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Efficiency Assessment of Resilience Engineering in Process Industries Using Data Envelopment Analysis Based on Type-2 Fuzzy Sets

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ABSTRACT Process industries have the talent of emerging high levels of turbulent behaviors and uncertainties, such as the leakage of toxic substances and explosive materials. Resilience engineering, as a novel approach, can run the effects of such actions. Resilience engineering factors involve culture, change management, knowledge acquisition, risk assessment, readiness, plasticity, reportage, the obligation of a top manager, consciousness, safety procedures, incident survey, employee participation, and competence. The present study aims to investigate resilience engineering in process industries and analyze its efficiency using the data envelopment analysis (DEA) technique. Since there are high levels of uncertainty in the factors, Type-2 fuzzy sets that have a high capability of considering uncertainty is used to analyze the efficiency. The results of this work, which is the first case in evaluating the efficiency of resilience engineering in process industries by DEA and Type-2 fuzzy sets, indicate a robust approach for analyzing the efficiency and identifying the opportunities in process industries.

INDEX TERMS Process industries, resilience engineering, data envelopment analysis, Type-2 fuzzy sets.

I. INTRODUCTION

The significance of risk management has increased due to the catastrophic events in complex socio-technical systems (CSS) [1]. Process industries, as a CSS, faced with out-of-control disturbances, which can be led to catastrophic events. Resilience engineering (RE) helps the system to investigate disturbances and provides tools for preventive risk management.

Resilience is defined as the ability to retrieve the operation of a process before, during, and after a disturbance, such that it can keep on its normal state after a disaster [2]. The concept of RE has been studied in different scopes such as psychology, economics, and social science. RE has been used in several studies, including safety science, cognition, technology, reliability engineering, and system safety [3]. Most areas studying the RE include aviation, healthcare, chemical and petrochemical industries, nuclear power plants, and railway [3]. Most of these studies focused on risk management and extensively on the risks of personnel and safety-related processes. Other studies were conducted on different types of risks, such as financial services systems, terrorist operations in chemical factories, and natural disasters.

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Many studies have been conducted on resilience. Van de Vorm *et al.* [4] conducted a systematic and limited review focusing on how resilience has been used at organizational, individual, and work team levels. Re and Macchi [5] analyzed the evolution of Erik Hollnagel's hypotheses on safety being created by his academic trend. McDonald [6] used three criteria for evaluating safety management theories such as RE, including the centralization of application (the level of the system raised by the theory), the power of theory (the amount of system prediction and control allowed by the idea), and technological preparedness (the amount of theory testing and evaluation). According to McDonald's commitment to systemic thinking, RE has a better performance in centralization, while more emphasis is needed on two other criteria. Madni & Jackson [1] have introduced the main goals for RE: supporting the management of the balance between safety and productivity, measuring resilience, developing mechanisms to promote resilience in organizations. A study by Back *et al.* [7] stated that RE theory should be implemented at three levels of the individual, team, and organizational. While empirical studies on RE often describe workshop workers' activities, researchers often emphasize the recognition of factors for designing work systems; Such elements are the result of organizational resilience, which affects individual and team resilience. In a study by Dolif *et al.* [8], being conducted according to meteorological activities to predict heavy rains, several organizational and technical problems that cause decision-making difficulties were identified.

By reviewing the studies conducted in RE, the following discussion and analysis can be presented. Three points are essential. First, most studies have proposed conceptual approaches, and not many quantitative models have been developed. As such, measuring resilience remains a challenge. Second, the resilience approach's implementation is less developed in industry sections, especially process industries, which potentially have catastrophic risks. Thus, quantitative methods for implementing resilience in process industries have been neglected. Third, approaches to dealing with uncertainty in resilience are less developed. For example, process industries face several disturbances with high levels of uncertainty. How to deal with and model such uncertainties is less worked. In this study, an attempt is made to provide a mathematical model based on DEA in case of uncertainty to measure efficiency in process industries to fill less covered areas. In the following, by introducing Type-2 fuzzy numbers that can consider a high level of uncertainty, the relationship between applied background and Type-2 fuzzy numbers is described. In this paper, the DEA method is used, which also introduces fuzzy sets and the DEA approach.

Investigating the studies on resilience engineering indicates that mathematical programming can be used for measuring the efficiency of process industries. DEA is a mathematical programming technique to estimate efficiency. The first DEA model named CCR, was provided by Charnes, Cooper, and Rhodes in 1987 [9]. In this model, the goal was measuring and comparing the comparative efficiency of municipalities, universities, medical centers, etc. with several similar inputs and outputs. After presenting the first DEA model, other models were provided by the researchers, including the BCC (Banker-Charnes-Cooper), FDH (free disposal hull), and SBM (slack-based measure) models. Since there are few quantitative studies on resilience engineering, DEA can be used for measuring efficiency. On the other hand, the process industry's uncertainty makes it possible to use a fuzzy approach for modeling.

Zadeh presented the theory of Type-1 fuzzy sets in 1965 [10]. Type-1 fuzzy sets have a definite membership value in the range [0-1]. The development of a Type-1 fuzzy set becomes a Type-2 fuzzy set. The membership function of Type-2 fuzzy sets is a fuzzy number being bounded in the interval [0-1]. The applications of Type-2 fuzzy sets were studied in many research pieces, such as [11]–[13].

The application background of Type-2 fuzzy sets can be divided into three main categories: manufacturing, service, and information and communication technology (ICT) [14]. The main focus in manufacturing is to provide the best quality of goods that can be released at the best possible time, with the lowest cost considering given constraints. Type-2 fuzzy applications in manufacturing include these domains: control of mobile robots, control of converters, plant monitoring and detection, speed control of diesel engines, coiler temperature prediction, etc. Service systems are designed to meet the needs, wants, and desires of customers. The application of Type-2 fuzzy sets in service operations and industries are included in these areas: medical diagnosis, traffic control, electricity distribution, management of water demand, knowledge presentation, etc. In many businesses, ICT has been evolving into a way of doing business. Domains of Type-2 fuzzy sets applications in ICT consist of pattern recognition, weather forecasting, image processing, information retrieval, etc. A substantial amount of uncertainties available in the study that can be handle by Type-2 fuzzy sets.

Also, to show the appropriateness of applying the fuzzy and DEA method, we present related literature. Fuzzy sets help the DEA method to consider uncertain input and output data [15]. Fuzzy DEA has been extensively studied in theory and methodology. We can divide the applications of the fuzzy DEA into two categories of efficiency analysis and decision making. In the efficiency analysis approach, some examples include measuring the university libraries' fuzzy efficiency, electric distribution companies, hydrogen energy technologies, innovation activities in the electric equipment industry, reverse supply chain, chemical industry, cement industry, railways, mining industry, and hospitals. In decision making and alternative improvement category, some instances include developing fuzzy DEA and grey analysis to evaluate semiconductor companies, combining the fuzzy DEA with a fuzzy measure to assess considerable large-cap equity mutual funds, integrating fuzzy DEA and the standard indicators in the wireless communication industry, and evaluating and improving the efficiency of a safety management system in powerplants.

As mentioned before, quantitative studies on resilience engineering can expand its applications in the real world. By studying the literature of resilience, it can be seen that many researchers developed the concept without expanding a quantitative model [16-18]. So, introducing a mathematical model can fill this gap. Meanwhile, when we want to apply resilience engineering in the real world, especially in process industries, we confront a level of uncertainty. In conclusion, this study's motivation is to develop a quantitative viewpoint through DEA mathematical programming, considering the Type-2 fuzzy approach.

In this study, first, the resilience engineering factors are discussed. Then by designing a questionnaire, the required data are collected from an oil refinery in Iran, a critical process industry. Due to the uncertainty in the collected data, the DEA method is used with Type-2 fuzzy sets to analyze the refinery units' efficiency. It means that due to the significant uncertainty in the assessment of resilience factors, we used Type-2 fuzzy sets. Type-2 fuzzy sets have more power in modeling uncertainties than Type-1 fuzzy sets. For this purpose, the new concept of credibility in Type-2 fuzzy sets and chance-constrained are employed. Also, this can be considered as the novelty of this study. Finally, the results are verified by performing the sensitivity analysis, and a comparative analysis validates the presented method. The new approach presented in this study is innovative in the sense of its application in refineries. Furthermore, it has good robustness for analyzing the efficiency of resilience.

