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Adaptive Robust Failure Compensation Control for Servo System Driven by Twin Motors

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ABSTRACT In the context of failure control of servo system driven by twin motors, there are still no available results to compensate unknown actuator failures which seem inevitable in practice by adaptive backstepping technique. Therefore, to rise the reliability of the system, we aim at addressing such a problem by proposing an adaptive robust actuator failure compensation control scheme based on backstepping technique for servo system driven by twin motors. Unlike the traditional backstepping, the estimation of unknown coefficient of intermediate state variable is introduced in coordinate changes. In addition, matching and non-matching uncertainties have been fully considered in the controller design. Simulation results show that the designed controller can ensure the boundedness of all the signals no matter actuator failures occur or not.

INDEX TERMS Servo system, adaptive control, twin motors, actuator failure.

I. INTRODUCTION

The servo system has been widely used in many fields including industry, military and vehicle [1] etc. To get higher movement performances, servo system driven by twin motors is usually used and the performance of such a servo system has been received extensive attention [2]–[4]. Unknown nonlinearities in input signal such as backlash and dead zone are deeply studied [4]. It is clear that the purpose is to eliminate their influences and to rise the controlled performance.

Besides improving the performance of the system under partial control information missing [5], [6], more and more attention has been paid to the problem of system reliability including the studies on fault detect, fault tolerant control etc [7]–[17]. Actuator failure is an common fault and seems inevitable in practice control systems. Such failures which may lead to instability or even catastrophic accidents are often uncertain in time, value and pattern. The reliable control becomes more difficult under such unknown actuator failures. To address this problem, several schemes have been proposed in recent twenty years. Compared to other methods, an adaptive approach [7]–[14] can handle system parametric uncertainties by online estimating unknown parameters with update laws. As a promising approach, backstepping technique [8]–[10] and [18]–[20] has been widely used in the controller design of a class of nonlinear systems. In [8]–[10], backstepping technique is used in the design of adaptive failure compensation scheme for a class of nonlinear systems. Transient performance was guaranteed by using prescribed performance bound in [8] while backlash hysteresis existing in practice of actuator failure systems was studied deeply in [9]. In [10], an important result which removes the assumption on finite failure number was established. However, in the context of the failure control of servo system driven by twin motors [2], there is still no results available based on adaptive approaches. In this note, we aim to address such a problem and propose an adaptive failure compensation control scheme technique for servo system driven by twin motors. Clearly, unknown parameter $a = \frac{K_L}{J_L}$ of x_3 in (8) makes traditional backstepping technique become incapable in the design of adaptive controller. Our idea is to introduce the estimation of $\vartheta = \frac{1}{a}$ in the virtual control design and the coordinate changes (14). And then, we use *proj* to guarantee this estimation ϑ being bounded. To compensate the effects caused by matching and non-matching uncertainties ω_1 and ω_2, the property of inequality about $tanh(\cdot)$ is used. Finally, an adaptive robust compensation control scheme is proposed and all uncertainties caused by unknown parameters, matching and non-matching terms and unknown actuator failures can be compensated successfully.

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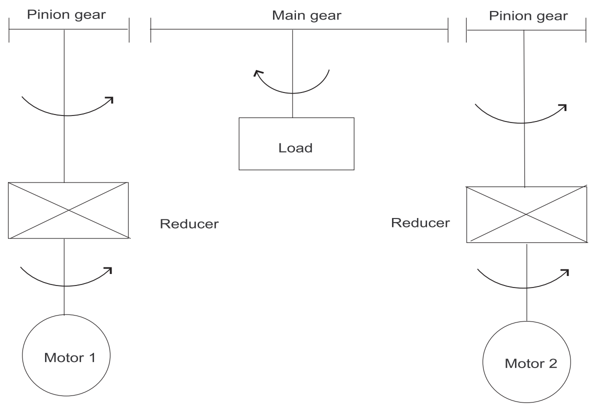

FIGURE 1. Structure of the servo system driven by twin motors.

The main contributions of this paper are as follows: (1) The control problem is investigated for servo system driven by twin motros with unknown parameters, matching and non-matching uncertainties, unknown actuator failures. (2) In addition, we use inequality of $tanh(\cdot)$ to handle the effects caused by matching and non-matching uncertainties. (3) Moreover, unlike the traditional backstepping technique the estimation of $\vartheta = \frac{1}{a}$ is introduced in coordinate changes to compensate unknown parameter a as the coefficient of x_3. It is shown that the proposed adaptive robust controller can ensure the boundedness of all the signals of the closed loop whether or not actuator failures occur.

The rest of the paper is organized as follows. In section 2, we formulate the servo system driven by twin motors with unknown actuator failures. An robust adaptive control scheme and the main results about stability are proposed in section 3. The simulation studies is given in section 4. Finally, the paper is concluded in section 5.

II. MODELS AND PROBLEM STATEMENT

We consider the servo system driven by twin motors [2] shown in Figs.1. The mathematical model can be described as follows

$$\begin{aligned}\dot{\theta}_L(t) &= \omega_L(t); \quad \dot{\theta}_j(t) = \omega_j(t)\\ \dot{\omega}_L(t) &= \frac{K_L(\theta_1(t)-\theta_L(t)) + K_L(\theta_2(t)-\theta_L(t))}{J_L} + w_1(t)\\ \dot{\omega}_j(t) &= \frac{K_{Tj}u_j(t) + K_L(\theta_j(t)-\theta_L(t))}{J_{mj}} + w_{2j}(t) \end{aligned} \tag{1}$$

where $\theta_L(t), \omega_L(t), J_L, K_L$ are angle position, angular velocity, moment of inertia from load transformed into motor side, stiffness coefficient of transmission mechanism, respectively. The effect caused by load is not directly shown in above system model (1). Instead, it has been considered in parameters $\theta_L(t), \omega_L(t), J_L$. Parameters $\theta_j(t)$, $\omega_j(t)$, J_{mj}, K_{Tj}, $u_j(t)$ $(j = 1, 2)$ are angle position, angular velocity, moment of inertia, electromagnetic torque coefficient and control signal of jth motor, respectively. $w_1(t), w_{2j}(t) (j = 1, 2)$ are unknown nonlinear modeling errors and denote matching, non-matching uncertainties. The effects generated by reducers and the backlash caused by gears are ignored in the establishment of the mathematical model of the servo system driven by twin motors shown in Figs.1. So the mathematical model (1) is an ideal model.

