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Measure Vulnerability of Metro Network Under Cascading Failure

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ABSTRACT Metro network is the important lifeline in modern cities. Its stability and reliability are critical for guaranteeing the urban residents' efficient commuting and continuous operation of urban functions. A series of disruptions show the vulnerability of metro network, and even the breakdown of a single node is sufficient to collapse the entire network due to cascading failure. However, the vulnerability assessments of metro network in the previous studies neglect the impact of cascading failure on the drop of network service performance. This paper measures the vulnerability of metro network by capturing the demand loss and travel delay under cascading failure. First, a load-capacity based model is developed to describe the dynamic process of cascading failures. Then, a weighted composite index composed of demand loss and travel time delay under cascading failure is used to measure metro network vulnerability. Finally, taking Shanghai metro network as an example, five attack scenarios are simulated to investigate cascading failure process and network vulnerability. The results reveal that cascading failures result in severe consequences in the metro network. The vulnerability of metro network without considering cascading failure is underestimated. The decrease in the tolerance parameter leads to the increase in metro network vulnerability. Random attack on one node is the most sensitive to the tolerance parameter. Node betweenness, cascading failure path and the initial load on the overloaded nodes also affect metro network vulnerability. This study provides a new perspective for understanding vulnerability of metro network, and also provides insights for improving operation reliability and stability of the network, as well as for designing emergency strategies to protect the network.

INDEX TERMS Metro network, vulnerability, complex network theory, cascading failure, numerical simulation.

I. INTRODUCTION

With the advantages of large capacity, punctuality and fast speed, metro network is the lifeline of metropolitan cities, playing a prominent role in alleviating traffic congestion in metropolis areas. During the year 2018 in Shanghai, 3.71 billion trips were made via urban rail transit (i.e., around 10.17 million trips per day), of which metro network accounted for approximately 90% [1]. Urban residents are highly dependent on metro network for their daily commuting. However, metro networks are subject to recurring service disruption, mainly due to mechanical or electrical failure, adverse weather, and sudden increase in travel demand [2]. For instance, on January 14, 2020, an equipment failure between Hongtu Avenue and Changqingcheng station led to service disruption of Metro Line 2 in Wuhan, and a large number of passengers were stranded at the stations. On August 10, 2020, Metro Line 16 encountered a power failure caused by lightning strikes, which led to service delay and overcrowding in Shanghai due to passengers rerouting their paths in the disrupted network. On April 9, 2020, due to a geological disaster caused by days of rain, the rail track between Futian and Xinxiu station of Metro Line 2 in Shenzhen was delayed for 50 minutes.

The above disruptions show the vulnerability of metro network, and even the breakdown of a single node is sufficient to collapse the entire network. The study on the vulnerability of metro network can provide insights for improving operation reliability and stability of the network, as well as for designing emergency strategies to protect the network. There is no common definition of vulnerability. The widely used definition proposed by Berdica [3] states that "vulnerability in the road transportation system is a susceptibility to

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incidents that can result in considerable reductions in road network serviceability''. This definition is equally valid for other modes of transport, including metro network.

Vulnerability assessment of metro network usually follows one of the two methods: topology-based analysis or system-based analysis [4]. Topology-based analysis is based on graph-topological measures, originating in the theory of complex network. Assuming a failure of single component in the network, the topology-based vulnerability is assessed by analyzing the structure of the graph model, and disregarding dynamic effects of the performance within the system [5]. The widely used graph-based measure indicators include network efficiency, degree, betweenness and so on [6]–[10]. Although the graph-based analysis has the advantage of limited data hungriness, it cannot capture the behavioral response and the dynamic effect of the disruption. System-based analysis can overcome these limitations by focusing on the interaction of demand and supply. Hence, system-based analysis is more helpful to provide insights to maintain the network reliability than graph-based analysis. This paper belongs to the category of system-based analysis.

Rodríguez-Núñez and García-Palomares [11] presented that ''the closure of a link in public transport network can have two distinctly different outcomes: (i) the network is separated into two non-connected components, or (ii) some travellers have to make a detour to reach their destinations''. Accordingly, the following two measure indicators are widely used in system-based vulnerability assessment of public transport network: (i) the unsatisfied demand, i.e., the number of trips that cannot be carried out and (ii) the increase in average travel time assuming that the affected travellers make the fastest possible detour [11]–[13]. However, the previous studies do not take into account another important impact of the disruption on the network, that is, cascading failure, whereby the failures of certain stations cause the failures of other stations due to passenger flow redistribution. Due to the heavy passenger flow and the fact that passengers usually make several transfers to reach their destinations, cascading failure occurs frequently in peak hours in metro network, leading to the drop in network service performance.

Cascading failure affects passenger behavior and travel time, and therefore has a direct impact on system-based measure indicators in vulnerability assessment. Hence, it is essential to take into account cascading failure when measuring vulnerability of metro network. Very few studies capture the vulnerability measures under cascading failure. This paper aims to fill this gap by measuring vulnerability of metro network from the system-based perspective by taking into account cascading failure. The contributions of this paper are as follows:

(1) A cascading failure model is developed, in which the overloaded node is not removed from the network, and the load on the attacked node is redistributed to the adjacent node following a path-based rule.

(2) A weighted composite index composed of demand loss and travel delay under cascading failure is used to measure vulnerability of metro network. Taking Shanghai metro network as an example, several conclusions are obtained from the results of five attack scenarios.

The remainder of this paper is organized as follows. Section 2 reviews the relevant previous studies on metro network vulnerability assessment and cascading failure model. After that, the graph model of metro network is developed in Section 3. Section 4 presents the developed cascading failure model of metro network. Section 5 proposes the weighted composite index to measure metro network vulnerability. Section 6 provides a case study of Shanghai metro network. The conclusions and implications drawn from the findings, as well as the future research are presented in Section 7.

II. LITERATURE REVIEW

A. METRO NETWORK VULNERABILITY

Mattsson and Jenelius [4] have provided a detailed review on transport network vulnerability before 2015. Hence, we will review the literature from then in the perspective of system-based analysis.

In the field of system-based analysis, multiple vulnerability measures have been proposed to capture different aspects in previous studies. To capture the impact of disruption on service level and the importance of components, measures such as travel delay and demand loss are adopted. Rodríguez-Núñez and García-Palomares [11] measured vulnerability of public transport network by analyzing the impact of the disruption on riding times and the number of missed trips with a full scan approach implemented in GIS. Szymula and Bešinović [5] proposed a new mathematical formulation to identify the most critical combination of links in railway network. The impact of the disruption on passengers and trains was explicitly captured by the passenger traveling time via rerouting, cancelling or short-turning the trains. Ma *et al.* [14] examined the impact of rainstorms on public transport network vulnerability composed of the ground bus and metro network. A comprehensive indicator composed of node scale, network efficiency and passenger flow in maximum connected sub-graph was proposed.

