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Robust Design of a Closed-Loop Supply Chain Considering Multiple Recovery Options and Carbon Policies Under Uncertainty

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ABSTRACT Increasing global warming, climate change and stringent governmental legislations are driving industry practitioners and decision makers to implement various strategies to reduce carbon emissions. One of the effective approaches to mitigate carbon emissions is the implementation of closed-loop supply chain (CLSC). The key motivation for considering multiple recovery options in the CLSC is to capture the remaining economic value and to reduce carbon emissions in the collection and recovery operations. Customer's willingness to return used product depends on the acquisition price and nearness to the collection center. This research proposes a deterministic mixed-integer linear programming (MILP) model for a multi-period and multi-product CLSC network under carbon pricing and carbon trading policies consideration. The model includes different acquisition price for returned products and multiple recovery options. Further, the study takes into consideration uncertainty in procurement cost, demand, and quantity of returned products. A robust optimization approach is adopted to address uncertainty in network parameters. Numerical results show that the proposed model captures trade-offs between total cost and carbon emission. Overall, the study reveals that the carbon trading policy incurs relatively lower total cost compared to the carbon pricing policy. Repair and recycling activities in the reverse supply chain contribute significantly to the total cost and carbon emission. This study provide evidence that it is possible to achieve an optimal CLSC network with reduced carbon emission at a moderate total supply chain cost. The proposed model could be used to guide firms to choose an appropriate budget of uncertainty toward achieving a robust supply chain network.

INDEX TERMS Sustainable manufacturing, closed-loop supply chain, mixed-integer linear programming, carbon policies, multi recovery options, robust optimization, uncertainty, reverse logistics.

I. INTRODUCTION

Growing environmental concerns, stringent governmental legislations and increasing social expectation have motivated Original Equipment Manufacturers (OEMs) to pay attention to reverse logistics in addition to forward supply chain (SC) network. OEMs realize that the issue of recovering used products through remanufacturing and recycling would improve environmental sustainability as well as increase revenue by (i) selling recovered products into the secondary markets, and (ii) reusing of recovered parts and materials in the forward flow of the network. This could lead to saving in production cost and improve their competitiveness. The successful establishment of a closed loop supply chain (CLSC) network not only requires appropriate reverse logistics infrastructures but also needs efficient collection and recovery plan of used products.

The prime importance of used products recovery is for: (i) promoting environmental sustainability, and (ii) maximizing value creation of products beyond their useable life cycle. Toward this end, there is a need to have a plan to increase the quantity and quality of returning used products.

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Incentives and promotional offers in acquisition price, cash rebate, discounts and product exchange are examples of factors that may influence the decision for product returns [1], [2]. One may assume that majority of returning used products from customers are of low-quality category. Only a small portion of them might belong to medium and high-quality categories. Generally, the cost of product recovery increases as the quality of returned products decreases. Firms are expecting to earn more revenue from high quality returns and less revenue from low quality returns.

Authorities in European countries and Japan have set mandatory requirement for firms to meet specified targets of recovering used products. Firms which fail to meet this target would be penalized and may be perceived negatively by potential customers [3]. Various governments and policy makers around the globe have introduced more stringent carbon emission policies and new initiatives. For example, Saudi Arabia has pledged to reduce GHG emissions up to 130 million tons of carbon dioxide equivalent annually by 2030 through economic diversification and adaptation. United Nations and European Unions have introduced different carbon regulatory mechanisms to reduce carbon and other GHG emissions. Carbon pricing and carbon trading policies are also widely adopted by many countries [4]–[6]. Under carbon pricing policy, penalty is incurred through taxes based on price per unit of emitted carbon. Whereas under carbon trading policy, policy makers set a carbon emission limit (cap) on firms. If a firm emits carbon less than its prescribed limit, it can sell the unused carbon credits in a carbon trading market. On the other hand, a firm needs to buy carbon credits if it emits more that the prescribed limit. Currently, there are more than 20 platforms for carbon trading in the world and more than 40 countries have adopted the carbon pricing policy.

Since the last decade, concerns due to uncertainties from various sources (external, internal) have prompted researchers to consider uncertainty in their supply chain network design (SCND) planning decisions to avoid sub-optimal or infeasible solutions. Various modelling approaches have been proposed by researchers to address the uncertainties, such as stochastic approach, fuzzy method, and robust optimization, among others. Robust optimization is not only helpful to study real-world problems where there is not enough historical data to estimate the probability distribution of uncertain parameters, but also provides a framework to handle the uncertainty of parameters in optimization problems that could immunize the optimal solution for any realization of the uncertainty in a given bounded uncertainty set [7]–[9].

This research began by proposing a deterministic MILP model for a generic CLSC network with multiple recovery options, returned products with different quality levels and carbon emission considerations. Then, the model was enhanced to include uncertainty in procurement cost, product demand, and quantity of return products at customer zones to be closer to the real-world situations. Further, this study explored the effect on total supply chain cost and carbon emission for the models under carbon pricing and carbon trading policies. Sensitivity analysis was performed on various parameters to analyze the robustness of the model and to gain useful managerial insights.

The rest of the paper is organized as follows: A review of literature is provided in Section II. Section III presents development of the proposed deterministic MILP model. Robust counterpart model under polyhedral uncertainty set is developed in Section IV. Computational results as well as sensitivity analysis are discussed in Section V. Finally, conclusions are presented in Section VI.

II. LITERATURE REVIEW

The literature review is divided into the following: (A) integration of carbon policies in SCND decisions, (B) used products recoveries and incentive strategies, (C) uncertainty considerations in CLSC network design, and (D) existing CLSC models.

A. INTEGRATION OF CARBON POLICIES IN SCND DECISIONS

Recently, few researchers have considered carbon footprint in their supply chain (SC) related decisions. Benjaafar *et al.* [10] investigated the effect of carbon footprint on SC operational decisions and proposed optimization models integrating carbon emission regulations on economic order quantity decisions. Palak *et al.* [11] addressed the impact of carbon policies on inventory management decision in a biofuel SC. Diabat *et al.* [12] investigated the effect of carbon trading policy on SCND decisions of a CLSC network, whereas Fahimnia *et al.* [13] proposed a MILP model for a CLSC network and analyzed the effect of carbon tax policy on SCND and planning decisions. Jin *et al.* [14] proposed mathematical models for a forward SC logistics of a major retailer (Walmart USA) considering carbon footprint and transportation mode selection.

Transportation is one of the key generators of carbon. Thus, selection of an efficient transportation mode for logistic activities is vital to minimize carbon emissions [15]. One of the earliest studies incorporating carbon footprint in transport mode selection for CLSC network planning decisions is by Paksoy *et al.* [16]. Zeballos *et al.* [17] extended the work of Paksoy *et al.* [16] and proposed a stochastic model for a multi-period CLSC network design problem under the supply and demand uncertainty.

B. USED PRODUCT RECOVERIES AND INCENTIVE STRATEGIES

Acquisition price for used products is an important aspect in the SCND and planning decisions. It is a financial incentive offers to customers for returning their used products. Guide *et al.* [18] assessed the significance of financial incentive strategy in reverse logistics covering collecting and recovering of the used products from customers. Subsequently, Guide *et al.* [19] examined the implications of financial incentives on the acquisition of used products. They proposed an economic model for buying back used cellular phones with focus on the remanufacturing process They assert that the quality of used phones should determine the buyback price since it affects the remanufacturing efforts. Aras and Aksen [20] emphasized that a firm's major incentive in buying back used products is the residual value that can be procured by different salvaging methods. They proposed a quality-dependent incentive based on a primary reverse logistics network with few simple assumptions.

Das and Dutta [21] proposed a product exchange strategy to maximize the collection of used products and to improve recovery process in a CLSC network. A mathematical model was developed by incorporating consumer's utility. They argued that an individual customer decision to return used products is depending on the financial incentive offered. Masoudipour *et al.* [22] addressed the issue of return rates for recovery processes in reverse logistic network. They proposed a conditional quality-based divisional policy using zero-waste strategy instead of considering commonly used predetermined return rates.

All the above works focused only on economical prospective, either maximizing profit or minimizing cost. There has been lack of research integrating environment perspective into the supply chain decision makings. The primary motivation for the reverse supply chain operation should be to maximize the life of a product and to increase the environmental sustainability. As such, there is a need to integrate environmental perspective, carbon emission policies into CLSC network design.

C. UNCERTAINTY CONSIDERATIONS IN CLSC NETWORK DESIGN

Uncertainties in supply chain could be attributed to changes in government policies, facility disruptions, machine breakdown, uncertain material delivery, dynamic demand and product return, price and cost volatility, and unpredictable weather condition, among others. Ignoring these uncertainties in SCND and planning decisions may lead to infeasible or sub-optimal solutions [7]. In recent years, researchers have focused on developing CLSC networks with uncertainty considerations. Several techniques have been adopted to model such uncertainties in the CLSC network design and planning problems such as stochastic programming, chance constrained methods, fuzzy sets, and robust optimization.

Few researchers investigated uncertainty in CLSC configuration using stochastic programming approaches [23]–[25]. Salema *et al.* [23] developed stochastic scenario-based programming approach to represent uncertainties in demand and return rate. They proposed a MILP model for a generic reverse network problem. Pishvaee *et al.* [24] also formulated a MILP model and proposed a scenario-based stochastic approach to represent parameters uncertainty. Mohammed *et al.* [25] proposed a stochastic model for a CLSC network considering various carbon policies under demand and returned products uncertainty. Vahdani *et al.* [26] proposed a novel bi-objective mathematical programming formulation for configuring optimal facilities in a CLSC network and used fuzzy numbers to represent uncertain parameters.

A limited number of researchers have considered robust optimization methodology to deal with uncertainty in CLSC network design problems. Pishvaee *et al.* [7] developed a robust twin of the deterministic MILP model using box uncertainty set to overcome parameters uncertainty. Gao and Ryan [27] addressed multi-periods CLSC network design problem under uncertainty using stochastic scenario-based approach. They also implemented robust optimization under box uncertainty set. Bertsimas and M. Sim [28], [29] noted that polyhedral uncertainty set provides flexibility to attain desired robustness as well as to prevail the worst-case scenario. As such, this robust optimization technique should also be explored.

D. EXISTING CLSC MODELS

There have been several previous works involved CLSC models addressing various related issues. Diabat *et al.* [12] and Paksoy *et al.* [16] investigated CLSC by considering carbon pricing and carbon trading. Choudhary *et al.* [30] and Xu *et al.* [31] considered various carbon policies and formulated MILP models. Aras and Aksen [20] formulated a nonlinear MILP model considering quality dependent financial incentives. Aras and Aksen [20] and Masoudipour *et al.* [22] incorporated financial incentive mechanism for acquiring used products from customers. Masoudipour *et al.* [22] addressed the issue of return rates in reverse logistic network and proposed a conditional quality-based segmentation policy. Few researchers studied parameters uncertainty in CLSC network and proposed solution approaches based on fuzzy programming [32], stochastic scenario-based approach [33], and robust optimization [7], [8], [34], [35], among others. The main limitations of above works are (a) lacking in environmental consideration to reduce carbon emission; (b) studied only one type of quality level of return used products, and (c) no mechanism in reverse network to maximize the collection of used products. Issues such as multiple recovery options, financial incentives for returning used products, and carbon emissions consideration are relatively ignored [36], [37]. This paper aims to address these limitations by considering uncertainty issues associated with supply, demand, and quantity of used products at customer zones to make the model more realistic and closer to the real-world environment.

III. MATHEMATICAL MODEL

Problem description is presented in Section A and a deterministic MILP model formulation for a generic CLSC network design problem is elaborated in Section B.

A. PROBLEM DESCRIPTION

Fig. 1 illustrates the CLSC network configuration considered in this study. It consists of raw materials (A) and parts (B) suppliers, manufacturing centers (I), distribution

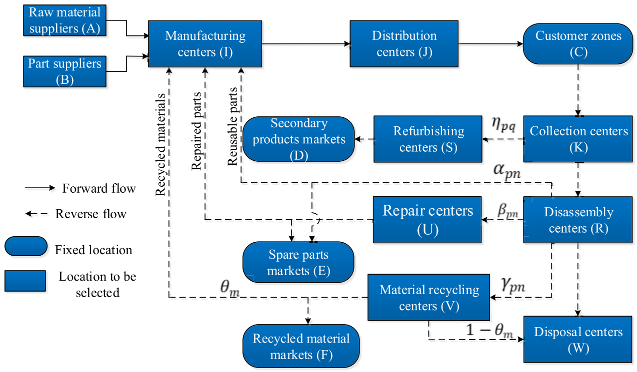


FIGURE 1. Configuration of the CLSC Network.

centers (J), and customer zones (C) in the forward flow network. Meanwhile, the reverse flow network is made of collection centers (K), refurbishing centers (S), disassembly centers (R), repair centers (U), recycling centers (V), disposal centers (W), secondary product markets (D), spare part markets (E) and recycled material markets (F). Physical configuration of the supply chain network is an important strategic and tactical decisions. The strategic decisions involve determining the optimal number and capacity of the facilities. Meanwhile, the tactical decisions involve selection of transportation channels, quantities of raw material to purchase, production rate, distribution route, buffer inventories at each facility and flow quantities between the facilities, among others.

Manufacturing centers acquire raw materials and components from the reverse flow network (recycled material, repaired parts, reusable parts) and procure additional quantity from raw materials and components suppliers. Then, these items are transformed into finished products. The finished products are then transported from distribution centers to customer zones to fulfil demand requirements. In the reverse flow, collection centers acquire returned products having different quality levels from customer zones. This study assumes a drop-off strategy where the customers have innate inclination to return the used products. However, this inclination depends on the acquisition price and the closeness to the collection center. A different acquisition price is offered based on the quality level of the returned products. Some proportion of returned products at the collection centers are sent to refurbishing centers for recovery process. The refurbished products will be sent to secondary products market to fulfil market demand. The remaining returned products at the collection centers are sent to disassembly centers for further operations. At the disassembly centers, products are disassembled, cleaned, tested, and sorted into reusable, repairable, recyclable and disposal categories depending on their quality levels.

The recovered components are either sent to: (i) manufacturing centers for use in producing new products, or (ii) offered to spare parts markets. Reusable parts of different quality levels are to be repaired at the repair centers. The repaired parts are either sent to the manufacturing centers or spare parts markets. Meanwhile, those of low quality or end of life materials are sent to the recycling centers. The recycled materials can serve both the manufacturing centers and the recycled material markets. The useless items and materials are transported to the disposal centers for earth filling.

It is critical for manufacturers to investigate the impact of carbon emissions associated with various decisions in the supply chain. This study focuses on carbon pricing and carbon trading policies since they are the most adopted policies by many countries. The primary challenge of carbon pricing policy is how to impose a financial penalty per unit carbon emitted such that the total cost is minimized, while the carbon emission is reduced. Meanwhile, the primary challenges with the carbon trading policy are: (i) how to allocate carbon emission cap to a firm, and (ii) how to penalize firms which exceed the emission quota limit and do not wish to buy additional carbon credits.

