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A Dynamic Linguistic Decision Making Approach for a Cryptocurrency Investment Scenario

ROMINA TORRES[1], (Member, IEEE), MIGUEL A. SOLIS[1], RODRIGO SALAS[2], (Senior Member, IEEE), AND AURELIO F. BARIVIERA[3]

[1]Facultad de Ingeniería, Universidad Andres Bello, Viña del Mar-Santiago 7500971, Chile
[2]Escuela de Ingeniería C. Biomédica, Universidad de Valparaíso, Valparaíso 2362905, Chile
[3]Department of Business, Universitat Rovira i Virgili, 43204 Reus, Spain

Corresponding author: Romina Torres (romina.torres@unab.cl)

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ABSTRACT Cryptocurrencies have been receiving the sustained attention of investors since 2009. These new investment vehicles are digitally native, meaning that they are traded exclusively on 24/7 digital platforms. Consequently, they offer an excellent scenario to test the Efficient Market Hypothesis, by developing algorithm-based trading strategies. Such strategies aim to beat the market. It has been previously reported that daily returns do not exhibit long range dependence. However, daily volatility in major cryptocurrencies is highly persistent. Therefore, buy/hold/sell decision support systems could be able to capture such market inefficiency. This is especially important for investors interested in periodically trading a set of cryptocurrencies, in order to maximize their wealth. This paper presents a dynamic linguistic decision making approach for building decision models to support cryptocurrency investors in buy/hold/sell decisions. This approach exhibits a good computational performance for obtaining recommendations based on quantitative data. Moreover, this procedure is able to identify some inefficient cryptocurrency behaviors which are not captured by traditional econometric techniques. Our results uncover arbitrage opportunities that outperform buy-and-hold or random strategies.

INDEX TERMS Cryptocurrency, linguistic decision models, multi-period multi-attribute decision making.

I. INTRODUCTION

In 2009 a white paper authored under the pseudonym of Satoshi Nakamoto was published that set the basis for blockchain, a new paradigm in peer-to-peer transactions [1]. The foremost product that emerged from blockchain is bitcoin, and by extension, other cryptocurrencies. By using cryptocurrencies, users are able to exchange value digitally without third party oversight [2]. Their existence is possible thanks to the blockchain technology that consists of a distributed system that logs transaction records on linked blocks and store them on an encrypted digital ledger. The records in the blockchain system are spread across a network of replicated databases that are always synchronized. As of October 2020, there are more than 7000 cryptocurrencies, traded in 31000 online venues, with a total market capitalization of $391 billion, and daily transactions exceeding $94 billion [3]. Bitcoin is the biggest player in this market, accounting for 65% of the total market capitalization at the time of writing. Despite the large number of cryptocoins, most studies focus their attention on Bitcoin, rather than on the other coins [4]. Cryptocurrencies have become an investment vehicle for millions of people around the world. Not all of them are sophisticated investors. In fact there are many small investors spread around the globe.

To a great extent, economic literature is focused precisely on the study of the weak form of the Efficient Market Hypothesis (EMH). According to the EMH, a market is weakly informationally efficient if prices fully reflect the information contained in past prices [5]. Several papers found that the behavior of daily returns (and some power transformations of returns) are close to a random walk [6], [7], which is consistent with the EMH. However, the behavior of the long memory is not constant across time, challenging the EMH [8]. In addition, it was found that bitcoin exhibits highly persistent volatility [4], [9]. Thus, there could be predictable components in its time series. In spite of the relevance for market practitioners, there are very few empirical

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papers on cryptocurrency price forecasting. For instance, Bayesian Neural Networks have been used [10] to forecast bitcoin return and volatility. They included some macroeconomic data as explanatory variables, in addition to lagged price and volume figures. However, as several authors have shown [11], [12], cryptocurrencies are detached from the main traditional assets and real economy proxies. Recently, Atsalakis *et al.* [13] used a PATSOS neuro-fuzzy controller forecasting system, in order to predict price changes in four cryptocurrencies. In spite of their satisfactory results compared to the buy-and-hold strategy, two drawbacks in their methodology were detected. On one hand, they only compared their results to the buy-and-hold strategy, without considering a ''chance hit'', as a result of random selection. On the other hand, they forecasted each cryptocurrency independently. Indeed, many papers have analyzed the random walk properties of cryptocurrency prices. However, this is not the situation that investors typically face. They are usually interested in selecting the ''best'' set of cryptocurrencies, instead of having a buy or sell recommendation for only one. A full discussion of literature research lines can be found in two recent reviews by [14] and [15].

It has been found that the standard deviation of cryptocurrency returns is ten times greater than that of traditional assets (stocks and bonds) [12]. In addition, other authors have reported strong persistence in return volatility [4], [9]. Based on such results, our research question is whether there exists some model that could capture such inefficiency. In order to fill this gap, we propose a realistic decision making model, based only on cryptocurrencies' own variables. We model the investor as a person who wants to maximize his or her wealth by periodically trading a fixed set of cryptocurrencies. The investor has a predetermined set of cryptocurrencies, among which he or she must decide to buy or sell.

This paper is related to decision making under uncertain conditions over time, where some factors may influence decision makers when evaluating alternatives at different instants of time. To overcome this issue, Wang and Li [16] introduced the power Bonferroni mean in order to capture the interrelationships among input arguments and mitigate the influence of unreasonable aggregation values. Moreover, Wan *et al.* [17] has proposed a hybrid Shapley Choquet integral to capture the interactive characteristics of criteria. Torres *et al.* [18] have proposed a time-based hesitant fuzzy information aggregation operator to manage hesitancy due to changing environments.

In recent years, multiple attribute decision making (MADM) models [19] have been successfully and widely used to support decision making in multiple areas. A plethora of MADM problems can be found in various everyday areas, such as investment decision making [20], [21] or personnel evaluation [22], to only name a few. For instance, Torres *et al.* [18] expanded MADM to support runtime decisions for a group of hesitant decision makers with a data-driven approach, which was successfully used for ranking web service replacements during runtime [23]. Mao *et al.* [24] used probabilistic linguistic multi-attribute group decision making to select the appropriate financial technologies to cooperate for banks. Recently, Xu *et al.* [25] have successfully applied the probabilistic linguistic group decision making method for selecting a suitable car sharing platform.

The aim of this paper is to show the suitability of multi-period multi-attribute models (MP-MADM) [26] in order to convey useful information to advise a cryptocurrency investor or day trader.

The contribution of this work is twofold:

- On the one side, as far as we know, this is the first time that multi-period decision making models have been applied to the cryptocurrency market. Thus, our proposal advances human understanding of the model, because the MP-MADM is a linguistic decision model where each attribute is modeled as a linguistic variable with a set of linguistic values (such as ''high'' or ''low''). Moreover, investors may easily use our research to derive different linguistic decision models, each one using different strategies.
- Also, we find that there are arbitrage opportunities in this novel market.

