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Event-Triggered Mixed Non-Fragile and Measurement Quantization Filtering Design for Interval Type-2 Fuzzy Systems

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ABSTRACT This paper is concerned with the problem of event-triggered non-fragile H_∞ filter design for interval type-2 (IT2) fuzzy systems subject to output quantization. The nonlinear plant is efficiently described by an IT2 fuzzy model, and the lower and upper membership functions with weighting coefficients are employed to characterize parameter uncertainties in the plant. To enhance filter adaptability, an IT2 non-fragile fuzzy filter method is proposed to acquire available information for the unknown state of the plant, where the multiplicative gain variations is taken into account in the filter analysis design process. In order to avoid continuous communications and save limited bandwidth, a dynamic event-based mechanism is employed to adopt the limited communications links. Then, based on the Lyapunov theory together with the inequality technique, a filtering system with event-based mechanism and measurement quantization is analyzed and constructed. Moreover, the obtained sufficient conditions for system analysis are given in the form of linear matrix inequalities (LMIs). Finally, a numerical simulation is provided to verify the effectiveness of the proposed filter design strategy.

INDEX TERMS Event-triggered scheme, interval type-2 fuzzy systems, non-fragile filtering, quantization.

I. INTRODUCTION

In the field of theoretical research and engineering applications, Takagi-Sugeno(T-S) fuzzy model is proposed to deal with system nonlinearities, which was first established in [1], [2]. It describes the nonlinear systems by average weighted summation of some local linear sub-models. By using of this replaceable model of nonlinear systems, the controller can be designed by utilizing the parallel distributed compensation scheme. This structure gives a general alternative method for the analysis and synthesis of nonlinear systems, which is called as the type-1 T-S fuzzy model. Recently, fruitful issues were reported for type-1 fuzzy systems. Fault detector and controller design was solved in [3]; fault estimation and fault-tolerant control of switched fuzzy systems was addressed in [4]; event-triggered fuzzy controller design and its application was given in [5]; fuzzy adaptive event-triggered control was considered in [6].

It is well known that the uncertainty is inevitable for various systems, which is caused by parameter uncertainty, modeling error, conversion error etc. In order to describe and capture uncertainties existing in nonlinear plants, an interval type-2 (IT2) T-S fuzzy logic was given to extend type-1 fuzzy version in [7], [8]. It adopts a lower and upper membership functions approach to deal with uncertain parameters existing in membership functions and has more better description ability and performance than type-1 T-S fuzzy logic system. Compared with type-1 T-S fuzzy model, the superiority

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of IT2 T-S fuzzy systems has been shown in many fields. Based on dissipativity performance of nonlinear discrete-time systems, a novel reliable IT2 fuzzy filter was constructed in [9]; H_∞ norm and Hankel-norm model reduction problem for high-order IT2 fuzzy systems were addressed in [10], [11], respectively. A new triple-integral Lyapunov-Krasovskii functional was provided in [12], where sufficient condition of controller design was developed by second-order Bessel-Legendre integral inequality. By employed input delay conversion method, H_∞ fuzzy networked sampled data control problem was solved in [13]. After designing the integral sliding mode surface, an adaptive fuzzy sliding mode controller of uncertain nonlinear systems was constructed to make the systems stable in [14]. Based on the framework of IT2 T-S fuzzy logic, the sampled-data tracking output-feedback control of polynomial fuzzy-model-based system was presented in [15]. Some sampled-data control results were obtained for IT2 systems in [16], [17]. Subjecting to time-varying delay and external disturbances, the reliable control problem of finite-time discrete-time uncertain IT2 systems was concerned with in [18].

Since the state information is not always available and known for the systems being studied, filter design or state estimation is required to estimate system state through an available measurement way in computer application, signal processing, network communications and control field. Until now, considerable research has focused on the problem of filtering analysis and design, for example, [19]–[21] and the references therein. On the other hand, increasing efforts have been made for type-1 and type-2 fuzzy filtering problem. Through the T-S fuzzy framework, a robust fault detection filtering design was proposed for nonlinear switched stochastic systems in [22]. By the event-triggered scheme, the authors in [23] have focused on fault detection filtering problem for nonlinear systems under the network environment. Using two-dimensional Roesser T-S fuzzy model, the authors in [24] have addressed positive filtering construction with induced ℓ_1 performance. By taking full advantage of the relationship information between multiple delay terms and the current system states, a novel delays dependent partitioning technique was employed to obtain H_∞ filter for discrete-time fuzzy systems in [25]. For tackling random link failures of sensor network, a distributed reliable IT2 fuzzy filter problem was carried out to have system states in [26]. Considered data packet dropouts and quantization, an IT2 fuzzy filter was got for a class of nonlinear networked systems in [27]. Based on an adaptive event-triggered method, a novel IT2 fuzzy filter was implemented to deal with asynchronously and imperfectly matched membership functions in [28]. By using of asynchronous adaptive event-triggered method, the tracking control problem of multi-agent systems was investigated in [29]. Considered the physical phenomenon of quantization and random network attacks, the fuzzy finite time controller was designed for nonlinear interconnected systems in [30]. In the framework of network environment and IT2 fuzzy model, an adaptive event-based fault detection filter was obtained by applying an improved Wirtinger-based integral inequality and reciprocally convex inequality in [31]. In the field of science or engineering, non-fragile performance is used to describe the robustness to the system uncertainty, which was reported in [39], [40]. It should be pointed out that fault-tolerant control [41], [42], tracking control [43] and fractional order systems [44] have always been hot research topic in this field. How to solve these topics of IT2 fuzzy systems will be one of our future research directions.

As a result of the above discussion, the research in filtering problem for IT2 T-S fuzzy systems with quantization and time-delay should be both valuable and meaningful for theoretical research and practical applications, which gives us a motivation to carry out this work. In this paper, the problem of event-triggered non-fragile H_∞ filtering for IT2 T-S fuzzy systems with quantization and time-delay is investigated. In the framework of IT2 fuzzy sets, both lower and upper of membership function is adopted to catch systems uncertainty. Furthermore, the information of both time-delay and quantization are taken into account in filtering system analysis and syntheses. With such information, the event-based mechanism is introduced to improve network communication applicability. Based on Lyapunov functional theory, sufficient conditions are acquired to determine the stability of the filtering error system. And then, the filter design process is given in terms of LMIs. Since the multiplicative gain variations is considered separately in each local sub-plant, the filter adaptability can be enhanced and the stability criteria are more relaxed. The main innovations and contributions obtained in this paper are as follows:

1) To improve the accuracy of nonlinear system modeling, an interval type 2 (IT2) fuzzy model is constructed by using of upper and lower membership functions to handle the uncertainties caused by time delay, quantization error and gain fluctuations.
2) In order to enhance the adaptability of the filter designed in this paper, a non-fragile filter process is given to deal with multiplicative gain variations. In addition, the data to be released was quantized by a logarithmic quantizer to satisfy the requirements of signal normalization.
3) By employing inequality technique and slack matrix method, the solvability problem of filter design is transformed into a convex optimization problem. Moreover, a simulation is given to reveal the effectiveness of the obtained results.

The remainder of this paper is organized as follows. In Section II, system description and preliminaries is presented. In Section III, the filter error systems analysis issue is discussed by the Lyapunov stability theory and the filter construction conditions are derived. In Section IV, the numerical simulation is provided to verify the effectiveness of obtained method. The conclusion is given in Section V.

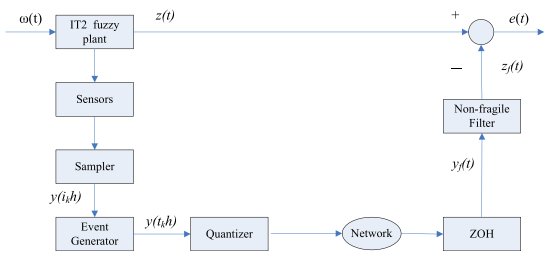

FIGURE 1. System Diagram.

Notation: All notations used throughout this article are really standard, which can be found detailed explanations in the corresponding math books. The superscripts "T" and "(-1)" denotes matrix transposition and matrix inverse, respectively. The notations "I" and "0" are donated as identity matrix and zero matrix. The notations $P > 0$, $P < 0$ means a positive, negative definite matrix P, respectively. $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix. $*$ represents a symmetric term in a matrix. Matrices whose dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

In this paper, the structure of interval type-2 (IT2) fuzzy system under network framework is shown in Figure 1, where the plant is represented by the following IT2 fuzzy system with r rules.

Rule i: IF $\theta_1(x(t))$ is $\tilde{M}_{i1}$, $\theta_2(x(t))$ is $\tilde{M}_{i2}$, ..., AND $\theta_p(x(t))$ is $\tilde{M}_{ip}$, THEN

$$\begin{cases} \dot{x}(t) = \mathcal{A}_i x(t) + \mathcal{B}_i \omega(t) \\ y(t) = \mathcal{C}_i x(t) + \mathcal{D}_i \omega(t) \\ z(t) = \mathcal{L}_i x(t) \end{cases} \tag{1}$$

where $x(t) \in R^{n_x}$ is the state vector; $y(t) \in R^{n_y}$ is the system measurement output; $z(t) \in R^{n_z}$ is the signal to be estimated; $\omega(t) \in R^{n_\omega}$ is the external disturbance, which belongs to $L_2[0,\infty)$; $\mathcal{A}_i$, $\mathcal{B}_i$, $\mathcal{C}_i$, $\mathcal{D}_i$ and $\mathcal{L}_i$ are system matrices with suitable dimensions. $\theta(x(t)) = \left[\theta_1(x(t)), \theta_2(x(t)), \ldots, \theta_p(x(t))\right]^T$ denotes premise variables, p is the number of fuzzy sets; $\tilde{M}_{i\partial}$ stands for fuzzy sets, $i \in \{1, 2, \ldots, r\}$, $\partial \in \{1, 2, \ldots, p\}$. The trigger intensity of rule i can be expressed as the following interval value set:

$$\mathbb{M}_i(x(t)) = [\mathcal{M}_i^1(x(t)), \mathcal{M}_i^2(x(t))] \tag{2}$$

where

$$\begin{aligned} \mathcal{M}_i^1(x(t)) &= \prod_{\partial=1}^{p} \mu^1_{\tilde{M}_{i\partial}}(\theta_\partial(x(t))) \geq 0, \\ \mathcal{M}_i^2(x(t)) &= \prod_{\partial=1}^{p} \mu^2_{\tilde{M}_{i\partial}}(\theta_\partial(x(t))) \geq 0, \\ \mu^2_{\tilde{M}_{i\partial}}(\theta_\partial(x(t))) &\geq \mu^1_{\tilde{M}_{i\partial}}(\theta_\partial(x(t))) \geq 0, \\ \mathcal{M}_i^2(x(t)) &\geq \mathcal{M}_i^1(x(t)) \geq 0. \end{aligned}$$

with $\mu^1_{\tilde{M}_{i\partial}}(\theta_\partial(x(t))) \in [0,1]$ and $\mu^2_{\tilde{M}_{i\partial}}(\theta_\partial(x(t))) \in [0,1]$ are called the lower and upper membership functions, respectively.