B. DETERMINISTIC MILP MODEL FORMULATION

In this section, we formulate a deterministic MILP model for the CLSC network configuration given in Fig. 1. The objective of the model is to minimize the total supply chain cost and carbon emissions by determining; (i) optimal number of potential facilities to be opened, (ii) optimal acquisition price of used products for different quality levels, (iii) an efficient transportation mode between the facilities, (iv) optimal purchasing quantities, production quantities, collected quantities and transportation frequency between the facilities, and (v) an efficient environmental protection technology at manufacturing centers. The following assumptions are adopted in the formulation of the MILP model:

- The number and capacity of facilities in the network are known in advance.
- The number of customer zones and secondary markets are fixed and predefined.
- The purchasing cost, customer demand, quantity of returned products at customer zones are assumed to be uncertain.
- At the beginning of planning horizon, distribution centers have enough products for next time periods to satisfy customers' demand.
- Penalty cost is incurred for not satisfying demand requirement.
- Used products are categorized as high, medium, and low-quality levels.
- The quantity of components and materials recovered are depending on the returns' quality levels.
- Components and materials are considered as brand-new after going through repairing and recycling processes. Their costs are cheaper as compared to fresh procurement from suppliers.
- The unit costs of recovery activities are quality dependent.

1) MODEL NOTATIONS

Based on the above problem description and assumptions, the following are the model notations covering sets and indices, parameters, and decision variables.

a: SETS AND INDICES

a	Potential location for raw materials supplier index, $a=1,2\ldots A$
b	Potential location for parts supplier index, $b=1,2\ldots B$
c	Fixed location for customer zone index, $c=1,2\ldots C$
d	Secondary market for products index, $d=1,2\ldots D$
e	Secondary market for spare part index, $e=1,2\ldots E$
f	Secondary market for recycled materials index, $f=1,2\ldots F$
g	Transport mode index, $g=1,2\ldots G$
i	Potential location for manufacturing center index, $i=1,2\ldots I$
j	Potential location for distribution center index, $j=1,2\ldots J$
k	Potential location for collection center, $k=1,2\ldots K$
l	Index for incentive level for collecting used product offered to customers, $l=1,2\ldots L$
m	Raw material type index, $m=1,2\ldots M$
n	Component type Index, $n=1,2\ldots N$
o	Production technology index, $o=1,2\ldots O$
p	Product type index, $p=1,2\ldots P$
q	Quality level of used product index, $q=1,2\ldots Q$
r	Potential location for disassembly center index, $r=1,2\ldots R$
s	Potential location for refurbishing center index, $s=1,2\ldots S$
t	Time period in the planning horizon index, $t=1,2\ldots T$
u	Potential location for repair center index, $u=1,2\ldots U$
v	Potential location for recycling center index, $v=1,2\ldots V$
w	Potential location for disposal center index, $w=1,2\ldots W$

b: PARAMETERS

DC^t_{cp}, Demand for new product p at customer zone c in period t

RC^t_{cpq}, Quantity of available used product p of quality q at customer zone c in period t

DD^t_{dp}, Demand for refurbished product p at secondary products market d in period t

DE^t_{en}, Demand for part n at spare parts market e in period t

DF^t_{fm}, Demand for raw material m at secondary materials market f in period t

rr^t_{cpql}, Probability of return of used product p with respect to quality q and price level l at customer zone c in period t

$pi^t_{cpql,}$ Incentive price offered for used product p with respect to quality q and price level l at customer zone c in period t

vm_m, Required space to store a unit of material m

vn_n, Required space to store a unit of part n

vp_p, Required space to store a unit of product p

ti_{po}, Time required to process (manufacture) a unit of product p using technology o

ts_p, Time required to process (refurbish) a unit of product p

tr_p, Time required to process (dismantle) a unit of product p

tu_n, Time required to process (repair) a unit of part n

tv_m, Time required to process (recycle) a unit of material m

ωm_m, Unit weight of material m

ωn_n, Unit weight of part n

ωp_p, Unit weight of product p

W_g, Maximum load capacity of transport mode g, in tons

φ_{pm}, Amount of material m in each unit of product p

ϕ_{pn}, Amount of part n in each unit of product p

η_{pq}, Proportion of returned product p with quality level q transported from collection center to refurbishing center

α_{pnq}, Proportion of reusable part n in the returned product p with quality level q transported from disassembly center to manufacturing center and spare parts market

β_{pnq}, Proportion of repairable part n in product p having q quality level

γ_{pmq}, Proportion of recyclable material m in product p having q quality level

θ_m, Proportion of recoverable material m at recycling center transported to manufacturing center and recycled material market

caa_a, Storage capacity of supplier a, in m^3

cab_b, Storage capacity of supplier b, in m^3

cai_i, Processing capacity of manufacturing center i, in hours

caj_j, Storage capacity of distribution center j, in m^3

cak_k, Storage capacity of collection center k, in m^3

car_r, Processing capacity of disassembly center r, in hours

cas_s, Processing capacity of refurbishing center s, in hours

cau_u, Processing capacity of repair center u, in hours

cav_v, Processing capacity of recycling center v in hours

caw_w, Storage capacity of disposal center w in m^3

fa_a, Fixed cost for choosing raw material supplier a

fb_b, Fixed cost for selecting part supplier b

fi^o_i, Fixed cost for opening a manufacturing center at location i with technology o

fj_j, Fixed cost for opening a distribution center at location j

fk_k, Fixed cost for opening the collection center k

fr_r, Fixed cost for opening a disassembly center at location r

fs_s, Fixed cost for opening a refurbishing center at location s

fu_u, Fixed cost for opening a repair center at location u

fv_v, Fixed cost for opening a recycling center at location v

fw_w, Fixed cost for opening a disposal center at location w

cpm_{am}, Procurement cost per unit of raw material m from supplier a

cpn_{bn}, Procurement cost per unit of part n from supplier b

cm^o_{ip}, Production cost per unit of product p at manufacturing center i using technology o

ch_{jp}, Inventory holding cost per unit of product p at distribution center j

ck_{kpq}, Processing cost per unit of return product p with quality level q at collection center k

cs_{spq}, Refurbishing cost per unit of product p with quality level q at refurbishing center s

cr_{rpq}, Dismantling cost per unit of returned product p with quality level q at disassembly center r

cu_{unq}, Repairing cost per unit of part n with quality level q at repair center u

cv_{vmq}, Unit recycling cost per unit volume of material m with quality level q at recycling center v

cw_{w}, Unit disposal cost at disposal center w

ρ^{t}_{cp}, Shortage cost per unit of non-satisfied demand of product p at customer zone c in period t

sd_{dp}, Selling price of refurbished product p at secondary products market d

se_{en}, Selling price of component n at spare parts market e

sf_{fm}, Selling price of recycled material m at material market f

sr_{in}, Per unit monetary saving resulted from using recovered part n at production plant i

su_{in}, Per unit monetary saving resulted from using repaired part n at production plant i

sv_{im}, Per unit monetary saving resulted from using recycled material m at production plant i

tai_{aimg}, Shipping cost per unit of raw material m from supplier a to manufacturing center i using transport mode g

tbi_{bing}, Cost of transporting per unit of part n from supplier b to manufacturing center i using transport mode g

tij_{ijpg}, Cost of transporting per unit of product p from manufacturing center i to distribution center j using transport mode g

tjc_{jcpg}, Cost of transporting per unit of product p from distribution center j to customer zone c using transport mode g

tks_{kspg}, Cost of transporting per unit of returned product p from collection center k to refurbishing center s using transport mode g

tkr_{krpg}, Cost of transporting per unit of returned product p from collection center k to disassembly center r using transport mode g

tsd_{sdpg}, Cost of transporting per unit of refurbished product p from refurbishing center s to secondary product market d using transport mode g

tri_{ring}, Cost of transporting per unit of reusable part n from disassembly center r to manufacturing center i using transport mode g

tre_{reng}, Cost of transporting per unit of reusable part n from disassembly center r to spare parts market e using transport mode g

tru_{rung}, Cost of transporting per unit of repairable part n from disassembly center r to repair center u using transport mode g

trv_{rvmg}, Cost of transporting per unit of recyclable material m from disassembly center r to recycling center v using transport mode g

trw_{rwg}, Cost of transporting per unit of non-recoverable from disassembly center r to disposal center w using transport mode g

tue_{ueng}, Cost of transporting per unit of part n from repair center u to spare parts market e using transport mode g

tui_{uing}, Cost of transporting per unit of part n from repair center u to manufacturing center i using transport mode g

tvf_{vfmg}, Cost of transporting per unit of recycled material m from recycling center v to recycled materials market f using transport mode g

tvi_{vimg}, Cost of transporting per unit of recycled material m from recycling center v to manufacturing center i using transport mode g

tvw_{vwmg}, Cost of transporting per unit of non-recyclable material m from recycling center v to disposal center w using transport mode g

eai_{aimg}, Estimated carbon emission in kg due to transporting a unit of raw material m from supplier a to manufacturing center i using transport mode g

ebi_{bing}, Estimated carbon emission in kg due to transporting a unit of part n from supplier b to manufacturing center i using transport mode g

ep_{iop}, Estimated carbon emission in kg due to producing a unit of product p at manufacturing center i using technology o

eij_{ijpg}, Estimated carbon emission in kg for transporting a unit of product p from manufacturing center i to distribution center j using transport mode g

ej_{jp}, Estimated carbon emission in kg for handling a unit of product p at distribution center j

ejc_{jcpg}, Estimated carbon emission in kg for transporting a unit of product p from distribution center j to customer zone c using transport mode g

eks_{kspg}, Estimated carbon emission in kg for transporting a unit of recoverable product p from collection center k to refurbishing center s using transport mode g

ekr_{krpg}, Estimated carbon emission in kg for transporting a unit of returned product p from collection center k to disassembly center r using transport mode g

esd_{sdpg}, Estimated carbon emission in kg for transporting a unit of refurbished product p from refurbishing center s to secondary product market d using transport mode g

ere_{reng}, Estimated carbon emission in kg for transporting a unit of reusable part n from disassembly center r to spare parts market e using transport mode g

eri_{ring}, Estimated carbon emission in kg for transporting a unit of reusable part n from disassembly center r to manufacturing center i

eru_{rung}, Estimated carbon emission in kg for transporting a unit of part n from disassembly center r to repair center u

erv_{rvmg}, Estimated carbon emission in kg for transporting a unit of raw material m from disassembly center r to recycling center v

erw_{rwg}, Estimated carbon emission in kg for transporting a unit of non-recoverable materials and parts from disassembly center r to disposal center w using transport mode g

eui_{uing}, Estimated carbon emission in kg for transporting a unit part n from repair center u to manufacturing center i using transport mode g

eue_{ueng}, Estimated carbon emission in kg for transporting a unit of part n from repair center u to spare parts market e using transport mode g

ev_{vm}, Estimated carbon emission in kg for processing a unit of raw material m at recycling center v

evi_{vimg}, Estimated carbon emission in kg for transporting a unit of recycled material m from recycling center v to manufacturing center i using transport mode g

evf_{vfmg}, Estimated carbon emission in kg for transporting a unit of recycled material m from recycling center v to material market f using transport mode g

evw_{vwmg}, Estimated carbon emission in kg for transporting a unit of disposable material m from recycling center v to disposal center w using transport mode g

ew_w, Estimated carbon emission in kg for landfilling at disposal center w

ρ, Carbon tax rate per unit, \$/ton

π, Buying and selling price per unit of carbon in the carbon market, in \$/ton

C_{max}, Maximum permissible carbon emission (carbon cap) over the entire planning horizon, in ton

c: DECISION VARIABLES RELATED TO FACILITIES AND TRANSPORTATION MODE

ZA_a, 1 if raw material supplier a is selected, 0 otherwise

ZB_b, 1 if part supplier b is selected, 0 otherwise

ZI_i^o, 1 if manufacturing center i with technology o is opened, 0 otherwise

ZJ_j, 1 if distribution center j is opened, 0 otherwise

ZS_s, 1 if refurbishing center s is opened, 0 otherwise

ZR_r, 1 if disassembly center r is opened, 0 otherwise

ZU_u, 1 if repair center u is opened, 0 otherwise

ZV_v, 1 if recycling center v is opened, 0 otherwise

ZW_w, 1 if disposal center w is opened, 0 otherwise

Y_{aig}^t, 1 if transport mode g is selected between the material supplier a and the manufacturing center i in period t, 0 otherwise

Y_{big}^t, 1if transport mode g is selected between the part supplier b and the manufacturing center i in period t, 0 otherwise

Y_{ijg}^t, 1 if transport mode g is selected between the manufacturing center i and the distribution center j in period t, 0 otherwise

Y_{jcg}^t, 1 if transport mode g is selected between the distribution center j and customer zone c in period t, 0 otherwise

Y_{ksg}^t, 1 if transport mode g is selected between the collection center k and the refurbishing center s in period t, 0 otherwise

Y_{krg}^t, 1 if transport mode g is selected between the collection center k and the disassembly center r in period t, 0 otherwise

Y_{sdg}^t, 1 if transport mode g is selected between the refurbishing center s and secondary products market d in period t, 0 otherwise

Y_{reg}^t, 1 if transport mode g is selected between the disassembly center r and secondary parts market e in period t, 0 otherwise

Y_{rig}^t, 1 if transport mode g is selected between the disassembly center r and the manufacturing center i in period t, 0 otherwise

Y_{rug}^t, 1 if transport mode g is selected between the disassembly center r and the repair center u in period t, 0 otherwise

Y_{rvg}^t, 1 if transport mode g is selected between the disassembly center r and the recycling center v in period t, 0 otherwise

Y_{rwg}^t, 1 if transport mode g is selected between the disassembly center r and the disposal center w in period t, 0 otherwise

Y_{ueg}^t, 1 if transport mode g is selected between the repair center u and the secondary parts market e in period t, 0 otherwise

Y_{uig}^t, 1 if transport mode g is selected between the repair supplier u and the manufacturing center i in period t, 0 otherwise

Y_{vfg}^t, 1 if transport mode g is selected between the recycling center v and the secondary materials market f in period t, 0 otherwise

Y_{vig}^t, 1 if transport mode g is selected between the recycling center v and the manufacturing center i in period t, 0 otherwise

Y_{vwg}^t, 1 if transport mode g is selected between the recycling center v and the disposal center i in period t, 0 otherwise

QAI_{aimg}^t, Quantity of raw material m transported from supplier a to manufacturing center i using transport mode g in period t

QBI_{bing}^t, Quantity of part n transported from supplier b to manufacturing center i using transport mode g in period t

QI_{iop}^t, Quantity of product p manufactured in manufacturing center i using technology o in period t

QIJ_{ijpg}^t, Quantity of product p transported from manufacturing center i to distribution center j using transport mode g in period t

QJC_{jcpg}^t, Quantity of product p transported from distribution center j to customer zone c using transport mode g in period t

QJ_{jp}^t, Inventory level of product p at distribution center j in period t

δ_{cp}^t, Quantity of non-satisfied demand of product p for customer c in period t

QCK_{ckpq}^t, Quantity of returned product p with quality level q transported from customer zone c to collection center k in period t

QKS_{kspqg}^t, Quantity of returned product p with quality level q transported from customer zone c to refurbishing center s using transport mode g in period t

QSD_{sdpg}^t, Quantity of refurbished product p transported from refurbishing center s to secondary product market d using transport mode g in period t

QKR^t_{krpqg}, Quantity of returned product p with quality level q transported from collection center k to disassembly center r using transport mode g in period t

QRE^t_{reng}, Quantity of reusable part n transported from disassembly center r to spare parts market e using transport mode g in period t

QRI^t_{ring}, Quantity of reusable part n transported from disassembly center r to manufacturing center i using transport mode g in period t

QRU^t_{runqg}, Quantity of part n with quality level q transported from disassembly center r to repair center u using transport mode g in period t

QRV^t_{rvmqg}, Quantity of recycling material m with quality level q transported from disassembly center r to recycling center v using transport mode g in period t