This paper falls into the area that categorized as *tests for return predictability* [27]. Our model is able to identify some inefficient cryptocurrency behaviors not captured by traditional econometric techniques (e.g. autocorrelation or unit root analysis). The results uncover arbitrage opportunities that outperform buy-and-hold and random strategies. Therefore, we contribute to the literature by providing evidence of the partial inefficiency of this market. It is also important to note that our model is based solely on cryptocurrency data, and does not include macroeconomic data, as cryptocurrencies are detached from main traditional assets.

The rest of the paper is structured as follows. Section II introduces the Dynamic Linguistic Decision Making approach. Section III shows the results of applying our approach to an illustrative example and discuses the main findings of our study through several experiments. Finally, Section IV lays out the main conclusions.

II. DYNAMIC LINGUISTIC DECISION MAKING APPROACH

Multiple attribute decision-making consists of selecting the best alternative according to multiple relevant attributes. In many real situations, the problem consists of making decisions at regular intervals in time and, updating historical information [28]. Specifically, Xu [20] proposed the *Multi-Period Multi-Attribute Decision Making* approach (MP-MADM) where attribute weights and values are provided by decision maker(s) at different periods. Moreover, Torres *et al.* [29] expanded the MP-MADM to support a group of decision makers with a data-driven approach.

In this article, the proposed Dynamic Linguistic Decision Making (DLDM) scheme extends the MP-MADM with a data-driven approach. The application of the proposed dynamic linguistic decision making approach should follow the following 7 steps.

1) Step 1: Building the Linguistic Decision Model

The main goal is to support decision makers in selecting the best alternative $\mathcal{A}_i$ from the set of n possible alternatives $\mathcal{A}_1, \ldots, \mathcal{A}_i, \ldots, \mathcal{A}_n$. Each alternative $\mathcal{A}_i$, $i = 1, \ldots, n$, is evaluated according to the criterium or attribute G^j from the set of m attributes $G^1, \ldots, G^j, \ldots, G^m$ at different points of time. It considers the time window $\mathbf{t} = \{t_1, t_2, \ldots, t_p, \ldots, t_P\}$ of a total of P periods for the evaluations, where t_p is the timestamp at period p, $p = 1, 2, \ldots, P$.

2) Step 2: Normalization of the Evaluations

Let $x_i^j(t)$ be the evaluation score obtained for the alternative $\mathcal{A}_i$, according to the criterium G^j at time t. We assume that $x_i^j(t)$ is a positive value. To use the data-driven approach, we need to normalize the evaluations $x_i^j(t)$ into $\hat{x}_i^j(t)$ depending whether it corresponds to a benefit or a cost attribute. For the benefit attribute G^j at specific time t, normalization is computed for all the alternatives $\mathcal{A}_i$ as follows:

$$\hat{x}_i^j(t) = \frac{x_i^j(t)}{\max_{i=1..n} x_i^j(t)} \qquad \text{for } i = 1, \ldots, n \tag{1}$$

On the other hand, for the *cost* attribute G_j at specific time t, normalization is computed for all the alternatives $\mathcal{A}_i$ as follows:

$$\hat{x}_i^j(t) = \frac{\min_{i=1..n} x_i^j(t)}{x_i^j(t)} \qquad \text{for } i = 1, \ldots, n \tag{2}$$

3) Step 3: Computing the Linguistic Variables

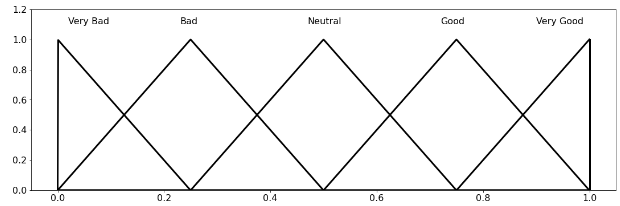
Linguistic variables are those whose values are words or sentences in a natural or artificial language. Thus, the attributes $G^1, \ldots, G^j, \ldots, G^m$ are modeled as linguistic variables. Each linguistic variable is composed of a set of linguistic terms s_q, where an additive linguistic evaluation scale $\overline{S}_Q$ is used [30], i.e.,

$$\overline{S}_Q = \{s_{-Q}, \ldots, s_0 = neutral, \ldots, s_Q\} \qquad Q \in \mathbb{N} \tag{3}$$

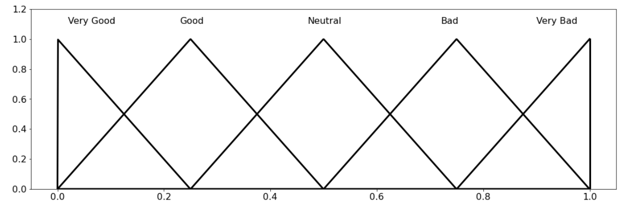
There are many functions for representing linguistic terms, the triangular, trapezoidal and gaussian shapes being the most common. These functions are defined parametrically and their values can be computed either from the data or by a group of experts. If the linguistic term is modeled with a triangular shape, the membership function is defined as follows:

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & \text{if } a \leq x < b \\ \dfrac{x-c}{b-c}, & \text{if } b \leq x < c \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

In this work, we have used statistical quantiles to obtain the values of the parameters of the triangular terms. The quantiles are cut points that divide the range in segments with



(a) Linguistic variable for a *benefit* attribute



(b) Linguistic variable for a *cost* attribute

FIGURE 1. Triangular linguistic terms for the benefit (1a) and cost (1b) linguistic variables.

an equal quantity of observations for each one. In this work, the attributes were normalized with equations (1) and (2), so the range is restricted to the interval [0, 1]. If the triangular shapes are selected for the K linguistic terms, then the peaks of the triangles (values b of equation (4)) were located at the following cut points: the minimum, the percentiles numbered $\frac{k}{K-1} \cdot 100$, $k = 1, \ldots, K-1$, and the maximum. The extreme points of the triangles (values a and c of equation (4))) were located at the peak of the next triangle (the left and right triangles). Figure 1 shows the linguistic variables of the benefit (figure 1a) and cost (figure 1b) types, both with 5 linguistic terms: *Very Bad*, *Bad*, *Neutral*, *Good* and *Very Good*.