According to fuzzy inference and weighting, the following IT2 fuzzy model can be obtained

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \tilde{\mathcal{M}}_i(x(t))[\mathcal{A}_i x(t) + \mathcal{B}_i \omega(t)] \\ y(t) = \sum_{i=1}^{r} \tilde{\mathcal{M}}_i(x(t))[\mathcal{C}_i x(t) + \mathcal{D}_i \omega(t)] \\ z(t) = \sum_{i=1}^{r} \tilde{\mathcal{M}}_i(x(t))\mathcal{L}_i x(t) \end{cases} \tag{3}$$

where

$$\begin{aligned} \tilde{\mathcal{M}}_i(x(t)) &= \mathfrak{G}_i^1(x(t))\mathcal{M}_i^1(x(t)) \\ &\quad + \mathfrak{G}_i^2(x(t))\mathcal{M}_i^2(x(t)) \geq 0, \\ \sum\nolimits_{i=1}^{r} \tilde{\mathcal{M}}_i(x(t)) &= 1, \\ 0 \leq \mathfrak{G}_i^1(x(t)) &\leq 1, \\ 0 \leq \mathfrak{G}_i^2(x(t)) &\leq 1, \\ \mathfrak{G}_i^1(x(t)) + \mathfrak{G}_i^2(x(t)) &= 1, \quad i \in 1, 2, \cdots, r. \end{aligned}$$

Remark 1: The uncertainties in nonlinear systems are described by nonlinear terms $\mathfrak{G}_i^1(x(t))$, $\mathfrak{G}_i^2(x(t))$ and interval value of membership function $\mathcal{M}_i^1(x(t))$, $\mathcal{M}_i^2(x(t))$, which avoids the vulnerability of the membership functions of type-1 fuzzy systems. $\mathfrak{G}_i^1(x(t))$ and $\mathfrak{G}_i^2(x(t))$ must exit, but needn't be known their exact value for some analyses.

Remark 2: Type-2 fuzzy sets have grades of membership that are themselves fuzzy. An Interval Type-2 Fuzzy Set is a special case of a Type-2 Fuzzy Set, which the membership functions of an element are an interval. Generally, an IT2 FS is said to be maximally uncertain because all of its secondary membership grades are the same value. If we plot all the primary memberships of IT2 fuzzy systems in a 2D graph, the area covered by those primary memberships is the Footprint of Uncertainty (FOU). The outer margin of the FOU is the upper bound of FOU, called the Upper Membership Function (UMF), and the lower margin of the FOU is the lower bound of FOU, called the Lower Membership Function (LMF). Two well-known approaches to construct upper bound and lower bound of FOU for IT2 fuzzy sets are Gaussian primary MF with uncertain standard deviation and Gaussian primary MF with uncertain mean.

Remark 3: For an IT2 fuzzy system, three kinds of fuzzifiers can be used: singleton, type-1 non-singleton, and IT2 non-singleton. Compared with a type-1 fuzzy systems, it is challenging to accurately determine the fuzzy rules of an IT2 fuzzy system. It should be pointed out that Hamrawi and Coupland introduced two ways to determine the fuzzy rule number of an IT2 fuzzy system in [38],

Now we aim to deal with the filter problem for the system in (3). In order to enhance the flexibility of filter design, it is reasonable to consider parameters change in the filter construction, j rule of filter is expressed in the following IF-THEN rule:

Rule j: IF $\sigma_1(x(t))$ is $\tilde{N}_{j1}$, $\sigma_2(x(t))$ is $\tilde{N}_{j2}$,..., AND $\sigma_q(x(t))$ is $\tilde{N}_{jq}$, THEN

$$\begin{cases} \dot{x}_f(t) = (\mathcal{A}_{fj} + \Delta\mathcal{A}_{fj})x_f(t) + (\mathcal{B}_{fj} + \Delta\mathcal{B}_{fj})\hat{y}_f(t) \\ z_f(t) = (\mathcal{C}_{fj} + \Delta\mathcal{C}_{fj})x_f(t) \end{cases} \tag{4}$$

where $x_f(t) \in R^{n_x}$ is the state vector of the filter; $\hat{y}_f(t) \in R^{n_y}$ denotes the input vector of the filter; $z_f(t) \in R^{n_z}$ is the estimated value; $\mathcal{A}_{fj}$, $\mathcal{B}_{fj}$ and $\mathcal{C}_{fj}$ are the parameter matrices of the filter to be designed; $\tilde{N}_{j\lambda}$ represents IT2 fuzzy set, $j = 1, 2, \ldots, r, \lambda = 1, 2, \ldots, q$; $\sigma(x(t)) = \left[\sigma_1(x(t)), \sigma_2(x(t)), \ldots, \sigma_q(x(t))\right]^T$ denotes premise variables; $\Delta\mathcal{A}_{fj}$, $\Delta\mathcal{B}_{fj}$ and $\Delta\mathcal{C}_{fj}$ are the uncertain parameter matrices of multiplicative gain variation, which satisfy $\Delta\mathcal{A}_{fj} = \mathcal{A}_{fj}\mathfrak{M}_{1j}\mathfrak{K}_1(t)\mathfrak{N}_{1j}$, $\Delta\mathcal{B}_{fj} = \mathcal{B}_{fj}\mathfrak{M}_{2j}\mathfrak{K}_2(t)\mathfrak{N}_{2j}$, and $\Delta\mathcal{C}_{fj} = \mathfrak{M}_{3j}\mathfrak{K}_3(t)\mathfrak{N}_{3j}\mathcal{C}_{fj}$, in which $\mathfrak{M}_{cj}(c = 1, 2, 3)$ and $\mathfrak{N}_{cj}(c = 1, 2, 3)$ are the known constant matrices; The uncertain functions satisfy $\mathfrak{K}_c^T(t)\mathfrak{K}_c(t) \leq I$, $c = 1, 2, 3$. The trigger intensity of rule j can be described as:

$$\mathbb{W}_j(x_f(t)) = [\mathcal{W}_j^1(x(t)), \mathcal{W}_j^2(x(t))]$$

where

$$\begin{aligned} \mathcal{W}_j^1(x(t)) &= \prod_{\lambda=1}^{q} \mu^1_{\tilde{N}_{j\lambda}}(\delta_\lambda(x(t))) \geq 0, \\ \mathcal{W}_j^2(x(t)) &= \prod_{\lambda=1}^{q} \mu^2_{\tilde{N}_{j\lambda}}(\delta_\lambda(x(t))) \geq 0, \\ \mu^2_{\tilde{N}_{j\lambda}}(\delta_\lambda(x(t))) &\geq \mu^1_{\tilde{N}_{j\lambda}}(\delta_\lambda(x(t))) \geq 0, \\ \mathcal{W}_j^2(x(t)) &\geq \mathcal{W}_j^1(x(t)) \geq 0. \end{aligned}$$

with $\mu^1_{\tilde{N}_{j\lambda}}(\delta_\lambda(x(t))) \in [0, 1]$ and $\mu^2_{\tilde{N}_{j\lambda}}(\delta_\lambda(x(t))) \in [0, 1]$.

Remark 4: For the time and physical space characteristics of the network environment, the multiplicative perturbation of filter parameters is considered to enhance the design flexibility. Moreover, it can be to extend to the form of additive parameter perturbation for the filter to be designed.

The overall model of IT2 fuzzy filter is given as:

$$\begin{cases} \dot{x}_f(t) = \sum_{j=1}^{r} \tilde{\mathcal{W}}_j(x(t))[\mathcal{A}_{\Delta fj}x_f(t) + \mathcal{B}_{\Delta fj}\hat{y}_f(t)] \\ z_f(t) = \sum_{j=1}^{r} \tilde{\mathcal{W}}_j(x(t))\mathcal{C}_{\Delta fj}x_f(t) \end{cases} \tag{5}$$

where $\tilde{\mathcal{W}}_j(x(t)) = \mathfrak{F}_j^2(x(t))\mathcal{W}_j^2(x(t)) + \mathfrak{F}_j^1(x(t))\mathcal{W}_j^1(x(t)) \geq 0$, $\sum_{i=1}^{r} \tilde{\mathcal{W}}_j(x(t)) = 1$, in which $\mathfrak{F}_j^1(x(t))$ and $\mathfrak{F}_j^2(x(t))$ are the nonlinear functions for the filter and satisfies $\mathfrak{F}_j^1(x(t)) + \mathfrak{F}_j^2(x(t)) = 1$ and $0 \leq \mathfrak{F}_j^1(x(t)), \mathfrak{F}_j^2(x(t)) \leq 1$. $\mathcal{A}_{\Delta fj} = \mathcal{A}_{fj} + \Delta\mathcal{A}_{fj}$, $\mathcal{B}_{\Delta fj} = \mathcal{B}_{fj} + \Delta\mathcal{B}_{fj}$, and $\mathcal{C}_{\Delta fj} = \mathcal{C}_{fj} + \Delta\mathcal{C}_{fj}$.

Remark 5: Although the plants and the filter have different premise variables, it does not lose generality to assume that they share the same number of fuzzy rule. Moreover, it can reduce the design complexity and calculation burden to a certain extent.

A. MODELING OF NONLINEAR NETWORKED COMMUNICATION LINK

1) EVENT-TRIGGERED MECHANISM

The main advantage of event-triggered scheme lies in reducing the burden of network resource, and only the required sampling data can be released or transmitted to other components in this way. In order to judge whether the sampling data is transmitted, the following event-based condition in this paper is employed:

$$e_k{}^T(t)\hat{\Theta}e_k(t) \geq \sigma y^T(i_k h)\hat{\Theta}y(i_k h) \tag{6}$$

where $\sigma \in [0, 1)$; $e_k(t) = y(i_k h) - y(t_k h)$ is the threshold error; $y(i_k h)$ represents the current sampling value; $y(t_k h)$ represents the last transmitted value; $i_k(h) = t_k(h) + \mathcal{L}h$, $\mathcal{L} \in \mathbb{N}$, $\hat{\Theta}$ is the positive definite trigger parameter matrix to be designed.

Remark 6: It can be seen that the trigger threshold in (6) has a relation with the scalar σ, the matrix $\hat{\Theta}$ and the sampling states. The condition in (6) determines the frequency of data transmission in the network environment.

2) DATA QUANTIZATION

For the networked control system, the sampled signal needs to be quantified in order to meet the requirements of the digital systems. In this paper, a logarithmic quantizer is given as:

$$\bar{y}(t) = \tilde{G}(y(t)) = \left[\tilde{G}_1(y_1(t))\tilde{G}_2(y_2(t)) \ldots \tilde{G}_{n_y}(y_{n_y}(t))\right]^T, \tag{7}$$

where $\bar{y}(t) \in R^{n_y}$ is the signal to be quantized. For each term $\tilde{G}_v(\cdot)$, $(1 \leq v \leq n_y)$, the set of quantization level is characterized as:

$$q_v = \left\{\pm u_l^{(v)}, u_l^{(v)} = \rho_v^l u_0^{(v)}, l = 0, \pm 1, \pm 2, \ldots\right\} \cup \{0\}, \tag{8}$$

where $0 < \rho_v < 1$ and $u_0^{(v)} > 0$.