To capture the drop of weighted network efficiency, the weight graph-based measures are used. Sun and Guan [2] analyzed betweenness centrality and passenger betweenness centrality, number of missed trips, weighted average path length, and weighted global efficiency considering relative disruption probability of each line to measure vulnerability of metro network from the line operation perspective. Xing *et al.* [10] also analyzed the weighted node degree, weighted shortest path and weighted node betweenness to measure public transport network vulnerability under random failure and malicious attack. Sun *et al.* [15] measure node vulnerability of metro network with the decrease of the weighted topological efficiency. Results from the case study of Shanghai Metro Network indicate that metro networks are generally vulnerable to the largest degree node-based attacks and the highest betweenness node-based attacks. Nian *et al.* [16] identified the optimal alignment corridor of a new metro line by considering network vulnerability.in which the vulnerability of quantified by the increase in passengers' travel time due to the closure of rail lines. To capture the transportation function loss, Zhang and Wang [17] defined the metro network function as the sum of the trains running on the rail, and measured the static and dynamic vulnerability of metro networks by the function loss of metro network. Zhang *et al.* [9] compared the vulnerability of Shanghai, Beijing and Guangzhou metro networks with network efficiency and functionality loss. In their study, the transport functionality of node is a self-defined function, where the initial transport functionalities of all the nodes are supposed to be 1, and the transport functionality of isolated node equals 0. To capture the societal welfare or losses, generalized cost measures are used. Yap *et al.* [18] quantified the societal costs of links to identify the most vulnerable links in the multi-level public transport network. Since flow bottlenecks are potent sources of vulnerability, Bell *et al.* [19] identified potential flow bottlenecks in the network with capacity weighted spectral partitioning, without reference to demand information or path assignments.

Although a variety of system-based measures have been developed to analyze vulnerability of public transit network, very few studies have considered the phenomenon of cascading failure in vulnerability assessment. Candelieri *et al.* [6] and Sun *et al.* [8] took into account cascading failure in public transport vulnerability assessment. In their work, the relative size of the largest connected component and the number of failed nodes were used as measure indicators and the system-based measures were not involved. Cascading failure causes the drop of network, which in turn affects the travel time and demand loss. This paper will capture the unsatisfied demand and the travel delay under cascading failure to measure vulnerability of metro network.

B. CASCADING FAILURE MODEL OF METRO NETWORK

Due to the serious threat to the entire network, cascading failures have been a hot topic in the research field of network survivability. Researchers have proposed many load-capacity cascading models in different kinds of actual networks, such as power grid network [20], [21], cyber-physical network [22], water network [23] and transport network [24], [25]. The most classic cascading model is the load-capacity model proposed by Motter *et al.* [26]. In this model, nodes have a certain capacity. When a node's load exceeds its capacity, the node fails, and the load on it will be redistributed to its neighbors according to specific rules, i.e., the failure load redistribution rule (FLDR) [27].

The differences in load-capacity models between all kinds of networks lies in failure mechanism and FLDR. Most of the previous studies assumed that once the node is overloaded, it will be removed from the network [25], [28]. Actually, in metro network, the overloaded node will be in a state of congestion. The failure mechanism depends on the operation process of different networks. For instance, in power grid network or water network, once the load of node exceeds its capacity, the node will be failed and removed from the network.

With regard to FLDR, Zhang *et al.* [27] summarized FLDR patterns into three categories: (i) FLDR pattern following average evacuation [29], (ii) FLDR pattern based on the proportion of adjacent node capacity [29], [30], (iii) FLDR pattern following user equilibrium evacuation [25]. In the field of public transport network, Zhang *et al.* [27] constructed a cascading failure model of weight public transit network, where the link prediction effect is considered in FLDR. He *et al.* [31] proposed a transfer-based path navigation strategy to describe the flow distribution rule of transit network. Liu *et al.* [32] improved the cascading failure model of metro network in previous studies by proposing an improved edge weight function to analyze the change of node state. Zhang *et al.* [33] established a cascading failure perspective-based mesoscopic reliability model for measuring PTN survivability. FLDR pattern is considered as a conscious dynamic game process following the user equilibrium rule. Zhang *et al.* [34] constructed a cascading failure model to measure dynamic functional vulnerability of transportation network. FLDR pattern is based on the proportion of adjacent node load. Zhang *et al.* [35] established a cascading failure model considering the self-organization effect in the interdependent PTN.

Actually, the overloaded node in metro network will be in a congested state, and it is not removed from the network. Besides, the passengers in metro network reroute their paths towards the direction of their destinations when their departure node is attacked, and the above FLDR does not fit metro network very well. It is necessary to analyze the failure process of metro network, and propose a cascading failure model that is more in line with the metro system.

III. GRAPH MODEL OF METRO NETWORK

Since metro lines have two-way traffic, metro network can be modeled using a directed graph $G = \{R, N, A\}$ consisting of routes R, nodes N and directed arcs A. The routes R represent lines in metro network. Each route $r \in R$ has two directions, i.e. up and down directions. The nodes N represent stations. The stations can be divided into transfer stations N_T and non-transfer stations N_{NT}. The arcs A represent the railway tracks connecting the adjacent stations. The adjacency matrix of networks is $D = (d_{ij})_{n\times n}$, representing a directed arc from station i and j, which is defined as $d_{ij} = \begin{cases} 1 (i,j) \in A \\ 0 (i,j) \notin A \end{cases}$ and $d_{ii} = 0$. The arc weights W represent the travel time on the arcs, which is expressed as $\mathrm{W} = \begin{cases} w_{ij}(i,j) \in A \\ 0(i,j) \notin A \end{cases}$, where w_{ij} is the travel time on arc $a_{ij} \in A$. Suppose P_{ij} is the shortest path of OD pair (i,j). Then, P_{ij} can be expressed as the sequence of stations in the path.

The graph model of a simple metro network is shown in FIGURE 1. In this network, the set of routes is $R = \{R_1, R_2, R_3\}$. The set of non-transfer stations is $N_{NT} = \{A, D, E, G, H, I\}$. The set of transfer stations is $N_T = \{B, C, F\}$. Suppose the weight of each arc is one. The shortest path from node A to H is $P_{AH} = \{A, B, F, H\}$.

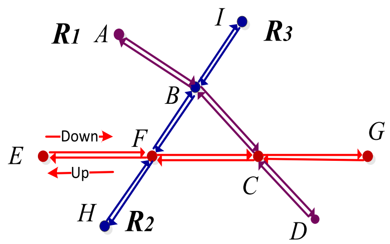

FIGURE 1. Graph model of metro network.

IV. SOME CONCEPTS AND NOTATIONS

A. TIME STEP

Cascading failure is a dynamic process. As proposed by Zhang *et al.* [27], a special time scale for dynamically modeling cascading failure needs to be introduced. The cascading failure process of metro network can be divided into discrete time steps expressed as $t\ :\ t\ =\ 0, 1, 2, 3, \ldots, |t|$. Here, the time step t represents a certain time length that is sufficiently long to complete passenger flow redistribution in network under cascading failure conditions. $t = 0$ represents the initial state of metro network before the network is attacked.

