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Mixed Far-Field and Near-Field Source Localization Using a Linear Electromagnetic-Vector-Sensor Array With Gain/Phase Uncertainties

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ABSTRACT This paper investigates the problem of mixed far-field (FF) and near-field (NF) source localization using a linear electromagnetic-vector-sensor array with gain/phase uncertainties. Firstly, several special fourth-order cumulant matrices are constructed, such that the shift invariance structure in the cumulant domain can be derived to estimate the DOA and polarization angles of each source at two electromagnetic vector sensors (EMVSs). Then, by computing the determinant of the coefficient matrix, the sources types can be classified with the prior knowledge of the number of both the FF and NF sources. On this basis, the range of NF sources and the DOAs of mixed sources at the phase reference point are captured subsequently. Finally, these estimates can be employed to generate the unknown gain/phase errors. Compared to the existing methods, the proposed one exploits both the spatial and polarization information of sources and provides a satisfactory parameters estimation performance under unknown phase/gain responses. Moreover, it does not need to perform any spectral search and not impose restriction on EMVSs placement, as well as realizes a more reasonable classification of the signal types. Simulations are carried out to verify the effectiveness of the proposed method.

INDEX TERMS Mixed source localization, direction-of-arrival (DOA), polarization, gain/phase uncertainties.

I. INTRODUCTION

In recent years, the problem of measuring spatial and polarization information of electromagnetic signals using vector sensor array has attracted increasing research, and numerous algorithms [1]–[8] have been developed for parameters estimation of far-field (FF) sources, whose wavefront is assumed to be a plane wave. However, when a source is located in the near-field (NF) region of an array, the wavefront must be characterized by both the DOA and range. The above-mentioned methods based on the far-field assumption are not applicable to this situation. Currently, a few methods have been developed to estimate the DOA, range and polarization parameters of near-field (NF) sources in [9]–[10]. However, in some practical applications, such as locating specific items in warehouses by using radio-frequency identification (RFID) tags [11], each item may be in the near-field or far-field of the RFID reader antenna array, and hence both FF and NF sources may coexist in such environment. In this case, the aforementioned algorithms [1]–[10] may fail to distinguish and locate the mixed sources.

To cope with this issue, some algorithms have been recently presented. In [12], a two-stage MUSIC (TSMUSIC) algorithm was firstly developed to localize mixed sources. However, it has a high computational complexity. Motivated by the shortcoming, He and Swamy [13] proposed a MUSIC-based one dimensional search (MBODS) method so as to obtain a lower computational cost than TSMUSIC algorithm. However, to distinguish the NF sources from the mixed sources, the MBODS algorithm resort to oblique projection

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technique, which would yield additional estimation errors. In [14], Xie *et al.* proposed an efficient mixed sources localization algorithm without estimating the source number, which can avoid the performance deterioration induced by erroneous source number estimation. In [15], a rank reduction (RARE) based algorithm for mixed sources localization in the presence of unknown mutual coupling was presented, which is effective for the classification and localization of mixed sources under unknown mutual coupling. In [16], [17], a two stage matrix differencing algorithm (TSMDA) was proposed, which achieves a more reasonable classification of the source types, alleviates the array aperture loss, as well as enhances the estimation accuracy of NF sources. In research [18], the use of hybrid second- and fourth-order statistics using the MUSIC technique has been considered, which offers a reasonable classification of the source types. However, it sets strict limits on the DOA intervals of the signals. In [19], a new subspace-based method called LOFNS was proposed for localization of the mixed FF and NF signals impinging on a symmetrical uniform linear arrays (ULA), which avoids the eigendecomposition and pair-matching processes. In [20], a novel localization algorithm via cumulant matrix reconstruction for mixed sources scenario was proposed, it avoids DOA search for NF sources and achieves a more reasonable classification of the source types. [21] investigates the localization of multiple near-field narrowband sources with a symmetric uniform linear array, and a new linear prediction approach based on the truncated singular value decomposition (LPATS) was proposed by taking an advantage of the anti-diagonal elements of the noiseless array covariance matrix. By exploiting the noncircular information of the signals, [22] proposed a novel localization method for mixed NF and FF sources using a symmetric uniform linear array (ULA). In [23], a new algorithm for mixed sources localization based on cross-cumulant was devised, which involves no DOA search and exhibits a higher localization accuracy. However, it still requires 1D-range search and suffers array aperture loss. Moreover, these methods in [12]–[23] restrict the array configurations to be ULA with inter-sensor spacing be within $\lambda/4$ [12]–[22] or $\lambda/8$ [23]. Inspired by the idea of nonuniform array, [24] proposed a localization algorithm for mixed sources using a symmetric double nested array (SDNA), which extends the array aperture and improves the localization accuracy. However, these mixed source localization methods [12]–[24] can only measure the spatial information of the reflected signals, whereas the polarization of electromagnetic signal is not taken into account.

Note that the above methods [1]–[24] are derived based on the assumption that the array is exactly known without uncertainties. However, the antenna arrays in practice are usually suffering from various uncertainties such as the unknown gains and phases caused by, say, the differences among the receivers utilized to demodulate and digitize the RF signals from the array elements [25]–[30]. This will results in significant distortion of the amplitude and phase of the signals received from the array, which will lead to a serious degradation of estimation accuracy or even failure of the methods in [1]–[24]. Recently, many methods have been proposed to discuss the problem of DOA estimation with unknown inter-sensor gain/phase responses [31]–[37]. In [33], a direction-of-arrival (DOA) estimation method for the uniform-circular array (UCA) in the presence of gain-phase errors was proposed. Later, aiming at solving problems in the Hadamard product-based method proposed specially for the UCA [33], a novel two-stage dimension reduction method (DRM) for the DOA estimation with the channel phase inconsistency was presented in [34]. In [35], an iterative algorithm was presented to estimate the DOAs and the gains/phases of the uncalibrated elements in partly calibrated array. Compared to [35], a computationally more efficient ESPRIT-like method was presented in [36] and further investigated in [37] by examining the conditions ensuring the uniqueness of DOA estimates and identifiability (i.e., the maximum number of sources that can be resolved), it has been verified in [37] that a partly calibrated ULA with M sensor elements is able to identify up to $L = M - 2$ DOAs. Note that these algorithms in [25]–[37] are efficient only in the FF source scenario with traditional scalar-array, while the problem of mixed source localization with polarization sensitive array under unknown gain/phase uncertainties has not been well investigated so far.

In view of the previous analyses, most existing algorithms face the following difficulties: 1) measuring the spatial and polarization parameters of mixed sources; 2) localizing the mixed sources successfully with polarization sensitive array in the presence of gain/phase errors; 3) classifying the FF and NF sources reasonably; 4) avoiding spectral search.

To solve these difficulties, we propose a novel algorithm for mixed source localization using a linear EMVS array with gain/phase uncertainties. By designing several special fourth-order cumulants and using the shift invariance properties in the cumulant domain, the estimation of DOA and polarization angles of each source at two EMVSs can be achieved. Then the signal types can be distinguished according to the determinant of coefficient matrix, and the location parameters of these sources can be calculated by least square method. After deriving the closed-form parameters estimation of the mixed sources, the array steering vectors can be estimated, which can be utilized to further estimate the gain/phase uncertainties. Our main contributions are listed as follows:

1) To the best of our knowledge, this is the first time that the DOA and polarization parameters of mixed sources are estimated using a linear EMVS array with unknown gains/phases uncertainties. The proposed algorithm is robust to the gain/phase errors and can estimate the unknown gains/phases as well.

2) The proposed algorithm can estimate the DOA and polarization angles of mixed sources as well as unknown gains/phases without knowing the position of each EMVS and has no restriction on EMVSs placement.

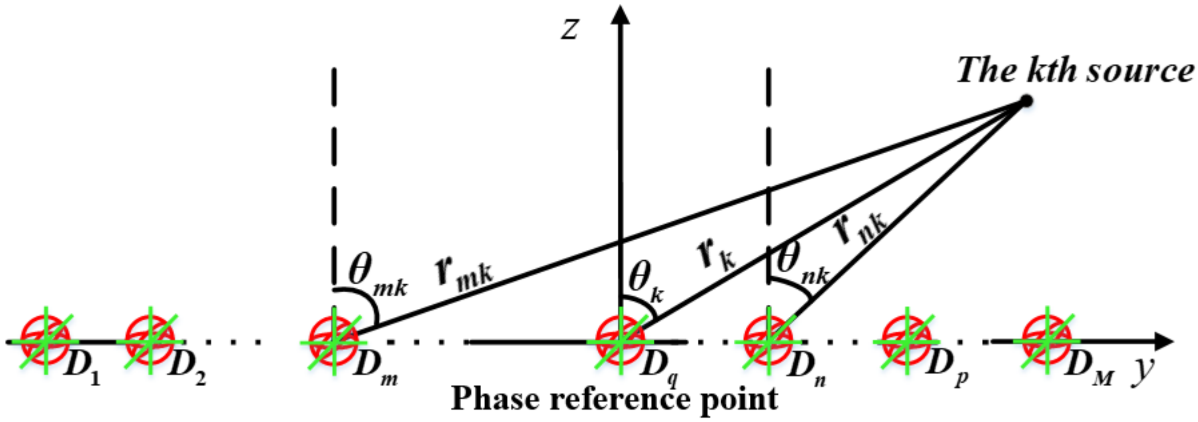

FIGURE 1. Linear EMVS array configuration.

3) The computational complexity of the proposed algorithm is analyzed, and the CRBs of the linear EMVS array with gain/phase uncertainties are derived as well.

The rest of the paper is organized as follows: Section 2 introduces the signal model. The proposed method is described in Section 3. In Section 4, simulations are conducted to validate the performance of our method. Section 5 draws the conclusion.

In the paper, the $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ and $\otimes$ denote the transpose, conjugate, conjugate transpose and kronecker product, respectively. $\angle\{\cdot\}$ denotes the argument of a complex number and $diag\{\cdot\}$ denotes a diagonal matrix. $\|\cdot\|$ and $\times$ denote the 2-norm and cross product, respectively.

II. SIGNAL MODEL

Consider K (NF or FF) narrowband and independent signal sources, including a mixture of K_1 FF sources and $K - K_1$ NF sources, impinging on a linear array with M EMVSs, each EMVS is composed of three identical, but orthogonally oriented, electrically short dipoles, plus three identical but orthogonally oriented magnetically small loops-all spatially collocated in a point-like geometry, as shown in Fig.1. We assume that all the EMVSs lie on the y-axis with their locations being $D_1, D_2, \ldots, D_M$. We first formulate the received signal for an ideal array without considering the unknown gain/phase uncertainties. With the qth EMVS being the phase reference point, the array output can be modeled as [12]:

$$\mathbf{X}(t) = \mathbf{AS}(t) + \mathbf{w}(t) \tag{1}$$

where $\mathbf{X}(t) = \left[x_{1,1}(t), x_{1,2}(t), x_{1,3}(t), x_{1,4}(t), x_{1,5}(t), x_{1,6}(t), \ldots, x_{M,6}(t)\right]^T$ is the array output vector, $\mathbf{S}(t) = [s_1(t), \ldots, s_K(t)]^T$ is the vector of the signal waveforms, $\mathbf{w}(t) = \left[w_{1,1}(t), w_{1,2}(t), w_{1,3}(t), w_{1,4}(t), w_{1,5}(t), w_{1,6}(t), \ldots, w_{M,6}(t)\right]^T$ is the array noise output vector. $\mathbf{A}$ is the $6M \times K$ array steering matrix of the mixed NF and FF sources, which is given by:

$$\begin{aligned}
&\mathbf{A} \\
&\quad = [\mathbf{a}(\theta_1, \gamma_1, \eta_1, r_1), \ldots, \mathbf{a}(\theta_K, \gamma_K, \eta_K, r_K)] \qquad (2)\\
&\mathbf{a}(\theta_k, \gamma_k, \eta_k, r_k) \\
&\quad = \left[\left[e^{j(D_1\alpha_k + D_1{}^2\beta_k)} \otimes \mathbf{c}_{1k}\right]^T, \right. \\
&\qquad \left. \ldots, \left[e^{j(D_M\alpha_k + D_M{}^2\beta_k)} \otimes \mathbf{c}_{Mk}\right]^T\right]^T \qquad (3)\\
&\alpha_k \\
&\quad = \frac{-2\pi \sin\theta_k}{\lambda}, \quad \beta_k = \frac{\pi\cos^2\theta_k}{\lambda r_k} \qquad (4)
\end{aligned}$$