In the second section of this study, the DEA method, Type-2 fuzzy sets, type reduction, and DEA with chance-constrained are presented. Moreover, resilience factors are introduced in this section. In the third section, the new approach presented in this study is described. In the fourth section, an example of the real case of an oil refinery in Iran is analyzed as a numerical example. Section 5 concludes the study.

II. PRELIMINARIES

A. DATA ENVELOPMENT ANALYSIS

The CCR model, which is the initial DEA model, is considered as a basic model for other DEA developments. The output-oriented DEA model aims to maximize outputs by keeping the inputs constant. The output-oriented model of CCR for assessing the efficiency of n decision-making units (DMU) is a model (1):

$$\begin{aligned} &\text{Max} \ \sum u_r y_{r\circ} \\ &\text{st:} \ \sum v_i x_{i\circ} = 1 \\ &\quad \frac{\sum u_r y_{rj}}{\sum v_i x_{ij}} \leq 1 \quad (j = 1, 2, \ldots, n) \\ &\quad u_r, v_i \geq 0 \end{aligned} \tag{1}$$

In model (1), u_r and v_i are the weight factors of inputs and outputs, respectively. The index j represents the DMU, and this model is used for evaluating DMU$_\circ$. $x_{1j}, x_{2j}, \ldots$ and $y_{1j}, y_{2j}, \ldots$ are the inputs and outputs, respectively. The first constraint indicates that inputs are fixed. The second constraint shows that the maximum efficiency is equal to one. The objective is to maximize the outputs. In this study, it is assumed that as the inputs change, the outputs change in the same proportion. Therefore, the CCR model is used [19].

B. TYPE-1 FUZZY SETS

$\tilde{A}$ is a Type-1 fuzzy set, since that its membership function is defined as follows [20]:

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) : \mu\tilde{A}(x) \in [0, 1], \forall x \in X \right\} \tag{2}$$

Zadeh [21] introduced the possibility theory for interpreting the uncertainty degree of fuzzy set members. Membership degree x in set $\tilde{A}(\mu_{\tilde{A}}(x))$ is called as the possibility that x belongs to the set. Possibility measure (Pos) for a fuzzy event $\{\tilde{\xi} \in B\}, B \subset R$ is explained as $\text{Pos}\{\tilde{\xi} \in B\} = \sup_{x \in B} \mu_{\tilde{\xi}}(x)$ and necessity measure (Nec) is described as $\text{Nec}\{\tilde{\xi} \in B\} = 1 - \text{Pos}\{\tilde{\xi} \in B^c\} = 1 - \sup_{x \in B^c} \mu_{\tilde{\xi}}(x)$, while generalized credibility measure (Cr) for $\{\tilde{\xi} \in B\}$ is defined as follow for a non-normal fuzzy variable [22]:

$$\begin{aligned} \text{Cr}\{\tilde{\xi} \in B\} &= \frac{1}{2} \left(\sup_{x \in R} \mu\tilde{\xi}(x) + \sup_{x \in B} \mu\tilde{\xi}(x) - \sup_{x \in B^C} \mu\tilde{\xi}(x) \right) \\ &= \frac{1}{2} \left(\sup_{X \in R} \mu\tilde{\xi}(x) + \text{Pos} - (1 - \text{Nec}) \right) \end{aligned} \tag{3}$$

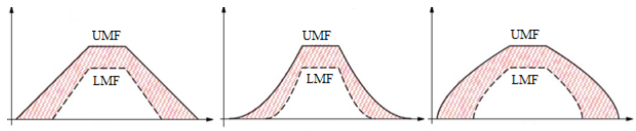

FIGURE 1. Type-2 trapezoidal fuzzy numbers.

C. TYPE-2 FUZZY SETS

Definition 1: A Type-2 fuzzy set $\tilde{\tilde{A}}$ is defined by a Type-2 membership function as follows:

$$\tilde{\tilde{A}} = \left\{ ((x, u); \mu_{\tilde{\tilde{A}}}(x, u)) \, | \forall x \in X, \forall u \in J_x \subseteq [0, 1], \ 0 \leq \mu_{\tilde{\tilde{A}}}(x, u) \leq 1 \right\} \tag{4}$$

where x is the domain of $\tilde{\tilde{A}}$, $\mu_{\tilde{\tilde{A}}}$ is linked to the membership function (the second membership function) of $\tilde{\tilde{A}}$ and J_x is an interval between zero and one, determining the initial membership function. A Type-2 fuzzy set can also be shown as follows:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x, u)/(x, u) \tag{5}$$

where we have $J_x \subseteq [0, 1]$ and $\int \int$ determines the union of acceptable values of x and u.

Definition 2: If $\tilde{\tilde{A}}$ is a Type-2 fuzzy set and all $\mu_{\tilde{\tilde{A}}}(x, u) = 1$, then $\tilde{\tilde{A}}$ is defined as an exceptional called interval Type-2 fuzzy set. An interval Type-2 fuzzy set $\tilde{\tilde{A}}$ can be considered as a specific type of Type-2 fuzzy sets as follow:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} 1/(x, u) \tag{6}$$

so that, $J_x \subseteq [0, 1]$.

Note: Interval Type-2 fuzzy sets are more general than other interval-valued fuzzy sets like interval-valued intuitionistic fuzzy sets.

Definition 3: The footprint of uncertainty (FOU) refers to a limited uncertainty area for the initial membership function which, is the overall union of initial membership functions. FOU is described by the upper and lower membership functions, UMF and LMF, respectively, that each of them is Type-1 fuzzy sets. An example of Type-2 trapezoidal fuzzy numbers is depicted in Fig. 1.

A Type-2 triangular fuzzy variable is an expansion of the Type-1 triangular fuzzy variable [23]. In a Type-2 triangular fuzzy variable, $\tilde{\xi} = (r_1, r_2, r_3; \theta_l, \theta_r)$, θ_l and θ_r indicate the spreads of primary possibilities.

Qin *et al.* [22] defined critical values (CV) for $\tilde{\xi}$ as optimistic CV (CV*), pessimistic CV (CV_*) and

CV as follows:

$$CV[\tilde{\xi}] = \sup_{\alpha\in[0,1]} [\alpha \wedge Pos\{\tilde{\xi} \geq \alpha\}] \quad (7)$$

$$CV_*[\tilde{\xi}] = \sup_{\alpha\in[0,1]} [\alpha \wedge Nec\{\tilde{\xi} \geq \alpha\}] \quad (8)$$

$$CV[\tilde{\xi}] = \sup_{\alpha\in[0,1]} [\alpha \wedge Cr\{\tilde{\xi} \geq \alpha\}] \quad (9)$$

Example 1. Suppose that ξ is a discrete random fuzzy variable and has the following possibility distribution:

$$\xi \sim \begin{pmatrix} r_1 & r_2 & \cdots & r_n \\ \mu_1 & \mu_2 & \cdots & \mu_n \end{pmatrix}$$

which, $r_i \in [0, 1]$ and $\max_{i=1}^{n} \mu_i = 1$. Let ξ has the subsequent possibility distribution:

$$\xi \sim \begin{pmatrix} 0.1 & 0.4 & 0.7 & 0.8 \\ 0.2 & 1 & 0.4 & 0.7 \end{pmatrix}$$

Then, according to the definition of Pos, Nec, and Cr in the previous section, we have:

$$Pos\{\xi \geq \alpha\} = \begin{cases} 1, & \text{if } \alpha \leq 0.4 \\ 0.7, & \text{if } 0.4 < \alpha \leq 0.8 \\ 0, & \text{if } 0.8 < \alpha \leq 1 \end{cases}$$

$$Nec\{\xi \geq \alpha\} = \begin{cases} 1, & \text{if } \alpha \leq 0.1 \\ 0.8, & \text{if } 0.1 < \alpha \leq 0.4 \\ 0, & \text{if } 0.4 < \alpha \leq 1 \end{cases}$$

and

$$Cr\{\xi \geq \alpha\} = \begin{cases} 1, & \text{if } \alpha \leq 0.1 \\ 0.9, & \text{if } 0.1 < \alpha \leq 0.4 \\ 0.35, & \text{if } 0.4 < \alpha \leq 0.8 \\ 0, & \text{if } 0.8 < \alpha \leq 1 \end{cases}$$