The structure of the quantizer associated with the quantization level set in (8) is as follows:

$$\tilde{G}_v(y_v(t)) = \begin{cases} u_l^v, & \text{if } \frac{1}{1+\delta_v}u_l^v < y_v(t) \leq \frac{1}{1-\delta_v}u_l^v, \\ 0, & \text{if } y_v(t) = 0, \\ -\tilde{G}_v(-y_v(t)), & \text{if } y_v(t) < 0. \end{cases} \tag{9}$$

where $\delta_v = \frac{1-\rho_v}{1+\rho_v}$, ρ_v represents quantized density.

Similar to mathematical process in [35], defining $\Delta_q = diag\{\Delta_1, \Delta_2, \ldots, \Delta_{n_y}\}$, $\Delta_{q_v} \in \left[-\delta_{qv}, \delta_{qv}\right]$, $v = 1, 2, \ldots, n_y$, the quantized sample signal, from Event Generator, can be described as:

$$\bar{y}(t_k h) = (I + \Delta_q)y(t_k h) \tag{10}$$

Without loss of generality, we assume that $\delta_{qv} = \delta$, it can be obtained that

$$\Delta_q^2 \leq \delta^2 I \tag{11}$$

Based on the aforementioned and considered the characteristics of ZOH, the input signal of the filter can be expressed as:

$$\hat{y}_f(t) = \bar{y}(t_k h) = (I + \Delta_q)y(t_k h), \quad t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$$

where τ_{t_k} represents the induced delay at time index t_k through network transmission, and $0 \le \tau_m \le \tau_{t_k} \le \bar{\tau}$, where τ_m and $\bar{\tau}$ represents the lower and upper of induced delay, respectively.

Taking the similar technique in [35], the holding zone can be described as follows.

$$\Omega = \left[t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}\right) = \bigcup_{i=0}^{m} \Omega_i, \tag{12}$$

where

$$\begin{cases} \Omega_0 = \left[t_k h + \tau_{t_k}, t_k h + h + \bar{\tau}\right) \\ \Omega_i = [t_k h + ih + \bar{\tau}, t_k h + (i+1)h + \bar{\tau}), \\ \Omega_m = \left[t_k h + mh + \bar{\tau}, t_{k+1}h + \tau_{t_{k+1}}\right) \\ i = 1, 2 \ldots, m-1 \end{cases}$$

Construct a function $\tau(t)$ as

$$\tau(t) = t - t_k h - ih, \quad t \in \Omega_i \tag{13}$$

It is obvious that

$$0 \le \tau_m \le \tau(t) \le \bar{\tau} + h$$

Let τ_M denote $h + \bar{\tau}$, and introduce a new vector

$$e_k(t) = y(t_k h + ih) - y(t_k h), \quad t \in \Omega_i \tag{14}$$

Therefore, the input signal of filter, $\hat{y}_f(t)$, can be represented by

$$\hat{y}_f(t) = (I + \Delta_q)[y(t - \tau(t)) - e_k(t)] \tag{15}$$

B. FILTERING ERROR SYSTEM

Define new vectors $\xi(t) = \left[x^T(t)\ x_f^T(t)\right]^T$, $\tilde{\omega}(t) = \left[\omega^T(t)\ \omega^T(t - \tau(t))\right]^T$. Taking the event-triggered mechanism (6) and quantizer (9) into account, the filtering error system can be obtained as:

$$\begin{cases} \dot{\xi}(t) = \bar{\mathcal{A}}_{ij}(t)\xi(t) + \bar{\mathcal{B}}_{\tau ij}(t)H\xi(t - \tau(t)) \\ \qquad + \bar{\mathcal{B}}_{\omega ij}(t)\tilde{\omega}(t) + \bar{\mathcal{B}}_{eij}(t)e_k(t) \\ e(t) = \bar{\mathcal{C}}_{ij}(t)\xi(t) \end{cases} \tag{16}$$

where $\bar{\mathcal{A}}_{ij}(t) = \tilde{\mathcal{A}}_{ij}(t) + \Delta\tilde{\mathcal{A}}_{ij}(t), \bar{\mathcal{B}}_{\tau ij}(t) = \tilde{\mathcal{B}}_{\tau ij}(t) + \Delta\tilde{\mathcal{B}}_{\tau ij}(t), \bar{\mathcal{B}}_{\omega ij}(t) = \tilde{\mathcal{B}}_{\omega ij}(t) + \Delta\tilde{\mathcal{B}}_{\omega ij}(t), \bar{\mathcal{B}}_{eij}(t) = \tilde{\mathcal{B}}_{eij}(t) + \Delta\tilde{\mathcal{B}}_{eij}(t), \bar{\mathcal{C}}_{ij}(t) = \tilde{\mathcal{C}}_{ij}(t) + \Delta\tilde{\mathcal{C}}_{ij}(t), H = [I \ \ 0]$,

$$\tilde{\mathcal{A}}_{ij}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\tilde{\mathcal{A}}_{ij}, \tilde{\mathcal{B}}_{\tau ij}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\tilde{\mathcal{B}}_{\tau ij},$$

$$\tilde{\mathcal{B}}_{\omega ij}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\tilde{\mathcal{B}}_{\omega ij}, \tilde{\mathcal{B}}_{eij}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\tilde{\mathcal{B}}_{eij},$$

$$\tilde{\mathcal{C}}_{ij}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\tilde{\mathcal{C}}_{ij}, \Delta\tilde{\mathcal{A}}_{ij}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\Delta\tilde{\mathcal{A}}_{ij},$$

$$\Delta\tilde{\mathcal{B}}_{\tau ij}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\Delta\tilde{\mathcal{B}}_{\tau ij}, \Delta\tilde{\mathcal{B}}_{\omega ij}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\Delta\tilde{\mathcal{B}}_{\omega ij},$$

$$\Delta\tilde{\mathcal{B}}_{eij}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\Delta\tilde{\mathcal{B}}_{eij},$$

$$\Delta\tilde{\mathcal{C}}_{ij}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\Delta\tilde{\mathcal{C}}_{ij},$$

$$\tilde{\mathcal{A}}_{ij} = \begin{bmatrix} \mathcal{A}_i & 0 \\ 0 & \mathcal{A}_{fj} \end{bmatrix}, \tilde{\mathcal{B}}_{\tau ij} = \begin{bmatrix} 0 \\ \mathcal{B}_{fj}(I + \Delta_q)\mathcal{C}_i \end{bmatrix},$$

$$\tilde{\mathcal{B}}_{\omega ij} = \begin{bmatrix} \mathcal{B}_i & 0 \\ 0 & \mathcal{B}_{fj}(I + \Delta_q)\mathcal{D}_i \end{bmatrix},$$

$$\tilde{\mathcal{B}}_{eij} = \begin{bmatrix} 0 \\ -\mathcal{B}_{fj}(I + \Delta_q) \end{bmatrix},$$

$$\tilde{\mathcal{C}}_{ij} = \begin{bmatrix} \mathcal{L}_i & -\mathcal{C}_{fj} \end{bmatrix}, \Delta\tilde{\mathcal{A}}_{ij} = \begin{bmatrix} 0 & 0 \\ 0 & \Delta\mathcal{A}_{fj} \end{bmatrix},$$

$$\Delta\tilde{\mathcal{B}}_{\tau ij} = \begin{bmatrix} 0 \\ \Delta\mathcal{B}_{fj}(I + \Delta_q)\mathcal{C}_i \end{bmatrix},$$

$$\Delta\tilde{\mathcal{B}}_{\omega ij} = \begin{bmatrix} 0 & 0 \\ 0 & \Delta\mathcal{B}_{fj}(I + \Delta_q)\mathcal{D}_i \end{bmatrix},$$

$$\Delta\tilde{\mathcal{B}}_{eij} = \begin{bmatrix} 0 \\ -\Delta\mathcal{B}_{fj}(I + \Delta_q) \end{bmatrix}, \Delta\tilde{\mathcal{C}}_{ij} = \begin{bmatrix} 0 & -\Delta\mathcal{C}_{fj} \end{bmatrix}.$$

The issue of non-fragile H_∞ filtering under event-based scheme in this article is addressed, which is summarized as follows:

1) Stable performance: The filtering error system in (16) with $\tilde{\omega}(t) = 0$ is asymptotically stable;
2) H_∞ performance: Under zero initial conditions and given a scalar $\gamma > 0$, for any disturbance $\tilde{\omega}(t) \neq 0$, the filtering error $e(t)$ satisfies $\|e(t)\|_2 < \gamma\|\tilde{\omega}(t)\|_2$, then the filtering error system (16) satisfies the given H_∞ performance index γ.

At the end of this section, the following definitions and lemmas need to be recalled, which paly a key role to obtain the main results in this paper.

Lemma 1 [32]: Given a scalar $a > 0$, vectors $p, q \in R^n$, and matrices W, M with suitable dimensions, such that

$$2pWM(t)Eq \le ap^TWW^Tp + a^{-1}q^TEE^Tq,$$

where $M^T(t)M(t) \le I$.

Lemma 2 [33]: Given the matrix $R > 0$ and the matrix of S_i and U_l with the appropriate dimensions, the following formula holds:

$$S_i^TRU_l + U_l^TRS_i \le S_i^TRS_i + U_l^TRU_l.$$

Lemma 3 [34]: For given suitable dimensions matrices V, H, E, and Q, there must exist a scalar $\varepsilon > 0$, such that

$$(V + HM(t)E)^T Q (V + HM(t)E) \leq V^T\left(Q^{-1} - \varepsilon HH^T\right)^{-1} V + \varepsilon^{-1} E^T E$$

where $M^T(t)M(t) \leq I$ and $Q^{-1} - \varepsilon HH^T > 0$.

Lemma 4 [36]: Given matrices $Q = Q^T$, X, and Y with given appropriate dimensions, if

$$Q + XM(t)Y + Y^T M(t)X^T < 0$$

hold for all $M(t)$ satisfies all of $M^T(t)M(t) \leq I$, then there must exist a scalar $\delta > 0$ such that

$$Q + \delta XX^T + \delta^{-1} Y^T Y < 0$$

III. MAIN RESULTS

A. H_∞ PERFORMANCE ANALYSIS

In this section, based on Lyapunov functional technique, the sufficient condition is presented such that the filtering error system (16) satisfies the asymptotically stability with a H_∞ index performance γ.

