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Couple-Group Consensus for First Order Discrete-Time Multiagent Systems With Competition-Cooperation and Input Saturation Constraints

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ABSTRACT Collaborative consensus control of multiagent systems has become an important research topic in the field of artificial intelligence in recent years. This paper deals with the couple-group consensus for the first order discrete-time systems with input saturation constraints. Compared to most previous studies, this article focuses on the control input constraints phenomenon in couple-group consensus problem. Based on the relationship between competition and cooperation, a new negotiation protocol is designed to achieve couple-group consensus. Furthermore, combining graph theory with Lyapunov theorem, the control gain condition is obtained. Finally, the results of numeric modeling show that the method is correctness and effectiveness.


INDEX TERMS Couple-group consensus, input saturation constraints, multiagent systems, cooperation-competition.

I. INTRODUCTION

In recent years, with the extensively application of multiagent systems, the coordination control has been widely discussed [1]–[6]. In the coordinated control of multiagent systems, each agent needs to communicate with other agents through receiving and broadcasting information among neighbors. Only in this way can all agents are able to reach a common state, such as position, phase, and velocity. The sophistication of the consensus problems essentially depend on the consensus control of the multiagent systems, such as formation control, tracking control, attitude alignment, etc. In order to study the consensus problems of the multiagent systems, the interactions among agents can be regarded as a network topology. Moreover, consensus control of the multiagent systems can be achieved under the appropriate protocols and algorithms. For verifying the convergence of multiagent

systems, some effective methods such as graph theory, matrix theory and Lyapunov stability theory are used.

At present, the interconnections among agents become more and more complex, and the idea of group consensus is to solve multitasks problems or decompose massive problems. It is more common to urge agents to achieve different goals in different sub-networks. The idea of this collective consensus has been attracted attention in [7]–[11]. However, the above work was to achieve group consensus on the premise of cooperation relationship. In fact, there are not only collaborative relationship but also competitive relationship among agents. It's a universal phenomenon in real world such as the hunting for intruders, race competition [12], [13]. The weights of edges were negative to represent the competitive relationship in [14]. Additionally, the switching topology of undirected graph and bidirectional graph networks were further explored in [15]. In [16], two control protocols were designed by using the properties of Laplacian matrix to achieve finite time consensus in antagonistic network. However, in the above studies, input constraints have not been considered.

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Since input constraints of the systems are inevitable, input saturation has attracted wide attention in control systems. In [17]–[20], the stabilization problem of linear systems with input saturation was proposed. And low-gain feedback was an effective method to design a family of linear control laws in a semi-global stabilization framework [21]. The characteristic of low-gain feedback is that given any bounded initial conditions, the closed-loop system can always be kept in the online domain by establishing small feedback gain parameters. The Semi-global consensus tracking problem for linear multi-agent systems with actuator saturation was studied in [21]–[23] by using low-gain feedback. At the same time, there were other researches on coordination tracking under saturation constraints. For example, in multiagent systems, the problems with input saturation coordination tracking for single integral system [24] and double-integrators [25]–[27] have been studied. Ref. [28] solved the global consensus tracking problem for high-order multiagent systems with input saturation. Using low-gain and high-gain feedback methods, the semi-global coordinated tracking of linear multiagent systems with input constraints was investigated in [29]. And the conditions of achieving global consensus for the first order discrete-time system and second system with input saturation constraints were given in [30], but there wasn't competitive-cooperative relationship among agents. Ref. [31] considered competition relationship between two groups. Ref. [32] designed a hybrid fuzzy control strategy for the goethite process and showed better control performance than PID controller. Different from our research, we mainly study the interaction between systems to achieve couple-group consensus. Competitive relationship exists in real life, especially in the field of multiagent systems, while previous research rarely takes into account the competitive relationship between two groups in multiagent systems via input constraints.

Thus, inspired by [30], [31], this paper proposes the following innovative ideas and methods.

- 1) Based on previous studies, this paper not only considers the competition-cooperation relationship among agents but also considers input constrains for first order discrete-time multiagent systems.
- 2) A novel control protocol is designed to realize couple-group consensus.
- 3) The sufficient condition of control parameter is obtained to ensure consensus under designed control protocol.