In order to simplify system model, we let

$$\begin{aligned} x_1 &= \theta_L(t); \quad x_2 = \omega_L(t)\\ x_{3j} &= \theta_j(t); \quad x_{4j} = \omega_j(t) \end{aligned} \tag{2}$$

Then we have

$$\begin{aligned}\dot{x}_1 &= x_2; \quad \dot{x}_{3j} = x_{4j}\\ \dot{x}_2 &= \frac{K_L(x_{31}-x_1) + K_L(x_{32}-x_1)}{J_L} + w_1(t)\\ \dot{x}_{4j} &= \frac{K_{Tj}u_j(t) + K_L(x_{3j}-x_1)}{J_{mj}} + w_{2j}(t)\end{aligned} \tag{3}$$

To obtain the strict feedback structure, letting

$$\begin{aligned} x_3 &= x_{31} + x_{32}\\ x_4 &= x_{41} + x_{42}\end{aligned} \tag{4}$$

Then we have

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= \frac{K_L x_3 - 2K_L x_1}{J_L} + w_1(t)\\ \dot{x}_3 &= x_4\\ \dot{x}_{4j} &= \frac{K_{Tj}u_j(t) + K_L(x_{3j}-x_1)}{J_{mj}} + w_{2j}(t)\end{aligned} \tag{5}$$

All interior parameters of these twin motors are the same. Namely, $K_{T1} = K_{T2}, J_{m1} = J_{m2}$. Then we have

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= \frac{K_L x_3 - 2K_L x_1}{J_L} + w_1(t)\\ \dot{x}_3 &= x_4\\ \dot{x}_{4j} &= \frac{K_{Tj}u_j(t) + K_L(x_{3j}-x_1)}{J_{mj}} + w_{2j}(t)\end{aligned} \tag{6}$$

Letting

$$a = \frac{K_L}{J_L}, \quad b_2 = \frac{K_L}{J_{mj}}, \quad \rho = \frac{K_{Tj}}{J_{mj}} \tag{7}$$

System model can be rewritten as

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= ax_3 - 2ax_1 + w_1(t)\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= \rho u_1(t) + \rho u_2(t) + b_2 x_3 - 2b_2 x_1 + w_2(t)\end{aligned} \tag{8}$$

where $w_2(t) = w_{21}(t) + w_{22}(t)$. We let $2a = b_1$ and $b = (b_1, b_2)^T$, then

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= ax_3 + (b_1, b_2)\begin{pmatrix}-x_1\\ 0\end{pmatrix} + w_1(t)\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= \rho\sum_{j=1}^{2} u_j(t) + (b_1, b_2)\begin{pmatrix}0\\ x_3 - 2x_1\end{pmatrix} + w_2(t)\end{aligned} \tag{9}$$

Namely,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= ax_3 + b^T f_1(x_1) + w_1(t) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \rho \sum_{j=1}^{2} u_j(t) + b^T f_4(x_1, x_3) + w_2(t) \end{aligned} \tag{10}$$

where known function

$$f_1 = \begin{pmatrix} -x_1 \\ 0 \end{pmatrix}; \quad f_4 = \begin{pmatrix} 0 \\ x_3 - 2x_1 \end{pmatrix}$$

Remark 1: The following difficults exist in the controller design.

- The unknown parameter $\vartheta = 1/a$ will be estimated to handle the unknown parameter a in term ax_3. In addition $b_1 = 2a$ is also estimated by designing an adaptive operator in the proposed adaptive control law to compensate the unknown effect caused by $2ax_1$.
- Compare to common triangular system, there is a nonzero coefficient composed by multiple parameter a in front of system state x_3 in the second state differential equation. Such an unknown parameter makes the controller design become more and more difficult and traditional backstepping becomes unavailable. We will eliminate the effect of this unknown parameters by designing an adaptive estimator of $\frac{1}{a}$ and introduce this estimator in the proposed controller.

As we all know, actuator failure is inevitable in the servo system driven by motors. Such as aging of electrical components and traction loss from input to output always exist. Similar to [8]- [10], failure of the jth actuator at time instant t_{jf} can be modeled as follows

$$\begin{aligned} u_j &= \sigma_j v_j + \bar{u}_j, \quad (\forall t \geq t_{jf}) \\ \sigma_j \bar{u}_j &= 0 \end{aligned} \tag{11}$$

where σ_j, $\bar{u}_j$, t_{jf} are unknown constants and $0 \leq \sigma_j \leq 1$. The signal v_j is the input of the $j-th$ actuator. An actuator with its input equal to its output, i.e. $u_j = v_j$ is regarded as a failure-free actuator. Easily, we can get the following situations:

- $\sigma_j = 1$,
  It indicates $u_j = v_j$. The jth actuator works normally.
- $0 < \sigma_j < 1$,
  It indicates $u_j = \sigma_j v_j$. The jth actuator is called partial loss of effectiveness.
- $\sigma_j = 0$,
  It indicates $u_j = \bar{u}_j$. The ith actuator is called total loss of effectiveness.

σ_j, $\bar{u}_j$ are unknown and can be seen as the time-dependent jump parameters. A specific failure corresponds to a group of values of σ_j, $\bar{u}_j$. With the failure model given in (11), system (10) can be rewritten as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= ax_3 + b^T f_1(x_1, x_2) + w_1(t) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \rho \sum_{j=1}^{2} (\sigma_j v_j + \bar{u}_j) + b^T f_4(x_1, x_2, x_3, x_4) + w_2(t) \end{aligned} \tag{12}$$

To design the adaptive control law, the following assumptions are made.

Assumption 1: There is at least one actuator being not total loss of effectiveness. Any actuator can change only from normal to partial failure or total failure and only fails once.

From Assumption 1, we know that there is a finite time instant T_f and no new failure will occur after T_f.

Assumption 2: Unknown parameters ρ, a, b lie in a known bounded set, respectively and these bounded sets don't include zero.

From (7) and the actual meanings of parameters K_L, J_L, J_{mj}, K_{Tj}, the *sign* of a, b, ρ are known.

Assumption 3: Unknown nonlinear function $w_1(t), w_2(t)$ are satisfied in the following bounded conditions

$$|w_1(t)| \leq \delta_1(x_1, x_2); \quad |w_2(t)| \leq \delta_2(x_1, x_2, x_3, x_4) \tag{13}$$

where $\delta_1(x_1, x_2)$, $\delta_2(x_1, x_2, x_3, x_4)$ are known functions.

Assumption 4: Reference signal $y_r(t)$ and its i-order $(i = 1, 2, \ldots, 4)$ derivatives are known and bounded.

III. DESIGN OF EVENT-TRIGGER CONTROLLER

The control objective is to design a robust adaptive failure compensation control scheme to guarantee all signals bounded under any failure of actuators and to realize the output signal $y = x_1$ tracking to the reference signal y_r effectively. To obtain a suitable control law and update laws for controller parameters based on the backstepping approach, we first make the following coordinate changes

$$\begin{aligned} z_1 &= x_1 - y_r; \\ z_2 &= x_2 - \alpha_1 - y_r^{(1)}; \\ z_3 &= x_3 - \hat{\vartheta}\alpha_2 - y_r^{(2)}; \\ z_4 &= x_4 - \alpha_3 - y_r^{(3)} \end{aligned} \tag{14}$$

where z_1 is tracking error and $\alpha_i (i = 1, 2, 3)$ is virtual control. $\hat{\vartheta}$ is the estimation of parameter $\vartheta = \frac{1}{a}$. Virtual control α_i is an actively constructed control variable. In step i, our objective is to design a virtual control law α_{i-1} which makes z_i tends to zero. Then we will give the recursive design steps by backstepping approaches.

Remark 2: Different from the general lower triangular system whose controller can be designed by using backstepping, an unknown parameter a as the coefficient of x_3 exists. Thus the traditional backstepping can not be used for the controller design. To solve this problem, an estimation of $\vartheta = 1/a$ is introduced in coordinate changes. Then the uncertainty caused by unknown parameter coefficient a can be compensated and backstepping technique can be carried out.