So, using Eqs. (7), (8), and (9), we have:

$$\begin{aligned} CV^*[\xi] &= \sup_{\alpha\in[0,1]} [\alpha \wedge Pos\{\xi \geq \alpha\}] \\ &= \sup_{\alpha\in[0,0.4]} [\alpha \wedge 1] \vee \sup_{\alpha\in(0.4,0.8]} [\alpha \wedge 0.7] \\ &\quad \vee \sup_{\alpha\in(0.8,1]} [\alpha \wedge 0] \\ &= 0.4 \vee 0.7 \vee 0 = 0.7 \\ CV_*[\xi] &= \sup_{\alpha\in[0,1]} [\alpha \wedge Nec\{\xi \geq \alpha\}] \\ &= \sup_{\alpha\in[0,0.1]} [\alpha \wedge 1] \vee \sup_{\alpha\in(0.1,0.4]} [\alpha \wedge 0.8] \\ &\quad \vee \sup_{\alpha\in(0.4,1]} [\alpha \wedge 0] \\ &= 0.1 \vee 0.4 \vee 0 = 0.4 \end{aligned}$$

and

$$\begin{aligned} CV[\xi] &= \sup_{\alpha\in[0,1]} [\alpha \wedge Cr\{\xi \geq \alpha\}] \\ &= \sup_{\alpha\in[0,0.1]} [\alpha \wedge 1] \vee \sup_{\alpha\in(0.1,0.4]} [\alpha \wedge 0.9] \\ &\quad \vee \sup_{\alpha\in(0.4,0.8]} [\alpha \wedge 0.35] \\ &\quad \vee \sup_{\alpha\in(0.8,1]} [\alpha \wedge 0] \\ &= 0.1 \vee 0.4 \vee 0.35 \vee 0 = 0.4 \end{aligned}$$

Also, it has to mention that $\mu_{\tilde{\xi}}(x)$ can be defined as [24]:

$$\mu_{\tilde{\xi}}(x) = \begin{cases} \left(\frac{x-r_1}{r_2-r_1} - \theta_l \frac{x-r_1}{r_2-r_1}, \frac{x-r_1}{r_2-r_1}, \frac{x-r_1}{r_2-r_1} + \theta_r \frac{x-r_1}{r_2-r_1}\right), & \text{if } x \in \left[r_1, \frac{r_1+r_2}{2}\right] \\ \left(\frac{x-r_1}{r_2-r_1} - \theta_l \frac{r_2-x}{r_2-r_1}, \frac{x-r_1}{r_2-r_1}, \frac{x-r_1}{r_2-r_1} + \theta_r \frac{r_2-x}{r_2-r_1}\right), & \text{if } x \in \left(\frac{r_1+r_2}{2}, r_2\right] \\ \left(\frac{r_3-x}{r_3-r_2} - \theta_l \frac{x-r_1}{r_3-r_2}, \frac{r_3-x}{r_3-r_2}, \frac{r_3-x}{r_3-r_2} + \theta_r \frac{x-r_2}{r_3-r_2}\right), & \text{if } x \in \left(r_2, \frac{r_2+r_3}{2}\right] \\ \left(\frac{r_3-x}{r_3-r_2} - \theta_l \frac{r_3-x}{r_3-r_2}, \frac{r_3-x}{r_3-r_2}, \frac{r_3-x}{r_3-r_2} + \theta_r \frac{r_3-x}{r_3-r_2}\right), & \text{if } x \in \left(\frac{r_2+r_3}{2}, r_3\right] \end{cases} \quad (10)$$

Due to the calculation complication of Type-2 fuzzy numbers, the proposed idea is the reduction of Type-2 fuzzy sets to Type-1 fuzzy sets [25]. Qin *et al.* [22] suggested a CV-based reduction method for reducing Type-2 fuzzy variables (T2FV) to Type-1 fuzzy variables (T1FV). Thus, if $\tilde{\xi}$ is T2FV with secondary possibility distribution function $\tilde{\mu}_{\tilde{\xi}}(x)$, then $CV^*[\tilde{\mu}_{\tilde{\xi}}(x)]$, $CV_*[\tilde{\mu}_{\tilde{\xi}}(x)]$, and $CV[\tilde{\mu}_{\tilde{\xi}}(x)]$ are used for obtaining T1FVs called optimistic, pessimistic, and CV reduction, respectively.

If $\tilde{\xi}$ is a Type-2 triangular fuzzy variable which explained as $\tilde{\xi} = (r_1, r_2, r_3; \theta_l, \theta_r)$, then using the optimistic CV reduction method, possibility distribution of ξ which is the reduction of $\tilde{\xi}$ can be defined as:

$$\mu_{\xi_{opt}}(x) = \begin{cases} \frac{(1+\theta_r)(x-r_1)}{r_2-r_1+\theta_r(x-r_1)}, & \text{if } x \in \left[r_1, \frac{r_1+r_2}{2}\right] \\ \frac{(1-\theta_r)x+\theta_r r_2 - r_1}{r_2-r_1+\theta_r(r_2-x)}, & \text{if } x \in \left(\frac{r_1+r_2}{2}, r_2\right] \\ \frac{(-1+\theta_r)x-\theta_r r_2+r_3}{r_3-r_2+\theta_r(x-r_2)}, & \text{if } x \in \left(r_2, \frac{r_2+r_3}{2}\right] \\ \frac{(1+\theta_r)(r_3-x)}{r_3-r_2+\theta_r(r_3-x)}, & \text{if } x \in \left(\frac{r_2+r_3}{2}, r_3\right] \end{cases} \quad (11)$$

Similarly, applying the pessimistic CV reduction method, the possibility distribution of ξ which is the reduction of $\tilde{\xi}$ can be defined as:

$$\mu_{\xi_{pes}}(x) = \begin{cases} \frac{(x-r_1)}{r_2-r_1+\theta_l(x-r_1)}, & \text{if } x \in \left[r_1, \frac{r_1+r_2}{2}\right] \\ \frac{x-r_1}{r_2-r_1+\theta_l(r_2-x)}, & \text{if } x \in \left(\frac{r_1+r_2}{2}, r_2\right] \\ \frac{r_3-x}{r_3-r_2+\theta_l(x-r_2)}, & \text{if } x \in \left(r_2, \frac{r_2+r_3}{2}\right] \\ \frac{r_3-x}{r_3-r_2+\theta_l(r_3-x)}, & \text{if } x \in \left(\frac{r_2+r_3}{2}, r_3\right] \end{cases} \quad (12)$$

Also employing the CV reduction method, the possibility distribution of ξ which is the reduction of $\tilde{\xi}$ can be defined as:

$$\mu_{\xi CV}(x) = \begin{cases} \dfrac{(1+\theta_r)(x-r_1)}{r_2-r_1+2\theta_r(x-r_1)}, & \text{if } x \in \left[r_1, \dfrac{r_1+r_2}{2}\right] \\ \dfrac{(1-\theta_l)x+\theta_l r_2 - r_1}{r_2-r_1+2\theta_l(r_2-x)}, & \text{if } x \in \left(\dfrac{r_1+r_2}{2}, r_2\right] \\ \dfrac{(-1+\theta_l)x-\theta_l r_2+r_3}{r_3+r_2+2\theta_l(x-r_2)}, & \text{if } x \in \left(r_2, \dfrac{r_2+r_3}{2}\right] \\ \dfrac{(1+\theta_r)(r_3-x)}{r_3-r_2+2\theta_r(r_3-x)}, & \text{if } x \in \left(\dfrac{r_2+r_3}{2}, r_3\right] \end{cases} \quad (13)$$

Example 2. Suppose $\tilde{\xi} = (\tilde{2}, \tilde{3}, \tilde{4}; 0.5, 1)$ and its support as Fig. 2. Let ξ_1, ξ_2, and ξ_3 are the reduction of $\tilde{\xi}$ acquired by the optimistic, pessimistic, and CV methods, respectively.

We have:

$$\mu_{\xi_1}(x) = \begin{cases} 2-\dfrac{2}{x-1}, & \text{if } x \in \left[2, \dfrac{5}{2}\right] \\ \dfrac{1}{4-x}, & \text{if } x \in \left(\dfrac{5}{2}, 3\right] \\ \dfrac{1}{x-2}, & \text{if } x \in \left(3, \dfrac{7}{2}\right] \\ 2-\dfrac{2}{5-x}, & \text{if } x \in \left(\dfrac{7}{2}, 4\right] \end{cases}$$

$$\mu_{\xi_2}(x) = \begin{cases} 2-\frac{4}{x}, & \text{if } x \in \left[2, \frac{5}{2}\right] \\ \dfrac{6}{5-x}-2, & \text{if } x \in \left(\dfrac{5}{2}, 3\right] \\ \dfrac{6}{x-1}-2, & \text{if } x \in \left(3, \dfrac{7}{2}\right] \\ 2-\dfrac{4}{6-x}, & \text{if } x \in \left(\dfrac{7}{2}, 4\right] \end{cases}$$

and

$$\mu_{\xi_3}(x) = \begin{cases} \dfrac{2(x-2)}{2x-3}, & \text{if } x \in \left[2, \dfrac{5}{2}\right] \\ \dfrac{x-1}{8-2x}, & \text{if } x \in \left(\dfrac{5}{2}, 3\right] \\ \dfrac{5-x}{2(x-2)}, & \text{if } x \in \left(3, \dfrac{7}{2}\right] \\ \dfrac{2(4-x)}{9-2x}, & \text{if } x \in \left(\dfrac{7}{2}, 4\right] \end{cases}$$

In the CV-based reduction method, the obtained T1FV is not necessarily normal but is a general fuzzy variable [25]. The following theorem is used for finding the equivalent forms of constraints for Type-2 triangular fuzzy variables.

