Research on Bus and Metro Transfer From Perspective of Hypernetwork–A Case Study of Xi'an, China (December 2020)

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ABSTRACT Aiming at the blank of hypernetwork in the empirical research field of the composite transportation network, taking Xi'an city as an example, this paper uses the ''HyperEdge'' to connect the bus network with the metro network and constructs a large-scale complex hypernetwork, hereinafter referred to as B-M hypernetwork. In this paper, the topological characteristics of a single hypernetwork are analogized to a complex hypernetwork, and the interaction mechanism among bus networks, metro networks, and B-M hypernetwork is studied. A multi-source least transferred algorithm based on the incidence matrix is proposed to solve any two nodes' minimum transfer times in a large complex hypernetwork. The multi-source least transferred algorithm and matrix transfer algorithm is applied to the B-M hypernetwork, and the random attack and intentional attack analyze the robustness of the B-M hypernetwork. The results show that the improved multi-source minimum transmission algorithm based on the incidence matrix is feasible and effective and can obtain superior computing power for large and complex hypernetworks.

INDEX TERMS Hypernetwork, topological characteristics, robustness, incidence matrix, deep optimization algorithm.

I. INTRODUCTION

Hypernetwork is an extraordinary complex network, which was first proposed and applied to the transportation system by Y. Sheffi in 1985 [1]. In 2002, A. Nagurney, an American scientist, spread the concept of ''hypernetwork'' to the fields of logistics and product trading and called the network higher than and beyond the existing network as the hypernetwork, which made the definition of hypernetwork clear [2]. Hypernetwork is widely used in transportation, supply chain, and logistics.

A. RESEARCH ON SUPPLY CHAIN BASED ON HYPERNETWORK

In the research of hypernetwork in the supply chain field, some scholars consider the market demand for building the

model of supply chain hypernetwork, using an algorithm to solve it, and then verify the model. Xu and Gao (2008) [3] studied the equivalent traffic network equilibrium model of supply chain network equilibrium model with elastic demand, expressed it as a nonlinear complementarity problem, and solved it by Levenberg Marquardt type method. Then, Xu and Gao (2010) [4] studied the supply chain network equilibrium model based on elastic demand and the equivalent transportation network equilibrium model based on hypernetwork, proposed a new semi-smooth Levenberg Marquardt algorithm, and proved that it is more effective than Newton method and modified projection method; Feng *et al.* (2014) [5] established a hypernetwork model of the closed-loop supply chain in demand market, including suppliers, manufacturers, retailers and consumers. The equilibrium conditions of the closed-loop supply chain were solved by using evolutionary variational inequality theory and dynamic prediction system, and numerical examples verified the rationality of the model;

Wang and Wang (2016) [6] established a hypernetwork model of reverse logistics supply chain with six decision makers, described the seasonal influence of demand on simple periodic function, and described the equilibrium state of the network by using the theory of variational inequality.

Some scholars combine supply chain hypernetwork with transportation network and construct the model considering transportation cost. Yamada *et al.* (2011) [7] proposed a hypernetwork equilibrium model combining the supply chain network with the transportation network. Considering freight forwarders and transportation network users' behavior, they determine the transportation costs generated in the supply chain network. Finally, they pointed out that improving the transportation network can improve the efficiency of the supply chain network; Yamada and Febri (2015) [8] proposed a discrete network from supply chain efficiency. The network design problem is used to optimize the design of the freight network. In the framework of mathematical programming with equilibrium constraints, a model including the supply chain and transportation network is established. The upper level determines the best action set for improving the transportation network, while the lower level is based on the hypernetwork balance of supply chain and multimodal transport. At the same time, new PSO variants are developed to approximate the upper layer. Finally, numerical experiments show that they have high performance and effective freight correlation effect.

Some scholars set up the hypernetwork model of the supply chain and describe all levels' behavior, such as suppliers, manufacturers, retailers, consumers, etcetera, derive the optimal conditions of decision-makers, and use the algorithm to solve the model, and then verify the model. Yang *et al.* (2006) [9] proposed modeling, analysis, and solution framework of supply chain hypernetwork in the e-commerce environment and established the control system using variational inequality. This paper describes the win-win situation of the supply chain hypernetwork model under the optimal conditions of manufacturer, logistics department alliance and demand market, and obtains the stationary point in this case by modified projection method. Finally, the model's effectiveness is verified by an example; Zhiping *et al.* (2009) [10] established a green supply chain return hypernetwork optimization model. Considering the supply chain equilibrium model at a lower level, considering the impact and pollution of goods transportation, goods packaging and goods processing on the supply chain at a higher level, the optimal conditions for manufacturers, distributors and consumers are established by using variational inequality and solved by modified projection method. Finally, the supply chain hypernetwork model's effectiveness is verified by an example; Then, Wang *et al.* (2010) [11] put forward a hypernetwork model of green reverse supply chain, which is composed of I manufacturers, J distributors, m markets and N recyclers. On this basis, the optimality conditions of manufacturers, distributors, consumers and recyclers are established by using variational inequality and solved by an improved projection method. Finally, the effectiveness of the model is verified by numerical examples.

After building the model based on the supply chain hypernetwork, researchers will use relevant algorithms to solve it. The algorithms mentioned in the above literature include the Levenberg Marquardt type method, modified projection method, dynamic projection system, genetic algorithm, Benders Decomposition algorithm, etcetera. After solving, the models and algorithms will be verified by numerical examples or empirical cases.

B. RESEARCH ON LOGISTICS IN HYPERNETWORK

Some scholars have explored the relationship between the logistics industry and other industries in logistics, some scholars have studied the related problems of logistics enterprises, and some scholars have studied some particular logistics directions. Suo and Wen (2009) [12] Combined with e-commerce, proposed a reverse Stackelberg model of the e-commerce logistics market based on hypernetwork theory. Taking profit, risk and credit as the elements of the utility function, they set up a sequence-dependent game process in the hypernetwork composed of transaction volume and credit, and proposed a reverse Stackelberg solution to maintain the existence and robustness of logistics market sequence; Zhu (2011) [13] transformed the coordination problem between the manufacturing industry and the logistics industry into a hypernetwork equilibrium problem composed of two networks, revealing the inseparable relationship between the two industries; Shang and Tan (2014) [14] studied the emergency logistics. Based on the hypernetwork theory, an emergency logistics hypernetwork model composed of three sub-networks of organization, information, and infrastructure is established. The hypernetwork capability of emergency logistics is analyzed in timeliness, quality, operation, and hypernetwork structure. The entropy evaluation model of hypernetwork capability of emergency logistics is established based on entropy theory and order degree, and time benefit, quality and operation are discussed A numerical example is given to prove the practicability of the model; Wang and Chen (2016) [15] considered the balance between logistics enterprises, built a logistics hypernetwork model of revenue and cost. Based on the hypernetwork, a three-layer logistics Hypernetwork Model of suppliers, distributors and consumers is established. To maximize profits, the objective function of revenue and cost is set. The market behavior of decision-makers of the hypernetwork model is expressed, and logistics network enterprises' equilibrium conditions are discussed. Finally, a case study of e-commerce logistics in Xi'an Amazon operation center is carried out.

After the establishment of the logistics hypernetwork model, researchers will use related algorithms to solve the problem. The algorithms mentioned above include the modified projection method, Euler algorithm, Floyd algorithm, etcetera. After solving, they will verify the model and algorithm according to numerical examples or empirical cases.

C. RESEARCH OF HYPERNETWORK IN TRAFFIC

When it comes to traffic, five modes of transportation are generally mentioned: highway, railway, waterway, aviation, and pipeline. Corresponding to the network research, they are highway network, railway network, waterway network, aviation network, pipeline network. Among them, highway networks, railway networks, aviation networks have been studied by many scholars, such as model construction and route optimization. In addition, as urban traffic congestion has become more and more severe in recent years, major cities have put forward the concept of giving priority to the development of urban public transport and accelerated the construction of bus (electric) vehicles (including trams) and urban rail transit facilities, providing essential travel services for the public, to solve the urban congestion problem effectively. In this context, many scholars also put the research perspective in the field of urban public transport. At present, the urban public transport network is mainly composed of bus networks and metro networks. Some scholars use the traditional complex network to study it, analyze the crucial stations and critical routes in urban public transport, and examine the robustness of urban public transport networks.

The research of hypernetwork in the field of transportation can be divided into two categories: one is to construct a hypernetwork model based on hypergraph theory, and to conduct empirical research on the existing bus network, rail network, aviation network, etcetera, and analyze their topological properties and network robustness; the other is to use the hypernetwork model to build a higher-level and more complex system, and explore its connection, or expand to weighted directional network, a more complex model is established to consider travel cost and travel preference, and is applied to solve other traffic problems.

