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An AC Mathematical Model for Solving Complex Restoration Problems in Radial Distribution Systems in a Treatable Runtime

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ABSTRACT A restoration problem in a radial electrical distribution system (EDS) is solved to restore de-energized loads downstream of sectors affected by a permanent fault, transferring loads among primary feeders through switching operations. As it is a computationally complex problem, its resolution by exact optimization techniques in a treatable runtime is a difficult task. The complexity is related to the number of switches available for solution. To reduce investment costs, a typical EDS has switches only in a portion of the branches, but the number of switches tends to be expressive in larger systems. In this context, less complex restoration problems have been successfully solved in the literature by exact methods, but they tend to be prohibitive in highly complex problems. In this paper, a mathematical model with an enhanced approach solves, through exact techniques, the restoration problem in a relatively large radial EDS with the aim of evaluating its ability to obtain optimal operationally reliable solutions with low computational effort. The problem is solved without simplifying the topological structure of the system and using only two types of binary variables. The model minimizes the demand not supplied with a minimum number of switching operations, allowing considering the influence of priority loads and remotely controlled switches, and ensures a feasible and radial operation. Tests were carried out on a radial 417-bus EDS adapted to have switches at strategic locations and the results show that the model effectively provides optimal solutions and can be applied for larger systems mainly in an offline or preventive way.

INDEX TERMS Large-scale distribution systems, mathematical modeling, service restoration.

I. INTRODUCTION

A radial electrical distribution system (EDS) is subject to supply outages due to contingencies. An unscheduled permanent fault in the EDS requires an urgent operational planning to attempt to restore the supply to the affected areas in a fast and safe way in order to improve the service reliability and reduce the negative impact in financial terms and in customers' satisfaction terms. The service restoration is made through switching operations, whereby the loads are reallocated in primary feeders. As it is a computationally complex multi-constrained combinatorial problem, solving it by exact techniques in a treatable runtime is a difficult task. Thus, heuristic methods have been the main alternatives to solve restoration problems in the literature, assuming that good quality solutions are satisfactory in an urgency context since they allow finding good quality solutions with low computational effort [1]–[18], which is of great importance especially for very complex problems.

Among the main heuristic methods that have been investigated for restoration in radial EDSs are heuristic rules based on expert knowledge [1], [2], a strategy of depth-first search on a decision binary tree [3], strategies that find a radial restoration topology through a constructive process of opening of branches on a meshed topology [4], [5], a heuristic that reconnects the de-energized areas in large-scale EDSs by minimal paths to the branches on the boundary with these areas [6], strategies of branches exchange besides the boundary of the de-energized area in large-scale EDSs [7], [8], a multi-agent approach [9], multi-objective optimization

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methods using an evolutionary approach and fuzzy sets [10] and using an evolutionary algorithm that incorporates specialized genetic operators and solves the problem in large-scale EDSs [11], and a NSGA-II proposal that optimizes four objective functions in a hierarchical structure [12]. Other studied heuristic approaches have been a four-stage method that combines reconfiguration with intentional islanding of distributed generators, allowing the restored EDS to operate in an islanded way with these generators [13], a method based on multi-agent systems using expert systems rules for autonomous restoration in active EDSs [14], a three-stage heuristic strategy, whereby the system operator participates in the second stage, filtering a set of feasible solutions generated in the first stage to be improved in the third stage through a local search process [15], a two-stage proposal that does the optimal island partitioning in a smart EDS for feasible operation with distributed generators [16], a strategy based on the weighted ideal point method that solves the problem with multiple objectives [17], and a proposal using simulated annealing and a local improvement strategy, where the search space is represented by a set of permutation vectors composed by the switches, and the time for the switching operations is estimated using a scheduling approach considering multiple dispatch teams [18]. Other important topics on restoration have been investigated in [19]–[21], whose proposals are an iterative method that generates alternative energizing path schemes for restoration after a blackout [19], an alternating direction method of multipliers that separates the restoration problem into two sub-problems to solve it iteratively [20], and a method based on the reinforcement learning technique that can address the self-healing and load management functions in advanced distribution management systems simultaneously or using the conventional strategy that addresses these two techniques separately [21].

Although exact restoration methods can provide optimal solutions, they are less used in the literature than heuristic methods and are effective for less complex problems. The main reason is that exact methods usually require a larger computational effort, and the execution time tends to be prohibitive when they are used to solve the problem in large EDSs with a very expressive number of switches. Solving restoration problems in a treatable runtime using exact techniques requires efficient and robustness methods. Thus, full mathematical models solved by exact techniques are relatively recent proposals for optimal restoration in radial EDSs [22]–[25]. Convexification techniques and efficient forms of representing the radiality [26]–[29] contributed to the proposal of mathematical models for restoration in radial EDSs. Linearization and search space reduction are examples of techniques applied to the development of these models to make them more efficient. This type of strategy is important because these models have binary variables, which make them integer-mixed programming problems and hence hard problems to solve. In [22], a second-order conic model minimizes the demand not supplied (DNS) and the number of switching operations in balanced EDSs, with the possibility of considering the influence of priority loads and the priority operation of remotely controlled switches. In [23], a linear model with relaxed power flow equations minimizes the DNS in a large EDS with low computational effort, but the configurations need to be validated with respect to their operational feasibility. In [24], a non-linear model minimizes the number of consumers not served during the switching sequence and on the final topology in unbalanced three-phase EDSs. In [25], two different models are proposed to minimize the energy not supplied: the first model uses predetermined time steps to represent the time horizon for restoration and the second one uses a route table and virtual energization agents to estimate the restoration time in unbalanced three-phase EDSs.

The main aim of this work is to enhance the approach in [22], where the model has binary variables representing the operational status of all branches, assuming that there are switches in all lines or that a section is simplified as a single bus in the sense that its internal configuration cannot change. A typical radial EDS has tie/sectionalizing switches installed only in a parcel of the lines, at points that allow a good coordination of the system and its operational optimization in face of urgent or short-term operational problems, and the arrangement of these switches delimits load sections usually composed of several consumer units connected internally by lines without a switching device. Thus, the number of candidate solutions increases greatly when the problem is solved considering switches in all lines, and the structure of the EDS is not fully represented when the sections are simplified as single elements. For the model in [22] to represent the sections in full, all buses must be considered and the variables that represent the status of closed branches without switches must be set to 1 before resolution. The strategy proposed in this work reduces the number of binary variables in relation to [22] and does not require additional variables to represent the load sections or the energization state of their internal elements, besides keeping the radiality constraint simple. In [25], the strategy used to represent the switchable lines and the load sections as well as to ensure radial configurations required a large number of binary variables to represent the energization state of the elements within those sections. The new approach considers the complete structure of the EDS and the real allocation of the switches without the need for previous adaptations as in [22], and the energization state of the elements within the sections is defined using only two types of binary variables, one associated with the buses and the other associated with the branches. The model with the new approach ensures the same operational safety and quality of service proposed in [22]. Thus, it can also be solved by exact techniques using efficient solvers and provide optimal operationally feasible solutions.