The rest of this paper is organized as follows. Section 2 lists some useful preliminaries, graph theory and problem formulations. Section 3 studies the first order discrete-time system and presents a sufficient condition of control parameters to achieve couple-group consensus. Several simulations are presented in Section 4 to verify the correctness of the theoretical results. Finally, the conclusion is drawn in section 5.

Note: In this paper, \mathbb{R} and $\mathbb{R}^{n \times n}$ stand for the one dimensional real space and the $n \times n$ real matrices respectively. I_n denotes n -dimensional identity matrix. Given a matrix A ,

A^T denotes its transpose and $\|A\|$ denotes its operator norm. Spectral radius of $n \times n$ real matrix A is denoted by $\rho(A) = \max\{|\lambda_i|, i = 1, 2, \dots, n\}$, where λ_i is a eigenvalue of A . The matrix $A > (\geq) 0$ or $A < (\leq) 0$ means that A is positive (positive semi-definite) or negative (negative semi-definite), which implies all the corresponding eigenvalues are positive (non-negative) or negative (non-positive). The Kronecker product is denoted by \otimes , which satisfies the following conditions

$$\begin{aligned} \|A \otimes I_N\| &= \|A\|, \\ (A + B) \otimes C &= A \otimes C + B \otimes C, \\ (A \otimes B)(C \otimes D) &= (AC) \otimes (BD), \\ \text{rank}(A \otimes B) &= \text{rank}(A) \text{rank}(B). \end{aligned}$$

II. PRELIMINARIES AND PROBLEM FORMULATION

A. PRELIMINARIES

In the beginning, the graph theory, interconnection topology and necessary mathematical notations are introduced.

The multiagent system with $N + M$ agents can be described by a weighted undirected graph $G = (V, \zeta, W)$, where $V = \{v_1, v_2, \dots, v_{N+M}\}$ denotes the node set indexed by a related agent set $I = \{1, 2, \dots, N + M\}$, $\zeta = V \times V$ denotes the edges set and $W = (w_{ij})_{(N+M) \times (N+M)} \in \mathbb{R}^{(N+M) \times (N+M)}$ denotes the adjacency matrix, where $w_{ij} > 0$ if $(j, i) \in \zeta$ and $w_{ij} = 0$ otherwise. Assuming that $w_{ij} = w_{ji}$ for $i \neq j$ and $w_{ii} = 0$ for $i \in I$. Let $N_i = \{j \in V : (j, i) \in \zeta\}$ be the set of neighbors of node i in G . When $i \neq j$, the Laplacian matrix is $L = (l_{ij})_{(N+M) \times (N+M)}$ with $l_{ij} = -w_{ij}$ and when $i = j$, $l_{ii} = \sum_{j=1, j \neq i}^{N+M} w_{ij}$. The eigenvalues of L can be denoted as $\lambda_i, i = 1, 2, 3, \dots, N + M$.

B. PROBLEM FORMULATION

A first order discrete-time multiagent system with input saturation constraints is considered. And there is cooperation-competition relationship among $N + M$ agents.

Assume that there are N agents in one group and M agents in the other one. So the network contains two sub-network $G_1 = (V_1, \zeta_1, W_1)$ and $G_2 = (V_2, \zeta_2, W_2)$, where $V_1 = \{v_1, v_2, \dots, v_N\}$ and $V_2 = \{v_{N+1}, v_{N+2}, \dots, v_{N+M}\}$ are node sets of two groups, ζ_1 and ζ_2 are the edge sets of two groups, W_1 and W_2 are adjacency matrices of G_1 and G_2 , respectively. $I_1 = \{1, 2, \dots, N\}$ and $I_2 = \{N + 1, N + 2, \dots, N + M\}$ are the index sets of two sub-networks. Note that $V = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$. The neighbors sets of two groups are $N_{1i} = \{v_j \in V_1 | (v_j, v_i) \in \varepsilon\}$ and $N_{2i} = \{v_j \in V_2 | (v_j, v_i) \in \varepsilon\}, \forall i \in I$.