4) Step 4: Obtaining the Temporal Linguistic Matrices

The behavior of cryptocurrencies changes over time, therefore the linguistic variables of G^j need to be calculated at every instant of time t. The normalized scores $\hat{x}_i^j(t)$ for each alternative $\mathcal{A}_i$ at time t are used to obtain the vector of membership evaluations $\mathbf{z}_i^j(t) = [z_1, z_2, \ldots, z_K]$, where $z_k = \mu_k^j(\hat{x}_i^j(t))$ is the membership function associated to the k-th linguistic term of the linguistic variable G^j. The linguistic value s_q associated with the evaluation $\hat{x}_i^j(t)$ is obtained as the dot product between the vector of membership evaluations $\mathbf{z}_i^j(t)$ and the vector $[-Q, \ldots, 0, \ldots, Q]$ or $[Q, \ldots, 0, \ldots, -Q]$ depending whether it corresponds to a benefit or a cost attribute, i.e.:

$$\begin{aligned} \text{Benefit: } q &= \mathbf{z}_i^j(t) \bullet [-Q, \ldots, 0, \ldots, Q] \\ \text{Cost: } q &= \mathbf{z}_i^j(t) \bullet [Q, \ldots, 0, \ldots, -Q] \end{aligned} \tag{5}$$

The linguistic decision matrices $\mathbf{S}_t$, $t = 1..P$, are calculated at each instant of time t, where the element s_{ij}^t located in the i-th row and the j-th column of the decision matrices $\mathbf{S}_t$, $t = 1..P$, corresponds to the linguistic evaluation obtained with equation (5) for the score $\mathbf{z}_i^j(t)$.

5) Step 5: Linguistic Information Aggregation of the Attributes

The linguistic aggregation process is carried out in two stages as follows. In the first stage, the linguistic information is aggregated in the dimension of the attributes at each instant of time. To accomplish this, a group of experts defines the weight vector $\mathbf{W} = [W_1, W_2, \ldots, W_m]$ that quantifies the importance of the attributes. To obtain the single decision matrix $\mathbf{R}$, the linguistic weighted averaging (LWA) operator is applied to aggregate the information of each row of the temporal decision matrices $\mathbf{S}_t$. The element r_{it} of the i-th row and t-th column of decision matrix $\mathbf{R}$ is computed as follows:

$$\begin{aligned} r_{it} &= \mathrm{LWA}(s_{i1}^t, s_{i2}^t, \ldots, s_{im}^t) \\ &= W_1 s_{i1}^t \oplus W_2 s_{i2}^t \oplus \ldots W_m s_{im}^t \end{aligned} \tag{6}$$

where $\oplus$ stands for the linguistic averaging operator [31] such that satisfies:

$$\begin{aligned} s_{q_1} \oplus s_{q_2} &= s_{q_1+q_2}, \\ &= s_{q_2+q_1}, \\ &= s_{q_2} \oplus s_{q_1}. \end{aligned}$$

6) Step 6: Dynamic Aggregation of the Temporal Information

Afterwards, in the second stage, the dynamic linguistic weighted averaging (DLWA) operator is used to aggregate the information of linguistic decision matrix $\mathbf{R}$ to obtain the final scores A_i for the alternatives $\mathcal{A}_i$, $i = 1..n$. The *DLWA* operator is defined as follows:

$$\begin{aligned} A_i &= \mathrm{DLWA}(r_{i1}, r_{i2}, \ldots, r_{iP}) \\ &= \omega_1 r_{i1} \oplus \omega_2 r_{i2} \oplus \cdots \oplus \omega_P r_{iP}, \end{aligned} \tag{7}$$

The key to the dynamism of the DLWA operator [32] is determining the weighting vector $\omega = [\omega_1, \omega_2, \ldots, \omega_P]$, where the basic unit-interval monotonic (BUM) function has been proposed [33]. Let $f : [0, 1] \rightarrow [0, 1]$ be a BUM function (where $f(0) = 0, f(1) = 1, f(x) \geq f(y)$ if $x > y$). The weighting vector ω is defined as

$$\omega_p = f\left(\frac{p}{P}\right) - f\left(\frac{p-1}{P}\right), \quad p = 1, 2, \ldots, P \tag{8}$$

where the sequence $\{\omega_p\}$ is a monotonic increasing (or decreasing) sequence, $\omega_{p+1} > \omega_p$ ($\omega_{p+1} < \omega_p$) with $p = 1, 2, \ldots, P-1$. For instance, Xu [32] proposed the function $f(x) = \frac{e^{\alpha x}-1}{e^{\alpha}-1}$, $\alpha > 0$, obtaining the following weighting vector:

$$\omega_p = \frac{e^{\frac{\alpha p}{P}} - 1}{e^{\alpha} - 1} - \frac{e^{\frac{\alpha(p-1)}{P}} - 1}{e^{\alpha} - 1} = \frac{e^{\frac{\alpha p}{P}} - e^{\frac{\alpha(p-1)}{P}}}{e^{\alpha} - 1} \tag{9}$$

7) Step 7: Ranking of the Alternatives

Finally, the alternatives are ranked from best to worst according to the score obtained by linguistic aggregation, i.e., $\mathcal{A}_{(1)} \succeq \mathcal{A}_{(2)} \succeq \ldots . \succeq \mathcal{A}_{(n)}$ such that the score is sorted as follows $A_{(1)} \geq A_{(2)} \geq \ldots . \geq A_{(n)}$ (Note that the sub-index (i) stands for the i-th position).

III. RESULTS

In this section, we first present an illustrative example with real data. Then we apply the dynamic linguistic decision making approach to a specific and bounded cryptocurrency investment scenario. We then perform several experiments using real data.

A. ILLUSTRATIVE EXAMPLE

In this example, an investor wants to invest 100 dollars in the most profitable cryptocurrency:

- $\mathcal{A}_1$: Bitcoin - BTC
- $\mathcal{A}_2$: Ethereum - ETH
- $\mathcal{A}_3$: Ripple - XRP
- $\mathcal{A}_4$: Litecoin - LTC
- $\mathcal{A}_5$: Bitcoin Cash - BCH.

From time to time, the investor has to decide if he or she ought to hold or changes his or her investment decision according to the market behavior. Considering that cryptocurrencies are unrelated to main traditional assets like stock and bond indices, or gold [11], [12], the investor must rely only on the information provided by the cryptocurrency market, and his or her own risk appraisal.

B. BUILDING THE DYNAMIC LINGUISTIC DECISION-MAKING MODEL FOR AN INVESTMENT SCENARIO

We begin by understanding the nature of the dataset and the kind of data for each attribute (categorical, numerical, and so on). In this case, given the business, the selected dataset will have only continuous and numerical values.

The first step consists of building the linguistic decision model. Given the nature of this market, eventually each investor may create his or her own model supporting his or her own strategy. In this study, the investor decides to consider the last five days, in order to rank alternatives daily and invest in the best possible alternative(s). We assume that once investors make the initial buy/hold/sell decision at a given time, they will assess their decision at least daily. For simplicity's sake, we assume that each attribute is equally important. Therefore, for the illustrative example, the requirement is stated as follows: *"the investor wants to invest in that cryptocurrency whose behavior along the variables day-profitability (more daily return), day variability (more fluctuation) and market capitalization (more market dominance) is the best possible from among the five most common cryptocurrencies"*.