B. PASSENGER ASSIGNMENT METHOD

Since the accessible paths of passengers in metro network are limited, it is assumed that passengers will choose the path with the shortest travel time. Hence, in this paper, it is assumed that the assignment of passengers on metro network follows the All-or-Nothing method.

C. NOTATIONS

The following notations are used throughout the paper unless otherwise specified. Since there is demand loss at each time step t, the passenger flow f_{ij} is defined as a dynamic variable that varies with time step t, that is $f_{ij}(t)$.

V. CASCADING FAILURE MODEL OF METRO NETWORK

A. INITIAL LOAD AND CAPACITY

The initial load $L_k\ (0)$ on node k at the time step $t\ =\ 0$ can be calculated by assigning passengers to their paths $P_{ij}\ (0)$ in metro network. The binary variable δ_k^{ij} is a node-path incidence indicator, where $\delta_k^{ij} = 1$ if node k is on the shortest path $P_{ij}\ (0)$, and $\delta_k^{ij}\ =\ 0$ otherwise. Hence, $L_k\ (0)$ can be calculated by Eq. (1).

$$L_k\ (0) = \sum\nolimits_{(i,j)\in OD} f_{ij}\ (0)\ \delta_k^{ij} \tag{1}$$

Here, $f_{\ ij}\ (0)$ represents the initial passenger flow of OD pair $(i, j) \in OD$ at $t = 0$ before the network is attacked.

The travel time $T_{ij}\ (t)$ for passengers traveling on the shortest path $P_{ij}\ (t)$ is composed of transferring time and riding time. The binary variable $\varphi_{ij,t}^{k1k2}$ is an arc-path indicator, where $\varphi_{ij,t}^{k1k2}\ =\ 1$ if arc $<\ k, k'\ >\in A$ is used by the shortest path $P_{ij}\ (t)$, and $\varphi_{ij,t}^{k1k2} = 0$ otherwise. The binary variable $\mu_{ij,t}^{k,l,l'}$ is a transfer pair-path indicator, where $\mu_{ij,t}^{k,l,l'}\ = 1$ if the transfer pair $(k, l, l')\ \in\ T$ is used by the shortest path $P_{ij}\ (t)$, and $\mu_{ij,t}^{k,l,l'}\ = 0$ otherwise. The travel time $T_{ij}\ (t)$ can be calculated

TABLE 1. Notations.

Sets	
R	Set of routes
N	Set of nodes
N_T	Set of transfer stations
N_{NT}	Set of non-transfer stations
A	Set of arcs
OD	Set of origin to destination (OD) pairs
TP	Set of transfer pairs
Indexes	
t	Index of time step,
i, j, k, m, q	Index of stations
(i, j)	OD pair whose origin node is i and destination node is j, $(i, j) \in OD$
(k, l, l')	Transfer pair for passengers transferring from route l to route l' at node k, $(k, l, l') \in TP$
$< m, k >$	Arc from node m to node k, $< m, k >\in A$
Parameters	
$L_k(t)$	Load of node k at time step t, $t = 0,1,2,3, \ldots, \lvert t \rvert$
C_k	Capacity of node k
β	Tolerance parameter
$P_{ij}(t)$	Shortest path of OD pair $(i, j) \in OD$ at time step t
$f_{ij}(t)$	Passenger flow of OD pair (i, j) at time step t
$w_{mk}(t)$	Travel time of arc $< m, k > \in A$ at time step t.
$tw_{i,r,r'}$	Transfer walking time for passengers transferring from route r to r' at node i.
$T_{ij}(t)$	Travel time for passengers traveling on the shortest path from node i to node j at time step t
TF	Total passenger flow in metro network
IA	Inaccessible flow in metro network because the node is attacked
$MT(t)$	Accumulated number of passengers whose travel time exceeds their acceptable threshold at time step t
$UN(t)$	Accumulated number of unsatisfied passengers at time step t
I	Proportion of missed trips when cascading failure ends
S	Change in travel time of the satisfied passengers when cascading failure ends
X	Total number of the overloaded nodes when cascading failure ends
V	Vulnerability of metro network in the attack scenario
γ_1, γ_2	Weight coefficients

by Eq. (2).

$$T_{ij}\ (t) = \sum\nolimits_{<k,k'>\in A} w_{k,k'}\ (t)\ \varphi_{ij,t}^{k,k'} + \sum\nolimits_{(k,l,l')\in TA} tw_{k,l,l'}\mu_{ij,t}^{k,l,l'} \tag{2}$$

In metro network, the capacity of a node is the maximum load that the station can handle. According to Motter *et al.* [26], Wu *et al.* [24], Zhong *et al.* [29], C_k is proportional to the initial load $L_k\ (0)$, that is,

$$C_k = (1 + \beta)L_k\ (0) \tag{3}$$

where β is the tolerance parameter.

B. NODE FAILURE CRITERION

If a node is removed from the network due to disruption, the flow balance is broken, which leads to flow redistribution in the network. At time step t, if the load of a node $L_k\ (t)$

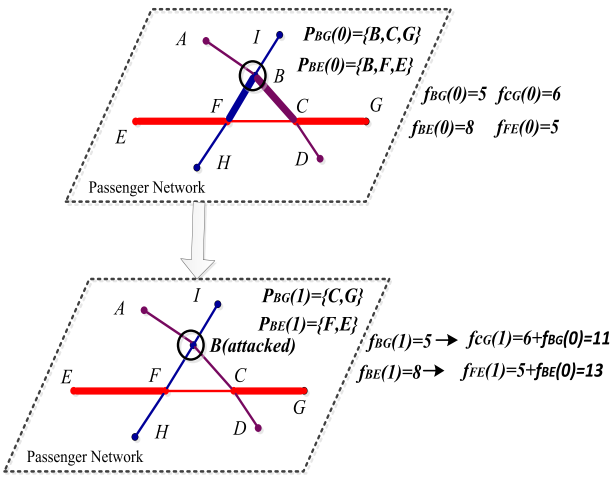

FIGURE 2. **Load redistribution of the attacked node.**

exceeds its capacity C_k, the node k is overloaded. The node "failure" in this paper is actually an overloaded state. The overloaded node is not removed from the network. However, the travel time on the link connecting the overloaded node will increase due to crowding when passengers are boarding and alighting. Suppose θ is the travel time penalty coefficient. If the node k is overloaded at time step t, the weight of the arcs connecting node k will be increased by $\theta \times \frac{L_k(t)}{C_k}$ times at time step $t+1$, as shown in Eq. (4).

$$w_{kj}(t+1) = \begin{cases} w_{ij}(t) \times (1+\theta \times \dfrac{L_k(t)}{C_k}) & if\ L_k(t) > C_k \\ w_{ij}(t) & if\ L_k(t) \leq C_k \end{cases} \tag{4}$$

C. LOAD REDISTRIBUTION OF THE ATTACKED NODE

Once a node is attacked, the passengers departing from the attacked node will have to shift to the adjacent stations. Different from the three FLDR patterns mentioned in Section II, a path-based FLDR pattern is proposed here according to the actual path decisions of passengers in the attack scenario, that is, the passengers will shift to the first non-attacked station of their initial shortest path before their departure node is attacked. The reason is that passengers will shift toward the node in the direction of their destinations. The metro network in FIGURE 2 is taken as a numerical example.