Consider a first order discrete-time multiagent system with $N + M$ identical agents whose dynamics are given by

$$x_i(k + 1) = Ax_i(k) + B\delta(u_i(k)), \quad i \in V, \quad (1)$$

where $x_i k \in \mathbb{R}^n, u_i(k) \in \mathbb{R}^m$ represent state and desired control signal of agent i , and $\delta(u_i(k)) = [\delta(u_{i1}(k)), \delta(u_{i2}(k)), \dots, \delta(u_{im}(k))]^T$, the matrix $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}$.

Each $\delta(\cdot)$ is the saturation function

$$\delta(u_{ih}(k)) = \begin{cases} 1, & u_{ih} \geq 1, \\ u_{ih}, & |u_{ih}| < 1, \\ -1, & u_{ih} \leq -1, \end{cases}$$

Remark 1: There are both competition and cooperation in the network. In other words, each agent cooperates with its neighbors in the same sub-network, while competes with its neighbors which lie in the other sub-network. In addition, there exist input saturation constraints $\delta(u_i)$ in the system (1), which obviously can influence the group consensus. When the saturation happens, the controlled object can't receive the desired control signal so that it causes the instability of the system (1).

Remark 2: Compared with [31], where only considers the influence of system matrix $A = I$ and $B = I$, the dynamics of agents is more complex in this paper. The system model in Ref [31] is a special circumstance of this paper. Moreover, Ref [31] only considered competition-cooperation relationship among agents. However, input saturation is inevitable in system. So it is essential to consider input saturation.

Remark 3: Ref. [30] considered single group consensus, and all agents reach consensus. In Ref [31], [33], not only cooperation relationship in single group but also competition relationship between couple group were considered. Couple-group consensus means that different groups interact with each other. Since competitive relationship exists in real life, it is meaningful to consider competition-cooperation relationship among agents.

Definition 1 [31]: If the states of the agents in G satisfy the following two conditions:

$$\begin{aligned} \lim_{k \rightarrow +\infty} \|x_i(k) - x_j(k)\| &= 0, \quad \forall i, j \in I_1 \\ \lim_{k \rightarrow +\infty} \|x_i(k) - x_j(k)\| &= 0, \quad \forall i, j \in I_2 \end{aligned}$$

then multiagent system (1) is said to reach group consensus asymptotically. Different groups will converge to the opposite value.

Assumption 1: The undirected graph G is connected.

Assumption 2: The pair (A, B) is controllable.

Assumption 3: $A^T A = I_n$.

Definition 2 [31]: $L = (l_{ij}) \in R^{(N+M) \times (N+M)}$ is defined as

$$l_{ij} = \begin{cases} w_{ij}, & i \in I_1, v_j \in N_{2i}, i \in I_2, v_j \in N_{1i}, \\ -w_{ij}, & i \in I_1, v_j \in N_{1i}, i \in I_2, v_j \in N_{2i}, \\ \sum_{k \neq i, k=1} |w_{ik}|, & i = j. \end{cases}$$

Lemma 1 [31]: In Definition 2, $L = (l_{ij}) \geq 0$ and $\text{rank}(L) = N + M - 1$, if G has a spanning tree.

Lemma 2 [30]: The matrix \bar{L} , as shown at the bottom of the page, is obtained through appropriate adjustment of Laplacian matrix $L = (l_{ij})$. The matrix \bar{L} has the following two properties:

- 1) all non-zero eigenvalues of \bar{L} have positive real parts;
- 2) \bar{L} has no zero eigenvalue, if and only if the graph G has a globally reachable node.

III. MAIN RESULTS

In this section, the first order discrete-time multiagent system with input constraints is investigated. Based on above assumptions, the sufficient condition is given to achieve couple-group consensus.