The second aim of this work is to apply the model with the new approach to solve the restoration problem in large EDSs in a more realistic way and evaluate its real ability in solving complex problems. The model in [22] required a few seconds to provide optimal solutions in critical fault scenarios in a radial 53-bus EDS with switches in all 61 branches, but it took many hours to provide optimal solutions in critical fault scenarios in a radial 417-bus EDS with switches in all 473 branches [23]. Although these EDSs are not very large in relation to the number of buses, the expressive number of switches makes the problem very complex. When tests are carried out with switches in all lines, the system becomes highly more complex than a typical EDS, which usually has around 10% to 25% of switchable lines [8], [30]. In [8], e.g., a 416-bus EDS has 446 branches and 97 switches, a percentage of 22%. In this paper, tests were performed on the radial 417-bus EDS adapted to have switches only at strategic locations for its operational optimization. Results show that the new model can efficiently solve the problem in even larger systems in the proposed context and can be applied satisfactorily to provide optimal solutions for very complex problems in an offline or preventive way.

Thus, the contributions of this work are briefly as follows:

- The formulation of an AC mathematical model that accurately represents the real structure of a radial EDS typically arranged in sections, and its application to provide optimal reliable solutions in large systems in a treatable runtime, as shown in the testing section;
- The structure of the EDS arranged in load sections is modeled without the need for an expressive number of binary variables to control the energization state of the internal elements of these sections and the radiality;
- The number of binary and continuous variables in the proposed model is significantly less compared to [22], and the new constraints modeled are mainly addressed to impose operating limits for energized/de-energized regions in the EDS under a restorative context;
- The model solves cases of single or multiple faults and allows considering the influence of priority loads and remotely controlled switches effectively with the same simplicity as [22].

II. FORMULATION OF THE RESTORATION PROBLEM

A mixed-integer second-order cone programming (MISOCP) mathematical model defines the switches to be operated for optimal restoration in balanced radial EDSs in scenarios of permanent faults. Consumer units and substations are represented by buses, and lines with or without switching devices are represented by branches. Only branches with an installed switch have a variable to represent their open/closed status, which means that the other branches have no variable status. Thus, sections delimited by switchable branches are represented by buses and closed branches without switches. This representation does not simplify the structure of the EDS and eliminates the need to identify the load sections individually. Areas directly affected by a permanent fault and switches indicated to isolate these areas are left out of the problem, representing that the directly affected areas remain insulated for repair and the other de-energized areas can be restored if there is enough capacity reserve in the supporting feeders, as well as switches to reallocate loads. The problem can be solved in scenarios of a single fault or multiple faults, requiring only the previous exclusion of the elements related to the areas to be isolated.

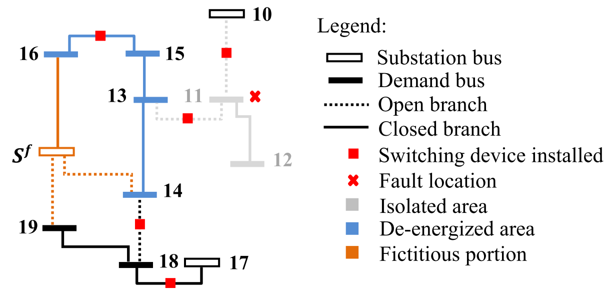

FIGURE 1. Example of connection of the fictitious substation.

A linear objective function minimizes the DNS with a minimum number of switching operations, complying with voltage and current limits, substation capacity, and radiality, which are fundamental constraints to obtain reliable radial configurations. Kirchhoff's two laws are modeled in the set of constraints. The set of constraints is based on a relaxed convex version [26] of the non-linear branch flow equations [31], on which the mathematical model in [22] is also based. The assessment of the objective function and the operating conditions is performed through these constraints. The model allows considering both the influence of priority loads and the priority operation of remotely controlled switches. Costs associated with the importance of the loads and with the switching operations are represented in the objective function by parameters to be chosen according to the characteristics of the EDS and the restoration objectives.

When some primary feeder is under fault, reallocating de-energized loads among other feeders can overload the system. In this context, the load shedding can be imperative to promote a safe operation. For this reason, the restoration problem is solved with the connectivity constraint relaxed and the strategy of using a fictitious substation and fictitious branches [22] was adopted to complement this constraint with regards to the number of branches to be closed to get a radial configuration. The fictitious substation connects de-energized sections through fictitious branches, keeping a radial operation. A single fictitious branch is closed between the fictitious substation and each de-energized parcel composed of one or more sections. This concept is shown in Figure 1, where the fictitious substation S^f connects two de-energized sections by closing only one fictitious branch: one section is formed by bus 16 and the second section is formed by buses 13, 14, and 15. Note that the switch in branch 15–16 links both de-energized sections and that it did not need to be opened.

III. MATHEMATICAL MODEL

The mathematical model is presented after the definition of its sets, parameters, and variables. The model consists of an explicit mathematical formulation that allows it to be solved by exact optimization techniques to provide optimal solutions to the problem. The new approach is highlighted in this section in contrast to the model in [22].

A. DEFINITION OF SETS, PARAMETERS, AND VARIABLES

The sets that represent the structure of the EDS available for restoration are Ω_b and Ω_l. The set Ω_b represents the buses of the EDS and the fictitious substation bus. The substation buses, including the fictitious one, are also represented by the set $\Omega_s \subset \Omega_b$. Thus, $|\Omega_b|$ is the total number of buses, and $|\Omega_s|$ is the total number of substation buses. The set Ω_l represents the branches of the EDS and is divided into two subsets, Ω_c and Ω_o. The subset Ω_c is composed of closed branches without switches, for which there are no decision variables representing their operating status; and Ω_o is composed of open or closed branches with switches that can be operated for restoration, whose operating status is represented by a binary variable. All branches with initially closed switches in Ω_o are candidates for opening and are represented by its subset $\Omega_o^{\uparrow}$. All branches with initially open switches in Ω_o are candidates for closure and are represented by its subset $\Omega_o^{\downarrow}$. In summary, the sets of branches are defined as follows: $\Omega_l = \Omega_c \cup \Omega_o$ and $\Omega_o = \Omega_o^{\uparrow} \cup \Omega_o^{\downarrow}$. The sections do not need to be identified, as the sets Ω_b and Ω_l define them completely in the model. The new approach is essentially related to the sets Ω_c and Ω_o, since in [22] all branches in Ω_l have binary decision variables representing their operating status.

The set Ω_h represents the fictitious branches with initially open switches to be closed exclusively to connect de-energized sections to the fictitious substation. For this connection, it is sufficient to have a fictitious branch between the fictitious substation and a bus in each section. So, the number of fictitious branches is equivalent to the number of sections. In [22], there is a fictitious branch for all buses, making Ω_h much larger than in the new proposal.