where α_k and β_k are called as the electric angles, λ is the signal wavelength, $\theta_k \in [0, \pi\,]$ and r_k are the DOA and the range of the kth signal at the phase reference point. $\mathbf{c}_{lk}$ denotes the response of the kth source at the lth EMVS, which can be represented by the 3×1 electric field vector and the 3×1 magnetic-field vector $\mathbf{h}_{lk}$:

$$\begin{aligned}
\mathbf{c}_{lk} &= \begin{bmatrix} \mathbf{e}_{lk} \\ \mathbf{h}_{lk} \end{bmatrix}
= \begin{bmatrix} c_1(\theta_{lk}, \gamma_k, \eta_k) \\ c_2(\theta_{lk}, \gamma_k, \eta_k) \\ c_3(\theta_{lk}, \gamma_k, \eta_k) \\ c_4(\theta_{lk}, \gamma_k, \eta_k) \\ c_5(\theta_{lk}, \gamma_k, \eta_k) \\ c_6(\theta_{lk}, \gamma_k, \eta_k) \end{bmatrix} \\
&= \underbrace{\begin{bmatrix} 0 & -1 \\ \cos\theta_{lk} & 0 \\ -\sin\theta_{lk} & 0 \\ -1 & 0 \\ 0 & -\cos\theta_{lk} \\ 0 & \sin\theta_{lk} \end{bmatrix}}_{\Theta(\theta_{lk})} \underbrace{\begin{bmatrix} \sin\gamma_k e^{j\eta_k} \\ \cos\gamma_k \end{bmatrix}}_{\mathbf{g}(\gamma_k, \eta_k)} \qquad (5)
\end{aligned}$$

where θ_{lk} denotes the DOA at the lth EMVS, $\gamma_k \in \left[0, \frac{\pi}{2}\right)$ and $\eta_k \in [-\pi, \pi)$ are the auxiliary polarization angle and polarization phase difference angle. It is noted that when the

kth source is a FF one, β_k is approximated by zero since the range approaches to ∞ (see [12] for details).

Taking the unknown inter-sensor gain/phase uncertainties into account, the steering vector should be rewritten as

$$\begin{aligned}\bar{\mathbf{a}}(\theta_k,\gamma_k,\eta_k,r_k) &= \left[\left[g_1 e^{j(D_1\alpha_k+D_1{}^2\beta_k)}\otimes\mathbf{c}_{1k}\right]^T,\ldots,\right.\\ &\qquad\left.\left[g_M e^{j(D_M\alpha_k+D_M{}^2\beta_k)}\otimes\mathbf{c}_{Mk}\right]^T\right]^T\\ &= \tilde{\mathbf{G}}\mathbf{a}(\theta_k,\gamma_k,\eta_k,r_k)\end{aligned}\tag{6}$$

where $g_m = \rho_m e^{j\varphi_m}$ $(m=1,\ldots,M)$ denotes the unknown gain/phase of the mth EMVS. ρ_m and φ_m are the gain and phase uncertainties, respectively. $\tilde{\mathbf{G}}$ is a $6M \times 6M$ diagonal matrix representing the gains/phases of the whole array and is given by

$$\tilde{\mathbf{G}} = blkdiag\{g_1\mathbf{I}_6,\ldots,g_M\mathbf{I}_6\} = diag\{\tilde{\mathbf{g}}\}\tag{7}$$

where $\mathbf{I}_6$ denotes the 6×6 identity matrix and $blkdiag\{\cdot\}$ constructs a block diagonal matrix from the bracketed matrices. $\tilde{\mathbf{g}}$ is the sensor gain/phase vector of the linear EMVS array

$$\tilde{\mathbf{g}} = [g_1,\ldots,g_1,g_2,\ldots,g_2,\ldots,g_M,\ldots,g_M]^T\tag{8}$$

Let the qth EMVS be the reference one, we have

$$g_q = 1\tag{9}$$

Consequently, the steering matrix is given by

$$\bar{\mathbf{A}} = \tilde{\mathbf{G}}\mathbf{A} = [\bar{\mathbf{a}}(\theta_1,\gamma_1,\eta_1,r_1),\ldots,\bar{\mathbf{a}}(\theta_K,\gamma_K,\eta_K,r_K)]\tag{10}$$

Therefore, the array output vector $\mathbf{X}(t)$ under unknown inter-sensor gain/phase responses can be modeled as

$$\mathbf{X}(t) = \tilde{\mathbf{G}}\mathbf{A}\mathbf{S}(t) + \mathbf{w}(t)\tag{11}$$

III. PROPOSED ALGORITHM

A. DOA AND POLARIZATION ESTIMATION OF ALL SOURCES AT THE mth EMVS

Firstly, the proposed algorithm begins with the fourth-order cumulant of the array outputs. By constructing several fourth order cumulant matrices, the shift invariant structure among the components of the mth EMVS in the cumulant domain can be derived, from which the estimation of DOA and polarization parameters of each source at the mth EMVS can be achieved. Define $u_{m,i,k} = c_i(\theta_{mk},\gamma_k,\eta_k)\, e^{j(D_m\alpha_k+D_m{}^2\beta_k)}$, then we have

$$\begin{aligned}&cum\left(x_{m,i}(t), x_{n,o}^*(t), x_{p,h}(t), x_{q,v}^*(t)\right)\\ &= cum\left\{\sum_{k=1}^K g_m u_{m,i,k}s_k(t), \left(\sum_{k=1}^K g_n u_{n,o,k}s_k(t)\right)^*\right.\\ &\qquad\left., \sum_{k=1}^K g_p u_{p,h,k}s_k(t), \left(\sum_{k=1}^K g_q u_{q,v,k}s_k(t)\right)^*\right\}\\ &= \sum_{k=1}^K g_m u_{m,i,k}\left(g_n u_{n,o,k}\right)^* g_p u_{p,h,k}\left(g_q u_{q,v,k}\right)^* c_{4,s_k}\\ &\quad (m,n,p,q = 1,2\ldots,M;\quad i,o,h,v = 1,2,3,4,5,6)\end{aligned}\tag{12}$$

where k denotes the index associated with the kth incident source, $c_{4,s_k} = cum\{s_k(t), s_k^*(t), s_k(t), s_k^*(t)\}$ is the kurtosis of the kth signal. Based on the above observation, the following cumulant matrices can be constructed.

$$\begin{aligned}\mathbf{R}_1 &= \sum_{o=1}^6 cum\left\{x_{m,1}(t), (x_{n,o}(t))^*, \mathbf{X}(t), \mathbf{X}^H(t)\right\} = \left(\tilde{\mathbf{G}}\mathbf{A}\right)\mathbf{\Psi}_1\left(\tilde{\mathbf{G}}\mathbf{A}\right)^H\\ \mathbf{R}_2 &= \sum_{o=1}^6 cum\left\{x_{m,2}(t), (x_{n,o}(t))^*, \mathbf{X}(t), \mathbf{X}^H(t)\right\} = \left(\tilde{\mathbf{G}}\mathbf{A}\right)\mathbf{\Psi}_2\left(\tilde{\mathbf{G}}\mathbf{A}\right)^H\\ \mathbf{R}_3 &= \sum_{o=1}^6 cum\left\{x_{m,3}(t), (x_{n,o}(t))^*, \mathbf{X}(t), \mathbf{X}^H(t)\right\} = \left(\tilde{\mathbf{G}}\mathbf{A}\right)\mathbf{\Psi}_3\left(\tilde{\mathbf{G}}\mathbf{A}\right)^H\\ \mathbf{R}_4 &= \sum_{o=1}^6 cum\left\{x_{m,4}(t), (x_{n,o}(t))^*, \mathbf{X}(t), \mathbf{X}^H(t)\right\} = \left(\tilde{\mathbf{G}}\mathbf{A}\right)\mathbf{\Psi}_4\left(\tilde{\mathbf{G}}\mathbf{A}\right)^H\\ \mathbf{R}_5 &= \sum_{o=1}^6 cum\left\{x_{m,5}(t), (x_{n,o}(t))^*, \mathbf{X}(t), \mathbf{X}^H(t)\right\} = \left(\tilde{\mathbf{G}}\mathbf{A}\right)\mathbf{\Psi}_5\left(\tilde{\mathbf{G}}\mathbf{A}\right)^H\\ \mathbf{R}_6 &= \sum_{o=1}^6 cum\left\{x_{m,6}(t), (x_{n,o}(t))^*, \mathbf{X}(t), \mathbf{X}^H(t)\right\} = \left(\tilde{\mathbf{G}}\mathbf{A}\right)\mathbf{\Psi}_6\left(\tilde{\mathbf{G}}\mathbf{A}\right)^H\end{aligned}\tag{13}$$

where

$$\mathbf{\Psi}_i = diag\left\{\sum_{o=1}^6 g_m u_{m,i,1}\left(g_n u_{n,o,1}\right)^* c_{4,s_1},\ldots,\sum_{o=1}^6 g_m u_{m,i,K}\left(g_n u_{n,o,K}\right)^* c_{4,s_K}\right\}\tag{14}$$

Based on the subspace theory, the $36M \times K$ signal subspace matrix $\mathbf{E}_{SL}$ of $\mathbf{R} = \left[\mathbf{R}_1^T,\mathbf{R}_2^T,\mathbf{R}_3^T,\mathbf{R}_4^T,\mathbf{R}_5^T,\mathbf{R}_6^T\right]^T$ can be expressed as

$$\mathbf{E}_{SL} = \begin{bmatrix}\tilde{\mathbf{G}}\mathbf{A}\mathbf{\Psi}_1\\ \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Psi}_2\\ \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Psi}_3\\ \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Psi}_4\\ \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Psi}_5\\ \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Psi}_6\end{bmatrix}\mathbf{T}\tag{15}$$

with $\mathbf{T}$ being a $K \times K$ invertible matrix, and $\mathbf{E}_{SL}$ is composed of the eigenvectors corresponding to the K largest singular values of $\mathbf{R}$. Let $\mathbf{E}_{SL}^{(i)}$ be the submatrice of $\mathbf{E}_{SL}$ from $6(i-1)M+1$ row to $6iM$ row, and $\mathbf{E}_{SL}^{(h)}$ be the partion of $\mathbf{E}_{SL}$ from $6(h-1)M+1$ row to $6hM$ row. From Eq. (15), we have

$$\begin{aligned}\mathbf{E}_{SL}^{(i)} &= \mathbf{J}^{(i)}\mathbf{E}_{SL} = \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Psi}_i\mathbf{T}\\ \mathbf{E}_{SL}^{(h)} &= \mathbf{J}^{(h)}\mathbf{E}_{SL} = \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Psi}_h\mathbf{T} = \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Psi}_i\mathbf{H}^{(i,h)}\mathbf{T}\end{aligned} \tag{16}$$

where $\mathbf{\Psi}_i$ and $\mathbf{\Psi}_h$ are two diagonal matrices related to $\mathbf{H}^{(i,h)}$, which is essentially the rotational invariant factor between the ith and hth components of the mth EMVS. $\mathbf{J}^{(i)} = \mathbf{e}_i \otimes \mathbf{I}_6$ is the selection matrix, in which $\mathbf{e}_i$ is a 1×6 row vector with the ith entry being 1 and 0 elsewhere. and $\mathbf{H}^{(i,h)}$ is of the following form:

$$\begin{aligned}\mathbf{H}^{(i,h)} &= diag\left\{\frac{\sum_{o=1}^{6} g_m u_{m,h,1}\left(g_n u_{n,o,1}\right)^* c_{4,s_1}}{\sum_{o=1}^{6} g_m u_{m,i,1}\left(g_n u_{n,o,1}\right)^* c_{4,s_1}}, \ldots, \right.\\ &\qquad \left.\frac{\sum_{o=1}^{6} g_m u_{m,h,K}\left(g_n u_{n,o,K}\right)^* c_{4,s_K}}{\sum_{s=1}^{6} g_m u_{m,i,K}\left(g_n u_{n,o,K}\right)^* c_{4,s_K}}\right\}\\ &= diag\left\{\frac{c_h\left(\theta_{m1},\gamma_1,\eta_1\right)}{c_i\left(\theta_{m1},\gamma_1,\eta_1\right)}, \ldots, \frac{c_h\left(\theta_{mK},\gamma_K,\eta_K\right)}{c_i\left(\theta_{mK},\gamma_K,\eta_K\right)}\right\}\end{aligned} \tag{17}$$