In the field of transportation empirical research, most scholars are based on a single network. In rail transit, empirical analysis is usually carried out on the mature urban rail network. According to the existing subway network, the hypernetwork model is established, and the model is described according to some topological characteristics to verify the robustness of the subway network. Some compare the model with the traditional spatial model. Wei and Ning (2018) [16] established the hypernetwork model of the Nanjing Metro network by using the hypernetwork method. The new model was described by hyperedge degree, node hyperdegree, and hyperedge hyperdegree and compared with the traditional spatial L and spatial P models. After that, Wei *et al.* (2019) [17] made statistics and classification on the faults of Nanjing Metro in recent three years and used relevant indicators to measure the hypernetwork of Nanjing Metro Robustness, explore the impact of intentional attack and random attack on the robustness of Nanjing Metro Network. In terms of the railway network, Mohmand and Wang (2014) [18] studied the structural characteristics of the Pakistan railway network and concluded that the network has small-world network characteristics and diversity.

According to the connectivity, the most important cities are determined to facilitate the identification of potential congestion points in the network; Suo and Guo (2018) [19] proposed an evolutionary model based on hypernetwork to describe the evolution mechanism of high-speed railway system, and deduced that the over degree distribution of railway station nodes follows the shift power-law distribution, and tested the influence of parameters on the model, and conducted an empirical study on China's Railway High-speed Railway.

Some scholars focus on solving the travel problems of actual residents or some traffic problems but ignore the attributes of the traffic network itself. Duan *et al.* (2009) [20] proposed dividing road networks into control areas based on a hypergraph model to solve the problems of when to coordinate intersections and how to generate control areas. The hermetic algorithm was used to divide the hypergraph into zones corresponding to and coordinated control areas, and a numerical example was carried out in combination with the actual situation in Beijing; Bingfeng *et al.* (2011) [21] and others put forward a description by cars, buses, bicycles and other different modes of transportation of superimposed and composite formation of city hypernetwork topology model of the multi-modal transport system of complex traveler choice behavior, including the mode choice and route choice carried on the thorough analysis, based on the analysis of traffic demand, considering the interference between different mode, different mode of generalized cost function is established and the link impedance function, put forward to describe classes of multimodal transport system user equilibrium and variational inequality model and use the diagonalization algorithm to solve.

For two or more traffic networks, most scholars are based on complex network empirical research. Mohmand and Wang (2013) [22] conducted a weighted complex network analysis on Pakistan's national highway network's travel routes. The network is responsible for handling 75% of road traffic, but it is inadequate, inferior, and unreliable. The highway network shows the characteristics of the small-world with various properties; Du *et al.* (2016) [23] studied the synchronization problem of the urban bus hypernetwork model using the external synchronization theory of coupled complex network. Zhang *et al.* (2018) [24] analyzed the network characteristics of three kinds of man and studied the vulnerability of man from two aspects of connectivity vulnerability and functional vulnerability by using two malicious attack methods; Ma *et al.* (2020) [25] based on complex network theory, proposed a robust model of two-layer bus metro network from two aspects of structure and function. Based on the urban traffic data of the Xicheng District of Beijing, based on the nonlinear load capacity model, this paper analyzes the cascading failures of the bus metro double-layer network in Xicheng District of Beijing.

It has to be mentioned that there are some limitations in using complex networks to study traffic networks. The related research of complex networks mainly describes the relationship between stations and lines in the network.

However, in real life, the interaction between lines is also the existence of the whole traffic network that can not be ignored. Hypernetwork happens to cover this aspect so that it can be more comprehensive research and analysis of the entire network.

Through the research and summary of the existing literature, we find that in the empirical research, the field of using hypernetwork to study composite transportation network is basically blank. With the considerable investment in rail transit construction in major cities, the urban bus system has gradually developed into a comprehensive transportation system dominated by ground bus and rail transit as the backbone. Therefore, based on the complexity of urban traffic networks, this study applies the hypernetwork theory to the study of the composite traffic network and explores the relationship between single and mixed networks. Taking Xi'an city as an example, this paper combines the ground bus station and metro network in Xi'an City, constructs the Xi'an bus metro hypernetwork model, analyzes the topological characteristics and vulnerability of the composite hypernetwork, and identifies the key stations and lines in the road network.

II. THEORETICAL BASIS

A. THE CONCEPT OF HYPERNETWORK

The existing concept of hypernetwork is divided into network-based hypernetwork and hypergraph-based hypernetwork. The former refers to mixing large-scale and complex connected networks, which is also called "multi-layer network" or "multi-level network"; the latter refers to the evolution of hypergraph edges into hyperedges, which can contain multiple nodes. Hypernetworks mentioned in this study refer to hypergraph-based hypernetworks.

B. HYPERGRAPH

Hypergraph is defined as a binary relation $H = (V, E^H)$, where $V = \{v_1, v_2, v_n\}$ is a finite set and (1) $E_i \neq \phi(1, 2, \ldots, m)$, (2) $U_{i=1}^m E_i = V$, $E^h = \{E_1, E_2, \ldots, E_{|E^h|}\}$. Each element in the set V is called the node of the hypergraph, and the element in E^h is called the hyperedge of the hypergraph. Each node can belong to more than one hyperedge, and each hyperedge can contain multiple nodes. If two nodes belong to one hyperedge simultaneously, the two nodes are adjacent; if the intersection of two hyperedges is not empty, then the two hyperedges are adjacent. Figure 1 is a simple hypergraph.

C. TOPOLOGICAL PROPERTIES OF HYPERNETWORKS

1) NODE DEGREE

In a hypernetwork, nodes are connected through hyperedges. The node degree D_i of node i is defined as the number of nodes directly adjacent to node i. Node degree describes the degree of connection between a node and other nodes. Node degree can be expressed by degree distribution, and the proportion of nodes with D_i in the hypernetwork to the total number of nodes in the network is called node degree distribution.
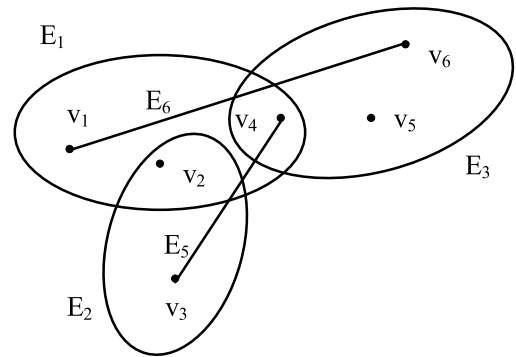

FIGURE 1. Schematic diagram of hypergraph.

2) NODE HYPERDEGREE

Node hyperdegree refers to the number of hyperedges containing the node, which is recorded as D_{Hi}. In a hypernetwork, the greater the node's degree, the greater its role and influence in the whole network, and vice versa. The node hyperdegree distribution can be expressed by probability distribution function $P(d_H)$, which refers to the proportion of nodes whose degree is d_H to the network's total number of nodes.

3) HYPEREDGE DEGREE

Hyperedge degree is the number of other hyperedges connected with the hyperedge, which is recorded as D_{Ei}. Hyperedge degree reflects the degree of connection between hyperedges.

4) HYPEREDGE HYPERDEGREE

Hyperedge hyperdegree refers to the number of nodes contained in the hyperedge.

5) AVERAGE DISTANCE LENGTH

The average distance of hypernetwork describes the connectivity between nodes.

For general hypernetworks, the average distance is

$$\langle D \rangle = \frac{\sum_{i \neq j} D_{ij}}{2 \left(2^N - 1 \right)} \qquad (1)$$

where D_{ij} is the distance between node i and node j, is the minimum number of hyperedges that i and j need to pass through; N is the number of all nodes in the hypernetwork.

For general hypernetworks, the formula's denominator is increased in the form of an exponential function, so the average distance calculated is relatively small. To better describe the connectivity of nodes in hypernetworks, the network efficiency E of general hypernetworks is defined as follows:

$$E = \ln \frac{1}{\langle D \rangle}. \qquad (2)$$

6) AGGLOMERATION COEFFICIENT

In a hypernetwork, the whole network's aggregation coefficient is defined as the ratio of the number of all closed three-point groups (triangles) to the total number
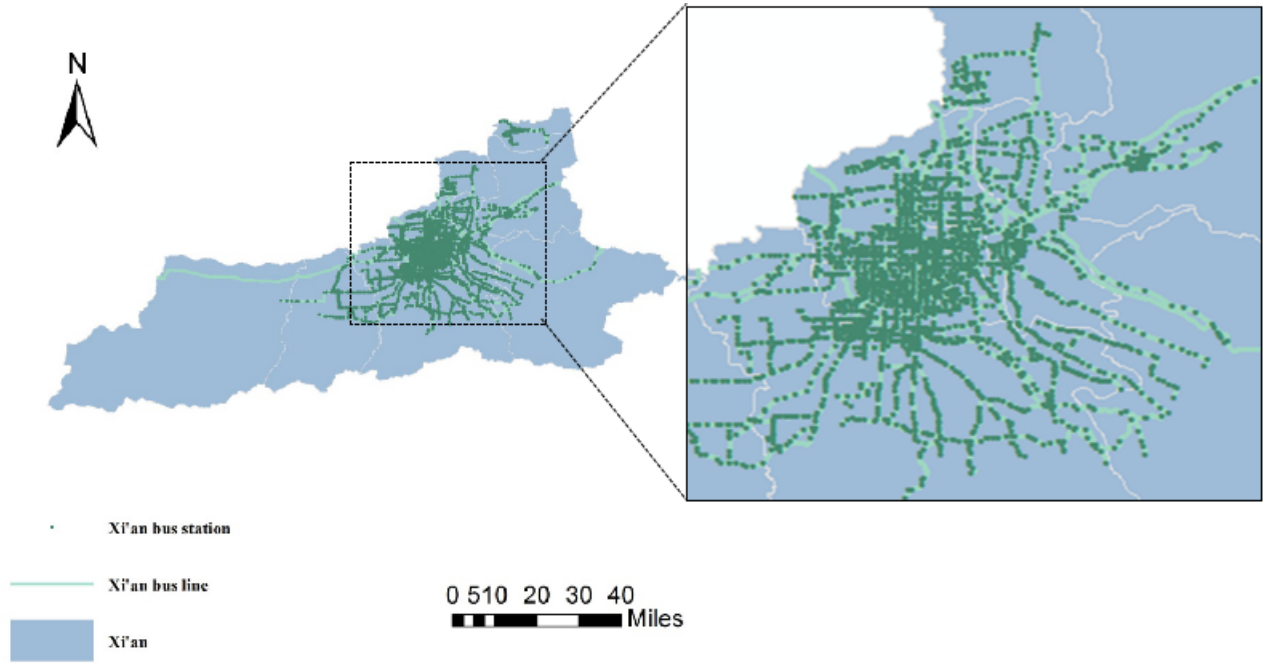
FIGURE 2. Geographical distribution of bus network in Xi'an.