The binary variables are $y_i\ \forall i \in \Omega_b$ and $x_{i,j}\ \forall i,j \in \Omega_o \cup \Omega_h$. The variable y_i indicates if a bus is energized or de-energized; 0 indicates an energized bus, and 1 indicates a de-energized bus. The variable $x_{i,j}$ indicates if the switch in the branch i,j is open or closed; 0 indicates an open switch, and 1 indicates a closed switch. Thus, for $i,j \in \Omega_o^{\downarrow}$ (candidate for closure), $x_{i,j} = 0$ indicates that the switch in the branch i,j remains open in the solution found, and $x_{i,j} = 1$ indicates that this switch was closed; for $i,j \in \Omega_o^{\uparrow}$ (candidate for opening), $x_{i,j} = 1$ indicates that the switch in the branch i,j remains closed in the solution found, and $x_{i,j} = 0$ indicates that this switch was opened; finally, for $i,j \in \Omega_h$ (candidate for closure), $x_{i,j} = 0$ indicates that the switch in the fictitious branch i,j remains open in the solution found, and $x_{i,j} = 1$ indicates that this switch was closed. In [22], y_i is also defined $\forall\ i \in \Omega_b$, but $x_{i,j}$ is defined $\forall\ i,j \in \Omega_l \cup \Omega_h$, which makes the problem more complex due to the greater number of candidate solutions.

The variables that represent the operational conditions of the EDS have the same definition in [22], since they need to be determined for all buses in Ω_b or all branches in Ω_l. The difference in the new proposal is how to limit them. $P_{i,j}$, $Q_{i,j}$, and $I_{i,j}^{sqr}\ \forall\ i,j \in \Omega_l$ are, respectively, the active and reactive power flows, and the square of the current magnitude in the branch i,j. $H_{i,j}\ \forall\ i,j \in \Omega_l \cup \Omega_h$ is the artificial power flow in the branch i,j. $V_i^{sqr}\ \forall\ i \in \Omega_b$ is the square of the voltage magnitude in bus i. P_i^G, Q_i^G, and $H_i^G\ \forall\ i \in \Omega_b$ are, respectively, the active, reactive, and artificial power generation in bus i. Lastly, $b_{i,j}\ \forall\ i,j \in \Omega_l$ is used to prevent Kirchhoff's second law from being fulfilled if the branch i,j is open or de-energized.

The parameters represent mainly data of the EDS and operating limits and have the same definition in [22], as they cover the whole EDS. They are defined as follows: P_i^D, Q_i^D, $\bar{S}_i^G$, and $\alpha_i\ \forall\ i \in \Omega_b$; $R_{i,j}$, $X_{i,j}$, $Z_{i,j}$, and $\bar{I}_{i,j}\ \forall\ i,j \in \Omega_l$; $\beta_{i,j}\ \forall\ i,j \in \Omega_o^{\downarrow}$; $\mu_{i,j}\ \forall\ i,j \in \Omega_o^{\uparrow}$; $\underline{V}$, $\bar{V}$, M, and S^f. P_i^D, and Q_i^D are the active and reactive power demands in bus i, respectively. $\bar{S}_i^G$ is the maximum generation of apparent power in substation bus i. α_i is the cost of not supplying the bus i per kW; $R_{i,j}$, $X_{i,j}$, $Z_{i,j}$, and $\bar{I}_{i,j}$ are the resistance, reactance, impedance, and maximum current capacity of the branch i,j, respectively. $\beta_{i,j}$ is the cost of closing the switch in the branch i,j. $\mu_{i,j}$ is the cost of opening the switch in the branch i,j. M is the maximum artificial power flow in the de-energized area, to be chosen as $M \geq |\Omega_b| - |\Omega_s|$. $\underline{V}$ and $\bar{V}$ are the minimum and maximum voltage magnitude, respectively. Lastly, S^f represents the fictitious substation bus.

B. NEW MISOCP MATHEMATICAL MODEL

The MISOCP model proposed as an enhanced approach of [22] is defined by the objective function (1) and the set of constraints (2)–(27).

The objective function (1) has three terms and comprises two optimization criteria considered to be non-conflicting. The most important objective, represented by the first term, is to minimize the load shedding on the restored topology. The last two terms represent the secondary objective, which is to minimize the number of switching operations. Although secondary, it is a significant objective, as the switching operations reflect costs and time to reconfigure the EDS. Parameters α_i, $\beta_{i,j}$, and $\mu_{i,j}$ must preserve the hierarchy between these two criteria and can be used to distinguish the priority loads and the most advantageous switches for operation, which allows considering the priority operation of remotely controlled switches. In this context, α_i is the cost of not serving the bus i, and $\beta_{i,j}$ or $\mu_{i,j}$ are the cost of operating the switch in the branch i,j. If the buses have the same importance, all parameters α_i must be set to the same value $\alpha > 0$; otherwise, α_i must reflect the importance of bus i for the service according to the characteristics of the EDS. If the objective is only to minimize the number of operated switches, all parameters $\beta_{i,j}$ and $\mu_{i,j}$ must be set to 1, but if objective is to minimize the operating costs, they must be set according to the characteristics of the EDS.

$$Min\ v = \sum_{i \in \Omega_b} \alpha_i P_i^D y_i + \sum_{i,j \in \Omega_o^{\downarrow}} \beta_{i,j}\, x_{i,j} + \sum_{i,j \in \Omega_o^{\uparrow}} \mu_{i,j} \left(1 - x_{i,j}\right) \tag{1}$$

It is important to note that the objective function has two objectives clearly distinct: one related to the load shedding and the other related to the switching operations. Thus, the first diagnosis is that the objective function (1) cannot be considered as a multi-objective function, since these parcels are not conflicting, but complementary. It should be noted mainly that these parcels are hierarchical, i.e., one criterion is more important than the other. This same phenomenon occurs when a linear programming problem is solved using the big M method of the simplex method, where the objective function has also two parcels, one formed by the original variables and the other by artificial variables. In the big M method, the fundamental idea is to minimize the parcel corresponding to the artificial variables first, discarding all these variables if possible, which means removing them from the base. Eventually, if the problem is infeasible, some artificial variables remain at the base, those with the lowest M values. The objective function (1) is formulated with this fundamental idea, whereby it tries preferably to eliminate the load shedding and, in a secondary way, to decrease the number of switching operations. Thus, the values chosen for the parameters α_i, $\beta_{i,j}$, and $\mu_{i,j}$ must respect the hierarchical logic, and their units must be chosen so that the units of each parcel of (1) are the same.