Furthermore, since $\mathbf{E}_{SL}^{(i)}$ and $\mathbf{E}_{SL}^{(h)}$ have full column rank, a unique non-singular matrix $\Omega^{(i,h)}$ exists such that

$$\mathbf{E}_{SL}^{(h)} = \mathbf{E}_{SL}^{(i)}\Omega^{(i,h)} \tag{18}$$

It can be easily derived from Eq. (16) that $\mathbf{E}_{SL}^{(h)} = \mathbf{E}_{SL}^{(i)}\mathbf{T}^{-1}\mathbf{H}^{(i,h)}\mathbf{T}$. Accordingly, we have

$$\Omega^{(i,h)} = \mathbf{T}^{-1}\mathbf{H}^{(i,h)}\mathbf{T} \tag{19}$$

which means that $\Omega^{(i,h)}$ and $\mathbf{H}^{(i,h)}$ are similar matrices, they have the same eigenvalues $\frac{c_h(\theta_{m1},\gamma_1,\eta_1)}{c_i(\theta_{m1},\gamma_1,\eta_1)}, \ldots, \frac{c_h(\theta_{mK},\gamma_K,\eta_K)}{c_i(\theta_{mK},\gamma_K,\eta_K)}$. This implies that $\mathbf{H}^{(i,h)}$ can be obtained once $\Omega^{(i,h)}$ is available. $\Omega^{(i,h)}$ can be solved by the least squares algorithm from Eq. (18) as

$$\Omega^{(i,h)} = \left(\mathbf{E}_{SL}^{(i)H}\mathbf{E}_{SL}^{(i)}\right)^{-1}\mathbf{E}_{SL}^{(i)H}\mathbf{E}_{SL}^{(h)} \tag{20}$$

From Eq. (5) and Eq. (17), we can see that the polarization components of $\mathbf{c}_{lk}$ satisfies the following relationship

$$c_s\left(\theta_{lk},\gamma_k,\eta_k\right) = c_1\left(\theta_{lk},\gamma_k,\eta_k\right)\cdot\left[\mathbf{H}^{(1,s)}\right]_k \quad (s=2,\ldots,6) \tag{21}$$

where $[\mathbf{H}^{(1,s)}]_k$ represents the kth diagonal element of $\mathbf{H}^{(1,s)}$. It can be seen from (19) that $\mathbf{H}^{(1,s)}$ can be calculated by performing eigendecomposition of $\Omega^{(1,s)}$. Then, based on the Maxwell equation, $\mathbf{H}^{(1,s)}$ can be utilized to obtain the pointing vector $\mathbf{\Gamma}_{mk}$ as follows

$$\begin{aligned}&\mathbf{e}_{mk} \times \mathbf{h}_{mk}\\ &\quad = \left\|c_1\left(\theta_{mk},\gamma_k,\eta_k\right)\right\|^2 \underbrace{\begin{bmatrix}1\\ \left[\mathbf{H}^{(1,2)}\right]_k\\ \left[\mathbf{H}^{(1,3)}\right]_k\end{bmatrix}}_{\mathbf{f}_{mk}} \times \underbrace{\begin{bmatrix}\left[\mathbf{H}^{(1,4)}\right]_k\\ \left[\mathbf{H}^{(1,5)}\right]_k\\ \left[\mathbf{H}^{(1,6)}\right]_k\end{bmatrix}}_{\mathbf{g}_{mk}}\end{aligned} \tag{22}$$

$$\begin{aligned}\mathbf{\Gamma}_{mk} &= \begin{bmatrix}\hat{u}_{mk}\\ \hat{v}_{mk}\\ \hat{w}_{mk}\end{bmatrix} = \begin{bmatrix}0\\ \sin\hat{\theta}_{mk}\\ \cos\hat{\theta}_{mk}\end{bmatrix}\\ &= \frac{\mathbf{e}_{mk} \times \mathbf{h}_{mk}}{\|\mathbf{e}_{mk}\|\cdot\|\mathbf{h}_{mk}\|} = \frac{\mathbf{f}_{mk} \times \mathbf{g}_{mk}}{\|\mathbf{f}_{mk}\|\cdot\|\mathbf{g}_{mk}\|}\end{aligned} \tag{23}$$

where $\|\cdot\|$ and $\times$ denote the 2-norm and cross product, respectively. According to Eq. (23), the DOA estimation of the kth signal at the mth EMVS can be derived as

$$\hat{\theta}_{mk} = \arctan\left\{\frac{\hat{v}_{mk}}{\hat{w}_{mk}}\right\} \tag{24}$$

According to Eq. (5), we can get

$$\hat{\mathbf{g}}\left(\gamma_{mk},\eta_{mk}\right) = \left[\Theta^H\left(\hat{\theta}_{mk}\right)\Theta\left(\hat{\theta}_{mk}\right)\right]^{-1}\Theta\left(\hat{\theta}_{mk}\right)\mathbf{c}_{mk} \tag{25}$$

Then, by substituting $\hat{\theta}_{mk}$ into the above equation, the corresponding polarization parameters can be estimated as

$$\begin{aligned}\hat{\gamma}_{mk} &= \arctan\left\{\frac{\left[\hat{\mathbf{g}}\left(\gamma_{mk},\eta_{mk}\right)\right]_1}{\left[\hat{\mathbf{g}}\left(\gamma_{mk},\eta_{mk}\right)\right]_2}\right\}\\ \hat{\eta}_{mk} &= \angle\left[\hat{\mathbf{g}}\left(\gamma_{mk},\eta_{mk}\right)\right]_1 - \angle\left[\hat{\mathbf{g}}\left(\gamma_{mk},\eta_{mk}\right)\right]_2\end{aligned} \tag{26}$$

B. DOA AND POLARIZATION ESTIMATION OF ALL SOURCES AT THE nth EMVS

Similarly, in order to construct the rotational invariant structure among the components of the nth EMVS in the cumulant domain, several cumulant matrices can be designed firstly, then the rotational invariant relationship among the elements of the nth EMVS can be derived, and hence the DOA and polarization parameters of the incident sources at the nth EMVS can be calculated. In specific, the fourth-order cumulant matrices can be designed as follows

$$\begin{aligned}\mathbf{C}_1 &= \sum_{o=1}^{6} cum\left\{x_{n,1}(t), \left(x_{m,o}(t)\right)^*, \mathbf{X}(t), \mathbf{X}^H(t)\right\}\\ &= \left(\tilde{\mathbf{G}}\mathbf{A}\right)\mathbf{\Phi}_1\left(\tilde{\mathbf{G}}\mathbf{A}\right)^H\\ \mathbf{C}_2 &= \sum_{o=1}^{6} cum\left\{x_{n,2}(t), \left(x_{m,o}(t)\right)^*, \mathbf{X}(t), \mathbf{X}^H(t)\right\}\\ &= \left(\tilde{\mathbf{G}}\mathbf{A}\right)\mathbf{\Phi}_2\left(\tilde{\mathbf{G}}\mathbf{A}\right)^H\\ \mathbf{C}_3 &= \sum_{o=1}^{6} cum\left\{x_{n,3}(t), \left(x_{m,o}(t)\right)^*, \mathbf{X}(t), \mathbf{X}^H(t)\right\}\\ &= \left(\tilde{\mathbf{G}}\mathbf{A}\right)\mathbf{\Phi}_3\left(\tilde{\mathbf{G}}\mathbf{A}\right)^H\end{aligned}$$

$$\begin{aligned}
\mathbf{C}_4 &= \sum_{o=1}^{6} cum\left\{x_{n,4}(t), \left(x_{m,o}(t)\right)^*, \mathbf{X}(t), \mathbf{X}^H(t)\right\} \\
&= \left(\tilde{\mathbf{G}}\mathbf{A}\right)\mathbf{\Phi}_4\left(\tilde{\mathbf{G}}\mathbf{A}\right)^H \\
\mathbf{C}_5 &= \sum_{o=1}^{6} cum\left\{x_{n,5}(t), \left(x_{m,o}(t)\right)^*, \mathbf{X}(t), \mathbf{X}^H(t)\right\} \\
&= \left(\tilde{\mathbf{G}}\mathbf{A}\right)\mathbf{\Phi}_5\left(\tilde{\mathbf{G}}\mathbf{A}\right)^H \\
\mathbf{C}_6 &= \sum_{o=1}^{6} cum\left\{x_{n,6}(t), \left(x_{m,o}(t)\right)^*, \mathbf{X}(t), \mathbf{X}^H(t)\right\} \\
&= \left(\tilde{\mathbf{G}}\mathbf{A}\right)\mathbf{\Phi}_6\left(\tilde{\mathbf{G}}\mathbf{A}\right)^H
\end{aligned} \tag{27}$$

where

$$\mathbf{\Phi}_i = diag\left\{\sum_{o=1}^{6} g_n u_{n,i,1}\left(g_m u_{m,o,1}\right)^* c_{4,s_1}, \ldots, \sum_{o=1}^{6} g_n u_{n,i,K}\left(g_m u_{m,o,K}\right)^* c_{4,s_K}\right\} \tag{28}$$

Then, the $36M \times K$ signal subspace matrix $\mathbf{E}_{SR}$ of $\mathbf{C} = \left[\mathbf{C}_1^T, \mathbf{C}_2^T, \mathbf{C}_3^T, \mathbf{C}_4^T, \mathbf{C}_5^T, \mathbf{C}_6^T\right]^T$ can be expressed as

$$\mathbf{E}_{SR} = \begin{bmatrix} \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Phi}_1 \\ \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Phi}_2 \\ \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Phi}_3 \\ \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Phi}_4 \\ \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Phi}_5 \\ \tilde{\mathbf{G}}\mathbf{A}\mathbf{\Phi}_6 \end{bmatrix}\mathbf{T} \tag{29}$$

Following the procedure of parameters estimation of the kth source at the mth EMVS, the Poynting vector of the kth signal at the nth EMVS can be achieved according to the following equations

$$\mathbf{J}^{(h)}\mathbf{E}_{SR} = \mathbf{J}^{(i)}\mathbf{E}_{SR}\mathbf{\Lambda}^{(i,h)} \tag{30}$$

$$\mathbf{\Lambda}^{(i,h)} = \mathbf{T}^{-1}\mathbf{Q}^{(i,h)}\mathbf{T} \tag{31}$$

$$\mathbf{\Lambda}^{(i,h)} = \left[\left(\mathbf{J}^{(i)}\mathbf{E}_{SR}\right)^H \mathbf{J}^{(i)}\mathbf{E}_{SR}\right]^{-1}\left(\mathbf{J}^{(i)}\mathbf{E}_{SR}\right)^H \mathbf{J}^{(h)}\mathbf{E}_{SR} \tag{32}$$

$$\begin{aligned}
\mathbf{\Gamma}_{nk} &= \begin{bmatrix} \hat{u}_{nk} \\ \hat{v}_{nk} \\ \hat{w}_{nk} \end{bmatrix} = \begin{bmatrix} 0 \\ \sin\hat{\theta}_{nk} \\ \cos\hat{\theta}_{nk} \end{bmatrix} \\
&= \frac{\begin{bmatrix} 1 \\ \left[\mathbf{Q}^{(1,2)}\right]_k \\ \left[\mathbf{Q}^{(1,3)}\right]_k \end{bmatrix} \times \begin{bmatrix} \left[\mathbf{Q}^{(1,4)}\right]_k \\ \left[\mathbf{Q}^{(1,5)}\right]_k \\ \left[\mathbf{Q}^{(1,6)}\right]_k \end{bmatrix}}{\left\| \begin{bmatrix} 1 \\ \left[\mathbf{Q}^{(1,2)}\right]_k \\ \left[\mathbf{Q}^{(1,3)}\right]_k \end{bmatrix} \right\| \cdot \left\| \begin{bmatrix} \left[\mathbf{Q}^{(1,4)}\right]_k \\ \left[\mathbf{Q}^{(1,5)}\right]_k \\ \left[\mathbf{Q}^{(1,6)}\right]_k \end{bmatrix} \right\|}
\end{aligned} \tag{33}$$

where

$$\begin{aligned}
\mathbf{Q}^{(i,h)} &= diag\left\{\frac{\sum_{o=1}^{6} g_n u_{n,h,1}\left(g_m u_{m,o,1}\right)^* c_{4,s_1}}{\sum_{o=1}^{6} g_n u_{n,i,1}\left(g_m u_{m,o,1}\right)^* c_{4,s_1}}, \ldots, \frac{\sum_{o=1}^{6} g_n u_{n,h,K}\left(g_m u_{m,o,K}\right)^* c_{4,s_K}}{\sum_{o=1}^{6} g_n u_{n,i,K}\left(g_m u_{m,o,K}\right)^* c_{4,s_K}}\right\} \\
&= diag\left\{\frac{c_h\left(\theta_{n1}, \gamma_1, \eta_1\right)}{c_i\left(\theta_{n1}, \gamma_1, \eta_1\right)}, \ldots, \frac{c_h\left(\theta_{nK}, \gamma_K, \eta_K\right)}{c_i\left(\theta_{nK}, \gamma_K, \eta_K\right)}\right\}
\end{aligned} \tag{34}$$