In this network, the passenger flow of OD pair (B, G) at time step $t = 0$ is $f_{BG}(0) = 5$, and its path before attack is $P_{BG}(0) = \{B, C, G\}$. The passenger flow of OD pair (B, E) is $f_{BE}(0) = 8$, and its path before attack is $P_{BE}(0) = \{B, F, E\}$, as shown by the thick lines in FIGURE 2. The passenger flows of OD pair (C, G) and (F, E) at time step t are $f_{CG}(0) = 6$ and $f_{FE}(0) = 5$, respectively.

At time step $t = 0$, node B is attacked and removed from the network. The passengers departing from node B will shift to its adjacent nodes at time step $t = 1$. The passenger flow $f_{BG}(0)$ of OD pair (B, G) will shift to node C, causing the passenger flow $f_{CG}(1)$ increase to be $f_{BG}(0) + f_{CG}(0) = 11$. The passenger flow $f_{BE}(0)$ of OD pair (B, E) will shift to node F, causing the passenger flow $f_{FE}(1)$ increase to be $f_{BE}(0) + f_{FE}(0) = 13$.

A binary variable ρ_{ij}^k is defined to denote whether node k is the first non-attacked node on the shortest path $P_{ij}(0)$. If node k is the first non-attacked node on the path $P_{ij}(0)$, then $\rho_{ij}^k = 1$,else $\rho_{ij}^k = 0$. Then, the passenger flow f_{kj} of OD pair (k, j) will increase to be:

$$f_{kj}(1) = f_{kj}(0) + \rho_{ij}^k \times f_{ij}(0) \tag{5}$$

VI. VULNERABILITY INDICATORS UNDER CASCADING FAILURE

The affected passengers under cascading failure in the attack scenario can be divided into two types: the unsatisfied passengers and the satisfied passengers. The demand loss can be measured by the number of unsatisfied passengers. The travel delay is measured by the increase in travel time of the satisfied passengers. A comprehensive index composed of the number of unsatisfied passengers and the change in travel time of satisfied passengers is constructed here to measure vulnerability of metro network.

A. THE NUMBER OF UNSATISFIED PASSENGERS

The number of unsatisfied passengers includes the following two types: (i) the number of passengers that cannot reach their destinations by the interrupted metro network, and (ii) the number of passengers whose travel time exceeds their acceptable threshold. Suppose ε is the acceptable coefficient of travel time. For the passenger flow $f_{ij}(t)$, the acceptable threshold can be defined as their initial travel time $T_{ij}(0)$ multiplied by $(1+\varepsilon)$, as shown in Eq. (6). Once the travel time of passengers exceeds their acceptable threshold, they will shift to other travel modes. Hence, at time step t, if the travel time of passenger flow $f_{ij}(t)$ satisfies the following Eq. (6), the passenger flow $f_{ij}(t)$ is determined as the unsatisfied passenger flow.

$$T_{ij}(t) > T_{ij}(0) \times (1+\varepsilon) \tag{6}$$

When cascading failure ends at time step $t = |t|$, the sum of unsatisfied passengers $UN(t)$ is the sum of inaccessible passenger flow IA and the total number of passengers $MT(t)$ whose travel time exceeds their acceptable threshold. The proportion of unsatisfied passengers can be calculated by Eq. (7).

$$I = \frac{UN(t)}{TF} = \frac{IA + MT(t)}{TF} \quad t = |t| \tag{7}$$

B. THE CHANGE IN TRAVEL TIME OF SATISFIED PASSENGERS

A binary variable $\rho_{ij,t}$ is defined to denote whether the passenger flow $f_{ij}(t)$ is the satisfied passenger flow or not at time step $t = |t|$. If the passenger flow $f_{ij}(t)$ is satisfied, $\rho_{ij,t} = 1$; else $\rho_{ij,t} = 0$. The change in travel time of satisfied passengers can be calculated by Eq.(8).

$$S = \frac{\rho_{ij,t} f_{ij}(t)(T_{ij}(t) - T_{ij}(0))}{T_{ij}(0)} \quad t = |t| \tag{8}$$

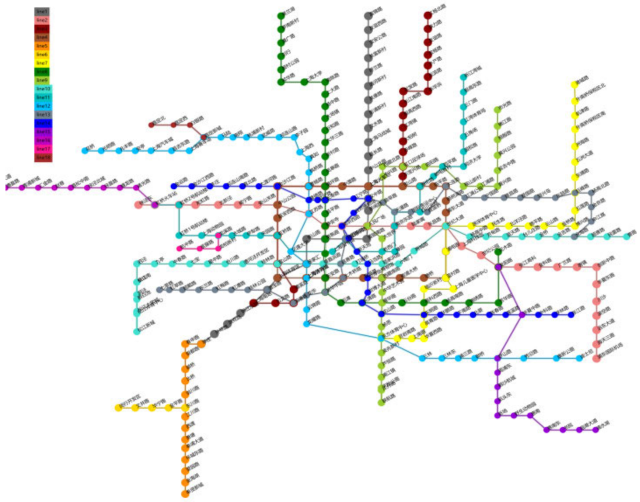

FIGURE 3. Shanghai metro network.

C. THE WEIGHTED COMPOSITE INDEX FOR VULNERABILITY ASSESSMENT

The weighted composite index for vulnerability assessment of metro network is composed of the number of unsatisfied passengers and the change in travel time of satisfied passengers, as shown in Eq. (9):

$$V = \gamma_1 \times I + \gamma_2 \times S \tag{9}$$

where γ_1 and γ_2 are weight coefficients

VII. CASE STUDIES OF SHANGHAI METRO NETWORK

A. METRO NETWORK AND PARAMETERS

As of August 2020, Shanghai metro network (SMN) has 17 lines, 416 stations, 60 transfer stations and 705 km mileage, as shown in Figure 3. The travel time on each section is obtained from the homepage of Shanghai Metro.[1] There are a total of 564 transfer pairs in SMN in this paper, and the average transfer walking time in the network is 3 minutes.[2] The passenger flow for each OD pair is obtained from metro operation company. The tolerance parameter β of node capacity is 0.2. The travel time penalty coefficient θ is 0.1. The acceptable coefficient of travel time ε is 0.5. The weight factors γ_1 and γ_2 are 0.6 and 0.4, meaning that the consequence of unsatisfied demand is more serious than that of travel time delay.

In order to compare the vulnerability of metro network under different attack strategies, the following five attack scenarios are set in this paper.

