sensitivity of the system reliability based on the mean number of system transient state transitions since its elements are the mean numbers of system states transitions, where each row represents unique initial conditions.

VI. SENSITIVITY AND ELASTICITY OF THE FUNDAMENTAL MATRIX

An absorbing Markov process is characterised by at least one recurrent state in the system, where the transition probability matrix takes the form given by (10).

$$\boldsymbol{P} = \begin{bmatrix} (\boldsymbol{Q}) & (\boldsymbol{M}) \\ (\boldsymbol{O}) & (\boldsymbol{I}) \end{bmatrix} \tag{10}$$

where $\boldsymbol{Q}$ is a n x n of transient probability states, $\boldsymbol{M}$ is an n x m matrix of m vectors comprising the system failure rate probabilities, and m is the number of recurrent states. The matrix $\boldsymbol{I}$ is the identity matrix of the order m that represents the number of recurrent system states. As demonstrated in [2], [3], the mean system number of states transitions is given by the elements of the fundamental matrix $\boldsymbol{N}$ in (11) [3], [14], [55].

$$\boldsymbol{N} = (\boldsymbol{I}-\boldsymbol{Q})^{-1} \tag{11}$$

In order to derive the responsiveness of the fundamental matrix $\boldsymbol{N}$, it is considered that $\boldsymbol{N}$ satisfies the identity given by (12).

$$\boldsymbol{I} = \boldsymbol{N}\boldsymbol{N}^{-1} \tag{12}$$

Now, differentiating both sides of (12) gives (13).

$$\boldsymbol{0} = (d\boldsymbol{N})\boldsymbol{N}^{-1} + \boldsymbol{N}\left(d\boldsymbol{N}^{-1}\right) \tag{13}$$

Reorganising (13) by applying the vec operator and Roth's theorem gives (14).

$$\mathrm{vec}\,\boldsymbol{0} = \left[(\boldsymbol{N}^{-1})^{\mathrm{T}}\otimes\boldsymbol{I}\right]d\mathrm{vec}\,\boldsymbol{N} + (\boldsymbol{I}\otimes\boldsymbol{N})\,d\mathrm{vec}\,\boldsymbol{N}^{-1} \tag{14}$$

Solving (14) for $d\mathrm{vec}\boldsymbol{N}$ simplifies to (15).

$$d\mathrm{vec}\,\boldsymbol{N} = \left[(\boldsymbol{N}^{-1})^{\mathrm{T}}\otimes\boldsymbol{I}\right]^{-1}(\boldsymbol{I}\otimes\boldsymbol{N})\,d\mathrm{vec}\,\boldsymbol{Q} \tag{15}$$

Applying the Kronecker product identity given by (16) and (17), (15) simplifies to (18) provided that the dimensions of the matrices satisfy the operators [55], [56].

$$(\boldsymbol{A}\otimes\boldsymbol{B})^{-1} = \boldsymbol{A}^{-1}\otimes\boldsymbol{B}^{-1} \tag{16}$$

$$(\boldsymbol{A}\otimes\boldsymbol{B})(\boldsymbol{C}\otimes\boldsymbol{D}) = (\boldsymbol{AC}\otimes\boldsymbol{BD}) \tag{17}$$

$$d\mathrm{vec}\boldsymbol{N} = \left(\boldsymbol{N}^{\mathrm{T}}\otimes\boldsymbol{N}\right)d\mathrm{vec}\boldsymbol{Q} \tag{18}$$

Using the identification theorem, (18) can be written in the form of (19)

$$\frac{d\,\mathrm{vec}\boldsymbol{N}}{d\,\mathrm{vec}\boldsymbol{Q}^{\mathrm{T}}} = \boldsymbol{N}^{\mathrm{T}} \otimes \boldsymbol{N} \tag{19}$$

Hence, if $\boldsymbol{N}$ is a function of a vector $\boldsymbol{R}$ of variables of interest, (19) can be extended to give (20) by using the chain rule; which is the responsiveness (i.e. sensitivity) of $\boldsymbol{N}$ to vector $\boldsymbol{R}$ [17], [18], [55].