Theorem 1: Consider the IT2 fuzzy systems and the fuzzy filter with quantizer and event-triggered mechanism subject to σ, for given scalars $\tau_M > 0$, $\varepsilon_i > 0 (i = 1, 2, \ldots, 9)$, and the membership function satisfies $\tilde{\mathcal{W}}_j - \ell_j \tilde{\mathcal{M}}_j \geq 0$ $(0 < \ell_j \leq 1)$. The filtering error system (16) is asymptotically stable with a performance index γ, if there exist matrices $P > 0$, $Q_1 > 0$, $R_1 > 0$, $\hat{\Theta} > 0$, $U_1 > 0$, and $X_i = X_i^T$, $(i = 1, 2, \ldots, r)$, such that

$$R_1^{-1} > 0, \tag{17}$$

$$\begin{bmatrix} I & \varepsilon_9 \tilde{M}_5 \\ * & \varepsilon_9 \end{bmatrix} > 0, \begin{bmatrix} R_1 & U_1^T \\ * & R_1 \end{bmatrix} \geq 0 \tag{18}$$

$$\Xi_{ij} - X_i < 0, \tag{19}$$

$$\ell_i \Xi_{ii} - \ell_i X_i + X_i < 0, \tag{20}$$

$$\ell_j \Xi_{ij} + \ell_i \Xi_{ji} - \ell_j X_i - \ell_i X_j + X_i + X_j < 0, \quad i < j, \tag{21}$$

where

$$\Xi_{ij} = \begin{bmatrix} \Xi_{ij}^{11} & \Xi_{ij}^{12} & \Xi_{ij}^{13} & \Xi_{ij}^{14} & \Xi_{ij}^{15} \\ * & \Xi_{ij}^{22} & \Xi_{ij}^{23} & 0 & \Xi_{ij}^{25} \\ * & * & \Xi_{ij}^{33} & 0 & 0 \\ * & * & * & \Xi_{ij}^{44} & 0 \\ * & * & * & * & \Xi_{ij}^{55} \end{bmatrix}$$

$$\begin{aligned}
\Xi_{ij}^{11} &= P\tilde{\mathcal{A}}_{ij} + \tilde{\mathcal{A}}_{ij}^T P + H^T(Q_1 - R_1)H + \varepsilon_1 P\tilde{M}_1\tilde{M}_1^T P \\
&\quad + \varepsilon_1^{-1}\tilde{N}_1^T\tilde{N}_1 + \varepsilon_2 P\tilde{M}_2\tilde{M}_2^T P + \varepsilon_3 P\tilde{M}_3\tilde{M}_3^T P \\
&\quad + \varepsilon_4 P\tilde{M}_4\tilde{M}_4^T P + \tilde{\mathcal{C}}_{ij}^T(I - \varepsilon_9\tilde{M}_5\tilde{M}_5^T)\tilde{\mathcal{C}}_{ij} + \varepsilon_9^{-1}\tilde{N}_5^T\tilde{N}_5 \\
&\quad + 4\tau_M^2(\tilde{\mathcal{A}}_{ij}^T H^T R_1 H\tilde{\mathcal{A}}_{ij} + \varepsilon_5^{-1}\tilde{N}_1^T\tilde{N}_1), \\
\Xi_{ij}^{12} &= P\tilde{\mathcal{B}}_{\tau ij} + H^T R_1^T - H^T U_1^T, \ \Xi_{ij}^{13} = H^T U_1^T, \\
\Xi_{ij}^{14} &= P\tilde{\mathcal{B}}_{eij}, \ \Xi_{ij}^{15} = P\tilde{\mathcal{B}}_{\omega ij}, \\
\Xi_{ij}^{22} &= \varepsilon_2^{-1}\tilde{N}_2^T\tilde{N}_2 - 2R_1 + U_1 + U_1^T \\
&\quad + 4\tau_M^2(\tilde{\mathcal{B}}_{\tau ij}^T H^T R_1 H\tilde{\mathcal{B}}_{\tau ij} + \varepsilon_6^{-1}\tilde{N}_2^T\tilde{N}_2) + \sigma\mathcal{C}_i^T\hat{\Theta}\mathcal{C}_i, \\
\Xi_{ij}^{23} &= R_1^T - U_1^T, \ \Xi_{ij}^{25} = \sigma\mathcal{C}_i^T\hat{\Theta}\begin{bmatrix} 0 & \mathcal{D}_i \end{bmatrix}, \ \Xi_{ij}^{33} = Q_1 - R_1, \\
\Xi_{ij}^{44} &= -\hat{\Theta} + \varepsilon_4^{-1}\tilde{N}_4^T\tilde{N}_4 + 4\tau_M^2(\tilde{\mathcal{B}}_{eij}^T H^T R_1 H\tilde{\mathcal{B}}_{eij} \\
&\quad + \varepsilon_8^{-1}\tilde{N}_4^T\tilde{N}_4), X_i = diag\{X_1^i, \underbrace{0, \ldots, 0}_{4}\}, X_1^i \\
&= diag\{\bar{X}_1^i, 0\} \\
\Xi_{ij}^{55} &= \sigma\begin{bmatrix} 0 & \mathcal{D}_i \end{bmatrix}^T\hat{\Theta}\begin{bmatrix} 0 & \mathcal{D}_i \end{bmatrix} + \varepsilon_3^{-1}\tilde{N}_3^T\tilde{N}_3 \\
&\quad + 4\tau_M^2(\tilde{\mathcal{B}}_{\omega ij}^T H^T R_1 H\tilde{\mathcal{B}}_{\omega ij} + \varepsilon_7^{-1}\tilde{N}_3^T\tilde{N}_3) - \gamma^2 I, \\
\tilde{M}_k(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\tilde{M}_k, k = 1, 2, \ldots, 5, \\
\tilde{N}_l(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\tilde{N}_l, l = 1, 2, \ldots, 5, \\
\tilde{M}_1 &= \begin{bmatrix} 0 \\ \mathcal{A}_{fj}\mathfrak{M}_{1j} \end{bmatrix}, \tilde{M}_2 = \begin{bmatrix} 0 \\ \rho\mathcal{B}_{fj}\mathfrak{M}_{2j} \end{bmatrix}, \\
\tilde{M}_3 &= \begin{bmatrix} 0 \\ \rho\mathcal{B}_{fj}\mathfrak{M}_{2j} \end{bmatrix}, \tilde{M}_4 = \begin{bmatrix} 0 \\ -\rho\mathcal{B}_{fj}\mathfrak{M}_{2j} \end{bmatrix}, \\
\tilde{M}_5 &= \mathfrak{M}_{3j}, \tilde{N}_1 = \begin{bmatrix} 0 & \mathfrak{N}_{1j} \end{bmatrix}, \tilde{N}_2 = \mathfrak{N}_{2j}\mathcal{C}_i, \\
\tilde{N}_3 &= \begin{bmatrix} 0 & \mathfrak{N}_{2j}\mathcal{D}_i \end{bmatrix}, \tilde{N}_4 = \mathfrak{N}_{2j}, \\
\tilde{N}_5 &= \begin{bmatrix} 0 & \mathfrak{N}_{3j}\mathcal{C}_{fj} \end{bmatrix}, \rho = I + \Delta_q,
\end{aligned}$$

Proof: The following Lyapunov-Krasovskii functional to be analyzed is established:

$$\begin{aligned}
V(t) &= V_1(t) + V_2(t) + V_3(t), \\
V_1(t) &= \xi^T(t)P\xi(t), \\
V_2(t) &= \int_{t-\tau_M}^{t}\xi^T(s)H^T Q_1 H\xi(s)ds, \\
V_3(t) &= \tau_M\int_{t-\tau_M}^{t}\int_{s}^{t}\dot{\xi}^T(v)H^T R_1 H\dot{\xi}(v)dvds.
\end{aligned} \tag{22}$$

Calculating the derivative of (22), it can be obtained that

$$\begin{aligned}
\dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t), \\
\dot{V}_1(t) &= 2\xi^T(t)P\dot{\xi}(t), \\
\dot{V}_2(t) &= \xi^T(t)H^T Q_1 H\xi(t) \\
&\quad - \xi^T(t-\tau_M)H^T Q_1 H\xi(t-\tau_M), \\
\dot{V}_3(t) &= \tau_M^2\dot{\xi}^T(t)H^T R_1 H\dot{\xi}(t) + G_3^1(t), \\
G_3^1(t) &= -\tau_M\int_{t-\tau_M}^{t}\dot{\xi}^T(s)H^T R_1 H\dot{\xi}(s)ds.
\end{aligned} \tag{23}$$

From (16), it yields

$$2\xi^T(t)P\dot{\xi}(t) = 2\xi^T(t)P\left[\iota_1(t) + \iota_2(t)\right]\chi(t) \tag{24}$$

where

$$\begin{aligned}
\chi(t) &= \begin{bmatrix} \xi^T(t) & \xi^T(t-\tau(t))H^T & \tilde{\omega}^T(t) & e_k^T(t) \end{bmatrix}^T, \\
\iota_2(t) &= \begin{bmatrix} \Delta\tilde{\mathcal{A}}_{ij}(t) & \Delta\tilde{\mathcal{B}}_{ij}(t) & \Delta\tilde{\mathcal{B}}_{\omega ij}(t) & \Delta\tilde{\mathcal{B}}_{eij}(t) \end{bmatrix}, \\
\iota_1(t) &= \begin{bmatrix} \tilde{\mathcal{A}}_{ij}(t) & \tilde{\mathcal{B}}_{ij}(t) & \tilde{\mathcal{B}}_{\omega ij}(t) & \tilde{\mathcal{B}}_{eij}(t) \end{bmatrix},
\end{aligned}$$

By employing Lemma 1, we can get

$$
\begin{aligned}
&2\xi^T(t)P\Delta\tilde{\mathcal{A}}_{ij}(t)\xi(t)\\
&= 2\xi^T(t)P\tilde{M}_1(t)\Re_1(t)\tilde{N}_1(t)\xi(t)\\
&\le \varepsilon_1\xi^T(t)P\tilde{M}_1(t)\tilde{M}_1^T(t)P\xi(t) + \varepsilon_1^{-1}\xi^T(t)\tilde{N}_1^T(t)\tilde{N}_1(t)\xi(t)
\end{aligned} \tag{25}
$$

where for any $\varepsilon_1 > 0$. Similarly, for arbitrary scalars $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, and $\varepsilon_4 > 0$, the following inequalities hold

$$
\begin{aligned}
&2\xi^T(t)P\Delta\tilde{\mathcal{B}}_{ij}(t)H\xi(t-\tau(t))\\
&\quad\le \varepsilon_2\xi^T(t)P\tilde{M}_2(t)\tilde{M}_2^T(t)P\xi(t)\\
&\qquad+\varepsilon_2^{-1}\xi^T(t-\tau(t))H^T\tilde{N}_2^T(t)\tilde{N}_2(t)H\xi(t-\tau(t))
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
&2\xi^T(t)P\Delta\tilde{\mathcal{B}}_{\omega ij}(t)\tilde{\omega}(t)\\
&\quad\le \varepsilon_3\xi^T(t)P\tilde{M}_3(t)\tilde{M}_3^T(t)P\xi(t)\\
&\qquad+\varepsilon_3^{-1}\tilde{\omega}^T(t)\tilde{N}_3^T(t)\tilde{N}_3(t)\tilde{\omega}(t)
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
&2\xi^T(t)P\Delta\tilde{\mathcal{B}}_{eij}(t)e_k(t)\\
&\quad\le \varepsilon_4\xi^T(t)P\tilde{M}_4(t)\tilde{M}_4^T(t)P\xi(t)\\
&\qquad+\varepsilon_4^{-1}e_k^T(t)\tilde{N}_4^T(t)\tilde{N}_4(t)e_k(t)
\end{aligned} \tag{28}
$$