Scenario 1 (Random Attack on One Node): Ten nodes are chosen randomly, and the cascading failure process when each node is attacked is simulated respectively. The average result is taken to measure the metro network vulnerability under the random attack on one node.

[1]The homepage is" http://www.shmetro.com/"

[2]It is calculated according to the transfer walking time data searched from Shanghai metro network on Baidu map " https:// map. baidu. com/ @13406401, 3526872,13z"

TABLE 2. The attacked nodes in each scenario.

Scenario No.	The attacked nodes
Scenario1	(1) West Nanjing Road ; (2) Luban Road ; (3) Linyi Xincun ; (4) Zhenping Road ; (5) People's Square ; (6) Longhua ; (7) Shanghai Railway Station ; (8) Yishan Road; (9) Century Park; (10) Changqing Road
Scenario 2	(1) Fengzhuang , South Yanggao Road , Changqing Road ; (2) Huangxing Park, Middle Huaxia Road, PanlongRoad ; (3) Pujiang Town,Nanjingeast Road, Shanghai Zoo; (4)Madang Road, North Jiangyang Road, Zhangjiang High-Technology Park ; (5) Lan'gao Road , Jiangpu Road ,Liuxing ; (6) Lujiazui , Donglan Road , Beicai ; (7) Hongqiao Airport Terminal 2, Minlei Road,Zhongchun Road (8) Yaohua Road ,Fengqiao Road , Sheshan (9) Shanghai Circus World , Zuibaichi Park , Zhaojiabang Road ; (10) Xujingbeicheng , Xincun Road , South Lingyan Road
Scenario 3	Century Avenue
Scenario 4	Century Avenue , Nanjingwest road and Hanzhong Road
Scenario 5	Century Avenue , Jiangsu road and Shanghai Railway Station.

Scenario 2 (Random Attack on Multi-Nodes): Ten groups of nodes are chosen to attack, each of which includes three nodes. The average result of the ten groups is taken to describe the metro network vulnerability under random attack on multi-nodes.

Scenario 3 (Deliberate Attack on One Node): with the largest degree and the largest betweenness. In SMN, the node with the largest degree and betweenness is Century

Avenue, and its degree and betweenness are 7 and 30915, respectively.

Scenario 4 (Deliberate Attack on Multi-Nodes With the Largest Degree): The top three nodes with the largest degree are Century Avenue, Nanjingwest road and Hanzhong Road.

Scenario 5 (Deliberate Attack on Multi-Nodes With the Largest Betweenness): The top three nodes with the largest betweenness are Century Avenue, Jiangsu road and Shanghai Railway Station.

The attacked nodes in each scenario are generated as follows:

B. RESULTS IN EACH SCENARIO

In Scenario 1, one of the nodes in the list shown in the Table 3 is removed at a time. The average number of overloaded nodes is 6. The average proportion of unsatisfied passengers is 12.03%. The average change in travel time of satisfied passengers is 0.37%. The vulnerability of metro network is 0.073.

In Scenario 2, one of the ten groups of nodes is removed at a time. The average number of overloaded nodes is 7. The average proportion of unsatisfied passengers is 13.19%. The average change in travel time of satisfied passengers is 0.24%. The vulnerability of metro network under random attack on one node is 0.080.

TABLE 3. Results of each scenario.

Scenario No.	$\|t\|$	X	I	S	V
Scenario 1	2	6	12.03%	0.37%	0.073
Scenario 2	2	7	13.19%	0.24%	0.080
Scenario 3	3	15	20.81%	0.19%	0.126
Scenario 4	4	17	37.27%	0.70%	0.226
Scenario 5	4	19	42.70%	1.12%	0.261

TABLE 4. vulnerability without considering cascading failure.

Scenario No.	I	S	V
Scenario1	6.25%	0.11%	0.038
Scenario 2	8.52%	0.16%	0.052
Scenario 3	19.20%	0.14%	0.116
Scenario 4	28.77%	0.33%	0.174
Scenario 5	34.25%	0.49%	0.207

In Scenario 3, the attack on Century Avenue leads to a total of 15 nodes being overloaded. The average proportion of unsatisfied passengers is 20.81%. The average change in travel time of satisfied passengers is 0.19%. The vulnerability of metro network is 0.126.

In Scenario 4, Century Avenue, Nanjingwest road and Hanzhong Road are removed from the network. The number of overloaded nodes is 17. The average proportion of unsatisfied passengers is 37.27%. The average change in travel time of satisfied passengers is 0.7%. The vulnerability of metro network under random attack on one node is 0.226.

In Scenario 5, the attack on Century Avenue, Jiangsu road and Shanghai Station results in a total of 19 nodes being overloaded, and the proportion of unsatisfied passengers is 42.70%. This means that the travel time of a total of 42.7% passengers increases by more than 1.5 times of their initial travel time. The above results are summarized in Table 3.

From the results in Table 3, it can be seen that the metro network vulnerability is in the following order: Scenario 5> Scenario 4> Scenario 3> Scenario 2> Scenario 1. It means that the deliberate attack on multi-nodes with the largest betweenness causes the highest vulnerability, and the random attack on one node leads to the lowest vulnerability.

By comparing the results of Scenarios 3~5 and Scenarios 1~2, it is obvious that the network vulnerability caused by deliberate attack is much higher than that of random attack, and deliberate attack causes more serious cascading failure than random attack. By comparing the results of Scenarios 4~5 and Scenario 3, Scenario 2 and Scenario 1, it is seen that the attack on multi-nodes leads to higher vulnerability of metro network than the attack on one node.

The results without considering cascading failure are shown in TABLE 4. It can be seen that cascading failure has an important impact on metro network vulnerability, and the vulnerability of metro network without considering cascading failure is underestimated.

TABLE 5. Results of each scenario when $\beta = 1.05$.

Scenario No.	$\|t\|$	X	I	S	V	ΔV
Scenario 1	5	37	42.22%	1.63%	0.260	256.16%
Scenario 2	3	18	26.71%	0.82%	0.164	105.00%
Scenario 3	2	33	29.79%	0.65%	0.181	43.65%
Scenario 4	3	12	53.18%	0.70%	0.325	43.81%
Scenario 5	1	14	48.70%	1.12%	0.299	14.56%

(Note: "ΔV" represents the percentage of network vulnerability increase when $\beta = 1.05$ compared with $\beta = 1.2$)

TABLE 6. Results OF each scenario when $\beta = 1.5$.