In order to realize couple-group consensus, we suppose that each agent cooperates with its neighbors in the same sub-network. If its neighbors lie in different sub-network, the agent will compete with them. For the system (1), a novel control protocols is designed as

$$u_i(k) = \begin{cases} -\eta B^T A \left[\sum_{v_j \in N_{1i}} w_{ij} (x_i(k) - x_j(k)) \right. \\ \left. + \sum_{v_j \in N_{2i}} w_{ij} (x_i(k) + x_j(k)) \right], & i \in I_1 \\ -\eta B^T A \left[\sum_{v_j \in N_{2i}} w_{ij} (x_i(k) - x_j(k)) \right. \\ \left. + \sum_{v_j \in N_{1i}} w_{ij} (x_i(k) + x_j(k)) \right], & i \in I_2 \end{cases} \quad (2)$$

where $\eta > 0$ is a control gain.

Remark 4: From [33], “-” represents cooperation relationship among agents and “+” represents the competitive relationship.

The system (1) can be written as the flowing form

$$x(k+1) = (I_{N+M} \otimes A)x(k) + (I_{N+M} \otimes B)\delta(u(k)), \quad (3)$$

where $x = [x_1, x_2, \dots, x_{N+M}]^T \in R^{(N+M) \times n}$, $\delta(u(k)) = [\delta(u_1(k)), \delta(u_2(k)), \dots, \delta(u_{N+M}(k))]^T \in R^{(N+M) \times m}$.

Theorem 1: Suppose that Assumption 1, 2 and 3 hold. The multiagent system (1) can achieve couple-group consensus if control gain $\eta \in \left(0, \frac{2}{\rho(L)\|B^T B\|}\right)$, where $\rho(L)$ is spectral radius, $\|B^T B\|$ denotes its operator norm.

Proof: Combine the control protocol (2) of agent i as well as the Definition 2, the control protocol $u(k)$ of the system dynamics (3) can be

$$u(k) = -\eta (L \otimes B^T A)x(k), \quad (4)$$

where $x(k)$ denote the sates of all agents.

$$\bar{L} = \begin{bmatrix} l_{2,2} - l_{1,2} & \cdots & l_{2,N} - l_{1,N} & \cdots & l_{2,N+M} - l_{1,N+M} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ l_{N,2} - l_{1,2} & \cdots & l_{N,N} - l_{1,N} & \cdots & l_{N,N+M} - l_{1,N+M} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ l_{N+M,2} - l_{1,2} & \cdots & l_{N+M,N} - l_{1,N} & \cdots & l_{N+M,N+M} - l_{1,N+M} \end{bmatrix} \in R^{(N+M-1) \times (N+M-1)}.$$

Consider the Lyapunov function

$$J(x(k)) = \frac{1}{2} x^T(k) (L \otimes I_n) x(k). \quad (5)$$

From Eq. (5), we have $J(x(k)) \geq 0$, $J(x(k)) = 0$ if and only if $x(k) = 0$.

Then combine with Eq.(3) and (5), the difference of $J(x(k))$ with respect to k , i.e.

$$\Delta J(x(k)) = J(x(k+1)) - J(x(k)) \quad (6)$$

Since $L = L^T$ for undirected graphs, we obtain

$$\begin{aligned} \Delta J(x(k)) &= J(x(k+1)) - J(x(k)) \\ &= \frac{1}{2} x^T(k+1) (L \otimes I_n) x(k+1) - \frac{1}{2} x^T(k) (L \otimes I_n) x(k) \\ &= \frac{1}{2} (Ax(k) + B\delta(u(k)))^T (L \otimes I_n) (Ax(k) + B\delta(u(k))) \\ &\quad - \frac{1}{2} x^T(k) (L \otimes I_n) x(k) \\ &= \frac{1}{2} \delta^T(u(k)) \left(L \otimes B^T A \right) x(k) + \frac{1}{2} x^T(k) \\ &\quad \times \left(L \otimes A^T B \right) \delta(u(k)) \\ &\quad + \frac{1}{2} \delta^T(u(k)) \left(L \otimes B^T B \right) \delta(u(k)) \\ &= -\frac{1}{\eta} \delta^T(u(k)) u(k) \\ &\quad + \frac{1}{2} \delta^T(u(k)) \left(L \otimes B^T B \right) \delta(u(k)) \\ &\leq -\delta^T(u(k)) \left(\frac{1}{\eta} I_{(N+M)n} - \frac{1}{2} \left(L \otimes B^T B \right) \right) \delta(u(k)) \\ &\leq 0. \end{aligned}$$

The last inequality comes from the property that $u^T \delta(u) \geq \delta(u)^T \delta(u)$. Combining with Eq.(6), we have $\Delta J(x(k)) \leq 0$, since $\eta \in \left(0, \frac{2}{\rho(L)\|B^T B\|} \right)$. $\Delta J(x(k)) = 0$ if and only if $\delta(u) = u = 0$, i.e. $(L \otimes B^T A)x = 0$.