Theorem 1 [22]: If $\tilde{\xi}_i = (r_1^i, r_2^i, r_3^i; \theta_{l,i}, \theta_{r,i})$ is a Type-2 triangular fuzzy variable and ξ_i is the reduced type of $\tilde{\xi}_i$ by CV reduction for $i = 1, 2, \ldots, n$, assuming $\xi_1, \xi_2, \ldots, \xi_n$ are mutually independent, and $k_i \geq 0$ then we have:

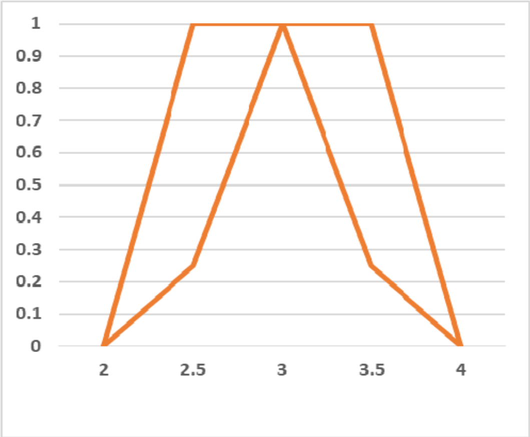

FIGURE 2. Support of $\tilde{\xi}$ in example 2.

(*i*) For generalized credibility level $\alpha\epsilon(0,0.5]$, if $\alpha\epsilon(0,0.25]$ then $\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i \leq t\} \geq \alpha$ will be equal to:

$$\sum_{i=1}^{n} \frac{(1-2\alpha+(1-4\alpha)\theta_{r,i})k_i r_1^i + 2\alpha k_i r_2^i}{1+(1-4\alpha)\theta_{r,i}} \leq t \quad (14)$$

And if $\alpha\epsilon(0.25,0.5]$ then $\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i \leq t\} \geq \alpha$ will be equal to:

$$\sum_{i=1}^{n} \frac{(1-2\alpha)k_i r_1^i + (2\alpha+(4\alpha-1)\theta_{l,i})k_i r_2^i}{1+(4\alpha-1)\theta_{l,i}} \leq t \quad (15)$$

(*ii*) For generalized credibility level $\alpha\epsilon(0.5,1]$, if $\alpha\epsilon(0.5,0.75]$ then $\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i \leq t\} \geq \alpha$ will be equal to:

$$\sum_{i=1}^{n} \frac{(2\alpha-1)k_i r_3^i + (2(1-\alpha)+(3-4\alpha)\theta_{l,i})k_i r_2^i}{1+(3-4\alpha)\theta_{l,i}} \leq t \quad (16)$$

And if $\alpha\epsilon(0.75,1]$, then $\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i \leq t\} \geq \alpha$ will be equal to:

$$\sum_{i=1}^{n} \frac{(2\alpha-1+(4\alpha-3)\theta_{r,i})k_i r_3^i + 2(1-\alpha)k_i r_2^i}{1+(4\alpha-3)\theta_{r,i}} \leq t \quad (17)$$

In this way, for constraint $\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i \geq t\} \geq \alpha$, we have [23]:

$$\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i \geq t\} \geq \alpha \Rightarrow \tilde{C}r\{\sum_{i=1}^{n} -k_i\xi_i \leq -t\} \geq \alpha \Rightarrow \tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i' \leq t'\} \geq \alpha \quad (18)$$

so that $\xi_i' = -\xi_i$ is reduced of $-\tilde{\xi}_i = (-r_3^i, -r_2^i, -r_1^i; \theta_{r,i}, \theta_{l,i})$ and $t' = -t$.

For the defuzzification of the T2FV, first, the CV-based reduction method is used to transform T2FV to T1FV, and then the centroid method [26] is used to obtain the crisp values [25].

D. DEA MODEL FORMULATION WITH TYPE-2 FUZZY VARIABLES

The CCR model with output orientation was introduced in section II. Now, it is supposed that inputs and outputs are T2FVs as:

$$\begin{aligned} &\text{Max } \Sigma u_r \tilde{y}_{ro} \\ &\text{st: } \sum v_i \tilde{x}_{io} = 1 \\ &\quad \sum u_r \tilde{y}_{rj} - \Sigma v_i \tilde{x}_{ij} \le 0 \quad (j = 1, 2, \ldots, n) \\ &\quad u_r, v_i \ge 0 \end{aligned} \tag{19}$$

where $\tilde{x}_{ij}$ is the Type-2 fuzzy input parameter for input i and DMU j, $\tilde{y}_{rj}$ is the Type-2 fuzzy output parameter for output r and DMU j ($\tilde{x}_{io}$, $\tilde{y}_{ro}$ are the input and output of target DMU, respectively). u_r and v_i are the weight of inputs and outputs, respectively.

Suppose that $\tilde{x}'_{ij}$ and $\tilde{y}'_{rj}$ are the reduced T1FVs of the T2FVs $\tilde{x}_{ij}$, $\tilde{y}_{rj}$ respectively, using a CV-based reduction method (may not be normalized). To solve the model (19), the chance-constrained programming method is applied. Chance-constrained programming in the fuzzy environment was used in many studies with Cr [24].

Since reduced fuzzy parameters $\tilde{x}'_{ij}$, $\tilde{y}'_{rj}$ may not be normalized, generalized credibility is used, and the chance-constrained programming model is formulated as:

$$\text{Max}(\text{Max}\,\bar{f}) \tag{20}$$

$$\text{st: } \tilde{Cr}\left\{\sum_{r=1}^{S} u_r \tilde{y}'_{r\circ} \ge \bar{f}\right\} \ge \alpha \tag{21}$$

$$\tilde{Cr}\left\{\sum_{i=1}^{m} v_i \tilde{x}'_{1} = 1\right\} \ge \beta \quad i = 1 \tag{22}$$

$$\tilde{Cr}\left\{\sum_{r=1}^{S} u_r \tilde{y}'_{rj} - \sum_{i=1}^{m} v_i \tilde{x}'_{1j} \le 0\right\} \ge \gamma_j; \quad (j = 1, 2, \ldots, n) \tag{23}$$

$$u_r, v_i \ge 0 \tag{24}$$

where Max $\bar{f}$ represents the maximum possible value with an objective equal to or more than it with generalized credibility of at least α $(0 \le \alpha \le 1)$. β and γ_j $(0 < \beta, \gamma_j \le 1)$ are the pre-determined generalized credibility levels of satisfaction for the second and third constraints.