Scenario No.	$\|t\|$	X	I	S	V	ΔV
Scenario 1	1	1	6.78%	0.13%	0.041	-43.84%
Scenario 2	1	2	3.09%	0.12%	0.066	-17.50%
Scenario 3	1	8	19.61%	0.11%	0.118	-6.35%
Scenario 4	1	5	30.89%	0.47%	0.187	-17.26%
Scenario 5	4	7	41.36%	0.75%	0.251	-3.83%

(Note: "ΔV" represents the percentage of network vulnerability increase when $\beta = 1.5$ compared with $\beta = 1.2$)

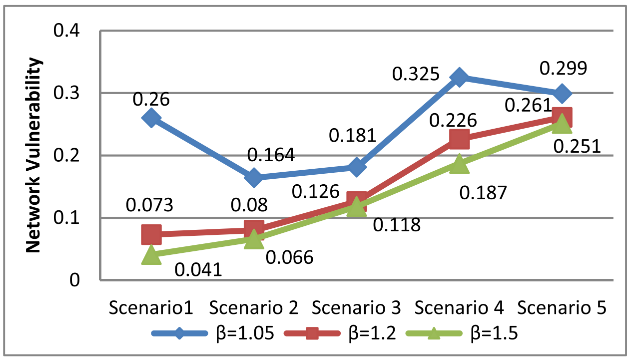

FIGURE 4. Network vulnerability with different tolerance parameters.

C. SENSITIVITY ANALYSIS OF TOLERANCE PARAMETER

In order to analyze the impact of the tolerance parameter on network vulnerability, the tolerance parameters β are saet to be 1.05 and 1.5, respectively. The results are shown in Tables 5 and 6, respectively. From Tables 3, 5~6, it is seen that the decrease in the tolerance coefficient increases the network vulnerability, while the increase in the tolerance coefficient reduces the network vulnerability.

Figure 4 shows the change trend of metro network vulnerability in each scenario with different tolerance parameters. Interestingly, it is found that the random attack on one node (Scenario 1) is the most sensitive to the tolerance parameter. When the tolerance parameter decreases to 1.05, the network vulnerability of metro network under random attack on one node increases by 256.16%. The deliberate attack on multi-nodes (Scenario 5) is the least sensitive to the tolerance parameter. Even when the tolerance parameter increases to 1.5, the network vulnerability in Scenario 5 only decreases by 3.83%.

TABLE 7. Results of scenarios 1 and 3.

Station	$\|t\|$	X	I	S	V
People's Square	3	14	31.49%	1.15%	0.193
Nanjingwest Road	5	14	28.41%	1.10%	0.175
Shanghai Railway station	3	14	20.07%	0.60%	0.123
Yishan Road	1	3	10.12%	0.30%	0.062
Zhenping Road	1	2	9.36%	0.05%	0.056
Century Park	0	0	8.94%	0.03%	0.053
Longhua	1	8	7.03%	0.21%	0.043
Luban Road	3	7	3.65%	0.12%	0.022
Changqing Road	1	2	0.80%	0.12%	0.005
Linyi Xincun	0	0	0.44%	0.01%	0.003
Century Avenue	1	8	20.81%	0.19%	0.1256

D. ANALYSIS OF THE INFLUENCING FACTORS OF NETWORK VULNERABILITY

Besides the attack strategies and tolerance parameter investigated above, the other factors affecting metro network vulnerability will also be explored here.

In Scenario 1, the results for attack of each node are listed in Table 7. Among the above attacked nodes, the removal of People's Square leads to the highest vulnerability with a value of 0.193. A total of 14 stations are overloaded. The proportion of unsatisfied passengers and the travel time delay of satisfied passengers are 31.49% and 1.15%, respectively, which are far higher than the average values. The removal of Linyi Xincun does not trigger cascading failure at all, and leads to the lowest vulnerability with a value of 0.003. This means that its removal has very slight impact on the number of unsatisfied passengers and travel time delay of the satisfied passengers.

The node degree and betweenness are shown in Table 8. By comparing the data in Tables 7 and 8, it is found that the network vulnerability does not strictly depend on node betweenness. Nevertheless, the node betweenness has a great impact on network vulnerability. For instance, Shanghai Railway Station has the largest betweenness, but its network vulnerability is lower than that of People's Square and Nanjingwest Road. The top three nodes with the largest betweenness are People's Square, Nanjingwest Road and Shanghai Railway station, the removal of which would lead to much higher network vulnerability than the other nodes, such as Yishan Road, Zhenping Road and so on.

Among the above nodes, the removal of Yishan Road or Zhenping Road will result in the metro network to be disconnected,as shown in FIGURE 5 and 6,respectively. The red circle in the figures show the disconnected point.

It is worth noting that the network vulnerability under the attack on People's Square is even higher than that of the attack on Century Avenue with the largest betweenness. That is to say, from the perspective of node individual level, the deliberate attack on the node with the largest betweenness does not always lead to the highest network vulnerability. The reason is that the attack on People's Square causes more serious cascading failure than the attack on Century Avenue.

TABLE 8. Node degree and betweenness for scenarios 1 and 3.

Station	Degree		Betweenness	
	Degree	No.	Betweenness	No.
Shanghai Railway Station	4	4	25844	3
Nanjingwest Road	6	2	22756	5
People's Square	6	2	22373	6
Yishan Road	5	3	12782	36
Zhenping Road	4	4	15633	19
Century Park	2	6	15591	20
Longhua	4	4	13119	32
Luban Road	2	6	4607	172
Changqing Road	4	4	6817	103
Linyi Xincun	2	6	2721	225
Century Avenue	8	1	309157	1

TABLE 9. Cascading failure path.

Station	Cascading failure path	Total load
People's Square	South Huangpi Road, Xinzha Road, Zhenping Road, Zhongtan Road , Tiantong Road, North Sichuan Road → Wulian Road, Lujiabang Road, Laoximen , Xiaonanmen, Shangcheng Road, Xintiandi → Wuning Road, Jiangning Road	1113409
Nanjingwest Road	Hanzhong Road, Baoshan Road, Dongbaoxing Road, Linping Road, Qufu Road, Xiaonanmen, Shangcheng Road, Xintiandi, Tiantong Road → Pudong Avenue, Yangshupu Road, Dalian Road, Hailun Road → Jinjing Road	197069
Shanghai Railway station	South Huangpi Road, Changshou Road, Qufu Road, Zhongxing Road, North Xizang Road,Tiantong Road, North Sichuan Road, International Cruise Terminal Station, Tilanqiao, Jiangning Road→ Nanjingwest Road →Yunshan Road, Jinqiao Road, Boxing Road, Wulian Road, Changshou road, North Sichuan Road	158568
Century Avenue	International Cruise Terminal Station,Tilanqiao,Aiguo Road, Fuxing Island, Donglu Road, North Yanggao Road, Jinjing Road, Shenjiang Road	34474

(Note: "Total load "represents the sum of initial load on the overload node in the cascading failure path)

The former results in 14 nodes overloaded, while the latter leads to 8 nodes overloaded. According to Eq. (4), more overloaded nodes mean a greater increase in the travel time in metro network.

In order to further analyze the impact of cascading failure on network vulnerability, the cascading failure path and the sum of the initial load on the overloaded node are included in the simulation process, as shown in Table 9.

It is found that the total load on the overloaded node in the cascading failure path when People's Square is attacked is much larger than that when Nanjingwest Road, Shanghai Railway station and Century Avenue are attacked. The cascading failure path and the initial load on the overloaded node also affect metro network vulnerability.