$QRW1^t_{rwng}$, Quantity of disposable part n transported from disassembly center r to disposal center w using transport mode g in period t

$QRW2^t_{rwmg}$, Quantity of disposable material m transported from disassembly center r to disposal center w using transport mode g in period t

QUE^t_{ueng}, Quantity of repaired part n transported from repair center u to spare parts market e using transport mode g in period t

QUI^t_{uing}, Quantity of repaired part n transported from repair center u to manufacturing center i using transport mode g in period t

QVF^t_{vfmg}, Quantity of recycled material m transported from recycling center v to raw material market f using transport mode g in period t

QVI^t_{vimg}, Quantity of recycled material m transported from recycling center v to manufacturing center i using transport mode g in period t

QVW^t_{vwmg}, Quantity of non-recyclable material m transported from recycling center v to disposal center w using transport mode g in period t

d: PARAMETERS AND DECISION VARIABLES RELATED TO ACQUISITION PRICE FOR COLLECTING USED PRODUCTS PARAMETERS

RC^t_{cpq}, Quantity of used product p of quality q at customer zone c in period t.

rr^t_{cpql}, Probability of return of used product p with respect to quality level q and price level l at customer zone c in period t

ri^t_{cpq}, Return rate of used product p with respect to quality level q at customer zone c in period t

ins^t_{cpql}, Incentive price offered for used product p with respect to quality level q and price level l at customer zone c in period t

Decision Variables:

Y^t_{cpql}, 1, if the incentive price level l is selected for product p with quality level q returned to customer zone c in period t, 0 otherwise

RL^t_{cpq}, Optimal return rate (%) of used product p with quality level q at customer zone c in period t

PL^t_{cpq}, Optimal incentive price offered for used product p having quality level q at customer zone c in period t

e: PARAMETERS AND DECISION VARIABLES RELATED SET COVERING OF CUSTOMER ZONES AND COLLECTION CENTERS PARAMETERS

fk_k, Fixed cost for opening collection center k

d_{ck}, Distance between customer zone c and collection center k

L_c, Distance limit within which a facility (collection center) can serve customer zone c

N_c, The set of eligible collection centers located within the distance limit and that can serve customer zone c ($N_c = \{k|d_{ck} \leq L_c\}$)

Decision Variables:

ZK_k, 1 if collection center k is opened, 0 otherwise

Y^t_{ck}, 1 if collection center k services customer zone c in period t, 0 otherwise

2) MODEL FORMULATION

a: OBJECTIVE FUNCTION

Minimize Total supply chain cost Z^D_1 = Fixed Cost (FC) + Procurement Cost (PC) + Manufacturing Cost (MC) + Inventory holding Cost at DC (HC) + Shortage Cost (SC) + Incentive Cost (IC) + Collection Cost (CC) + Refurbishing Cost (RfC) + Disassembly Cost (DiC) + Repair Cost (ReC) + Recycling Cost (RyC) + Disposal Cost (DsC) + Transportation Cost (TrC) - Total Revenue (Rev)

Fixed Cost (FC): Sum of fixed cost of opening and operating the potential facilities.

$$\begin{aligned} = &\sum_{a\in A} fa_a ZA_a + \sum_{b\in B} fb_b ZB_b + \sum_{i\in I}\sum_{o\in O} fi^o_i ZI^o_i \\ &+ \sum_{j\in J} fj_j ZJ_j + \sum_{k\in K} fk_k ZK_k + \sum_{s\in S} fs_s ZS_s + \sum_{r\in R} fr_r ZR_r \\ &+ \sum_{u\in U} fu_u ZU_u + \sum_{v\in V} fv_v ZV_v + \sum_{w\in W} fw_w ZW_w \end{aligned} \tag{1}$$

Procurement Cost (PC): Cost of purchasing raw materials quantities and components from raw material suppliers and components suppliers, respectively.

$$\begin{aligned} = &\sum_{a\in A}\sum_{i\in I}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} cpm_{am} QAI^t_{aimg} \\ &+ \sum_{b\in B}\sum_{i\in I}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} cpn_{bn} QBI^t_{bing} \end{aligned} \tag{2}$$

Manufacturing Cost (MC): Cost of manufacturing the products using environmentally protection technology.

$$= \sum_{i\in I}\sum_{p\in P}\sum_{o\in O}\sum_{t\in T} cm_{iop} QI^t_{iop} \tag{3}$$

Inventory Holding Cost at DC (HC): Cost of holding excess products at distribution centers.

$$= \sum_{j\in J}\sum_{p\in P}\sum_{t\in T} ch_{jp} QJ^t_{jp} \tag{4}$$

Shortage Cost (SC): Penalty cost for not satisfying demand of products at customer zones.

$$= \sum_{c\in C}\sum_{p\in P}\sum_{t\in T} \rho_{cp}^{t}\delta_{cp}^{t} \tag{5}$$

Incentive Cost (IC): Cost of financial incentives (acquisition price) based on different quality levels for acquiring used products from customer zones

$$= \sum_{c\in C}\sum_{p\in P}\sum_{q\in Q}\sum_{l\in L}\sum_{t\in T} ins_{cpql}^{t} rr_{cpql}^{t} Y_{cpql}^{t} RC_{cpq}^{t} \tag{6}$$

Collection Cost (CC): Cost of collecting returned products at collection centers.

$$= \sum_{c\in C}\sum_{k\in K}\sum_{p\in P}\sum_{q\in Q}\sum_{t\in T} ck_{kpq} QCK_{ckpq}^{t} \tag{7}$$

Refurbishing Cost (RfC): Cost of recovering used products at refurbishing centers.

$$= \sum_{k\in K}\sum_{s\in S}\sum_{p\in P}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} cs_{spq} QKS_{kspqg}^{t} \tag{8}$$

Disassembly Cost (DiC): Cost of disassembling and sorting of returned products into materials and components at disassembly centers.

$$= \sum_{k\in K}\sum_{r\in R}\sum_{p\in P}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} cr_{rpq} QKR_{krpqg}^{t} \tag{9}$$

Repair Cost (ReC): Cost of recovering components at repair centers.

$$= \sum_{r\in R}\sum_{u\in U}\sum_{n\in N}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} cu_{unq} QRU_{runqg}^{t} \tag{10}$$

Recycling Cost (RyC): Cost of recycling materials at recycling centers.

$$= \sum_{r\in R}\sum_{v\in V}\sum_{m\in M}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} cv_{vmq} QRV_{rvmqg}^{t} \tag{11}$$

Disposal Cost (DsC): Cost of disposing (land filling) components as well as materials at disposal centers.

$$\begin{aligned} = &\sum_{r\in R}\sum_{v\in V}\sum_{w\in W}\sum_{n\in N}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} cw_{w} \\ &\left(QRW1_{rwng}^{t} + QRW2_{rwmg}^{t} + QVW_{vwmg}^{t}\right) \end{aligned} \tag{12}$$

Transportation Cost (TrC): Cost of transporting materials, components, and products between the opened facilities using a transportation mode.

$$\begin{aligned} = &\sum_{a\in A}\sum_{i\in I}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} tai_{aimg} QAI_{aimg}^{t} \\ &+ \sum_{b\in B}\sum_{i\in I}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} tbi_{bing} QBI_{bing}^{t} \\ &+ \sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\sum_{g\in G}\sum_{t\in T} tij_{ijpg} QIJ_{ijpg}^{t} \\ &+ \sum_{j\in J}\sum_{c\in C}\sum_{p\in P}\sum_{g\in G}\sum_{t\in T} tjc_{jcpg} QJC_{jcpg}^{t} \\ &+ \sum_{k\in k}\sum_{s\in S}\sum_{p\in P}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} tks_{kspg} QKS_{kspqg}^{t} \\ &+ \sum_{s\in S}\sum_{d\in D}\sum_{p\in P}\sum_{g\in G}\sum_{t\in T} tsd_{sdpg} QSD_{sdpg}^{t} \\ &+ \sum_{k\in K}\sum_{r\in R}\sum_{p\in P}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} tkr_{krpg} QKR_{krpqg}^{t} \\ &+ \sum_{r\in R}\sum_{e\in E}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} tre_{reng} QRE_{reng}^{t} \\ &+ \sum_{r\in R}\sum_{i\in I}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} tri_{ring} QRI_{ring}^{t} \\ &+ \sum_{r\in R}\sum_{u\in U}\sum_{n\in N}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} tru_{rung} QRU_{runqg}^{t} \\ &+ \sum_{r\in R}\sum_{v\in V}\sum_{m\in M}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} trv_{rvmg} QRV_{rvmqg}^{t} \\ &+ \sum_{u\in U}\sum_{e\in E}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} tue_{ueng} QUE_{ueng}^{t} + \sum_{r\in R}\sum_{w\in W}\sum_{m\in M} \\ &\times \sum_{n\in N}\sum_{g\in G}\sum_{t\in T} trw_{rwg}(QRW1_{rwng}^{t} + QRW2_{rwmg}^{t}) \\ &+ \sum_{u\in U}\sum_{i\in I}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} tui_{uing} QUI_{uing}^{t} \\ &+ \sum_{v\in V}\sum_{f\in F}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} tvf_{vfmg} QVF_{vfmg}^{t} \\ &+ \sum_{v\in V}\sum_{i\in I}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} tvi_{vimg} QVI_{vimg}^{t} \\ &+ \sum_{v\in V}\sum_{w\in W}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} tvw_{vwmg} QVW_{vwmg}^{t} \end{aligned} \tag{13}$$

Total Revenue (Rev): Sum of the selling price of recovered materials, components, and products at secondary markets as well as the sum of monetary saving resulted from recovered materials, components, and products.

$$\begin{aligned} = &\sum_{s\in S}\sum_{d\in D}\sum_{p\in P}\sum_{g\in G}\sum_{t\in T} sd_{dp} QSD_{sdpg}^{t} \\ &+ \sum_{r\in R}\sum_{e\in E}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} se_{en} QRE_{reng}^{t} \\ &+ \sum_{u\in U}\sum_{e\in E}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} se_{en} QUE_{ueng}^{t} \\ &+ \sum_{v\in V}\sum_{f\in F}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} sf_{fm} QVF_{vfmg}^{t} \\ &+ \sum_{r\in R}\sum_{i\in I}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} sr_{in} QRI_{ring}^{t} \\ &+ \sum_{u\in U}\sum_{i\in I}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} su_{in} QUI_{uing}^{t} \\ &+ \sum_{v\in V}\sum_{i\in I}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} svi_{im} QVI_{vimg}^{t} \end{aligned} \tag{14}$$

In summary, the total supply chain cost,

$$\begin{aligned} Z_{1}^{D} = &FC + PC + MC + HC + SC + IC + CC + RfC \\ &+ DiC + ReC + RyC + DsC + TrC - Rev \end{aligned} \tag{15}$$

Total Carbon Emissions (Z_2^D): Sum of carbon emissions due to various processes at the facilities and sum of carbon emissions due to logistic activities between the facilities.

$$\begin{aligned}
&= \sum_{i\in I}\sum_{o\in O}\sum_{p\in P}\sum_{t\in T} ei_{iop}QI_{iop}^{t} \\
&\quad + \sum_{a\in A}\sum_{i\in I}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} eai_{aimg}QAI_{aimg}^{t} \\
&\quad + \sum_{b\in B}\sum_{i\in I}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} ebi_{bing}QBI_{bing}^{t} \\
&\quad + \sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\sum_{g\in G}\sum_{t\in T} eij_{ijpg}QIJ_{ijpg}^{t} \\
&\quad + \sum_{j\in J}\sum_{c\in C}\sum_{p\in P}\sum_{g\in G}\sum_{t\in T} ejc_{jcpg}QJC_{jcpg}^{t} \\
&\quad + \sum_{k\in k}\sum_{s\in S}\sum_{p\in P}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} eks_{kspg}QKS_{kspqg}^{t} \\
&\quad + \sum_{s\in S}\sum_{d\in D}\sum_{p\in P}\sum_{g\in G}\sum_{t\in T} esd_{sdpg}QSD_{sdpg}^{t} \\
&\quad + \sum_{k\in K}\sum_{r\in R}\sum_{p\in P}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} ekr_{krpg}QKR_{krpqg}^{t} \\
&\quad + \sum_{r\in R}\sum_{e\in E}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} ere_{reng}QRE_{reng}^{t} \\
&\quad + \sum_{r\in R}\sum_{i\in I}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} eri_{ring}QRI_{ring}^{t} \\
&\quad + \sum_{r\in R}\sum_{u\in U}\sum_{n\in N}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} eru_{rung}QRU_{runqg}^{t} \\
&\quad + \sum_{r\in R}\sum_{v\in V}\sum_{m\in M}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} erv_{rvmg}QRV_{rvmqg}^{t} \\
&\quad + \sum_{u\in U}\sum_{e\in E}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} eue_{ueng}QUE_{ueng}^{t} \\
&\quad + \sum_{u\in U}\sum_{i\in I}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} eui_{uing}QUI_{uing}^{t} \\
&\quad + \sum_{v\in V}\sum_{f\in F}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} evf_{vfmg}QVF_{vfmg}^{t} \\
&\quad + \sum_{v\in V}\sum_{i\in I}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} evi_{vimg}QVI_{vimg}^{t} \\
&\quad + \sum_{r\in R}\sum_{w\in W}\sum_{m\in M}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} \\
&\quad erw_{rwg}(QRW1_{rwng}^{t} + QRW2_{rwmg}^{t}) \\
&\quad + \sum_{v\in V}\sum_{w\in W}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} evw_{vwmg}QVW_{vwmg}^{t} \qquad (16)
\end{aligned}$$

Using Equation (15) and Equation (16), the total supply cost is formulated under carbon pricing policy and carbon trading policy as given in Equation (17) and Equation (18) respectively. The notation δ is the penalty cost per unit carbon emission generation, π is the carbon trading (buying and selling) price, and C_{max} is the maximum allowable carbon emissions (carbon cap).

The goal of carbon pricing policy is to

$$Minimize Z_1^D + \delta Z_2^D \qquad (17)$$

and the goal of carbon trading policy is to

$$Minimize Z_1^D + \pi (Z_2^D - C_{max}) \qquad (18)$$

b: CONSTRAINTS

The constraints for flow balance, set covering, pricing incentives, facility capacity, transportation mode, and technology selection are as follows.