To obtain the crisp equivalence, assume that $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ are the Type-2 triangular fuzzy sets being mutually independent defined as $\tilde{x}_{ij} = \left(x^1_{ij}, x^2_{ij}, x^3_{ij}; \theta_{l,ij}, \theta_{r,ij}\right)$ and $\tilde{y}_{rj} = \left(y^1_{rj}, y^2_{rj'}, y^3_{rj'}; \theta'_{l,rj}, \theta'_{r,rj}\right)$ also $\tilde{x}'_{ij}$ and $\tilde{y}'_{rj}$ are the reduction by CV reduction method, respectively. From Theorem 1, the formulation of the chance-constrained method can be turned to crisp equivalent parametric programming:

Case-I: for $0 < \alpha \le 0.25$ the equivalent parametric programming problem for (20)-(24) is (for proof, see Appendix):

$$\text{Max } \left[\sum_{r=1}^{s} \frac{(1 - 2\alpha + (1 - 4\alpha)\theta'_{l,ro})y^3_{ro}u_r + 2\alpha u_r y^2_{ro}}{1 + (1 - 4\alpha)\theta'_{l,ro}}\right] \tag{25}$$

$$\text{s.t. } \sum_{i=1}^{m} \frac{(1 - 2\beta + (1 - 4\beta)\theta_{r,ij})v_i x^1_{io} + 2\beta v_i x^2_{io}}{1 + (1 - 4\beta)\theta_{r,ij}} \le 1 \tag{26}$$

$$\sum_{i=1}^{m} \frac{(1 - 2\beta + (1 - 4\beta)\theta_{l,ij})v_i x^3_{io} + 2\beta v_i x^2_{io}}{1 + (1 - 4\beta)\theta_{l,ij}} \ge 1 \tag{27}$$

$$\sum_{r=1}^{s} \frac{(1 - 2\gamma_j + (1 - 4\gamma_j)\theta'_{r,rj})u_r y^1_{rj} + 2\gamma_j u_r y^2_{rj}}{1 + (1 - 4\gamma_j)\theta'_{r,rj}} \tag{28}$$

$$- \sum_{i=1}^{m} \frac{(1 - 2\gamma_j + (1 - 4\gamma_j)\theta_{r,ij})v_i x^1_{ij} + 2\gamma_j v_i x^2_{ij}}{1 + (1 - 4\gamma_j)\theta_{r,ij}} \le 0; \; j = 1, 2, \ldots, n$$

$$u_r, v_i \ge 0 \tag{29}$$

The equivalent parametric programming problem for the model (20)-(24) can be obtained for similar cases of $0.25 < \alpha \le 0.5$, $0.5 < \alpha \le 0.75$ and $0.75 < \alpha \le 1$ similarly.

III. SOLUTION

Model (25) – (29) is linear in decision variables u_r and v_i and can be optimized by conventional solvers like Lingo.

IV. NUMERICAL EXAMPLE

A. RESILIENCE FACTORS

The main resilience factors were extracted from several studies [27]–[29]. Six classic resilience factors introduced by Hollnagel [30] involve the obligation of a top manager, reportage, knowledge acquisition, consciousness, readiness, and plasticity, and other factors include just culture, change management, risk management/assessment, safety management system, accident investigation, employee participation, and competence. The reasons behind choosing these factors are: (1) they are cornerstones of creating resilience engineering; (2) they emphasize dynamic relation between employees, technology, and management; (3) they attempt to prevent focusing just on negative consequences [31]. A brief description of these factors is as follows:

1) TOP MANAGEMENT COMMITMENT

Top directors should address and resolve the concerns and issues related to human resource efficiency [28]. Improving safety, reducing costs, and increasing profits should be the main objectives of the organization.

2) REPORTING CULTURE

With reporting culture, which is an atmosphere of trust, employees have self-confidence in reporting the cases related to safety without the fear to blame [27].

3) LEARNING

The system should learn from past accidents. The lessons learned should be taught to others to know how to operate safely[28].

4) AWARENESS

Managers should allow employees to know about system failures to be aware of what happened, why it happened, and what was the effects. All employees should be aware of the defensive position of the system [28].

5) PREPAREDNESS

A system that effectively identifies the problems and is prepared to deal with them [27].

6) FLEXIBILITY

It is the design of more flexible processes to deal with unexpected accidents [29].

7) JUST CULTURE

The atmosphere of trust in such a way to encourage individuals to report the concerns and issues related to safety [29].

8) CHANGE MANAGEMENT

When some changes are made to equipment, procedures, personnel, or operations, the best method is used to ensure that the risks are under control [27].

9) RISK MANAGEMENT/ASSESSMENT

A moral system for assessing the possible hazards which can occur in the activities [27].

10) SAFETY MANAGEMENT SYSTEM

A systemic method for preventive hazards includes constructioning required organizational structures, definitions of responsibilities, policies, and guidelines [27].

11) ACCIDENT INVESTIGATION

The process of collecting and analyzing the accident data systematically [27].

12) EMPLOYEE PARTICIPATION

To what extent the employees participate in decision-making and safety-related planning [27].

13) COMPETENCE

To what extent the employees are capable of fulfilling their assigned activities [5].

B. CASE STUDY

Here is a numerical example of an oil refinery in Iran being considered as a critical process industry. The refinery produces petroleum products using crude oil and natural gas. The main products of this refinery include liquefied gas, gasoline, naphtha, jet fuel, kerosene, diesel fuel, mazut, and sulfur. The operational units of this refinery involve two areas A and B, utility, and storage tanks.

The units of this refinery are as follows according to their supervision:

- Distillation in vacuum atmosphere (DMU1)
- Viscosity reduction, liquefied gas, gasoline treatment, and light naphtha refinement (DMU2)
- Hydrocracker and hydrogen (DMU3)
- Sulfur recovery, gas refinement with amino, sour water refinement, contaminated caustic neutralization (DMU4)
- Naphta refinement and catalytic conversion, Kerosene refinement (DMU5)
- Distillation in atmosphere and vacuum, and liquefied gas (DMU6)
- Tanks (DMU7)
- Desalination unit (DMU8)
- Electricity and steam (DMU9)
- Fuel, air, and nitrogen (DMU10)
- Water intake pond and pumping (DMU11)
- Crude oil receipt unit (DMU12)
- Recycling (DMU13)

TABLE 1. Linguistic terms and corresponding T2FVs.

Linguistic terms	T2FVs
Very Low (VL)	(0,0.1,0.2;0.9,1)
Low (L)	(0.2,0.3,0.4;0.9,1)
Medium (M)	(0.4,0.5,0.6;0.9,1)
High (H)	(0.6,0.7,0.8;0.9,1)
Very High (VH)	(0.8,0.9,1;0.9,1)

TABLE 2. The percent of participants in different organizational positions, education, work experience, and employment status.

		Percent of participants
Organizational positions	Head of a unit	5
	Shifting unit	8
	Unit director	16
	Control room	30
	Yard officer	41
Education	Diploma	20
	Associate degree	21
	BSc	50
	MSc	9
Work experience	< 5 years	15
	5-10 years	15
	10-20 years	52
	>20 years	18
Employment status	Formal contract	71
	Fixed-term contract	16
	Supplier contract	13

C. DESIGNING THE QUESTIONNAIRE

In this study, each of the units described in the previous section is considered as a DMU. First, the biography of respondents has been considered, and the following items were specified:

i. The position of respondents in the organization
ii. The educational level of respondents
iii. The work experience of respondents
iv. Employment status

Each of the resilience factors mentioned in the previous sections has been brought up as some questions. The answers to these questions are linguistic terms, and the Likert scale includes Very Low, Low, Medium, High, and Very High are used. Table 1 shows the linguistic variables with the corresponding T2FV [32]:

Eighty-six employees working at the refinery completed the questionnaire. Table 2 indicates the demographic of respondents based on their organizational position, level of education, work experience, and employment status.

D. VALIDITY AND RELIABILITY OF THE QUESTIONAIRE

A questionnaire was designed using the literature review of resilience engineering ([19], [31]) and then evaluated with a consultation with four academic advisors and four specialists in the HSE section of the refinery. It was attempted to deal with all features of resilience engineering in the design of the questionnaire. For instance, a sample question of each factor is given in Table 3. The factors presented in the questionnaire attempt to evaluate the refinery from different aspects.

1) QUESTIONNAIRE RELIABILITY TEST

To assess the reliability of the questionnaire, Cronbach's alpha was measured. Cronbach's alpha coefficient was presented by Cronbach, 1951, and is a general tool for measuring questionnaire reliability. Theoretically, the coefficient fluctuates in the range of [0-1], and the utility is to gain a higher value since it shows the high compatibility between questions [33].

2) QUESTIONNAIRE VALIDITY TEST

Validity is a measure of the study results' accuracy and indicates how well the questionnaire accurately measured what it intended to measure. There are several kinds of validity containing content validity and construct validity. Content validity typically answers whether the designed tool encompasses all the significant aspects of the concept being measured.

Lawshe presented the content validity ratio (CVR) that was extensively applied to measure content validity [34]. In this way, some experts are used to answer each of the tool items in three ranges including "essential", "useful but not necessary", and "not essential". CVR is calculated based on expert opinion as:

$$\mathrm{CVR} = \frac{\mathrm{n_e} - \frac{\mathrm{N}}{2}}{\frac{\mathrm{N}}{2}} \tag{30}$$

where n_e is the number of panelists who have given the "essential" answer, and N is the count of panelists. CVR varies in the range of [−1,1]. The average CVR among all items is used as a general evaluation of content validity. Construct validity represents the level to which an assessment computes what it says. Here, the construct validity is investigated through exploratory factor analysis based on principal components.