$$\frac{d\,\mathrm{vec}\boldsymbol{N}}{d\,\mathrm{vec}\boldsymbol{R}^{\mathrm{T}}} = \left(\boldsymbol{N}^{\mathrm{T}} \otimes \boldsymbol{N}\right) \frac{d\,\mathrm{vec}\boldsymbol{Q}}{d\,\mathrm{vec}\boldsymbol{R}^{\mathrm{T}}} \tag{20}$$

Since the proportional effectiveness (elasticity) oy_if to x_j is given by (21), the elasticity of vectors $\boldsymbol{y}$ to $\boldsymbol{x}$ is given by (22) [55], [57].

$$\frac{\varepsilon y_i}{\varepsilon x_j} = \frac{x_j}{y_i}\frac{dy_i}{dx_j} \tag{21}$$

$$\frac{\varepsilon \boldsymbol{y}}{\mathcal{D}\boldsymbol{x}^{\mathrm{T}}} = \mathcal{D}(\boldsymbol{y})^{-1}\frac{d\boldsymbol{y}}{d\boldsymbol{x}^{\mathrm{T}}}\mathcal{D}\left(\boldsymbol{x}\right) \tag{22}$$

The notation $\mathcal{D}(\boldsymbol{X})$ is a square matrix with the elements of the vector $\boldsymbol{X}$ on the diagonal of the matrix (i.e. $a_{ij} = 0$ for $i \neq j$). Therefore, the elasticity of the fundamental matrix $\boldsymbol{N}$ to vector $\boldsymbol{R}$ comprising all lower-level parameters of interest is given by (23) [16], [55].

$$\frac{\varepsilon\,\mathrm{vec}\boldsymbol{N}}{\varepsilon \boldsymbol{R}^{\mathrm{T}}} = \mathcal{D}(\mathrm{vec}\boldsymbol{N})^{-1}\frac{d\,\mathrm{vec}\boldsymbol{N}}{d\boldsymbol{R}^{\mathrm{T}}}\mathcal{D}\left(\boldsymbol{R}\right) \tag{23}$$

The next section presents the lower-level parameter model of the state probability transition matrix of the 'one-out-of-two' protection scheme, as well as the derivation of sensitivity and elasticity of the system reliability based on the mean number of system state transitions to the repair efficiency and diagnostic coverage factors.

VII. MODELLING SENSITIVITY AND ELASTICITY TO REPAIR FACTORS: 'ONE-OUT-OF-TWO' SCHEME

Figure 2 depicts the state transition diagram of the 'one-out-of-two' system presented. In order to model the responsiveness of the system to repair and diagnostic coverage factors, the state transition probability matrix P that describes the state transitions of the system is remodelled as a function of low-level system parameters of interest (i.e. repair efficiency and diagnostic coverage factors) in its stochastic form given by (24) [1], [3].

$$\boldsymbol{P} = \begin{bmatrix} \frac{1-\boldsymbol{P}_{12}-\boldsymbol{P}_{13}}{\lambda_A+\lambda_B} & \frac{\lambda_A}{\lambda_A+\lambda_B} & \frac{\lambda_B}{\lambda_A+\lambda_B} & 0 \\ \frac{\mu_A e_{dcA} r_{effA}}{\mu_A e_{dcA} r_{effA} + (\lambda_B+\mu_A-\mu_A e_{edcA})} & \frac{1-\boldsymbol{P}_{21}-\boldsymbol{P}_{24}}{\mu_A e_{dcA} r_{effA} + (\lambda_B+\mu_A-\mu_A e_{edcA})} & 0 & \frac{\lambda_B+\mu_A(1-e_{edcA})}{\lambda_B+\mu_A\left(1-e_{edcA}\right)+\mu_A e_{dcA} r_{effA}} \\ \frac{\mu_B e_{dcB} r_{effB}}{\mu_B e_{dcB} r_{effB} + (\lambda_A+\mu_B-\mu_B e_{edcA})} & 0 & \frac{1-\boldsymbol{P}_{31}-\boldsymbol{P}_{34}}{\lambda_A+\mu_B\left(1-e_{edcB}\right)+\mu_B e_{dcB} r_{effB}} & \frac{\lambda_A+\mu_B(1-e_{edcB})}{\lambda_A+\mu_B\left(1-e_{edcB}\right)+\mu_B e_{dcB} r_{effB}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