Applying Jensen's inequality to the integral term in (23), we have that:

$$
\begin{aligned}
&-\tau_M\int_{t-\tau_M}^{t}\dot{\xi}^T(s)H^TR_1H\dot{\xi}(s)ds\\
&\le -\frac{\tau_M}{\tau(t)}\left(\int_{t-\tau(t)}^{t}\dot{\xi}(s)Hds\right)^T R_1\left(\int_{t-\tau(t)}^{t}\dot{\xi}(s)Hds\right)\\
&\quad-\frac{\tau_M}{\tau_M-\tau(t)}\left(\int_{t-\tau_M}^{t-\tau(t)}\dot{\xi}(s)Hds\right)^T R_1\left(\int_{t-\tau_M}^{t-\tau(t)}\dot{\xi}(s)Hds\right)
\end{aligned} \tag{29}
$$

By applying the reciprocal convex technique to (29), one can get

$$
\begin{aligned}
&-\tau_M\int_{t-\tau_M}^{t}\dot{\xi}^T(s)H^TR_1H\dot{\xi}(s)ds\\
&\qquad\le -\begin{bmatrix} H\xi(t)\\ H\xi(t-\tau(t))\\ H\xi(t-\tau_M)\end{bmatrix}^T T\begin{bmatrix} H\xi(t)\\ H\xi(t-\tau(t))\\ H\xi(t-\tau_M)\end{bmatrix}
\end{aligned} \tag{30}
$$

where

$$
T = \begin{bmatrix} R_1 & -R_1^T+U_1^T & -U_1^T\\ * & 2R_1-U_1-U_1^T & -R_1^T+U_1^T\\ * & * & R_1\end{bmatrix}
$$

Then, by using of Lemma 2, the term $\tau_M^2\dot{\xi}^T(t)H^TR_1H\dot{\xi}(t)$ in (23) can be written as

$$
\begin{aligned}
&\tau_M^2\dot{\xi}^T(t)H^TR_1H\dot{\xi}(t)\\
&\le 4\tau_M^2\xi^T(t)\bar{\mathcal{A}}_{ij}^T(t)H^TR_1H\bar{\mathcal{A}}_{ij}(t)\xi(t)\\
&\quad+4\tau_M^2\xi^T(t-\tau(t))H^T\bar{\mathcal{B}}_{ij}^T(t)H^TR_1H\bar{\mathcal{B}}_{ij}(t)H\xi(t-\tau(t))\\
&\quad+4\tau_M^2\tilde{\omega}^T(t)\bar{\mathcal{B}}_{\omega ij}^T(t)H^TR_1H\bar{\mathcal{B}}_{\omega ij}(t)\tilde{\omega}(t)\\
&\quad+4\tau_M^2e_k^T(t)\bar{\mathcal{B}}_{eij}^T(t)H^TR_1H\bar{\mathcal{B}}_{eij}(t)e_k(t)
\end{aligned} \tag{31}
$$

Further, applying Lemma 3 to (31), there exist some scalars $\varepsilon_k > 0\ (k = 5, 6, 7, 8)$ such that

$$
\begin{aligned}
&4\tau_M^2\xi^T(t)\bar{\mathcal{A}}_{ij}^T(t)H^TR_1H\bar{\mathcal{A}}_{ij}(t)\xi(t)\\
&\quad\le 4\tau_M^2\xi^T(t)\left\{\tilde{\mathcal{A}}_{ij}^T(t)H^T\mathbf{K}_1^{-1}H\tilde{\mathcal{A}}_{ij}(t)+\mathbf{Q_1}\right\}\xi(t)
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
&4\tau_M^2\xi^T(t-\tau(t))H^T\bar{\mathcal{B}}_{ij}^T(t)H^TR_1H\bar{\mathcal{B}}_{ij}(t)H\xi(t-\tau(t))\\
&\quad\le 4\tau_M^2\xi^T(t-\tau(t))H^T\tilde{\mathcal{B}}_{ij}^T(t)H^T\mathbf{K}_2^{-1}H\tilde{\mathcal{B}}_{ij}(t)H\xi(t-\tau(t))\\
&\qquad+4\tau_M^2\xi^T(t-\tau(t))H^T\varepsilon_6^{-1}\tilde{N}_2^T(t)\tilde{N}_2(t)H\xi(t-\tau(t))
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
&4\tau_M^2\tilde{\omega}^T(t)\bar{\mathcal{B}}_{\omega ij}^T(t)H^TR_1H\bar{\mathcal{B}}_{\omega ij}(t)\tilde{\omega}(t)\\
&\quad\le 4\tau_M^2\tilde{\omega}^T(t)\left\{\tilde{\mathcal{B}}_{\omega ij}^T(t)H^T\mathbf{K}_3{}^{-1}H\tilde{\mathcal{B}}_{\omega ij}(t)+\mathbf{Q_2}\right\}\tilde{\omega}(t)
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
&4\tau_M^2e_k^T(t)\bar{\mathcal{B}}_{eij}^T(t)H^TR_1H\bar{\mathcal{B}}_{eij}(t)e_k(t)\\
&\quad\le 4\tau_M^2e_k^T(t)\left\{\tilde{\mathcal{B}}_{eij}^T(t)H^T\mathbf{K}_4{}^{-1}H\tilde{\mathcal{B}}_{eij}(t)+\mathbf{Q_3}\right\}e_k(t)
\end{aligned} \tag{35}
$$

where

$$
\begin{aligned}
\mathbf{K1} &= R_1^{-1}-\varepsilon_5H\tilde{M}_1(t)\tilde{M}_1^T(t)H^T > 0,\\
\mathbf{K2} &= R_1^{-1}-\varepsilon_6H\tilde{M}_2(t)\tilde{M}_2^T(t)H^T > 0,\\
\mathbf{K3} &= R_1^{-1}-\varepsilon_7H\tilde{M}_3(t)\tilde{M}_3^T(t)H^T > 0,\\
\mathbf{K4} &= R_1^{-1}-\varepsilon_8H\tilde{M}_4(t)\tilde{M}_4^T(t)H^T > 0,\\
\mathbf{Q_1} &= \varepsilon_5^{-1}\tilde{N}_1^T(t)\tilde{N}_1(t),\\
\mathbf{Q_2} &= \varepsilon_7^{-1}\tilde{N}_3^T(t)\tilde{N}_3(t),\\
\mathbf{Q_3} &= \varepsilon_8^{-1}\tilde{N}_4^T(t)\tilde{N}_4(t).
\end{aligned}
$$

Adopting a similar approach, there exists a scalar $\varepsilon_9 > 0$ satisfying $I-\varepsilon_9\tilde{M}_5(t)\tilde{M}_5^T(t) > 0$, such that

$$
\begin{aligned}
e^T(t)e(t) &= \xi^T(t)\bar{\mathcal{C}}_{ij}^T(t)\bar{\mathcal{C}}_{ij}(t)\xi(t)\\
&\le \xi^T(t)\tilde{\mathcal{C}}_{ij}^T(t)\left[I-\varepsilon_9\tilde{M}_5(t)\tilde{M}_5^T(t)\right]^{-1}\tilde{\mathcal{C}}_{ij}(t)\xi(t)\\
&\quad+\varepsilon_9^{-1}\xi^T(t)\tilde{N}_5^T(t)\tilde{N}_5(t)\xi(t)
\end{aligned} \tag{36}
$$

By combining (23) to (36), and the event triggered condition, we can obtain

$$
\begin{aligned}
&\dot{V}(t)+e^T(t)e(t)-\gamma^2\tilde{\omega}^T(t)\tilde{\omega}(t)\\
&\qquad\le \sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\eta^T(t)\Xi_{ij}\eta(t)
\end{aligned} \tag{37}
$$

where

$$
\eta^T(t) = \left[\xi^T(t)\xi^T(t-\tau(t))H^T\xi^T(t-\tau_M)H^Te_k^T(t)\tilde{\omega}^T(t)\right]
$$

Resorting to a similar technique in [37], the following slack matrix is introduced

$$
\sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i(\tilde{\mathcal{M}}_j-\tilde{\mathcal{W}}_j)X_i = 0, X_i = X_i^T \quad (i = 1, 2, \ldots, r) \tag{38}
$$

According to (37) and (38), one can get

$$
\begin{aligned}
&\sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\eta^T(t)\Xi_{ij}\eta(t)\\
&=\sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i(\tilde{\mathcal{M}}_j-\tilde{\mathcal{W}}_j+\ell_j\tilde{\mathcal{M}}_j-\ell_j\tilde{\mathcal{M}}_j)\eta^T(t)X_i\eta(t)\\
&\quad+\sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{W}}_j\eta^T(t)\Xi_{ij}\eta(t)\\
&=\sum_{i=1}^{r}\tilde{\mathcal{M}}_i^2\eta^T(t)(\ell_i\Xi_{ii}-\ell_iX_i+X_i)\eta(t)\\
&\quad+\sum_{i=1}^{r-1}\sum_{j=i+1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{M}}_j\eta^T(t)(\ell_j\Xi_{ij}-\ell_jX_i+X_i)\eta(t)\\
&\quad+\sum_{i=1}^{r-1}\sum_{j=i+1}^{r}\tilde{\mathcal{M}}_i\tilde{\mathcal{M}}_j\eta^T(t)(\ell_i\Xi_{ji}-\ell_iX_j+X_j)\eta(t)\\
&\quad+\sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{\mathcal{M}}_i(\tilde{\mathcal{W}}_j-\ell_j\tilde{\mathcal{M}}_j)\eta^T(t)(\Xi_{ij}-X_i)\eta(t)
\end{aligned}
$$

where $\tilde{\mathcal{W}}_j-\ell_j\tilde{\mathcal{M}}_j\geq 0$ is satisfying for all of j. From (17) to (21), one has

$$\dot{V}(t)+e^T(t)e(t)-\gamma^2\tilde{\omega}^T(t)\tilde{\omega}(t)\leq 0 \tag{39}$$

From (39), by using of Schur Complement Lemma, it is easily concluded that the filtering error systems (16) with $\tilde{\omega}(t)=0$ meets the stability requirement.

Taking the integral from 0 to ∞ of both sides of (39), we get

$$\int_0^{\infty}e^T(t)e(t)dt<\int_0^{\infty}\gamma^2\tilde{\omega}^T(t)\tilde{\omega}(t)dt \tag{40}$$

Obviously, for any non-zero $\tilde{\omega}(t)\in L_2[0,\infty)$, we have $\|e(t)\|_2<\gamma\|\tilde{\omega}(t)\|_2$. Thus, the filtering error system (16) is asymptotically stable and satisfies the H_∞ performance. This ends the proof. □

B. FILTER DESIGN

In this sequel, based on Theorem 1, we will give a specific algorithm for filter parameter of (5).