of connected three-point groups (whether open or closed). For hypernetworks, the clustering coefficient is determined by the number of some hypertriangles and all paths with a length of 2. It represents the aggregation degree of the network and indicates whether the neighbor node i is adjacent. It can be expressed as:

$$C_2(H) = \frac{6 \times \text{the number of hypertrian gles}}{\text{number of paths with length 2}} \quad (3)$$

From the above formula, it can be obtained that:

$$C_2(H) = \frac{CW_3 - 6t}{W_2 - CW_2 - 6t} \quad (4)$$

The hyper triangle is a sequence of three different nodes and three different hyperedges $(v_i, E_p, v_j, E_q, v_k, E_r, v_k)$. The three nodes are connected with each other, and the path with a length of 2 is like the sequence of v_i, E_p, v_j, E_q, v_k. CW_3 is the total number of closed chains with a length of 3 in the hypernetwork; CW_2 is the total number of closed chains with a length of 2 in the hypernetwork; W_2 is the number of $v_i - v_j$ chains of length; t is the number of false hypertriangles.

7) GLOBAL EFFICIENCY
The global efficiency of hypernetworks is a measure to define the robustness of hypernetworks, which is measured by the average shortest path distance $<D>$ of hypernetworks. When a station or a line is attacked, the greater the $<D>$ is, the more significant the impact of the corresponding station or line on the network robustness. It can be seen that the station is an essential station of the hypernetwork, and the line is an important line of the hypernetwork. The formula for calculating the

global efficiency E of the hypernetwork is as follows:

$$E = \frac{1}{<D>} \quad (5)$$

III. GENERAL SITUATION OF XI'AN TRAFFIC NETWORK
A. OVERVIEW OF XI'AN BUS NETWORK
As of September 23, 2019, 361 bus lines and 2836 bus stations have been opened in Xi'an. We use E_1–E_{361} and v_1–v_{2836} to represent 361 bus lines and 2836 bus stations, respectively. Observing Xi'an's bus data, we can see that the number of bus stations is uneven. The longest line has 56 stations, and the minimum is only 6 stations. The distance range of bus lines is relatively large, ranging from 2.6 km to 46 km. The shorter the average distance between stations, the denser the distribution of stations. The longer the average distance between stations, the sparser the distribution of stations passing through remote suburbs. Figure 2 shows the geographical distribution of the Xi'an bus network. The bus station is considered a node. If there is a direct bus line between two stations, the two bus stations are connected. The longer line is the trunk line, which plays the role of connecting the main urban area of Xi'an, while the shorter line is usually the extension line connecting the main urban space and the outer suburbs.

Using MATLAB software combined with Xi'an bus data, the adjacency matrix between bus stations is obtained. Combined with the concept of the hypergraph, the Xi'an bus hypernetwork model is constructed for subsequent analysis.

B. OVERVIEW OF XI'AN METRO NETWORK
The Xi'an metro line data comes from the public data released by Xi'an Metro Co, Ltd. Table 1 shows the metro lines opened
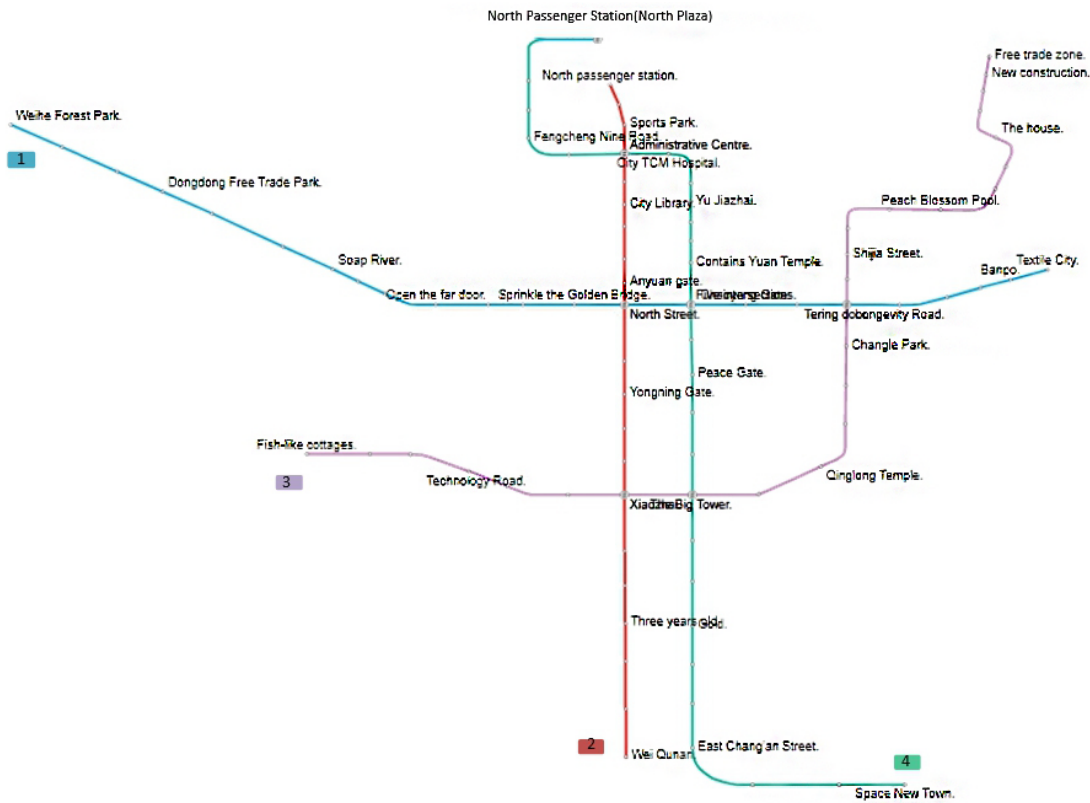
FIGURE 3. Planning map of Xi'an metro line.

TABLE 1. Opening sequence and performance of Xi'an metro.

Opening sequence	Number of stations	Line length(km)	Average length between stations(km)	Opening year
1	19	31.50	1.75	2013
2	21	26.30	1.31	2011
3	26	27.13	1.11	2016
4	28	35.20	1.30	2018

in Xi'an and its related parameters. From 2011 to 2020, Xi'an has opened four metro lines, namely No. 1, No. 2, No. 3, and No. 4. There are 88 metro stations, of which 6 are transfer stations. We use E_{362}–E_{365} and v_{2837}–v_{2924} to represent 4 metro lines and 88 metro stations, respectively. It can be seen from Table 1 that the number of stations of the Xi'an metro line is relatively uniform. The longest line is line 4 with 28 stations, and the shortest line is line 2 with 21 stations. The distance of the Xi'an metro line is relatively long, ranging from 26.30 km to 35.20 km.

Figure 3 is the planning map of Xi'an Metro. It can be seen that the Xi'an metro network is composed of four metro lines, and six transfer stations are connected to facilitate passengers to go to passenger stations, airports, and major commercial centers.

Similar to the bus hypernetwork, the metro station is used as the node by using MATLAB software. If there is a direct metro line between the two stations, the two stations are included with an ellipse to draw the metro hypernetwork model of Xi'an metro.

IV. CONSTRUCTION OF B-M HYPERNETWORK MODEL

The bus network and metro network are composed of different stations and lines. The principle of constructing the B-M hypernetwork is to ignore the mode of transportation, capacity and ticket price of bus and metro, and integrate the bus and metro network into a whole bus network. Its significance lies in the global network analysis, rather than through a single bus network or metro network research.