(a) Flow Balance Constraints in Forward Flow: Constraint (19) ensures that sum of the flow of raw materials from both raw materials suppliers and recycling centers entering at each manufacturing center equals to the sum of the quantity of finished products using environmentally friendly technology.

$$\begin{aligned}
&\sum_{a\in A}\sum_{g\in G} QAI_{aimg}^{t} + \sum_{v\in V}\sum_{g\in G} QVI_{vimg}^{t} \\
&\quad = \varphi_{pm}\sum_{o\in O} QI_{iop}^{t} \quad \forall i\in I,\ p\in P,\ m\in M,\ t\in T \qquad (19)
\end{aligned}$$

Constraint (20) ensures that sum of the flow of components from components suppliers, disassembly centers, and repair centers entering at each manufacturing center equals to the sum of the quantity of finished products using environmentally friendly technology.

$$\begin{aligned}
&\sum_{b\in B}\sum_{g\in G} QBI_{bing}^{t} + \sum_{r\in R}\sum_{g\in G} QRI_{ring}^{t} + \sum_{u\in U}\sum_{g\in G} QUI_{uing}^{t} \\
&\quad = \phi_{pn}\sum_{o\in O} QI_{iop}^{t} \quad \forall i\in I,\ p\in P,\ n\in N,\ t\in T \\
&\qquad (20)
\end{aligned}$$

Constraint (21) ensures that the sum of the finished products exiting from manufacturing center to distribution center equals the manufacturing quantity using production technology in each period.

$$\sum_{o\in O} QI_{iop}^{t} = \sum_{j\in J}\sum_{g\in G} QIJ_{ijpg}^{t} \quad \forall i\in I,\ p\in P,\ t\in T \quad (21)$$

Constraint (22) ensures that the sum of the entering flow of products to each distribution center is equal to the sum of the flow exiting from each distribution center and inventory of the current period.

$$\begin{aligned}
&QJ_{jp}^{t-1} + \sum_{i\in I}\sum_{g\in G} QIJ_{ijpg}^{t} = QJ_{jp}^{t} + \sum_{c\in C}\sum_{g\in G} QJC_{jcpg}^{t} \\
&\qquad \forall j\in J,\ p\in P,\ t\in T;\ where\ QJ_{jp}^{0} = 0 \qquad (22)
\end{aligned}$$

Constraint (23) ensures that the sum of the finished products exiting all distribution centers to each customer zone satisfies minimum level of customer's demand.

$$\begin{aligned}
&\sum_{j\in J}\sum_{g\in G} QJC_{jcpg}^{t} + \delta_{cp}^{t} \geq DC_{cp}^{t} \quad \forall c\in C,\ p\in P,\ t\in T \\
&\qquad (23)
\end{aligned}$$

(b) Flow Balance Constraints in Reverse Network: Constraints for flow balance in reverse network are as given in Constraints (24) to (38). Constraint (24) ensures that the entering flow of used products to each collection center from all customer zones equals to the sum of outgoing flow of used products to refurbishing centers and disassembly centers.

$$\sum_{c\in C} QCK^{t}_{ckpq} = \sum_{s\in S}\sum_{g\in G} QKS^{t}_{kspqg} + \sum_{r\in R}\sum_{g\in G} QKR^{t}_{krpqg} \quad \forall k\in K,\ p\in P,\ q\in Q, t\in T \tag{24}$$

Constraint (25) ensures that the proportion of incoming flow of returned products from customer zones equals to the exiting flow of recoverable products to refurbishing centers.

$$\sum_{c\in C} \eta_{pq} QCK^{t}_{ckpq} = \sum_{s\in S}\sum_{g\in G} QKS^{t}_{kspqg} \quad \forall k\in K,\ p\in P,\ q\in Q,\ t\in T \tag{25}$$

Constraint (26) ensures that the remaining proportion of returned products entering from all customer zones is equal to the exiting flow of returned products to disassembling centers (flow balance constraint).

$$\sum_{c\in C} \left(1-\eta_{pq}\right) QCK^{t}_{ckpq} = \sum_{r\in R}\sum_{g\in G} QKR^{t}_{krpqg} \quad \forall k\in K,\ p\in P,\ q\in Q,\ t\in T \tag{26}$$

Constraints (27) ensures that the entering flow of returned products at each refurbishing center from collection centers equals to the exiting flow of refurbished products to the secondary market.

$$\sum_{k\in K}\sum_{q\in Q}\sum_{g\in G} QKS^{t}_{kspqg} = \sum_{d\in D}\sum_{g\in G} QSD^{t}_{sdpg} \quad \forall s\in S,\ p\in P,\ t\in T \tag{27}$$

Constraints (28) satisfies minimum demand requirement of secondary products market.

$$\sum_{s\in S}\sum_{g\in G} QSD^{t}_{sdpg} \geq DD^{t}_{dp} \quad \forall d\in D,\ p\in P,\ t\in T \tag{28}$$

Constraint (29) ensures that returned products quantity with different quality level entering to each disassembly center is equal to the proportion of reusable parts (high quality i.e., as good as new) exiting to spare parts markets and manufacturing centers.

$$\sum_{k\in K}\sum_{p\in P}\sum_{q\in Q}\sum_{g\in G} \alpha_{pnq}\phi_{pn} QKR^{t}_{krpqg} = \sum_{e\in E}\sum_{g\in G} QRE^{t}_{reng} + \sum_{i\in I}\sum_{g\in G} QRI^{t}_{ring} \quad \forall r\in R,\ n\in N,\ t\in T \tag{29}$$

Constraint (30) ensures that returned products quantity with different quality level incoming to each disassembly center is equal to the proportion of repairable parts (under warranty) with quality level (high, medium, and low) outgoing to the repair centers for further processing.

$$\sum_{k\in K}\sum_{p\in P}\sum_{g\in G} \beta_{pnq}\phi_{pn} QKR^{t}_{krpq} = \sum_{u\in U}\sum_{g\in G} QRU^{t}_{runqg} \quad \forall r\in R,\ n\in N,\ q\in Q,\ t\in T \tag{30}$$

Constraint (31) ensures that returned products quantity with different quality level entering each disassembly center is equal to the proportion of recyclable materials with quality level exiting to the recycling centers.

$$\sum_{k\in K}\sum_{p\in P}\sum_{g\in G} \gamma_{pmq}\varphi_{pm} QKR^{t}_{krpqg} = \sum_{v\in V}\sum_{g\in G} QRV^{t}_{rvmqg} \quad \forall r\in R,\ m\in M,\ q\in Q,\ t\in T \tag{31}$$

Constraint (32) ensures that returned products quantity entering the disassembly center is equal to the sum of scrapped (non-repairable) parts exiting to the disposal centers.

$$\sum_{k\in K}\sum_{p\in P}\sum_{q\in Q}\sum_{g\in G} \left(1-\alpha_{pnq}-\beta_{pnq}\right)\phi_{pn} QKR^{t}_{krpqg} = \sum_{w\in W}\sum_{g\in G} QRW1^{t}_{rwng} \quad \forall r\in R,\ n\in N,\ t\in T \tag{32}$$

Constraint (33) ensures that returned products quantity entering the disassembly center is equal to the sum of scrapped (non-recyclable) materials flow exiting to the disposal centers.

$$\sum_{k\in K}\sum_{p\in P}\sum_{q\in Q}\sum_{g\in G} \left(1-\gamma_{pmq}\right)\varphi_{pm} QKR^{t}_{krpqg} = \sum_{w\in W}\sum_{g\in G} QRW2^{t}_{rwmg} \forall r\in R, m\in M, t\in T \tag{33}$$

Constraint (34) ensures that repairable parts of different quality levels entering repair centers are equal to the sum of repaired parts that fulfil the requirement of both spare parts market and manufacturing centers.

$$\sum_{r\in R}\sum_{q\in Q}\sum_{g\in G} QRU^{t}_{runqg} = \sum_{e\in E}\sum_{g\in G} QUE^{t}_{ueng} + \sum_{i\in I}\sum_{g\in G} QUI^{t}_{uing} \forall u\in U, n\in N, t\in T \tag{34}$$

Constraint (35) satisfies minimum demand requirements at spare parts market.

$$\sum_{r\in R}\sum_{g\in G} QRE^{t}_{reng} + \sum_{u\in U}\sum_{g\in G} QUE^{t}_{ueng} \geq DE^{t}_{en} \quad \forall e\in E,\ n\in N,\ t\in T \tag{35}$$

Constraints (36) and (37) state the entering and exiting flow of raw materials at each recycling center.

$$\sum_{r\in R}\sum_{q\in Q}\sum_{g\in G}(1-\theta_m)\,QRV^t_{rvmqg} = \sum_{w\in W}\sum_{g\in G}QVW^t_{vwmg} \quad \forall v\in V,\ m\in M,\ t\in T \tag{36}$$

$$\sum_{r\in R}\sum_{q\in Q}\sum_{g\in G}\theta_m QRV^t_{rvmqg} = \sum_{f\in F}\sum_{g\in G}QVF^t_{vfmg} + \sum_{i\in I}\sum_{g\in G}QVI^t_{vimg} \quad \forall v\in V,\ m\in M,\ t\in T \tag{37}$$

Constraint (38) is to satisfy demand of raw materials at materials market.

$$\sum_{v\in V}\sum_{g\in G}QVF^t_{vfmg} \geq DF^t_{fm} \quad \forall f\in F,\ m\in M,\ t\in T \tag{38}$$

(c) Set Covering Constraints: Logical constraints for locating collection centers proximity to customer zones are as given in Constraints (39) to (42). Constraint (39) ensures that all collection centers are assigned to at least one customer zone c where N_c is a set of eligible collection centers located within the distance limit that can service customer zone c.

$$\sum_{k\in N_c} ZK_k \geq 1, \quad \forall c\in C \tag{39}$$

Constraint (40) allows assignment only to those collection centers which are opened at location k.

$$ZK_k \geq Y^t_{ck} \quad \forall c\in C,\ k\in K,\ t\in T \tag{40}$$

Constraint (41) allows at most one collection center k to be assigned to each customer zone c in each period t. If $Y^t_{ck}=1$, it means that customer zone c is covered by collection center k within the distance limit, and 0 (not covered) otherwise.

$$\sum_{k\in K} Y^t_{ck} = 1, \quad \forall c\in C,\ t\in T \tag{41}$$

Constraint (42) forces $d_{ck'}$ to be equals to maximum distance where $d_{ck'}$ is the maximum distance between customer zone c and collection center k.

$$\sum_{k\in K} d_{ck}Y^t_{ck} = d_{ck'} \quad \forall c\in C,\ k'=1,2,3,..K \tag{42}$$

(d) Pricing Incentives Constraints: Financial incentives for collecting returned products based on three possible quality levels at customer zones are given in Constraints (43) – (46). Constraint (43) ensures that only one level of acquisition price is assigned to each customer zone in each period.

$$\sum_{l\in L} Y^t_{cpql} = 1, \quad \forall c\in C,\ p\in P,\ q\in Q,\ t\in T \tag{43}$$

Constraint (44) calculates the return rate of each returned product for customer zone in each period.

$$\sum_{l\in L} Y^t_{cpql}\,rr^t_{cpql} = ri^t_{cpq} \quad \forall c\in C,\ p\in P,\ q\in Q,\ t\in T \tag{44}$$

Constraint (45) calculates the quantity of each returned product with one definite quality level collected at collection centers from each customer zone in each period.

$$\sum_{k\in K} QCK^t_{ckpq} = ri^t_{cpq}RC^t_{cpq} \quad \forall c\in C,\ p\in P,\ q\in Q,\ t\in T \tag{45}$$

Constraint (46) ensures that used products from each customer zone are assigned to opened collection center in each period.

$$\sum_{p\in P}\sum_{q\in Q} QCK^t_{ckpq} \leq MY^t_{ck} \quad \forall c\in C,\ k\in K,\ t\in T \tag{46}$$

(e) Facilities Capacity Constraints: Logical constraints related to selecting suitable capacities of each potential facility are given by Constraints (47) – (56). Constraint (47) ensures that the volume of raw materials leaving a raw materials supplier does not exceed its capacity.

$$\sum_{i\in I}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} vm_m QAI^t_{aimg} \leq caa_a ZA_a \quad \forall a\in A \tag{47}$$

Constraint (48) ensures that the volume of components leaving a parts supplier does not exceed its capacity.

$$\sum_{i\in I}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} vn_n QBI^t_{bing} \leq cab_b ZB_b \quad \forall b\in B \tag{48}$$

Constraint (49) ensures that the total production time for all products at each manufacturing center with available technology over all periods does not exceed its maximum capacity.

$$\sum_{p\in P}\sum_{t\in T} ti_{po} QI^t_{iop} \leq cai^o_i ZI^o_i \quad \forall i\in I,\ o\in O \tag{49}$$

Constraint (50) ensures that the volume of finished products in previous period plus the volume of products entering the distribution center does not exceed its maximum capacity.

$$\sum_{p\in P}\sum_{t\in T} vp_p QJ^t_{jp} + \sum_{i\in I}\sum_{p\in P}\sum_{g\in G}\sum_{t\in T} vp_p QIJ^t_{ijpg} \leq caj_j ZJ_j \quad \forall j\in J \tag{50}$$

Constraints (51) to (56) belong to facility capacity of reverse SC network. Constraint (51) ensures that the volume of returned products entering from all customer zones to each collection center does not exceed its maximum capacity.

$$\sum_{c\in C}\sum_{p\in P}\sum_{q\in Q}\sum_{t\in T} vp_p QCK^t_{ckpq} \leq cak_k ZK_k \quad \forall k\in K \tag{51}$$

Constraint (52) ensures that the total processing time for recovering of returned products at each refurbishing center is not exceeding its maximum capacity.

$$\sum_{k\in K}\sum_{p\in P}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} ts_p QKS^t_{kspqg} \le cas_s ZS_s \quad \forall s\in S \quad (52)$$

Similarly, Constraints (53) to (55) ensure that the total processing time for recovering of components and raw materials at various facilities, namely, disassembly center, refurbishing center, repair center, and recycling center are not exceeding their respective maximum capacities.

$$\sum_{k\in K}\sum_{p\in P}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} tr_p QKR^t_{krpqg} \le car_r ZR_r \quad \forall r\in R \quad (53)$$

$$\sum_{r\in R}\sum_{n\in N}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} tu_n QRU^t_{runqg} \le cau_u ZU_u \quad \forall u\in U \quad (54)$$

$$\sum_{r\in R}\sum_{m\in M}\sum_{q\in Q}\sum_{g\in G}\sum_{t\in T} tv_m QRV^t_{rvmqg} \le cav_v ZV_v \quad \forall v\in V \quad (55)$$

Constraint (56) ensures that the total volume of scrapped parts and raw materials entering each disposal center are not exceeding its maximum capacity.

$$\sum_{r\in R}\sum_{n\in N}\sum_{g\in G} vn_n QRW1^t_{rwng} + \sum_{r\in R}\sum_{m\in M}\sum_{g\in G} vm_m QRW2^t_{rwmg} + \sum_{v\in V}\sum_{m\in M}\sum_{g\in G} vm_m QVW^t_{vwmg} \le caw_w ZW_w \quad \forall w\in W \quad (56)$$

(f) Transportation Mode Related Constraints: Constraints (57) to (74) ensure that there are no links between any two facilities with any transportation mode in any period without actual transportation links between them.