In this stage, the investors select the relevant features to be considered when an investment decision is made. For instance, we considered the following three features of the original dataset:

- G^1: day-profitability, obtained in terms of the ratio given by closing and opening prices for a certain cryptocurrency,
- G^2: day-variability, calculated in terms of the ratio of the highest and lowest price for a given cryptocurrency,

TABLE 1. Values of those attributes of the cryptocurrencies that are of interest. The table shows the values from day 1 to day 5.

Cryp.	Attribute					
		t_1	t_2	t_3	t_4	t_5
$\mathcal{A}_1$	G^1	1.01	1.03	1.00	1.00	0.94
	G^2	1.03	1.05	1.03	1.02	1.08
	G^3	25.96	25.99	25.99	26.00	25.93
$\mathcal{A}_2$	G^1	0.98	1.00	1.01	0.98	0.96
	G^2	1.03	1.02	1.03	1.02	1.05
	G^3	25.15	25.15	25.17	25.15	25.11
$\mathcal{A}_3$	G^1	0.98	1.00	1.11	0.95	0.97
	G^2	1.04	1.03	1.15	1.13	1.08
	G^3	24.30	24.29	24.39	24.35	24.31
$\mathcal{A}_4$	G^1	1.01	0.99	1.02	0.99	0.93
	G^2	1.04	1.04	1.04	1.03	1.09
	G^3	23.19	23.18	23.20	23.18	23.11
$\mathcal{A}_5$	G^1	0.99	1.00	1.02	0.99	0.95
	G^2	1.04	1.04	1.05	1.02	1.07
	G^3	23.80	23.80	23.81	23.79	23.74

TABLE 2. Normalized attributes values of cryptocurrencies of interest. The table shows the normalized values from day 1 to day 5.

Cryp.	Attribute					
		t_1	t_2	t_3	t_4	t_5
$\mathcal{A}_1$	G^1	1.00	1.00	0.90	1.00	0.97
	G^2	0.99	1.00	0.90	0.91	0.99
	G^3	1.00	1.00	1.00	1.00	1.00
$\mathcal{A}_2$	G^1	0.97	0.97	0.91	0.98	0.99
	G^2	0.99	0.97	0.90	0.90	0.97
	G^3	0.97	0.97	0.97	0.97	0.97
$\mathcal{A}_3$	G^1	0.97	0.97	1.00	0.95	1.00
	G^2	0.99	0.98	1.00	1.00	0.99
	G^3	0.94	0.93	0.94	0.94	0.94
$\mathcal{A}_4$	G^1	1.00	0.95	0.91	0.98	0.97
	G^2	1.00	0.99	0.90	0.91	1.00
	G^3	0.89	0.89	0.89	0.89	0.89
$\mathcal{A}_5$	G^1	0.98	0.96	0.91	0.98	0.99
	G^2	1.00	0.99	0.92	0.90	0.98
	G^3	0.92	0.92	0.92	0.92	0.92

- G^3: (*log* of) Market capitalization, which corresponds to the relative size of the cryptocurrency based on the price and circulating supply, where the latter is the best possible approximation to the number of coins that are circulating in the market and in public (electronic) wallets.

Table 1 shows an extract of the data, where we can see five cryptocurrencies in the first column $\mathcal{A}_i$ ($i = 1, 2, 3, 4, 5$) and in the second column we can see the values for three attributes G^1, G^2, G^3 over time t_k ($k = 1, 2, 3, 4, 5$).

The second step of our approach is to classify which attributes are benefits: the higher this attribute value is for a given alternative, the more desirable it is (considering only one dimension). In this case study, the attributes G^1, G^2 and G^3 were considered as benefit attributes. The reason that the day-variability (calculated in terms of the ratio of the highest to lowest price for a given cryptocurrency) is also considered as a benefit attribute is because it acts as a proxy for volatility or risk, and a decision maker might desire to manage risk.

However, from the point of view of a trader, high volatility is desirable. There are more profitable opportunities in markets with higher volatility, than in low volatility markets (such as sovereign bond markets), provided that the fluctuations can be predicted, at least to some extent. Given that cryptocurrencies constitute a relatively new and highly speculative market, traders tend to have a risk-tolerant profile and look for much higher returns than what could be achieved in, for example, the US stock or bond market.

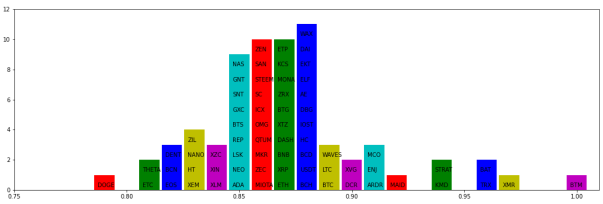
Table 2 shows the corresponding normalized values related to Table 1.

The third step consists of modeling each feature as a linguistic variable. This is the crucial part, as it facilitates human understanding of the model. Thus, each attribute (G^1, G^2, G^3) is modeled as a linguistic variable. For the sake of simplicity, we have selected the triangular membership functions for the additive linguistic scale of five terms: *"Very Bad"*, *"Bad"*, *"Neutral"*, *"Good"* and *"Very Good"*, i.e.,

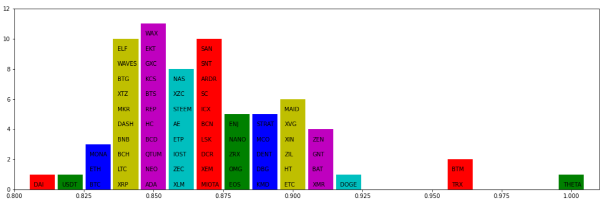
$$\overline{S}_2 = \{s_{-2} = Very\ Bad, s_{-1} = Bad, s_0 = Neutral,\\ s_1 = Good, s_2 = Very\ Good\},$$

Figure 2 shows the histograms of the distributions of the normalized values of the 68 most well-known digital assets for the (2a) G^1 day-profitability, (2b) G^2 day-variability, and (2c) G^3 (log-of) market capitalization attributes. As explained in the previous section, the parameters of the triangular membership functions were obtained using statistical quantiles, whereas for the $\overline{S}_2$ scale, quartiles were used. Figure 3 shows the corresponding linguistic variables with additive triangular linguistic terms of the cryptocurrencies for the (3a) G^1 day-profitability, (3b) G^2 day-variability, and (3c) G^3 (log-of) market capitalization attributes.

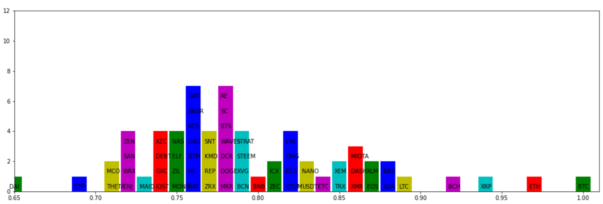
The fourth step consists of obtaining the temporal Linguistic Matrices. The linguistic decision matrices S_1, S_2, S_3, S_4 and S_5 are shown in a compact form in Table 3, which elements were obtained using equation (5) for the five selected cryptocurrencies.



