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Characterizations for Matrices of Dominant Support Parameters in Soft Sets

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ABSTRACT The matrix of dominant support parameters plays an important role in solving the normal and pseudo parameter reduction problems for soft sets. This article aims to make a fundamental investigation on the properties of matrix of dominant support parameters. Firstly, we obtain some basic structural and quantitative properties. Then we give some retrieving algorithms and filling algorithms for computing the initial soft sets and the matrix itself by using only part of the matrix. Next we propose some characterization theorems to check which kind of set-valued matrices can be induced by a soft set as its matrix of dominant support parameters. Finally, we make a comparison between the matrix of dominant support parameters and the soft discernibility matrix. It's shown that the matrix of dominant support parameters has its own characteristic and can represent the soft discernibility matrix in a simple way. An alternative and simple procedure for computing the order relations with the matrix of dominant support parameters is brought in.

INDEX TERMS Soft set, matrix of dominant support parameters, normal parameter reduction, soft discernibility matrix.

I. INTRODUCTION

A. SOFT SET AND CHOICE VALUE

In 1999 Molodtsov initiated the theory of soft set, which represented a new mathematical tool for dealing with uncertainties and vagueness [1]. The soft set theory has been studied algebraically [2]–[11] and topologically [12]–[15], and it has also been combined with types of vague concepts such as fuzzy model set [16]–[24] and rough set [25]–[27]. A hypersoft set was studied in [28]. The theory of soft sets has been shown useful for decision making in various fields [18], [29]–[34].

A soft set can be regarded as a 0-1 valued information system [35]. It can be represented by a 0-1 valued tabular or a 0-1 valued matrix. In a soft set over U, the choice value of an object has been defined as its number of supporting parameters, i.e., the sum of its corresponding row in the tabular or matrix representation of the soft set [1]. The dicision making scheme of a soft set is to give a rank of the objects by the choice values of the objects. The object with maximum number of supporting parameters is the dicision making result.

B. NORMAL PARAMETER REDUCTION OF SOFT SET AND THE MATRIX OF DOMINANT SUPPORT PARAMETERS

When there exist lots of parameters in a soft set, we need to figure out a kind of subsets of parameters, which is named as a normal parameter reduction [36]. Each combination of parameters of this kind contributes the same to each object. That is to say, once we have deleted this subset of parameters, each object should lose the same amount for the choice value. As a result, the rank of objects will not change.

It is important to give optimal algorithms for normal parameter reductions of soft sets. Many researchers have made contributions to this problem [37]–[40]. A method for integrating all normal parameter reductions of a soft set into a propositional logic formula is proposed in [41]. Ma *et al.* [42] pointed out an important property of normal parameter reduction of soft sets, by which the workload for finding candidates can be reduced.

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The matrix of dominant support parameters was brought in and investigated in [41], [43], [44]. It has been proved in [43] that the parameter reduction problems of soft sets can be translated as 0-1 linear programming problems. It was shown that by using part of the matrix of dominant support parameters [43], the conditions for a normal or pseudo parameter reduction can be represented by some linear constraints among local parameters.

C. COMPARISION AMONG MATIX OF DOMINANT SUPPORT PARAMETERS AND SEVERAL KINDS OF MATRICES FOR 0-1 TABLES IN DIFFERENT FIELDS

0-1 information systems do appear in many fields. It can be used to record information in computer systems. Black and white pictures can be represented by 0-1 matrices or tables. In formal conceptual exploration [45], a formal context (U, A, I) can be represented by a 0-1 information system. In rough set theory [46], if an information system (U, A, D, f) satisfies: $\forall a \in A$, $|D_a| = 2$, i.e., D_a is two-valued, then such an information system can be shown as a 0-1 information system. In graph theory [47], a classical graph (V, E) can be represented by a 0-1 table or matrix too, which is named as adjacency matrix. In soft set theory, we often use 0-1 table to represent a soft set. However, we must make it clear that given the same 0-1 information system, it may have different meanings or senses in the corresponding areas. What's more, confronted with different fields, we have different aims or research goals or applications. As a result, there exist different methods for dealing with the related 0-1 information systems.

In rough set theory, given a 0-1 valued information system, we can define its discernbility matrix D [48]. For an arbitrary entry $D(i, j)$, it records the set of attributes, each one of which can be used to distinguish the object u_i and u_j. For general information systems, we can also define its discernbility matrix D. And the discernbility matrix D is very important for the attribute reduction problems of rough sets. In ordered information system, a dominance relation was proposed by [49]. In formal concept analysis theory, the 0-1 information system is called the relation matrix. It plays an important role in the attribute reduction problem of concept lattice theory.

The matrix of dominant support parameters for a soft set is different from the discernbility matrix (or dominance relation) of a general (ordered) information system [49]. For each pair of objects, it not only gets rid of the parameters for which the objects have the same value, but also makes a detailed classification for the left ones. This operation helps us to get the essence for the parameter reduction problems. In other words, the matrix of dominant support parameters maintains the advantage information of each object.

Using the idea of discernbility matrix, [50] proposed similar concepts such as soft discernibility and weighted soft discernibility in soft sets, which was shown useful in the decision making of soft set. We will make a detailed analysis between the matrix of dominant support parameters and soft discernibility matrix in Section V.

D. MAIN QUESTIONS TO BE INVESTIGATED

The parameter reduction problems are quite different from that of information systems in rough set theory or that of formal concept analysis. So it becomes an important task for researchers to learn and develop these important properties or characterizations for matrices of dominant support parameters. It has been discussed in [43], but it is not enough or systematic. A lot of questions need to be investigated. We make a list of them as follows:

(1) From the point view of knowledge representation, what's the logical relationship among the entries of the matrix of dominant support parameters?

(2) Given a matrix of dominant support parameters, how can we figure out the initial soft set?

(3) Given a set-valued square matrix, how can we check or determinate whether it is the matrix of dominant support parameters of a certain soft set?

These questions are fundamental but important for the development of soft set theory. The remainder of this article is organized as follows. Section II introduces basic concepts such as the soft set and the matrix of dominant support parameters. Then fundamental structural and quantitative properties are proposed in section III. With these properties we can learn much more from the matrix of dominant support parameters. In section IV, algorithms for retrieving the soft set itself by using the given matrix of dominant support parameters are given, and then the logical formulas for the relationship among the entries are thoroughly discussed. At last, characterization theorems for the matrix of dominant support parameters are brought in. In section V we will make a comparison between the matrix of dominant support parameters and the soft discernibility matrix. Finally, we come to a conclusion of this article and outlook for potential future work.

II. PRELIMINARIES

In this article, suppose $U = \{u_1, u_2, \cdots, u_n\}$ is a finite set of objects, E is a set of parameters. For example, the attributes in information systems can be taken as parameters. $\wp(U)$ means the power set of U, $|A|$ means the cardinality of set A. If $B \subseteq A$, then B^C means the complementary set of B. By [1] and [43] we have basic concepts about soft sets shown in Definition 2.1 and Definition 2.2.

Definition 2.1 (Soft Set): A soft set on U is a pair $S = (F, A)$, where

(i) A is a subset of E;

(ii) $F : A \to \wp(U)$, $\forall e \in A, F(e)$ means the subset of U corresponding with parameter e. We also use $F(u, e) = 1$ ($F(u, e) = 0$) to mean than u is (not) an element of $F(e)$, i.e., $u \in F(e)$ ($u \notin F(e)$).

Definition 2.2 (Support Set of Parameters for Objects): Let $S = (F, A)$ be a soft set over U. $\forall u \in U$, define the support set of parameters for u as the set $\{e \in A | F(u, e) = 1\}$, denoted by $supp(u)$.

Definition 2.3 (Choice Value Function): Let $S = (F, A)$ be a soft set over U. The function $\sigma_S : U \to \mathbb{N}$ defined by

$$\begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array}\left(\begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ \emptyset & \{e_2\} & \{e_2,e_3,e_7,e_8\} & \{e_3,e_8\} & \{e_3,e_8\} & \{e_2,e_8\} \\ \{e_5,e_6\} & \emptyset & \{e_3,e_7,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_6,e_8\} \\ \{e_5,e_6\} & \emptyset & \emptyset & \{e_5,e_6\} & \{e_5,e_6\} & \{e_6\} \\ \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ \{e_1,e_5\} & \{e_1\} & \{e_1,e_3,e_7\} & \{e_1,e_3,e_5\} & \{e_1,e_3,e_5\} & \emptyset \end{array}\right)$$

FIGURE 1. A sketch map for the Main Diagonal Empty Property with soft set in TABLE 1.

$\sigma_S(u) = |supp(u)| = \sum_{e\in A} F(u, e)$ is called the choice value function of S.

We write σ_S as σ for short if the underlying soft set S is explicit.