Constraints (2)–(4) impose Kirchhoff's first law for each bus $i \in \Omega_b$. Active, reactive, and fictitious power balances are satisfied according to y_i. If $y_i = 0$, the active and reactive demands are supplied by a real substation through P_i^G and Q_i^G in (2) and (3), and there is no artificial demand in (4). If $y_i = 1$, the active and reactive demands are supplied by the fictitious generator $\left(P_i^D + Q_i^D\right) y_i$ in (2) and (3), and the artificial demand is supplied by the fictitious substation through H_{Sf}^G in (4). Constraints (5)–(7) complement (2)–(4) regarding that supply, as they allow the active/reactive power generation only in real substation buses and the artificial power generation only in the fictitious substation bus.

$$\sum_{j,i \in \Omega_l} P_{j,i} - \sum_{i,j \in \Omega_l} \left(P_{i,j} + R_{i,j} I_{i,j}^{sqr}\right) + P_i^G = P_i^D (1 - y_i) \qquad \forall\, i \in \Omega_b \qquad (2)$$

$$\sum_{j,i \in \Omega_l} Q_{j,i} - \sum_{i,j \in \Omega_l} \left(Q_{i,j} + X_{i,j} I_{i,j}^{sqr}\right) + Q_i^G = Q_i^D (1 - y_i) \qquad \forall\, i \in \Omega_b \qquad (3)$$

$$\sum_{j,i \in \Omega_l \cup \Omega_h} H_{j,i} - \sum_{i,j \in \Omega_l \cup \Omega_h} H_{i,j} + H_i^G = y_i \qquad \forall\, i \in \Omega_b \qquad (4)$$

$$P_i^G = 0 \qquad \forall\, i \in (\Omega_b - \Omega_s) \cup S^f \qquad (5)$$

$$Q_i^G = 0 \qquad \forall\, i \in (\Omega_b - \Omega_s) \cup S^f \qquad (6)$$

$$H_i^G = 0 \qquad \forall\, i \in \Omega_b, i \neq S^f \qquad (7)$$

A radial operation with the EDS fully connected is defined with $|\Omega_b| - |\Omega_s|$ closed branches. The connectivity is ensured by constraints (2)–(7), as there must be a path between each load bus and a substation bus to supply its demand. The radial operation is ensured by the constraint (8), which defines the number of branches to be closed as a necessary condition for radiality. The branches in Ω_c are permanently closed, so $|\Omega_b| - |\Omega_s| - |\Omega_c|$ is the number of branches to be additionally closed in $\Omega_o \cup \Omega_h$. Thus, the solutions are connected and radial due to the set of constraints (2)–(8), as proven in [28], with the aid of the fictitious components. Constraints (8) differs from [22] regarding which variables to choose for closing, since $x_{i,j}$ is not defined for Ω_c.

$$\sum_{i,j \in \Omega_o \cup \Omega_h} x_{i,j} = |\Omega_b| - |\Omega_s| - |\Omega_c| \qquad (8)$$

Constraint (9) imposes Kirchhoff's second law for each loop formed by a branch $i, j \in \Omega_l$ when $b_{i,j} = 0$. This condition is fulfilled when the branches are energized. If the branch is de-energized or open, $b_{i,j} \neq 0$ in (9), relaxing the fulfillment of this law. The value that $b_{i,j}$ must assume is defined in (10) and (11). In (10), y_i identifies in which area the branch $i, j \in \Omega_c$ is closed. In (11), $x_{i,j}$ identifies if the branch $i, j \in \Omega_o$ is open or closed, and the area in which the branch is when it is closed is identified by y_i and y_j. The buses at the ends of a closed branch are both energized or both de-energized, whereas an open branch can be additionally between one energized bus and another de-energized bus. Thus, $y_i = y_j = 0$ or $y_i = y_j = 1$ if the branch i, j is closed, and y_i and y_j can be indefinitely 0 or 1 if the branch i, j is open.

$$V_i^{sqr} - V_j^{sqr} = 2\left(R_{i,j} P_{i,j} + X_{i,j} Q_{i,j}\right) + Z_{i,j}^2 I_{i,j}^{sqr} + b_{i,j} \qquad \forall\, i,j \in \Omega_l \qquad (9)$$

$$\left|b_{i,j}\right| \le \left(\bar{V}^2 - \underline{V}^2\right) y_i \qquad \forall\, i,j \in \Omega_c \qquad (10)$$

$$\left|b_{i,j}\right| \le \left(\bar{V}^2 - \underline{V}^2\right) \left(1 - x_{i,j} + y_i + y_j\right) \qquad \forall\, i,j \in \Omega_o \qquad (11)$$

Constraint (12) complements (9) and is a relaxed version of the non-linear relation among power flows, voltage, and current, which is originally an equality relation. The inequality '≥' turns the non-linear problem into a second-order conic problem. Although the two problems define different feasible regions, the optimal solution of the conic problem is also the optimal solution of the non-linear problem if (12) is active in the solution found, as proven in [29]. When (12) is not active in the solution found, it is important to verify if the error in the approximation is large.

$$V_j^{sqr} I_{i,j}^{sqr} \ge P_{i,j}^2 + Q_{i,j}^2 \qquad \forall\, i,j \in \Omega_l \qquad (12)$$

Constraints (13)–(16) impose operating limits for the operational feasibility of the EDS in terms of security and quality of the service. Constraint (13) limits the apparent power generation in substation buses, (14) keeps the voltage magnitude in energized buses within the lower and upper limits allowed, and constraints (15) and (16) prevent overload in energized branches. Constraints (17) and (18) limit the artificial power flow in the de-energized area.

$$\left(P_i^G\right)^2 + \left(Q_i^G\right)^2 \le \left(\bar{S}_i^G\right)^2 \qquad \forall\, i \in \Omega_s, i \neq S^f \qquad (13)$$

$$\underline{V}^2 (1 - y_i) \le V_i^{sqr} \le \bar{V}^2 (1 - y_i) \qquad \forall\, i \in \Omega_b \qquad (14)$$

$$0 \le I_{i,j}^{sqr} \le \bar{I}_{i,j}^2(1 - y_i) \qquad \forall\, i,j \in \Omega_c \qquad (15)$$
$$0 \le I_{i,j}^{sqr} \le \bar{I}_{i,j}^2 x_{i,j} \qquad \forall\, i,j \in \Omega_o \qquad (16)$$
$$|H_{i,j}| \le M y_i \qquad \forall\, i,j \in \Omega_c \qquad (17)$$
$$|H_{i,j}| \le M x_{i,j} \qquad \forall\, i,j \in \Omega_o \cup \Omega_h \qquad (18)$$

Constraints (19)–(24) reduce the feasible region without eliminating the optimal solution and speed up the solution of the model, providing good lower limits for the relaxed problems needed for the solution of the integer programming problem. Constraints (19)–(22) limit the power flows to $\bar{V}\bar{I}_{i,j}$ or 0, whereas (23) and (24) prevent a closed branch from connecting an energized bus and a de-energized bus simultaneously. Constraint (25) sets $y_{S^f} = 1$ to keep S^f in the de-energized area.

$$|P_{i,j}| \le \bar{V}\bar{I}_{i,j}(1 - y_i) \qquad \forall\, i,j \in \Omega_c \qquad (19)$$
$$|Q_{i,j}| \le \bar{V}\bar{I}_{i,j}(1 - y_i) \qquad \forall\, i,j \in \Omega_c \qquad (20)$$
$$|P_{i,j}| \le \bar{V}\bar{I}_{i,j}\, x_{i,j} \qquad \forall\, i,j \in \Omega_o \qquad (21)$$
$$|Q_{i,j}| \le \bar{V}\bar{I}_{i,j}\, x_{i,j} \qquad \forall\, i,j \in \Omega_o \qquad (22)$$
$$|y_i - y_j| \le (1 - x_{i,j}) \qquad \forall\, i,j \in \Omega_o \cup \Omega_h \qquad (23)$$
$$y_i = y_j \qquad \forall\, i,j \in \Omega_c \qquad (24)$$
$$y_{S^f} = 1 \qquad (25)$$

Constraints (10), (11), and (15)–(24) differ from [22] regarding how to identify open or de-energized branches. In the new approach, where there is no variable $x_{i,j}$ for the branches in Ω_c, y_i identifies if these branches are energized or de-energized. Finally, constraints (26) and (27) define $x_{i,j}$ and y_i as binary variables.

$$x_{i,j} \in \{0, 1\} \qquad \forall\, i,j \in \Omega_o \cup \Omega_h \qquad (26)$$
$$y_i \in \{0, 1\} \qquad \forall\, i,j \in \Omega_b \qquad (27)$$

The model provides the optimal configuration, defining the final status of all switches, the energization status of all buses, and the operational conditions of the EDS in terms of current, voltage, and power generation and flow, ensuring topologies connected, radial and operationally feasible. The switching operations are identified on the optimal solution by comparing the initial and final status of each switch initially candidate to be operated for restoration.