(a) Day Profitability

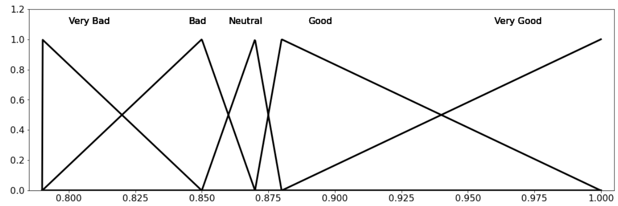


(b) Day Variability

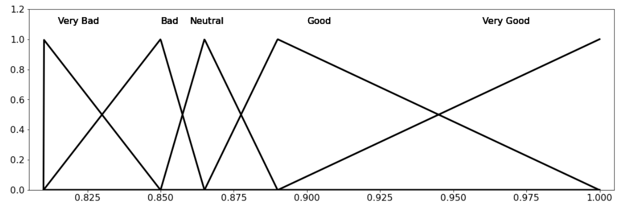


(c) Log of the Market Capitalization

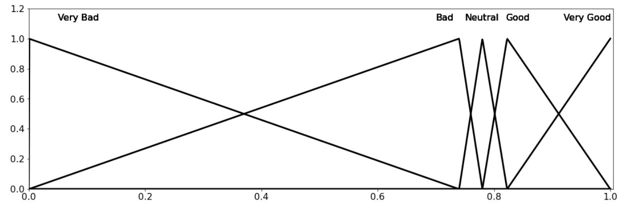
FIGURE 2. Histograms of the distributions of the normalized values for the (2a) G^1 day-profitability, (2b) G^2 day-variability, and (2c) G^3 (log-of) market capitalization attributes of 68 cryptocurrencies.



(a) Day Profitability



(b) Day Variability



(c) Log of the Market Capitalization

FIGURE 3. Linguistic Variables with additive triangular linguistic terms of cryptocurriencies for the (3a) G^1 day-profitability, (3b) G^2 day-variability, and (3c) G^3 (log-of) market capitalization attributes.

The fifth step of our approach consists of aggregating the linguistic information in the attribute dimension. Again, we assume that each attribute is equally important, i.e., $W = [W_1 = \frac{1}{3}, W_2 = \frac{1}{3}, W_3 = \frac{1}{3}]$. The *LWA* operator given by equation (6) is applied to each row of the matrices $\mathbf{S}_t$, $t = 1, \ldots, 5$, of Table 3 to obtain the decision matrix $\mathbf{R}$. The resulting matrix $\mathbf{R}$ is given in Table 4. Some examples of how the elements of the linguistic decision matrix $\mathbf{R}$ were computed are given below:

$$\begin{aligned}
\text{Alternative } \mathcal{A}_1 \text{ at } t_1 : r_{11} &= \frac{1}{3}s_2 \oplus \frac{1}{3}s_{1.92} \oplus \frac{1}{3}s_2 \\
&= s_{1.97} \\
\text{Alternative } \mathcal{A}_3 \text{ at } t_2 : r_{32} &= \frac{1}{3}s_{1.78} \oplus \frac{1}{3}s_{1.86} \oplus \frac{1}{3}s_{1.61} \\
&= s_{1.75} \\
\text{Alternative } \mathcal{A}_5 \text{ at } t_4 : r_{54} &= \frac{1}{3}s_{1.87} \oplus \frac{1}{3}s_{1.71} \oplus \frac{1}{3}s_{1.56} \\
&= s_{1.71}
\end{aligned}$$

In the sixth step, the DLWA operator is used to aggregate the temporal information from the decision matrix $\mathbf{R}$ to obtain the final scores for each alternative $\mathcal{A}_i$, $i = 1, \ldots, n$. If we consider the whole five-day period, the weight vector ω for the temporal aggregation is obtained from equation (9) with $\alpha = 0.5$ and $P = 5$. In this case, the values correspond to $\omega = (0.15, 0.16, 0.18, 0.2, 0.22)$. The final scores for each alternative at time $t = 5$ are:

$$\begin{aligned}
A_1 &= 0.15s_{1.97} \oplus 0.16s_2 \oplus 0.18s_1 \oplus 0.2s_{1.91} \\
&\quad \oplus 0.22s_{1.81} = s_{1.58} \\
A_2 &= 0.15s_{1.83} \oplus 0.16s_{1.80} \oplus 0.18s_{1.21} \oplus 0.2s_{1.81} \\
&\quad \oplus 0.22s_{1.77} = s_{1.52} \\
A_3 &= 0.15s_{1.78} \oplus 0.16s_{1.75} \oplus 0.18s_{1.89} \oplus 0.2s_{1.78} \\
&\quad \oplus 0.22s_{1.85} = s_{1.64} \\
A_4 &= 0.15s_{1.79} \oplus 0.16s_{1.65} \oplus 0.18s_{1.10} \oplus 0.2s_{1.67} \\
&\quad \oplus 0.22s_{1.65} = s_{1.42} \\
A_5 &= 0.15s_{1.79} \oplus 0.16s_{1.73} \oplus 0.18s_{1.19} \oplus 0.2s_{1.71} \\
&\quad \oplus 0.22s_{1.72} = s_{1.47}
\end{aligned}$$

Finally, in the last step and according to the scores, the alternatives are sorted from best to worst $\mathcal{A}_3 = XRP \succ \mathcal{A}_1 = BTC \succ \mathcal{A}_2 = ETH \succ \mathcal{A}_5 = LTC \succ \mathcal{A}_4 = BCH$ at time $t = 5$.

When the investor is willing to run this procedure every day, the temporal weighting vector will have a different length depending on the number of days data is collected or the time window selected. Then, values for $\omega(t_k)$ when $\alpha = 0.5$ in the BUM function are given by

$$\begin{aligned}
\omega_1 &= (0.61), \\
\omega_2 &= (0.34, 0.44), \\
\omega_3 &= (0.24, 0.28, 0.33),
\end{aligned}$$

TABLE 3. Linguistic Decision Matrices for each instance of time.