$$Y^t_{aig} \le \sum_{m\in M} QAI^t_{aimg} \quad \forall a\in A,\ i\in I,\ g\in G,\ t\in T \quad (57)$$

$$Y^t_{big} \le \sum_{n\in N} QBI^t_{bing} \quad \forall b\in B,\ i\in I,\ g\in G,\ t\in T \quad (58)$$

$$Y^t_{ijg} \le \sum_{p\in P} QIJ^t_{ijpg} \quad \forall i\in I,\ j\in J,\ g\in G,\ t\in T \quad (59)$$

$$Y^t_{jcg} \le \sum_{p\in P} QJC^t_{jcpg} \quad \forall j\in J,\ c\in C,\ g\in G,\ t\in T \quad (60)$$

$$Y^t_{ksg} \le \sum_{p\in P}\sum_{q\in Q} QKS^t_{kspqg} \quad \forall k\in K,\ s\in S,\ g\in G,\ t\in T \quad (61)$$

$$Y^t_{krg} \le \sum_{p\in P}\sum_{q\in Q} QKR^t_{krpqg} \quad \forall k\in K,\ r\in R,\ g\in G,\ t\in T \quad (62)$$

$$Y^t_{sdg} \le \sum_{p\in P} QSD^t_{sdpg} \quad \forall s\in S,\ d\in D,\ g\in G,\ t\in T \quad (63)$$

$$Y^t_{reg} \le \sum_{n\in N} QRE^t_{reng} \quad \forall r\in R,\ e\in E,\ g\in G,\ t\in T \quad (64)$$

$$Y^t_{rig} \le \sum_{n\in N} QRI^t_{ring} \quad \forall r\in R,\ i\in I,\ g\in G,\ t\in T \quad (65)$$

$$Y^t_{rug} \le \sum_{n\in N}\sum_{q\in Q} QRU^t_{runqg} \quad \forall r\in R,\ u\in U,\ g\in G,\ t\in T \quad (66)$$

$$Y^t_{rvg} \le \sum_{m\in M}\sum_{q\in Q} QRV^t_{rvmqg} \quad \forall r\in R,\ v\in V,\ g\in G,\ t\in T \quad (67)$$

$$Y^t_{rwg} \le \sum_{n\in N} QRW1^t_{rwng} \quad \forall r\in R,\ w\in W,\ g\in G,\ t\in T \quad (68)$$

$$Y^t_{rwg} \le \sum_{m\in M} QRW2^t_{rwmg} \quad \forall r\in R,\ w\in W,\ g\in G,\ t\in T \quad (69)$$

$$Y^t_{ueg} \le \sum_{n\in N} QUE^t_{ueng} \quad \forall u\in U,\ e\in E,\ g\in G,\ t\in T \quad (70)$$

$$Y^t_{uig} \le \sum_{n\in N} QUI^t_{uing} \quad \forall u\in U,\ i\in I,\ g\in G,\ t\in T \quad (71)$$

$$Y^t_{vfg} \le \sum_{m\in M} QVF^t_{vfmg} \quad \forall v\in V,\ f\in F,\ g\in G,\ t\in T \quad (72)$$

$$Y^t_{vig} \le \sum_{m\in M} QVI^t_{vimg} \quad \forall v\in V,\ i\in I,\ g\in G,\ t\in T \quad (73)$$

$$Y^t_{vwg} \le \sum_{m\in M} QVW^t_{vwmg} \quad \forall v\in V,\ w\in W,\ g\in G,\ t\in T \quad (74)$$

Constraints (75) to (92) ensure that there are no shipments between any two non-linked facilities using any transportation mode in any periods.

$$\sum_{m\in M} \omega m_m QAI^t_{aimg} \le W_g Y^t_{aig} \quad \forall a\in A,\ i\in I,\ g\in G,\ t\in T \quad (75)$$

$$\sum_{n\in N} \omega n_n QBI^t_{bing} \le W_g Y^t_{big} \quad \forall b\in B,\ i\in I,\ g\in G,\ t\in T \quad (76)$$

$$\sum_{p\in P} \omega p_p QIJ^t_{ijpg} \le W_g Y^t_{ijg} \quad \forall i\in I,\ j\in J,\ g\in G,\ t\in T \quad (77)$$

$$\sum_{p\in P} \omega p_p QJC^t_{jcpg} \le W_g Y^t_{jcg} \quad \forall j\in J,\ c\in C,\ g\in G,\ t\in T \quad (78)$$

$$\sum_{p\in P}\sum_{q\in Q} \omega p_p QKS^t_{kspqg} \le W_g Y^t_{ksg} \quad \forall k\in K,\ s\in S,\ g\in G,\ t\in T \quad (79)$$

$$\sum_{p\in P}\sum_{q\in Q} \omega p_p QKR^t_{krpqg} \le W_g Y^t_{krg} \quad \forall k\in K,\ r\in R,\ g\in G,\ t\in T \quad (80)$$

$$\sum_{p\in P} \omega p_p QSD^t_{sdpg} \le W_g Y^t_{sdg} \quad \forall s\in S,\ d\in D,\ g\in G,\ t\in T \quad (81)$$

$$\sum_{n\in N} \omega n_n QRE^t_{reng} \leq W_g Y^t_{reg} \quad \forall r\in R,\ e\in E,\ g\in G,\ t\in T \tag{82}$$

$$\sum_{n\in N} \omega n_n QRI^t_{ring} \leq W_g Y^t_{rig} \quad \forall r\in R,\ i\in I,\ g\in G,\ t\in T \tag{83}$$

$$\sum_{n\in N}\sum_{q\in Q} \omega n_n QRU^t_{runqg} \leq W_g Y^t_{rug} \quad \forall r\in R,\ u\in U,\ g\in G,\ t\in T \tag{84}$$

$$\sum_{m\in M}\sum_{q\in Q} \omega m_m QRV^t_{rvmqg} \leq W_g Y^t_{rvg} \quad \forall r\in R,\ v\in V,\ g\in G,\ t\in T \tag{85}$$

$$\sum_{n\in N} \omega n_n QRW1^t_{rwng} \leq W_g Y^t_{rwg} \quad \forall r\in R,\ w\in W,\ g\in G,\ t\in T \tag{86}$$

$$\sum_{m\in M} \omega m_m QRW2^t_{rwmg} \leq W_g Y^t_{rwg} \quad \forall r\in R,\ w\in W,\ g\in G,\ t\in T \tag{87}$$

$$\sum_{n\in N} \omega n_n QUE^t_{ueng} \leq W_g Y^t_{ueg} \quad \forall u\in U,\ e\in E,\ g\in G,\ t\in T \tag{88}$$

$$\sum_{n\in N} \omega n_n QUI^t_{uing} \leq W_g Y^t_{uig} \quad \forall u\in U,\ i\in I,\ g\in G,\ t\in T \tag{89}$$

$$\sum_{m\in M} \omega m_m QVF^t_{vfmg} \leq W_g Y^t_{vfg} \quad \forall v\in V,\ f\in F,\ g\in G,\ t\in T \tag{90}$$

$$\sum_{m\in M} \omega m_m QVI^t_{vimg} \leq W_g Y^t_{vig} \quad \forall v\in V,\ i\in I,\ g\in G,\ t\in T \tag{91}$$

$$\sum_{m\in M} \omega m_m QVW^t_{vwmg} \leq W_g Y^t_{vwg} \quad \forall v\in V,\ w\in W,\ g\in G,\ t\in T \tag{92}$$

(g) Technology Selection Constraint: Constraint (93) ensures that only one technology type is selected at each activated manufacturing center in a particular period.

$$\sum_{o\in O} ZI^o_i \leq 1 \quad \forall i\in I \tag{93}$$

Constraint (94) ensures that if there are no items to produce at a production plant, then there will be no manufacturing center with appropriate technology activated.

$$QI^t_{iop} \leq ZI^o_i \sum_{c\in C} D^t_{cp} \quad \forall i\in I,\ o\in O,\ p\in P,\ t\in T \tag{94}$$

IV. ROBUST MODEL FORMULATION

A polyhedral robust equivalent of the proposed deterministic MILP model is formulated to deal with various uncertainties in the CLSC network. We follow the robust optimization formulation presented in Bertsimas and Sim [28], [29] (refer to Appendix) to handle uncertainties associated with procurement cost of raw materials and components, product demand, and availability of used products at customer zones. First, the robust counterpart of the procurement cost is formulated to represent uncertainty in procurement cost. The uncertain procurement cost of raw materials cpm_{am} takes a value in an interval $\left[\overline{cpm}_{am}-\widehat{cpm}_{am}, \overline{cpm}_{am}+\widehat{cpm}_{am}\right]$ where $\overline{cpm}_{am}$ is the nominal value and $\widehat{cpm}_{am}$ is the maximum deviation from its nominal value. Whereas procurement cost of components cpn_{bn} takes a value in an interval $\left[\overline{cpn}_{bn}-\widehat{cpn}_{bn}, \overline{cpn}_{bn}+\widehat{cpn}_{bn}\right]$. The robust counterpart of the procurement cost is as shown below.

$$\begin{aligned} RC_{PC} = {} & \sum_{a\in A}\sum_{i\in I}\sum_{m\in M}\sum_{g\in G}\sum_{t\in T} \overline{cpm}_{am} QAI^t_{aimg} \\ & + \sum_{b\in B}\sum_{i\in I}\sum_{n\in N}\sum_{g\in G}\sum_{t\in T} \overline{cpn}_{bn} QBI^t_{bing} \\ & + \sum_{a\in A}\sum_{m\in M} p^1_{am} \\ & + \sum_{b\in B}\sum_{n\in N} p^2_{bn} + \lambda^0\Gamma^M + \lambda^1\Gamma^N \end{aligned} \tag{95}$$

$$\lambda^0 + p^1_{am} \geq \widehat{cpm}_{am} QAI^t_{aimg} \quad \forall a\in A,\ i\in I,\ m\in M,\ g\in G,\ t\in T \tag{96}$$

$$\lambda^1 + p^2_{bn} \geq \widehat{cpn}_{bn} QBI^t_{bing} \quad \forall b\in B,\ i\in I,\ n\in N,\ g\in G,\ t\in T \tag{97}$$

Uncertainty in product demand D^t_{cp} takes the range $\left[\bar{D}^t_{cp}-\hat{D}^t_{cp}, \bar{D}^t_{cp}+\hat{D}^t_{cp}\right]$, then the robust counterpart of demand constraint (23) is as follows.

$$\sum_{j\in J}\sum_{g\in G} QJC^t_{jcpg} + \delta^t_{cl} \geq \bar{D}^t_{cp} + \Gamma^D \hat{D}^t_{cp} \quad \forall c\in C,\ p\in P,\ t\in T \tag{98}$$

Similarly, uncertainty in availability of used products at customer zones R^t_{cpq} takes the range $\left[\bar{R}^t_{cpq}-\hat{R}^t_{cpq}, \bar{R}^t_{cpq}+\hat{R}^t_{cpq}\right]$ then the robust counterpart of it is as follows.

$$\begin{aligned} RC_{IC} = {} & \sum_{c\in C}\sum_{p\in P}\sum_{q\in Q}\sum_{l\in L}\sum_{t\in T} ins^t_{cpql} rr^t_{cpql} Y^t_{cpql} R\bar{C}^t_{cpq} \\ & + \sum_{c\in C}\sum_{p\in P}\sum_{q\in Q}\sum_{t\in T} p^3_{cpqt} + \lambda^2\Gamma^R \end{aligned} \tag{99}$$

$$\lambda^2 + p^3_{cpqt} \geq ins^t_{cpql} rr^t_{cpql} Y^t_{cpql} \hat{R}^t_{cpq} \quad \forall c\in C,\ p\in P,\ q\in Q,\ l\in L,\ t\in T \tag{100}$$

$$\sum_{k\in K} QCK^t_{ckpq} \geq ri^t_{cpq}\bar{R}^t_{cpq} + p^4_{cpqt} + \lambda^3_{cpqt}\Gamma^R \quad \forall c\in C,\ p\in P,\ q\in Q,\ t\in T \tag{101}$$

$$\lambda^3_{cpqt} + p^4_{cpqt} \geq ri^t_{cpq}\hat{R}^t_{cpq} \quad \forall c\in C,\ p\in P,\ q\in Q,\ t\in T \tag{102}$$

$$\sum_{k\in K} QCK^t_{ckpq} \leq ri^t_{cpq}\bar{R}^t_{cpq} + p^5_{cpqt} + \lambda^4_{cpqt}\Gamma^R$$

$$\forall c \in C,\ p \in P,\ q \in Q,\ t \in T \qquad (103)$$

$$\lambda^{4}_{cpqt} + p^{5}_{cpqt} \geq ri^{t}_{cpq}\hat{R}^{t}_{cpq}$$
$$\forall c \in C,\ p \in P,\ q \in Q,\ t \in T \qquad (104)$$

The robust model given by polyhedral uncertainty set is corresponding to the following robust MILP problem under carbon pricing and carbon trading policies. Equation (105) shows that the procurement cost (PC) and incentive cost (IC) in Equation (15) are replaced by RC_{PC} of Equation (95) and RC_{IC} of Equation (99) respectively. The total supply chain cost becomes

$$\begin{aligned} Z^{R}_{1} = {} & FC + RC_{PC} + MC + HC + SC + RC_{IC} \\ & + CC + RfC + DiC + ReC + +RyC \\ & + DsC + TrC - Rev \end{aligned} \qquad (105)$$

The total carbon emission becomes

$$Z^{R}_{2} = Z^{D}_{2} \qquad (106)$$

and the robust MILP model considering carbon pricing policy becomes

$$Minimize\ Z^{R}_{1} + \delta Z^{R}_{2} \qquad (107)$$

Subject to constraints (19) to (22), (24) to (44), (46) to (94), (96) to (98), and (100) to (104).

The robust MILP model considering carbon trading policy becomes

$$Minimize\ Z^{R}_{1} + \pi(Z^{R}_{2} - C_{max}) \qquad (108)$$

Subject to constraints (19) to (22), (24) to (44), (46) to (94), (96) to (98), and (100) to (104).

The proposed model is coded in GAMS/CPLEX version 24.7.4 using Branch and Bound algorithm on a laptop Intel core i7, 2.8 GHz, and 16 GB of RAM.

V. RESULTS AND DISCUSSION

Previous researchers such as Pishvaee *et al.* [7], Kisomi *et al.* [8], Keyvanshokooh *et al.* [34], Rad and Nahavandi [39], Zarbakhshnia *et al.* [40], Govindan *et al.* [41], Amin and Baki [42], and Ghomi-Avili *et al.* [43] have adopted numerical experiments and sensitivity analysis to test the validity of their proposed models. Similarly, this paper implemented this approach to study the impact of recovery options, carbon emission policies, and uncertainty on the SCND. This section presents the results obtained by the robust model while considering the two carbon regulatory policies. Section A discusses the data and test instances, and Section B provides sensitivity analysis.

A. DATA AND TEST INSTANCES

This study adopted reference datasets that combine information gathered from the literature together with realistic assumptions for CLSC network instances as in Paksoy *et al.* [16]. Two randomly generated test instances of different set sizes as given in Table 1 were used to validate the proposed model. The notation represents entities as in Fig. 1 and the description of each set is provided in Section III.

TABLE 1. Test instances' size.

Test Instance	*A*	*B*	*I*	*J*	*C*	*D*	*E*	*F*	*K*	*R*	*S*	*U*	*V*	*W*	*M*	*N*	*O*	*P*	*Q*	*L*	*G*	*T*
1	3	3	2	3	5	2	2	2	3	2	2	2	2	2	3	3	2	2	3	4	3	2
2	4	4	3	5	7	3	3	3	5	3	3	3	3	3	3	3	2	2	3	4	3	4

Specifically, Test Instance 1 consists of three raw materials suppliers ($A = 3$), three parts suppliers ($B = 3$), two manufacturing centers ($I = 2$), three distribution centers ($J = 3$), and five customer zones ($C = 5$) in the forward flow network. Manufacturers procure three types of raw materials ($M = 3$) and components ($N = 3$) in the required quantities. Some procurement is from the reverse network and the remaining are procured from potential suppliers in the forward network. Then, these items are processed into any of the two alternative products ($P = 2$). Finished products are then shipped to customer zones via distribution centers.