The transient states of the system are described by the matrix $\boldsymbol{Q}$ defined in (10) and given by (25) [14], [55].

$$\boldsymbol{Q} = \begin{bmatrix} \frac{1-\boldsymbol{P}_{12}-\boldsymbol{P}_{13}}{\lambda_A+\lambda_B} & \boldsymbol{P}_{12} & \boldsymbol{P}_{13} \\ \frac{\mu_A e_{dcA} r_{effA}}{\mu_A e_{dcA} r_{effA} + (\lambda_B+\mu_A-\mu_A e_{edcA})} & \frac{1-\boldsymbol{P}_{21}-\boldsymbol{P}_{24}}{\mu_A e_{dcA} r_{effA} + (\lambda_B+\mu_A-\mu_A e_{edcA})} & 0 \\ \frac{\mu_B e_{dcB} r_{effB}}{\mu_B e_{dcB} r_{effB} + (\lambda_A+\mu_B-\mu_B e_{edcB})} & 0 & \frac{1-\boldsymbol{P}_{31}-\boldsymbol{P}_{34}}{\lambda_A+\mu_B\left(1-e_{edcB}\right)+\mu_B e_{dcB} r_{effB}} \end{bmatrix} \tag{25}$$

The derivation of major sub-functions of the transient state matrix sensitivity to the vector $\boldsymbol{R}$ is presented next to offer comprehension of the approach and the organisation of the vector elements. The organisation of the individual vectors is presented to ensures accurate interpretation and analysis of the results. As discussed earlier, the use of vector arrangement method requires the matrix $\boldsymbol{Q}$ to be transformed into a vector using the vec operator, of which the vector organisation of $\boldsymbol{Q}$ is given by (26) [54], [55].

$$\mathrm{vec}\boldsymbol{Q} = \begin{bmatrix} \frac{1-\boldsymbol{P}_{12}-\boldsymbol{P}_{13}}{\lambda_A+\lambda_B} \\ \frac{\mu_A e_{dcA} r_{effA}}{\mu_A e_{dcA} r_{effA} + (\lambda_B+\mu_A-\mu_A e_{edcA})} \\ \frac{\mu_B e_{dcB} r_{effB}}{\mu_B e_{dcB} r_{effB} + (\lambda_A+\mu_B-\mu_B e_{edcB})} \\ \boldsymbol{P}_{12} \\ \frac{1-\boldsymbol{P}_{21}-\boldsymbol{P}_{24}}{\mu_A e_{dcA} r_{effA} + (\lambda_B+\mu_A-\mu_A e_{edcA})} \\ 0 \\ \boldsymbol{P}_{13} \\ 0 \\ \frac{1-\boldsymbol{P}_{31}-\boldsymbol{P}_{34}}{\lambda_A+\mu_B\left(1-e_{edcB}\right)+\mu_B e_{dcB} r_{effB}} \end{bmatrix} \tag{26}$$

In order to simplify the equations and the computation effort, the following expressions are defined.

$$P_{21}NM = \mu_A e_{dcA} r_{effA} \quad (27)$$

$$P_{21}DN = \mu_A e_{dcA} r_{effA} + (\lambda_B + \mu_A - \mu_A e_{edcA}) \quad (28)$$

$$P_{31}NM = \mu_B e_{dcB} r_{effB} \quad (29)$$

$$P_{31}DN = \mu_B e_{dcB} r_{effB} + (\lambda_A + \mu_B - \mu_B e_{edcB}) \quad (30)$$

Delimitate vector $\boldsymbol{R}$ to comprise the factors of interest to which the responsiveness of the mean number of state transitions represented by the transient state matrix $\boldsymbol{Q}$ are analysed [55], [57], given by (31).

$$\boldsymbol{R} = \begin{bmatrix} r_{effA} \\ r_{effB} \\ e_{dcA} \\ e_{dcB} \end{bmatrix} \quad (31)$$

Thus, the sensitivity of the transient state matrix to the elements of vector $\boldsymbol{R}$ (i.e. $\frac{d\text{vec}\boldsymbol{Q}}{d\boldsymbol{R}^T}$) is given by (32).