In this section, the results of the validity and reliability of the questionnaires are reported. For the measurement of the questionnaire's reliability, Cronbach's alpha was computed. None of the indicators have been deleted. All indices have an acceptable reliability level, and their α value is at least 0.7. The results of the reliability test are reported in Table 4.

Validities of content and construct are assessed to ensure that the questions are justifiable and consistent. CVR index was used to determine content validity. The CVR average value of items was computed as an index for the general content validity test. As well, the values of CVR are reported in Table 4. The results show that content validity is appropriate.

TABLE 3. Sample of questions.

Factor	Sample question
Change management	If a job-related change to the work guidelines occurs, I will be well and timely informed.
Risk management/ assessment	All employees are aware of the dangers they face.
Incident survey	The accidents that take place in my workplace are analyzed.
Culture	If there are any safety issues in my workplace, I can talk to my supervisor about it.
Reportage	Near misses can be announced without concerns
Employee participation	Employees contribute insights to others.
Flexibility	If the process fails for any reason, it can recover quickly and return to its steady state.
Obligation of managers	My chief is informed about work hazards.
Consciousness	In an emergency situation, the necessary procedures are easy to implement.
Readiness	The refinery's safety culture and rules are appropriate and can be used in the future.
Hazard procedures	At the refinery, safety performance is part of the employee performance appraisal system.
Knowledge acquisition	At the refinery, retraining courses are conducted regularly and timely.
Competence	The staff has the skills necessary to do the work.

TABLE 4. Cronbach's alpha and CVR.

Item	Cronbach's alpha	CVR	Item	Cronbach's alpha	CVR
1	0.8535	0.875	8	0.9244	0.893
2	0.7771	0.950	9	0.7223	0.833
3	0.7379	0.917	10	0.7962	0.875
4	0.8403	0.844	11	0.7727	0.813
5	0.7719	0.950	12	0.8084	0.917
6	0.7226	0.833	13	0.7903	0.938
7	0.8269	0.917			

For the assessment of construct validity, exploratory factor analysis was performed using principal component analysis with varimax rotation. The eigenvalues test and screen test consequences disclosed thirteen factors for resilience engineering. The Kaiser-Meyer-Olkin (KMO) statistics and Bartlett's test p-value were computed to display that the correlation matrix among at least some variables was positively correlated, and the data were sufficient to perform factor analysis. The results of KMO statistics and Bartlett's test p-value are depicted in Table 5. The values of the KMO statistics and Bartlett's test p-value show that the correlation matrix among the variables is positively correlated and that the data is sufficient to perform factor analysis. The factor analysis test for resilience engineering identifies thirteen factors. The factor loading values indicate the construct validity of the measures employed in this survey.

E. INPUT AND OUTPUT VARIABLES

In this study, the number of human resources in each DMU (x_1) and the complexity index of each unit (x_2) are used as

TABLE 5. Construct validity results.

Item		Loading	Item		Loading
Change management	Q1	0.988	Flexibility	Q1	0.896
	Q2	0.819		Q2	0.802
	Q3	0.913		Q3	0.798
	Q4	0.991		Q4	0.896
	Q5	0.918		Q5	0.984
	Q6	0.826		Q6	0.987
	Q7	0.825	Top management commitment	Q1	0.976
	Q8	0.822		Q2	0.979
Risk management/ assessment	Q1	0.973		Q3	0.963
	Q2	0.814		Q4	0.940
	Q3	0.983		Q5	0.867
	Q4	0.867		Q6	0.909
	Q5	0.794		Q7	0.925
Accident investigation	Q1	0.827	Awareness	Q1	0.888
	Q2	0.829		Q2	0.680
	Q3	0.999		Q3	0.937
Just culture	Q1	0.875	Preparedness	Q1	0.909
	Q2	0.835		Q2	0.775
	Q3	0.809		Q3	0.968
	Q4	0.983		Q4	0.997
	Q5	0.878	Safety Management System	Q1	0.902
	Q6	0.758		Q2	0.763
	Q7	0.750		Q3	0.965
	Q8	0.966		Q4	0.997
Reporting culture	Q1	0.968	Learning	Q1	0.841
	Q2	0.979		Q2	0.804
	Q3	0.776		Q3	0.804
	Q4	0.930		Q4	0.962
	Q5	0.999		Q5	0.987
Employee participation	Q1	0.889		Q6	0.964
	Q2	0.680	Competence	Q1	0.955
	Q3	0.936		Q2	0.755
				Q3	0.988
				Q4	0.940
Concept: Resilience engineering (KMO = 0.639, Bartlett's test p-value<0.000), Number of factors = 13					

fixed inputs. The complexity index determines the criticality degree of the process of each unit in comparison with others. Output variables involve resilience factors. Table 6 indicates the input and output data:

F. NUMERICAL RESULT

Here, the suggested approach is illustrated numerically. For this purpose, a case study of an oil refinery has been used, as mentioned previously.

F.1) In the first section, all outputs are considered as a Type-2 fuzzy number. Because there is a level of uncertainty associated with the measurement of resilience factors, we considered the outputs as a Type-2 fuzzy number that has a high capability of modeling uncertainty. To solve the mathematical programming model, the defuzzified values of outputs are calculated. For this purpose, firstly, the CV

TABLE 6. Inputs and outputs.

DMU	Outputs													Inputs	
	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	x_1	x_2
1	M	M	H	M	M	H	M	M	M	M	M	M	H	34	0.909
2	M	H	H	H	H	H	H	H	M	H	H	H	H	36	1
3	M	H	H	H	H	H	H	H	M	M	H	M	H	45	0.909
4	M	H	H	H	H	H	H	H	M	H	H	H	H	64	0.909
5	M	H	H	M	M	H	M	M	M	M	M	H	H	40	1
6	M	H	H	H	H	H	H	H	H	M	M	M	H	40	0.952
7	M	H	H	H	H	H	H	H	H	H	M	M	H	59	1
8	M	M	M	M	M	M	M	M	H	M	M	M	H	42	1
9	H	H	H	H	H	VH	H	H	H	H	H	H	H	11	1
10	H	H	VH	H	H	VH	H	VH	H	H	H	H	VH	14	1
11	M	M	H	M	M	M	M	M	M	M	M	M	H	22	1
12	M	H	H	H	H	M	H	M	M	M	M	M	H	26	1
13	H	H	H	H	H	H	H	H	H	H	H	H	H	16	1

reduction method is applied to reduce T2FVs $\tilde{y}_{rj}$ to T1FVs, and then the centroid method [35] ($\sum_x x\,\mu_{\tilde{A}}(x)/\sum_x \mu_{\tilde{A}}(x)$) is used to obtain the crisp values. Such crisp values are shown as y^c_{rj} and are received as:

$$''\text{Very Low}'' \equiv y^c_{rj} = 1, \quad ''\text{Low}'' \equiv y^c_{rj} = 3,$$
$$''\text{Medium}'' \equiv y^c_{rj} = 5$$
$$''\text{High}'' \equiv y^c_{rj} = 7, \quad ''\text{Very High}'' \equiv y^c_{rj} = 9$$

Using the model (1) and such crisp values, the optimal solution is obtained by Lingo software and is presented in table 7:

F.2) To illustrate the model presented in this study, the mathematical model is regarded with Type-2 fuzzy outputs. For the chance-constrained programming model (25)-(29), we took pre-determined general credibility levels $\alpha = \beta = \gamma_j = 0.9$. So, the equivalent parametric programming problem is as:

$$\text{Max}\left[\sum_{r=1}^{13}\frac{(0.8+0.6\theta_{1,i})\,u_r y^3_{r\circ}+0.2u_r y^2_{r\circ}}{1+0.6\theta_{1,i}}\right] \tag{31}$$

$$\text{s.t.}\ \sum_{i=1}^{2} v_i x_i = 1 \tag{32}$$

$$\sum_{r=1}^{13}\frac{(0.8+0.6\theta_{r,ij})\,u_r y^3_{rj}+0.2u_r y^2_{rj}}{1+0.6\theta_{r,rj}} \tag{33}$$

$$-\sum_{i=1}^{2}\frac{(0.8+0.6\theta_{r,ij})\,v_i x^3_{ij}+0.2v_i x^2_{ij}}{1+0.6\theta_{r,ij}}$$

$$\leq 0;\ j = 1, 2, \ldots, 13$$

$$u_r \geq 0;\quad r = 1, 2, \ldots, 13$$

$$v_i \geq 0;\quad i = 1, 2 \tag{34}$$

Solving this model using Lingo software, the efficiency scores are obtained as Table 8.