Example 2.1 [43]: TABLE 1 represents a soft set $S = (F, E)$ over objects domain $U = \{u_1, u_2, \cdots, u_6\}$ and parameters domain $E = \{e_1, e_2, \cdots, e_8\}$, where $F(e_1) = \{u_6\}$, $F(e_2) = \{u_1, u_4, u_5\}$, $F(e_3) = \{u_1, u_2, u_6\}$, $F(e_4) = U$, $F(e_5) = \{u_2, u_3, u_6\}$, $F(e_6) = \{u_2, u_3\}$, $F(e_7) = \{u_1, u_2, u_4, u_5, u_6\}$, $F(e_8) = \{u_1, u_2\}$. σ_S represents the choice value function of soft set S.

Definition 2.4 [41], [43], [44] (Dominant Support Parameters): Given a soft set $S = (F, A)$ over U, $\forall u_i, u_j \in U$, define $D_{i\leftarrow j} = supp(u_i) - supp(u_j)$. We call $D_{i\leftarrow j}$ the set of **dominant support parameters** of u_i over u_j.

Example 2.2: Consider the soft set $S = (F, A)$ given in Table 1. By Definition 3.1 and Fig. 1 it is easy to get that $D_{1\leftarrow 2} = supp(u_1) - supp(u_2) = \{e_2\}$, $D_{2\leftarrow 1} = supp(u_2) - supp(u_1) = \{e_5, e_6\}$, $D_{1\leftarrow 6} = supp(u_1) - supp(u_6) = \{e_2, e_8\}$, $D_{6\leftarrow 1} = supp(u_6) - supp(u_1) = \{e_1, e_5\}$.

Definition 2.5 [43] (Matrix of Dominant Support Parameters): Given a soft set $S = (F, A)$ over U, $|U| = n$. We call matrix $D_S = [D_{i\leftarrow j}]_{n\times n}$ as **the matrix of dominant support parameters** for soft set S. We will also use $D_S(i, j)$ for $D_{i\leftarrow j}$.

Example 2.3 [43]: Consider the soft set $S = (F, A)$ given in Table 1, then by Definition 3.2 we have D_S which is shown in Fig.1 (here for the convenience of readers we list the objects u_i, $i = 1, 2, \cdots, 6$).

TABLE 1. Tabular representation of a soft set $S = (F, A)$.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	σ_S
u_1	0	1	1	1	0	0	1	1	5
u_2	0	0	1	1	1	1	1	1	6
u_3	0	0	0	1	1	1	0	0	3
u_4	0	1	0	1	0	0	1	0	3
u_5	0	1	0	1	0	0	1	0	3
u_6	1	0	1	1	1	0	1	0	5

III. FUNDAMENTAL STRUCTURAL AND QUANTITATIVE PROPERTIES FOR ENTRIES OF MATRIX OF DOMINANT SUPPORT PARAMETERS

A. STRUCTURAL PROPERTIES OR RELATIONS FOR ENTRIES OF MATRIX OF DOMINANT SUPPORT PARAMETERS

By *the lemma 3.1* in [43], we have the following Theorem 3.1. We add its proof and summarize the properties here. As a result, it becomes more systematically together with other properties in this subsection.

Theorem 3.1: Given a soft set $S = (F, A)$ over U, $|U| = n$. Then D_S is a set-valued matrix which satisfies the following properties:

(i) (**Main Diagonal Empty Property**) the main diagonal elements are all empty subsets, i.e., $D_S(i, i) = \emptyset$, $i = 1, 2, \cdots, n$;

(ii) (**Symmetry Disjoint Property**) the entries on symmetry positions with respect to the main diagonal are disjoint, i.e., $D_S(i, j) \cap D_S(j, i) = \emptyset$, $1 \leq i \neq j \leq n$;

(iii) (**Transition Disjoint Property**) $\forall i, j, k = 1, 2, \cdots, n$, $i \neq j, j \neq k, i \neq k, D_S(i, j) \cap D_S(j, k) = \emptyset$.

Proof: (i) $\forall i = 1, 2, \cdots, n$, $\forall e \in A$, $F(u_i, e) = F(u_i, e)$, so $D_S(i, i) = \emptyset$.

(ii) $\forall i, j = 1, 2, \cdots, n$, $i \neq j$, $\forall e \in A$, if $e \in D_S(i, j)$, then $F(u_i, e) = 1$, $F(u_j, e) = 0$, so $e \notin D_S(j, i)$; if $e \in D_S(j, i)$, then $F(u_j, e) = 1$, $F(u_i, e) = 0$, so $e \notin D_S(i, j)$. Hence $D_S(i, j) \cap D_S(j, i) = \emptyset$.

(iii) $\forall i, j, k = 1, 2, \cdots, n$, $i \neq j, j \neq k, i \neq k$, $\forall e \in A$, if $e \in D_S(i, j)$, then $F(u_i, e) = 1$, $F(u_j, e) = 0$, so $e \notin D_S(j, k)$; if $e \in D_S(j, k)$, then $F(u_j, e) = 1$, $F(u_k, e) = 0$, so $e \notin D_S(i, j)$. Hence $D_S(ij) \cap D_S(j, k) = \emptyset$.

For a better understanding of the above three fundamental properties, by using the soft set given by Example 2.1 and its matrix of dominant support parameters shown in Example 2.3, we give three figures Fig.1 to Fig. 3.

Theorem 3.2 (Identical-Column-Row Properties for Matrix of Dominant Support Parameters): Given a soft set $S = (F, A)$ over U, $|U| = n$. Then D_S is a set-valued matrix which satisfies the following properties:

(i) (**Identical-Column-Row Disjoint Property**) once one parameter appears in the i_{th} column, then it can't appear in the i_{th} row, and vice versa, i.e., $\forall i = 1, 2, \cdots, n$,

$$(\bigcup_{k=1}^{n} D_S(i, k)) \cap (\bigcup_{k=1}^{n} D_S(k, i)) = \emptyset. \tag{3.1}$$

(ii) (**Identical-Column-Row Union Property**) $\forall i = 1, 2, \cdots, n$, $(\bigcup_{k=1}^{n} D_S(i, k)) \cup (\bigcup_{k=1}^{n} D_S(k, i)) = E - \{e|F(e) = \emptyset \text{ or } F(e) = U\}$.

$$D_S = \begin{array}{ccccccc} & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ u_1 & \emptyset & \{e_2\} & \{e_2,e_3,e_7,e_8\} & \{e_3,e_8\} & \{e_3,e_8\} & \{e_2,e_8\} \\ u_2 & \{e_5,e_6\} & \emptyset & \{e_3,e_7,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_6,e_8\} \\ u_3 & \{e_5,e_6\} & \emptyset & \emptyset & \boxed{\{e_5,e_6\}} & \{e_5,e_6\} & \{e_6\} \\ u_4 & \emptyset & \{e_2\} & \boxed{\{e_2,e_7\}} & \emptyset & \emptyset & \{e_2\} \\ u_5 & \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ u_6 & \{e_1,e_5\} & \{e_1\} & \{e_1,e_3,e_7\} & \{e_1,e_3,e_5\} & \{e_1,e_3,e_5\} & \emptyset \end{array}$$

FIGURE 2. A sketch map for Symmetry Disjoint Property with soft set in TABLE 1.

$$D_S = \begin{array}{ccccccc} & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ u_1 & \emptyset & \{e_2\} & \{e_2,e_3,e_7,e_8\} & \{e_3,e_8\} & \{e_3,e_8\} & \{e_2,e_8\} \\ u_2 & \{e_5,e_6\} & \emptyset & \boxed{\{e_3,e_7,e_8\}} & \{e_3,e_5,e_6,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_6,e_8\} \\ u_3 & \{e_5,e_6\} & \emptyset & \emptyset & \boxed{\{e_5,e_6\}} & \{e_5,e_6\} & \{e_6\} \\ u_4 & \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ u_5 & \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ u_6 & \{e_1,e_5\} & \{e_1\} & \{e_1,e_3,e_7\} & \{e_1,e_3,e_5\} & \{e_1,e_3,e_5\} & \emptyset \end{array}$$

FIGURE 3. A sketch map for the Transition Disjoint Property with soft set in TABLE 1.

$$D_S = \begin{array}{ccccccc} & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ u_1 & \emptyset & \{e_2\} & \{e_2,e_3,e_7,e_8\} & \{e_3,e_8\} & \{e_3,e_8\} & \{e_2,e_8\} \\ u_2 & \{e_5,e_6\} & \emptyset & \{e_3,e_7,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_6,e_8\} \\ u_3 & \{e_5,e_6\} & \emptyset & \emptyset & \{e_5,e_6\} & \{e_5,e_6\} & \{e_6\} \\ u_4 & \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ u_5 & \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ u_6 & \{e_1,e_5\} & \{e_1\} & \{e_1,e_3,e_7\} & \{e_1,e_3,e_5\} & \{e_1,e_3,e_5\} & \emptyset \end{array}$$

FIGURE 4. A sketch map for the Identical-Column-Row Disjoint Property with soft set in TABLE 1.