$$\frac{dvec\boldsymbol{Q}}{d\boldsymbol{R}_1} = \begin{bmatrix} 0 \\ \frac{\mu_A e_{dcA}(P_{21}DN - P_{21}NM)}{P_{21}DN^2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (32)$$

$$\frac{dvec\boldsymbol{Q}}{d\boldsymbol{R}_2} = \begin{bmatrix} 0 \\ 0 \\ \frac{\mu_B e_{dcB}(P_{31}DN - P_{31}NM)}{P_{31}DN^2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (33)$$

$$\frac{dvec\boldsymbol{Q}}{d\boldsymbol{R}_3} = \begin{bmatrix} 0 \\ \frac{\mu_A((P_{21}DNr_{effA} - P_{21}NM\left(r_{effA} - 1\right))}{P_{21}DN^2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (34)$$

$$\frac{dvec\boldsymbol{Q}}{d\boldsymbol{R}_4} = \begin{bmatrix} 0 \\ 0 \\ \frac{\mu_B\left((P_{31}DNr_{effB} - P_{31}NM\left(r_{effB} - 1\right)\right)}{P_{31}DN^2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (35)$$

Applying (20) and (23) gives the sensitivity and elasticity of the system reliability (i.e. mean states transitions) to the elements of the vector $\boldsymbol{R}$, respectively. Therefore, (32) to (35) gives the sensitivity of the transient state matrix $\boldsymbol{Q}$ to the individual factors of interest contained in vector $\boldsymbol{R}$. The next section presents the results and discussion of the sensitivity and elasticity of the system mean state transitions to repair and diagnostic coverage factors.

VIII. RESULTS AND DISCUSSIONS

In order to gain insights about the system behaviour while easing the analysis effort, the following assumptions are made:

a) The resources that are used to support and maintain subsystems A and B are not the same. In addition, the repair efficiency of subsystem A is assumed to be 95% while that of subsystem B is assumed to be 70%.
b) The system is fully functional at the beginning of the simulation.

The selection of repair efficiency factors is informed by the case study results of the eigenvalue analysis method in section IV and is intended to enable observation of the system's response at different levels of repair factors [2], [6]. The choice of factors demonstrates that the method used in this paper can compute different repair efficiencies. The diagnostic coverage levels of the cases studies presented here have been presented in Table 3 of section IV. MATLAB/Simulink is used for the modelling, computation and presentation of the results. The system state transitions are depicted by S_{xy}, where x represent the initial state of the system and y is the state into which the system transitioned. The fully functional system state is represented by state S-1 and depicted by S_{1y} in the simulation results.

A. CASE STUDY C-1

In this case study, the diagnostic coverage factors of the individual subsystems are assumed to be 90% and 99% for subsystem A and subsystem B, respectively. The sensitivity of the system to the repair efficiency factors is depicted in Figure 5(a). It is noticeable that the system is more sensitive to the repair efficiency of subsystem A, which has a lower system diagnostic coverage compared to subsystem B even though the repair efficiency of subsystem A is 95%. The incremental change of subsystem A repair efficiency (r_{effA}) causes the mean system state transition to increase

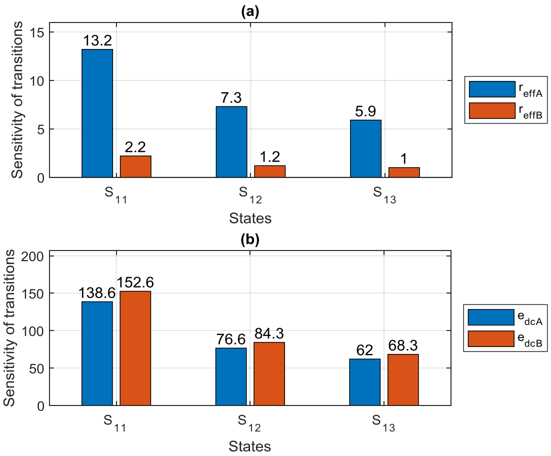

FIGURE 5. Sensitivity analysis of repair and diagnostic coverage factors of case study C-1.