Theorem 2: Consider the IT2 fuzzy systems and the fuzzy filter with quantizer and event-triggered mechanism subject to σ, for given scalars $\tau_M>0$, $\varepsilon_i>0(i=1,2,\ldots,9)$, and the membership function satisfies $\tilde{\mathcal{W}}_j-\ell_j\tilde{\mathcal{M}}_j\geq 0$ $(0<\ell_j\leq 1)$. The filtering error system (16) is asymptotically stable with a performance index γ, if there exist matrices $P=\begin{bmatrix}P_1 & -P_2\\ -P_2 & P_2\end{bmatrix}>0$, $Q_1>0$, $R_1>0$, $\hat{\Theta}>0$, $U_1>0$, and $\tilde{X}_i=\tilde{X}_i^T$, $(i=1,2,\ldots,r)$ satisfy the following inequalities:

$$2P_2-R_1>0, \tag{41}$$

$$\begin{bmatrix}I & \varepsilon_9\tilde{M}_5\\ * & \varepsilon_9\end{bmatrix}>0,\ \begin{bmatrix}R_1 & U_1^T\\ * & R_1\end{bmatrix}\geq 0 \tag{42}$$

$$\tilde{\Xi}_{ij}-\tilde{X}_i<0, \tag{43}$$

$$\ell_i\tilde{\Xi}_{ii}-\ell_i\tilde{X}_i+\tilde{X}_i<0, \tag{44}$$

$$\ell_j\tilde{\Xi}_{ij}+\ell_i\tilde{\Xi}_{ji}-\ell_j\tilde{X}_i-\ell_i\tilde{X}_j+\tilde{X}_i+\tilde{X}_j<0,\quad i<j, \tag{45}$$

where

$$\tilde{\Xi}_{ij}=\begin{bmatrix}\tilde{\Xi}_{ij}^{11} & \tilde{\Xi}_{ij}^{12} & \tilde{\Xi}_{ij}^{13} & \tilde{\Xi}_{ij}^{14} & \tilde{\Xi}_{ij}^{15} & \tilde{\Xi}_{ij}^{16}\\ * & \tilde{\Xi}_{ij}^{22} & \tilde{\Xi}_{ij}^{23} & 0 & 0 & \tilde{\Xi}_{ij}^{26}\\ * & * & \tilde{\Xi}_{ij}^{33} & 0 & 0 & 0\\ * & * & * & \tilde{\Xi}_{ij}^{44} & 0 & 0\\ * & * & * & * & \tilde{\Xi}_{ij}^{55} & 0\\ * & * & * & * & * & \tilde{\Xi}_{ij}^{66}\end{bmatrix},$$

$$\tilde{\Xi}_{ij}^{11}=\begin{bmatrix}\Phi_{ij}^{11} & \Phi_{ij}^{12} & \mathcal{L}_i^T & \Phi_{ij}^{14} & 0\\ * & \Phi_{ij}^{22} & -\tilde{\mathcal{C}}_{fj}^T & \Phi_{ij}^{24} & \Phi_{ij}^{25}\\ * & * & \Phi_{ij}^{33} & 0 & 0\\ * & * & * & \Phi_{ij}^{44} & 0\\ * & * & * & * & -\varepsilon_9\end{bmatrix}$$

with

$$
\begin{aligned}
\Phi_{ij}^{11}&=Q_1-R_1+P_1\mathcal{A}_i+\mathcal{A}_i^TP_1+4\tau_M^2\mathcal{A}_i^TR_1\mathcal{A}_i,\\
\Phi_{ij}^{14}&=\left[-\sqrt{\varepsilon_1}\tilde{\mathcal{A}}_{fj}\mathfrak{M}_{1j}-\rho\sqrt{\varepsilon_2}\chi_1-\rho\sqrt{\varepsilon_3}\chi_1\rho\sqrt{\varepsilon_4}\chi_1\right],\\
\Phi_{ij}^{12}&=-\tilde{\mathcal{A}}_{fj}-\mathcal{A}_i^TP_2,\ \chi_1=\tilde{\mathcal{B}}_{fj}\mathfrak{M}_{2j},\\
\Phi_{ij}^{22}&=\tilde{\mathcal{A}}_{fj}+\tilde{\mathcal{A}}_{fj}^T+(\varepsilon_1^{-1}+4\varepsilon_5^{-1}\tau_M^2)\mathfrak{N}_{1j}^T\mathfrak{N}_{1j},\ \Phi_{ij}^{24}=-\Phi_{ij}^{14},\\
\Phi_{ij}^{25}&=\mathcal{C}_{fj}^T\mathfrak{N}_{3j}^T,\ \Phi_{ij}^{33}=-I+\varepsilon_9\mathfrak{M}_{3j}\mathfrak{M}_{3j}^T,\\
\Phi_{ij}^{44}&=diag\{-I,-I,-I,-I\},
\end{aligned}
$$

$$\tilde{\Xi}_{ij}^{12}=\begin{bmatrix}-\rho\tilde{\mathcal{B}}_{fj}\mathcal{C}_i+R_1^T-U_1^T\\ \rho\tilde{\mathcal{B}}_{fj}\mathcal{C}_i\\ 0\\ 0\\ 0\end{bmatrix},\ \tilde{\Xi}_{ij}^{13}=\begin{bmatrix}U_1^T\\ 0\\ 0\\ 0\\ 0\end{bmatrix},$$

$$\tilde{\Xi}_{ij}^{14}=\begin{bmatrix}\rho\tilde{\mathcal{B}}_{fj}\\ -\rho\tilde{\mathcal{B}}_{fj}\\ 0\\ 0\\ 0\end{bmatrix},\ \tilde{\Xi}_{ij}^{15}=\begin{bmatrix}P_1\mathcal{B}_i\\ -P_2\mathcal{B}_i\\ 0\\ 0\\ 0\end{bmatrix},$$

$$\tilde{\Xi}_{ij}^{16}=\begin{bmatrix}-\rho\tilde{\mathcal{B}}_{fj}\mathcal{D}_i\\ \rho\tilde{\mathcal{B}}_{fj}\mathcal{D}_i\\ 0\\ 0\\ 0\end{bmatrix},$$

$$
\begin{aligned}
\tilde{\Xi}_{ij}^{22}&=(\varepsilon_2^{-1}+4\tau_M^2\varepsilon_6^{-1})\mathfrak{N}_{2j}^T\mathfrak{N}_{2j}-2R_1+U_1+U_1^T\\
&\quad+\sigma\mathcal{C}_i^T\hat{\Theta}\mathcal{C}_i,\\
\tilde{\Xi}_{ij}^{23}&=R_1^T-U_1^T,\ \tilde{\Xi}_{ij}^{26}=\sigma\mathcal{C}_i^T\hat{\Theta}\mathcal{D}_i,\ \tilde{\Xi}_{ij}^{33}=-Q_1-R_1,\\
\tilde{\Xi}_{ij}^{44}&=(\varepsilon_4^{-1}+4\tau_M^2\varepsilon_8^{-1})\mathfrak{N}_{2j}^T\mathfrak{N}_{2j}-\hat{\Theta},\ \tilde{\Xi}_{ij}^{55}\\
&=4\tau_M^2\mathcal{B}_i^TR_1\mathcal{B}_i-\gamma^2I,\ \tilde{X}_i=diag\{\bar{X}_1^i,\underbrace{0,\ldots,0}_{10}\},\\
\tilde{\Xi}_{ij}^{66}&=(\varepsilon_3^{-1}+4\tau_M^2\varepsilon_7^{-1})\mathcal{D}_i^T\mathfrak{N}_{2j}^T\mathfrak{N}_{2j}\mathcal{D}_i+\sigma\mathcal{D}_i^T\hat{\Theta}\mathcal{D}_i-\gamma^2I,
\end{aligned}
$$

and, the filter parameters are as follows:

$$\mathcal{A}_{fj} = P_2^{-1}\tilde{\mathcal{A}}_{fj}, \mathcal{B}_{fj} = P_2^{-1}\tilde{\mathcal{B}}_{fj}, \mathcal{C}_{fj} = \tilde{\mathcal{C}}_{fj}. \tag{46}$$

Proof: Give $P_1 > P_2 > 0$, and construct $P = \begin{bmatrix} P_1 & -P_2 \\ -P_2 & P_2 \end{bmatrix}$.

According to Schur Complement Lemma, we have $P_1 - P_2P_2^{-1}P_2 = P_1 - P_2 > 0$ to ensure $P > 0$. Pre-multiplying and post-multiplying (17) by P_2, it can be obtained that $P_2R_1^{-1}P_2 > 0$. Then, by using $-P_2R_1^{-1}P_2 \leq -2P_2 + R_1$, we can get (41).

From Theorem 1, we have:

$$\begin{aligned}
P\tilde{\mathcal{A}}_{ij} &= \begin{bmatrix} P_1\mathcal{A}_i & -P_2\mathcal{A}_{fj} \\ -P_2\mathcal{A}_i & P_2\mathcal{A}_{fj} \end{bmatrix}, \\
P\tilde{\mathcal{B}}_{\tau ij} &= \begin{bmatrix} -P_2\mathcal{B}_{fj}\rho\mathcal{C}_i \\ P_2\mathcal{B}_{fj}\rho\mathcal{C}_i \end{bmatrix}, \\
P\tilde{\mathcal{B}}_{eij} &= \begin{bmatrix} P_2\mathcal{B}_{fj}\rho \\ -P_2\mathcal{B}_{fj}\rho \end{bmatrix}, \\
P\tilde{\mathcal{B}}_{\omega ij} &= \begin{bmatrix} P_1\mathcal{B}_i & -P_2\mathcal{B}_{fj}\rho\mathcal{D}_i \\ -P_2\mathcal{B}_i & P_2\mathcal{B}_{fj}\rho\mathcal{D}_i \end{bmatrix}, \\
P\tilde{M}_1 &= \begin{bmatrix} -P_2\mathcal{A}_{fj}\mathfrak{M}_{1j} \\ P_2\mathcal{A}_{fj}\mathfrak{M}_{1j} \end{bmatrix}, P\tilde{M}_2 = \begin{bmatrix} -\rho P_2\mathcal{B}_{fj}\mathfrak{M}_{2j} \\ \rho P_2\mathcal{B}_{fj}\mathfrak{M}_{2j} \end{bmatrix} \\
P\tilde{M}_3 &= \begin{bmatrix} -\rho P_2\mathcal{B}_{fj}\mathfrak{M}_{2j} \\ \rho P_2\mathcal{B}_{fj}\mathfrak{M}_{2j} \end{bmatrix} P\tilde{M}_4 = \begin{bmatrix} \rho P_2\mathcal{B}_{fj}\mathfrak{M}_{2j} \\ -\rho P_2\mathcal{B}_{fj}\mathfrak{M}_{2j} \end{bmatrix}
\end{aligned} \tag{47}$$