(iii) (**Identical-Column-Row Partition Property**) $\forall i = 1, 2, \cdots, n$, $(\bigcup_{k=1}^{n} D_S(i,k))$ and $(\bigcup_{k=1}^{n} D_S(k,i))$ is a partition of $E - \{e|F(e) = \emptyset \text{ or } F(e) = U\}$.

Proof: (i) Suppose $e \in (\bigcup_{k=1}^{n} D_S(i,k)) \cap (\bigcup_{k=1}^{n} D_S(k,i)) \neq \emptyset$, then $\exists k_1, k_2$ such that $e \in D_S(i, k_1)$ and $e \in D_S(k_2, i)$. Hence $F(u_i, e) = 1, F(u_{k_1}, e) = 0; F(u_{k_2}, e) = 1, F(u_i, e) = 0$, which is a contradiction.

(ii) On one hand, since if $e \in \{e|F(e) = \emptyset \text{ or } F(e) = U\}$, it is easy to see that $\forall k_1, k_2, F(u_i, e) = F(u_{k_1}, e) = F(u_{k_2}, e)$, $e \notin D_S(i, k_1)$ and $e \notin D_S(k_2, i)$. Thus $(\bigcup_{k=1}^{n} D_S(i,k)) \cup (\bigcup_{k=1}^{n} D_S(k,i)) \subseteq E - \{e|F(e) = \emptyset \text{ or } F(e) = U\}$.

On the other hand, if $e \in E - \{e|F(e) = \emptyset \text{ or } F(e) = U\}$, then $\exists k_1, k_2$ such that $F(u_{k_1}, e) = 1, F(u_{k_2}, e) = 0$. If $F(u_i, e) = 1$, then $e \in D_S(i, k_2)$; else $e \in D_S(k_1, i)$. So $E - \{e|F(e) = \emptyset \text{ or } F(e) = U\} \subseteq (\bigcup_{k=1}^{n} D_S(i,k)) \cup (\bigcup_{k=1}^{n} D_S(k,i))$.

(iii) By (i) and (ii).

Fig. 4 shows an example of the Identical-Column-Row Disjoint Property. The union of entries $D(4, 1)$ to $D(4, 6)$ (framed with strong line) is disjoint with that of entries $D(1, 4)$ to $D(6, 4)$ (surrounded with dotted wire).

TABLE 2 shows the Identical-Column-Row Disjoint Property and the Identical-Column-Row union Property with the D_S in Example 2.3.

By Theorem 3.2 we have

Corollary 3.1 (Symmetry Disjoint Extension Rules): Suppose D_S is the matrix of dominant support parameters for a soft set $S = (F, A)$. $\forall i, j = 1, 2, \cdots, n-1, i \neq j$, then

$$D_S(i,j) \cap \bigcup_{k=1}^{|U|} D_S(k,i) = \emptyset; \tag{3.2}$$

$$D_S(j,i) \cap \bigcup_{k=1}^{|U|} D_S(k,j) = \emptyset, \tag{3.3}$$

and

$$D_S(i,j) \cap \bigcup_{k=1}^{|U|} D_S(j,k) = \emptyset; \tag{3.4}$$

$$D_S(j,i) \cap \bigcup_{k=1}^{|U|} D_S(i,k) = \emptyset. \tag{3.5}$$

TABLE 2. Tabular explanation for the Identical-Column-Row Partition Property with soft set in TABLE 1.

	$\alpha = (\bigcup_{k=1}^{n} D_S(i,k))$	$\beta = \bigcup_{k=1}^{n} D_S(k,i)$	$\alpha \cap \beta$	$\alpha \cup \beta$
$i=1$	$\{e_2,e_3,e_7,e_8\}$	$\{e_1,e_5,e_6\}$	$\emptyset$	$E-\{e_4\}$
$i=2$	$\{e_3,e_5,e_6,e_7,e_8\}$	$\{e_1,e_2\}$	$\emptyset$	$E-\{e_4\}$
$i=3$	$\{e_5,e_6\}$	$\{e_1,e_2,e_3,e_7,e_8\}$	$\emptyset$	$E-\{e_4\}$
$i=4$	$\{e_2,e_7\}$	$\{e_1,e_3,e_5,e_6,e_8\}$	$\emptyset$	$E-\{e_4\}$
$i=5$	$\{e_2,e_7\}$	$\{e_1,e_3,e_5,e_6,e_8\}$	$\emptyset$	$E-\{e_4\}$
$i=6$	$\{e_1,e_3,e_5,e_7\}$	$\{e_2,e_8\}$	$\emptyset$	$E-\{e_4\}$

$$D_S = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array}$$

$$D_S = \begin{array}{c|cccccc} & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ \hline u_1 & \emptyset & \{e_2\} & \{e_2,e_3,e_7,e_8\} & \{e_3,e_8\} & \{e_3,e_8\} & \{e_2,e_8\} \\ u_2 & \{e_5,e_6\} & \emptyset & \{e_3,e_7,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_6,e_8\} \\ u_3 & \{e_5,e_6\} & \emptyset & \emptyset & \{e_5,e_6\} & \{e_5,e_6\} & \{e_6\} \\ u_4 & \emptyset & \{e_2\} & \boxed{\{e_2,e_7\}} & \emptyset & \emptyset & \{e_2\} \\ u_5 & \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ u_6 & \{e_1,e_5\} & \{e_1\} & \{e_1,e_3,e_7\} & \{e_1,e_3,e_5\} & \{e_1,e_3,e_5\} & \emptyset \end{array}$$

FIGURE 5. A second sketch map for Corollary 3.1.

$$D_S = \begin{array}{c|cccccc} & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ \hline u_1 & \emptyset & \{e_2\} & \{e_2,e_3,e_7,e_8\} & \{e_3,e_8\} & \{e_3,e_8\} & \{e_2,e_8\} \\ u_2 & \{e_5,e_6\} & \emptyset & \{e_3,e_7,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_6,e_8\} \\ u_3 & \{e_5,e_6\} & \emptyset & \emptyset & \{\boxed{e_5},e_6\} & \{\boxed{e_5},e_6\} & \{e_6\} \\ u_4 & \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ u_5 & \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ u_6 & \{e_1,e_5\} & \{e_1\} & \{e_1,e_3,e_7\} & \{e_1,e_3,\boxed{e_5}\} & \{e_1,e_3,\boxed{e_5}\} & \emptyset \end{array}$$

FIGURE 6. A sketch map for Submatrix Diagonal Vertices Rule with soft set in TABLE 1.

Corollary 3.1 comes specially to symmetrical positions. It tells us that for arbitrary pair of symmetrical entries $D_S(i,j)$ and $D_S(j,i)$, $D_S(i,j)$ is not only disjoint with $D_S(j,i)$ but also disjoint with any entries on the j_{th} row or i_{th} column; and $D_S(j,i)$ is not only disjoint with $D_S(i,j)$ but also disjoint with any entries on the i_{th} row or j_{th} column. In Fig. 5, $D(4,3) = \{e_2,e_7\}$, it is easy to see that $\{e_2,e_7\}$ (The one framed with solid wire) is disjoint with any entry in the 4th column which is surrounded by dotted line.

Theorem 3.3 (Submatrix Diagonal Vertices Rule): Suppose D_S is the matrix of dominant support parameters for a soft set $S=(F,A)$. $\forall k \in \{1,2,\cdots,|A|\}$, If $e_k \in D_S(i_1,j_1)$, $e_k \in D_S(i_2,j_2)$, and $i_1 \neq i_2, j_1 \neq j_2$, then

$$e_k \in D_S(i_1,j_2), e_k \in D_S(i_2,j_1). \tag{3.6}$$

Proof: Since $e_k \in D_S(i_1,j_1)$, $e_k \in D_S(i_2,j_2)$, we have $F(u_{i_1},e)=1, F(u_{j_1},e)=0$; $F(u_{i_2},e)=1, F(u_{j_2},e)=0$. Therefore $e_k \in D_S(i_1,j_2)$, $e_k \in D_S(i_2,j_1)$.

The **Submatrix Diagonal Vertices Rule** tells us that if e appears both in one pair of diagonal vertices of a submatrix, then e also appears both in the other pair of diagonal vertices of the same submatrix. See Fig. 6 for an example. We see that $e_5 \in D(3,4) \cap D(6,5)$, then consider the subdiagonal line of the submatrix which consists of $D(3,4), D(6,5), D(6,4), D(3,5)$ (corresponding to these entries with colored elements). We can see that e_5 appears in both $D(6,4)$ and $D(3,5)$.

Theorem 3.4 (Single-Point-Induced Vertical and Horizontal Rule): Given a soft set $S=(F,A)$ over U, $|U|=n$. D_S is its matrix of dominant support parameters. Then $\forall i,j,k = 1,2,\cdots,n, i\neq j, i\neq k, j\neq k$, if $e \in D_S(k,j)$, we have $e \in D_S(i,j)$ or $e \in D_S(k,i)$, i.e., logically

$$e \in D_S(k,j) \to (e \in D_S(i,j) \vee e \in D_S(k,i)) \equiv \top,$$
$$\neg(e \in D_S(i,j)) \wedge \neg(e \in D_S(k,i)) \to \neg(e \in D_S(k,j)) \equiv \top. \tag{3.7}$$

Here $\top$ means the tautology formular.