Note that $\mathbf{Q}^{(i,h)}$ can be estimated by performing eigendecomposition of $\mathbf{\Lambda}^{(i,h)}$. Therefore, the DOA of the kth source at the nth EMVS can be achieved as

$$\hat{\theta}_{nk} = \arctan\left\{\frac{\hat{v}_{nk}}{\hat{w}_{nk}}\right\} \tag{35}$$

From Eq. (5), we can get

$$\hat{\mathbf{g}}\left(\gamma_{nk}, \eta_{nk}\right) = \left[\Theta^H\left(\hat{\theta}_{nk}\right)\Theta\left(\hat{\theta}_{nk}\right)\right]^{-1}\Theta^H\left(\hat{\theta}_{nk}\right)\mathbf{c}_{nk} \tag{36}$$

Then the corresponding polarization parameters can be obtained as

$$\begin{aligned}
\hat{\gamma}_{nk} &= \arctan\left\{\frac{\left[\hat{\mathbf{g}}\left(\gamma_{nk}, \eta_{nk}\right)\right]_1}{\left[\hat{\mathbf{g}}\left(\gamma_{nk}, \eta_{nk}\right)\right]_2}\right\} \\
\hat{\eta}_{nk} &= \angle\left[\hat{\mathbf{g}}\left(\gamma_{nk}, \eta_{nk}\right)\right]_1 - \angle\left[\hat{\mathbf{g}}\left(\gamma_{nk}, \eta_{nk}\right)\right]_2
\end{aligned} \tag{37}$$

C. PARAMETERS PAIR MATCHING

Note that there may exist mismatch between the parameters estimates of the kth source at the mth EMVS and nth EMVS, thus the match pairing operation needs to be conducted. It is known that the DOAs of the kth source at the mth EMVS and nth EMVS are distinct, while the polarization parameters at the two sensors are approximately the same. This fact can be easily utilized to pair $\hat{\theta}_{mk}, \hat{\theta}_{nk}, \hat{\gamma}_{mk}, \hat{\gamma}_{nk}, \hat{\eta}_{mk}, \hat{\eta}_{nk}$ successfully. Furthermore, since several independent eigendecompositions are performed in this section, which may lead to mismatch of the eigenvalues obtained from different eigendecomposition. In specific, $\left[\mathbf{H}^{(i,h)}\right]_k$ or $\left[\mathbf{Q}^{(i,h)}\right]_k$ $(i, h = 1, 2, \ldots, 6)$ in different $\mathbf{H}^{(i,h)}$ or $\mathbf{Q}^{(i,h)}$ may not correspond to the same target, but their is no mismatch between $\left[\mathbf{H}^{(i,h)}\right]_k$ and its corresponding eigenvector, as well as $\left[\mathbf{Q}^{(i,h)}\right]_k$ and its corresponding eigenvector. Herein, we take the kth row of $\mathbf{T}_H{}^{(i,h)}$ and $\mathbf{T}_Q{}^{(i,h)}$ as the the eigenvectors related to $\left[\mathbf{H}^{(i,h)}\right]_k$ and $\left[\mathbf{Q}^{(i,h)}\right]_k$, respectively. Thus, different $\left[\mathbf{H}^{(i,h)}\right]_k$ or $\left[\mathbf{Q}^{(i,h)}\right]_k$ can be paired for the same source by pairing the orthogonal rows of $\mathbf{T}_H{}^{(i,h)}$ or $\mathbf{T}_Q{}^{(i,h)}$. Take, for example, $\left[\mathbf{H}^{(1,2)}\right]_k$ and $\left[\mathbf{H}^{(1,3)}\right]_k$. Let k denotes the row index of the matrix element with the largest absolute value in the fth column of the matrix $\left\{\mathbf{T}_H{}^{(1,2)}\left[\mathbf{T}_H{}^{(1,3)}\right]^{-1}\right\}$.

Then the kth row of ${\mathbf{T}_H}^{(1,2)}$ must correspond to the fth row of ${\mathbf{T}_H}^{(1,3)}$. Now, $\left\{\left[\mathbf{H}^{(1,2)}\right]_k, k=1,\ldots,K\right\}$ and $\left\{\left[\mathbf{H}^{(1,3)}\right]_k, k=1,\ldots,K\right\}$ have been correctly paired. In a similar way, $\left[\mathbf{H}^{(i,h)}\right]_k$ and $\left[\mathbf{Q}^{(i,h)}\right]_k$ can all be paired successfully. For more details of the pair matching operation, please refer to [3].

D. LOCATION ESTIMATES OF MIXED SOURCES AT PHASE REFERENCE POINT AND CLASSIFICATION OF SIGNAL TYPES

In this subsection, the main goal is to distinguish the signal types and locate the mixed sources. Based on the DOA estimation of the kth source at the mth and nth EMVSs, the signal types can be classified by computing the determinant of the coefficient matrix, and the location estimates can be obtained by the least square method subsequently.

With the DOA estimation of the kth source at the mth and nth EMVSs, the following equations hold according to the array geometry in Fig. 1.

$$\begin{aligned} r_{mk}\sin\hat{\theta}_{mk} - r_{nk}\sin\hat{\theta}_{nk} &= D_n - D_m \\ r_{mk}\cos\hat{\theta}_{mk} - r_{nk}\cos\hat{\theta}_{nk} &= 0 \end{aligned} \tag{38}$$

Write in matrix form, Eq. (38) can be rewritten as

$$\underbrace{\begin{bmatrix} \sin\hat{\theta}_{mk} & -\sin\hat{\theta}_{nk} \\ -\cos\hat{\theta}_{mk} & \cos\hat{\theta}_{nk} \end{bmatrix}}_{\boldsymbol{\Pi}_k} \underbrace{\begin{bmatrix} r_{mk} \\ r_{nk} \end{bmatrix}}_{\mathbf{R}_k} = \underbrace{\begin{bmatrix} D_n - D_m \\ 0 \end{bmatrix}}_{\Upsilon_k} \tag{39}$$

where $\boldsymbol{\Pi}_k$ is the coefficient matrix, which is composed of the DOAs of the kth target at the mth and nth sensors. $\mathbf{R}_k$ is the unknown range matrix of the kth source.

When the kth source is a FF one, θ_{mk} is approximately equal to θ_{nk}, that is to say, $\boldsymbol{\Pi}_k$ almost becomes a singular matrix. Thus, the following equation holds for FF source

$$\det(\boldsymbol{\Pi}_k) \approx 0 \tag{40}$$

Therefore, the FF sources can be determined by selecting the signals corresponding to the K_1 minimum values of $|\det(\boldsymbol{\Pi}_k)|$ $(k=1,\ldots,K)$. For the kth FF source, let the range $\hat{r}_k$ be ∞, its DOA and polarization parameters can be estimated as

$$\hat{\theta}_k = \frac{\hat{\theta}_{mk}+\hat{\theta}_{nk}}{2}, \quad \hat{\gamma}_k = \frac{\hat{\gamma}_{mk}+\hat{\gamma}_{nk}}{2}, \quad \hat{\eta}_k = \frac{\hat{\eta}_{mk}+\hat{\eta}_{nk}}{2} \tag{41}$$

When the kth source is a NF one, i.e., $\theta_{mk} \neq \theta_{nk}$, $\boldsymbol{\Pi}_k$ would be a a full rank matrix, thus the remaining sources corresponding to the largest $K-K_1$ values of $|\det(\boldsymbol{\Pi}_k)|$ $(k=1,\ldots,K)$ are regarded as the NF ones, and the range matrix of the kth NF source can be estimated as

$$\hat{\mathbf{R}}_k = \begin{bmatrix} \hat{r}_{mk} \\ \hat{r}_{nk} \end{bmatrix} = {\boldsymbol{\Pi}_k}^{-1}\boldsymbol{\Upsilon}_k \tag{42}$$

Define $\{x_k, y_k\}$ as the location of the kth NF source, it is obvious that the following equation holds.

$$\begin{aligned} x_k &= \hat{r}_{nk}\sin\hat{\theta}_{nk} + D_n = \hat{r}_{mk}\sin\hat{\theta}_{mk} + D_m \\ y_k &= \hat{r}_{nk}\cos\hat{\theta}_{nk} = \hat{r}_{mk}\cos\hat{\theta}_{mk} \end{aligned} \tag{43}$$

Thus, the location of the kth NF source can be computed by

$$\begin{aligned} \hat{x}_k &= \frac{\left(\hat{r}_{nk}\sin\hat{\theta}_{nk} + D_n\right) + \left(\hat{r}_{mk}\sin\hat{\theta}_{mk} + D_m\right)}{2} \\ \hat{y}_k &= \frac{\hat{r}_{nk}\cos\hat{\theta}_{nk} + \hat{r}_{mk}\cos\hat{\theta}_{mk}}{2} \end{aligned} \tag{44}$$

From Eq. (44), the DOA, range and polarization parameters of the kth NF source at the phase reference point can be obtained as

$$\begin{aligned} \hat{\theta}_k &= \arctan\left\{\frac{\hat{y}_k}{\hat{x}_k}\right\}, \quad \hat{r}_k = \sqrt{\hat{x}_k^2 + \hat{y}_k^2} \\ \hat{\gamma}_k &= \frac{\hat{\gamma}_{mk}+\hat{\gamma}_{nk}}{2}, \quad \hat{\eta}_k = \frac{\hat{\eta}_{mk}+\hat{\eta}_{nk}}{2} \end{aligned} \tag{45}$$

E. UNKNOWN GAIN/PHASE ESTIMATION

Now we perform EVD on the covariance matrix $\mathbf{R}_x = E\left\{\mathbf{X}(t)\mathbf{X}^H(t)\right\}$:

$$\mathbf{R}_x = \mathbf{U}_s\Sigma_s\mathbf{U}_s^H + \mathbf{U}_n\Sigma_n\mathbf{U}_n^H \tag{46}$$

where $\Sigma_s \in \mathbb{C}^{K\times K}$ and $\Sigma_n \in \mathbb{C}^{(6M-K)\times K}$ are the diagonal matrices containing the K largest and $(6M-K)$ smallest eigenvalues of $\mathbf{R}_x$, respectively. $\mathbf{U}_s \in \mathbb{C}^{6M\times K}$ and $\mathbf{U}_n \in \mathbb{C}^{6M\times(6M-K)}$ are composed of the eigenvectors of $\mathbf{R}_x$ corresponding to the K largest and $(6M-K)$ smallest eigenvalues, respectively.