In this study, the specific methods of constructing the B-M hypernetwork are described as follows:

A. UNDIRECTED UNWEIGHTED HYPERNETWORKS

Taking each station as the node of the hypernetwork and each line as the hyperedge of the hypernetwork, we set up a hypernetwork model of Xi'an bus and metro. Travelers can arrive at station j from station i, and they can also arrive at station i from station j in the same way, due to the neglect of transportation mode, capacity and ticket price of bus and metro. Therefore, it can be considered that the constructed network is an undirected and unweighted simple hypernetwork.

B. TRANSFER STATION DESCRIPTION

Generally speaking, the metro runs faster than the bus. Therefore, in real life, for the same distance or the same starting point and ending point, passengers generally prefer the metro with high speed and short time. After studying the
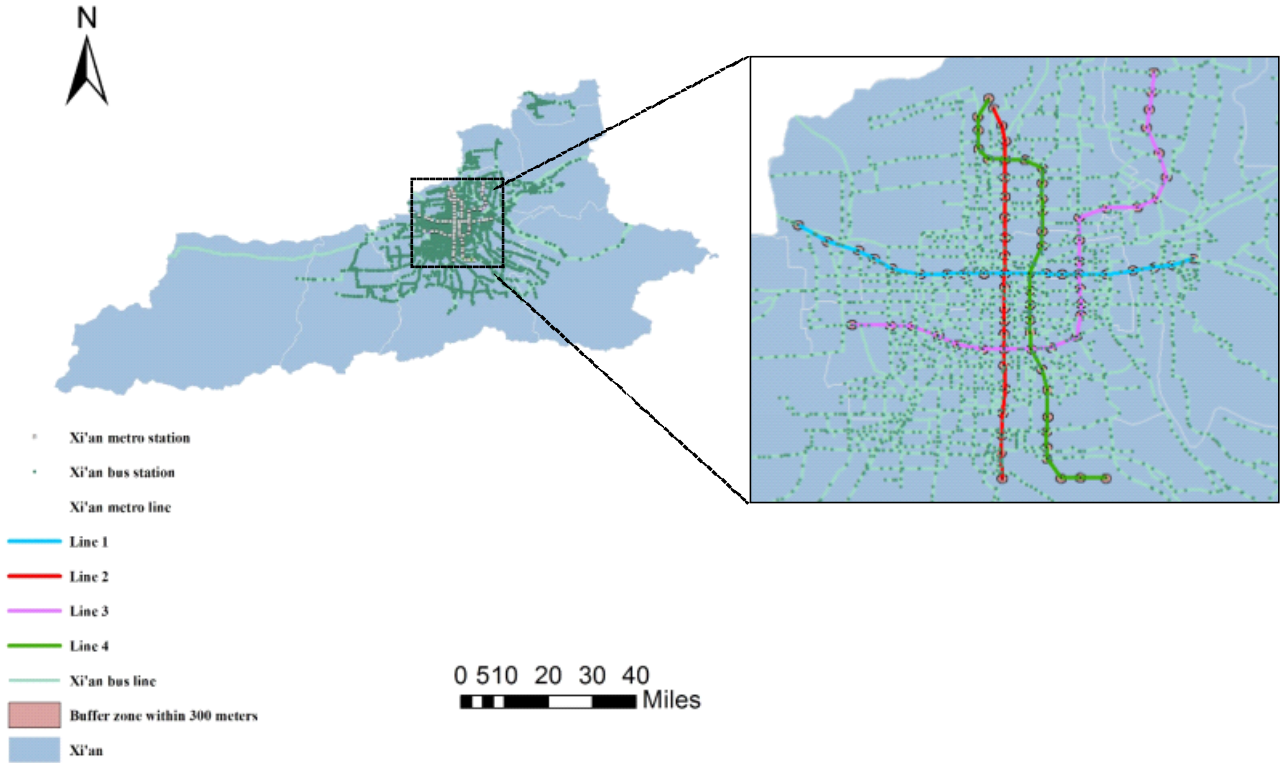
FIGURE 4. Geographical distribution of buffer zone.

relevant literature, we found that the available selection range of connecting the two networks through some stations is 200m, 500m, etcetera in the paper of large-scale composite transportation network. Considering that ordinary people's walking speed is 1m/s, and generally speaking, the walking time range of transfer between different public transport networks acceptable to pedestrians is 5-10 minutes. According to the calculation, the bus station and metro station's acceptable distance is [300m], [600m].

Besides, in the ArcGIS platform, the distance between the two stations in Xi'an city's base map is the spatial straight-line distance. However, bus stations and metro stations are often connected by overpass or underground passage in real life so that the actual pedestrian transfer distance will be slightly larger than the straight-line distance between stations in the ArcGIS base map. Considering this, we take the linear distance between Xi'an bus station and metro station as the lower limit of [300m], [600m], that is, [300m].

Add the base map of Xi'an bus network and metro network to the ArcGIS platform, and takes each metro station as the center, and draws a circle with a radius of 300m as the buffer zone, as shown in Figure 4. If a bus station falls into the buffer zone, it is regarded as the metro station's transfer station. The other bus stations that are not within the buffer zone, that is, residents will not transfer between bus and metro through these stations, but they are also indirectly connected to the metro network through the virtual hyperedge. A metro station
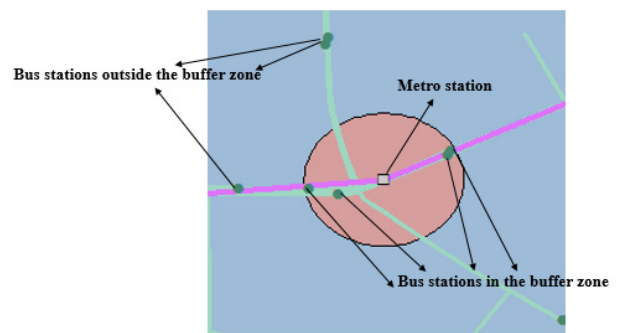

FIGURE 5. Schematic diagram of buffer zone.

may have several transfer bus stations at the same time, or there may be no transfer bus station in this buffer zone, as shown in Figure 5. In the hypernetwork, the connecting edge between the metro station and its transfer bus station is also regarded as the hyperedge, through which the bus network and the metro network are connected.

Using ArcGIS, the bus stations within 300m of each metro station are obtained, and the attribute table information is derived. We got 102 transfer lines altogether and named "transfer No.1", "transfer No.2", "transfer No.3", ..., "transfer No. 102", which is indicated by $E_{366}-E_{467}$. These 102 transfer lines are also hyperedge. Table 2 lists some bus stations' names connected with six important transfer stations in the metro network by virtual "hyperedge".

TABLE 2. Bus stations within 300m buffer zone of transfer metro station.

Serial number	Metro Station	Number of bus stations	Bus stations
1	Tonghua gate	4	Middle section of Wannian Road East 2nd Ring Road Jinkang intersection [Jinhua Road] Dongerhuan Changle intersection [light industry market] Xinhe intersection of East 2nd Ring Road [Jinhua Road]
2	Xiaozhai	2	Xiaozhai [Xiaozhai North] Xiaozhai
3	North Street	1	North Street
4	Wulu Road	1	Wulu Road
5	Big Wild Goose Pagoda	1	Dayan Pagoda North Square
6	Administrative center	0	—
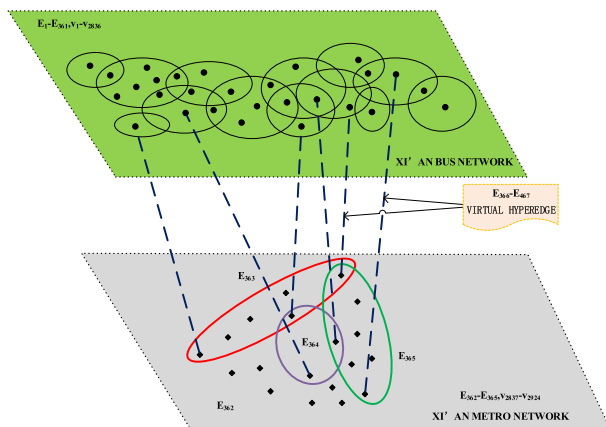
FIGURE 6. Schematic diagram of Xi'an B-M hypernetwork.

FIGURE 7. Degree distribution and cumulative probability distribution of node degree.

C. CONSTRUCTION OF INCIDENCE MATRIX

Incidence matrix is a matrix used to represent the relationship between nodes and edges in a network., a hypernetwork based on a simple hypergraph has a unique incidence matrix. In the Xi'an B-M hypernetwork model, the incidence matrix represents the relationship between nodes and hyperedges. In this model, it involves two networks: the bus network and the metro network. Simultaneously, a virtual hyperedge is constructed by using "Hyperedge" to connect the two networks. Therefore, the incidence matrix of Xi'an B-M hypernetwork model consists of six parts: the incidence matrix composed of bus stations and bus lines (2836*361), the incidence matrix composed of bus stations and metro lines (2836*4), the incidence matrix composed of bus stations and virtual hyperedge (2836*102), and the incidence matrix composed of metro stations and bus lines (88*361), the incidence matrix consisting of the metro station and metro line (88*4), and the incidence matrix composed of metro stations and virtual hyperedge (88*102), as shown in "(6)".