The Pearson correlation coefficient was calculated as 0.969 in the results of this section and section F.1 using

TABLE 7. Efficiency scores with crisp values.

DMU	Efficiency score	DMU	Efficiency score
1	0.831438	8	0.938018
2	0.955377	9	1
3	1	10	1
4	1	11	0.763556
5	0.937708	12	0.95059
6	1	13	0.99
7	0.941		

TABLE 8. Efficiency scores with Type-2 fuzzy outputs.

DMU	Efficiency score	DMU	Efficiency score
1	0.908245	8	0.9368013
2	0.9550211	9	0.9989636
3	0.998721	10	0.9989696
4	0.9987259	11	0.841882
5	0.9372683	12	0.9493656
6	0.9987208	13	0.9917398
7	0.9427944		

TABLE 9. Comparison analysis results.

DMU	Crisp	Type-1	Type-2
1	0.831438	0.890565	0.908245
2	0.955377	0.959574	0.955021
3	1	1	0.998721
4	1	1	0.998726
5	0.937708	0.953407	0.937268
6	1	1	0.998721
7	0.941	1	0.942794
8	0.938018	0.914052	0.936801
9	1	1	0.998964
10	1	1	0.99897
11	0.763556	0.809524	0.841882
12	0.95059	0.956482	0.949366
13	0.99	1	0.99174

Minitab software, indicating the high correlation and verification of the model. Furthermore, the proposed model in section F.2 is much less computational volume than the model in section F.1. Also, this model has a higher degree of resolution. As observed in section F.1, five DMUs have an efficiency score of one, and other models like the Andersen-Petersen [36] should distinguish between them, which needs a higher computational volume.

More importantly, in the proposed model, the fuzzy data is preserved considering the DEA model's outputs as Type-2 fuzzy numbers; But by converting Type-2 to Type-1 fuzzy variable and then to crisp values, data fuzziness will be missed.

G. COMPARATIVE ANALYSIS

To show the rationality and effectiveness of the proposed method, we make a comparative analysis with classical approaches. First, we consider the crisp data, and then the classical Type-1 fuzzy numbers [37] are studied. Table 9 shows the results.

Fig. 3 shows the correlation matrix between the proposed method and classical crisp and Type-1 approaches. It can be seen that the proposed method has a high correlation

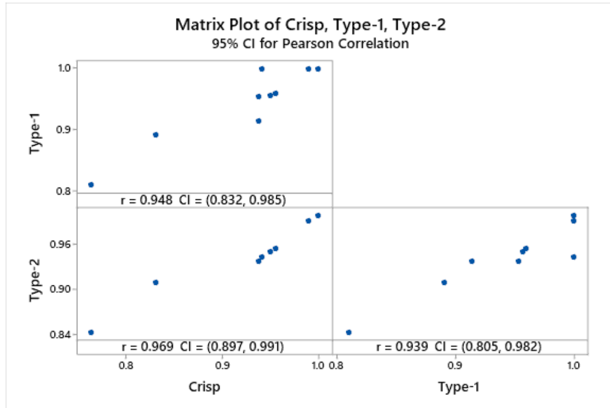

FIGURE 3. Correlation between the proposed and classical methods.

TABLE 10. Sensitivity analysis results.

α	0.9			0.8	0.9	0.95
γ_j	0.8	0.9	0.95	0.9		
	Eff.	Eff.	Eff.	Eff.	Eff.	Eff.
DMU1	0.933	0.908	0.902	0.881	0.908	0.917
DMU2	0.981	0.955	0.948	0.929	0.955	0.964
DMU3	1.028	0.999	0.991	0.972	0.999	1.008
DMU4	1.027	0.999	0.991	0.972	0.999	1.008
DMU5	0.963	0.937	0.930	0.912	0.937	0.946
DMU6	1.027	0.999	0.991	0.972	0.999	1.008
DMU7	0.969	0.943	0.936	0.917	0.943	0.951
DMU8	0.943	0.937	0.936	0.911	0.937	0.945
DMU9	1.026	0.999	0.993	0.977	0.999	1.008
DMU10	1.026	0.999	0.993	0.977	0.999	1.008
DMU11	0.865	0.842	0.835	0.816	0.842	0.851
DMU12	0.976	0.949	0.942	0.924	0.949	0.958
DMU13	1.019	0.992	0.984	0.965	0.992	1.001

with classical models. Meanwhile, it has more discrimination levels than others. This can validate the proposed method.

H. SENSITIVITY ANALYSIS

To illustrate the validity and logical rightness of the proposed model, a sensitivity analysis is given for the numerical results presented in section F.2. By changing the credibility levels in the model (25) - (29), the changes in efficiency scores are shown in Table 10 and Fig. 4:

Fig. 4 shows that the efficiency scores reduce by increasing γ_j (credibility levels of constraints) as α (credibility level of the objective function) is constant. The reason is that with the increase of γ_j, the defuzzified coefficients of u_r and v_i in constraint (28) are also increased, and as a result, the ratio of outputs to inputs, which is the efficiency score, increases. Then, the efficiency scores increase by increasing α with constant values of γ_j. The reason is that the defuzzified coefficients of objectives grow with credibility level α.

1) MANAGERIAL INSIGHT

The analysis enables managers to recognize and specify the factors that have the most significant impact on resilience engineering efficiency; thereby, they can increase their organization's efficiency. In fact, by incorporating the uncertainties described in the modeling of resilience engineering efficiency analysis, managers can identify their organization's strengths and weaknesses and take steps to improve them.

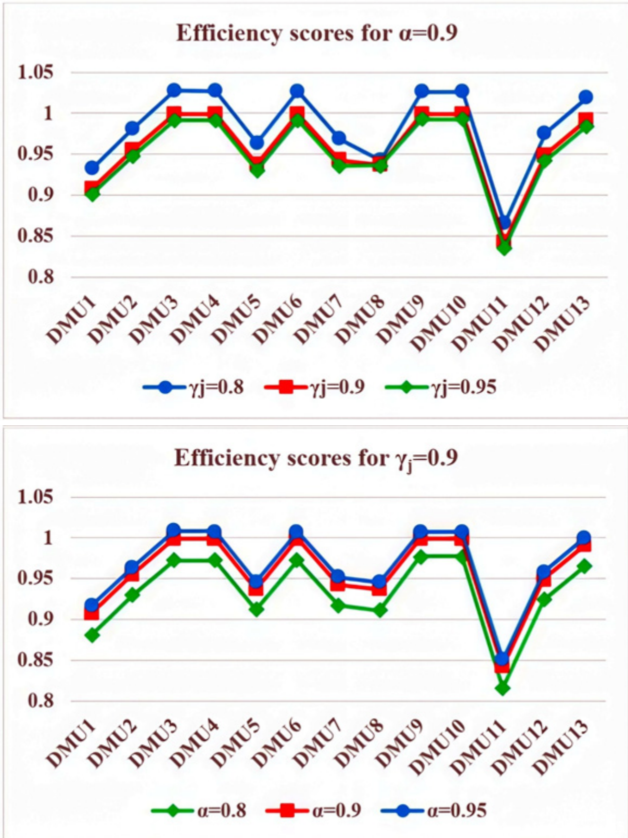

FIGURE 4. Sensitivity analysis for different α and γ.

2) MERITS AND LIMITATIONS

The merit of the proposed method is to enable managers to develop a managerial dashboard based on our proposed approach to their organization's situation. Also, they can run the model periodically and identify the weaknesses and strengths of their safety system and draw a road map to improve their resiliency. One of the limitations of this research is to consider some of the essential factors of resilience engineering; However, several factors have been introduced in related studies. In this paper, we try to view the most commonly used aspects. Another limitation concerns data collection and analysis of their validity and reliability. This study is being implemented in an oil refinery as critical process industry, and its application in other similar industries needs extreme caution.

3) FUTURE STUDY

Our recommendation for further research is to consider the network structure of resilience engineering. It can be modeled, and as a result, the efficiency scores are comparable. Also, to develop the proposed method, the Pythagorean fuzzy sets can be applied. These are efficient and recent math tools to consider the uncertainty that can reduce negative information's impact [38-39].