With the DOA, range, and polarization angles estimation of the mixed sources, the orthogonality between the noise subspace $\mathbf{U}_n$ and steering vectors $\bar{\mathbf{a}}\left(\hat{\theta}_k, \hat{\gamma}_k, \hat{\eta}_k, \hat{r}_k\right)$ $(k=1,\ldots,K)$ can be utilized to get the gain/phase estimation. Then we have

$$\begin{bmatrix} \mathbf{U}_n^H\tilde{\mathbf{G}}\mathbf{a}\left(\hat{\theta}_1,\hat{\gamma}_1,\hat{\eta}_1,\hat{r}_1\right) \\ \vdots \\ \mathbf{U}_n^H\tilde{\mathbf{G}}\mathbf{a}\left(\hat{\theta}_K,\hat{\gamma}_K,\hat{\eta}_K,\hat{r}_K\right) \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{6M-K,1} \\ \vdots \\ \mathbf{0}_{6M-K,1} \end{bmatrix} \tag{47}$$

$\tilde{\mathbf{G}}\mathbf{a}\left(\hat{\theta}_k,\hat{\gamma}_k,\hat{\eta}_k,\hat{r}_k\right)$ could be reformulated as

$$\tilde{\mathbf{G}}\mathbf{a}\left(\hat{\theta}_k,\hat{\gamma}_k,\hat{\eta}_k,\hat{r}_k\right) = diag\left\{\mathbf{a}\left(\hat{\theta}_k,\hat{\gamma}_k,\hat{\eta}_k,\hat{r}_k\right)\right\}\tilde{\mathbf{g}} \tag{48}$$

Thus, Eq. (47) can be transformed as

$$\begin{aligned} &\underbrace{\begin{bmatrix} \mathbf{U}_n^H diag\left\{\mathbf{a}\left(\hat{\theta}_1,\hat{\gamma}_1,\hat{\eta}_1,\hat{r}_1\right)\right\} \\ \vdots \\ \mathbf{U}_n^H diag\left\{\mathbf{a}\left(\hat{\theta}_K,\hat{\gamma}_K,\hat{\eta}_K,\hat{r}_K\right)\right\} \end{bmatrix}}_{\mathbf{W}}\tilde{\mathbf{g}} \\ &= \left[\mathbf{W}_1 \vdots \mathbf{W}_2 \vdots \mathbf{W}_3\right]\begin{bmatrix} \tilde{\mathbf{g}}_1 \\ \mathbf{1}_6 \\ \tilde{\mathbf{g}}_3 \end{bmatrix} \\ &= \underbrace{\left[\mathbf{W}_1 \vdots \mathbf{W}_3\right]}_{\mathbf{W}_{13}}\begin{bmatrix} \tilde{\mathbf{g}}_1 \\ \tilde{\mathbf{g}}_3 \end{bmatrix} + \mathbf{W}_2\mathbf{1}_6 = \mathbf{0}_{K(6M-K),1} \end{aligned} \tag{49}$$

where $\mathbf{1}_6$ is a 6×1 vector with all ones and $\mathbf{0}_{K(6M-K),1}$ denotes a $K(6M-K)$ column vector with all zeros. $\mathbf{W}_1$ and $\mathbf{W}_3$ are composed of the left $6(q-1)$ columns and right $6(M-q)$ columns of $\mathbf{W}$, respectively. $\mathbf{W}_2$ consists of the middle six columns of $\mathbf{W}$. Then we can obtain the least square solution of Eq. (49) as

$$\tilde{\mathbf{g}}_{13} = -\left(\mathbf{W}_{13}{}^H \mathbf{W}_{13}\right)^{-1} \mathbf{W}_{13}{}^H \mathbf{W}_2 \mathbf{1}_6 \tag{50}$$

As a result, the array gains/phases can be estimated as

$$\hat{g}_n = \begin{cases} \dfrac{\sum\limits_{l=1}^{6} \tilde{\mathbf{g}}_{13}\left(6(n-1)+l\right)}{6} & n < q \\ \dfrac{\sum\limits_{l=1}^{6} \tilde{\mathbf{g}}_{13}\left(6(n-2)+l\right)}{6} & n > q \end{cases} \tag{51}$$

F. IMPLEMENTATION OF THE PROPOSED ALGORITHM

In this subsection, the proposed algorithm is summarized. In the previous subsections, we use true covariance matrices and their corresponding subspace matrices for simplicity. However, in practice, the covariance matrix $\mathbf{R}_x$ is usually unavailable. In cases of finite snapshots, the array covariance matrix can be approximately computed by

$$\hat{\mathbf{R}}_x = \frac{1}{L} \sum_{l=1}^{L} \mathbf{x}(t_l)\, \mathbf{x}^H(t_l) \tag{52}$$

where L is the total number of snapshots. Consequently, the presented method for mixed sources localization using linear EMVS array with gain/phase uncertainties is summarized as follows.

Step 1: Construct the fourth-cumulant matrices according to Eq. (13) and Eq. (27).

Step 2: Estimate the DOAs and polarization parameters of the mixed sources at the mth EMVS and nth EMVS from Eq. (16)-Eq. (26) and Eq. (30)-Eq. (37), respectively.

Step 3: Pair the parameters estimates at the mth and nth EMVSs for the same target.

Step 4: Classify the signal types by computing the coefficient matrix of Eq. (39).

Step 5: Estimate the parameters of the mixed sources at the phase reference point according to Eq. (41)-Eq. (43).

Step 6: Estimate the covariance matrix $\hat{\mathbf{R}}_x$ by Eq. (52).

Step 7: Eigendecompose $\hat{\mathbf{R}}_x$ to generate its noise subspace $\hat{\mathbf{U}}_n$.

Step 8: Obtain the unknown gain/phase errors from Eq. (49)-Eq. (51).

G. COMPUTATIONAL COMPLEXITY

The main computations of the proposed method include: (a) construction of twelve $6M \times K$ fourth-order cumulant matrices $\mathbf{R}_i$ and $\mathbf{C}_i$ $(i = 1, \ldots, 6)$ with $\mathcal{O}(12 \cdot 9(6M)^2 L)$ flops, (b) eigendecomposition to obtain $\mathbf{E}_{SL}$ and $\mathbf{E}_{SR}$ with $\mathcal{O}(\frac{8}{3}(36M)^3)$ flops, (c) estimation of $\mathbf{H}^{(1,2)}$, $\mathbf{H}^{(1,3)}$, $\mathbf{H}^{(1,4)}$, $\mathbf{H}^{(1,5)}$, $\mathbf{H}^{(1,6)}$, $\mathbf{Q}^{(1,2)}$, $\mathbf{Q}^{(1,3)}$, $\mathbf{Q}^{(1,4)}$, $\mathbf{Q}^{(1,5)}$ and $\mathbf{Q}^{(1,6)}$ with $\mathcal{O}(20K^2(6M) + 10\left(\frac{4}{3}K^3\right))$ flops, (d) construction of covariance matrix $\hat{\mathbf{R}}_x$ with $\mathcal{O}((6M)^2 L)$ flops, (e) eigendecomposition of $\hat{\mathbf{R}}_x$ with $\mathcal{O}(\frac{4}{3}(6M)^3)$ flops. Overall, the major computational load of the proposed algorithm is $\mathcal{O}(12 \cdot 9(6M)^2 L + \frac{8}{3}(36M)^3 + 20K^2(6M) + (6M)^2 L + \frac{4}{3}(6M)^3)$ flops.

IV. SIMULATION RESULTS

In this section, simulations are conducted to validate the performance of the proposed algorithm and the results are compared with one representative existing approach in [21] and the related Cramer-Rao bound (CRB). In our simulations, we consider a linear array composed of $M = 6$ EMVSs with $D_1 = -3\lambda, D_2 = 0, D_3 = 5\lambda, D_4 = 5.5\lambda, D_5 = 6\lambda, D_6 = 6.25\lambda$. For the cross-cumulant based algorithm in [21], a 36-element ULA composed of scalar sensors with spacing $\frac{\lambda}{8}$ is considered. Hence, the number of sensor elements is the same for all the two algorithms. The gain/phase vector is chosen as $\tilde{\mathbf{g}} = \left[0.8218e^{j\frac{\pi}{3}}\mathbf{1}_6^T, \mathbf{1}_6^T, 1.0413e^{-j\frac{\pi}{4}}\mathbf{1}_6^T, 0.7209e^{j\frac{\pi}{5}}\mathbf{1}_6^T, 1.2999 e^{j\frac{\pi}{6}}\mathbf{1}_6^T, 0.5434e^{-j\frac{\pi}{2}}\mathbf{1}_6^T\right]^T$. 100 Monte Carlo experiments are conducted to obtain the results, and the following root mean squard error (RMSE) is defined as

$$RMSE = \sqrt{\frac{1}{100K} \sum\nolimits_{k=1}^{K} \sum\nolimits_{j=1}^{100} E\left[(\tilde{\alpha}_k - \alpha_k)^2\right]}$$

where $\tilde{\alpha}_k$ is the estimated α_k, k denotes the source number and j denotes the trial number.

In the first experiment, the proposed algorithm is used to deal with one FF source and one NF source under gain/phase uncertainties. The DOA, range and polarization parameters $(\theta, r, \gamma, \eta)$ of the two sources are $(20^\circ, 5\lambda, 40^\circ, 80^\circ)$ and $(50^\circ, \infty, 45^\circ, 60^\circ)$, respectively.

A. RMSE VERSUS SNR

The number of snapshots is set as 1000. When the SNR varies from 0dB to 40dB, the RMSEs of parameters estimations versus SNR is shown in Fig. 2. As it can be seen, the proposed algorithm significantly outperforms the cross-cumulant based algorithm [21]. For the proposed method, it can robustly estimate the DOA and range of mixed sources under the gain-phase errors. However, owing to the existence of unknown gains/phases, the algorithm in [21] cannot localize mixed sources effectively. Moreover, our method can also accurately estimate the polarization angles and unknown gains/phases. Note that the RMSE of the proposed method does not approach the CRB effectively, the main reason is that it only uses the information inside of the EMVS but does not make use of the array aperture for parameters estimation.

B. RMSE VERSUS SNAPSHOTS

The SNR is fixed at 20 dB. When snapshot number varies from 300 to 3000, the RMSEs of the parameters estimations is shown in Fig. 3. Again, it is seen that the proposed algorithm still has obvious advantages over the method in [21] for all available snapshots.

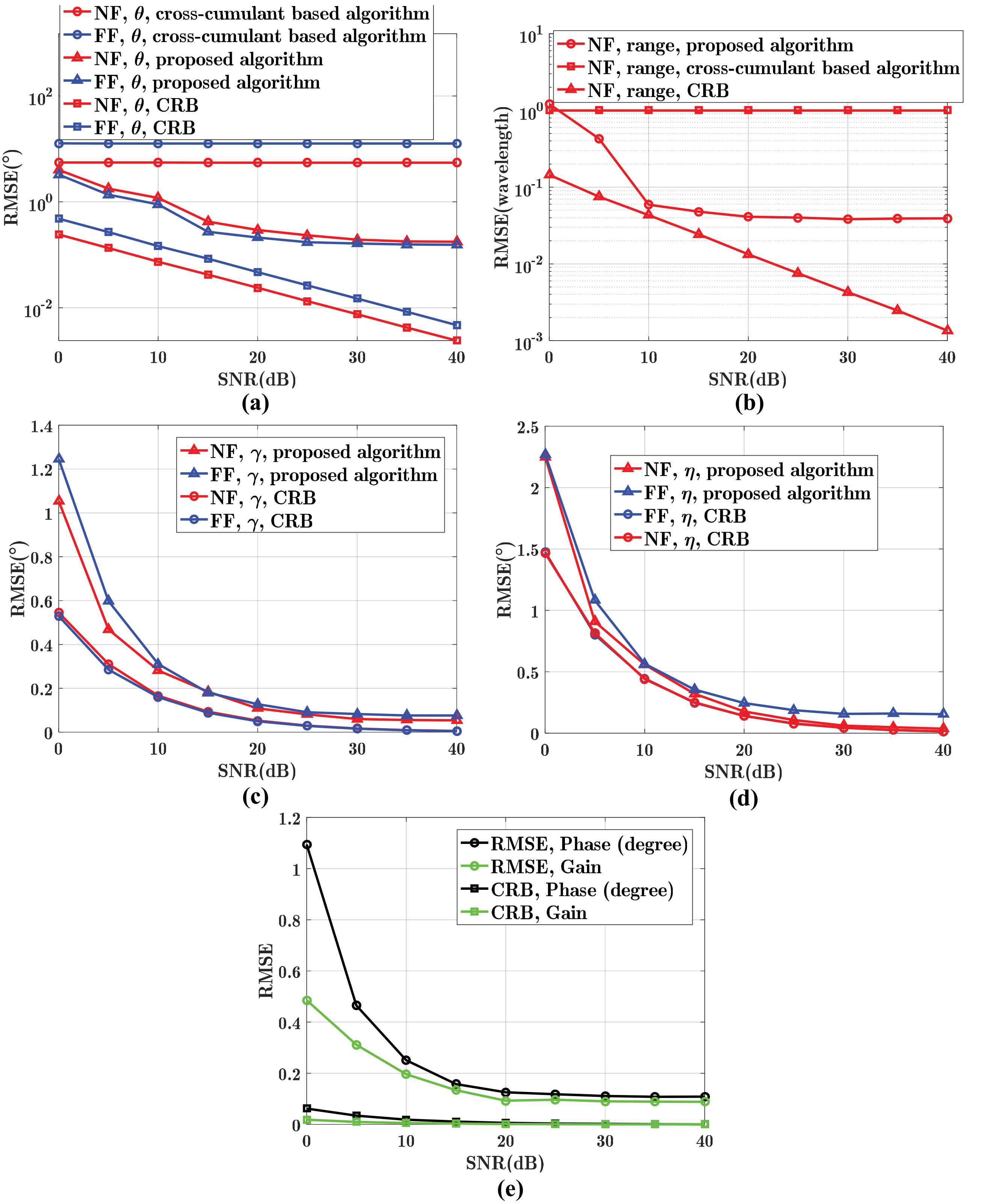

FIGURE 2. RMSEs of parameters estimates for one FF and one NF source versus SNR.