In the reverse flow network, the used products from customer zones are of three quality levels ($Q = 3$: high, medium, low). Each quality level can be assigned any of the four alternative financial incentive values ($L = 4$). Three collection centers ($K = 3$) can acquire used products and classify them according to their quality levels. High quality products are sent to any of the two refurbishing centers ($S = 2$). Recovered products are forwarded to any of the two secondary product markets ($D = 2$) to satisfy secondary products demand. The remaining products (medium and low-quality levels) are forwarded to any of the two disassembly centers ($R = 2$) for further operations. At the disassembly centers, products are disassembled into components and materials, which then will be cleaned, tested, and sorted for reuse, repair, recycling, and disposal operations. The recovered components are sent either to the manufacturing centers or to two alternatives spare parts markets ($E = 2$). Reusable components with medium quality level are repaired at any of the two repair centers ($U = 2$). Reusable components recovered after repair operation are sent to manufacturers or any of the two spare parts markets. The recovered low-quality materials are sent to any of the two recycling centers ($V = 2$). The recycled materials can serve either the manufacturers or the material markets ($F = 2$). The useless materials and components are transported to any of the two disposal centers ($W = 2$) for earth filling. Two time-periods ($T = 2$) of one year each is used in the planning horizon.

As for the logistic activities, this study considers three transportation modes ($G = 3$) having different capacity, transportation cost, and amount of emitted carbon as summarized in Table 2 [33]. It is assumed that light trucks emit less carbon and incur more cost per unit shipment. In contrast, heavy trucks incur less cost per unit shipment and emit more carbon. These assumptions are adopted from Zeballos *et al.* [17], Gao and Ryan [27], and Haddadsisakht and Ryan [33].

TABLE 2. Transportation modes and related data.

Transportation Mode	Capacity W_g (tons)	Cost ($ per unit transported)	CO_2 emission (kg per unit)
Light truck	5000	0.0240	0.012
Medium truck	10000	0.0225	0.018
Heavy duty truck	14000	0.0210	0.025

Table 3 summarizes input parameters used to represent various instances of the problem. The range of parameter values are based on references [13], [17], [33] and [38]. For each uncertain parameter, we varied the budget of uncertainty between zero and its maximum value.

B. SENSITIVITY ANALYSIS

Sensitivity analysis was performed to explore the model robustness to changes/uncertainty in parameters of the model. First, we investigate the effect of uncertainty in the selected operational parameters namely, manufacturing cost, shortage cost, collection cost, recovery cost, disassembly cost, repair cost, recycle cost, and transportation cost. To examine their effects, one cost at a time is multiplied by some constant coefficients refers to as change coefficients. When the change coefficient equals to one, the total cost and carbon emission reach their nominal values.

Second, we analyze the effect of changes in carbon tax for carbon pricing policy, and simultaneous changes in carbon cap and carbon market price for carbon trading policy. Finally, we investigate the sensitivity of the objective function to variation in conservatism degrees (budget of uncertainty) for raw material procurement cost (Γ_m), component procurement cost (Γ_n) product demand (Γ_d), and returned product (Γ_r). Table 4 presents numerical results for the two test instances in terms of carbon emissions, total SC cost, and CPU time for the carbon pricing model and the carbon trading model. Note that the total cost and carbon emissions at conservatism degrees, $\Gamma_m = \Gamma_n = \Gamma_d = \Gamma_r = 0$ are for the deterministic models. Further discussion on budget of uncertainty is provided in the following subsection (V).

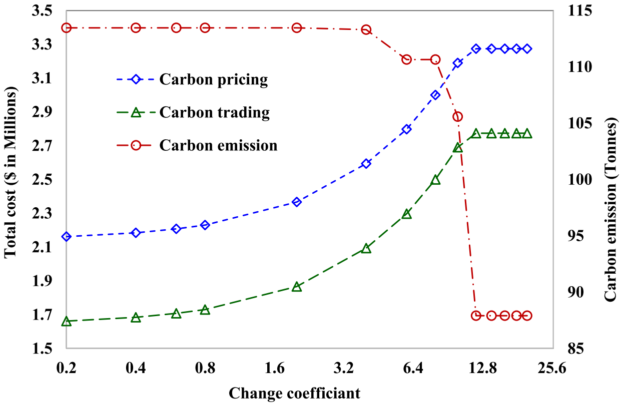


FIGURE 2. Sensitivity of total cost and carbon emission to variation of manufacturing cost under carbon pricing and carbon trading policies.

1) MANUFACTURING COST

The sensitivity of manufacturing cost on the total supply chain cost and carbon emission is demonstrated in Fig. 2. We vary the change coefficient for manufacturing cost between 0.2 to 25.6. The results show that as the manufacturing cost increases, the total supply chain cost also steadily increases until it stabilizes after 12.8 for both carbon pricing and carbon trading polices. Both carbon policies generated similar amount of carbon. This could be attributed to our assumption that the carbon tax rate and the carbon market price are the same. The carbon emission remains consistently high from the beginning until the change coefficient exceeds 3.2 where it shifted down suddenly and becomes constant after 12.8. This result suggests that facilities within the supply chain network continue to fulfill demand until a limit is met and resulted in a sudden reduction in carbon emission. The gap between the total costs for the two carbon policies indicates the relative cost saving that can be gained by adopting the carbon trading policy.

2) SHORTAGE COST

We vary the change coefficient for shortage cost between 0 to 2 and evaluate its effect on the total cost and carbon emission as illustrated in Fig. 3. The result indicates that, initially the total supply chain cost is more sensitive to changes in shortage cost compared to the carbon emission. However, beyond 0.4, the carbon emission drastically increases, and the supply chain total cost starts to stabilize. This suggests that higher shortage cost would lead to more carbon emission and higher total cost. The carbon emission curve is the same for both carbon policies due to the same reason as in Fig.2. A constant gap between the total cost under carbon pricing and carbon trading policies could be attributed to the carbon emission limit (carbon cap) imposed in the carbon trading policy. The sudden turning point for total cost and carbon

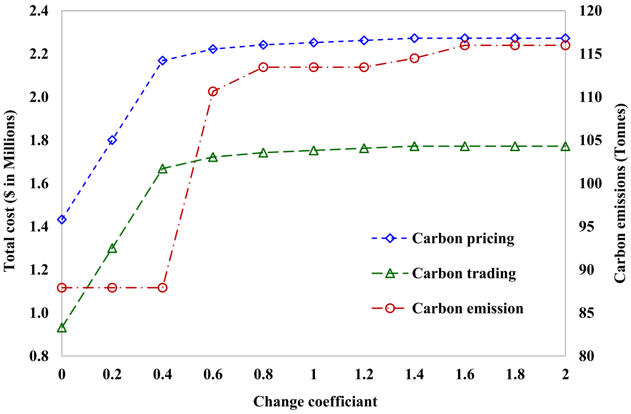


FIGURE 3. Sensitivity of total cost and carbon emission to variation of shortage cost under carbon pricing and carbon trading policies.

TABLE 3. **Parameter values used in the test instances.**

Parameter	Values
D_{cl}^{t}	Uniform (200,400)
R_{clq}^{t}	Uniform (150,350)
DD_{dp}^{t}	Uniform (20,40)
DE_{en}^{t}	Uniform (30,60)
DF_{fm}^{t}	Uniform (30,60)
vm_m	Uniform (1,5)
vn_n	Uniform (1,5)
vp_p	Uniform (12,16)
ts_p	Uniform (3,5)
tr_p	Uniform (4,6)
tu_n	Uniform (1,3)
ωm_m	Uniform (0.2,0.5)
ωn_n	Uniform (0.5,1)
ωp_p	Uniform (4,6)
φ_{pm}	Uniform (3,4)
ϕ_{pn}	Uniform (3,4)
θ_m	80%
fa_a	Uniform (1200,1500)
fb_b	Uniform (1200,1500)
fp_{po}^{t}	Uniform (10000,20000)
fd_{d}^{t}	Uniform (4000,6000)
fk_{k}^{t}	Uniform (4000,6000)
fr_r	Uniform (1000,1200)
fs_s	Uniform (1500,1800)
fu_u	Uniform (2000,2400)
caa_a	Uniform (10000,15000)
cab_b	Uniform (10000,15000)
cap_{po}^{t}	Uniform (50000,60000)
cad_{d}^{t}	Uniform (6000,7000)
cak_{k}^{t}	Uniform (2000,3000)
car_r	Uniform (4000,5000)
cas_s	Uniform (1000,2000)
cau_u	Uniform (10000,20000)
cav_v	Uniform (40000, 60000)
caw_w	Uniform (50000, 60000)
cpm_{am}	Uniform (1,2)
cpn_{bn}	Uniform (3,4)
cmp_{phl}^{t}	Uniform (21, 24)
chp_{pn}^{t}	Uniform (2, 4)
chq_{ql}^{t}	Uniform (2, 5)

Parameter	Values
ccc_{kl}^{t}	Uniform (6, 9)
crc_{rn}^{t}	Uniform (7, 9)
cdw_{wn}^{t}	Uniform (2, 4)
sd_{dp}	Uniform (18,20)
se_{en}	Uniform (13,14)
sf_{fm}	Uniform (8,9)
sr_{in}	Uniform (2,3)
su_{in}	Uniform (2,3)
sv_{im}	Uniform (1,2)

Parameter	Values at quality levels		
	q_1	q_2	q_3
ck_{kpq}	3	2	1
cs_{spq}	5	4	3
cr_{rpq}	3	2	1
cu_{unq}	4	3	2
cv_{vmq}	4	3	2
η_{pq}	0.7	0.4	0.1
α_{pnq}	0.9	0.6	0.3
β_{pnq}	0.9	0.7	0.3
γ_{pmq}	0.9	0.7	0.3

For quality level, q_1

Parameter	Values at financial incentive levels			
	l_1	l_2	l_3	l_4
rr_{cpql}^{t}	0	0.8	0.9	1
ins_{cpql}^{t}	0	15	16	17

For quality level, q_2

Parameter	Values at financial incentive levels			
	l_1	l_2	l_3	l_4
rr_{cpql}^{t}	0	0.8	0.9	1
ins_{cpql}^{t}	0	10	11	12

For quality level, q_3

Parameter	Values at financial incentive levels			
	l_1	l_2	l_3	l_4
rr_{cpql}^{t}	0	0.8	0.9	1
ins_{cpql}^{t}	0	5	6	7

emission between change coefficient 0.3 to 0.6 requires further study to clarify this phenomenon.

3) OPERATIONAL COSTS OF REVERSE NETWORK

This study investigated the sensitivity of total cost with respect to change coefficient for operational costs in reverse network covering recovery, disassembly, repair, and recycle activities. As shown in Fig. 4 (a) and Fig. 4 (b), we vary the operational cost change coefficient between 0.1 and 25.6. These figures show that both carbon pricing and carbon trading policies demonstrate the same trend. When the operational costs are low, there is little effect to the total supply chain cost. Beyond the change coefficient 1.6, the repair and recycling costs contribute significantly to the total supply chain cost. However, the contribution of recovery and disassembly costs remain low. The above finding could be attributed to our assumptions that unit cost for repairing and recycling are higher than the unit cost for disassembly and recovery operations. Moreover, the assumed fraction of repairable components and recycling materials are also higher. The above results confirm the findings as reported by Paksoy *et al.* [16] and Zeballos *et al.* [17]. Overall, the carbon trading policy incurs relatively lower total cost compared to the carbon pricing policy. This finding suggests the buying and selling feature in carbon trading policy leads to a lower total supply chain cost.

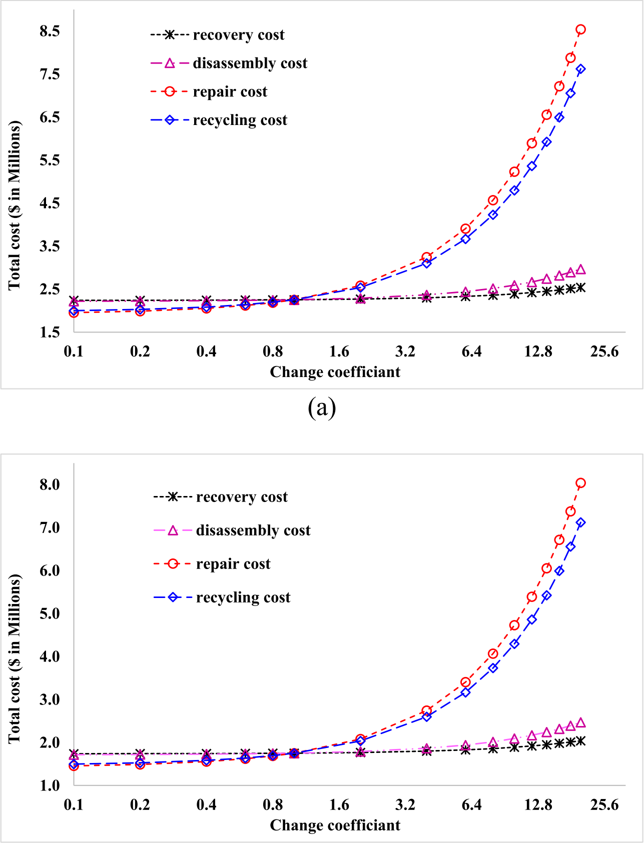
4) EMISSION RATES AND CARBON MARKETS

The sensitivity of total cost to variation of carbon emission rate for three transportation modes is examined in this section. The transportation modes are light truck (Mode 1), medium-size truck (Mode 2) and heavy-duty truck (Mode 3) as given in Table 2.

Carbon Emission Rate and Transportation Modes: Fig. 5(a) and Fig. 5(b) show the sensitivity of total cost to variation of carbon emission rate for three transportation modes.

TABLE 4. Numerical results and model runtime at various conservatism degrees.

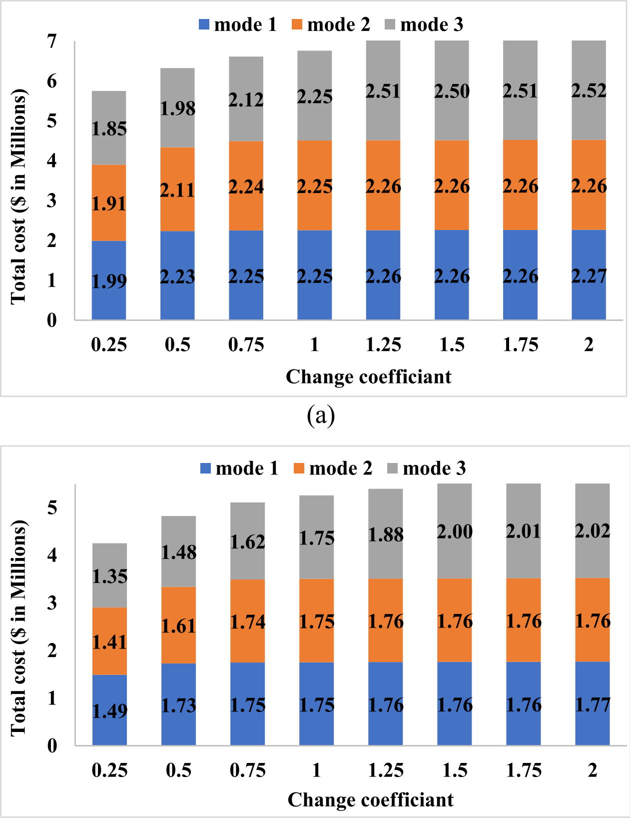
Test instance	Budget uncertainty sets of various uncertain parameters				Carbon pricing model			Carbon trading model		
	Γ_m	Γ_n	Γ_d	Γ_r	Carbon emissions (kgs)	Total SC cost ($)	CPU Time (sec)	Carbon emissions (kgs)	Total SC cost ($)	CPU Time (sec)
1	0	0	0	0	14864.9	325020.1	24	14864.9	275020.1	18
	3	3	4	12	23683.4	990088.5	35	23683.4	940088.5	31
	3	3	8	24	23723.5	2057995.2	37	23723.5	2007995.2	34
	6	6	12	36	25296.2	3228748.3	49	25296.2	3250281.8	42
	6	6	16	48	25993.8	4353549.2	60	25993.8	4276641.7	53
	9	9	20	60	26043.9	5504853.7	73	26043.9	5454853.7	67
2	0	0	0	0	21267.0	831150.8	13	21267.0	781150.8	11
	3	3	16	48	28076.2	7693954.3	270	28076.2	7580773.7	255
	6	6	32	96	28507.1	14768747.1	435	28507.1	14625587.3	348
	9	9	48	144	29000.3	21864725.8	630	29000.3	21814725.8	450
	12	12	64	192	29287.9	38614195.7	754	29287.9	38604725.8	633
	15	15	80	240	30268.4	48122607.0	760	30268.4	48110112.5	667



(a)

(b)

FIGURE 4. Sensitivity of total cost to variation of operational cost in reverse network under (a) carbon pricing policy and (b) carbon trading policy.