Next, we will prove $(L \otimes B^T A)x = 0$ if and only if $x_i = x_j, i, j \in I_1$ or I_2 . Note that $(L \otimes B^T A)x = 0$ implies that $(\bar{L} \otimes B^T A)p = 0$, where the matrix \bar{L} is introduced in Lemma 2, $p = \begin{bmatrix} p_i^T; p_j^T \end{bmatrix}^T, p_i = x_i - x_1, i \in \{2, 3, \dots, N\}, p_j = x_j - x_1, j \in \{N+1, N+2, \dots, N+M\}$. It is obviously that the eigenvalues of \bar{L} are positive from Lemma 2, i.e. $\text{rank}(\bar{L}) = N + M - 1$.

It is worth noticing that $(\bar{L} \otimes B^T A)p = 0$ equals to $p^T (\bar{L} \otimes A^{-1}B) = 0$ from Assumption 3. From system (1), we have $p(k+1) = (I_{N+M-1} \otimes A)p(k) = 0$. Then, combining with Eq. (4), we get

$$\left(\bar{L} \otimes B^T A \right) p(k+1) = \left(\bar{L} \otimes B^T A^2 \right) p(k) = 0.$$

Obviously, $p^T(k) (\bar{L} \otimes A^{-2}B) = 0$. Then, continuing to iterate, we obtain

$$p^T(k) (\bar{L} \otimes A^{-q}B) = 0, \quad q = 2, 3, 4, \dots, n+1.$$

Hence,

$$p^T(k) \left(\bar{L} \otimes A^{-(n+1)} [A^n B \dots AB, B] \right) = 0. \quad (7)$$

Note that $\text{rank} [A^n B \dots AB, B] = n$ since Assumption 2 is satisfied. From Assumption 3 we know the matrix A is non-singular. So $\text{rank} (A^{-(n+1)} [A^n B \dots AB, B]) = n$. Then $\text{rank} (\bar{L} \otimes A^{-(n+1)} [A^n B \dots AB, B]) = \text{rank}(\bar{L})$, $\text{rank} (A^{-(n+1)} [A^n B \dots AB, B]) = (N + M - 1)n$.

Therefore, the solution of Eq.(7) is $p = 0$, i.e. $p_i = 0, p_j = 0$, so $\Delta J(x(k)) = 0$ if and only if $x_i = x_j, i, j \in I_1$ or I_2 . Thus, from Definition 2 the system (1) can asymptotically reach the group consensus.

Corollary 1: If the system (1) describes as the following form

$$x_i(k+1) = x_i(k) + \delta(u_i(k)), \quad i \in V. \quad (8)$$

The system (8) reach the group consensus if $\eta \in \left(0, \frac{2}{\rho(L)} \right)$.

IV. SIMULATION

In this section, three examples are provided to verify the effectiveness of the proposed theoretical results.

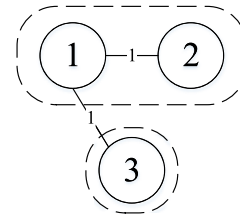


FIGURE 1. The topological graph of example 1.

Example 1: Consider a multiagent system, where the agent 1 and agent 2 are in the first group and the agent 3 is in second group. The topology graph is shown in Fig. 1, where the coupling strength between each pair of agents is 1 and the corresponding Laplacian matrix is

$$L_1 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad (9)$$

By Eq. (9) the max eigenvalue of the Laplacian matrix L_1 is 3.