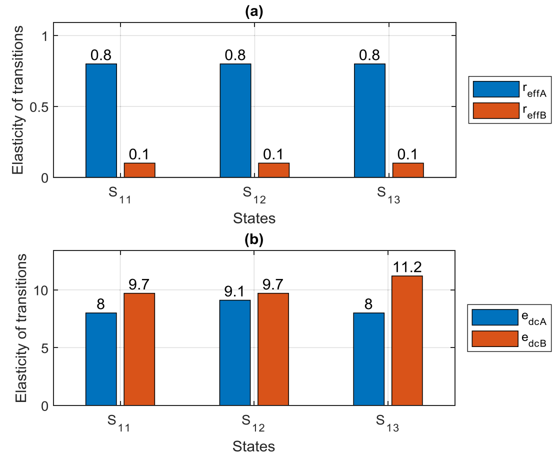

FIGURE 6. Elasticity analysis of repair and diagnostic coverage factors of case study C-1.

by 13, 7 and 6, for transitions into states S-1, S-2 and S-3, respectively; of which the total is the sum of transitions in states S_{1y}. Hence, it is more beneficial to increase the repair efficiency of subsystem A compared to that of subsystem B since it could only improve the state transitions by 2, 1 and 1, for transitions into states S-1, S-2 and S-3, respectively. The high number of system state transitions into state S-2 compared to S-3 is expected since subsystem A has a higher failure rate than subsystem B, as well as a higher repair efficiency factor considering that the diagnostic capabilities of the subsystems are relatively high.

In contrast, the diagnostic coverage factors have more impact on the mean system state transitions, as depicted in Figure 5(b). The incremental change in diagnostic coverage of the subsystem (e_{edcB}) causes the mean system state transitions to increase by 152, 84 and 68 transitions into states S-1, S-2 and S-3, respectively; of which the total is the sum of transitions given that the initial state is S-1. The sensitivity of the system to the factors indicates that diagnostic coverage is a critical factor in the determination of system reliability performance.

Figure 6 depicts the elasticities of the system state transitions to both the repair efficiency and system diagnostic coverage factors. The elasticity magnitudes of the repair efficiency factors r_{effA} and r_{effB} are inelastic as depicted in Figure 6(a). It is noticeable that improving the repair efficiency of subsystem A is beneficial than that of subsystem B since the elasticity of the mean system state transition is 0.8 for subsystem A, whereas that of subsystem B is 0.1.

Figure 6(b) depicts the elasticity of the system to the diagnostic coverage factors. The diagnostic coverage factors are perfectly elastic at around 8 for subsystem A, and 9.7 for subsystem B. Thus, it is beneficial to increase the diagnostic coverage factor of subsystem B because of its low repair efficiency factor. In context, the reliability of the system can be improved by concentrating on the performance of subsystem B because it is the most elastic factor with the highest overall incremental system state transitions.

B. CASE STUDY C-2

The diagnostic coverage factors of the individual subsystem A and subsystem B are assumed to be 90% and 60% respectively. Figure 7(a) depicts the sensitivity of the system reliability to the repair efficiency factors. It is noticeable that the system is more sensitive to the repair efficiency of subsystem B, which is lower by 25% compared to that of subsystem A at 95%.

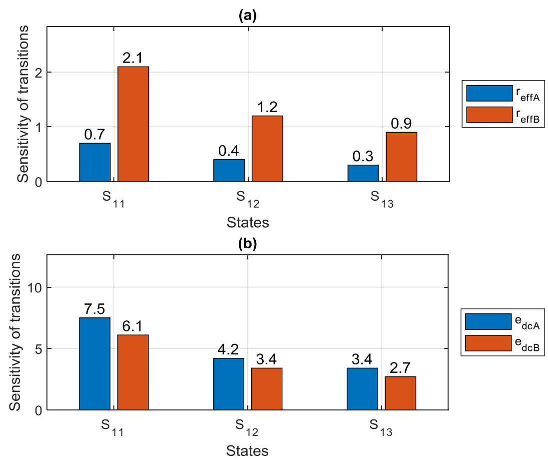

FIGURE 7. Sensitivity analysis of repair and diagnostic coverage factors of case study C-2.

The incremental change in repair efficiency of subsystem B (r_{effB}) causes the mean system state transition into states S-1, S-2 and S-3 to increase by 2, 1 and 1, respectively; of which the total is the sum of states transitions given that the initial system state is S-1. Hence, the individual subsystem failure rates maintain their influence regardless of the reduced system diagnostic coverage of 60% for subsystem B. Nevertheless, the sensitivity of the system to the system repair efficiency factors has significantly reduced, which signifies the level of impact imposed by the diagnostic coverage of the system.