IV. TESTS AND RESULTS

The model (1)–(27) was used to define optimal post-fault configurations on the 417-bus EDS [32], adapted in this paper to have switches in 26% of the branches. The model was programmed in AMPL [33] and solved with CPLEX [34] using a computer with an Intel i7-4770 processor. The stopping criterion was a maximum gap of 0% to guarantee optimal solutions.

A. TEST SYSTEM

The 417-bus test system has 3 substation buses, 13 feeders, 473 branches and, under normal conditions, feeds 414 buses with the radial base topology shown in Figure 2. It was adapted to have 125 switches, 66 normally closed and 59 normally open, identified in Figure 2. Tie switches were allocated in all normally open branches on base topology and influence the allocation of the sectionalizing switches due to radiality. Information about the 66 sections formed with this allocation is shown in Table 1, and the fault scenarios in each section are described in Table 2.

Data concerning the maximum current capacity were generated in [35], and the following additional changes have been made in this paper in relation to [35] for greater operational consistency, up to the feasibility limits: the demands in buses 12, 81, 90, 109, 125, 159, 168, 170, 175, 178, 195, 305, 307, 310, 370, 380, and 383 were changed for $5+j3$ kVA (17 buses, in total), and the demands in buses 1, 3, 4, 8, 15, 16, 20, 27, 32, 37, 38, 43, 49, 53, 59, 75, 91, 116, 117, 118, 119, 122, 123, 124, 126, 127, 135, 157, 167, 186, 188, 189, 196, 197, 198, 199, 217, 233, 238, 242, 258, 261, 289, 344, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 371, 372, 373, 374, 375, 378, 379, 381, 382, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, and 413 were reset to 0 (88 buses, in total), as they are originally pass buses in [32] that had a demand slightly greater than 0 in the database in [35].

With the new demands, the 417-bus EDS normally operates supplying 27440 kW and 13285 kVAr, the lowest voltage magnitude observed is 0.93 p.u. in bus 30, and the active losses on the base topology are 709.39 kW. The nominal voltage is 10.0 kV, and the lower and upper limits of voltage magnitude assumed correspond to 0.90 and 1.0 p.u., respectively. Complete original data are in [35] and at http://www.feis.unesp.br/#!/lapsee (Downloads section).

B. ANALYZED FAULT SCENARIOS

Single permanent faults on the 417-bus EDS were analyzed in sections 263, 83, 1, 227, 207, 250, 214, and 283, located mainly at the beginning of primary feeders, near substations, representing very critical scenarios, as shown in Table 2. For instance, section 263, located at feeder 02 and delimited by switches 416–273, 262–263, and 263–264, consists of buses 273, 274, 275, and 263, demands 400 kW and 193 kVAr, and is supplied by the substation 416. A fault in section 263, which keeps isolated without supply throughout the restorative state, also interrupts supply to the downstream sections 260, 264, 342, 128, 134, 136, 159, 129, and 133, totalizing 49 buses and a recoverable DNS equal to 3289 kW and 1590 kVAr to be restored. Results of these tests are shown in Table 3 and were obtained without considering priority loads or priority switches, using the following values for the parameters of the objective function (1): all parameters α_i, representing costs per kW, were set to 0.1, and all parameters $\beta_{i,j}$ and $\mu_{i,j}$, representing costs per switching operation, were set to 1.

In 7 cases (87.5%), there was no load shedding, i.e., all recoverable DNS, which is the sum of loads downstream of the isolated section, was restored. The loads were restored with up to 4 switching operations in 6 cases (75%).