From the condition is given in Corollary 1, let $\eta = 0.09$, $A = B = I_2$. Simulation results shown in Fig.1 verify the correctness of the Corollary 1. In addition, from Fig.2 the agent 1, 2, which located in the same group converge to same value meanwhile the agent 3 converges to the opposite value since competition relationship exists in system. Fig. 3 shows that input u of each agent converges to zero when $t \rightarrow +\infty$. Clearly, the input saturation $\delta(u)$ of agents belongs to $[-1, 1]$ in Fig.3.

Example 2: Consider the multiagent system with six agents, where agent 1, 2 and 3 are in the same group and the rest of agents are located in other group. The topology graph is shown in Fig. 4, where the coupling strength between each

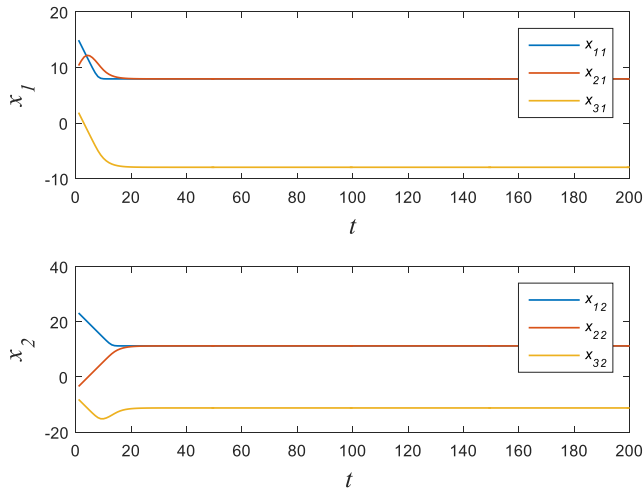


FIGURE 2. The trajectories of the agents.

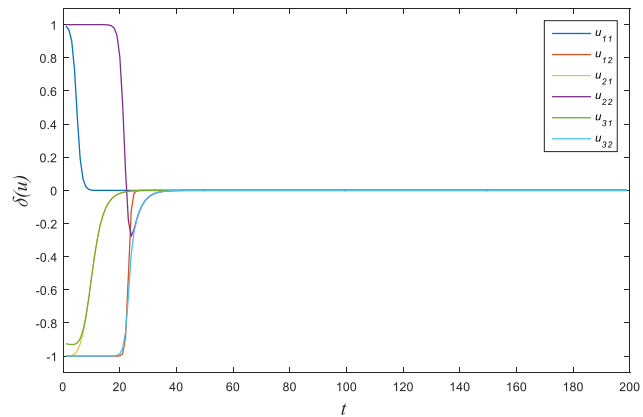


FIGURE 3. Saturated input $\delta(u_{ih}(k))$.

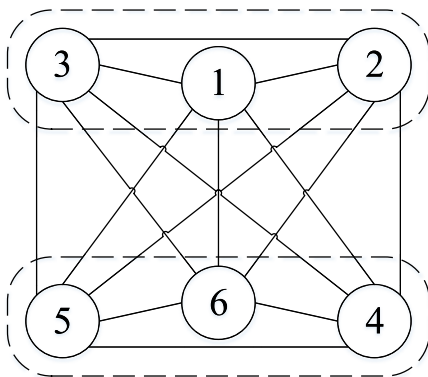


FIGURE 4. The topological graph of example 2.

pair of agents is 1. Then, the corresponding Laplacian matrix is

$$L_2 = \begin{bmatrix} 5 & -1 & -1 & 1 & 1 & 1 \\ -1 & 5 & -1 & 1 & 1 & 1 \\ -1 & -1 & 5 & 1 & 1 & 1 \\ 1 & 1 & 1 & 5 & -1 & -1 \\ 1 & 1 & 1 & -1 & 5 & -1 \\ 1 & 1 & 1 & -1 & -1 & 5 \end{bmatrix} \quad (10)$$