The incremental change in subsystem A diagnostic coverage (e_{edcA}) causes the mean system state transitions to increase by only 7, 4 and 3 in states S-1, S-2 and S-3, respectively. Contrasting the incremental change of subsystem A diagnostic coverage (e_{edcA}), the incremental change of subsystem B diagnostic coverage (e_{edcB}) causes the mean system state transitions to increase by only 6, 3 and 3, respectively. The difference in repair efficiency of the subsystems, as well as their failure rates, causes the difference in the effectiveness of the two diagnostic coverage factors. Again, the sensitivity of the system to the factors indicates that diagnostic coverage is the most critical factor in the determination of system performance.

Figure 8 depicts the elasticities of the system reliability to both the repair efficiency and diagnostic coverage factors. The magnitude of the elasticity of both the repair efficiency factors r_{effA} and r_{effB} indicate that the factors are inelastic as depicted in Figure 8(a).

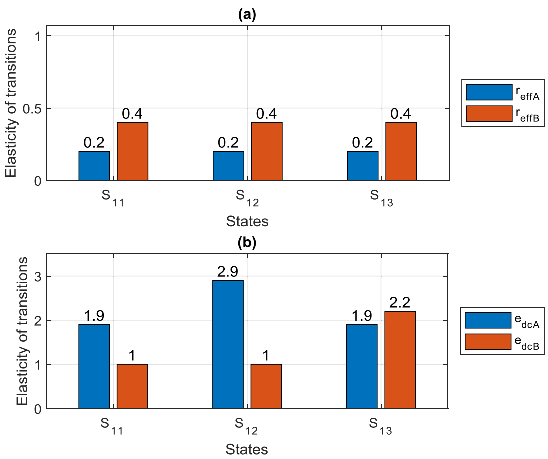

FIGURE 8. **Elasticity analysis of repair and diagnostic coverage factors of case study C-2.**

Improving the repair efficiency of subsystem B is proportionally beneficial than that of subsystem A since the elasticity of the mean system state transition is 0.4 for subsystem B, whereas that of subsystem A is 0.2. Figure 8(b) depicts the elasticity of the system to the diagnostic coverage factors. The diagnostic coverage factors are elastic at 2, 3 and 2 transitions for states S-1, S-2 and S-3 in subsystem A, respectively and 1, 1 and 2 for states S-1, S-2 and S-3 for subsystem B. Thus, it is again beneficial to improve the diagnostic coverage factor of subsystem A because of its high repair efficiency factor.

C. CASE STUDY C-3

This case study assumes that the diagnostic coverage factor of the individual subsystems is 60%. Figure 9(a) depicts the sensitivity of the system to the repair efficiency factors. In contrast to the diagnostic coverage level of C-1 and C-2 case studies, the incremental change in repair efficiency factors does not affect the mean system state transitions. Further, even though subsystem A has a higher repair efficiency than subsystem B, the responsiveness of the two subsystems is relatively identical because of the low level of system diagnostic coverage. This observation suggests that whatever the level

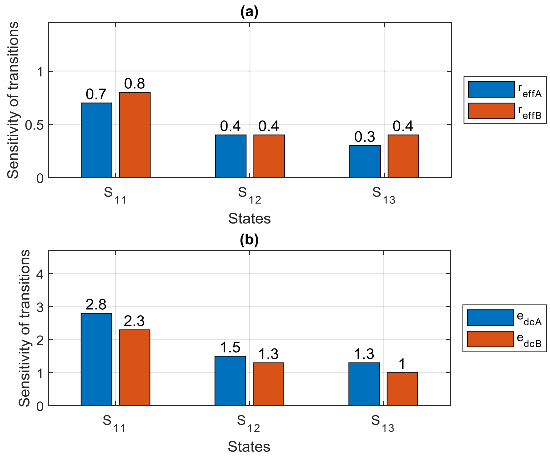

FIGURE 9. **Sensitivity analysis of repair and diagnostic coverage factors of case study C-3.**

of repair efficiency, the level of unidentified system faults is very high for the repairs to be significant. Hence the increase of the repair efficiency factors in not useful, and therefore not beneficial.