Proof: If $e \in D_S(k,j)$, we have $F(u_k,e)=1, F(u_j,e)=0$. Since $i \neq k$, if $F(u_i,e)=1$, then $e \in D_S(i,j)$; else $F(u_i,e)=0$, we have $e \in D_S(k,i)$.

$$D_S = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ \emptyset & \{e_2\} & \{e_2,e_3,e_7,e_8\} & \{e_3,e_8\} & \{e_3,e_8\} & \{e_2,e_8\} \\ \{e_5,e_6\} & \emptyset & \{e_3,e_7,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_6,e_8\} \\ \{e_5,e_6\} & \emptyset & \emptyset & \{e_5,e_6\} & \{e_5,e_6\} & \{e_6\} \\ \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ \{e_1,e_5\} & \{e_1\} & \{e_1,e_3,e_7\} & \{e_1,e_3,e_5\} & \{e_1,e_3,e_5\} & \emptyset \end{array}$$

FIGURE 7. **A sketch map for Single-Point-Induced Vertical and Horizontal Rule with soft set in TABLE 1.**

$$D_S = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ \emptyset & \{e_2\} & \{e_2,e_3,e_7,e_8\} & \{e_3,e_8\} & \{e_3,e_8\} & \{e_2,e_8\} \\ \{e_5,e_6\} & \emptyset & \{e_3,e_7,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_6,e_8\} \\ \{e_5,e_6\} & \emptyset & \emptyset & \{e_5,e_6\} & \{e_5,e_6\} & \{e_6\} \\ \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ \{e_1,e_5\} & \{e_1\} & \{e_1,e_3,e_7\} & \{e_1,e_3,e_5\} & \{e_1,e_3,e_5\} & \emptyset \end{array}$$

FIGURE 8. **A sketch map for Corollary 3.2.**

Fig. 7 gives us an example for Theorem 3.4 with the D_S in Example 2.3. We can see that $e_5 \in \{e_5, e_6\} = D(2, 1)$. Then by Theorem 3.4 we have: $e_5 \in \{e_5, e_6\} = D(3, 1)$ but $e_5 \notin D(2, 3)$; $e_5 \in \{e_5, e_6\} = D(6, 1)$ but $e_5 \notin D(2, 6)$; $e_5 \notin D(4, 1)$ but $e_5 \in D(2, 4)$; $e_5 \notin D(5, 1)$ but $e_5 \in D(2, 5)$.

According to the Theorem 3.3, the following corollary 3.2 and corollary 3.3 can be induced.

Corollary 3.2 (Two-Points-Induced Consistent Distribution Rule for Corresponding Columns): Suppose D_S is the matrix of dominant support parameters for a soft set $S = (F, A)$. $\forall k \in \{1, 2, \cdots, |A|\}$, if $e_k \in D_S(i_1, j_1)$, $e_k \in D_S(i_2, j_2)$, and $j_1 \neq j_2$, then $\forall K \in \{1, 2, \cdots, |U|\}$,

$$e_k \in D_S(K, j_1) \textit{ if and only if } e_k \in D_S(K, j_2). \tag{3.8}$$

Corollary 3.2 tells us that if e appears in two different columns, then for arbitrary pair of entries of this two columns at the same row, e doesn't appear or e appears two times.

Fig. 8 gives us an example for Corollary 3.2 with the D_S in Example 2.3. It can be implied that $e_2 \in \{e_2, e_7\} = D(4, 3)$ and $e_2 \in \{e_2\} = D(5, 2)$. Then by Corollary 3.2: for these pairs $D(i, 2)$ and $D(i, 3)$ $(i = 1, 4, 5)$ which are surrounded by solid lines, $e \in D(i, 2) \cap D(i, 3)$; for these pairs $D(i, 2)$ and $D(i, 3)$ $(i = 2, 3, 6)$ which are surrounded by dotted lines, $e \notin D(i, 2)$ and $e \notin D(i, 3)$.

Corollary 3.3 (Two-Points-Induced Consistent Distribution Rule for Corresponding Rows): Suppose D_S is the matrix of dominant support parameters for a soft set $S = (F, A)$. $\forall k \in \{1, 2 \cdots, |A|\}$, if $e_k \in D_S(i_1, j_1)$, $e_k \in D_S(i_2, j_2)$, and $i_1 \neq i_2$, then $\forall K \in \{1, 2, \cdots, |U|\}$,

$$e_k \in D_S(i_1, K) \textit{ if and only if } e_k \in D_S(i_2, K). \tag{3.9}$$

B. QUANTITATIVE PROPERTIES FOR THE ENTRIES OF THE MATRIX OF DOMINANT SUPPORT PARAMETERS

Given a soft set $S = (F, A)$ over U, $|U| = n$. D_S is the matrix of dominant support parameters. In this subsection we will bring in some definitions and then discuss their properties.

1) TIMES THAT e_k APPEARS IN D_S

Definition 3.1: Given a soft set $S = (F, A)$ over U, $|U| = n$. D_S is the matrix of dominant support parameters, $\forall e_k \in A$, denote the times that e_k appears in D_S by $ND_S(e_k)$, i.e.,

$$ND_S(e_k) = \sum_{i=1}^{n}\sum_{j=1}^{n} |D_S(i, j) \cap \{e_k\}|. \tag{3.10}$$

Theorem 3.5: Given a soft set $S = (F, A)$ over U, $|U| = n$. D_S is the matrix of dominant support parameters, $\forall e_k \in A$, then $ND_S(e_k) =$

$$|\{u \in U|F(u, e_k) = 1\}| \cdot |\{u \in U|F(u, e_k) = 0\}|, \tag{3.11}$$

i.e.,

$$|\{u \in U|F(u, e_k) = 1\}| \cdot (|U| - |\{u \in U|F(u, e_k) = 1\}|).$$

Proof: $\forall e_k \in A$, it suffices to get the number of pairs of $F(u, e_k) = 1$ and $F(v, e_k) = 0$. By the knowledge of **Permutation and Combination**, it is easy to see $\forall e_k \in A$, $ND_S(e_k) = |\{u \in U|F(u, e_k) = 1\}| \cdot |\{u \in U|F(u, e_k) = 0\}|$. And obviously $ND_S(e_k) = |\{u \in U|F(u, e_k) = 1\}| \cdot (|U| - |\{u \in U|F(u, e_k) = 1\}|) = |\{u \in U|F(u, e_k) = 1\}| \cdot |\{u \in U|F(u, e_k) = 0\}|$.

Corollary 3.4: Given a soft set $S = (F, A)$ over U, $|U| = n$. D_S is the matrix of dominant support parameters, $\forall e_k \in A$,

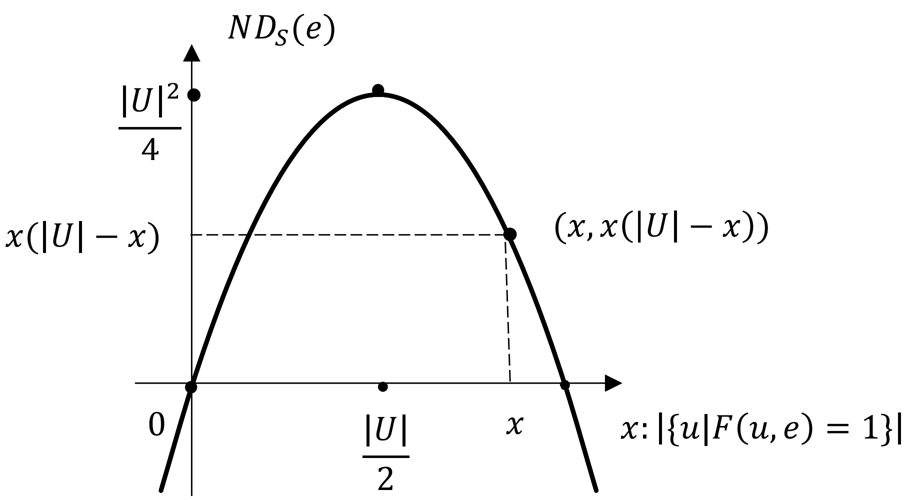

FIGURE 9. A sketch map for Corollary 3.5.