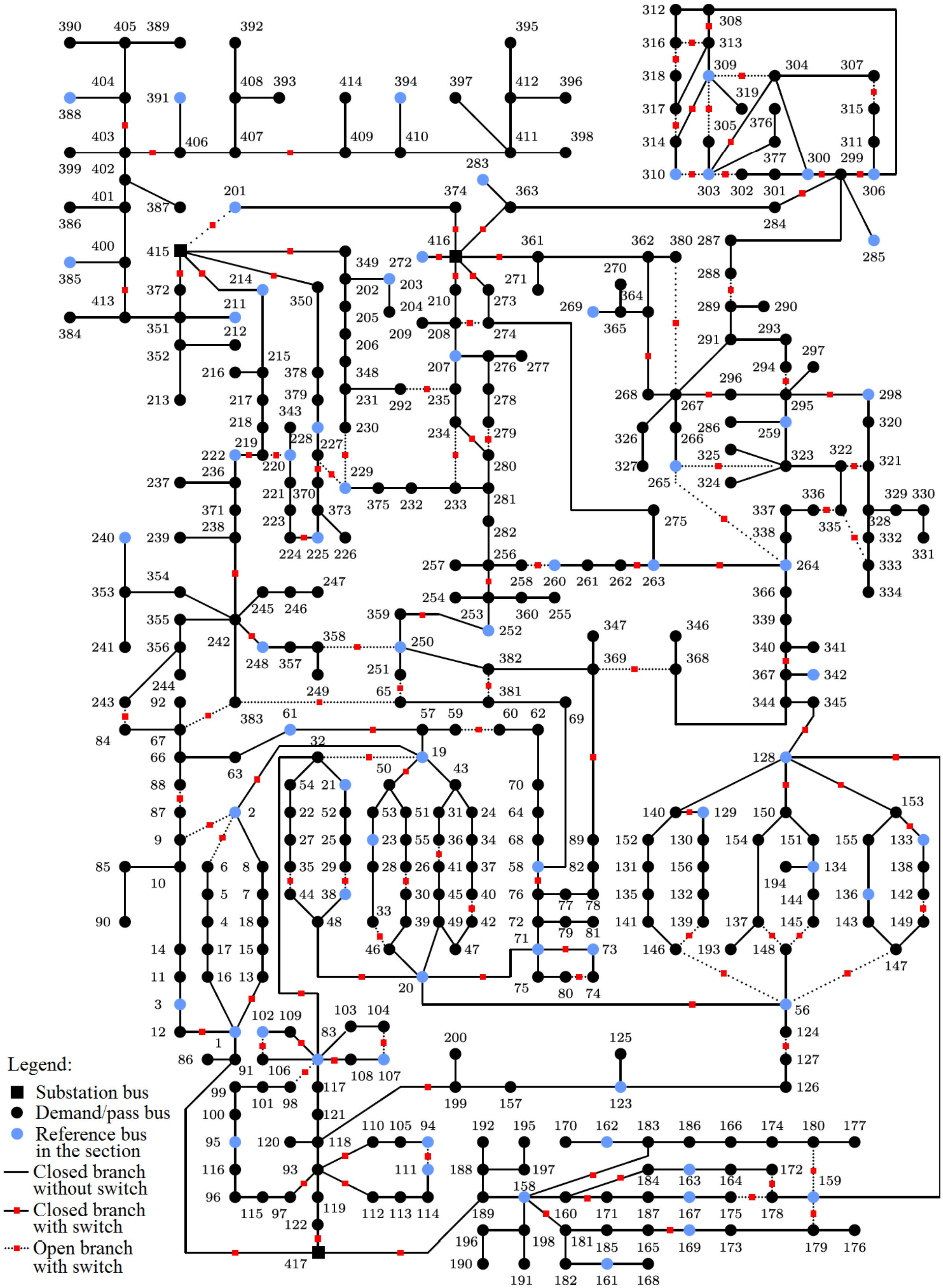

FIGURE 2. Base topology of the radial 417-bus EDS adapted from [32].

TABLE 1. Information about the 66 sections on the 417-bus EDS.

Feeder♦	Section*	Switches located on the border*	Buses present in section	Demand (kW)	Demand (kVAr)
■ 01	**272**	416-272	272	56.0	27.0
■ 02	263	416-273, 262-263, 263-264	273, 274, 275, 263	400.0	193.0
	260	262-263, **260-258**	262, 261, 260	69.0	34.0
	264	263-264, 340-367, **264-265**, **335-336**	264, 338, 337, 336, 366, 339, 340, 341	542.0	262.0
	342	340-367, 128-345, **368-369**	367, 342, 344, 345, 368, 346	64.0	30.0
	128	128-345, 159-128, 128-153, 128-150, 140-129, **139-146**, **56-146**	128, 140, 152, 131, 135, 141, 146	488.0	236.0
	134	128-150, **148-137**, **145-148**	150, 154, 137, 193, 151, 134, 194, 144, 145	660.0	319.0
	136	128-153, 153-133, **147-56**, **142-149**	153, 155, 136, 143, 147, 149	483.0	233.0
	159	159-128, **159-179**, **159-180**, **172-178**, **175-178**	159, 178	5.0	3.0
	129	140-129, **139-146**	129, 130, 156, 132, 139	463.0	224.0
	133	153-133, **142-149**	133, 138, 142	515.0	249.0
■ 03	83	122-417, 93-112, 93-110, 97-93, 199-118, 83-32, 83-108, 109-83, **83-98**, **104-107**, **106-102**	122, 119, 93, 118, 120, 121, 117, 83, 103, 104, 106	673.0	325.0
	94	93-110, **94-111**	110, 105, 94	321.0	155.0
	111	93-112, **94-111**	112, 113, 114, 111	309.0	149.0
	95	97-93, **83-98**	97, 115, 96, 116, 95, 100, 99, 101, 98	512.0	248.0
	123	199-118, **124-127**	199, 200, 157, 123, 125, 126, 127	57.0	28.0
	21	83-32, **19-32**, **35-44**, **38-29**	32, 21, 52, 25, 29, 54, 22, 27, 35,	922.0	446.0
	107	83-108, **104-107**	108, 107	60.0	28.0
	102	109-83, **106-102**	109, 102	76.0	37.0
■ 04	1	417-91, 1-12, 1-13, **6-2**	91, 86, 1, 16, 17, 4, 5, 6	263.0	127.0
	2	1-13, 2-19, **6-2**, **2-9**	13, 15, 18, 7, 8, 2	303.0	146.0
	3	1-12, **2-9**, **87-88**	12, 3, 11, 14, 10, 9, 87, 85, 90	526.0	255.0
	19	2-19, 19-50, 57-61, **19-32**, **40-42**, **41-36**, **60-59**	19, 43, 24, 34, 37, 40, 31, 36, 57, 59	637.0	307.0
	23	19-50, **26-30**, **33-46**	50, 51, 55, 26, 53, 23, 28, 33	745.0	360.0
	61	57-61, **87-88**, **84-243**, **383-67**	61, 63, 66, 88, 67, 84, 92	422.0	203.0
■ 05	227	415-350, 227-370, **229-227**	350, 378, 379, 228, 227	308.0	149.0
	225	227-370, 225-224	370, 373, 225, 226	225.0	110.0
	220	225-224, **220-219**	224, 223, 221, 220, 343	580.0	281.0
■ 06	**202**	415-349, **229-230**, **292-235**	349, 202, 203, 204, 205, 206, 348, 231, 230, 292	1,015.0	492.0
■ 07	**201**	416-374, **201-415**	374, 201	202.0	98.0
■ 08	207	210-416, 280-234, **274-208**, **292-235**, **234-233**, **279-280**	210, 208, 209, 207, 235, 234, 276, 277, 278, 279	1,809.0	877.0
	229	280-234, 253-256, **279-280**, **234-233**, **229-227**, **229-230**, **258-260**	280, 281, 233, 232, 375, 229, 282, 256, 257, 258	1,110.0	538.0
	252	253-256, 359-252	253, 252, 254, 360, 255	175.0	86.0
	250	359-252, 89-369, **358-250**, **65-251**, **381-382**, **368-369**	359, 250, 251, 382, 369, 347	41.0	20.0
	71	89-369, 76-58, 71-73, 20-71, **74-80**	89, 82, 78, 77, 76, 72, 79, 81, 71, 75, 80	794.0	385.0
	58	76-58, **60-59**, **381-382**, **65-251**, **65-383**	58, 68, 64, 70, 62, 60, 69, 381, 65	728.0	351.0
	73	71-73, **74-80**	73, 74	168.0	82.0
	20	20-71, 20-48, 20-56, **33-46**, **26-30**, **40-42**, **41-36**	20, 46, 39, 30, 49, 47, 42, 45, 41	631.0	306.0
	38	20-48, **35-44**, **38-29**	48, 44, 38	199.0	96.0
	56	20-56, **56-146**, **147-56**, **148-137**, **145-148**, **124-127**	56, 124, 148	63.0	30.0
■ 09	214	415-214, 219-222, **220-219**	214, 215, 216, 217, 218, 219	434.0	210.0
	222	219-222, 238-242	222, 236, 237, 371, 238, 239	398.0	193.0
	240	238-242, 242-248, **84-243**, **65-383**, **383-67**	242, 355, 356, 243, 244, 383, 354, 353, 240, 241, 245, 246, 247	256.0	125.0
	248	242-248, **358-250**	248, 357, 358, 249	149.0	72.0
■ 10	158	417-189, 158-181, 158-183, 160-171, 160-184	189, 188, 192, 197, 195, 158, 160, 198, 191, 196, 190	233.0	113.0
	161	158-181, 165-169	181, 182, 161, 168, 185, 165	493.0	240.0
	162	158-183, **159-180**	183, 162, 170, 186, 166, 174, 180, 177	581.0	283.0
	163	160-184, **172-178**	184, 163, 164, 172	333.0	162.0
	167	160-171, **175-178**	171, 187, 167, 175	199.0	97.0
	169	165-169, **159-179**	169, 173, 179, 176	399.0	193.0
■ 11	211	415-372, 413-400	372, 351, 211, 352, 212, 213, 413, 384	121.0	59.0
	385	413-400, 403-404, 403-406	400, 385, 401, 386, 402, 387, 403, 399	124.0	60.0
	388	403-404	404, 388, 405, 389, 390	179.0	87.0
	391	403-406, 407-409	406, 391, 407, 408, 392, 393	58.0	28.0
	394	407-409	409, 414, 410, 394, 411, 397, 398, 412, 395, 396	128.0	62.0
■ 12	269	361-416, 268-364, **380-267**	361, 271, 362, 380, 364, 365, 269, 270	102.0	50.0
	265	268-364, 267-296, **380-267**, **323-265**, **264-265**, **295-294**, **289-288**	268, 267, 266, 265, 326, 327, 291, 289, 290, 293, 294	1,446.0	700.0
	259	267-296, 298-295, **323-265**, **322-321**, **333-335**, **335-336**, **295-294**	296, 295, 297, 259, 286, 323, 324, 325, 322, 335	1,376.0	667.0
	298	298-295, **322-321**, **333-335**	298, 320, 321, 328, 329, 330, 331, 332, 333, 334	930.0	450.0
■ 13	283	416-363, 299-284	363, 283, 284	35.0	16.0
	285	299-284, 299-300, 306-299, **289-288**	299, 285, 287, 288	387.0	186.0
	300	299-300, 303-304, **302-303**, **309-304**, **307-315**	300, 301, 302, 304, 307	715.0	347.0
	306	306-299, 313-308, **307-315**, **313-316**, **316-318**	306, 311, 315, 308, 312, 316	266.0	128.0
	303	303-304, **302-303**, **310-303**, **305-309**	303, 305, 377, 376	191.0	92.0
	309	313-308, 309-314, **313-316**, **309-304**, **305-309**, **317-314**, **316-318**	313, 309, 319, 317, 318	250.0	121.0
	310	309-314, **317-314**, **310-303**	314, 310	38.0	19.0