In the matrix, if the node m is on the hyperedge n, then the value of (v_m, E_n) in the matrix is 1; otherwise, it is 0. Therefore, the Xi'an B-M hypernetwork model's incidence
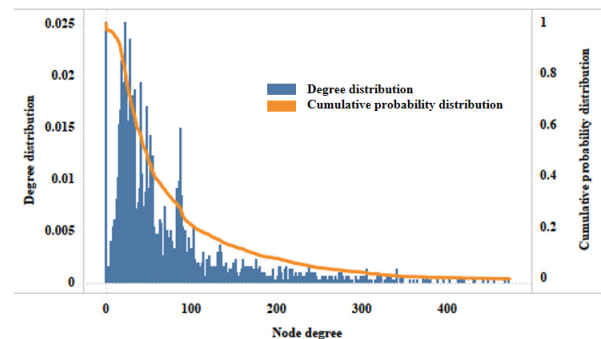
Therefore, there are 467 hyperedges and 2924 nodes in the Xi'an B-M hypernetwork. Figure 6 is the schematic diagram of Xi'an B-M hypernetwork.

matrix is a 0-1 matrix with 2924 rows and 467 columns.

$$M_{2924*467} = \begin{bmatrix} [\quad]_{2836*361} & [\quad]_{2836*4} & [\quad]_{2836*102} \\ [\quad]_{88*361} & [\quad]_{88*4} & [\quad]_{88*102} \end{bmatrix} \quad (6)$$

V. ANALYSIS OF TOPOLOGICAL CHARACTERISTICS OF HYPERNETWORKS

According to the Xi'an B-M hypernetwork model's incidence matrix, we can calculate the topological characteristics of the Xi'an B-M hypernetwork. Besides, we compare the three hypernetworks of Xi'an Bus hypernetwork, Xi'an Metro hypernetwork, and Xi'an B-M hypernetwork, and explore the influence of "Hyperedge" on the bus and metro network.
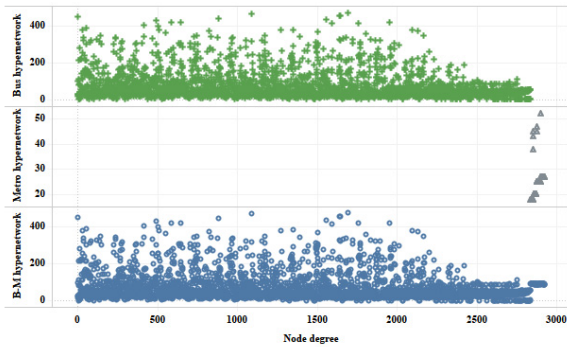
A. NODE DEGREE

Node degree refers to the number of other stations adjacent to a station, reflecting its passenger flow and transfer capacity. According to the calculation of incidence matrix M, the average node degree value of Xi'an B-M hypernetwork is 71, which indicates that each station can transfer to 71 different stations on average. According to the node degree of hypernetwork, the key stations are shown in Table 3. The station with the largest node degree is Nanshaomen, with a node degree of 472. The degree distribution and cumulative probability distribution of node degree in Xi'an B-M hypernetwork model are shown in Figure 7. The statistical results show that the cumulative probability distribution of node degree follows an exponential distribution, the exponential function

TABLE 3. Top 5 stations with node degree in Xi'an B-M hypernetwork model.

Serial number	Node label	The name of the station	Node degree
1	V_{1694}	Nanshaomen	472
2	V_{1093}	Municipal Library	467
3	V_{1640}	Gaoxin road science and technology intersection	455
4	V_{1649}	Beiguan	454
5	v_1	Xinjiamiao bus dispatching station	447

TABLE 4. Top 5 stations with node hyperdegree in Xi'an B-M hypernetwork model.

Serial number	Node label	The name of the station	Node hyperdegree
1	V_{1694}	Nanshaomen	838
2	V_{1640}	Gaoxin road science and technology intersection	819
3	v_1	Xinjiamiao bus dispatching station	787
4	V_{1649}	Beiguan	776
5	V_{1596}	Yuxiang gate	744

FIGURE 8. Schematic diagram of node degree comparison of three hypernetworks.

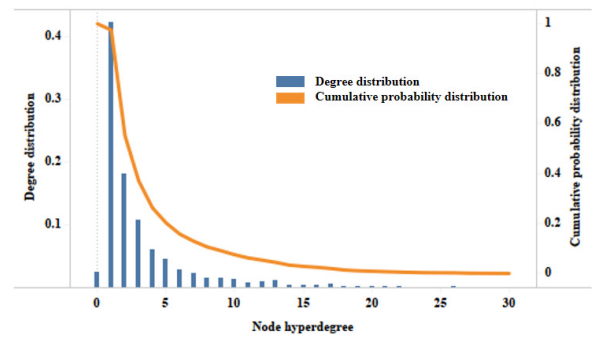**FIGURE 9.** Degree distribution and cumulative probability distribution of node hyperdegree.

fitted is as follows: $y = 1.1495e^{-0.015x}$. According to the exponential distribution nature, most of the nodes have the same number of connections, and few nodes are much higher or lower than the average, and the difference is not as evident as power-law distribution. This shows that the load planning of the B-M hypernetwork is relatively uniform and can meet the passenger travel demand.

The node degrees of 2836 bus stations in the bus hypernetwork and 88 metro stations in the metro hypernetwork are calculated respectively, and compared with the node degrees of 2924 nodes in the B-M hypernetwork, as shown in Figure 8. After the use of ''Hyperedge'' to connect the bus hypernetwork and the metro hypernetwork, the node degree of 2836 bus stations changed little, only 102 stations increased by 1, and the node degree value of other stations did not change; however, the change ratio of node degree of 88 metro stations was more extensive, and the difference was in the interval [23], [72]. In the metro hypernetwork, the node degree of each station is related to the metro line. The node degrees of the stations above the four metro lines are 18, 20, 25, 27 respectively. Among them, the node degrees of six transfer stations are relatively large, which are 38, 43, 45, 45, 47, 52, respectively. In the B-M hypernetwork, the node degree values of 88 metro stations are between [87], [92].

Whether it is the slight change of node degree of 2836 bus stations or the significant change of node degree of 88 subway stations, they are all caused by 102 ''hyperedges'' constructed by us.

B. NODE HYPERDEGREE

Node hyperdegree refers to the number of lines passing through a station. According to the calculation of incidence matrix M, the average node hyperdegree value of Xi'an B-M hypernetwork is 91, which means that 91 lines are passing through each station on average. Corresponding to the node degree, the station with the largest node hyperdegree is Nanshaomen, and its node hyperdegree is 838. According to the hypernetwork node hyperdegree, the key stations are shown in Table 4. The degree distribution and cumulative probability distribution of node hyperdegree in Xi'an B-M hypernetwork model are shown in Figure 9. The statistical results show that the cumulative probability distribution of node hyperdegree follows an exponential distribution, the exponential function fitted is as follows: $y = 1.1353e^{-0.241x}$.

From the cumulative probability distribution of node hyperdegree, we can see that for the same node, if it is located in a large number of lines, this kind of node is relatively small in the hypernetwork, which also conforms to the ''pareto principle'' in life. The stations that often carry a lot of passenger flow are usually a small part, and most of the rest are mainly responsible for the connection.

The relationship between node degree and node hyperdegree is shown in Figure 10. As shown in Figure 10, the node hyperdegree of a station with a large node degree is also large.

Similar to the node degree, the node excesses of 2836 bus stations and 88 metro stations in the three hypernetworks are calculated and compared, as shown in Figure 11. Because the node degree of the same station is directly proportional
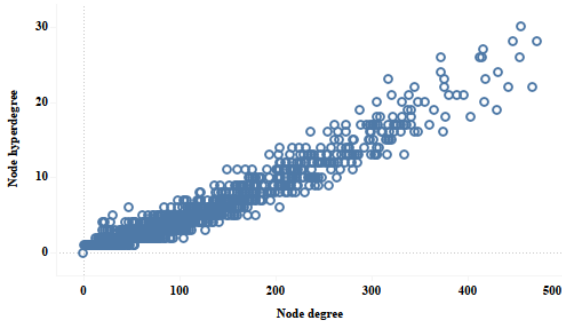
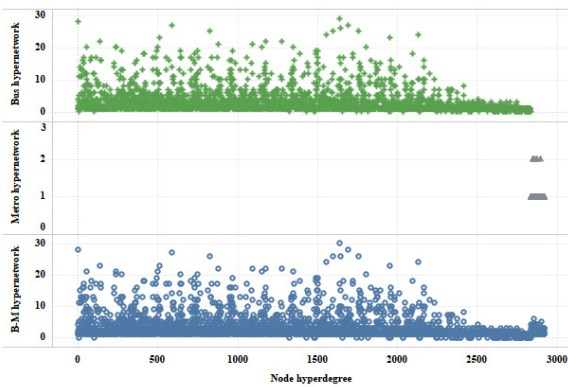**FIGURE 10.** Relation graph of node degree and node hyperdegree.

FIGURE 11. Schematic diagram of node hyperdegree comparison of three hypernetworks.

FIGURE 12. Degree distribution and cumulative probability distribution of hyeregde degree.