Decision Matrix	Crypt.	Attribute G^1	G^2	G^3
S_1	$\mathcal{A}_1$	s_2	$s_{1.92}$	s_2
	$\mathcal{A}_2$	$s_{1.75}$	$s_{1.92}$	$s_{1.83}$
	$\mathcal{A}_3$	$s_{1.75}$	$s_{1.92}$	$s_{1.66}$
	$\mathcal{A}_4$	s_2	s_2	$s_{1.38}$
	$\mathcal{A}_5$	$s_{1.83}$	s_2	$s_{1.55}$
S_2	$\mathcal{A}_1$	s_2	s_2	s_2
	$\mathcal{A}_2$	$s_{1.78}$	$s_{1.79}$	$s_{1.83}$
	$\mathcal{A}_3$	$s_{1.78}$	$s_{1.86}$	$s_{1.61}$
	$\mathcal{A}_4$	$s_{1.64}$	$s_{1.93}$	$s_{1.39}$
	$\mathcal{A}_5$	$s_{1.71}$	$s_{1.93}$	$s_{1.56}$
S_3	$\mathcal{A}_1$	s_0	s_1	s_2
	$\mathcal{A}_2$	$s_{0.8}$	s_1	$s_{1.83}$
	$\mathcal{A}_3$	s_2	s_2	$s_{1.67}$
	$\mathcal{A}_4$	$s_{0.8}$	$s_{1.10}$	$s_{1.39}$
	$\mathcal{A}_5$	$s_{0.8}$	$s_{1.20}$	$s_{1.56}$
S_4	$\mathcal{A}_1$	s_2	$s_{1.74}$	s_2
	$\mathcal{A}_2$	$s_{1.87}$	$s_{1.71}$	$s_{1.83}$
	$\mathcal{A}_3$	$s_{1.69}$	s_2	$s_{1.67}$
	$\mathcal{A}_4$	$s_{1.87}$	$s_{1.74}$	$s_{1.39}$
	$\mathcal{A}_5$	$s_{1.87}$	$s_{1.71}$	$s_{1.56}$
S_5	$\mathcal{A}_1$	$s_{1.56}$	$s_{1.87}$	s_2
	$\mathcal{A}_2$	$s_{1.85}$	$s_{1.62}$	$s_{1.83}$
	$\mathcal{A}_3$	s_2	$s_{1.87}$	$s_{1.67}$
	$\mathcal{A}_4$	$s_{1.56}$	s_2	$s_{1.39}$
	$\mathcal{A}_5$	$s_{1.85}$	$s_{1.75}$	$s_{1.56}$

TABLE 4. Linguistic decision matrix R after applying the LWA operator.

	t_1	t_2	t_3	t_4	t_5
$\mathcal{A}_1$	$s_{1.97}$	s_2	s_1	$s_{1.91}$	$s_{1.81}$
$\mathcal{A}_2$	$s_{1.83}$	$s_{1.80}$	$s_{1.21}$	$s_{1.81}$	$s_{1.77}$
$\mathcal{A}_3$	$s_{1.78}$	$s_{1.75}$	$s_{1.89}$	$s_{1.78}$	$s_{1.85}$
$\mathcal{A}_4$	$s_{1.79}$	$s_{1.65}$	$s_{1.10}$	$s_{1.67}$	$s_{1.65}$
$\mathcal{A}_5$	$s_{1.79}$	$s_{1.73}$	$s_{1.19}$	$s_{1.71}$	$s_{1.72}$

$$\omega_4 = (0.18, 0.21, 0.23, 0.26),$$
$$\omega_5 = (0.15, 0.16, 0.18, 0.2, 0.22).$$

Table 5 shows the final results of the application of the DLDM approach to the first 5 time periods, where the best alternatives are highlighted. Additionally, Table 6 shows the ranking of the alternatives from best to worst according to the score obtained through linguistic aggregation, i.e., $\mathcal{A}_{(1)} \succeq \mathcal{A}_{(2)} \succeq \ldots . \succeq \mathcal{A}_{(5)}$ such that the score is sorted as follows $A_{(1)} \geq A_{(2)} \geq \ldots . \geq A_{(5)}$.

TABLE 5. Final score of each alternative $\mathcal{A}_i$ for each period of time $[t_1, t_P]$ using the DLWA operator.

Period	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$	Best $\mathcal{A}$
t_1	$s_{1.20}$	$s_{1.11}$	$s_{1.08}$	$s_{1.09}$	$s_{1.09}$	$\mathcal{A}_1 = BTC$
$[t_1, t_2]$	$s_{1.55}$	$s_{1.41}$	$s_{1.37}$	$s_{1.33}$	$s_{1.37}$	$\mathcal{A}_1 = BTC$
$[t_1, t_3]$	$s_{1.36}$	$s_{1.34}$	$s_{1.53}$	$s_{1.25}$	$s_{1.30}$	$\mathcal{A}_3 = XRP$
$[t_1, t_4]$	$s_{1.50}$	$s_{1.46}$	$s_{1.59}$	$s_{1.36}$	$s_{1.41}$	$\mathcal{A}_3 = XRP$
$[t_1, t_5]$	$s_{1.58}$	$s_{1.52}$	$s_{1.64}$	$s_{1.42}$	$s_{1.47}$	$\mathcal{A}_3 = XRP$

TABLE 6. Ranking of best alternatives using our approach with the DLWA operator.