In the second experiment, we consider the situation where two NF sources are impinging on the above array with $(\theta, r, \gamma, \eta)$ being $(20, 5\lambda, 40, 80)$ and $(50^\circ, 9\lambda, 45^\circ, 60^\circ)$, respectively.

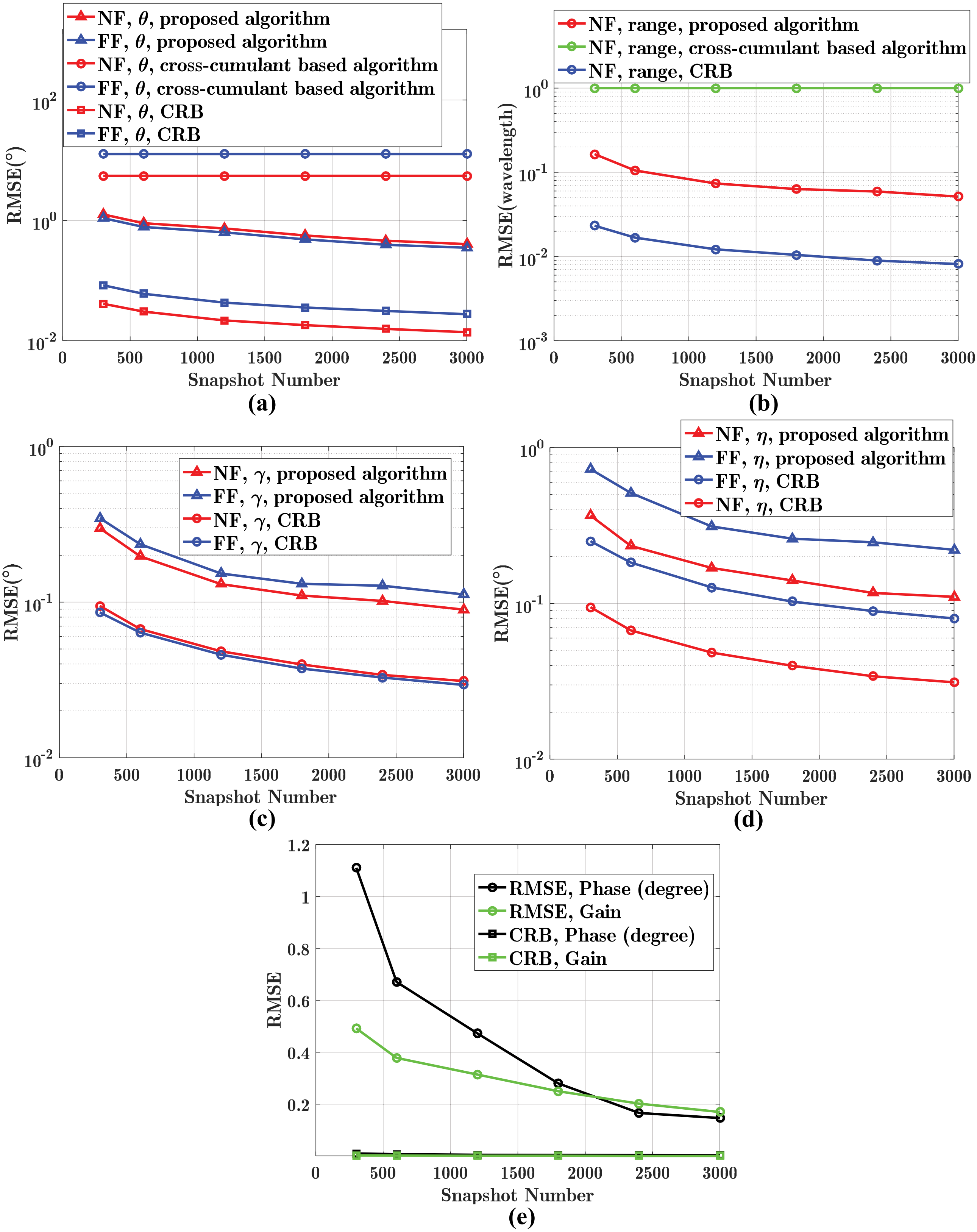

FIGURE 3. RMSEs of parameters estimates for one FF and one NF source versus snapshots.

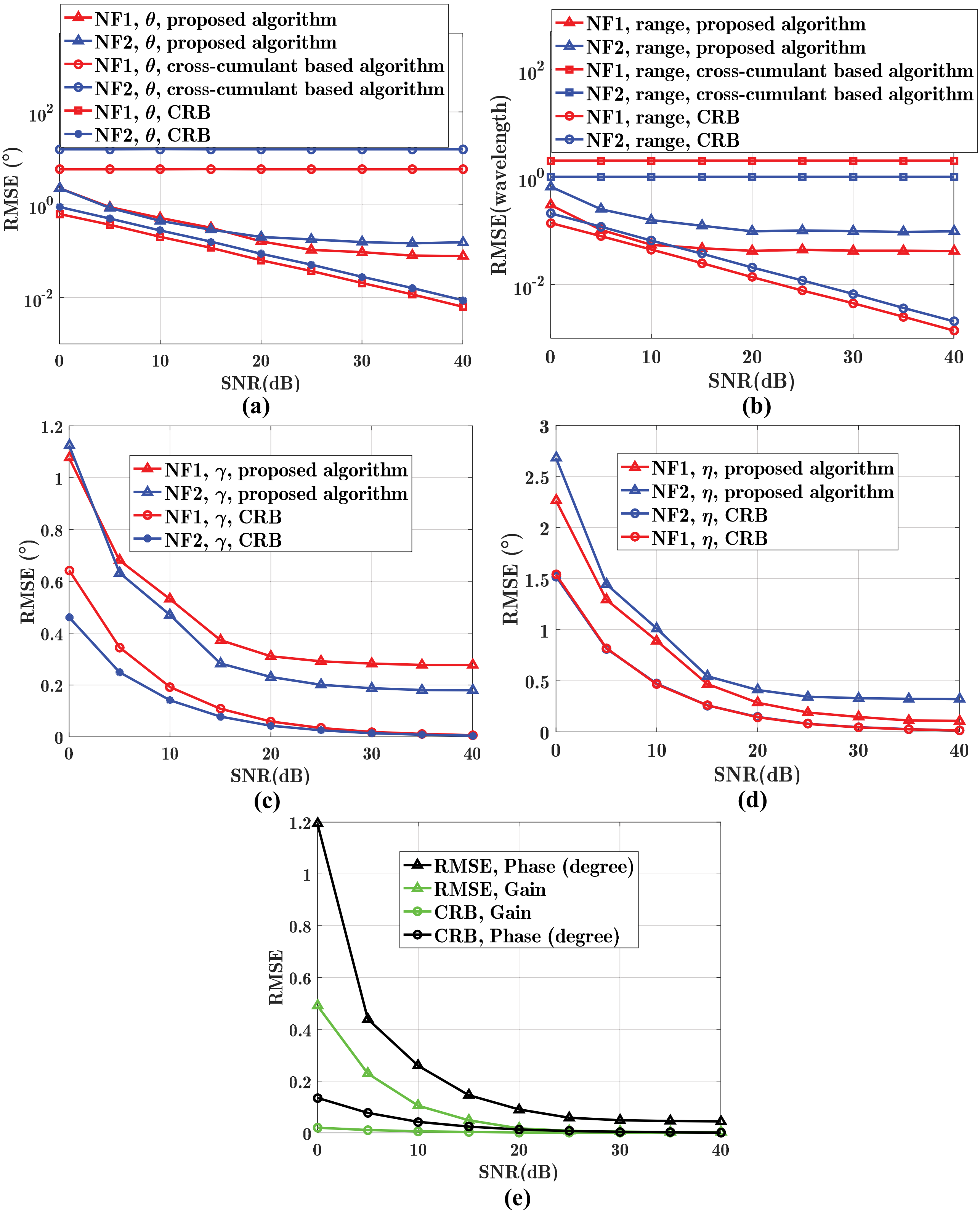

FIGURE 4. RMSEs of parameters estimates for two NF sources versus SNR.

1) RMSE VERSUS SNR

The RMSEs of parameters estimates using the proposed algorithm is presented in Fig. 4. In addition, the cross-cumulant based method [21] and the related CRB are also given for comparison. We can see that in the pure NF scenario, the proposed algorithm significantly outperforms the other method.

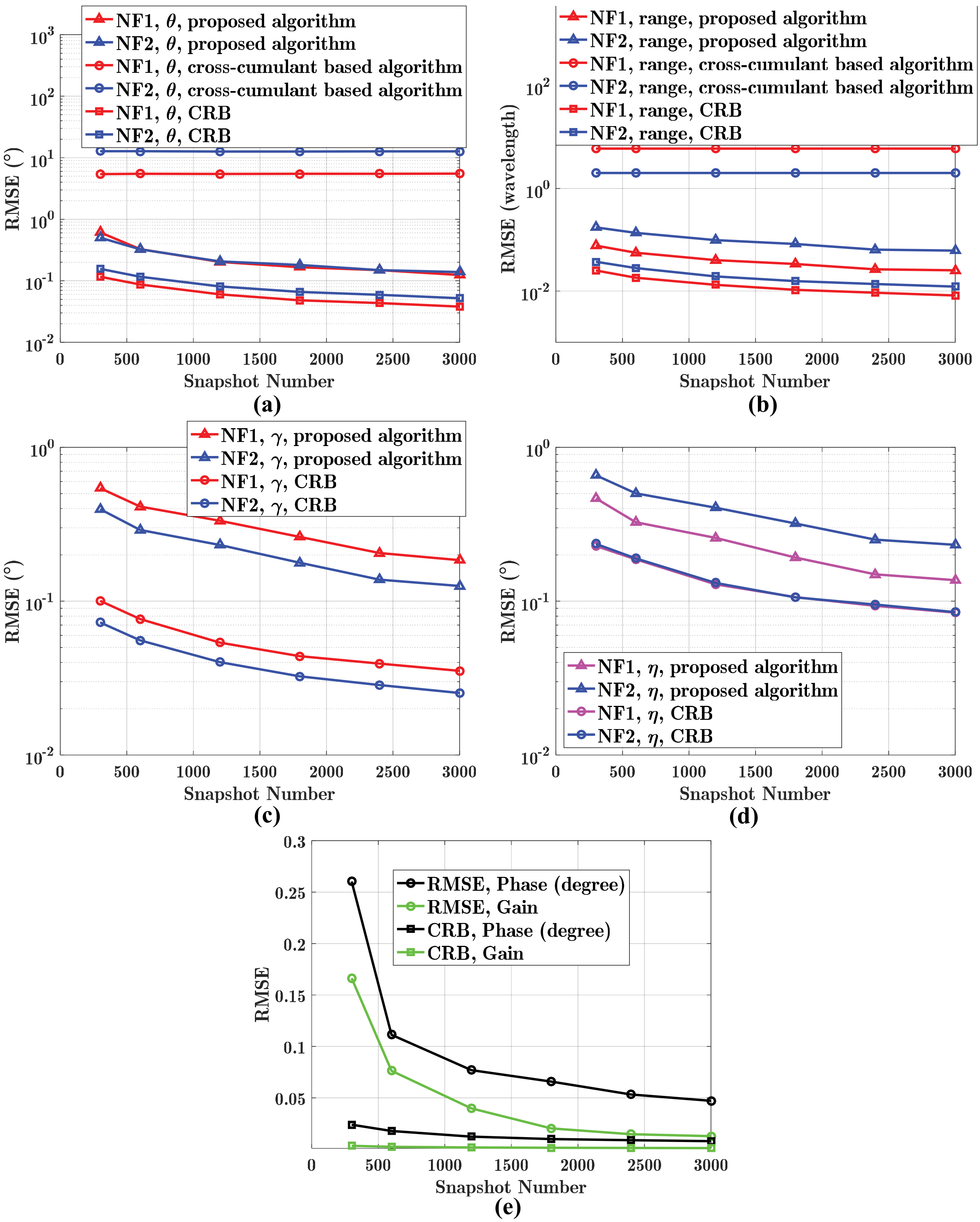

FIGURE 5. RMSEs of parameters estimates for two NF sources versus snapshots.