The incremental change in subsystem A diagnostic coverage (e_{dcA}) causes the mean system state transitions to increase by only 3, 1 and 1, respectively. Also, the incremental change of subsystem B diagnostic coverage (e_{dcB}) causes the mean system state transitions to increase by only 2, 1 and 1, respectively. The similarity in the behaviour of the subsystems is attributed to the low diagnostic coverage of the subsystems, which accounts for a high level of unidentified system faults. Hence, the diagnostic coverages of the subsystems highly influence the performance of the system.

The elasticities of the system to both the repair and diagnostic coverage factors are depicted in Figure 10. The magnitude of the elasticity of both the repair factors of subsystem A and B (r_{effA} and r_{effB}) indicate that the factors are inelastic as depicted in Figure 10(a). Improving the repair efficiency of subsystem A yields similar results as that of subsystem B since the elasticities of the mean system state transition for

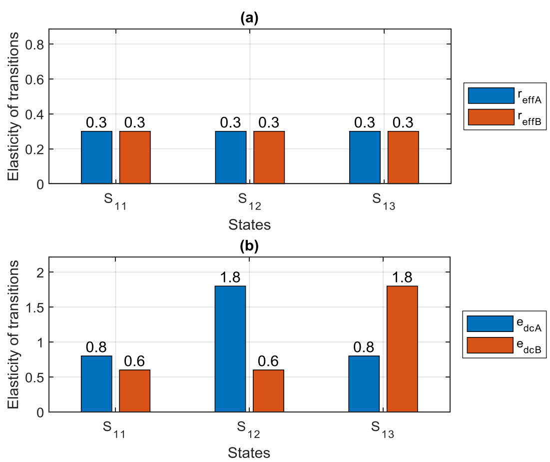

FIGURE 10. **Elasticity analysis of repair and diagnostic coverage factors of case study C-3.**

subsystem A are relatively equal to that of subsystem B, given the equally low diagnostic coverage of the subsystems at 60%.

Figure 10(b) depicts the elasticity of the system to the diagnostic coverage factors. The diagnostic coverage factors are no longer elastic for both subsystem A and subsystem B. Consequently, it is equally beneficial to improve the diagnostic coverage factors of both the subsystems. The much-reduced effectiveness of the factors is attributed to the much lower diagnostic coverage of 60% on the individual subsystems.

IX. CONCLUSION

The incremental responsiveness of the system to repair efficiency and diagnostic coverage factors can be accurately determined using sensitivity and elasticity analysis studies. This method enables the incremental impact of the factors to the reliability of the system to be investigated beyond the dynamical behaviour of the system at the subsystem level based on the mean system state transitions; which in turn enables objective system optimisation. Even though the system is dependent on both the factors, the system is perfectly inelastic to the repair efficiency factors. Hence, the benefits as a result of increasing the repair efficiency factors are not significant for a given level of system performance, especially for high system diagnostic coverage factors. Thus, the diagnostic coverage of the system is the most critical of the two factors, with higher elasticity as the factor approaches 100%.

Nevertheless, both sensitivity and elasticity decrease rapidly as the diagnostic coverage of the system decrease. Notably, from case study C-1 to case study C-2, the sensitivity of system reliability to the diagnostic coverage of subsystem B is reduced by 96% when the diagnostic coverage of subsystem B is 60%. Similarly, a magnitude of 63% reduction in sensitivity of system reliability to the subsystem A diagnostic coverage was observed in the C-3 case study as the subsystem A diagnostic coverage was reduced to 60%. The elasticities give the proportional effect of the impact of the factors on the respective case studies. The incremental adjustment of repair efficiency in small magnitudes proved not to be effective at low system diagnostic coverage levels where the level of unidentified system faults is high. Even though the repair efficiency of subsystem A is 25% higher than that of subsystem B, the non-responsiveness of the system is noticeable by the relatively equal system sensitivity of magnitudes in case study C-3. Thus, it is concluded that more emphasis must be put on the diagnostic coverage of the system because it is embedded in the design of the system, which can be expensive and practically not achievable once the system is commissioned and is running.

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