Step 1: From system model (12) and coordinate transformation (14), we have

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 - y_r^{(1)} \\ &= x_2 - y_r^{(1)} \\ &= z_2 + \alpha_1 \end{aligned} \tag{15}$$

Considering the following Lyapunov function

$$V_1 = \frac{1}{2} z_1^2 \tag{16}$$

The derivative of Lyapunov function is

$$\begin{aligned} \dot{V}_1 &= z_1 \dot{z}_1 \\ &= z_1(z_2 + \alpha_1) \end{aligned} \tag{17}$$

Virtual control α_1 can be chosen as

$$\alpha_1 = -c_1 z_1 \tag{18}$$

where c_1 is a positive constant. Then we have

$$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2 \tag{19}$$

Step 2: From system model, we know that the derivative of z_2 is

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_1 - y_r^{(2)} \\ &= a x_3 + b^T f_1(x_1, x_2) + w_1(t) - \dot{\alpha}_1 - y_r^{(2)} \end{aligned} \tag{20}$$

Clearly, we know α_1 is dependent on variables x_1 and y_r. So we can get

$$\begin{aligned} \dot{z}_2 &= a x_3 + b^T f_1(x_1, x_2) + w_1(t) \\ &\quad -(\frac{\partial \alpha_1}{\partial x_1}\dot{x}_1 + \frac{\partial \alpha_1}{\partial y_r} y_r^{(1)}) - y_r^{(2)} \\ &= a x_3 + b^T f_1(x_1, x_2) + w_1(t) \\ &\quad -(\frac{\partial \alpha_1}{\partial x_1} x_2 + \frac{\partial \alpha_1}{\partial y_r} y_r^{(1)}) - y_r^{(2)} \\ &= a(z_3 + \hat{\vartheta}\alpha_2 + y_r^{(2)}) + b^T f_1(x_1, x_2) + w_1(t) \\ &\quad -(\frac{\partial \alpha_1}{\partial x_1} x_2 + \frac{\partial \alpha_1}{\partial y_r} y_r^{(1)}) - y_r^{(2)} \end{aligned} \tag{21}$$

With (14) we can get

$$\begin{aligned} \dot{z}_2 = a z_3 + a\hat{\vartheta}\alpha_2 + a y_r^{(2)} + b^T f_1(x_1, x_2) + w_1(t) \\ -(\frac{\partial \alpha_1}{\partial x_1} x_2 + \frac{\partial \alpha_1}{\partial y_r} y_r^{(1)}) - y_r^{(2)} \end{aligned} \tag{22}$$

Note that $\tilde{\vartheta} = \vartheta - \hat{\vartheta}$, then $a\hat{\vartheta}\alpha_2 = a(\vartheta - \tilde{\vartheta})\alpha_2 = \alpha_2 - a\tilde{\vartheta}\alpha_2$

$$\begin{aligned} \dot{z}_2 = a z_3 + \alpha_2 - a\tilde{\vartheta}\alpha_2 + a y_r^{(2)} + b^T f_1(x_1, x_2) + w_1(t) \\ -(\frac{\partial \alpha_1}{\partial x_1} x_2 + \frac{\partial \alpha_1}{\partial y_r} y_r^{(1)}) - y_r^{(2)} \end{aligned} \tag{23}$$

Considering the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{|a|}{2\Gamma_\vartheta}\tilde{\vartheta}^2 + \frac{1}{2\Gamma_a}\tilde{a}^2 + \frac{1}{2}\tilde{b}^T \Gamma_b^{-1}\tilde{b} \tag{24}$$

where $\hat{a}$ is the estimation of parameter a and $\hat{b}$ is the estimation of parameter b. $\tilde{a} = a - \hat{a}$ and $\tilde{b} = b - \hat{b}$ denote estimation errors. $\Gamma_\vartheta, \Gamma_a$ are positive constants and Γ_b is a positive definite design matrix. With (19) and (23), the derivative of V_2 is

$$\begin{aligned} \dot{V}_2 &= -c_1 z_1^2 + z_1 z_2 + z_2 \dot{z}_2 - \frac{|a|}{\Gamma_\vartheta}\tilde{\vartheta}\dot{\hat{\vartheta}} - \frac{1}{\Gamma_a}\tilde{a}\dot{\hat{a}} - \tilde{b}^T \Gamma_b^{-1}\dot{\hat{b}} \\ &= -c_1 z_1^2 + z_1 z_2 + z_2\Big(a z_3 + \alpha_2 - a\tilde{\vartheta}\alpha_2 + a y_r^{(2)} \\ &\quad + b^T f_1(x_1, x_2) + w_1(t) - (\frac{\partial \alpha_1}{\partial x_1} x_2 + \frac{\partial \alpha_1}{\partial y_r} y_r^{(1)}) \\ &\quad - y_r^{(2)}\Big) - \frac{|a|}{\Gamma_\vartheta}\tilde{\vartheta}\dot{\hat{\vartheta}} - \frac{1}{\Gamma_a}\tilde{a}\dot{\hat{a}} - \tilde{b}^T \Gamma_b^{-1}\dot{\hat{b}} \end{aligned} \tag{25}$$

Choosing α_2 as

$$\begin{aligned} \alpha_2 &= -c_2 z_2 - z_1 - \hat{a} y_r^{(2)} - \hat{b}^T f_1(x_1, x_2) \\ &\quad + (\frac{\partial \alpha_1}{\partial x_1} x_2 + \frac{\partial \alpha_1}{\partial y_r} y_r^{(1)}) + y_r^{(2)} \\ &\quad - \delta_1(x_1, x_2) tanh(\frac{\delta_1(x_1, x_2) z_2}{\varepsilon_1}) \end{aligned} \tag{26}$$

where c_2 is a positive constant. So we have

$$\begin{aligned} \dot{V}_2 &= -\sum_{i=1}^{2} c_i z_i^2 + a z_2 z_3 - a\tilde{\vartheta}\alpha_2 z_2 \\ &\quad + \tilde{a} y_r^{(2)} z_2 + \tilde{b}^T f_1(x_1, x_2) z_2 + w_1(t) z_2 \\ &\quad - \delta_1(x_1, x_2) z_2 tanh(\frac{\delta_1(x_1, x_2) z_2}{\varepsilon_1}) \\ &\quad - \frac{|a|}{\Gamma_\vartheta}\tilde{\vartheta}\dot{\hat{\vartheta}} - \frac{1}{\Gamma_a}\tilde{a}\dot{\hat{a}} - \tilde{b}^T \Gamma_b^{-1}\dot{\hat{b}} \\ &\le -\sum_{i=1}^{2} c_i z_i^2 + a z_2 z_3 - a\tilde{\vartheta}\alpha_2 z_2 + \tilde{a} y_r^{(2)} z_2 \\ &\quad + \tilde{b}^T f_1(x_1, x_2) z_2 + |w_1(t) z_2| \\ &\quad - \delta_1(x_1, x_2) z_2 tanh(\frac{\delta_1(x_1, x_2) z_2}{\varepsilon_1}) \\ &\quad - \frac{|a|}{\Gamma_\vartheta}\tilde{\vartheta}\dot{\hat{\vartheta}} - \frac{1}{\Gamma_a}\tilde{a}\dot{\hat{a}} - \tilde{b}^T \Gamma_b^{-1}\dot{\hat{b}} \\ &\le -\sum_{i=1}^{2} c_i z_i^2 + a z_2 z_3 + |\delta_1(x_1, x_2) z_2| \\ &\quad - \delta_1(x_1, x_2) z_2 tanh(\frac{\delta_1(x_1, x_2) z_2}{\varepsilon_1}) \\ &\quad - \frac{|a|}{\Gamma_\vartheta}\tilde{\vartheta}(\dot{\hat{\vartheta}} + sign(a)\Gamma_\vartheta z_2 \alpha_2) \\ &\quad - \frac{1}{\Gamma_a}\tilde{a}(\dot{\hat{a}} - \Gamma_a y_r^{(2)} z_2) \\ &\quad - \tilde{b}^T \Gamma_b^{-1}(\dot{\hat{b}} - \Gamma_b f_1(x_1, x_2) z_2) \end{aligned} \tag{27}$$