TABLE 3. $ND_S(e_k)$ values for the soft set $S=(F,A)$ in TABLE 1.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
$\alpha: \lvert\{u \in U \lvert F(u,e_k)=1\}\rvert$	1	3	3	6	3	2	5	2
$\beta: \lvert\{u \in U \lvert F(u,e_k)=0\}\rvert$	5	3	3	0	3	4	1	4
$\alpha \cdot \beta$	5	9	9	0	9	8	5	8
$\lvert\lvert\{u \lvert F(u,e_k)=1\}\rvert - \frac{\lvert U\rvert}{2}\rvert$	2	0	0	3	0	1	2	1
$ND_S(e_k)$	5	9	9	0	9	8	5	8

then $ND_S(e_k)=\bigvee_{i=1}^{m} ND_S(e_i)$ if and only if

$$||\{u|u \in F(e_k)\}| - \frac{|U|}{2}| = \bigwedge_{i=1}^{m} ||\{u|u \in F(e_i)\}| - \frac{|U|}{2}|.$$

Corollary 3.5: Given a soft set $S=(F,A)$ over U, $|U|=n$. D_S is the matrix of dominant support parameters, $\forall e_k \in A$, then $\forall k=1,2,\cdots,m$,

$$ND_S(e_k) \leq \frac{n^2}{4}. \tag{3.12}$$

Fig. 9 shows us the underlying reason with a quadratic function image for Corollary 3.4 and Corollary 3.5.

Example 3.1: TABLE 3 shows the $ND_S(e_k)$ values for the soft set S represented by TABLE 1.

By Theorem 3.5 we can get

Corollary 3.6: Assume D_S is the matrix of dominant support parameters for the soft set $S=(F,A)$ over U, $|U|=n$. $\forall i,j=1,2,\cdots|A|$, if $ND_S(e_i)$ and $ND_S(e_j)$ are both prime numbers larger than 1, then

$$NDS(e_i)=NDS(e_j)=|U|-1. \tag{3.13}$$

Example 3.2: By TABLE 3 $ND_S(e_1)=ND_S(e_7)=5$, 5 is a prime number larger than 1. It is easy to testify that $5=|U|-1$, where $|U|=6$.

Corollary 3.7: Given a soft set $S=(F,A)$ over U, $|U|=n$. D_S is the matrix of dominant support parameters, denote the sum of cardinalities of all entries in D_S by $N(D_S)$, then

$$N(D_S)=\sum_{e_k \in A} |\{u \in U|F(u,e_k)=1\}| \cdot(|U|-|\{u \in U|F(u,e_k)=1\}|). \tag{3.14}$$

2) TIMES THAT e_k APPEARS IN A PAIR OF COLUMN AND ROW IN D_S

Definition 3.2: Given a soft set $S=(F,A)$ over U, $|U|=n$. D_S is the matrix of dominant support parameters, $\forall e_k \in A$, denote the times that e_k appears in the i_{th} row and j_{th} column of D_S by $ND_S(e_k,i,j)$, i.e.,

$$ND_S(e_k,i,j)=\sum_{t=1}^{n}|D_S(i,t)\cap\{e_k\}|+\sum_{t=1}^{n}|D_S(t,j)\cap\{e_k\}| \tag{3.15}$$

Corollary 3.8: Given a soft set $S=(F,A)$ over U, $|U|=n$. D_S is the matrix of dominant support parameters, $\forall i,j=1,2,\cdots,n$, $i \neq j$, $\forall k=1,2,\cdots,m$, if $e_k \in D_S(i,j)$, then

$$ND_S(e_k,i,j)=|U|-1. \tag{3.16}$$

Example 3.3: Consider the soft set $S=(F,A)$ given in TABLE 1, $D_S|_{e_3}$ in expression (3.17) maintains the details

TABLE 4. Some $ND_S(e_k, i, j)$ values for D_S of the soft set $S = (F, A)$ in TABLE 1.

(e_k, i, j)	$(e_1, 6, 1)$	$(e_2, 4, 2)$	$(e_3, 2, 3)$	$(e_5, 3, 4)$	$(e_6, 3, 5)$	$(e_7, 6, 3)$	$(e_8, 2, 6)$
$ND_S(e_k, i, j)$	5	5	5	5	5	5	5

especially for e_3 when we remove all other parameters in D_S. It can be checked that $ND_S(e_3, i, j)$ where $e_3 \in D_S(i, j)$ satisfies Corollary 3.8.

$$D_S|_{e_3} = \left(\begin{array}{c|cccccc} & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ u_1 & \emptyset & \emptyset & \{e_3\} & \{e_3\} & \{e_3\} & \emptyset \\ u_2 & \emptyset & \emptyset & \{e_3\} & \{e_3\} & \{e_3\} & \emptyset \\ u_3 & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ u_4 & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ u_5 & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ u_6 & \emptyset & \emptyset & \{e_3\} & \{e_3\} & \{e_3\} & \emptyset \end{array}\right). \tag{3.17}$$

IV. APPLICATIONS AND CHARACTERIZATIONS OF D_S WITH CERTAIN ENTRIES OF THE MATRIX OF DOMINANT SUPPORT PARAMETERS

In this section, all soft sets mentioned have no $\emptyset$ or U approximations. This is to say the tabular representations of these soft sets have no one column which has only 0 or has only 1.

A. RETRIEVING OF SOFT SETS WITH JTH ROW AND JTH COLUMN THE MATRIX OF DOMINANT SUPPORT PARAMETERS

As shown in the above section, the entries of the matrix of dominant support parameters for the same soft set have connections. Actually, we need only part of these entries to regain the initial soft set.

Theorem 4.1 (Retrieving Algorithm 1 for S With the First Row and the First Column of D_S): Given the first row and the first column of D_S for soft set $S = (F, A)$ over U, then $\forall k = 1, 2, \cdots, |A|, \forall i = 1, 2, \cdots, |U|$,

$$F(u_i, e_k) = \begin{cases} 0, & e_k \in D_{1\leftarrow i}; \\ 1, & e_k \in D_{i\leftarrow 1}; \\ \bigvee_{K\neq i} e_k \in D_{1\leftarrow K}, & e_k \notin D_{1\leftarrow i} \wedge e_k \notin D_{i\leftarrow 1}. \end{cases} \tag{4.1}$$

Proof: (i) If $e_k \in D_{1\leftarrow i}$, then $F(u_1, e_k) = 1$ and $F(u_i, e_k) = 0$.

(ii) If $e_k \in D_{i\leftarrow 1}$, then $F(u_1, e_k) = 0$ and $F(u_i, e_k) = 1$.

(iii) If $e_k \notin D_{1\leftarrow i} \wedge e_k \notin D_{i\leftarrow 1}$, then $F(u_1, e_k) = F(u_i, e_k)$. Since it is assumed that $F(e_k) \neq \emptyset$ and $F(e_k) \neq U$, $\exists J$ such that $F(u_1, e_k) \neq F(u_J, e_k)$. If $F(u_1, e_k) = 1$, then $F(u_J, e_k) = 0$, $e_k \in D_{1\leftarrow K}$, so $F(u_1, e_k) = F(u_i, e_k) = \bigvee_{K\neq i} e_k \in D_{1\leftarrow K}$. If $F(u_1, e_k) = 0$, then the logical value of formula $\bigvee_{K\neq i} e_k \in D_{1\leftarrow K}$ is equal to 0, $F(u_i, e_k) = 0$.

Corollary 4.1: Given two soft sets $S = (F_1, A)$ and $T = (F_2, A)$ over U, $|U| = n$. Then $S = T$ if and only if $D_S(1, :) = D_T(1, :)$ and $D_S(:, 1) = D_T(:, 1)$.

Similarly, we can also use the *Jth* row and the *Jth* column of D_S for regaining soft set $S = (F, A)$.

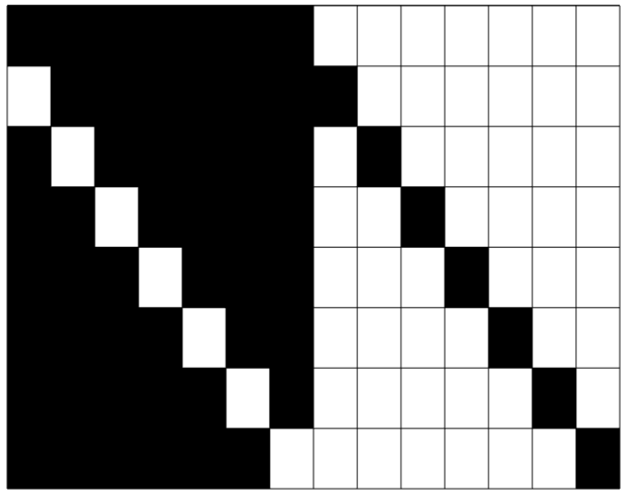

FIGURE 10. The soft set induced by $D(1, :)$ and $D(:, 1)$ in Example 4.1.

Theorem 4.2 (Retrieving Algorithm 2 for S With the Jth Row and Jth Column of D_S): Given the *Jth* row and the *Jth* column of D_S for soft set $S = (F, A)$ over U, then $\forall k = 1, 2, \cdots, |A|, \forall i = 1, 2, \cdots, |U|$,

$$F(u_i, e_k) = \begin{cases} 0, & e_k \in D_{J\leftarrow i}; \\ 1, & e_k \in D_{i\leftarrow J}; \\ \bigvee_{K\neq i} e_k \in D_{J\leftarrow K}, & e_k \notin D_{J\leftarrow i} \wedge e_k \notin D_{i\leftarrow J}. \end{cases} \tag{4.2}$$

Proof: It is similar with that of Theorem 4.1, we omit it here.