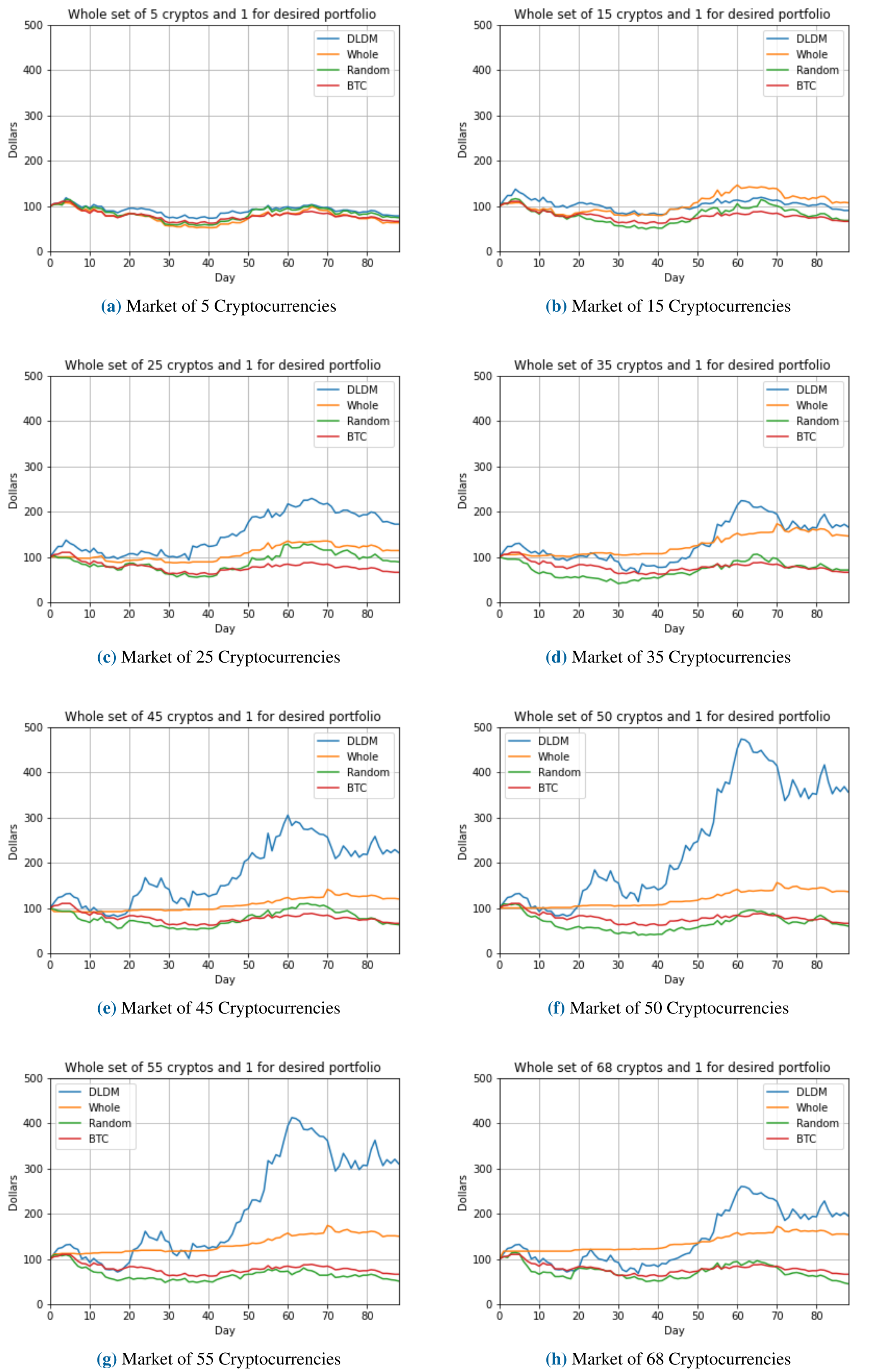
	$\mathcal{A}_{(1)}$	$\mathcal{A}_{(2)}$	$\mathcal{A}_{(3)}$	$\mathcal{A}_{(4)}$	$\mathcal{A}_{(5)}$
Day 1	**BTC**	ETH	BHC	LTC	XRP
Day 2	**BTC**	ETH	XRP	BCH	LTC
Day 3	**XRP**	BTC	ETH	BCH	LTC
Day 4	**XRP**	BTC	ETH	BCH	LTC
Day 5	**XRP**	BTC	ETH	BCH	LTC

TABLE 7. Dollars obtained when selling at closing value with different investment strategies: Bitcoin, Whole set, DLDM with the best alternative, DLDM with the best two alternatives, and random selection.

Day	BTC	Whole	DLDM-1 (Best crypt)	DLDM-2 (2-Crypt)	Random
1	105	104	**105**	104	102
2	**106**	104	**106**	104	103
3	**110**	105	106	106	101
4	110	107	**118**	112	102
5	110	106	**112**	109	101

Table 7 shows a portfolio performance comparison between a fixed, equally-distributed portfolio composed of all cryptocurrencies (Whole), another portfolio built from random decisions (Random), two portfolios demonstrating our DLDM approach with the selection of the best (DLDM-1) and the best two assets (DLDM-2), and the bitcoin-holding strategy (BTC). The table illustrates the initial investment of 100 dollars for buying a given digital asset at opening price on the first day, and shows the (rounded) equivalent money when selling at closing price that same day. In the following days the investor does the entire procedure over again, moving the corresponding time window. This performance is compared with another investor who decides to invest only in the most popular cryptocurrency (bitcoin in this case) every day, noting that transactions fees are excluded for sake of simplicity. Note that for this simple case study, the profit obtained with the investment made with DLDM approach outperforms the profit of both bitcoin-holding and random choice.

It is important to emphasize that the ranking shown in Table 6 gives the best prediction according to the procedure presented for time step t_k, but corresponding data also needs the closing price at that time step, so Table 7 presents the investment results when the algorithm suggestion is applied



(a) Market of 5 Cryptocurrencies

(b) Market of 15 Cryptocurrencies

(c) Market of 25 Cryptocurrencies

(d) Market of 35 Cryptocurrencies

(e) Market of 45 Cryptocurrencies

(f) Market of 50 Cryptocurrencies

(g) Market of 55 Cryptocurrencies

(h) Market of 68 Cryptocurrencies

FIGURE 4. Daily wealth obtained following our DLDM approach, selecting the best alternative compared to the benchmark strategies. The experiments consider markets with 5, 15, 25, 35, 45, 50, 55, and 68 cryptocurrencies.

TABLE 8. Rate of return (in percentage with respect to the initial investment) of our approach and benchmark strategies. Our approach considers different numbers of cryptocurrencies in the reference set, and selects the best one each day.

	5 crypt.	15 crypt.	25 crypt.	35 crypt.	45 crypt.	50 crypt.	55 crypt.	68 crypt.
DLDM-1 (Best-Crypt)	**−22%**	−10%	**72%**	**66%**	**122%**	**256%**	**210%**	**95%**
Whole set naïve diversification	−37%	**7%**	14%	46%	20%	36%	50%	54%
Random (seed=500)	−45%	−2%	1%	−3%	−10%	−4%	−14%	3%
Random (seed=1000)	−26%	−32%	−11%	−29%	20%	−40%	−49%	−55%
Bitcoin buy-and-hold	−34%	−34%	−34%	−34%	−34%	−34%	−34%	−34%

on t_{k+1}. Then, for the sake of clarity, the Table shows that when BTC has been chosen for investment on Day 1 at opening price for 100 dollars, the equivalent sum of dollars when selling at closing price that day is 105 dollars, which is used the following day for buying BTC at opening price, or the cryptocurrency suggested by available data up to the day before, if contemplating investment. In order to allow for our experiments to be reproduced, implementation can be found in a Jupyter Notebook.[1]

C. EXPERIMENTS UNDER SEVERAL MARKET SCENARIOS

In this section we carry out several experiments with the approach proposed above, using data from 01/03/2018 until 27/05/2018. Data was obtained from [3]. The experiments differ from each other in the number of cryptocurrencies taken into account as the referential set (market), as well as the number of eligible cryptocurrencies to buy or sell. In other words, our model works as follows: each day we select a portfolio with the M best cryptocurrencies to buy, among the N of the whole set, and we sell the M we had from the previous day. We update our decision on a daily basis, during the 88 days of the experiment. At the end of the period, we compare the final wealth generated by our approach with three benchmarks: random selection, buy-and-hold naïve diversification, and bitcoin buy-and-hold. The rationale for the selection of such benchmarks is the following:

- Random selection: If the market fulfills the EMH, then no forecast could systematically outperform a random selection of financial assets.
- Buy-and-hold naïve diversification: instead of buying and selling every day, we set at the beginning of our experiment an equally weighted portfolio with the N cryptocurrencies under consideration, and we hold this portfolio until the end of the experiment.
- Bitcoin buy-and-hold: Considering that bitcoin is the most well-known and liquid cryptocurrency, and that previous studies have focused on bitcoin forecasting, we compare our model with that of the traditional buy-and-hold strategy with this cryptocurrency.