Note that the following properties of function $tanh(\cdot)$

$$0 \le |\chi| - \chi tanh(\frac{\chi}{\varepsilon}) \le 0.2785\varepsilon \qquad (\forall \varepsilon > 0) \tag{28}$$

Well, as we know, all continuous approximate functions of $sign(\cdot)$ have similar property, for example $sg(\cdot)$ in [21]. With

(28), we can get

$$|\delta_1(x_1,x_2)z_2| - \delta_1(x_1,x_2)z_2\, tanh(\frac{\delta_1(x_1,x_2)z_2}{\varepsilon_1}) \le 0.2785\varepsilon_1$$

Then we have

$$\dot V_2 \le -\sum_{i=1}^{2} c_i z_i^2 + a z_2 z_3 + 0.2785\varepsilon_1 - \frac{|a|}{\Gamma_\vartheta}\tilde\vartheta(\dot{\hat\vartheta} - \tau_1^\vartheta) - \frac{1}{\Gamma_a}\tilde a(\dot{\hat a} - \tau_1^a) - \tilde b^T \Gamma_b^{-1}(\dot{\hat b} - \tau_1^b) \quad (29)$$

where

$$\tau_1^\vartheta = -sign(a)\Gamma_\vartheta z_2\alpha_2$$
$$\tau_1^a = \Gamma_a y_r^{(2)} z_2$$
$$\tau_1^b = \Gamma_b f_1(x_1,x_2) z_2 \quad (30)$$

Step 3: The derivative of z_3 is

$$\begin{aligned}\dot z_3 &= \dot x_3 - \dot{\hat\vartheta}\alpha_2 - \hat\vartheta\dot\alpha_2 - y_r^{(3)}\\ &= z_4 + \alpha_3 + y_r^{(3)} - \dot{\hat\vartheta}\alpha_2 - \hat\vartheta\dot\alpha_2 - y_r^{(3)}\\ &= z_4 + \alpha_3 - \dot{\hat\vartheta}\alpha_2 - \hat\vartheta(\sum_{i=1}^{2}\frac{\partial\alpha_2}{\partial x_i}\dot x_i\\ &\quad + \sum_{i=0}^{2}\frac{\partial\alpha_2}{\partial y_r^{(i)}}y_r^{(i+1)} + \frac{\partial\alpha_2}{\partial\hat b}\dot{\hat b} + \frac{\partial\alpha_2}{\partial\hat a}\dot{\hat a})\\ &= z_4 + \alpha_3 - \dot{\hat\vartheta}\alpha_2 - \hat\vartheta\Big(\frac{\partial\alpha_2}{\partial x_1}x_2 + \frac{\partial\alpha_2}{\partial x_2}(a x_3\\ &\quad + w_1(t) + b^T f_1(x_1,x_2)) + \sum_{i=0}^{2}\frac{\partial\alpha_2}{\partial y_r^{(i)}}y_r^{(i+1)}\\ &\quad + \frac{\partial\alpha_2}{\partial\hat b}\dot{\hat b} + \frac{\partial\alpha_2}{\partial\hat a}\dot{\hat a}\Big)\end{aligned} \quad (31)$$

Considering the following Lyapunov function

$$V_3 = V_2 + \frac{1}{2}z_3^2 \quad (32)$$

The derivative of V_3 is

$$\dot V_3 \le -\sum_{i=1}^{2} c_i z_i^2 + a z_2 z_3 + 0.2785\varepsilon_1 - \frac{|a|}{\Gamma_\vartheta}\tilde\vartheta(\dot{\hat\vartheta} - \tau_1^\vartheta) - \frac{1}{\Gamma_a}\tilde a(\dot{\hat a} - \tau_1^a) - \tilde b^T\Gamma_b^{-1}(\dot{\hat b} - \tau_1^b) + z_3\dot z_3 \quad (33)$$

Following we analyze the term $z_3\dot z_3$

$$\begin{aligned} z_3\dot z_3 &= z_3\Big(z_4 + \alpha_3 - \dot{\hat\vartheta}\alpha_2 - \hat\vartheta\Big(\frac{\partial\alpha_2}{\partial x_1}x_2 + \frac{\partial\alpha_2}{\partial x_2}(a x_3\\ &\quad + b^T f_1(x_1,x_2) + w_1(t)) + \sum_{i=0}^{2}\frac{\partial\alpha_2}{\partial y_r^{(i)}}y_r^{(i+1)}\\ &\quad + \frac{\partial\alpha_2}{\partial\hat b}\dot{\hat b} + \frac{\partial\alpha_2}{\partial\hat a}\dot{\hat a}\Big)\Big)\end{aligned} \quad (34)$$

Then α_3 can be chosen as

$$\begin{aligned}\alpha_3 &= -c_3 z_3 - \hat a z_2 + \hat\vartheta\Big(\frac{\partial\alpha_2}{\partial x_1}x_2 + \sum_{i=0}^{2}\frac{\partial\alpha_2}{\partial y_r^{(i)}}y_r^{(i+1)}\Big)\\ &\quad + \hat\vartheta\frac{\partial\alpha_2}{\partial x_2}(\hat a x_3 + \hat b^T f_1(x_1,x_2)) + \hat\vartheta\frac{\partial\alpha_2}{\partial\hat b}\tau_2^b\\ &\quad + \hat\vartheta\frac{\partial\alpha_2}{\partial\hat a}\tau_2^a - \bar\delta_1(x_1,x_2,x_3)tanh(\frac{\bar\delta_1(x_1,x_2,x_3)z_3}{\varepsilon_2})\\ &\quad + \tau_2^\vartheta\alpha_2\end{aligned}$$

where

$$\bar\delta_1(x_1,x_2,x_3) \ge |\hat\vartheta\frac{\partial\alpha_2}{\partial x_2}w_1(t)| \quad (35)$$

Remark 3: We will use $proj(\cdot)$ to guarantee $\hat\vartheta$ being bounded. So above bound function $\bar\delta_1(x_1,x_2,x_3)$ can be found easily.