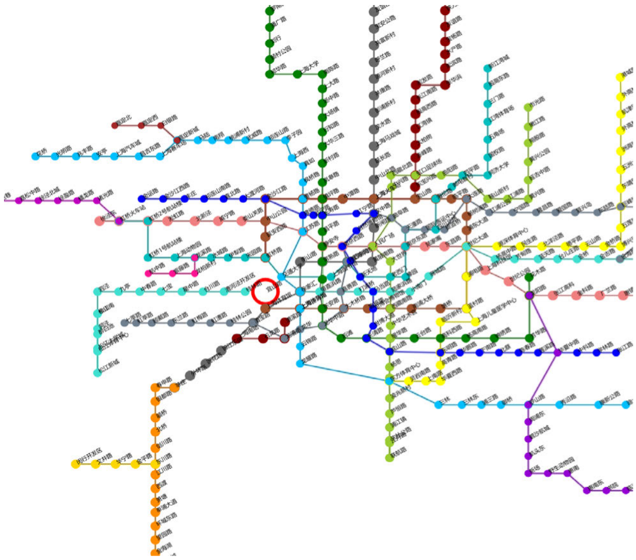

FIGURE 5. Shanghai metro network when Yishan Road is removed.

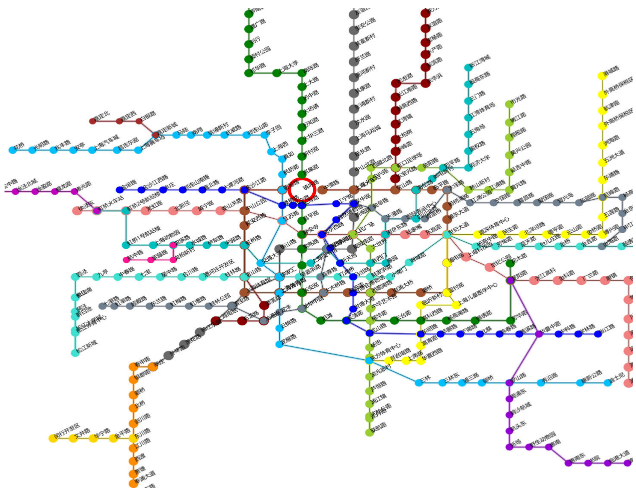

FIGURE 6. Shanghai metro network when Zhenping Road is removed.

VIII. CONCLUSION

Metro network vulnerability is defined to analyze the impact of cascading failure on the drop of service performance of network in this paper. A cascading failure model is presented, in which the overloaded node is not removed from the network, and the load on the attacked node is redistributed to the adjacent node following a path-based rule. Then, a weighted composite index composed of demand loss and travel delay under cascading failure is developed to measure metro network vulnerability. Taking Shanghai metro network as a case study, the following conclusions are obtained:

(1) Cascading failure has important impact on metro network vulnerability, and the vulnerability of metro network without considering cascading failure is underestimated.

(2) From the perspective of average vulnerability level, the deliberate attack on multi-nodes with the largest betweenness causes the highest vulnerability and the random attack on one node leads to the lowest vulnerability. However, from the perspective of node individual level, the deliberate attack on the node with the largest degree and betweenness does not always lead to the highest network vulnerability.

(3) The decrease in the tolerance coefficient increases the network vulnerability, while the increase in the tolerance coefficient reduces the network vulnerability. The random attack on one node is the most sensitive to the tolerance parameter.

(4) Node betweenness, cascading failure path and the initial load on the overloaded node also affect metro network vulnerability.

This paper focused on metro network vulnerability in different scenarios. Our future work will evaluate the probability of node attack to measure vulnerability of the whole metro network without any premise of scenario. Besides, strategies to improve metro network resilience and reduce network vulnerability will be further explored.

REFERENCES

[1] China Urban Rail Transit Association, "Urban rail transit 2018 annual statistics and analysis report," (in Chinese), *Urban Rail Transit*, no. 4, pp. 16–34, 2019.

[2] D. J. Sun and S. Guan, "Measuring vulnerability of urban metro network from line operation perspective," *Transp. Res. Part A, Policy Pract.*, vol. 94, pp. 348–359, Dec. 2016, doi: 10.1016/j.tra.2016.09.024.

[3] K. Berdica, "An introduction to road vulnerability: What has been done, is done and should be done," *Transp. Policy*, vol. 9, no. 2, pp. 117–127, 2002, doi: 10.1016/S0967-070X(02)00011-2.

[4] L.-G. Mattsson and E. Jenelius, "Vulnerability and resilience of transport systems–a discussion of recent research," *Transp. Res. Part A, Policy Pract.*, vol. 81, pp. 16–34, Nov. 2015, doi: 10.1016/j.tra.2015.06.002.

[5] C. Szymula and N. Besinovic, "Passenger-centered vulnerability assessment of railway networks," *Transp. Res. Part B, Methodol.*, vol. 136, pp. 30–61, Jun. 2020, doi: 10.1016/j.trb.2020.03.008.

[6] A. Candelieri, B. G. Galuzzi, I. Giordani, and F. Archetti, "Vulnerability of public transportation networks against directed attacks and cascading failures," *Public Transp.*, vol. 11, no. 1, pp. 27–49, Jun. 2019, doi: 10.1007/s12469-018-00193-7.

[7] Q. Ye and H. Kim, "Assessing network vulnerability of heavy rail systems with the impact of partial node failures," *Transportation*, vol. 46, no. 5, pp. 1591–1614, Oct. 2019, doi: 10.1007/s11116-018-9859-6.

[8] L. Sun, Y. Huang, Y. Chen, and L. Yao, "Vulnerability assessment of urban rail transit based on multi-static weighted method in Beijing, China," *Transp. Res. Part A, Policy Pract.*, vol. 108, pp. 12–24, Feb. 2018, doi: 10.1016/j.tra.2017.12.008.

[9] J. Zhang, S. Wang, and X. Wang, "Comparison analysis on vulnerability of metro networks based on complex network," *Phys. A, Stat. Mech. Appl.*, vol. 496, pp. 72–78, Apr. 2018, doi: 10.1016/j.physa.2017.12.094.

[10] Y. Xing, J. Lu, S. Chen, and S. Dissanayake, "Vulnerability analysis of urban rail transit based on complex network theory: A case study of Shangha i metro," *Public Transp.*, vol. 9, no. 3, pp. 501–525, Oct. 2017, doi: 10.1007/s12469-017-0170-2.

[11] E. Rodríguez-Nunez and J. C. García-Palomares, "Measuring the vulnerability of public transport networks," *J. Transp. Geography*, vol. 35, pp. 50–63, Feb. 2014, doi: 10.1016/j.jtrangeo.2014.01.008.

[12] E. Jenelius, T. Petersen, and L.-G. Mattsson, "Importance and exposure in road network vulnerability analysis," *Transp. Res. Part A, Policy Pract.*, vol. 40, no. 7, pp. 537–560, Aug. 2006, doi: 10.1016/j.tra.2005.11.003.