Furthermore, new variables are defined:

$$\tilde{\mathcal{A}}_{fj} = P_2\mathcal{A}_{fj}, \tilde{\mathcal{B}}_{fj} = P_2\mathcal{B}_{fj}, \tilde{\mathcal{C}}_{fj} = \mathcal{C}_{fj}. \tag{48}$$

By Schur Complement Lemma, (19) is equivalent to the following inequality

$$\begin{bmatrix} \phi_{ij}^{11} & \phi_{ij}^{12} & \phi_{ij}^{13} & \phi_{ij}^{14} & \phi_{ij}^{15} \\ * & \Xi_{ij}^{22} & \Xi_{ij}^{23} & 0 & \Xi_{ij}^{25} \\ * & * & \Xi_{ij}^{33} & 0 & 0 \\ * & * & * & \Xi_{ij}^{44} & 0 \\ * & * & * & * & \Xi_{ij}^{55} \end{bmatrix} - \tilde{X}_i < 0, \tag{49}$$

where

$$\begin{aligned}
\phi_{ij}^{11} &= \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} \\ * & \phi_{22} & 0 & 0 \\ * & * & \phi_{33} & 0 \\ * & * & * & -\varepsilon_9 \end{bmatrix}, \\
\phi_{11} &= P\tilde{\mathcal{A}}_{ij} + \tilde{\mathcal{A}}_{ij}^T P + H^T(Q_1 - R_1)H \\
&\quad + \varepsilon_1^{-1}\tilde{N}_1^T\tilde{N}_1 + 4\tau_M^2(\tilde{\mathcal{A}}_{ij}^T H^T R_1 H\tilde{\mathcal{A}}_{ij} \\
&\quad + \varepsilon_5^{-1}\tilde{N}_1^T\tilde{N}_1), \phi_{12} = \tilde{\mathcal{C}}_{ij}^T, \\
\phi_{13} &= \left[\sqrt{\varepsilon_1}P\tilde{M}_1\sqrt{\varepsilon_2}P\tilde{M}_2\sqrt{\varepsilon_3}P\tilde{M}_3\sqrt{\varepsilon_4}P\tilde{M}_4\right],
\end{aligned}$$

$$\phi_{14} + \tilde{N}_5^T, \phi_{22} = -I + \varepsilon_9\tilde{M}_5\tilde{M}_5^T, \phi_{33} = \Phi_{ij}^{44},$$

$$\begin{aligned}
\phi_{ij}^{12} &= \left[\left(\Xi_{ij}^{12}\right)^T \ 0\ 0\ 0\right]^T, \\
\phi_{ij}^{13} &= \left[\left(\Xi_{ij}^{13}\right)^T \ 0\ 0\ 0\right]^T, \\
\phi_{ij}^{14} &= \left[\left(\Xi_{ij}^{14}\right)^T \ 0\ 0\ 0\right]^T, \\
\phi_{ij}^{15} &= \left[\left(\Xi_{ij}^{15}\right)^T \ 0\ 0\ 0\right]^T.
\end{aligned}$$

Based on the filtering error system (16), we have

$$\begin{aligned}
\tilde{\mathcal{A}}_{ij}^T H^T R_1 H\tilde{\mathcal{A}}_{ij} &= \begin{bmatrix} \mathcal{A}_i^T R_1\mathcal{A}_i & 0 \\ 0 & 0 \end{bmatrix}, \\
\tilde{\mathcal{B}}_{\omega ij}^T H^T R_1 H\tilde{\mathcal{B}}_{\omega ij} &= \begin{bmatrix} \mathcal{B}_i^T R_1\mathcal{B}_i & 0 \\ 0 & 0 \end{bmatrix}, \\
\tilde{\mathcal{B}}_{\tau ij}^T H^T R_1 H\tilde{\mathcal{B}}_{\tau ij} &= 0, \tilde{\mathcal{B}}_{eij}^T H^T R_1 H\tilde{\mathcal{B}}_{eij} = 0.
\end{aligned} \tag{50}$$

By substituting (47), (48) and (50) into inequalities (49), we can derive (43). Similarly, (44) and (45) can be obtained. This ends the proof. □

It is worth noting that there are uncertainties Δ_q in the term ρ for Theorem 2. In order to facilitate the subsequent simulation analysis, we give the standard linear matrix inequalities (LMIs) form in the following theorem.

Theorem 3: Consider the IT2 fuzzy systems and the fuzzy filter with quantizer and event-triggered mechanism subject to σ, for given scalars $\tau_M > 0$, $\delta > 0$, $\varepsilon_i > 0 (i = 1, 2, \ldots, 10)$, and the membership function satisfies $\mathcal{W}_j - \ell_j\mathcal{M}_j \geq 0\ (0 < \ell_j \leq 1)$. The filtering error system (16) is asymptotically stable with a performance index γ, if there exist matrices $P = \begin{bmatrix} P_1 & -P_2 \\ -P_2 & P_2 \end{bmatrix} > 0$, $Q_1 > 0$, $R_1 > 0$, $\hat{\Theta} > 0$, $U_1 > 0$, and $\bar{X}_i = \bar{X}_i^T$, $(i = 1, 2, \ldots, r)$ satisfy the following inequalities:

$$2P_2 - R_1 > 0, \tag{51}$$

$$\begin{bmatrix} I & \varepsilon_9\tilde{M}_5 \\ * & \varepsilon_9 \end{bmatrix} > 0, \begin{bmatrix} R_1 & U_1^T \\ * & R_1 \end{bmatrix} \geq 0 \tag{52}$$

$$\Psi_{ij} - \bar{X}_i < 0, \tag{53}$$

$$\ell_i\Psi_{ii} - \ell_i\bar{X}_i + \bar{X}_i < 0, \tag{54}$$

$$\ell_j\Psi_{ij} + \ell_i\Psi_{ji} - \ell_j\bar{X}_i - \ell_i\bar{X}_j + \bar{X}_i + \bar{X}_j < 0, \quad i < j. \tag{55}$$

where

$$\Psi_{ij} = \begin{bmatrix} \bar{\Xi}_{ij} & \delta\Sigma_1^T & \varepsilon_{10}\Sigma_2^T \\ * & -\varepsilon_{10}I & 0 \\ * & * & -\varepsilon_{10}I \end{bmatrix},$$

$$\bar{\Xi}_{ij} = \begin{bmatrix} \bar{\Xi}_{ij}^{11} & \bar{\Xi}_{ij}^{12} & \tilde{\Xi}_{ij}^{13} & \bar{\Xi}_{ij}^{14} & \bar{\Xi}_{ij}^{15} & \bar{\Xi}_{ij}^{16} \\ * & \tilde{\Xi}_{ij}^{22} & \tilde{\Xi}_{ij}^{23} & 0 & 0 & \bar{\Xi}_{ij}^{26} \\ * & * & \tilde{\Xi}_{ij}^{33} & 0 & 0 & 0 \\ * & * & * & \tilde{\Xi}_{ij}^{44} & 0 & 0 \\ * & * & * & * & \tilde{\Xi}_{ij}^{55} & 0 \\ * & * & * & * & * & \tilde{\Xi}_{ij}^{66} \end{bmatrix},$$

$$\bar{\Xi}_{ij}^{11} = \begin{bmatrix} \Phi_{ij}^{11} & \Phi_{ij}^{12} & \mathcal{L}_i^T & \bar{\Phi}_{ij}^{14} & 0 \\ * & \Phi_{ij}^{22} & -\tilde{\mathcal{C}}_{fj}^T & \bar{\Phi}_{ij}^{24} & \Phi_{ij}^{25} \\ * & * & \Phi_{ij}^{33} & 0 & 0 \\ * & * & * & \Phi_{ij}^{44} & 0 \\ * & * & * & * & -\varepsilon_9 \end{bmatrix},$$

with

$$\bar{\Phi}_{ij}^{14} = \left[-\sqrt{\varepsilon_1}\tilde{\mathcal{A}}_{fj}\mathfrak{M}_{1j} - \sqrt{\varepsilon_2}\chi_1 - \sqrt{\varepsilon_3}\chi_1\sqrt{\varepsilon_4}\chi_1\right],$$

$$\bar{\Xi}_{ij}^{12} = \begin{bmatrix} \chi_2 \\ \tilde{\mathcal{B}}_{fj}\mathcal{C}_i \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{\Xi}_{ij}^{14} = \begin{bmatrix} \tilde{\mathcal{B}}_{fj} \\ -\tilde{\mathcal{B}}_{fj} \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\bar{\Xi}_{ij}^{16} = \begin{bmatrix} -\tilde{\mathcal{B}}_{fj}\mathcal{D}_i \\ \tilde{\mathcal{B}}_{fj}\mathcal{D}_i \\ 0 \\ 0 \\ 0 \end{bmatrix}, \bar{X}_i = diag\{\bar{X}_1^i, \underbrace{0, \ldots, 0}_{12}\},$$

$$\chi_2 = -\tilde{\mathcal{B}}_{fj}\mathcal{C}_i + R_1^T - U_1^T, \Sigma_1 = \left[\tilde{\mathcal{B}}_{fj}^T \ \tilde{\mathcal{B}}_{fj}^T \ O_{1\times 11}\right],$$

$$\Sigma_2 = \left[\chi_3 \ \chi_4\right], \bar{\Phi}_{ij}^{24} = -\bar{\Phi}_{ij}^{14},$$

$$\chi_3 = \left[O_{1\times 4} \ \sqrt{\varepsilon_2}\mathfrak{M}_{2j} \ \sqrt{\varepsilon_3}\mathfrak{M}_{2j} \ -\sqrt{\varepsilon_4}\mathfrak{M}_{2j}\right],$$

$$\chi_4 = \left[0 \ \mathcal{C}_i \ 0 \ -I \ 0 \ \mathcal{D}_i\right],$$

and, the filter parameters are constructed as follows:

$$\mathcal{A}_{fj} = P_2^{-1}\tilde{\mathcal{A}}_{fj}, \mathcal{B}_{fj} = P_2^{-1}\tilde{\mathcal{B}}_{fj}, \mathcal{C}_{fj} = \tilde{\mathcal{C}}_{fj}. \tag{56}$$

Proof: According to Theorem 2, we have

$$\tilde{\Xi}_{ij} - \tilde{X}_i = \bar{\Xi}_{ij} - \tilde{X}_i + \Sigma_1^T\Delta_q\Sigma_2 + \Sigma_2^T\Delta_q\Sigma_1 < 0 \tag{57}$$

Applying Lemma 4 to (57), there exists a scalar $\varepsilon_{10} > 0$ such that

$$\bar{\Xi}_{ij} - \tilde{X}_i + \varepsilon_{10}^{-1}\Sigma_1^T\delta^2\Sigma_1 + \varepsilon_{10}\Sigma_2^T\Sigma_2 < 0 \tag{58}$$

Then, (51) to (55) are obtained by Schur Complement Lemma. This completes the proof. □