* Terminal sections and open switches on the base topology are shown in bold; ♦ Feeder connected to the substation: ■ 415 ■ 416 ■ 417

TABLE 2. Information about fault scenarios in each section on the 417-bus EDS.

Isolated section*	Downstream de-energized sections	Recoverable DNS ♦ (kW)	(kVAr)	Isolated section*	Downstream de-energized sections	Recoverable DNS ♦ (kW)	(kVAr)
272	-	-	-	#250	71, 58, 73, 20, 38, 56	2,583.0	1,250.0
#263	260, 264, 342, 128, 134, 136, 159, 129, 133	3,289.0	1,590.0	71	58, 73, 20, 38, 56	1,789.0	865.0
260	-	-	-	58	-	-	-
264	342, 128, 134, 136, 159, 129, 133	2,678.0	1,294.0	73	-	-	-
342	128, 134, 136, 159, 129, 133	2,614.0	1,264.0	20	38, 56	262.0	126.0
128	134, 136, 159, 129, 133	2,122.1	1,025.0	38	-	-	-
134	-	-	-	56	-	-	-
136	133	515.0	249.0	#214	222, 240, 248	803.0	390.0
159	-	-	-	222	240, 248	405.0	197.0
129	-	-	-	240	248	149.0	72.0
133	-	-	-	248	-	-	-
#83	94, 111, 95, 123, 21, 107, 102	2,257.0	1,091.0	158	161, 162, 163, 167, 169	2,005.0	975.0
94	-	-	-	161	169	399.0	193.0
111	-	-	-	162	-	-	-
95	-	-	-	163	-	-	-
123	-	-	-	167	-	-	-
21	-	-	-	169	-	-	-
107	-	-	-	211	385, 388, 391, 394	489.0	237.0
102	-	-	-	385	388, 391, 394	365.0	177.0
#1	2, 3, 19, 23, 61	2,633.0	1,271.0	388	-	-	-
2	19, 23, 61	1,804.0	870.0	391	394	128.0	62.0
3	-	-	-	394	-	-	-
19	23, 61	1,167.0	563.0	269	265, 259, 298	3,752.0	1,817.0
23	-	-	-	265	259, 298	2,306.0	1,117.0
61	-	-	-	259	298	930.0	450.0
#227	225, 220	805.0	391.0	298	-	-	-
225	220	580.0	281.0	#283	285, 300, 306, 303, 309, 310	1,847.0	893.0
220	-	-	-	285	300, 306, 303, 309, 310	1,460.0	707.0
202	-	-	-	300	303	191.0	92.0
201	-	-	-	306	309, 310	288.0	140.0
#207	229, 252, 250, 71, 58, 73, 20, 38, 56	3,909.0	1,894.0	303	-	-	-
229	252, 250, 71, 58, 73, 20, 38, 56	2,799.0	1,356.0	309	310	38.0	19.0
252	250, 71, 58, 73, 20, 38, 56	2,624.0	1,270.0	310	-	-	-

* Fault scenarios with results presented in this paper are preceded by #. ♦ Sum of loads in downstream de-energized sections.

TABLE 3. Summary of the obtained results.

Isolated section	Switches to be open	Switches to be closed	Sections out of service after restoration	Load shedding (kW)	(kVAr)	Runtime (s)
263	128–345	258–260, 159–179, 335–336	-	0	0	163
83	76–58, 20–56	65–383, 35-44, 56-146, 124-127	95, 111, 94, 107, 102	1,278.0	617.0	162
1	2–19, 57–61, 20–71	33–46, 2–9, 87–88, 84–243, 124–127	-	0	0	369
227	-	220–219	-	0	0	19
207	89–369	124–127, 358–250	-	0	0	108
250	-	124–127	-	0	0	97
214	20–71	65–383, 124–127	-	0	0	46
283	-	289–288	-	0	0	144

In three of them, only 1 switch was indicated for operation, which means that the de-energized loads in each case were transferred to a single support feeder without overloading it.

The solution with load shedding corresponds to the fault in section 83 at feeder 03. In the proposed solution, sections 123 and 21, which sum 979 kW and 474 kVAr, were restored with 6 switching operations, and sections 95, 111, 94, 107, and 102, which sum 1278 kW and 617 kVAr, were not restored because there are no tie switches that could connect them to other feeders or to energized sections on feeder 03. The restored sections were transferred to feeders 02 and 08 as follows: by opening the sectionalizing switch 76–58 on feeder 08 and closing the tie switch 65–383 between feeders 08 and 09, section 58 was transferred from feeder 08 to feeder 09, relieving the feeder 08 to receive section 21 from feeder 03, whose transfer was made closing the tie switch 35–44; by opening the sectionalizing switch 20–56 on feeder 08 and closing the tie switches 124–127 between feeders 03 and 08 and 56–146 between feeders 08 and 02, section 123 at feeder 03 was connected to section 56 at feeder 08 and both were transferred to feeder 02.