TABLE 5. Top 5 lines with hyperedge degree in Xi'an B-M hypernetwork model.

Serial number	Hyperedge label	The name of the line	Hyperedge degree
1	E_{296}	Line 711	158
2	E_{188}	Line 262	151
3	E_{292}	Line 701	150
4	E_1	Second ring line 1	149
5	E_2	Second ring line 2	149
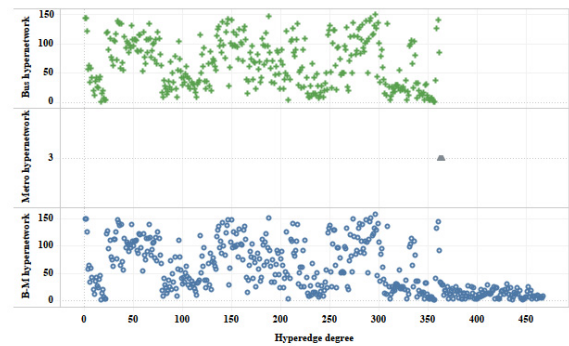
FIGURE 13. Schematic diagram of hyperedge degree comparison of three hypernetworks.

to the node overrun, the change of each station's overrun value is very similar to that of the node degree value. After using the "Hyperedge" to connect the bus hypernetwork and the metro hypernetwork, only 102 of the 2836 bus stations have increased the node degree value by 1, while the node degree value of other stations has not changed; while the change difference of node overrun value of 88 metro stations is different in the interval [0, 4]. In the metro hypernetwork, there are only two values of node hyperdegree: 1 and 2. The node hyperdegree of the six transfer stations is 2, and the others are 1. In the B-M hypernetwork, the node hyperdegree of 88 metro stations is in the interval [1, 6].

C. HYPEREDGE DEGREE

Hyperedge degree refers to the number of other lines passing through the same station as a certain line. According to the calculation of incidence matrix M, the average value of hyperedge degree is 56, which means that all stations of each line can transfer to 56 other lines on average. The crucial lines obtained from the hyperedge degree are shown in Table 5. The degree distribution and cumulative probability distribution of hyperedge degree in Xi'an B-M hypernetwork model are shown in Figure 12. The results show that the cumulative probability distribution of hyperedge degree obeys the linear distribution, and the fitting function is as follows: $y = -0.0062x + 0.826$. The hyperedge degree reflects the overall transfer capacity of a certain line.
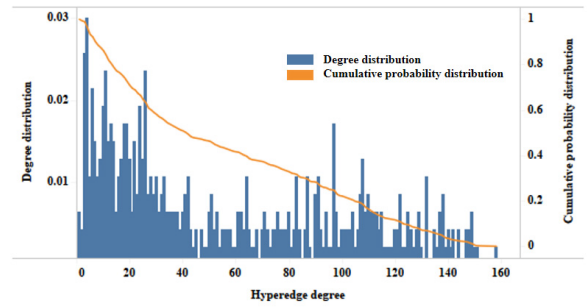
The hyperedge degrees of 361 bus lines in the bus hypernetwork and 4 metro lines in the metro hypernetwork are calculated respectively, and the hyperedge degrees of 361 bus lines and 4 metro lines corresponding to 467 hyperedges in the B-M hypernetwork are compared, as shown in Figure 13. After the use of "Hyperedge" to connect the bus hypernetwork and the metro hypernetwork, the hyperedge degree of 361 bus lines changes very little, and the change range is only between [0, 14], while the hyperedge degree of four metro lines changes relatively large. In the Metro hypernetwork, the out of edge values of 4 metro lines are all 3; in the B-M hypernetwork, the out of edge values of 4 metro lines become 33, 32, 29, 29 respectively.

For 361 bus lines and 4 metro lines, the increase of outliers is similar to the rise of node degree and node overrun of each station, which is caused by the virtual

TABLE 6. Top 5 lines with hyperedge hyperdegree in Xi'an B-M hypernetwork model.

Serial number	Hyperedge label	The name of the line	Hyperedge hyperdegree
1	E_{296}	Line 711	551
2	E_{274}	Line 600	534
3	E_2	Second ring line 2	508
4	E_1	Second ring line 1	504
5	E_{212}	Line 301	472
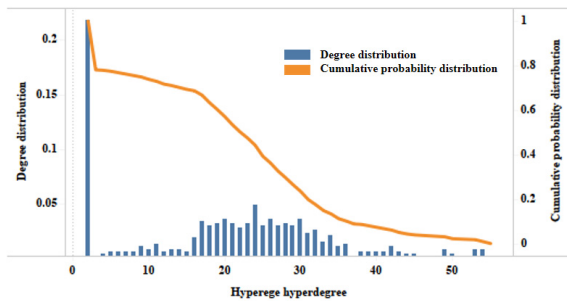
FIGURE 14. Degree distribution and cumulative probability distribution of hyperegde hyperdegree.

FIGURE 15. Schematic diagram of hyperedge hyperdegree comparison of three hypernetworks.

"Hyperedge" constructed. For the added part, it can be divided into two categories. For bus lines, one is the metro line, the other is the constructed virtual " hyperedge"; for the metro line, one is the bus line, and the other is the created virtual "hyperedge".

D. HYPEREDGE HYPEREDGE

Hyperedge hyperdegree refers to the number of stations contained in a certain line. According to the calculation of incidence matrix M, the average hyperedge hyperdegree is 151, that is, each line comprises 151 stations on average. The crucial lines obtained from the hyperedge hyperdegree are shown in Table 6. The line with the largest hyperedge hyperdegree is bus line 711, with a hyperedge hyperdegree of 472. The degree distribution and cumulative probability distribution of hyperedge hyperdegree in Xi'an B-M hypernetwork model are shown in Figure 14. The statistical results show that the cumulative probability distribution of hyperedge hyperdegree accords with the linear distribution, as shown in Figure 14, and the fitting function is as follows: $y = -0.02x + 0.8987$, $R^2 = 0.9501$. HyperEdge hyperdegree can reveal the area and population size served by a certain line.

Similar to the hyperedge degree, the hyperedge hyperdegree of 361 bus lines and 4 metro lines in the three hypernetworks is calculated and compared, as shown in Figure 15. Compared with the data, it is found that no matter 361 bus lines or 4 metro lines, their overrun values have no change. It is mainly because the concept of hyperedge hyperdegree refers to the number of stations on a certain line. In the three

hypernetworks, the relative positions of bus stations and bus routes have not changed, and the relative positions of metro stations and metro lines have not changed. Also, considering the existing traffic network, its change is almost impossible. Therefore, the change of hyperedge hyperdegree of 361 bus lines and 4 metro lines is zero.

E. AVERAGE DISTANCE

The average distance describes the average degree of separation between nodes in a hypernetwork. According to the average distance calculation formula in the general hypernetwork, the average length of the Xi'an B-M hypernetwork is 1.9199. That is, any two nodes can be reached through 1.9199 lines on average.

Similarly, the bus network's average shortest distance is 4.2939, and that of the metro is 1.7168. Compared with the B-M hypernetwork, it can be seen that after the two hypernetworks are connected, the average shortest distance relative to the bus hypernetwork decreases from 4.2939 to 1.9199, while the average shortest distance relative to the metro hypernetwork increases from 1.7168 to 1.9199.

From the perspective of urban traffic development, the construction of metro stations and the opening of metro lines greatly improve residents' travel convenience. And the existence of buses seems to increase residents' travel and transfer. Due to the long investment period and high investment cost, the return on investment in remote urban areas is far lower than that in urban centers. The existence of buses makes up for residents' difficulties to get to the surrounding city by metro.

F. CLUSTERING COEFFICIENT

The clustering coefficient of hypernetwork indicates the closeness of the connection between stations and lines. Generally speaking, the nodes in the hypernetwork are compact, and the clustering of the network is between [0, 1]. Through calculation, we got the clustering coefficient of Xi'an B-M hypernetwork is 0.7077, which is relatively large compared with the median value of 0.5. This shows that the Xi'an B-M hypernetwork's overall density is good, and the possibility of distance connection between nodes through hyperedge connection is relatively large. Compared with the network of

the same scale, the clustering property of the hypernetwork is very high.

We also calculate the clustering coefficient of bus hypernetwork and metro hypernetwork and get 0.7031 and 0.9659, respectively.

It is worth mentioning that the clustering coefficient of metro networks in the relevant literature is relatively small because most of them use the traditional complex network to construct the network model and subsequent analysis, such as spatial L models, spatial P models, etcetera. In the complex network, mainly describes the relationship between points or lines but largely ignores the relationship between stations and lines. But hypernetworks make up for that.

It is easier to understand the difference by mapping it into the unique adjacency matrix corresponding to the network. In traditional complex networks, only two nodes are directly connected, and there is no other node in the middle. The element in the corresponding adjacency matrix is 1; otherwise, it is 0. However, in a hypernetwork, if some nodes are on the same line, the corresponding adjacency matrix elements formed by any two of these nodes are 1. Mapping to this paper, if two stations belong to the same line, then from one station to another station is not required to transfer, can be directly arrived.