Corollary 4.2: Given two soft sets $S = (F_1, A)$ and $T = (F_2, A)$ over U, $|U| = n$. Then $\forall J = 1, 2, \cdots, |U|$, $S = T$ if and only if $D_S(J, :) = D_T(J, :)$ and $D_S(:, J) = D_T(:, J)$.

The retrieving algorithms enable us to construct soft sets or 0-1 ordered information systems which are supposed or required to be under certain constraints. It's very interesting to construct different distribution of subsets of parameters on the first row and the first column, and see what kind of information systems we get.

Example 4.1: $D_S(1, 2) = \{1\}$; $D_S(1, 3) = \{2\}$; $D_S(1, 4) = \{3\}$; $D_S(1, 5) = \{4\}$; $D_S(1, 6) = \{5\}$; $D_S(1, 7) = \{6\}$; $D_S(1, 8) = \{7\}$; $D_S(2, 1) = \{8\}$; $D_S(3, 1) = \{9\}$; $D_S(4, 1) = \{10\}$; $D_S(5, 1) = \{11\}$; $D_S(6, 1) = \{12\}$; $D_S(7, 1) = \{13\}$; $D_S(8, 1) = \{14\}$, then we can get the induced soft set $S = (F, A)$ over U shown in Fig. 10, where $A = \{1, 2, \cdots, 14\}$, $U = \{u_1, u_2, \cdots, u_8\}$.

Example 4.2: $D_S(1, 2) = \{1\}$; $D_S(1, 3) = \{1, 2\}$; $D_S(1, 4) = \{1, 2, 3\}$; $D_S(1, 5) = \{1, 2, 3, 4\}$; $D_S(1, 6) = \{1, 2, 3, 4, 5\}$; $D_S(1, 7) = \{1, 2, 3, 4, 5, 6\}$; $D_S(1, 8) = \{1, 2, 3, 4, 5, 6, 7\}$; $D_S(2, 1) = \{8\}$; $D_S(3, 1) = \{8, 9\}$;

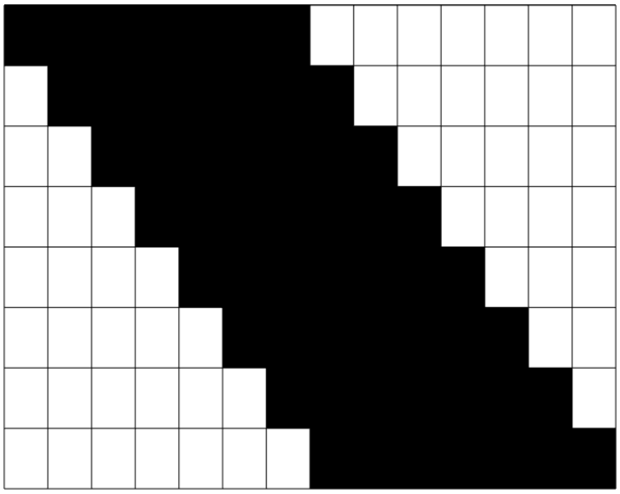

FIGURE 11. The soft set $S = (F, A)$ over U induced by the Retrieving Algorithm 1 in Example 4.2.

$D_S(4, 1) = \{8, 9, 10\}$; $D_S(5, 1) = \{8, 9, 10, 11\}$; $D_S(6, 1) = \{8, 9, 10, 11, 12\}$; $D_S(7, 1) = \{8, 9, 10, 11, 12, 13\}$; $D_S(8, 1) = \{8, 9, 10, 11, 12, 13, 14\}$, then we can get the induced soft set $S = (F, A)$ over U shown in Fig. 11, where $A = \{1, 2, \cdots, 14\}$, $U = \{u_1, u_2, \cdots, u_8\}$.

B. FILLING ALGORITHMS OF D_S WITH PART OF THE MATRIX OF DOMINANT SUPPORT PARAMETERS

According to the Definition 2.5 and the **Retrieving Algorithm 2**, we can get the following algorithm which can be used to compute the rest of D_S with the Jth row and the Jth column of D_S:

The Filling Algorithm 1 for D_S with the Jth row and the Jth column of D_S computes the rest of D_S one by one, it costs a lot. It can be proved that $\forall i, j$, $D_{i\leftarrow j}$ can be represented by the entries in the 1st row and the 1st column of D_S as follows:

Theorem 4.3 (Filling Algorithm 2 for D_S With the First Row and First Column of D_S): Given the 1st row and the 1st column of D_S for soft set $S = (F, A)$ over U, then $\forall i, j = 1, 2, \cdots, |U|$,

$$D_{i\leftarrow j} = \alpha \cap \beta^C, \tag{4.3}$$

where

$$\alpha = D^C_{1\leftarrow i} \cap (D_{i\leftarrow 1} \cup \bigcup_{k=1}^{|U|} D_{1\leftarrow k}), \textit{ i.e.,}$$

$$\alpha = (\bigcup_{k=1}^{|U|} D_{1\leftarrow k} - D_{1\leftarrow i}) \cup D_{i\leftarrow 1}, \tag{4.4}$$

and

$$\beta = D^C_{1\leftarrow j} \cap (D_{j\leftarrow 1} \cup \bigcup_{k=1}^{|U|} D_{1\leftarrow k}), \textit{ i.e.,}$$

$$\beta = (\bigcup_{k=1}^{|U|} D_{1\leftarrow k} - D_{1\leftarrow j}) \cup D_{j\leftarrow 1}. \tag{4.5}$$

TABLE 5. Filling Algorithm 1 for D_S with the Jth row and the Jth column of D_S.

Input: the Jth row and the Jth column of D_S for soft set $S = (F, A)$ over U.
Output: $e_k \in D_{i\leftarrow j}, \forall k = 1, 2, \cdots, |A|, \forall i, j = 1, 2, \cdots, |U|$.
Step 1 Compute $F(u_i, e_k)$ and $F(u_j, e_k)$ by the Retrieving Algorithm 2;
Step 2 $e_k \in D_{i\leftarrow j}$ if and only if $F(u_i, e_k) = 1$ and $F(u_j, e_k) = 0$.

Proof: (1) We prove $D_{i\leftarrow j} \subseteq \alpha \cap \beta^C$.

Suppose $e \in D_{i\leftarrow j}$, then $F(u_i, e) = 1$ and $F(u_j, e) = 0$. If $F(u_1, e) = 0$, then $e \in \alpha$. If $F(u_1, e) = 1$, then $e \notin D_{1\leftarrow i}$. Since $F(e) \neq \emptyset$ and $F(e) \neq U$, $e \in \bigcup_{k=1}^{|U|} D_{1\leftarrow k}) - D_{1\leftarrow i}$. Next we try to prove that $e \notin (\bigcup_{k=1}^{|U|} D_{1\leftarrow k} - D_{1\leftarrow j}) \cup D_{j\leftarrow 1}$, i.e., $e \notin (\bigcup_{k=1}^{|U|} D_{1\leftarrow k} - D_{1\leftarrow j})$ and $e \notin D_{j\leftarrow 1}$.

- If $e \in (\bigcup_{k=1}^{|U|} D_{1\leftarrow k} - D_{1\leftarrow j}$, then $e \in \bigcup_{k=1}^{|U|} D_{1\leftarrow k}$ and $e \notin D_{1\leftarrow j}$. Hence by $e \in \bigcup_{k=1}^{|U|} D_{1\leftarrow k}$, there exists K, $F(u_1, e) = 1$, $F(u_K, e) = 0$. Therefore $e \in D_{1\leftarrow j}$, which is a contradiction because $e \notin D_{1\leftarrow j}$.
- Since $F(u_j, e) = 0$, $e \notin D_{j\leftarrow 1}$.

(2) We prove $\alpha \cap \beta^C \subseteq D_{i\leftarrow j}$.

If $e \in \alpha \cap \beta^C$, then $e \in \alpha$, $e \in \beta^C$. Hence $e \in (\bigcup_{k=1}^{|U|} D_{1\leftarrow k} - D_{1\leftarrow j})^C$ and $e \in D^C_{j\leftarrow 1}$. By $e \in \alpha$, we have two situations:

- $e \in D_{i\leftarrow 1}$. Thus $F(u_i, e) = 1$, $F(u_1, e) = 0$. It suffices to show that $F(u_j, e) = 0$. We prove it by contrary. If $F(u_j, e) = 1$, then $e \in D_{j\leftarrow 1}$. That is a contradiction.
- $e \in (\bigcup_{k=1}^{|U|} D_{1\leftarrow k} - D_{1\leftarrow i})$. So $F(u_1, e) = 1$ and $e \notin D_{1\leftarrow i}$. So $F(u_i, e) = 1$. It suffices to show that $F(u_j, e) = 0$. We prove it by contrary. If $F(u_j, e) = 1$, then $e \notin D_{1\leftarrow j}$. So $e \in (\bigcup_{k=1}^{|U|} D_{1\leftarrow k} - D_{1\leftarrow j})$. That's a contradiction.