The fastest resolution took 19 seconds and corresponds to the fault in section 227 at feeder 05, where the de-energized sections 225 and 220 were transferred to feeder 09 by closing the tie switch 220–219. The other solutions with only one switch operated correspond to the faults in sections 250 at feeder 08 and 283 at feeder 13. In first case, sections 71, 58, 73, 20, 38, and 56 were transferred to feeder 03 by closing the tie switch 124–127. In second case, sections 285, 300, 306, 303, 309, and 310 were transferred to feeder 12 by closing the tie switch 289–288.

The second fault on feeder 08 was simulated in section 207, upstream of section 250. In this case, 3 switching operations were needed for restoring the de-energized loads, an additional 1326 kW and 644 kVAr compared to the fault in section 250, representing an average increase of 51.4%. In the proposed solution, the sectionalizing switch 89–369 on feeder 08 was opened, dividing the de-energized sections into two blocks, and the closure of the tie switch 124–127 transferred the same sections 71, 58, 73, 20, 38, and 56 to feeder 03, whereas the closure of the tie switch 358–250 transferred sections 229, 252, and 250 to feeder 09. This test shows the sensibility and robustness of the model to avoid overloading the system.

The solution for the fault scenario in section 214 at feeder 09 also required 3 switching operations, but it took less than half the runtime than the case previously analyzed, whose recoverable DNS is almost 5 times bigger. The opening of the sectionalizing switch 20–71 on feeder 08 followed by the closing of the tie switch 124–127 between feeders 08 and 03 transferred sections 20, 38, and 56 at feeder 08 to feeder 03, and the closing of the tie switch 65–383 between feeders 08 and 09 restored the sections 222, 240, and 248.

In the fault case in section 263 at feeder 02, whose solution had 4 switching operations, by opening the sectionalizing switch 345–128 on feeder 02, the de-energized sections were divided into two blocks; then section 260 was transferred to feeder 08 by closing the tie switch 258–260, sections 264 and 342 were transferred to feeder 12 by closing the tie switch 335–336, and sections 128, 134, 136, 159, 129, and 133 were transferred to feeder 10 by closing the tie switch 159–179. It should be noted that downstream of switch 345–128 there are 15 candidate switches, generating thousands of possible combinations to reconnect the transferred sections to feeder 10, but only switch 159–179 was operated. This shows the effect of minimizing the switching operations in the objective function.

The most onerous case corresponds to the fault scenario in section 1 at feeder 04, whose solution took approximately 6 minutes and required 8 switching operations. The de-energized loads were transferred to feeders 03 and 09 as follows: sections 2, 3, and 61 were transferred to feeder 09 by opening the sectionalizing switches 2–19 and 57–61 on feeder 04, closing the tie switches 2–9 between sections 2 and 3 and 87–88 between sections 3 and 61 on feeder 04, and closing the tie switch 84–243 between feeders 04 and 09; and sections 19 and 23 were transferred together with sections 20, 38, and 56 at feeder 08 to feeder 03 by closing the tie switch 33–46 between feeders 04 and 08, opening the sectionalizing switch 20–71 on feeder 08, and closing the tie switch 124–127 between feeders 08 and 03.

Although 6 minutes is an onerous time for service restoration in real time, the resolution time was less than 3 minutes in 87.5% of cases, which is acceptable, considering achieving an optimal configuration after very critical faults for a relatively large system and a very complex problem using exact techniques. In addition, faster computers can decrease the resolution time and make it more treatable. Another important aspect to be observed is that there is a preventive operational control in the distribution control centers that allows anticipating solutions to restoration problems based on information about the operating conditions of the system obtained by the management systems. In this context, the time to obtain preventive solutions is less urgent than solving the problem in real time, and this allows focusing on achieving optimal solutions.

All solutions were found with a gap of 0% and were confirmed being operationally feasible as defined in (13)–(16) and radial as defined in (8). The equality in (12) was satisfied in all solutions, which means that the constraint (12) is active in the found solutions and that they are optimal for both the model (1)–(27) and the equivalent non-linear model with (12) not relaxed.

V. CONCLUSION

A MISOCP mathematical model has been proposed to define optimal post-fault configurations in balanced radial EDSs. The model is based on [22] and consists of spot improvements related to the representation of the EDS. The new approach is important mainly by reducing the number of binary variables in the new model. The model is proposed to solve the problem without simplifying the topological structure of the EDS typically arranged in load sections. As a typical EDS has switches only at strategic locations for its operational optimization, several sections are formed on the EDS without switching devices allocated in their internal configurations. This characteristic favors the simplification of these sections as single elements to facilitate the problem resolution since the solution basically consists of deciding which switches must be operated for restoration, not including decisions regarding the internal structure of the sections. A classical representation in the literature is simplifying the topology of the EDS in load blocks and switchable lines [25], [30]. However, it is necessary to comply with operational requirements within the sections; thus the restoration methods must allow the assessment of the operational conditions of the EDS in full. In general, heuristic methods decide the switching operations and evaluate the operating conditions of the system separately, whose evaluation process is carried out after solving a power flow problem for solution proposals with the complete topological structure or by means of heuristics that avoid overloading the feeders. In mathematical models solved by exact techniques, the process of deciding the switches and evaluating the operation of the EDS is unique; thus, the complete topological structure is more accurate and realistic to allow a better assessment of the operational conditions. The new approach differs from [22] because the new model represents the complete topological structure of the EDS and the real allocation of the switches, disregarding the need for previous adaptations before solving the restoration problem. The improvements applied in contrast to [22] were highlighted in section that presents the proposed mathematical model.

The strategy presented in this work is especially relevant because the number of binary variables that control the energization state of the different elements within the sections is also significantly less than other proposals in the literature [25]. A problem with an expressive number of binary variables is a difficult problem to solve. Another aspect that can make the problem more complex is the way of representing the radiality constraint. The new model keeps the radiality constraint simple and does not require any additional variables to ensure radial configurations in addition to those that represent the status of the switches.

The new model was used to solve the restoration problem after very critical faults near substations in a relatively large EDS with the main aim of evaluating its ability to obtain optimal operationally reliable solutions in highly complex problems with low computational effort. The more important information about the complexity of the problem is related to the number of switches available for restoration, and the analyzed scenarios were simulated on a radial 417-bus EDS with 473 branches, adapted to have 125 switches. The size of the test system and the number of switches allocated make the problem very complex. Therefore, the system used and the tests chosen are adequate to show the satisfactory performance of the model to solve highly complex problems. Results show that the model is robust and can be effective in solving complex restoration problems in systems larger than the test EDS, mainly in an offline or preventive way. Solutions were obtained by exact techniques using commercial solvers and are optimal in terms of maximum supply and minimum number of switching operations through a hierarchical objective function. The longest resolution time was approximately 6 minutes and the shortest one was 19 seconds, and there was no load shedding in both cases. In addition, faster computers can make the runtimes even more treatable; however, as complexity increases, resolution times can become prohibitive, which is a characteristic common to all mathematical models that solve highly complex combinatorial problems through exact techniques. In cases of prohibitive application in real time, but not prohibitive for offline solutions, the model can be used to define preventive restoration proposals.

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