Moreover, it should be noted that the minimum value of the performance index γ can be obtained by solving the following convex optimization problem:

$$min\,\gamma \text{ subject to inequalities (51) to (55).}$$

IV. NUMERICAL SIMULATION

In this sequel, a simulation example is provided to show the effectiveness of the obtained method in this article. Consider the IT2 T-S fuzzy networked system (3) with two rules, and the system parameter matrices are given as follows:

$$\mathcal{A}_1 = \begin{bmatrix} -3.1 & 10 \\ -1 & -10 \end{bmatrix}, \mathcal{B}_1 = \begin{bmatrix} 0.9 \\ -0.2 \end{bmatrix}, \mathcal{D}_1 = 0.1,$$

$$\mathcal{C}_1 = \begin{bmatrix} -0.1 & -0.2 \end{bmatrix}, \mathcal{L}_1 = \begin{bmatrix} 1 & -0.5 \end{bmatrix},$$

$$\mathcal{A}_2 = \begin{bmatrix} -0.1 & 10 \\ -1 & -10 \end{bmatrix}, \mathcal{B}_2 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \mathcal{D}_2 = 0.1,$$

$$\mathcal{C}_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \mathcal{L}_2 = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}.$$

The membership functions of the system (3) and filter (5) are selected as follows:

$$\mathcal{M}_1^1(x_1(t)) = 1 - \frac{1}{1 + e^{\frac{(x_1(t))^2 - 0.25}{2}}},$$

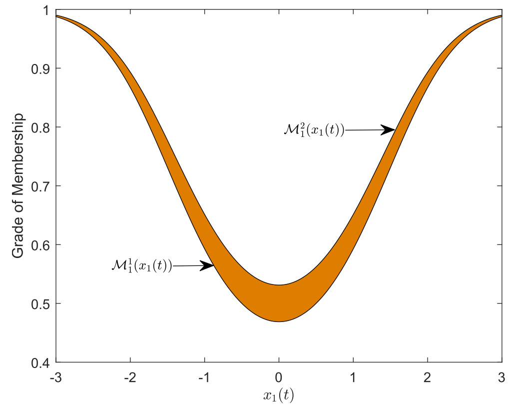

FIGURE 2. FOUs for $\mathcal{M}_1(x_1(t))$.

TABLE 1. Obtained minimum performance index γ for different τ_M.

τ_M	0.1	0.3	0.5	0.7	0.9
γ	0.6987	0.7570	0.7593	0.8025	0.8482

$$\mathcal{M}_1^2(x_1(t)) = 1 - \frac{1}{1 + e^{\frac{(x_1(t))^2 + 0.25}{2}}},$$

$$\mathcal{M}_2^1(x_1(t)) = 1 - \mathcal{M}_1^2(x_1(t)), \mathfrak{G}_i^1(x(t)) = \sin^2(x_1(t)),$$

$$\mathcal{M}_2^2(x_1(t)) = 1 - \mathcal{M}_1^1(x_1(t)), \mathfrak{G}_i^2(x(t)) = \cos^2(x_1(t)),$$

$$\mathcal{W}_1^1(x_1(t)) = 0.3 + 0.4 * e^{-\frac{(x_1(t))^2}{2}},$$

$$\mathcal{W}_1^2(x_1(t)) = 0.3 + 0.4 * e^{-\frac{(x_1(t))^2}{7}},$$

$$\mathcal{W}_2^1(x_1(t)) = 1 - \mathcal{W}_1^2(x_1(t)), \mathfrak{F}_j^1(x(t)) = 0.5,$$

$$\mathcal{W}_2^2(x_1(t)) = 1 - \mathcal{W}_1^1(x_1(t)), \mathfrak{F}_j^2(x(t)) = 0.5, i, j \in \{1, 2\}.$$

Assuming that the sampling period of the system is $h = 0.01s$, the matrices of filter parameter uncertainties and scalars are chosen as follows:

$$\mathfrak{M}_{11} = \mathfrak{M}_{12} = [0.1 - 0.3], \mathfrak{M}_{21} = \mathfrak{M}_{22} = -0.02,$$

$$\mathfrak{M}_{31} = \mathfrak{M}_{32} = 0.01, \mathfrak{N}_{11} = \mathfrak{N}_{12} = [0.10.01],$$

$$\mathfrak{K}_1(t) = \mathfrak{K}_2(t) = \mathfrak{K}_3(t) = \sin(t), \mathfrak{N}_{21} = \mathfrak{N}_{22} = 0.02,$$

$$\mathfrak{N}_{31} = \mathfrak{N}_{32} = 0.01, \varepsilon_1 = 0.08, \varepsilon_2 = 0.05, \varepsilon_3 = 0.6,$$

$$\varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 0.7, \varepsilon_7 = \varepsilon_8 = \varepsilon_9 = 0.06, \varepsilon_{10} = 10.$$

In order to better describe the uncertainty of the membership function for IT2 fuzzy systems, the FOUs of $\mathcal{M}_1(x_1(t))$ and $\mathcal{M}_2(x_1(t)$ are depicted in Figure 2 and Figure 3, respectively.

We assume the initial state is $x_0 = \left[0.1 \ -0.1\right]^T$, $\sigma = 0.2$, $\ell_1 = 0.7$, $\ell_2 = 0.5$, $\delta = 0.1$ and the external noise is

$$\omega(t) = \begin{cases} 0.1, & 0 \le t \le 1; \\ -0.1, & 1 < t \le 2; \\ 0, & other. \end{cases} \tag{59}$$

By solving Theorem 3, we derive the minimum performance indexes under different delay upper bounds in Table 1.

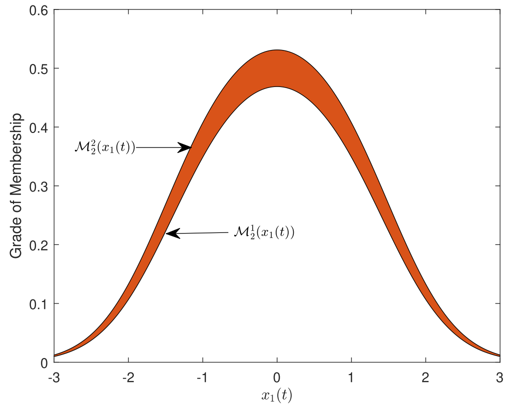

FIGURE 3. FOUs for $\mathcal{M}_2(x_1(t))$.

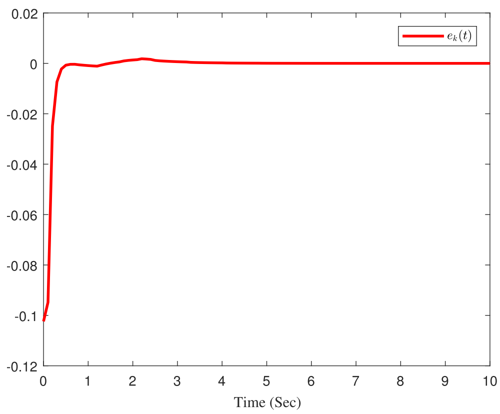

FIGURE 4. Response of filtering error.

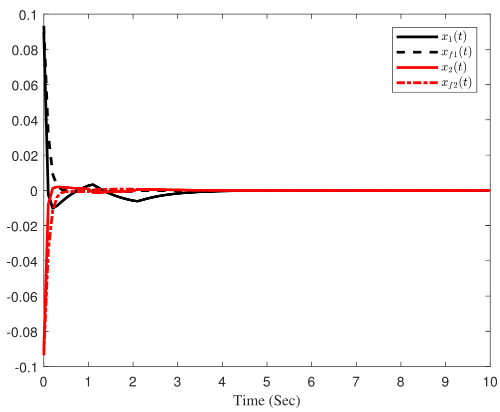

FIGURE 5. The states of filter and original system.

TABLE 2. Triggered times and transmission rate for different σ.

σ	0	0.2	0.4	0.6	0.8
Triggered times	100	25	18	15	12
Transmisson rate	100%	25%	18%	15%	12%

It can be seen that γ becomes bigger with an increasing the upper bound of delay. Table 2 provides the numbers of triggered and transmission rate for different triggered parameter σ. One can observe from Table 2 that the transmission rate

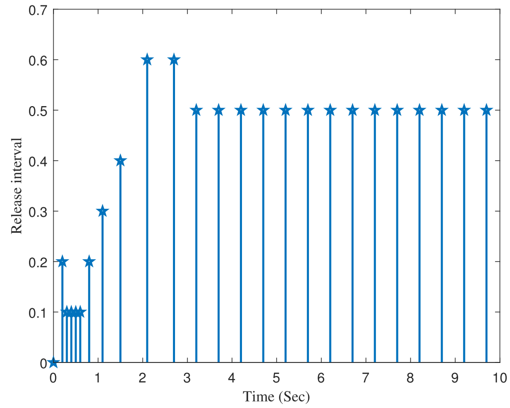

FIGURE 6. The curve of triggered instants and intervals.

and triggered times decrease with the increase of σ, and when $\sigma = 0$, the event-triggered mechanism changes into the time-triggered case.

Solving Theorem 3 with $\tau_M = 0.2$, the parameters of the event-triggered mechanism and the filter are obtained as follows:

$$\begin{aligned}
\mathcal{A}_{f1} &= \begin{bmatrix} -7.1838 & -4.2681 \\ 4.7704 & -6.6837 \end{bmatrix}, \mathcal{B}_{f1} = \begin{bmatrix} 1.2988 \\ -1.0512 \end{bmatrix}, \\
\mathcal{A}_{f2} &= \begin{bmatrix} -6.6736 & 4.5226 \\ 4.5488 & -6.9496 \end{bmatrix}, \mathcal{B}_{f2} = \begin{bmatrix} 1.3634 \\ -1.1250 \end{bmatrix}, \\
\mathcal{C}_{f1} &= \begin{bmatrix} 0.2459 & -0.1826 \end{bmatrix}, \mathcal{C}_{f2} = \begin{bmatrix} 0.1380 & -0.1745 \end{bmatrix}, \\
\hat{\Theta} &= 1.7628, \gamma = 0.7241
\end{aligned}$$

Figure 4 depicts the filtering error. The evolutions of $x(t)$ and $x_f(t)$ are shown in Figure 5. Figure 6 gives the triggered instants and intervals. As can be seen from Fig. 6, only 25 times of all sampled data are transmitted to the next node in the simulation time. From Figure 4 to Figure 6, it can be drawn the conclusion that the designed filter is effective and the event-based mechanism can save communication resources.

V. CONCLUSION

In this paper, a non-fragile fuzzy filter has been investigated for a class of nonlinear networked system. Due to the network framework, an event-based mechanism was employed to alleviate the communication load and save the limited bandwidth. In addition, the data to be released was quantized by a logarithmic quantizer to satisfy the requirements of signal normalization. Consequently, a method was proposed to construct the corresponding filter with H_∞ performance index γ. An example has been provided to demonstrate the usefulness of the proposed approach.

In the future, the study of event-based mechanism under the network framework can be extended to fault diagnosis and fault tolerance for IT2 fuzzy systems. Moreover, the research on the practical application of IT2 fuzzy systems deserves more attention.

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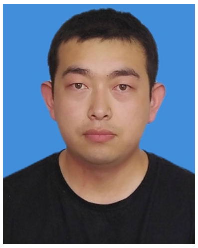

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