With (33)-(35), we have

$$\begin{aligned}\dot V_3 &\le -\sum_{i=1}^{3} c_i z_i^2 + \tilde a z_2 z_3 - (\dot{\hat\vartheta} - \tau_2^\vartheta)\alpha_2 z_3 + z_3 z_4\\ &\quad + 0.2785\varepsilon_1 - \frac{|a|}{\Gamma_\vartheta}\tilde\vartheta(\dot{\hat\vartheta} - \tau_1^\vartheta) - \frac{1}{\Gamma_a}\tilde a(\dot{\hat a} - \tau_1^a)\\ &\quad - \tilde b^T\Gamma_b^{-1}(\dot{\hat b} - \tau_1^b) - \hat\vartheta z_3\frac{\partial\alpha_2}{\partial x_2}(\tilde a x_3 + \tilde b^T f_1)\\ &\quad - \hat\vartheta\frac{\partial\alpha_2}{\partial\hat b}z_3(\dot{\hat b} - \tau_2^b) - \hat\vartheta\frac{\partial\alpha_2}{\partial\hat a}z_3(\dot{\hat a} - \tau_2^a) + |z_3\bar\delta_1|\\ &\quad - z_3\bar\delta_1(x_1,x_2,x_3)tanh(\frac{\bar\delta_1(x_1,x_2,x_3)z_3}{\varepsilon_2})\end{aligned} \quad (36)$$

Then

$$\begin{aligned}\dot V_3 &\le -\sum_{i=1}^{3} c_i z_i^2 - (\dot{\hat\vartheta} - \tau_2^\vartheta)\alpha_2 z_3 + z_3 z_4\\ &\quad + 0.2785(\varepsilon_1 + \varepsilon_2) - \frac{|a|}{\Gamma_\vartheta}\tilde\vartheta(\dot{\hat\vartheta} - \tau_1^\vartheta)\\ &\quad - \frac{1}{\Gamma_a}\tilde a(\dot{\hat a} - \tau_1^a) - \tilde b^T\Gamma_b^{-1}(\dot{\hat b} - \tau_1^b)\\ &\quad - \hat\vartheta z_3\frac{\partial\alpha_2}{\partial x_2}(\tilde a x_3 + \tilde b^T f_1(x_1,x_2)) + \tilde a z_2 z_3\\ &\quad - \hat\vartheta\frac{\partial\alpha_2}{\partial\hat b}z_3(\dot{\hat b} - \tau_2^b) - \hat\vartheta\frac{\partial\alpha_2}{\partial\hat a}z_3(\dot{\hat a} - \tau_2^a)\\ &\le -\sum_{i=1}^{3} c_i z_i^2 - (\dot{\hat\vartheta} - \tau_2^\vartheta)\alpha_2 z_3 + z_3 z_4\\ &\quad + 0.2785(\varepsilon_1 + \varepsilon_2) - \frac{|a|}{\Gamma_\vartheta}\tilde\vartheta(\dot{\hat\vartheta} - \tau_1^\vartheta)\\ &\quad - \hat\vartheta\frac{\partial\alpha_2}{\partial\hat a}z_3(\dot{\hat a} - \tau_2^a) - \frac{1}{\Gamma_a}\tilde a(\dot{\hat a} - \tau_1^a - \Gamma_a z_2 z_3\\ &\quad + \Gamma_a\hat\vartheta z_3\frac{\partial\alpha_2}{\partial x_2}x_3) - \hat\vartheta\frac{\partial\alpha_2}{\partial\hat b}z_3(\dot{\hat b} - \tau_2^b)\\ &\quad - \tilde b^T\Gamma_b^{-1}(\dot{\hat b} - \tau_1^b + \Gamma_b\hat\vartheta z_3\frac{\partial\alpha_2}{\partial x_2}f_1(x_1,x_2))\end{aligned} \quad (37)$$

where

$$\begin{aligned}\tau_2^{\vartheta} &= \tau_1^{\vartheta}\\ \tau_2^{a} &= \tau_1^{a} + \Gamma_a z_2 z_3 - \Gamma_a \hat{\vartheta} z_3 \frac{\partial \alpha_2}{\partial x_2} x_3\\ \tau_2^{b} &= \tau_1^{b} - \Gamma_b \hat{\vartheta} z_3 \frac{\partial \alpha_2}{\partial x_2} f_1(x_1, x_2)\end{aligned} \tag{38}$$

So we have

$$\begin{aligned}\dot{V}_3 \le & -\sum_{i=1}^{3} c_i z_i^2 - (\dot{\hat{\vartheta}} - \tau_2^{\vartheta})\alpha_2 z_3 + z_3 z_4\\ & +0.2785(\varepsilon_1 + \varepsilon_2) - \frac{|a|}{\Gamma_{\vartheta}} \tilde{\vartheta}(\dot{\hat{\vartheta}} - \tau_2^{\vartheta})\\ & -\hat{\vartheta}\frac{\partial \alpha_2}{\partial \hat{a}} z_3(\dot{\hat{a}} - \tau_2^{a}) - \frac{1}{\Gamma_a}\tilde{a}(\dot{\hat{a}} - \tau_2^{a})\\ & -\hat{\vartheta}\frac{\partial \alpha_2}{\partial \hat{b}} z_3(\dot{\hat{b}} - \tau_2^{b}) - \tilde{b}^T \Gamma_b^{-1}(\dot{\hat{b}} - \tau_2^{b})\end{aligned} \tag{39}$$

Step 4: The derivative of z_4 is

$$\begin{aligned}\dot{z}_4 &= \dot{x}_4 - \dot{\alpha}_3 - y_r^{(4)}\\ &= \rho\sum_{j=1}^{2}(\sigma_j \nu_j + \bar{u}_j) + b^T f_4(x_1, x_2, x_3, x_4)\\ &\quad +w_2(t) - \dot{\alpha}_3 - y_r^{(4)}\end{aligned} \tag{40}$$

The derivative of α_3 is

$$\begin{aligned}\dot{\alpha}_3 = & \frac{\partial \alpha_3}{\partial x_1} x_2 + \frac{\partial \alpha_3}{\partial x_2}(a x_3 + b^T f_1(x_1, x_2) + w_1(t))\\ & +\frac{\partial \alpha_3}{\partial x_3} x_4 + \sum_{i=0}^{3} \frac{\partial \alpha_3}{\partial y_r^{(i)}} y_r^{(i+1)} + \frac{\partial \alpha_3}{\partial \hat{\vartheta}}\dot{\hat{\vartheta}}\\ & +\frac{\partial \alpha_3}{\partial \hat{a}}\dot{\hat{a}} + \frac{\partial \alpha_3}{\partial \hat{b}}\dot{\hat{b}}\end{aligned} \tag{41}$$

Different from traditional backstepping technique, in the following we will give the control law and adaptive update laws of unknown parameters. Similar to [7]-[9], the structure of adaptive failure compensation controller can be written as

$$\nu_j = sign(\rho)\kappa^T \varpi$$

where κ is a desired parametric vector and ϖ is a known vector. Both are 3 dimensional vectors. They can be described as

$$\kappa = (\kappa_1, \kappa_{21}, \kappa_{22})^T; \quad \varpi = (\varpi_1, \varpi_{21}, \varpi_{22})^T \tag{42}$$

Because κ is unknown, it can not be directly used in the ν_j design. Instead, we use its estimation to generate the input signal ν_j. Then we can get