On the other hand it can be seen that our proposed method is robust to the unknown gains/phases in the estimation of DOA and range, while there exists serious performance degradation for the cross-cumulant based method [21]. Moreover, it can be found that the proposed algorithm can well estimate the polarization angles and unknown sensor gains/phases as well. Additionally, it is worth noting that there exists a gap between the RMSEs of the proposed method and the corresponding CRBs, the reason is the same as that of the first experiment.

2) RMSE VERSUS SNAPSHOTS

The SNR is fixed at 20dB. When the number of snapshots varies from 300 to 3000, the RMSEs of parameters estimations versus number of snapshots are plotted in Fig. 5. It can be observed that the simulation results are similar to those of Figs. 4 and the proposed method gains obvious advantages over the cross-cumulant based method [21].

V. CONCLUSION

An efficient robust localization algorithm is proposed in this paper to localize the mixed FF and NF sources using a linear EMVS array with gain/phase uncertainties. Compared with some existing methods, the proposed approach can measure both the sources' spatial and polarization information, and is able to achieve high-accuracy estimation performance in the presence of unknown inter-sensor gain/phase responses. Moreover, it avoids the spectral search and has no restriction on EMVSs placement, as well as realizes a more reasonable classification of the signal types.

APPENDIX A

Considering a zero mean complex Gaussian vector $\mathbf{X}_{(t)}$, which has the following covariance matrix:

$$\mathbf{R}_x = \tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H + \sigma^2\mathbf{I}_{6M} \tag{53}$$

where $\tilde{\mathbf{G}}$ is a $6M \times 6M$ diagonal matrix representing the gains/phases of the whole array, $\bar{\mathbf{A}}$ is the steering matrix, $\mathbf{R}_s$ is the covariance matrix of the incident signals.

The unknown parameter vector in $\mathbf{R}_x$ can be defined as

$$\boldsymbol{\varepsilon} = \left[\boldsymbol{\theta}^T, \mathbf{r}^T, \boldsymbol{\gamma}^T, \boldsymbol{\eta}^T, \mathbf{g}^T, \boldsymbol{\varphi}^T\right]^T \tag{54}$$

where $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_K]^T$, $\mathbf{r} = \left[r_{K_1+1}, \ldots, r_K\right]^T$, $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_K]^T$, $\boldsymbol{\eta} = [\eta_1, \ldots, \eta_K]^T$, $\boldsymbol{\rho} = [\rho_1, \ldots, \rho_1, \ldots, \rho_{M-1}, \ldots, \rho_{M-1}]^T \in \mathbb{C}_{6(M-1)\times 1}$, $\boldsymbol{\varphi} = [\varphi_1, \ldots, \varphi_1, \ldots, \varphi_{M-1}, \ldots, \varphi_{M-1}]^T \in \mathbb{C}_{6(M-1)\times 1}$. Suppose that the number of snapshots is L, then the general form of the (m, n)-th element in the Fisher information matrix (FIM) can be written as [15]

$$\mathbf{F}_{mn} = L \cdot tr\left\{\mathbf{R}_x^{-1}\frac{\partial \mathbf{R}_x}{\partial \varepsilon_m}\mathbf{R}_x^{-1}\frac{\partial \mathbf{R}_x}{\partial \varepsilon_n}\right\} \tag{55}$$

A. DERIVATIVES WITH RESPECT TO DOA

The partial derivative of the covariance matrix with the DOA θ_m is given by

$$\frac{\partial \mathbf{R}_x}{\partial \theta_m} = \tilde{\mathbf{G}}\dot{\mathbf{A}}_{\theta_m}\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H + \tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\dot{\mathbf{A}}_{\theta_m}^H\tilde{\mathbf{G}}^H \tag{56}$$

where $\dot{\mathbf{A}}_{\theta_m} = \frac{\partial \mathbf{A}}{\partial \theta_m}$. Substituting (56) into (55), we have

$$\begin{aligned}\mathbf{F}_{\theta_m\theta_n} &= 2L \cdot \mathrm{Re}\left\{tr\left(\tilde{\mathbf{G}}\dot{\mathbf{A}}_{\theta_m}\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_{\theta_n}\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\right)\right.\\ &\quad\left. + tr\left(\tilde{\mathbf{G}}\dot{\mathbf{A}}_{\theta_m}\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\dot{\mathbf{A}}_{\theta_n}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\right)\right\}\end{aligned} \tag{57}$$

Observe that

$$\dot{\mathbf{A}}_{\theta_m} = \dot{\mathbf{A}}_{\theta}\mathbf{e}_m\mathbf{e}_m^T \tag{58}$$

where the unit vector $\mathbf{e}_m$ is the mth column vector of the identity matrix, and $\dot{\mathbf{A}}_\theta$ is the matrix of derivatives defined by

$$\dot{\mathbf{A}}_\theta = \sum_{m=1}^{K}\frac{\partial \mathbf{A}}{\partial \theta_m} \tag{59}$$

using (58), equation (57) becomes

$$\begin{aligned}\mathbf{F}_{\theta_m\theta_n} &= 2L\cdot\mathrm{Re}\\ &\quad\times\left\{tr\left(\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\mathbf{e}_m\mathbf{e}_m^T\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\mathbf{e}_n\mathbf{e}_n^T\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\right)\right.\\ &\quad\left.+tr\left(\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta e_m\mathbf{e}_m^T\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\mathbf{e}_n\mathbf{e}_n^T\dot{\mathbf{A}}_\theta^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\right)\right\}\\ &= 2L\cdot\mathrm{Re}\\ &\quad\times\left\{\left(\mathbf{e}_m^T\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\mathbf{e}_n\mathbf{e}_n^T\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\mathbf{e}_m\right)\right.\\ &\quad\left.+\left(\mathbf{e}_m^T\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\mathbf{e}_n\mathbf{e}_n^T\dot{\mathbf{A}}_\theta^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\mathbf{e}_m\right)\right\}\end{aligned} \tag{60}$$

Hence,

$$\begin{aligned}\mathbf{F}_{\boldsymbol{\theta\theta}} &= 2L\cdot\mathrm{Re}\left\{\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\right)\odot\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\right)^T\right.\\ &\quad\left.+\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\right)\odot\left(\dot{\mathbf{A}}_\theta^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\right)^T\right\}\end{aligned} \tag{61}$$

Herein, $\odot$ denotes the Hadamard product.

B. DERIVATIVES WITH RESPECT TO RANGE

Let

$$\dot{\mathbf{A}}_r = \sum_{m=K_1+1}^{K}\frac{\partial \mathbf{A}}{\partial r_m} \tag{62}$$

where K_1 is the number of FF source, it can be similarly obtained that

$$\begin{aligned}\mathbf{F}_{\mathbf{rr}} &= 2L\cdot\mathrm{Re}\left\{\bar{\mathbf{H}}\left[\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_r\right)\right.\right.\\ &\quad\odot\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_r\right)^T\\ &\quad\left.\left.\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\right)\odot\left(\dot{\mathbf{A}}_r^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_r\right)^T\right]\bar{\mathbf{H}}^T\right\}\end{aligned} \tag{63}$$

where $\bar{\mathbf{H}}$ is a $(K-K_1)\times K$ matrix with its (m,n)th entry being

$$[\bar{\mathbf{H}}]_{m,n}=\begin{cases}1, & \text{if } n=K_1+m\\ 0, & \text{otherwise}\end{cases} \quad (64)$$

C. DERIVATIVES WITH RESPECT TO γ

Similarly we obtain

$$\begin{aligned}&\mathbf{F}_{\gamma\gamma}\\ &=2L\cdot\mathrm{Re}\left\{\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\gamma\right)\odot\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\gamma\right)^T\right.\\ &\quad\left.+\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\right)\odot\left(\dot{\mathbf{A}}_\gamma^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\gamma\right)^T\right\} \quad (65)\end{aligned}$$

where

$$\dot{\mathbf{A}}_\gamma=\sum_{m=1}^{K}\frac{\partial\mathbf{A}}{\partial\gamma_m} \quad (66)$$

D. DERIVATIVES WITH RESPECT TO η

In a similar way we obtain

$$\begin{aligned}&\mathbf{F}_{\eta\eta}\\ &=2L\cdot\mathrm{Re}\left\{\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\eta\right)\odot\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\eta\right)^T\right.\\ &\quad\left.+\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\right)\odot\left(\dot{\mathbf{A}}_\eta^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\eta\right)^T\right\} \quad (67)\end{aligned}$$

where

$$\dot{\mathbf{A}}_\eta=\sum_{m=1}^{K}\frac{\partial\mathbf{A}}{\partial\eta_m} \quad (68)$$

E. DERIVATIVES WITH RESPECT TO GAIN

Repeating the same set of considerations leading to (61) we obtain

$$\begin{aligned}\mathbf{F}_{\rho_i\rho_j}&=2L\cdot\mathrm{Re}\left\{tr\left(\dot{\bar{\mathbf{A}}}_{\rho_i}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\dot{\bar{\mathbf{A}}}_{\rho_j}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\right)\right.\\ &\quad\left.+\,tr\left(\dot{\bar{\mathbf{A}}}_{\rho_i}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\bar{\mathbf{A}}\mathbf{R}_s\dot{\bar{\mathbf{A}}}_{\rho_j}^H\mathbf{R}_x^{-1}\right)\right\} \quad (69)\end{aligned}$$

where

$$\dot{\bar{\mathbf{A}}}_{\rho_j}=\frac{\partial\bar{\mathbf{A}}}{\partial\rho_j}=\mathbf{e}_j\mathbf{e}_j^T\mathbf{P}\mathbf{A} \quad (70)$$

Here $\mathbf{P}$ is a diagonal matrix containing the exponents of the sensors' phases, which is of the following form

$$\begin{aligned}[\mathbf{P}]_{6M\times 6M}=diag\{&1,\ldots 1,e^{j\varphi_1},\ldots,e^{j\varphi_1},\\ &e^{j\varphi_{M-1}},\ldots,e^{j\varphi_{M-1}}\} \quad (71)\end{aligned}$$

Note that since the gain of the first EMVS is assumed known and therefore $7\leqslant j\leqslant 6M$. We therefore define the $6(M-1)\times 6M$ matrix

$$[\bar{\mathbf{Q}}]_{i,j}=\begin{cases}1 & \text{if } j=i+6\\ 0 & \text{otherwise}\end{cases} \quad (72)$$

Using $\bar{\mathbf{Q}}$ we can write

$$\begin{aligned}&\mathbf{F}_{\rho\rho}\\ &=2L\cdot\mathrm{Re}\left\{\bar{\mathbf{Q}}\left[\left(\mathbf{P}\mathbf{A}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\right)\odot\left(\mathbf{P}\mathbf{A}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\right)^T\right.\right.\\ &\quad\left.\left.+\left(\mathbf{P}\mathbf{A}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\bar{\mathbf{A}}\mathbf{R}_s\mathbf{A}^H\mathbf{P}^H\right)\odot\left(\mathbf{R}_x^{-1}\right)^T\right]\bar{\mathbf{Q}}^T\right\} \quad (73)\end{aligned}$$

F. DERIVATIVES WITH RESPECT TO PHASE

Repeating the considerations leading to (60) we obtain

$$\begin{aligned}\mathbf{F}_{\varphi_i\varphi_j}&=2L\cdot\mathrm{Re}\left\{tr\left(\dot{\bar{\mathbf{A}}}_{\varphi_i}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\dot{\bar{\mathbf{A}}}_{\varphi_j}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\right)\right.\\ &\quad\left.+\,tr\left(\dot{\bar{\mathbf{A}}}_{\varphi_i}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\bar{\mathbf{A}}\mathbf{R}_s\dot{\bar{\mathbf{A}}}_{\varphi_j}^H\mathbf{R}_x^{-1}\right)\right\} \quad (74)\end{aligned}$$

where

$$\dot{\bar{\mathbf{A}}}_{\varphi_j}=\frac{\partial\bar{\mathbf{A}}}{\partial\varphi_j}=j\mathbf{e}_j\mathbf{e}_j^T\bar{\mathbf{A}} \quad (75)$$