According to the adjacent matrix of Xi'an Metro hypernetwork and the calculation formula of the clustering coefficient, we use MATLAB software to program and finally get the clustering coefficient of Xi'an Metro hypernetwork is 0.9659, which is in line with the characteristics of a small-world network. Considering that there are only 88 metro stations and 4 metro lines in Xi'an metro network, such a conclusion is not questionable.

Compared with bus hypernetwork and B-M hypernetwork, Metro hypernetwork itself is a small world network with super high concentration. Compared with the clustering coefficient of the B-M hypernetwork of 0.7077, after the two hypernetworks are connected, the clustering coefficient is slightly increased compared with the bus hypernetwork, while compared with the Metro hypernetwork, the clustering coefficient is significantly reduced. This is mainly due to the rapid increase in the number of stations and lines after the network connection.

In this part, we mainly calculate the static index of Xi'an B-M hypernetwork topology characteristics and get the crucial stations and critical lines in the whole network. In the next part of the robustness analysis, we can determine the sequence of intentional attacks according to this order.

VI. ROBUSTNESS ANALYSIS OF XI'AN B-M HYPERNETWORK

The robustness of hypernetworks is one of the essential dynamic properties of hypernetworks. Robustness refers to the anti-interference, firmness, and stability of the hypernetwork. If a system maintains its function when disturbed, the system is robust.

A. DESIGN OF ATTACK STRATEGY

In the traditional complex network-related research, attack methods are often divided into random attack and deliberate attack. Random attack refers to the situation of unexpected accidents and bad weather in the regular operation of the system. Deliberate attack relates to road construction, attack and damage to the station or line according to certain requirements. Based on the unique incidence matrix of Xi'an B-M hypernetwork, four attack strategies can be designed: random station attack, random line attack, deliberate station attack, and deliberate line attack. Under these four attack strategies, the global efficiency E of the whole hypernetwork is calculated.

The four attack strategies are mapped into the incidence matrix. The attacked stations/lines in random station attack and random line attack are determined by generating random numbers. The order of attacked stations in deliberate station attack is determined by combining node degree and node hyperdegree. The attack line order in deliberate line attack is determined by combining hyperedge degree and hyperedge hyperdegree. The corresponding node/hyperedge is deleted from the incidence matrix M to form a new hypernetwork incidence matrix M_1, and the global efficiency E_1 of the hypernetwork is calculated again; the above steps are repeated until all the stations/lines in the hypernetwork are deleted, and the influence of each station/line fault on the global efficiency of the hypernetwork is observed.
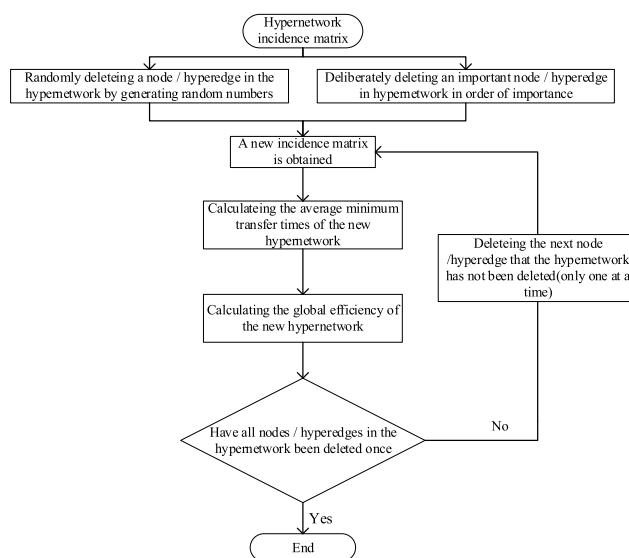
The attack strategy flow chart is shown in Figure 16:

FIGURE 16. Flow chart of attack strategy.

B. ALGORITHM DESIGN

To calculate the global efficiency of the hypernetwork, we first need to get the minimum transfer times between any two stations, which is equivalent to seeking the shortest path from multiple sources. Floyd algorithm is used to solve it. The traditional Floyd algorithm is based on the graph

adjacency matrix to calculate the shortest path between the vertices, and the time complexity of the Floyd algorithm is $O\,(N^3)$, so it is not suitable to operate a large number of data. There are 2924 nodes and 467 hyperedges in the Xi'an B-M hypernetwork. If the traditional Floyd algorithm is used, its time complexity will reach $O(2924^3)$ and $O\,(467^3)$. Therefore, we improve the Floyd algorithm based on a similar traffic composite hypernetwork. In this paper, we need to consider the minimum number of the transfer. In addition to the connection between stations, we should also consider the relationship between stations and lines as well as the relationship between lines. Any station belongs to one or more lines, and there is an intersection or separation relationship between lines. The incidence matrix of hypernetwork can be used to summarize these relations.

We use the Floyd algorithm's core idea to calculate the minimum number of multi-source transfers in the hypernetwork incidence matrix. When seeking the minimum transfer times from node v_a to node v_b, we first judge whether node v_a and node v_b belong to the same hyperedge. If node v_a and node v_b belong to the same hyperedge, the number of transfer can be directly recorded as 0; if node v_a and node v_b does not belong to the same hyperedge, it is necessary to cross the hyperedge to find the hyperedge where node v_a is and the hyperedge where node v_b is located. It needs several public nodes to intersect them. The number of shared nodes found through the scheme with the least common nodes is the minimum transfer times from node v_a to node v_b.

After getting the minimum transfer times of any two nodes, we can get the average minimum transfer times of the whole hypernetwork and then get the global efficiency of the hypernetwork. When designing an attack, whether it is a random attack or a deliberate attack, it is only necessary to delete the row of data in the original incidence matrix of the hypernetwork when attacking the station, while only the data of the line in the original incidence matrix of the hypernetwork can be deleted when attacking the line. Then we calculate the global efficiency of the new supernetwork according to the multi-source minimum transmission algorithm.

C. ROBUSTNESS ANALYSIS

According to equation (5), we calculate the global efficiency of the whole hypernetwork is 0.5209. According to the attack strategy flow chart in Figure 16, random attack and deliberate attack are carried out on Xi'an B-M hypernetwork, respectively.

In the hypernetwork, both the bus network and the metro network exist in reality. Due to a large amount of capital investment in the establishment process and the dependence of passengers in the operation process, all metro stations of the same metro line cannot be destroyed or stationped at the same time; similarly, in the bus network, because there are 2836 bus stations and 361 bus lines, each bus station does not only belong to one bus line. If a bus line is completely removed from the road network, the impact on the whole network is vast. Besides, many bus lines have the same
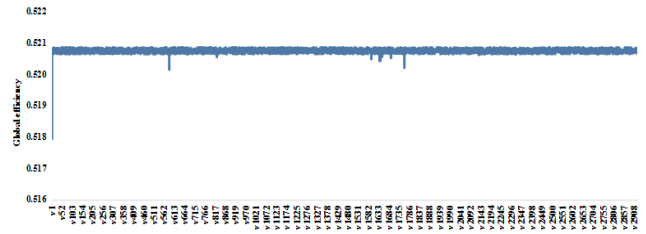
FIGURE 17. Change of global efficiency of hypernetwork after randomly deleting nodes.
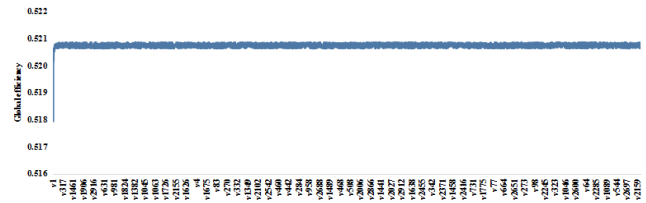
FIGURE 18. Change of global efficiency of hypernetwork after deliberately deleting nodes.

route in real life and pass through several stations with the same order and number. When such bus lines are stationped, the impact on the whole network is minimal. The virtual lines connected by the ''Hyperedge'' of the hypernetwork are all within 300 meters of each metro station. In the site selection and design of metro stations, it is often the grid with large passenger flow that will be considered to build metro stations. The bus stations within the corresponding scope are also bus stations with large passenger flow in the bus network, so passengers will not abandon such virtual lines. To sum up, it is not significant to attack the line in Xi'an B-M hypernetwork. Therefore, in this paper, we only shoot the stations, random attack and intentional attack.

1) RANDOM ATTACK

After random attack on the station, the global efficiency change of the hypernetwork after randomly deleting nodes is shown in Figure 17. As can be seen from Figure 17, the hypernetwork efficiency at the triangle broken line fluctuates wildly. For example, V_1 (xinjiamiao bus dispatching station), v_{588} (people's paradise), v_{1596} (Yuxiang gate), v_{1640} (Science and technology intersection of Gaoxin Road), v_{1694} (Nanshaomen), v_{1761} (North Street) and other stations are at the three corners broken line. If the node at the triangle broken line breaks down, the global efficiency of the hypernetwork will fluctuate significantly, which will add many unexpected states to the traffic network of Xi'an city. Therefore, we should pay attention to these stations, do an excellent job of emergency prevention and protection in case of sudden failure, prevent these stations from loss, and avoid large disturbance in the hypernetwork.