Control Laws:

$$\nu_j = sign(\rho)\hat{\kappa}^T \varpi \tag{43}$$

where $\hat{\kappa}$ be its estimation. Let T_i is the time instant of actuator failure occurrence and set Q_{iT} denotes the actuators of total failure in interval $(T_i, T_{i+1}](i = 0, 1, \cdots, f)$ and $Q_{iT} \cup \bar{Q}_{iT} = 1, 2$. Let $T_f = +\infty$ and $T_0 = 0$. In time interval $(T_0, T_1]$, all actuators work normally. In interval $(T_i, T_{i+1}]$, vectors κ and ϖ should be chosen to satisfy

$$\sum_{j \in \bar{Q}_{iT}} \rho\sigma_j \nu_j = \alpha - \sum_{j \in Q_{iT}} \rho\bar{u}_j, \quad t \in (T_i, T_{i+1}] \tag{44}$$

By fixing ϖ as

$$\varpi_1 = \alpha; \quad \varpi_{2j} = 1(j = 1, 2) \tag{45}$$

κ can be chosen as

$$\begin{aligned}\kappa_1 &= \frac{1}{\sum_{j \in \bar{Q}_{iT}} |\rho|\sigma_j}; \quad \kappa_{2j} = \frac{\rho\bar{u}_j}{\sum_{j \in \bar{Q}_{iT}} |\rho|\sigma_i}(j \in Q_{iT});\\ \kappa_{2j} &= 0, \quad (j \in \bar{Q}_{iT})\end{aligned} \tag{46}$$

α can be regarded as the virtual control signal in this step and chosen as

$$\begin{aligned}\alpha = & -c_4 z_4 - z_3 + y_r^{(4)} + \frac{\partial \alpha_3}{\partial x_1} x_2 + \frac{\partial \alpha_3}{\partial x_3} x_4\\ & +\sum_{i=0}^{3} \frac{\partial \alpha_3}{\partial y_r^{(i)}} y_r^{(i+1)} + \frac{\partial \alpha_3}{\partial x_2}\hat{b}^T f_1 + \frac{\partial \alpha_3}{\partial x_2} x_3 \hat{a} - \hat{b}^T f_4\\ & +\frac{\partial \alpha_3}{\partial \hat{\vartheta}}\tau_3^{\vartheta} + \frac{\partial \alpha_3}{\partial \hat{a}}\tau_3^{a} + \frac{\partial \alpha_3}{\partial \hat{b}}\tau_3^{b}\\ & -\bar{\delta}_2(x_1, x_2, x_3, x_4) tanh(\frac{\bar{\delta}_2(x_1, x_2, x_3, x_4) z_4}{\varepsilon_3})\\ & +\hat{\vartheta}(\Gamma_b f_4 - \Gamma_b \frac{\partial \alpha_3}{\partial x_2} f_1)\frac{\partial \alpha_2}{\partial \hat{b}} z_3 - \hat{\vartheta}\Gamma_a \frac{\partial \alpha_3}{\partial x_2} x_3 \frac{\partial \alpha_2}{\partial \hat{a}} z_3\end{aligned} \tag{47}$$

where

$$\bar{\delta}_2(x_1, x_2, x_3, x_4) \ge |w_2(t) - \frac{\partial \alpha_3}{\partial x_2} w_1(t)| \tag{48}$$

and **update laws** are

$$\begin{aligned}\dot{\hat{b}} &= proj(\tau_3^{b}); \quad \tau_3^{b} = \tau_2^{b} + \hat{\vartheta}(\Gamma_b f_4 z_4 - \Gamma_b \frac{\partial \alpha_3}{\partial x_2} f_1 z_4)\\ \dot{\hat{a}} &= proj(\tau_3^{a}); \quad \tau_3^{a} = \tau_2^{a} - \hat{\vartheta}\Gamma_a \frac{\partial \alpha_3}{\partial x_2} x_3 z_4\\ \dot{\hat{\vartheta}} &= proj(\tau_3^{\vartheta}); \quad \tau_3^{\vartheta} = \tau_2^{\vartheta}\\ \dot{\hat{\kappa}} &= -\Gamma_\kappa \varpi z_4 - \Gamma_\kappa l_\kappa(\hat{\kappa} - \kappa_0)\end{aligned} \tag{49}$$

where Γ_κ is a positive definite matrix. l_κ is a positive constant and κ_0 is a constant vector. $proj(\cdot)$ denotes a projection operator. It can guarantee that the estimations $\hat{b}, \hat{a}, \hat{\vartheta}$ are all bounded. We choose Lyapunov function in time interval $(T_i, T_{i+1}]$

$$V = V_3 + \frac{1}{2}z_4^2 + \sum_{j \in \bar{Q}_{iT}} \frac{\sigma_j|\rho|}{2}\tilde{\kappa}^T \Gamma_\kappa^{-1}\tilde{\kappa} \tag{50}$$

where $\tilde{\kappa} = \kappa - \hat{\kappa}$. Γ_κ is a positive definite matrix. The derivative of V is

$$\begin{aligned}\dot{V} \le & -\sum_{i=1}^{3} c_i z_i^2 - (\dot{\hat{\vartheta}} - \tau_2^{\vartheta})\alpha_2 z_3 + z_3 z_4 + 0.2785(\varepsilon_1\\ & +\varepsilon_2) - \frac{|a|}{\Gamma_{\vartheta}}\tilde{\vartheta}(\dot{\hat{\vartheta}} - \tau_2^{\vartheta}) - \hat{\vartheta}\frac{\partial \alpha_2}{\partial \hat{a}} z_3(\dot{\hat{a}} - \tau_2^{a})\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{\Gamma_a}\tilde{a}(\dot{\hat{a}}-\tau_2^a)-\hat{\vartheta}\frac{\partial\alpha_2}{\partial\hat{b}}z_3(\dot{\hat{b}}-\tau_2^b)\\
&-\tilde{b}^T\Gamma_b^{-1}(\dot{\hat{b}}-\tau_2^b)+z_4\Big(\rho\sum_{j=1}^{2}(\sigma_j\nu_j+\bar{u}_j)\\
&+b^Tf_4(x_1,x_2,x_3,x_4)+w_2(t)-y_r^{(4)}\\
&-\frac{\partial\alpha_3}{\partial x_1}x_2-\frac{\partial\alpha_3}{\partial x_2}(ax_3+b^Tf_1(x_1,x_2)\\
&+w_1(t))-\frac{\partial\alpha_3}{\partial x_3}x_4-\sum_{i=0}^{3}\frac{\partial\alpha_3}{\partial y_r^{(i)}}y_r^{(i+1)}-\frac{\partial\alpha_3}{\partial\hat{\vartheta}}\dot{\hat{\vartheta}}\\
&-\frac{\partial\alpha_3}{\partial a}\dot{\hat{a}}-\frac{\partial\alpha_3}{\partial b}\dot{\hat{b}}\Big)-\sum_{j\in\bar{Q}_{iT}}\frac{\sigma_j|\rho|}{2}\tilde{\kappa}^T\Gamma_\kappa^{-1}\dot{\hat{\kappa}} \qquad (51)
\end{aligned}
$$