At last, it is easy to check by set theory that

$$\alpha = D^C_{1\leftarrow i} \cap (D_{i\leftarrow 1} \cup \bigcup_{k=1}^{|U|} D_{1\leftarrow k})$$
$$= (\bigcup_{k=1}^{|U|} D_{1\leftarrow k} - D_{1\leftarrow i}) \cup D_{i\leftarrow 1},$$

and

$$\beta = D^C_{1\leftarrow j} \cap (D_{j\leftarrow 1} \cup \bigcup_{k=1}^{|U|} D_{1\leftarrow k})$$
$$= (\bigcup_{k=1}^{|U|} D_{1\leftarrow k} - D_{1\leftarrow j}) \cup D_{j\leftarrow 1}.$$

Generally, we get

Corollary 4.4 (Filling Algorithm 3 for D_S With the Jth Row and Jth Column of D_S): Given the Jth row and the Jth column of D_S for soft set $S = (F, A)$ over U, then $\forall i, j = 1, 2, \cdots, |U|$,

$$D_{i\leftarrow j} = \alpha \cap \beta^C, \tag{4.6}$$

$$D_S = \begin{array}{cc} & \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} & \left(\begin{array}{cccccc} \emptyset & \{e_2\} & \boxed{\{e_2,e_3,e_7,e_8\}} & \boxed{\{e_3,e_8\}} & \{e_3,e_8\} & \{e_2,e_8\} \\ \{e_5,e_6\} & \emptyset & \{e_3,e_7,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_3,e_5,e_6,e_8\} & \{e_6,e_8\} \\ \boxed{\{e_5,e_6\}} & \emptyset & \emptyset & \{e_5,e_6\} & \{e_5,e_6\} & \{e_6\} \\ \boxed{\emptyset} & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ \emptyset & \{e_2\} & \{e_2,e_7\} & \emptyset & \emptyset & \{e_2\} \\ \{e_1,e_5\} & \{e_1\} & \{e_1,e_3,e_7\} & \{e_1,e_3,e_5\} & \{e_1,e_3,e_5\} & \emptyset \end{array}\right) \end{array}$$

FIGURE 12. A sketch map for Filling Algorithm 4.

$$D_S = \begin{array}{cc} & \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} & \left(\begin{array}{ccccc} \emptyset & \{e_1\} & \{e_1,e_3,e_4,e_5\} & \{e_4,e_5\} & \{e_1,e_5\} \\ \{e_6,e_7\} & \emptyset & \{e_3,e_4,e_5\} & \{e_4,e_5,e_6,e_7\} & \{e_5,e_7\} \\ \{e_6,e_7\} & \emptyset & \emptyset & \{e_6,e_7\} & \{e_7\} \\ \emptyset & \{e_1\} & \{e_1,e_3\} & \emptyset & \{e_1\} \\ \{e_2,e_6\} & \{e_2\} & \{e_2,e_3,e_4\} & \{e_2,e_4,e_6\} & \emptyset \end{array}\right) \end{array}.$$

FIGURE 13. The matrix of dominant support parameters for the soft set in TABLE 6.

where

$$\alpha = D^C_{J\leftarrow i} \cap (D_{i\leftarrow J} \cup \bigcup_{k=1}^{|U|} D_{J\leftarrow k}), i.e.,$$

$$\alpha = (\bigcup_{k=1}^{|U|} D_{J\leftarrow k} - D_{J\leftarrow i}) \cup D_{i\leftarrow J}, \tag{4.7}$$

and

$$\beta = D^C_{J\leftarrow j} \cap (D_{j\leftarrow J} \cup \bigcup_{k=1}^{|U|} D_{J\leftarrow k}), i.e.,$$

$$\beta = (\bigcup_{k=1}^{|U|} D_{J\leftarrow k} - D_{J\leftarrow j}) \cup D_{j\leftarrow J}. \tag{4.8}$$

Theorem 4.4 (Filling Algorithm 4 for D_S With the First Row and First Column of D_S): Given the 1st row and the 1st column of D_S for soft set $S = (F, A)$ over U, then $\forall i, j = 1, 2, \cdots, |U|$,

$$D_{i\leftarrow j} = (D_{1\leftarrow j} - D_{1\leftarrow i}) \cup (D_{i\leftarrow 1} - D_{j\leftarrow 1}). \tag{4.9}$$

Proof: (1) We prove that $D_{i\leftarrow j} \subseteq (D_{1\leftarrow j} - D_{1\leftarrow i}) \cup (D_{i\leftarrow 1} - D_{j\leftarrow 1})$. Suppose $e \in D_{i\leftarrow j}$, then $F(u_i, e) = 1$ and $F(u_j, e) = 0$. If $e \in D_{1\leftarrow j} - D_{1\leftarrow i}$, then $e \in (D_{1\leftarrow j} - D_{1\leftarrow i}) \cup (D_{i\leftarrow 1} - D_{j\leftarrow 1})$. Otherwise $e \notin D_{1\leftarrow j} - D_{1\leftarrow i}$, thus we have two possible situations:

- $e \notin D_{1\leftarrow j}$, then we have $F(u_1, e) = 0$, so $e \in (D_{i\leftarrow 1} - D_{j\leftarrow 1})$.
- $e \in D_{1\leftarrow j}$ and $e \in D_{1\leftarrow i}$, then we have $F(u_1, e) = 1$, $F(u_i, e) = 0$, that's a contradiction, since we have $e \in D_{i\leftarrow j}$.

So $D_{i\leftarrow j} \subseteq (D_{1\leftarrow j} - D_{1\leftarrow i}) \cup (D_{i\leftarrow 1} - D_{j\leftarrow 1})$.

(2) We prove that $(D_{1\leftarrow j} - D_{1\leftarrow i}) \cup (D_{i\leftarrow 1} - D_{j\leftarrow 1}) \subseteq D_{i\leftarrow j}$. Suppose $e \in (D_{1\leftarrow j} - D_{1\leftarrow i}) \cup (D_{i\leftarrow 1} - D_{j\leftarrow 1})$, then we have two possible situations:

- If $e \in D_{1\leftarrow j} - D_{1\leftarrow i}$, then $F(u_1, e) = 1$, $F(u_j, e) = 0$, $F(u_i, e) = 1$. Thus $e \in D_{i\leftarrow j}$.
- If $e \in D_{i\leftarrow 1} - D_{j\leftarrow 1}$, then $F(u_i, e) = 1$, $F(u_1, e) = 0$, $F(u_j, e) = 0$. Thus $e \in D_{i\leftarrow j}$.

According to the Filling Algorithm 4, we need only four values $D_{1\leftarrow j}, D_{1\leftarrow i}, D_{i\leftarrow 1}, D_{j\leftarrow 1}$ to figure out $D(i, j)$. Notice that we need at least the first row and the first collumn of D_S if we use the Filling Algorithm 2.

Fig. 13 gives an example for the Filling Algorithm 4. Take $D_S(3, 4) = \{e_5, e_6\}$ for an example (circled with red line), $D_S(3, 4) = (D_S(1, 4) - D_S(1, 3)) \cup (D_S(3, 1) - D_S(4, 1)) = (\{e_3, e_8\} - \{e_2, e_3, e_7, e_8\}) \cup (\{e_5, e_6\} - \emptyset) = \emptyset \cup \{e_5, e_6\} = \{e_5, e_6\}$.