2) DELIBERATE ATTACK

When attacking the station intentionally, the single node with a larger node degree and node hyperdegree is first
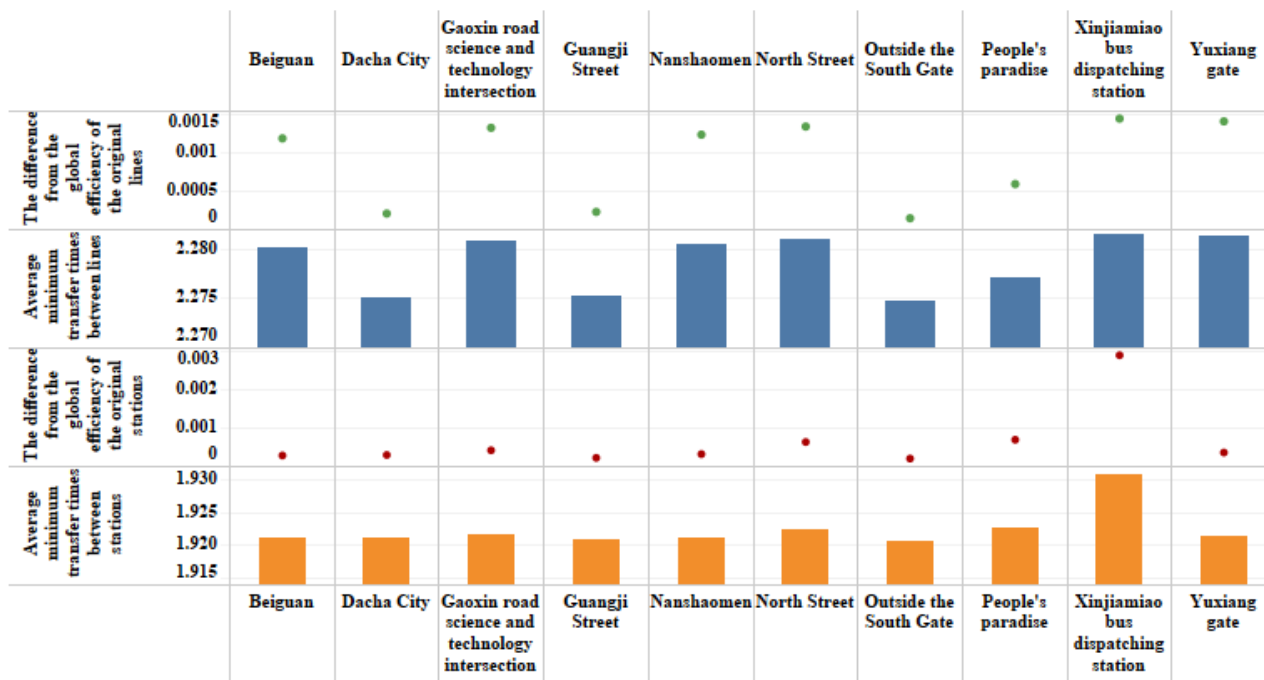
FIGURE 19. The impact of attacking important stations on the average minimum transfer times and global efficiency between stations and lines.

attacked. For example, when attacking v_{1640} (Gaoxin road science and technology intersection), the hypernetwork efficiency decreases from 0.5209 to 0.5204; when attacking v_{1694} (Nanshaomen), the hypernetwork efficiency decreases from 0.5209 to 0.5205. Obviously, compared with the failure of v_{1694}, the failure of v_{1640} has a more significant impact on the efficiency of the whole hypernetwork. The changes of hypernetwork efficiency after deliberately attacking nodes with high node degree and node hyperdegree are shown in Figure 18: v_1 (xinjiamiao bus dispatching station), v_{588} (Minyuan Park), v_{1761} (North Street), v_{1640} (Gaoxin Road intersection), v_{1596} (Yuxiang gate), v_{1694} (Nanshaomen), v_{826} (Dacha City), v_{1649} (Beiguan), v_{2133} (Guangji Street) and v_{1557} (outside the South Gate).

The failure of these ten stations has the most significant impact on the efficiency decline of hypernetwork. These ten stations are undoubtedly the most essential and critical stations in Xi'an B-M hypernetwork. The failure or damage of these stations will make the efficiency of Xi'an B-M hypernetwork decline the most. In operation management, disaster prevention and control should be carried out in advance for these stations to avoid damage and minimize loss.

VII. CONCLUSION

With the concept of giving priority to the development of urban public transport has been widely implemented in major cities, and the comprehensive transportation system with the ground bus as the main body and rail transit as the backbone has achieved good results in some large cities, major cities have also begun to vigorously develop rail transit, with huge investment in rail transit construction.

This paper takes Xi'an bus network and metro network as the research object, based on the hypernetwork theory, taking bus station and metro station as nodes, bus line and metro line as hyperedge, and constructs bus hypernetwork model and metro hypernetwork model. According to the definition of "Hyperedge" in the hypernetwork, that is, the existing or virtual edges of the nodes are hyperedges in the hypernetwork, the virtual "Hyperedge" is constructed to connect the public transport network and the railway network effectively, and then the Xi'an B-M hypernetwork model is built.

By the start date of the study, 361 bus lines and 4 metro lines have been opened in Xi'an, 2836 bus stations and 88 metro stations have been built. Among them, Metro Line 1, line 2, line 3, and line 4 are the main lines, supplemented by bus lines, forming a topological structure radiating from the central urban area to the surrounding suburbs, which plays a vital role in the whole of Xi'an city.

For B-M hypernetwork, it does not merely overlap bus hypernetwork and Metro hypernetwork but connects the two networks through virtual "walking" lines through some public or adjacent stations. This kind of "walking" dotted line is often manifested as pedestrian overpass or underpass in the city. After constructing three hypernetwork models, we analyze the interaction mechanism between urban public transport and metro by comparing six static topological characteristics of node degree, node overrun, hyperedge degree, average shortest distance and aggregation coefficient, and obtain the critical stations and key lines in B-M hypernetwork in a certain sense. It is found that the reasonable and efficient connection and complementary advantages of metro

and public transport can improve the overall efficiency of the public transport system and significantly improve the quality of travel and life of residents. The metro can complete a larger scale of passenger transport, breakthrough certain physical space restrictions, and make the connection between any two places in the city more direct; the ground bus is more flexible, which can cover the areas that the metro can not reach.

After that, we analyze the dynamic topological characteristics of the B-M hypernetwork and test its overall stability. The average minimum transmission time and global efficiency are used to evaluate the robustness of the B-M hypernetwork under random attack and deliberate attack. The crucial sites' attack order is determined according to the importance of the sites obtained in the last part. In fact, the transfer between stations in the network can be simplified as the transfer between lines. On this basis, we improve the traditional node attack and hyperedge attack. This paper mainly attacks the nodes in the hypernetwork, combined with the unique incidence matrix of the hypernetwork, carries on the matrix transformation, obtains the influence of the route corresponding to the site on the whole hypernetwork when being attacked, so as to better study the relationship between stations and lines in the network.

In order to reduce the computational complexity, we propose a multi-source minimum transmission algorithm based on the incidence matrix. Using this algorithm, we get the initial minimum number of inter-node transmission is 1.9199, and the initial minimum number of transfers between lines is 2.2740. Then, according to the random attack and deliberate attack on the station, the average minimum transmission time and global efficiency between stations are calculated, and the transmission relationship between lines is also calculated. The influence of random attack on the network is more significant than that of random attack.

Figure 19 shows the changes in average minimum transmission time and global efficiency of Xi'an B-M hypernetwork when 10 important sites are deliberately attacked. From the point of view of station line relationship, it can be divided into the following four categories: the change of average minimum transfer times between stations after the attack, the change of average minimum transfer times between lines after attack, the change of global efficiency between stations after attack and the change of global efficiency between lines after attack. It can be found that when high importance sites are deliberately attacked, the influence between lines in the hypernetwork is different. We can infer that when evaluating the importance of a node in a network, we should not only consider the importance of the station in all stations, but also consider the importance of its line in all lines.

According to the research results, the key stations and important lines in the whole B-M hypernetwork can be determined. In the daily operation and management of these vital bus stations and metro stations, disaster prevention and control should be carried out in advance to avoid

and reduce losses, which also provides certain management enlightenment for supporting urban traffic planning. It can be extended to other cities and even the whole country to study the interaction mechanism of various traffic modes in a city and analyze its overall robustness. The verification of Xi'an B-M hypernetwork shows that the algorithm has the same applicability for similar large-scale complex hypernetworks.

This study is only the first step for us to study transportation hypernetwork. After that, we will add the factors such as passenger volume, ticket price and travel time to weight the hypernetwork so that our research is closer to reality and has more practical significance. In the future, more metro and bus lines will be opened in Xi'an. How to make the bus and metro better cooperation, further shorten the diameter of the hypernetwork, reduce the number of passengers transfer in the composite network, improve the transport efficiency of the urban public transport network, and better play the advantages of public transport and metro, is also our future research direction.

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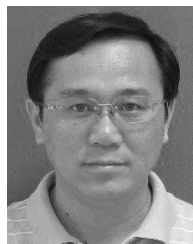
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