Note that

$$
\rho\sum_{j=1}^{2}(\sigma_j\nu_j+\bar{u}_j)=\sum_{j\in\bar{Q}_{iT}}\rho\sigma_j\nu_j+\sum_{j\in Q_{iT}}\rho\bar{u}_j
$$

With (45), we have

$$
\rho\sum_{j=1}^{2}(\sigma_j\nu_j+\bar{u}_j)=\alpha
$$

Then with virtual control α given in (48), we have

$$
\begin{aligned}
\dot{V}\le&-\sum_{i=1}^{4}c_iz_i^2-(\dot{\hat{\vartheta}}-\tau_2^{\vartheta})\alpha_2z_3+0.2785(\varepsilon_1+\varepsilon_2\\
&+\varepsilon_3)-\frac{|a|}{\Gamma_\vartheta}\tilde{\vartheta}(\dot{\hat{\vartheta}}-\tau_2^{\vartheta})-\hat{\vartheta}\frac{\partial\alpha_2}{\partial\hat{a}}z_3(\dot{\hat{a}}-(\tau_2^a\\
&-\hat{\vartheta}\Gamma_a\frac{\partial\alpha_3}{\partial x_2}x_3z_4))-\frac{1}{\Gamma_a}\tilde{a}(\dot{\hat{a}}-(\tau_2^a-\hat{\vartheta}\Gamma_a\frac{\partial\alpha_3}{\partial x_2}x_3\\
&z_4))-\hat{\vartheta}\frac{\partial\alpha_2}{\partial\hat{b}}z_3(\dot{\hat{b}}-(\tau_2^b+\hat{\vartheta}(\Gamma_bf_4z_4-\Gamma_b\frac{\partial\alpha_3}{\partial x_2}f_1\\
&z_4)))-\tilde{b}^T\Gamma_b^{-1}(\dot{\hat{b}}-(\tau_2^b+\hat{\vartheta}(\Gamma_bf_4z_4-\Gamma_b\frac{\partial\alpha_3}{\partial x_2}f_1\\
&z_4)))-\frac{\partial\alpha_3}{\partial\hat{\vartheta}}z_4(\dot{\hat{\vartheta}}-\tau_2^{\vartheta})-\frac{\partial\alpha_3}{\partial a}z_4(\dot{\hat{a}}-(\tau_2^a-\hat{\vartheta}\Gamma_a\\
&\frac{\partial\alpha_3}{\partial x_2}x_3z_4))-\frac{\partial\alpha_3}{\partial b}z_4(\dot{\hat{b}}-(\tau_2^b+\hat{\vartheta}(\Gamma_bf_4z_4-\Gamma_b\\
&\frac{\partial\alpha_3}{\partial x_2}f_1z_4)))-\sum_{j\in\bar{Q}_{iT}}\frac{\sigma_j|\rho|}{2}\tilde{\kappa}^T\Gamma_\kappa^{-1}(\dot{\hat{\kappa}}+\Gamma_\kappa\varpi z_4) \qquad (52)
\end{aligned}
$$

With update laws (50), we can get

$$
\begin{aligned}
\dot{V}\le-\sum_{i=1}^{4}c_iz_i^2+\sum_{j\in\bar{Q}_{iT}}\frac{\sigma_j|\rho|}{2}\tilde{\kappa}^Tl_\kappa(\hat{\kappa}-\kappa_0)\\
+0.2785\sum_{i=1}^{3}\varepsilon_i \qquad (53)
\end{aligned}
$$

Note that

$$
\tilde{\kappa}^Tl_\kappa(\hat{\kappa}-\kappa_0)\le-\frac{1}{2}l_\kappa||\tilde{\kappa}||^2+\frac{1}{2}l_\kappa||\kappa-\kappa_0||^2 \qquad (54)
$$

So

$$
\begin{aligned}
\dot{V}\le-\sum_{i=1}^{4}c_iz_i^2+\sum_{j\in\bar{Q}_{iT}}\frac{\sigma_j|\rho|}{2}(-\frac{1}{2}l_\kappa||\tilde{\kappa}||^2\\
+\frac{1}{2}l_\kappa||\kappa-\kappa_0||^2)+0.2785\sum_{i=1}^{3}\varepsilon_i \qquad (55)
\end{aligned}
$$

Namely,

$$
\dot{V}\le-\sum_{i=1}^{4}c_iz_i^2-\sum_{j\in\bar{Q}_{iT}}\frac{\sigma_j|\rho|}{4}l_\kappa||\tilde{\kappa}||^2+\Pi_i \qquad (56)
$$

where

$$
\Pi_i=0.2785\sum_{i=1}^{3}\varepsilon_i+\sum_{j\in\bar{Q}_{iT}}\frac{\sigma_j|\rho|}{4}l_\kappa||\kappa-\kappa_0||^2 \qquad (57)
$$

Theorem 1: Consider the servo system driven by twin motors (1), with unknown parameters and unknown actuator failures described by (11). Under the Assumption 1 to Assumption 4 and with the control laws (43)-(48) and the update laws (49), all signals of the closed-loop system are bounded.

Proof: From (56), in time interval $[0, T_1]$ signals $\tilde{\kappa}$, z_i are bounded. Note that $\tilde{\vartheta}$, $\tilde{a}$, $\tilde{b}$ are bounded due to the *proj*$(\cdot)$ operator in update laws. Therefor we can get V is bounded in interval $[0, T_1]$.

Note that the difference between $V(T_1^+)$ and $V(T_1^-)$ is only the coefficients in front of the term $\tilde{\kappa}^T\Gamma_\kappa\tilde{\kappa}$. Since all the possible jumping on κ are bounded, $V(T_1^+)$ is bounded, then $V(T_2^-)$ is bounded. Similar to the above analysis, we can get $V(T_{j+1}^-)$ is bounded from the bound of $V(T_j^+)$. Also in time interval (T_f, ∞), it can be shown that $V(t)$ is bounded. Then we have z, $\tilde{\vartheta}$, $\tilde{\kappa}$, $\tilde{b}$, $\tilde{a}$ bounded in $[0, \infty]$. Further more, all signals are bounded including virtual control $\alpha_i(i = 1, 2, 3)$, α state x_i and input signal ν_i. □□

IV. SIMULATION STUDIES

Now we consider the servo system driven by twin motors given in (12). Unknown modelling errors ω_1 and ω_2 are given as

$$
\omega_1=0.1sin(x_1+2x_2^2);\quad \omega_2=0.1cos(x_4) \qquad (58)
$$

In simulation, the parameters of the servo system are taken as $J_{mi} = 6kg\cdot m^2(i = 1, 2)$, $K_{Ti} = 0.04N\cdot m/A(i = 1, 2)$, $J_L = 6kg\cdot m^2$, $K_L = 0.16N\cdot m/A$. The design parameters are selected as $c_1 = 35$, $c_2 = 5$, $c_3 = 5$, $c_4 = 2$, $\varepsilon_1 = \varepsilon_2 = 1$, $\Gamma_a = \Gamma_\vartheta = 0.001$, $\Gamma_\kappa = 0.001E_3$, $\Gamma_b = 0.001E_2$, where E_2, E_3 denotes identity matrix.