V. CONCLUSION

Resilience engineering is a novel approach in the safety management area, which helps organizations return to a normal state quickly after a crisis. But there is a level of complexity in measuring the efficiency of organizations in the field of resilience engineering. Accordingly, the authors examined the application of resilience engineering through efficiency analysis. In this order, the uncertainty according to resilience engineering was considered, and therefore an instrument was proposed to measure and assess the efficiency of decision-making units. Also, to extract useful information from uncertain data, the T2FV was applied. Our contribution is to develop a mathematical programming approach to assess resilience engineering's efficiency in process industries. Also, because process industries disposed of a high uncertainty level, we used Type-2 fuzzy sets to confront it. In this regard, the proposed instrument can be used as a useful tool to measure the resilience efficiency of process industries, such as refineries, petrochemical plants, etc. The results show a robust tool to analyze and evaluate resilience efficiency.

VI. LIST OF ABBREVIATIONS AND SYMBOLS

A. ABBREVIATIONS

Abbreviation	Meaning
BCC	Banker-Charnes-Cooper
CCR	Charnes-Cooper-Rhodes
Cr	Credibility measure
CSS	Complex socio-technical systems
CV	Critical values
CV^*	Optimistic CV
CV_*	Pessimistic CV
CVR	Content validity ratio
DEA	Data envelopment analysis
DMU	Decision-making unit
FDH	Free disposal hull
FOU	Footprint of uncertainty
KMO	Kaiser-Meyer-Olkin
LMF	Lower membership function
Nec	Necessity measure
Pos	Possibility measure
SBM	Slack-based measure
T1FV	Type-1 fuzzy variable
T2FV	Type-2 fuzzy variable
UMF	Upper membership function

B. SYMBOLS

Symbol	Meaning
$\tilde{A}$	Type-1 fuzzy set
$\tilde{\tilde{A}}$	Type-2 fuzzy set
$\bar{f}$	Objective value
J_x	Initial membership function
$K(k_i)$	A constant value
N	Count of panelists
n_e	Number of panelists
r_1, r_2, r_3	Left, center, and right point of triangular fuzzy variable
(r_1^i, r_2^i, r_3^i)	Left, center, and right point of triangular fuzzy variable
t	A constant value

u_r	Weight factor of inputs
v_i	Weight factor of outputs
x_{ij}	i^{th} input for j^{th} DMU
$\tilde{x}_{ij}$	Fuzzy value of the i^{th} input for j^{th} DMU
$\tilde{x}'_{ij}$	Reduced T1FVs of T2FVs $\tilde{x}_{ij}$
$\left(x^1_{ij}, x^2_{ij}, x^3_{ij}\right)$	Left, center, and right point of triangular fuzzy variable
y_{rj}	r^{th} output for j^{th} DMU
$\tilde{y}_{rj}$	Fuzzy value of the r^{th} output for j^{th} DMU
$\tilde{y}'_{rj}$	Reduced T1FVs of T2FVs $\tilde{y}_{ij}$
$\left(y^1_{rj}, y^2_{rj}, y^3_{rj}\right)$	Left, center, and right point of triangular fuzzy variable
α	Generalized credibility levels of satisfaction
β	Generalized credibility levels of satisfaction
γ_j	Generalized credibility levels of satisfaction
θ_l, θ_r	The spread of primary possibilities
$\left(\theta'_{l,ij}, \theta'_{r,rj}\right)$	The spread of primary possibilities
$\left(\theta_{l,ij}, \theta_{r,rj}\right)$	The spread of primary possibilities
$\left(\theta_{l,i}, \theta_{r,j}\right)$	The spread of primary possibilities
$\tilde{\mu}$	The secondary possibility distribution function
$\mu_{\tilde{A}}$	The first membership function
$\mu_{\tilde{\tilde{A}}}$	The second membership function
ξ	Reduced type of $\tilde{\xi}$
$\tilde{\xi}$	Fuzzy event

APPENDIX

Proof of the deterministic forms (25)-(29):

Since x'_{ij} and y'_{ij} are the reduced types of Type-2 triangular fuzzy variables $x_{ij} = (x^1_{ij}, x^2_{ij}, x^3_{ij}; \theta_{l,ij}, \theta_{r,ij})$ and $y_{rj} = (y^1_{rj}, y^2_{rj}, y^3_{rj}; \theta'_{l,rj}, \theta'_{r,rj})$ from the CV-reduction method, respectively and $u_r, v_i \geq 0$, the theorem 1 is used for $0 < \alpha \leq 0.25$. $\tilde{C}r\{\sum_{r=1}^{s} u_r\tilde{y}'_{ro} \geq \bar{f}\} \geq \alpha$ is equal to:

$$\tilde{C}r\{\sum_{r=1}^{s} -u_r\tilde{y}'_{ro} \leq -\bar{f}\} \geq \alpha \tag{A.1}$$

or

$$\sum_{r=1}^{s} \frac{(1-2\alpha+(1-4\alpha)\theta'_{l,ro})y^3_{ro}u_r + 2\alpha u_r y^2_{ro}}{1+(1-4\alpha)\theta'_{l,ro}} \geq \bar{f} \tag{A.2}$$

So, for $0 < \alpha \leq 0.25$, the maximization of Max $\bar{f}$ such that $\tilde{C}r\{\sum_{r=1}^{s} u_r\tilde{y}'_{ro} \geq \bar{f}\} \geq \alpha$ is equal to solve:

$$\text{Max}\ [\sum_{r=1}^{s} \frac{(1-2\alpha+(1-4\alpha)\theta'_{l,ro})y^3_{ro}u_r + 2\alpha u_r y^2_{ro}}{1+(1-4\alpha)\theta'_{l,ro}}] \tag{A.3}$$

From these two equations, the objective function (25) is obtained. Thus, the objective function can be obtained similarly to other values α.

Besides, the crisp forms of constraint (22), i.e., $\tilde{C}r\{\sum_{i=1}^{m} v_i\tilde{x}'_{io} = 1\} \geq \beta$ or it's equivalent $\tilde{C}r\{\sum_{i=1}^{m} v_i\tilde{x}'_{io} \leq 1\} \geq \beta$ and $\tilde{C}r\{\sum_{i=1}^{m} v_i\tilde{x}'_{io} \geq 1\} \geq \beta$ are obtained from theorem 1 where x'_{io} is the CV reduction of $\tilde{x}_{io} = (x^1_{io}, x^2_{io}, x^3_{io}; \theta_{l,io}, \theta_{r,io})$ and obtained as follows:

$$\sum_{i=1}^{m} \frac{(1-2\beta+(1-4\beta)\theta_{r,ij})v_i x^1_{io} + 2\beta v_i x^2_{io}}{1+(1-4\beta)\theta_{r,ij}} \leq 1 \text{ if } 0 < \beta \leq 0.25 \tag{A.4}$$

and

$$\sum_{i=1}^{m} \frac{(1-2\beta+(1-4\beta)\theta_{l,ij})v_i x^3_{io} + 2\beta v_i x^2_{io}}{1+(1-4\beta)\theta_{l,ij}} \geq 1 \text{ if } 0 < \beta \leq 0.25 \tag{A.5}$$

It can be equally calculated for other values β.

Also, the crisp form of constraints (23), i.e., $\tilde{C}r\{\sum_{r=1}^{s} u_r\tilde{y}'_{rj} - \sum_{i=1}^{m} v_i\tilde{x}'_{ij} \leq 0\} \geq \gamma_j; (j = 1, 2, \ldots, n)$ where $\tilde{x}'_{ij}$ and $\tilde{y}'_{rj}$ are the CV reduction of $x_{ij} = (x^1_{ij}, x^2_{ij}, x^3_{ij}; \theta_{l,ij}, \theta_{r,ij})$ and $\tilde{y}_{rj} = (y^1_{rj}, y^2_{rj}, y^3_{rj}; \theta'_{l,rj}, \theta'_{r,rj})$ is obtained from theorem 1 as follows:

$$\sum_{r=1}^{s} \frac{(1-2\gamma_j+(1-4\gamma_j)\theta'_{r,rj})u_r y^1_{rj} + 2\gamma_j u_r y^2_{rj}}{1+(1-4\gamma_j)\theta'_{r,rj}} - \sum_{i=1}^{m} \frac{(1-2\gamma_j+(1-4\gamma_j)\theta_{r,ij})v_i x^1_{ij} + 2\gamma_j v_i x^2_{ij}}{1+(1-4\gamma_j)\theta_{r,ij}} \leq 0; \quad j = 1, 2, \ldots, n \text{ if } 0 < \gamma_j \leq 0.25 \tag{A.6}$$

The expressions for other values γ_j are obtained similarly.

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