Figure 4 shows the comparative performance of daily wealth obtained following our DLDM approach, selecting the best alternative compared to benchmark strategies. In our experiments, the size of the referential set of the market

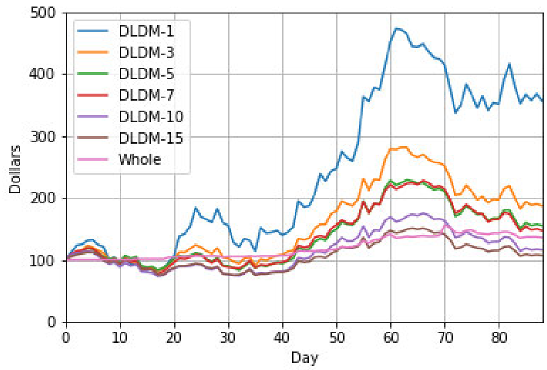


FIGURE 5. Daily wealth obtained following our DLDM approach with different quantities of selected cryptocurrencies in the portfolio compared to benchmark strategies. The experiments consider a market of 50 cryptocurrencies.

varied between 5 and 68 cryptocurrencies. We can see that our DLDM strategy shows the best performance for most of the period, and for the different market sizes. However, for the small market size (15 cryptocurrencies or less in the referential set), all strategies saw an economic loss at the end of the period (the end of day 88), which could be due to the enormous dominance of bitcoin, such that the final wealth achieved following any investment decision is similar to that generated just by holding bitcoin. However, when we expand the quantity of eligible cryptocurrencies, the relative importance of bitcoin vanishes, and the benefit of diversification rises. Indeed, for the larger market size (larger than 15 cryptocurrencies in the referential set) our approach achieves significantly larger final wealth *vis-à-vis* our benchmarks. The biggest difference was achieved when a market of size 50 cryptocurrencies was considered.

Figure 5 shows the impact on the performance of the portfolio generated with the DLDM strategy as the number of selected cryptocurrencies selected is increased. It can be seen that performance diminishes as more cryptocurrencies are selected, approximating the performance of the conservative strategy that considers the entire data set.

We believe that our approach will prove useful to investors who want to trade in the cryptocurrency market, by selecting the best buy option for each day. The inclusion of more cryptocurrencies in our referential set, allows for capturing superior returns from ''secondary'' currencies. This is particularly true, when we consider that the bitcoin buy-and- hold is usually the least attractive strategy.

[1] Available at http://jupyterhub.innovacionyrobotica.com

Table 8 shows the rate of return (as a percentage with respect to the original investment) for all 88 days, in experiments considering markets of 5, 15, 25, 35, 45, 50, 55 and 68 cryptocurrencies. We can observe that the performance of our approach is significantly better than any other strategy in most scenarios. Although random selection could find a good portfolio by chance, it is very unstable, with a lot of variability depending on the random seed.

Although the proposed approach is not predictive, since it gives the best choice for a given time-step based on data up to that moment, investment performance results show the equivalent amount in dollars obtained when an investor (or day trader) holds the current best decision on the following time-step (the following day, in this case study).

IV. CONCLUSION

Daily transactions executed in the Cryptocurrency market now total almost $100 billion. Thus, this is a market generating a high volume of data at high velocity, which demands data-driven decision support systems.

In this work, we present an approach for investors that need to support their cryptocurrency buy/hold/sell decisions over time. We show the suitability of the approach to facilitating human understanding of the model (explainable models). Moreover, our study shows that using our approach obtains better results than either naïve diversification or a buy-and-hold strategy, as shown in the case study where including more cryptocurrencies than just the five largest ones helps achieve a greater final total.

In general terms, we found that there are arbitrage opportunities in this novel market, and that this approach is able to exploit them in a better way, outperforming buy- and-hold or random strategies. This work also contributes evidence of the partial inefficiency of this market to the literature.

Although the case study gives the best choice for investing on a given cryptocurrency, the reader should note that decisions based on data until a given day are obtained with data including the closing value for that day. As a result, there is no guarantee that the best option for investment holds until the following day.

As future work, the proposed *Dynamic Linguistic Decision Making* approach can be applied to other uncertainty environments such as the selection of a hybrid portfolio of cryptocurrencies and traditional assets, personnel evaluation, disaster management or healthcare management.

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ROMINA TORRES (Member, IEEE) received the B.S. and M.Sc. degrees in informatics engineering and the Dr.Eng. degree in informatics from Federico Santa María Technical University (UTFSM), Valparaíso, Chile, in 2001, 2003, and 2014, respectively. From 2001 to 2008, she worked with major international companies, such as Motorola and Software AG. Since 2013, she has been a Professor with the Engineering Faculty, Andres Bello University (UNAB), Viña del Mar, Chile. She is currently the Director of the computer science and informatics engineering master programs. Her research interests include intelligent systems and their applications for decision making in several interdisciplinary areas.

MIGUEL A. SOLIS received the B.S. and M.Sc. degrees in electronics engineering and the Dr.Eng. degree in informatics from Federico Santa María Technical University (UTFSM), Chile, in 2011, 2012, and 2017, respectively. He was an Interim Academic with the Department of Computing and Systems Engineering, Universidad Católica del Norte. He has been the Head of the Robotics and Automation Engineering Program, Universidad Andrés Bello, since 2020. He has taught hands-on courses on Robotics Fundamentals and the Internet of Things. His research interests include adaptive control and decision-making, and cognitive robotics.

RODRIGO SALAS (Senior Member, IEEE) received the B.S. and M.Sc. degrees in informatics engineering and the Dr.Eng. degree in informatics from Federico Santa María Technical University (UTFSM), Chile, in 2001, 2002, and 2010, respectively. From 2002 to 2004, he was a Research Assistant with the Informatics Department, UTFSM. Since 2004, he has been with the Universidad de Valparaíso, where he is currently a Full Professor of the Biomedical Engineering School and also teaches in Data Mining, Probability and Statistics, and Machine Learning. He is also a main Researcher of the Center of Research and Development in Health Engineering (CINGS-UV). His research interests include computational intelligence, data science, decision support systems, intelligent systems, and their applications to finance, air pollution, healthcare, and medicine.

AURELIO F. BARIVIERA received the B.S. degree in finance from the National University of La Plata, Argentina, in 1995, the master's diploma degree in law, economics, and politics from the European Union from the University of Padua, Italy, in 1999, and the Ph.D. degree in economics from Universitat Rovira i Virgili, Spain, in 2005.

In 1997, he was a Student Researcher with SRI International in Arlington, VA, USA. He is currently an Associate Professor of mathematics for economists with Universitat Rovira i Virgili. His research interests include time series analysis, econophysics, complex systems, and information theory applied to econometrics.