The initial values are chosen as $x_1(0) = x_2(0) = x_3(0) = x_4(0) = 0$, and $\hat{a} = 0$, $\hat{b} = 0$, $\hat{\vartheta} = 0$, $\hat{\kappa} = 0$. The reference signal is set as $y_r = 0.5sint$. To illustrate the effectiveness of this proposed control scheme, the following four cases are considered.

Case 1. All actuators work normally during the operation of servo system. Namely no failure occurs. Figs.2-3 show

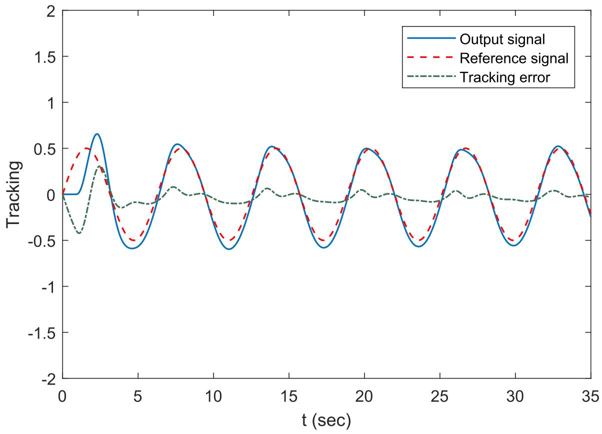

FIGURE 2. **Tracking (Case 1).**

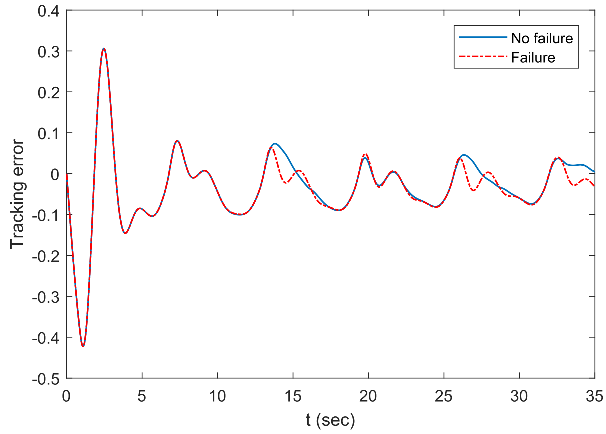

FIGURE 5. **Tracking errors (Case 1 and Case 2).**

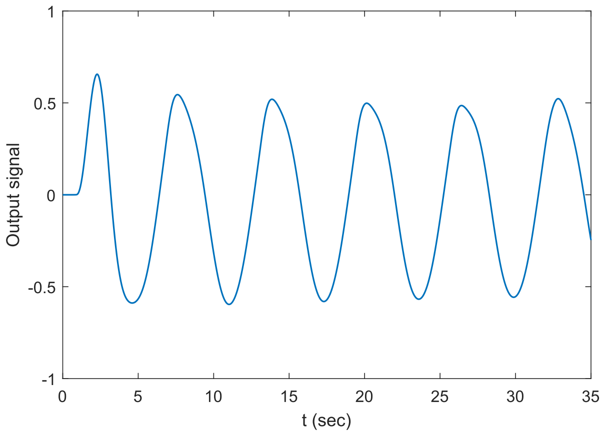

FIGURE 3. **Output signal (Case 1).**

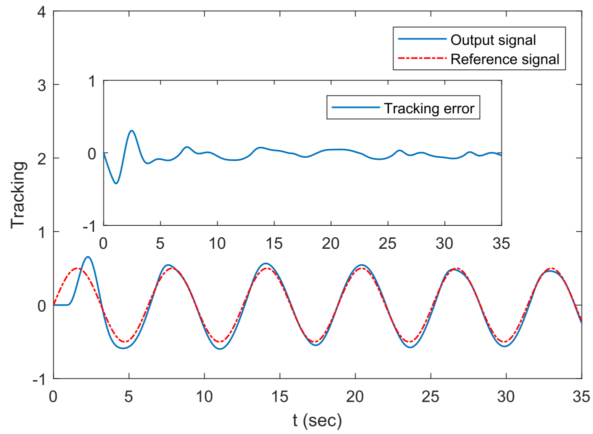

FIGURE 6. **Tracking errors (Case 3).**

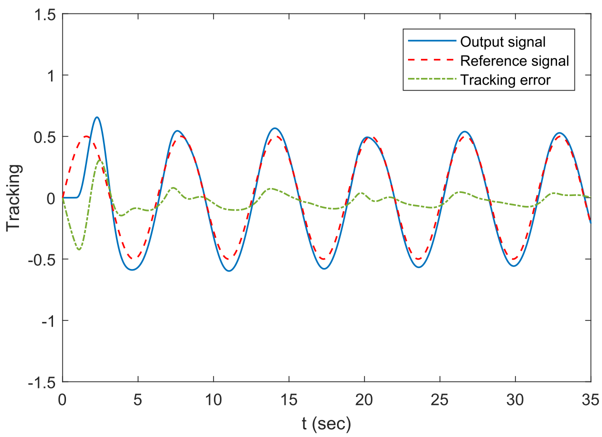

FIGURE 4. **Tracking (Case 2).**

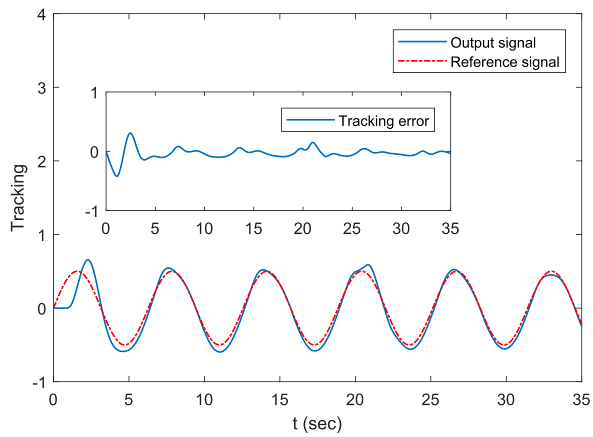

FIGURE 7. **Tracking errors (Case 4).**

the simulation results. The tracking performance including tracking error is shown in Fig.2 and the output signal is given in Fig.3.

Case 2. We suppose at $t = 10$, the first actuator loses its effectiveness by an unknown value 90%. The respective has been shown in Fig.4 including tracking error and output signal. Fig.5 shows the compare of the tracking error between these first two cases.

Case 3. We suppose at $t = 10$, the first actuator is stuck at an unknown value 30. The tracking error and output signal are shown in Fig.6.

Case 4. A comprehensive failures is considered in this case. We suppose the first actuator loses its effectiveness by an unknown value 50% at unknown time instant $t = 10$ and the second actuator is stuck at an unknown value 20 at $t = 20$. The tracking error and output signal are shown in Fig.7.

Based on the above simulation results, we can get that the tracking performance of the system can be achieved successfully whether or not failures occur.

V. CONCLUSION

An adaptive robust control scheme is proposed by using backstepping techniques to compensate unknown actuator failures for servo system driven by twin motors. The boundedness of all signals of closed-loop system can be guaranteed whether or not failures occur. The simulation studies also have verified the established theoretical results.

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