[13] H. Liu and X. Rong, "Vulnerability assessment of road traffic network," *J. Traffic Transp. Eng.*, vol. 16, no. 2, pp. 94–99, 2018.

[14] F. Ma, F. Liu, K. Yuen, P. Lai, Q. Sun, and X. Li, "Cascading failures and vulnerability evolution in bus–metro complex bilayer networks under rainstorm weather conditions," *Int. J. Environ. Res. Public Health*, vol. 16, no. 3, p. 329, Jan. 2019, doi: 10.3390/ijerph16030329.

[15] D. Sun, Y. Zhao, and Q.-C. Lu, "Vulnerability analysis of urban rail transit networks: A case study of shanghai, China," *Sustainability*, vol. 7, no. 6, pp. 6919–6936, May 2015, doi: 10.3390/su7066919.

[16] G. Nian, F. Chen, Z. Li, Y. Zhu, and D. J. Sun, ''Evaluating the alignment of new metro line considering network vulnerability with passenger ridership,'' *Transportmetrica A, Transp. Sci.*, vol. 15, no. 2, pp. 1402–1418, Nov. 2019, doi: 10.1080/23249935.2019.1599080.

[17] J. Zhang and M. Wang, ''Transportation functionality vulnerability of urban rail transit networks based on movingblock: The case of nanjing metro,'' *Phys. A, Stat. Mech. Appl.*, vol. 535, Dec. 2019, Art. no. 122367, doi: 10.1016/j.physa.2019.122367.

[18] M. D. Yap, N. van Oort, R. van Nes, and B. van Arem, ''Identification and quantification of link vulnerability in multi-level public transport networks: A passenger perspective,'' *Transportation*, vol. 45, no. 4, pp. 1161–1180, Jul. 2018, doi: 10.1007/s11116-018-9892-5.

[19] M. G. H. Bell, F. Kurauchi, S. Perera, and W. Wong, ''Investigating transport network vulnerability by capacity weighted spectral analysis,'' *Transp. Res. Part B, Methodol.*, vol. 99, pp. 251–266, May 2017, doi: 10.1016/j.trb.2017.03.002.

[20] W. Liao, S. Salinas, M. Li, P. Li, and K. A. Loparo, ''Cascading failure attacks in the power system: A stochastic game perspective,'' *IEEE Internet Things J.*, vol. 4, no. 6, pp. 2247–2259, Dec. 2017.

[21] A. Azzolin, L. Dueñas-Osorio, F. Cadini, and E. Zio, ''Electrical and topological drivers of the cascading failure dynamics in power transmission networks,'' *Rel. Eng. Syst. Saf.*, vol. 175, pp. 196–206, Jul. 2018.

[22] C. Zhang, X. Xu, and H. Dui, ''Analysis of network cascading failure based on the cluster aggregation in cyber-physical systems,'' *Reliab. Eng. Syst. Saf.*, vol. 202, Oct. 2020, Art. no. 106963, doi: 10.1016/j.ress.2020.106963.

[23] Q. Shuang, M. Zhang, and Y. Yuan, ''Node vulnerability of water distribution networks under cascading failures,'' *Reliab. Eng. Syst. Saf.*, vol. 124, pp. 132–141, Apr. 2014, doi: 10.1016/j.ress.2013.12.002.

[24] J. J. Wu, Z. Y. Gao, and H. J. Sun, ''Effects of the cascading failures on scale-free traffic networks,'' *Physica A, Stat. Mech. Appl.*, vol. 378, no. 2, pp. 505–511, 2007, doi: 10.1016/j.physa.2006.12.003.

[25] J. J. Wu, H. J. Sun, and Z. Y. Gao, ''Cascading failures on weighted urban traffic equilibrium networks,'' *Phys. A, Stat. Mech. Appl.*, vol. 386, no. 1, pp. 407–413, Nov. 2007, doi: 10.1016/j.physa.2007.08.034.

[26] A. E. Motter and Y. Lai, ''Cascade-based attacks on complex networks,'' *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 66, no. 6, pp. 1–5, 2002, doi: 10.1103/PhysRevE.66.065102.

[27] L. Zhang, J. Lu, B. Fu, and S. Li, ''A cascading failures model of weighted bus transit route network under route failure perspective considering link prediction effect,'' *Phys. A, Stat. Mech. Appl.*, vol. 523, pp. 1315–1330, Jun. 2019, doi: 10.1016/j.physa.2019.04.122.

[28] B. Schäfer, D. Witthaut, M. Timme, and V. Latora, ''Dynamically induced cascading failures in power grids,'' *Nature Commun.*, vol. 9, no. 1, Dec. 2018, doi: 10.1038/s41467-018-04287-5.

[29] L. Zhang, B.-B. Fu, and Y.-X. Li, ''Cascading failure of urban weighted public transit network under single station happening emergency,'' *Procedia Eng.*, vol. 137, pp. 259–266, 2016, doi: 10.1016/j.proeng.2016.01.267.

[30] J. Wang, C. Jiang, and J. Qian, ''Robustness of interdependent networks with different link patterns against cascading failures,'' *Phys. A, Stat. Mech. Appl.*, vol. 393, pp. 535–541, Jan. 2014, doi: 10.1016/j.physa.2013.08.031.

[31] T. He, N. Zhu, Z. Hou, and G. Xiong, ''A novel cascading failure model on city transit network,'' in *Proc. 6th Int. Conf. Mach., Mater., Environ., Biotechnol. Comput.*, Tianjin, China, Jun. 2016.

[32] L. Zhang, B. Fu, and Y. Li, ''Cascading failure of urban weighted public transit network under single station happening emergency,'' *Procedia Eng.*, vol. 137, pp. 259–266, 2016, doi: 10.1016/j.proeng.2016.01.267.

[33] L. Zhang, J. Lu, M. Long, S.-B. Li, and J. Zhou, ''A cascading failures perspective based mesoscopic reliability model of weighted public transit network considering congestion effect and user equilibrium evacuation,'' *Math. Problems Eng.*, vol. 2018, pp. 1–13, May 2018, doi: 10.1155/2018/9292375.

[34] J. Zhang and M. Wang, ''Transportation functionality vulnerability of urban rail transit networks based on movingblock: The case of Nanjing metro,'' *Phys. A, Stat. Mech. Appl.*, vol. 535, Dec. 2019, Art. no. 122367, doi: 10.1016/j.physa.2019.122367.

[35] L. Zhang, J. Lu, B.-B. Fu, S.-B. Li, Y.-X. Li, and M. Long, ''A quantitatively controllable mesoscopic reliability model of an interdependent public transit network considering congestion, time-delay interaction and self-organization effects,'' *Nonlinear Dyn.*, vol. 96, no. 2, pp. 933–958, Apr. 2019, doi: 10.1007/s11071-019-04831-y.

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