(a)

(b)

FIGURE 5. Sensitivity of total cost to variation of carbon emission rate by transportation mode under (a) carbon pricing policy and (b) carbon trading policy.

We varied the change coefficient between 0.25 and 2.0. The result indicates that heavy duty truck (Mode 3) contributes relatively more to total supply chain cost compared to the other modes when the change coefficient exceeds 1.0, despite its cost per unit transported being the lowest. As expected, heavy duty truck emits more carbon per unit compared to the light truck and medium-size truck. The same trend is observed for both carbon policies. However, operating under carbon pricing policy results in relatively more carbon emission compared to carbon trading policy for all modes of transportation.

Carbon Tax Rate (Carbon Pricing): We studied the effect of changes in carbon tax rate between \$0 - \$50 per kg to the total cost and carbon emission. Fig. 6 reveals that the total cost increases linearly as the carbon tax rate increases.

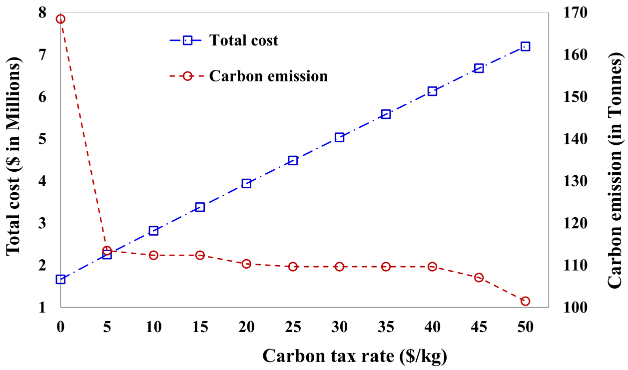


FIGURE 6. **Sensitivity of total cost and carbon emission to variation of carbon tax rate (carbon pricing policy).**

Conversely, carbon emission is extremely sensitive to an increase in carbon tax rate especially at initial stage. Further increase in carbon tax rate beyond \$5/kg results in only slight reduction in carbon emission. This result suggests that carbon tax is an effective mechanism to curb carbon emission in the supply chain. This finding is in line with other researchers [4], [13], [31].

Carbon Market Trading Price: The sensitivity of total cost to variation of carbon cap for various carbon market prices (\$5, \$10, \$15, and \$20) is shown in Fig. 7. We vary the carbon cap between 0 to 200 tons. The graph shows that as the carbon cap increases, the total supply chain cost decreases significantly especially during high carbon trading price. Higher carbon cap is relatively more effective for cost control during high carbon price compared to during low carbon price. This result suggests that at low capping, supply chain operation is very costly especially during high carbon price. Conversely, the total cost is significantly lower at high carbon caping particularly during high carbon price. This implies that under carbon trading policy, it is costly for industry to operate with low carbon capping especially when the carbon trading price is high. Industry need to spend more money to run supply chain activities. Meanwhile, it is relatively more profitable to operate at high carbon cap especially when the carbon trading price is high.

Fig. 8 depicts the sensitivity of carbon emission to variation in carbon trading price. We vary the carbon price between \$5 - \$20 per unit. The result shows that an increase in carbon market trading price leads to a decrease in carbon emission. This suggests that firms are motivated to sell carbon quota rather than consumed all the allocate quotas especially when the carbon trading price is high. This may also be achieved through improving productivity of the supply chain operations. The above finding supports argument in Palak *et al.* [11] and Fahimnia [13] who noted that carbon trading policy is more attractive and favorable to many countries. Overall, our study reveals that carbon trading policy generates less carbon emission and incurs lower total cost compared to carbon pricing policy. This finding is in line with Choudhary *et al.* [30] and Xu *et al.* [31].

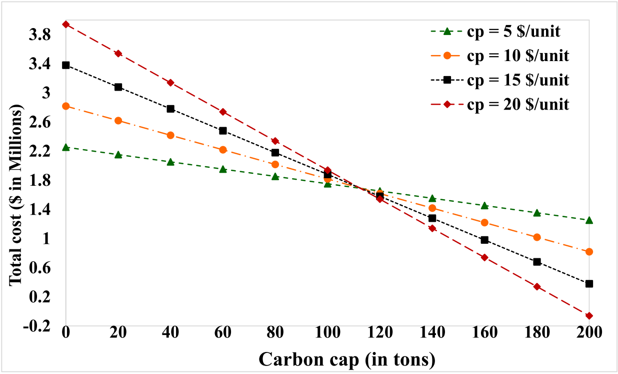


FIGURE 7. **Sensitivity of total cost to variation of carbon cap.**

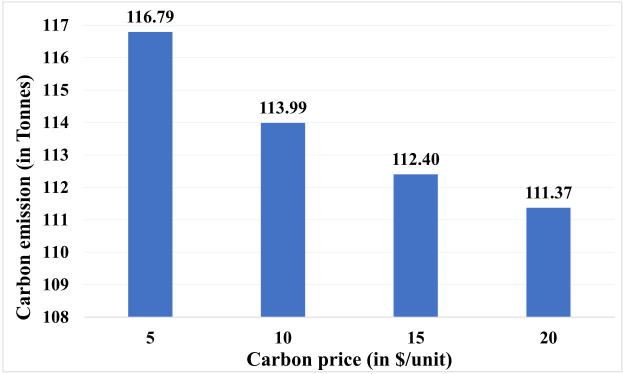


FIGURE 8. **Sensitivity of carbon emission to variation of carbon trading price.**

5) BUDGET OF UNCERTAINTY

The effect of uncertainty was investigated by altering values of conservatism degree of procurement costs (raw materials and components), product demand, and returned products. The variability is taken as 10%, 20%, 30%, and 40% of the nominal values which define the radius of the polyhedral uncertainty set. We vary the respective conservatism degree from zero (nominal) to its maximum value (worst case). As shown in Fig. 9 and Fig. 10 the maximum conservatism degree for uncertain procurement price of materials and components is $|A| \times |M| = 9$ and $|B| \times |N| = 9$, respectively. Similarly, for uncertain demand $|C| \times |P| \times |T| = 20$ and returned products $|C| \times |P| \times |Q| \times |T| = 60$ as shown in Fig. 11 and 12, respectively.

As noted earlier, Table 4 summarizes the performance of the models on two test instances. Generally, as conservatism degree increases, the total cost and carbon emission are

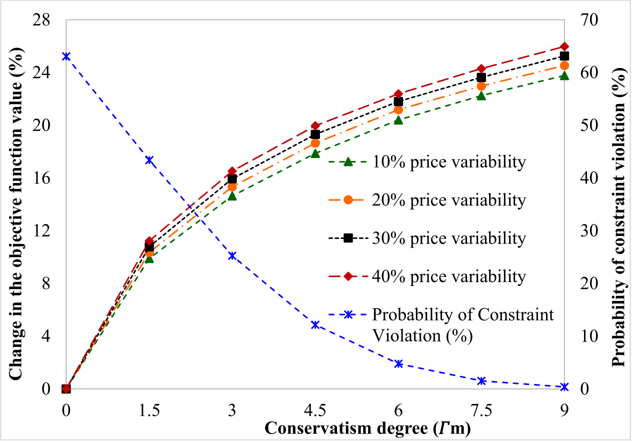


FIGURE 9. Effect of price variability of raw materials as a function of Γ_m and prob. constraint violation on the objective function value.

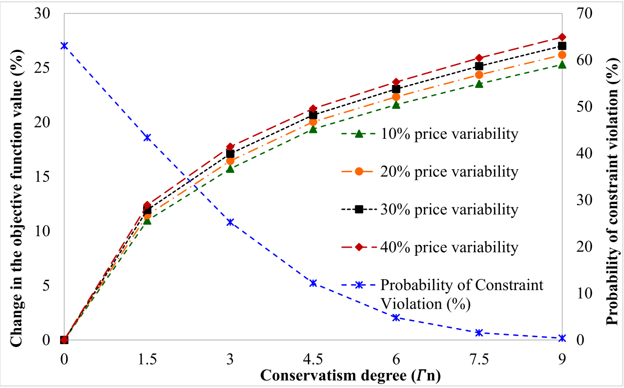


FIGURE 10. Effect of price variability of components as a function of Γ_n and prob. constraint violation on the objective function value.

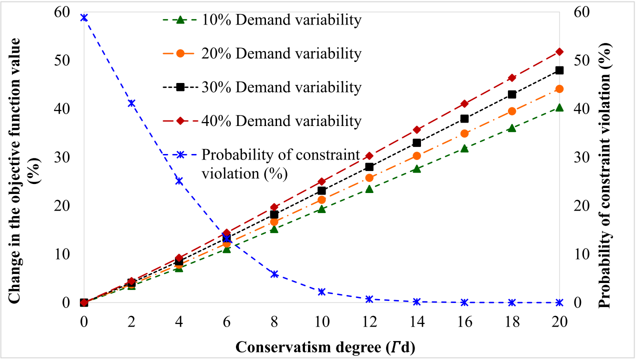


FIGURE 11. Effect of demand variability as a function of Γ_d and prob. constraint violation on the objective function value.

getting higher to hedge against the uncertainties. Fig. 9 to Fig. 12 show how different value of conservatism degrees effect on the total cost under carbon trading policy. The total costs are represented in terms of normalized deviation (% change) instead of their optimal values. The relative increase in optimal total cost is calculated as $Z_r - Z_n)/Z_n$ where Z_n and Z_r represent the objective function of the nominal model and the robust model, respectively.

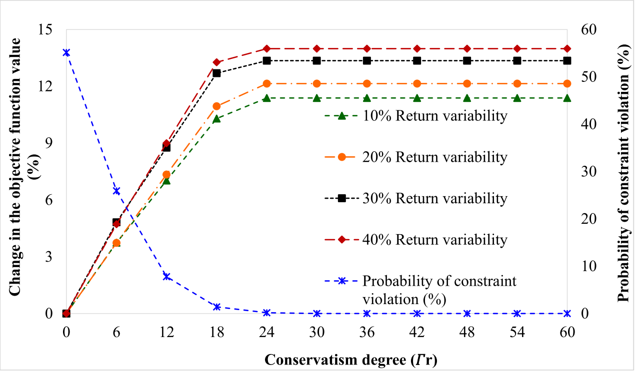


FIGURE 12. Effect of return variability as a function of Γ_r and prob. constraint violation on the objective function value.

Specifically, Fig. 9 and Fig. 10 illustrate that increase in conservatism degree for raw material price (Γ_m) and component price (Γ_n) lead to non-linear increases in the total cost (objective function value). Meanwhile, Fig. 11 shows that the total cost increases linearly to the increase in conservatism degree for product demand (Γ_d). Fig. 12 reveals that when the conservatism degree for return products (Γ_r) reaches 24 point, the objective function reaches its maximum value. Further increase in Γ_r has no effect. Overall, this study suggests that higher uncertaintywould lead to increase in the network complexity. More efforts are needed to balance among variabilities in procurement prices, product demand and product returns. Firms need to bear additional costs to accommodate such dynamic scenarios.Besides, Table 4 also shows that as the budget of uncertainty increases, so does the CPU processing time.

One approach for choosing suitable degree of conservatism is by referring to the probability of constraint violation curve. The probability of constraint violation is computed according to Equation (A7), as given in the Appendix with the assumption that all uncertain parameters follow symmetric distribution. This method can guide decision makers to vary the degree of conservatism where the probability of constraint violation remains less than a specified value for each uncertain parameter. Fig. 9 to Fig. 12 show that the probabilities of constraint violations are decreasing rapidly as the conservatism degrees increase especially prior to 7.5 point.

VI. CONCLUSION

This paper proposes a deterministic MILP model for a generic CLSC network problem under multiple recovery options and carbon policies consideration. Carbon pricing and carbon trading policies are integrated into the proposed model where the strategic and tactical decisions were simultaneously investigated. Customer's willingness to return used product depends on both acquisition price and nearness to the collection center. A different acquisition price is offered based on the quality level of the returned products. Selection of transportation mode is also considered in the model. Further, a robust optimization model is proposed to deal with uncertainty in procurement cost, demand, and returned products.

Sensitivity analyses reveal that parameters of the forward SC network, the manufacturing cost and shortage cost moderately influence the total cost and carbon emission under both carbon policies. As the manufacturing cost increases, the total supply chain cost also steadily increases until it stabilizes. The supply chain activities continue to fulfill demand until a penalty limit is met. The study also reveals that higher shortage cost would lead to more carbon emission and higher total cost.

The recovery and disassembly activities of the reverse flow supply chain network have small contribution to the total cost. Whereas repair and recycling activities significantly contribute the total cost and carbon emission under both carbon policies. Regarding the transportation mode, heavy duty truck contributes relatively more to the total supply chain cost despite its lowest cost per unit transported. Carbon pricing policy generates relatively more carbon compared to carbon trading policy for all modes of transportation. Whereas in carbon trading policy, higher carbon cap is relatively more effective for cost control during high carbon price compared to during low carbon price. The flexibility feature in the carbon trading policy (selling and buying) positively promotes firms to generate less carbon than their allowable limits. Overall, the study reveals that the carbon trading policy incurs relatively lower total cost compared to the carbon pricing policy.

The robust model incurs higher total cost than the deterministic model due to larger solution space to accommodate uncertainties in the supply chain network. However, this study provide evidence that it is possible to achieve an optimal CLSC network with reduced carbon emission at a moderate total supply chain cost. The proposed model could be used to guide firms to choose an appropriate budget of uncertainty toward achieving a robust supply chain network. A further investigation is needed for an in-depth analysis on sudden turning points with respect to changes in some design parameters. This study could also be extended by incorporating social constraints and operational disruptions.

APPENDIX

ROBUST OPTIMIZATION

A brief overview of robust counterpart optimization adopted from Bertsimas and Sim [28], [29] is presented here. Let us consider a linear program (LP) in Equation (A1):

$$\textit{Minimize } cx;\ \textit{subject to}:\ Ax \le b,\ x \ge 0 \qquad \text{(A1)}$$

In the above formulation, let us assume only elements of matrix A are subjected to uncertainty i.e., consider uncertainty in a specific row i of A and a set of coefficients in row i (J_i) is exposed to uncertainty. Each element $a_{ij}, j \in J_i$ is formulated as a symmetric and bounded independent random variable $\tilde{a}_{ij}, j \in J_i$ using value in an interval $\left[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}\right]$ where a_{ij} is the nominal value and $\hat{a}_{ij}$ is the maximum deviation from this nominal value. Then, the LP in Equation (A1) becomes:

$$\textit{Minimize } cx;\ \textit{subject to}:\ \max_{\forall \tilde{a}_{ij} \in J_i} \left(\sum\nolimits_j \tilde{a}_{ij} x_j\right) \le b_j \qquad \forall i,\ x \ge 0 \qquad \text{(A2)}$$

A scaled deviation $z_{ij} = \left(\tilde{a}_{ij} - a_{ij}\right) / \hat{a}_{ij}$ is associated with uncertain data $\tilde{a}_{ij}$, that obeys an unknown but bounded symmetric distribution that always belongs to the interval [-1, 1].