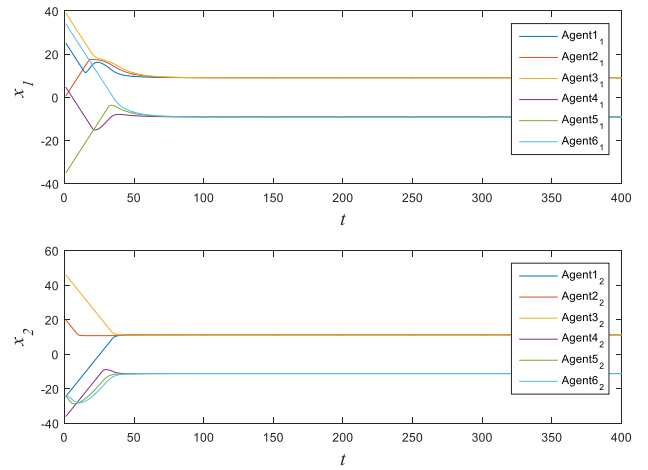


FIGURE 5. The trajectories of four agents.

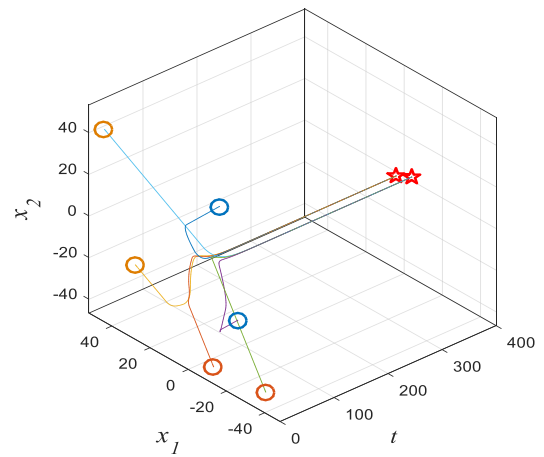


FIGURE 6. The 3D graphs of six agents' trajectories.

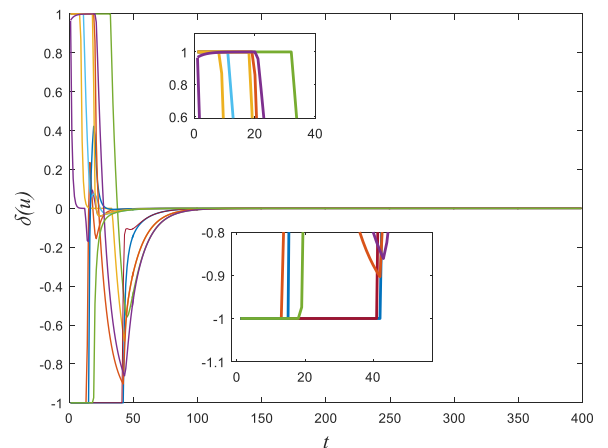


FIGURE 7. Input constraints of all agents.

The max eigenvalue derived of (10) is six. Let $\eta = 0.3, A = I_2, B = I_2$. Clearly η satisfies the condition of the Corollary 1. The trajectories of the six agents are shown in Fig. 5, which indicates that all agents converge to two adverse subgroups. Obviously, all agents achieve couple-group consensus since states of agents where in same

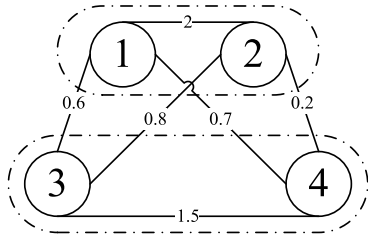


FIGURE 8. The topological graph of example 3.

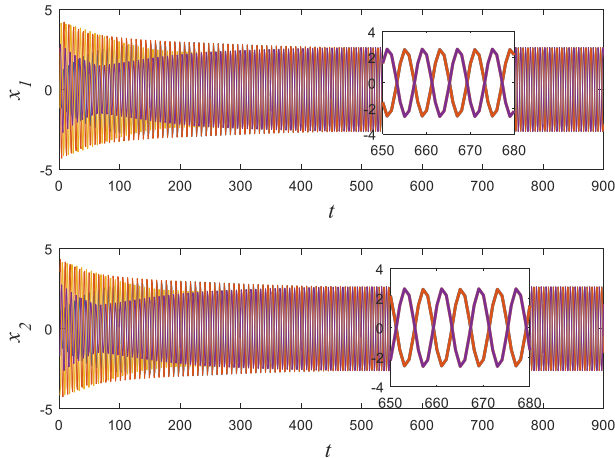


FIGURE 9. The trajectories of four agents.