Substituting (75) in (74) we obtain

$$\begin{aligned}\mathbf{F}_{\varphi\varphi}&=2L\cdot\mathrm{Re}\left\{\bar{\mathbf{Q}}\left[-\left(\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\right)\odot\left(\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\right)^T\right.\right.\\ &\quad\left.\left.+\left(\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\right)\odot\left(\mathbf{R}_x^{-1}\right)^T\right]\bar{\mathbf{Q}}^T\right\} \quad (76)\end{aligned}$$

G. DERIVATIVES WITH RESPECT TO DOA-RANGE CROSS TERMS

In an analogous manner, we can get

$$\begin{aligned}&\mathbf{F}_{\theta\mathbf{r}}\\ &=2L\cdot\mathrm{Re}\left\{\left[\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_r\right)\odot\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\right)^T\right.\right.\\ &\quad\left.\left.\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\right)\odot\left(\dot{\mathbf{A}}_r^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\right)^T\right]\bar{\mathbf{H}}^T\right\}\\ &\qquad\qquad (77)\end{aligned}$$

H. DERIVATIVES WITH RESPECT TO DOA-γ CROSS TERMS

In a similar way we obtain

$$\begin{aligned}\mathbf{F}_{\theta\gamma}&=2L\cdot\mathrm{Re}\left\{\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\gamma\right)\right.\\ &\quad\odot\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\right)^T\\ &\quad\left.+\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\right)\odot\left(\dot{\mathbf{A}}_\gamma^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\right)^T\right\}\\ &\qquad\qquad (78)\end{aligned}$$

I. DERIVATIVES WITH RESPECT TO DOA-η CROSS TERMS

In an analogous manner, we can get

$$\begin{aligned}&\mathbf{F}_{\theta\eta}\\ &=2L\cdot\mathrm{Re}\left\{\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\eta\right)\odot\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\right)^T\right.\\ &\quad\left.+\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\right)\odot\left(\dot{\mathbf{A}}_\eta^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\right)^T\right\}\\ &\qquad\qquad (79)\end{aligned}$$

J. DERIVATIVES WITH RESPECT TO DOA-GAIN CROSS TERMS

Similarly we get

$$\mathbf{F}_{\theta\rho} = 2L \cdot \mathrm{Re}\left\{\left[\left(\mathbf{R}_s\bar{\mathbf{A}}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\right)\odot\left(\mathbf{PAR}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\right)^T + \left(\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\bar{\mathbf{A}}\mathbf{R}_s\mathbf{A}^H\mathbf{P}^H\right)\odot\left(\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\right)^T\right]\bar{\mathbf{Q}}^T\right\} \tag{80}$$

K. DERIVATIVES WITH RESPECT TO DOA-PHASE CROSS TERMS

Similarly, $\mathbf{F}_{\theta\varphi}$ is given by

$$\mathbf{F}_{\theta\varphi} = 2L \cdot \mathrm{Re}\left\{\left[j\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\right)\odot\left(\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\right)^T - j\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\right)\odot\left(\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\theta\right)^T\right]\bar{\mathbf{Q}}^T\right\} \tag{81}$$

L. DERIVATIVES WITH RESPECT TO RANGE-γ CROSS TERMS

In a similar manner, $\mathbf{F}_{\mathbf{r}\gamma}$ can be derived as

$$\mathbf{F}_{\mathbf{r}\gamma} = 2L \cdot \mathrm{Re}\left\{\bar{\mathbf{H}}\left[\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\gamma\right)\odot\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_r\right)^T \left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\right)\odot\left(\dot{\mathbf{A}}_\gamma^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_r\right)^T\right]\right\} \tag{82}$$

M. DERIVATIVES WITH RESPECT TO RANGE-η CROSS TERMS

Similarly we get

$$\mathbf{F}_{\mathbf{r}\eta} = 2L \cdot \mathrm{Re}\left\{\bar{\mathbf{H}}\left[\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\eta\right)\odot\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_r\right)^T \left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\right)\odot\left(\dot{\mathbf{A}}_\eta^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_r\right)^T\right]\right\} \tag{83}$$

N. DERIVATIVES WITH RESPECT TO RANGE-GAIN CROSS TERMS

Similarly, $\mathbf{F}_{\mathbf{r}\rho}$ can be written as

$$\mathbf{F}_{\mathbf{r}\rho} = 2L \cdot \mathrm{Re}\left\{\bar{\mathbf{H}}\left[\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\right)\odot\left(\mathbf{PAR}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_r\right)^T \left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\mathbf{A}^H\mathbf{P}^H\right)\odot\left(\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_r\right)^T\right]\bar{\mathbf{Q}}^T\right\} \tag{84}$$

O. DERIVATIVES WITH RESPECT TO RANGE-PHASE CROSS TERMS

Similarly, $\mathbf{F}_{\mathbf{r}\varphi}$ can be written as

$$\mathbf{F}_{\mathbf{r}\varphi} = 2L \cdot \mathrm{Re}\left\{\bar{\mathbf{H}}\left[j\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\right)\odot\left(\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_r\right)^T - j\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\right)\odot\left(\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_r\right)^T\right]\bar{\mathbf{Q}}^T\right\} \tag{85}$$

P. DERIVATIVES WITH RESPECT TO γ-η CROSS TERMS

Similarly, $\mathbf{F}_{\gamma\eta}$ is given by

$$\mathbf{F}_{\gamma\eta} = 2L \cdot \mathrm{Re}\left\{\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\eta\right)\odot\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\gamma\right)^T + \left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\mathbf{A}\mathbf{R}_s\right)\odot\left(\dot{\mathbf{A}}_\eta^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\gamma\right)^T\right\} \tag{86}$$

Q. DERIVATIVES WITH RESPECT TO γ-GAIN CROSS TERMS

In a similar way, we obtain

$$\mathbf{F}_{\gamma\rho} = 2L \cdot \mathrm{Re}\left\{\left[\left(\mathbf{R}_s\bar{\mathbf{A}}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\right)\odot\left(\mathbf{PAR}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\gamma\right)^T + \left(\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\bar{\mathbf{A}}\mathbf{R}_s\mathbf{A}^H\mathbf{P}^H\right)\odot\left(\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\gamma\right)^T\right]\bar{\mathbf{Q}}^T\right\} \tag{87}$$

R. DERIVATIVES WITH RESPECT TO γ-PHASE CROSS TERMS

Similarly, $F_{\gamma\varphi}$ can be derived as

$$\mathbf{F}_{\gamma\varphi} = 2L \cdot \mathrm{Re}\left\{\left[j\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\right)\odot\left(\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\gamma\right)^T - j\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\right)\odot\left(\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\gamma\right)^T\right]\bar{\mathbf{Q}}^T\right\} \tag{88}$$

S. DERIVATIVES WITH RESPECT TO η-GAIN CROSS TERMS

In a similar way, we get

$$\mathbf{F}_{\eta\rho} = 2L \cdot \mathrm{Re}\left\{\left[\left(\mathbf{R}_s\bar{\mathbf{A}}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\right)\odot\left(\mathbf{PAR}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\eta\right)^T + \left(\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\bar{\mathbf{A}}\mathbf{R}_s\mathbf{A}^H\mathbf{P}^H\right)\odot\left(\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\eta\right)^T\right]\bar{\mathbf{Q}}^T\right\} \tag{89}$$

T. DERIVATIVES WITH RESPECT TO η-PHASE CROSS TERMS

In a similar way, $F_{\eta\varphi}$ is given by

$$\mathbf{F}_{\eta\varphi} = 2L \cdot \mathrm{Re}\left\{\left[j\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\right)\odot\left(\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\eta\right)^T - j\left(\mathbf{R}_s\mathbf{A}^H\tilde{\mathbf{G}}^H\mathbf{R}_x^{-1}\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\right)\odot\left(\mathbf{R}_x^{-1}\tilde{\mathbf{G}}\dot{\mathbf{A}}_\eta\right)^T\right]\bar{\mathbf{Q}}^T\right\} \tag{90}$$

U. DERIVATIVES WITH RESPECT TO PHASE-GAIN CROSS TERMS

In a similar way, $F_{\varphi\rho}$ is given by

$$\mathbf{F}_{\varphi\rho} = 2L \cdot \mathrm{Re}\left\{\left[j\left(\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\right)\odot\left(\mathbf{P}\mathbf{A}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\right)^T + j\left(\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{R}_x^{-1}\bar{\mathbf{A}}\mathbf{R}_s\bar{\mathbf{A}}^H\mathbf{P}^H\right)\odot\left(\mathbf{R}_x^{-1}\right)^T\right]\bar{\mathbf{Q}}^T\right\} \tag{91}$$

V. THE CRAMER-RAO LOWER BOUND FOR DOA, RANGE, POLARIZATION PARAMETERS, GAIN AND PHASE

Taking the above components together, we can formulate the FIM as the following expression

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{\theta\theta} & \mathbf{F}_{\theta\mathbf{r}} & \mathbf{F}_{\theta\gamma} & \mathbf{F}_{\theta\eta} & \mathbf{F}_{\theta\rho} & \mathbf{F}_{\theta\varphi} \\ \mathbf{F}_{\mathbf{r}\theta} & \mathbf{F}_{\mathbf{rr}} & \mathbf{F}_{\mathbf{r}\gamma} & \mathbf{F}_{\mathbf{r}\eta} & \mathbf{F}_{\mathbf{r}\rho} & \mathbf{F}_{\mathbf{r}\varphi} \\ \mathbf{F}_{\gamma\theta} & \mathbf{F}_{\gamma\mathbf{r}} & \mathbf{F}_{\gamma\gamma} & \mathbf{F}_{\gamma\eta} & \mathbf{F}_{\gamma\rho} & \mathbf{F}_{\gamma\varphi} \\ \mathbf{F}_{\eta\theta} & \mathbf{F}_{\eta\mathbf{r}} & \mathbf{F}_{\eta\gamma} & \mathbf{F}_{\eta\eta} & \mathbf{F}_{\eta\rho} & \mathbf{F}_{\eta\varphi} \\ \mathbf{F}_{\rho\theta} & \mathbf{F}_{\rho\mathbf{r}} & \mathbf{F}_{\rho\gamma} & \mathbf{F}_{\rho\eta} & \mathbf{F}_{\rho\rho} & \mathbf{F}_{\rho\varphi} \\ \mathbf{F}_{\varphi\theta} & \mathbf{F}_{\varphi\mathbf{r}} & \mathbf{F}_{\varphi\gamma} & \mathbf{F}_{\varphi\eta} & \mathbf{F}_{\varphi\rho} & \mathbf{F}_{\varphi\varphi} \end{bmatrix} \tag{92}$$

Subsequently, the CRBs can be defined as:

$$CRB_\theta = \sqrt{\frac{1}{K}\sum_{i=1}^{K}\left[\mathbf{F}^{-1}\right]_{ii}} \tag{93}$$

$$CRB_r = \sqrt{\frac{1}{K-K_1}\sum_{i=K+1}^{2K-K_1}\left[\mathbf{F}^{-1}\right]_{ii}} \tag{94}$$

$$CRB_\gamma = \sqrt{\frac{1}{K}\sum_{i=2K-K_1+1}^{3K-K_1}\left[\mathbf{F}^{-1}\right]_{ii}} \tag{95}$$

$$CRB_\eta = \sqrt{\frac{1}{K}\sum_{i=3K-K_1+1}^{4K-K_1}\left[\mathbf{F}^{-1}\right]_{ii}} \tag{96}$$

$$CRB_\rho = \sqrt{\frac{1}{K}\sum_{i=4K-K_1+1}^{4K-K_1+6(M-1)}\left[\mathbf{F}^{-1}\right]_{ii}} \tag{97}$$

$$CRB_\varphi = \sqrt{\frac{1}{K}\sum_{i=4K-K_1+6(M-1)+1}^{4K-K_1+12(M-1)}\left[\mathbf{F}^{-1}\right]_{ii}} \tag{98}$$

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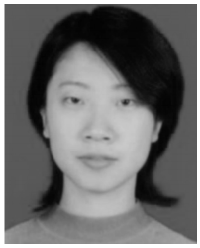

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