The budget of uncertainty $\Gamma_i \in [0, |J_i|]$ is a maximum number of parameters that can deviate from their nominal values for each constraint i. Further, the average scaled deviation of uncertain parameters for constraint i is bounded as $\sum_{j \in J_i} |z_{ij}| \le \Gamma_i, \forall i$. It plays an important role in adjusting the degree of conservatism against the robustness. If $\Gamma_i = 0$, no protection against uncertainty, if $\Gamma_i = J_i$, a complete protection of i^{th} constraint against the worst-case realization of uncertain parameters. If $\Gamma_i \in (0, |J_i|)$, the decision makers consider a tradeoff between conservatism and cost of the solution against the level of protection as well as constraint violation [34]. Let the set J_i is defined as $J_i = \left\{\tilde{a}_{ij} | \tilde{a}_{ij} = a_{ij} + \hat{a}_{ij} z_{ij}, \forall i, j, z \in Z\right\}$ where $Z = \left\{z | \sum_{j \in 1}^{n} z_{ij} \le \Gamma_i, |z_{ij}| \le 1, \forall i\right\}$. Restating each constraint i as $\sum_j \tilde{a}_{ij} x_j = \sum_j \left(a_{ij} + \hat{a}_{ij} z_{ij}\right) x_j = \sum_j a_{ij} x_j + \sum_j \hat{a}_{ij} z_{ij} x_j$, the LP can be reformulated as shown in Equation (A3).

$$\textit{Minimize } cx;\ \textit{subject to}: \sum_j a_{ij} x_j + \max_{z_{ij} \in Z_i} \left(\sum_j \hat{a}_{ij} z_{ij} x_j\right) \le b_j \qquad \forall i,\ x \ge 0 \qquad \text{(A3)}$$

The lower level problem $\max_{z_{ij} \in Z_i} \left(\sum_j \hat{a}_{ij} z_{ij} x_j\right)$ for a given vector x^* is corresponding to

$$\textit{maximize} \left(\sum_j \hat{a}_{ij} z_{ij} x_j^*\right);\ \textit{subject to}: \sum_j z_{ij} \le \Gamma_i \qquad \forall i,\ 0 \le z_{ij} \le 1 \quad \forall j \in J_i \qquad \text{(A4)}$$

By introducing the dual variables λ_i and μ_{ij}, the dual of LP in Equation (A1) is:

$$Min \Gamma_i \lambda_i + \sum_{j \in J_i} \mu_{ij},\ s.t. \lambda_i + \mu_{ij} \ge \hat{a}_{ij} x_j^* \qquad \forall i,\ j \in J_i,\ \lambda_i \ge 0,\ \mu_{ij} \ge 0 \quad \forall i,\ j \in J_i \qquad \text{(A5)}$$

The dual in Equation (A5) is applied to LP in Equation (A2) to obtain robust counterpart of LP in Equation (A1):

$$\textit{Minimize } cx;\ \textit{subject to}:\ a_i x + \Gamma_i \lambda_i + \sum_{j \in J_i} \mu_{ij} \le b_i \forall i,\ \lambda_i + \mu_{ij} \ge \hat{a}_{ij} x_j^* \qquad \forall i,\ j \in J_i,\ \lambda_i \ge 0,\ \mu_{ij} \ge 0 \quad \forall i,\ j \in J_i \qquad \text{(A6)}$$

For the robust counterpart model in Equation (A6), this methodology provides an effective fashion to determine probability bounds for the constraint violation. Let x_{ij}^* be the solution of the robust optimization model. The probability that the i^{th} constraint is violated can be approximated by

$$Pr\left(\sum_{j} \hat{a}_{ij} z_{ij} x_j^* > b_j\right) \le 1 - \Phi\left(\frac{\Gamma_i - 1}{\sqrt{|J_i|}}\right), \qquad \text{(A7)}$$

where $\Phi(\theta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\theta} exp\left(-\frac{y^2}{2}\right) dy$

Equation (A7) is the cumulative distribution function of a standard normal, for all i. An interesting feature of this bound is to provide a way of assigning appropriate budget parameters to the different constraints, considering only a probability level that can be intuitive for an expert.

REFERENCES

[1] A. Yamzon, V. Ventura, P. Guico, and C. Sy, "Optimal planning of incentive-based quality in closed-loop supply chains," *Clean Technol. Environ. Policy*, vol. 18, no. 5, pp. 1415–1431, Feb. 2016.

[2] P. Dutta, D. Das, F. Schultmann, and M. Fröhling, "Design and planning of a closed-loop supply chain with three way recovery and buy-back offer," *J. Cleaner Prod.*, vol. 135, pp. 604–619, Nov. 2016.

[3] M. Jeihoonian, M. K. Zanjani, and M. Gendreau, "Accelerating benders decomposition for closed-loop supply chain network design: Case of used durable products with different quality levels," *Eur. J. Oper. Res.*, vol. 251, no. 3, pp. 830–845, Jun. 2016.

[4] A. Zakeri, F. Dehghanian, B. Fahimnia, and J. Sarkis, "Carbon pricing versus emissions trading: A supply chain planning perspective," *Int. J. Prod. Econ.*, vol. 164, pp. 197–205, Jun. 2015.

[5] Y. Jiang, B. Li, X. Qu, and Y. Cheng, "A green vendor-managed inventory analysis in supply chains under carbon emissions trading mechanism," *Clean Technol. Environ. Policy*, vol. 18, no. 5, pp. 1369–1380, Jun. 2016.

[6] Q. Bai and M. Chen, "The distributionally robust newsvendor problem with dual sourcing under carbon tax and cap- and-trade regulations," *Comput. Ind. Eng.*, vol. 98, pp. 260–274, Aug. 2016.

[7] M. S. Pishvaee, M. Rabbani, and S. A. Torabi, "A robust optimization approach to closed-loop supply chain network design under uncertainty," *Appl. Math. Model.*, vol. 35, no. 2, pp. 637–649, Feb. 2011.

[8] M. S. Kisomi, M. Solimanpur, and A. Doniavi, "An integrated supply chain configuration model and procurement management under uncertainty: A set-based robust optimization methodology," *Appl. Math. Model.*, vol. 40, nos. 17–18, pp. 7928–7947, Sep. 2016.

[9] F. Mohammed, A. Hassan, and S. Z. Selim, "Robust optimization for closed-loop supply chain network design considering carbon policies under uncertainty," *Int. J. Ind. Eng. Theory Appl. Pract.*, vol. 25, no. 4, pp. 526–558, Oct. 2018.

[10] S. Benjaafar, Y. Li, and M. Daskin, "Carbon footprint and the management of supply chains: Insights from simple models," *IEEE Trans. Autom. Sci. Eng.*, vol. 10, no. 1, pp. 99–116, Jan. 2013.

[11] G. Palak, S. D. Eksioglu, and J. Geunes, "Analyzing the impacts of carbon regulatory mechanisms on supplier and mode selection decisions: An application to a biofuel supply chain," *Int. J. Prod. Econ.*, vol. 154, pp. 198–216, Aug. 2014.

[12] A. Diabat, T. Abdallah, A. Al-Refaie, D. Svetinovic, and K. Govindan, "Strategic closed-loop facility location problem with carbon market trading," *IEEE Trans. Eng. Manag.*, vol. 60, no. 2, pp. 398–408, May 2013.

[13] B. Fahimnia, J. Sarkis, F. Dehghanian, N. Banihashemi, and S. Rahman, "The impact of carbon pricing on a closed-loop supply chain: An Australian case study," *J. Cleaner Prod.*, vol. 59, pp. 210–225, Nov. 2013.

[14] M. Jin, N. A. Granda-Marulanda, and I. Down, "The impact of carbon policies on supply chain design and logistics of a major retailer," *J. Cleaner Prod.*, vol. 85, pp. 453–461, Dec. 2014.

[15] X. Chen and X. Wang, "Effects of carbon emission reduction policies on transportation mode selections with stochastic demand," *Transp. Res. E, Logistics Transp. Rev.*, vol. 90, pp. 196–205, Jun. 2016.

[16] T. Paksoy, T. Bektaş, and E. Özceylan, "Operational and environmental performance measures in a multi-product closed-loop supply chain," *Transp. Res. E, Logistics Transp. Rev.*, vol. 47, no. 4, pp. 532–546, Jul. 2011.

[17] L. J. Zeballos, C. A. Méndez, A. P. Barbosa-Povoa, and A. Q. Novais, "Multi-period design and planning of closed-loop supply chains with uncertain supply and demand," *Comput. Chem. Eng.*, vol. 66, pp. 151–164, Jul. 2014.

[18] V. D. R. Guide and L. N. Wassenhove, "Managing product returns for remanufacturing," *Prod. Oper. Manage.*, vol. 10, no. 2, pp. 142–155, Jan. 2009.

[19] V. D. R. Guide, R. H. Teunter, and L. N. Van Wassenhove, "Matching demand and supply to maximize profits from remanufacturing," *Manuf. Service Oper. Manage.*, vol. 5, no. 4, pp. 303–316, Oct. 2003.

[20] N. Aras and D. Aksen, "Locating collection centers for distance- and incentive-dependent returns," *Int. J. Prod. Econ.*, vol. 111, no. 2, pp. 316–333, Feb. 2008.

[21] D. Das and P. Dutta, "Design and analysis of a closed-loop supply chain in presence of promotional offer," *Int. J. Prod. Res.*, vol. 53, no. 1, pp. 141–165, Jan. 2015.

[22] E. Masoudipour, H. Amirian, and R. Sahraeian, "A novel closed-loop supply chain based on the quality of returned products," *J. Cleaner Prod.*, vol. 151, pp. 344–355, May 2017.

[23] M. I. G. Salema, A. P. Barbosa-Povoa, and A. Q. Novais, "An optimization model for the design of a capacitated multi-product reverse logistics network with uncertainty," *Eur. J. Oper. Res.*, vol. 179, no. 3, pp. 1063–1077, Jun. 2007.

[24] M. S. Pishvaee, F. Jolai, and J. Razmi, "A stochastic optimization model for integrated forward/reverse logistics network design," *J. Manuf. Syst.*, vol. 28, no. 4, pp. 107–114, Dec. 2009.

[25] F. Mohammed, S. Z. Selim, A. Hassan, and M. N. Syed, "Multi-period planning of closed-loop supply chain with carbon policies under uncertainty," *Transp. Res. D, Transp. Environ.*, vol. 51, pp. 146–172, Mar. 2017.

[26] B. Vahdani, R. Tavakkoli-Moghaddam, M. Modarres, and A. Baboli, "Reliable design of a forward/reverse logistics network under uncertainty: A robust-M/M/c queuing model," *Transp. Res. E, Logistics Transp. Rev.*, vol. 48, no. 6, pp. 1152–1168, Nov. 2012.

[27] N. Gao and S. M. Ryan, "Robust design of a closed-loop supply chain network for uncertain carbon regulations and random product flows," *EURO J. Transp. Logistics*, vol. 3, no. 1, pp. 5–34, Jun. 2014.

[28] D. Bertsimas and M. Sim, "Robust discrete optimization and network flows," *Math. Program.*, vol. 98, nos. 1–3, pp. 49–71, Sep. 2003.

[29] D. Bertsimas and M. Sim, "The price of robustness," *Oper. Res.*, vol. 52, no. 1, pp. 35–53, Feb. 2004.

[30] A. Choudhary, S. Sarkar, S. Settur, and M. K. Tiwari, "A carbon market sensitive optimization model for integrated forward–reverse logistics," *Int. J. Prod. Econ.*, vol. 164, pp. 433–444, Jun. 2015.

[31] Z. Xu, S. Pokharel, A. Elomri, and F. Mutlu, "Emission policies and their analysis for the design of hybrid and dedicated closed-loop supply chains," *J. Cleaner Prod.*, vol. 142, pp. 4152–4168, Jan. 2017.

[32] A. Mohajeri and M. Fallah, "A carbon footprint-based closed-loop supply chain model under uncertainty with risk analysis: A case study," *Transp. Res. D, Transp. Environ.*, vol. 48, pp. 425–450, Oct. 2016.

[33] A. Haddadsisakht and S. M. Ryan, "Closed-loop supply chain network design with multiple transportation modes under stochastic demand and uncertain carbon tax," *Int. J. Prod. Econ.*, vol. 195, pp. 118–131, Jan. 2018.

[34] E. Keyvanshokooh, S. M. Ryan, and E. Kabir, "Hybrid robust and stochastic optimization for closed-loop supply chain network design using accelerated benders decomposition," *Eur. J. Oper. Res.*, vol. 249, no. 1, pp. 76–92, Feb. 2016.

[35] M. Yavari and M. Geraeli, "Heuristic method for robust optimization model for green closed-loop supply chain network design of perishable goods," *J. Cleaner Prod.*, vol. 226, pp. 282–305, Jul. 2019.

[36] J. Kim, B. D. Chung, Y. Kang, and B. Jeong, "Robust optimization model for closed-loop supply chain planning under reverse logistics flow and demand uncertainty," *J. Cleaner Prod.*, vol. 196, pp. 1314–1328, Sep. 2018.

[37] A. Hassanpour, J. Bagherinejad, and M. Bashiri, "A robust leader-follower approach for closed loop supply chain network design considering returns quality levels," *Comput. Ind. Eng.*, vol. 136, pp. 293–304, Oct. 2019.

[38] M. Marufuzzaman, S. D. Eksioglu, and Y. E. Huang, "Two-stage stochastic programming supply chain model for biodiesel production via wastewater treatment," *Comput. Oper. Res.*, vol. 49, pp. 1–17, Sep. 2014.

[39] R. Sadeghi Rad and N. Nahavandi, "A novel multi-objective optimization model for integrated problem of green closed loop supply chain network design and quantity discount," *J. Cleaner Prod.*, vol. 196, pp. 1549–1565, Sep. 2018.

[40] N. Zarbakhshnia, H. Soleimani, M. Goh, and S. S. Razavi, "A novel multi-objective model for green forward and reverse logistics network design," *J. Cleaner Prod.*, vol. 208, pp. 1304–1316, Jan. 2019.

[41] K. Govindan, P. C. Jha, and K. Garg, "Product recovery optimization in closed-loop supply chain to improve sustainability in manufacturing," *Int. J. Prod. Res.*, vol. 54, no. 5, pp. 1463–1486, Mar. 2016.

[42] S. H. Amin and F. Baki, "A facility location model for global closed-loop supply chain network design," *Appl. Math. Model.*, vol. 41, pp. 316–330, Jan. 2017.

[43] M. Ghomi-Avili, S. G. J. Naeini, R. Tavakkoli-Moghaddam, and A. Jabbarzadeh, "A fuzzy pricing model for a green competitive closed-loop supply chain network design in the presence of disruptions," *J. Cleaner Prod.*, vol. 188, pp. 425–442, Jul. 2018.

[44] K. M. R. Hoen, T. Tan, J. C. Fransoo, and G. J. van Houtum, "Effect of carbon emission regulations on transport mode selection under stochastic demand," *Flexible Services Manuf. J.*, vol. 26, nos. 1–2, pp. 170–195, Jun. 2014.

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