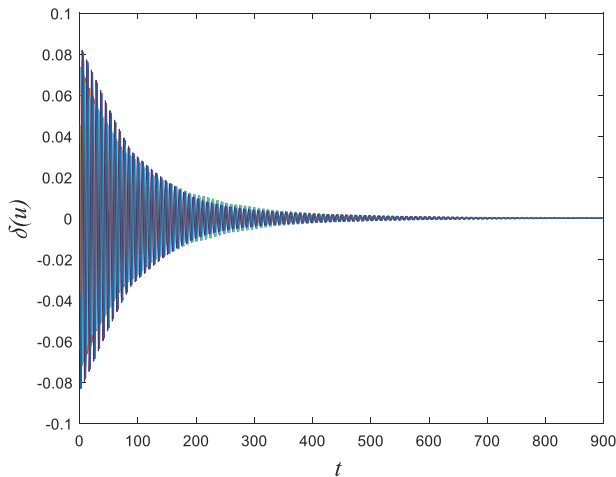


FIGURE 10. The input saturation constraints.

group to converge same value. The state of another group converges to the opposite value since competition-cooperation relationship between two groups. It reflects the trajectories of all agents completely in Fig.6. Six small circles represent the initial states of all agents and two five-pointed stars represent the final states of all agents. From the Fig. 6, we know that the group consensus can be achieved. Each of the input constraint shown in Fig. 7 converges to zero as $t \rightarrow +\infty$ and all input constraints $\delta(u) \in [-1, 1]$.

Example 3: Consider a multiagent system with four agents described by the graph in Fig.8. Let $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \end{bmatrix}$. Obviously, A satisfies the Assumption 3. In order to achieve couple-group

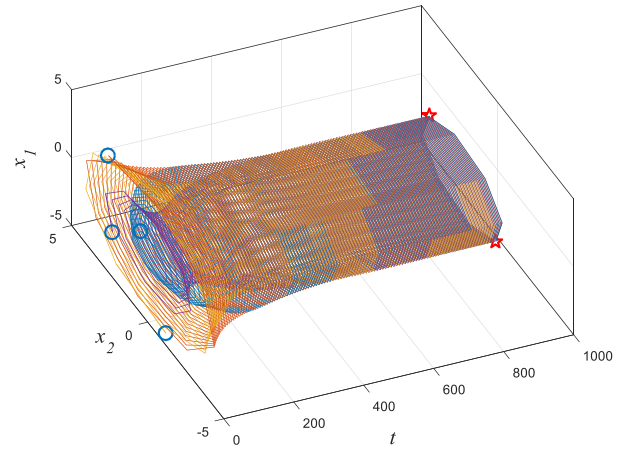


FIGURE 11. The 3D graph of all agents.

consensus, the control gain η is set to 0.3. The trajectories of the four agents are shown in Fig. 9. The results show that all agents converge to two subgroups. It means that they achieve group consensus. And all input constraints shown in Fig. 10 converge to zero when $t \rightarrow +\infty$. Four circles represent the initial states of four agents and two stars represent the final states of all agents in Fig.11. Obviously, the multi-agent system achieves couple-group consensus finally.

V. CONCLUSION

In this paper, the couple-group consensus is investigated for the first order discrete-time multiagent system with input constraints. Based on the idea of competition and cooperation, a novel control protocol is proposed to achieve group consensus. For multiagent system under undirected topology, the sufficient condition is given to ensure group consensus. The Lyapunov stability theory, the knowledge of the input saturation constraints and the matrix theory are used to achieve the asymptotic group consensus. It's worth noticing that the condition in this paper also satisfies the case which exists only input saturation constraints. The simulations of numeric modeling show that the system can achieve couple-group consensus. It shows that the condition of control gain is correctness for couple-group consensus.

DECLARATION OF INTERESTS STATEMENT

All authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

DATA AVAILABILITY STATEMENT

All data in the article are available.

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