Generally, the following corollary is true:

Corollary 4.5 (Filling Algorithm 5 for D_S With the Jth Row and Jth Column of D_S): Given the *Jth* row and the *Jth* column of D_S for soft set $S = (F, A)$ over U, then $\forall i, j = 1, 2, \cdots, |U|$, we have

$$D_{i\leftarrow j} = (D_{J\leftarrow j} - D_{J\leftarrow i}) \cup (D_{i\leftarrow J} - D_{j\leftarrow J}). \tag{4.10}$$

Corollary 4.6: Given the 1*st* row and the 1*st* column of D_S for soft set $S = (F, A)$ over U, then $\forall i, j = 1, 2, \cdots, |U|$, we have

$$|D_{i\leftarrow j}| = |D_{1\leftarrow j} - D_{1\leftarrow i}| + |D_{i\leftarrow 1} - D_{j\leftarrow 1}|. \tag{4.11}$$

$$
\left(
\begin{array}{ccccccc}
 & C_1 & C_2 & C_3 & C_4 & C_5 & GET \\
C_1 & \emptyset & \{e_1\} & \{e_1,e_3,e_4,e_5\} & \{e_4,e_5\} & \{e_1,e_5\} & 9 \\
C_2 & \{e_6,e_7\} & \emptyset & \{e_3,e_4,e_5\} & \{e_4,e_5,e_6,e_7\} & \{e_5,e_7\} & 11 \\
C_3 & \{e_6,e_7\} & \emptyset & \emptyset & \{e_6,e_7\} & \{e_7\} & 5 \\
C_4 & \emptyset & \{e_2\} & \{e_1,e_3\} & \emptyset & \{e_1\} & 4 \\
C_5 & \{e_2,e_6\} & \{e_2\} & \{e_2,e_3,e_4\} & \{e_2,e_4,e_6\} & \emptyset & 9 \\
LOSE & 6 & 3 & 12 & 11 & 6 &
\end{array}
\right)
$$

FIGURE 14. **The GET and LOSE values with respect to the matrix of dominant support parameters for the soft set in TABLE 6.**

$$
\left(
\begin{array}{cccccc}
- & SC.(C_1) & SC.(C_2) & SC.(C_3) & SC.(C_4) & SC.(C_5) \\
SC.(C_1) & 0 & -5 & 10 & 10 & 0 \\
SC.(C_2) & 5 & 0 & -15 & 15 & 5 \\
SC.(C_3) & -10 & 15 & 0 & 0 & -10 \\
SC.(C_4) & -10 & -15 & 0 & 0 & -10 \\
SC.(C_5) & 0 & -5 & 10 & 10 & 0
\end{array}
\right)
$$

FIGURE 15. **Menus operation results of the SCORE values for the objects in TABLE 6.**

Proof: Since $(D_{1\leftarrow j} - D_{1\leftarrow i}) \cap (D_{i\leftarrow 1} - D_{j\leftarrow 1}) = \emptyset$, by Theorem 4.4, $|D_{i\leftarrow j}| = |D_{1\leftarrow j} - D_{1\leftarrow i}| + |D_{i\leftarrow 1} - D_{j\leftarrow 1}|$.

C. CHARACTERIZATION THEOREMS OF THE MATRIX OF DOMINANT SUPPORT PARAMETERS

Theorem 4.5 (Characterization Theorem I With the First Row and the First Column of D): Given the domain of objects U, $|U| = n$. $A = \{e_1, e_2, \cdots, e_m\}$. Let D be a set-valued $n \times n$ matrix, i.e., $\forall i, j = 1, 2, \cdots n$, $D(i, j) \subseteq A$. Then D is the matrix of dominant support parameters for a soft set $S = (F, A)$ if and only if the following conditions are satisfied:

(i) $D(1, 1) = \emptyset$;

(ii) $(\bigcup_{k=1}^{n} D_{1\leftarrow k}) \cup (\bigcup_{k=1}^{n} D_{k\leftarrow 1}) = A$, $(\bigcup_{k=1}^{n} D_{1\leftarrow k}) \cap (\bigcup_{k=1}^{n} D_{k\leftarrow 1}) = \emptyset$, $i = 1, 2, \cdots, n$;

(iii) $\forall i, j = 1, 2, \cdots, |U|$, $D(i, j) = \alpha \cap \beta^C$, where

$$\alpha = D_{1\leftarrow i}^C \cap (D_{i\leftarrow 1} \cup \bigcup_{k=1}^{|U|} D_{1\leftarrow k}),$$

and

$$\beta = D_{1\leftarrow j}^C \cap (D_{j\leftarrow 1} \cup \bigcup_{k=1}^{|U|} D_{1\leftarrow k}).$$

Proof: Theorem 4.5 is implied by Theorem 3.1, Theorem 3.2 and Theorem 4.3 (Filling Algorithm 2).

Generally, by Theorem 3.1, Theorem 3.2 and Corollary 4.4 (Filling Algorithm 3) we can derive the following theorem.

Theorem 4.6 (Characterization Theorem II With the Jth Row and Jth Column of D): Given the domain of objects U, $|U| = n$. $A = \{e_1, e_2, \cdots, e_m\}$. Let D be a set-valued $n \times n$ matrix, i.e., $\forall i, j = 1, 2, \cdots n$, $D(i, j) \subseteq A$. Then D is the matrix of dominant support parameters for a soft set $S = (F, A)$ if and only if the following conditions are satisfied:

(i) $D(J, J) = \emptyset$;

(ii) $(\bigcup_{k=1}^{n} D_{J\leftarrow k}) \cup (\bigcup_{k=1}^{n} D_{k\leftarrow J}) = A$, $(\bigcup_{k=1}^{n} D_{J\leftarrow k}) \cap (\bigcup_{k=1}^{n} D_{k\leftarrow J}) = \emptyset$, $i = 1, 2, \cdots, n$;

(iii) $\forall i, j = 1, 2, \cdots, |U|$, $D(i, j) = \alpha \cap \beta^C$, where

$$\alpha = D_{J\leftarrow i}^C \cap (D_{i\leftarrow J} \cup \bigcup_{k=1}^{|U|} D_{J\leftarrow k}),$$

and

$$\beta = D_{J\leftarrow j}^C \cap (D_{j\leftarrow J} \cup \bigcup_{k=1}^{|U|} D_{J\leftarrow k}).$$

Theorem 4.7 (Characterization Theorem III With the First Row and the First Column of D): Given the domain of objects

U, $|U| = n$. $A = \{e_1, e_2, \cdots, e_m\}$. Let D be a set-valued $n \times n$ matrix, i.e., $\forall i, j = 1, 2, \cdots n$, $D(i,j) \subseteq A$. Then D is the matrix of dominant support parameters for a soft set $S = (F, A)$ if and only if the following conditions are satisfied:

(i) $D(1, 1) = \emptyset$;

(ii) $(\bigcup_{k=1}^{n} D_{1\leftarrow k}) \cup (\bigcup_{k=1}^{n} D_{k\leftarrow 1}) = A$, $(\bigcup_{k=1}^{n} D_{1\leftarrow k}) \cap (\bigcup_{k=1}^{n} D_{k\leftarrow 1}) = \emptyset$, $i = 1, 2, \cdots, n$;

(iii) $\forall i, j = 1, 2, \cdots, |U|$, the following equality holds:

$$D_{i\leftarrow j} = (D_{1\leftarrow j} - D_{1\leftarrow i}) \cup (D_{i\leftarrow 1} - D_{j\leftarrow 1}).$$

Proof: Theorem 4.7 is implied by Theorem 3.1, Theorem 3.2 and Theorem 4.4 (The Filling Algorithm 4).

Generally, by Theorem 3.1, Theorem 3.2 and Corollary 4.5 (The Filling Algorithm 5) we get

Theorem 4.8 (Characterization Theorem IV With the Jth Row and Jth Column of D): Given the domain of objects U, $|U| = n$. $A = \{e_1, e_2, \cdots, e_m\}$. Let D be a set-valued $n \times n$ matrix, i.e., $\forall i, j = 1, 2, \cdots n$, $D(i,j) \subseteq A$. Then D is the matrix of dominant support parameters for a soft set $S = (F, A)$ if and only if the following conditions are satisfied:

(i) $D(J, J) = \emptyset$;

(ii) $(\bigcup_{k=1}^{n} D_{J\leftarrow k}) \cup (\bigcup_{k=1}^{n} D_{k\leftarrow J}) = A$, $(\bigcup_{k=1}^{n} D_{J\leftarrow k}) \cap (\bigcup_{k=1}^{n} D_{k\leftarrow J}) = \emptyset$, $i = 1, 2, \cdots, n$;

(iii) $\forall i, j = 1, 2, \cdots, |U|$, it holds that

$$D_{i\leftarrow j} = (D_{J\leftarrow j} - D_{J\leftarrow i}) \cup (D_{i\leftarrow J} - D_{j\leftarrow J}).$$

Theorem 4.9: Given the domain of objects U, $|U| = n$. $A = \{e_1, e_2, \cdots, e_m\}$. Denote the set of dominant support parameters generated by all soft sets $S = (F, A)$ over U satisfying our assumptions (i.e, $\forall e \in A, F(e) \neq \emptyset$) and $F(e) \neq U$) as $D_S(U, A)$, then

$$|D_S(U, A)| = (2^{|U|} - 2)^{|A|}. \tag{4.12}$$

Proof: First we need to divide A into two parts, and we have $2^{|A|}$ ways for doing this. Each way can be represented by a pair $B \subseteq A$ (appearing on the first column) and $B^C \subseteq A$ (appearing on the first row). For the first column, there exist $(2^{|U|-1} - 1)^{|B|}$ ways. Similarly, for the fist row, there are $(2^{|U|-1} - 1)^{|B^C|}$ ways. So in total,

$$|D_S(U, A)| = 2^{|A|} * ((2^{|U|-1} - 1)^{|B|}) * (2^{|U|-1} - 1)^{|B^C|}.$$

So

$$|D_S(U, A)| = (2^{|U|} - 2)^{|A|}.$$

Corollary 4.8: Suppose U is a set of objects, A is a set of parameters. D is a randomly generated set-valued matrix of size $|U| \times |U|$ with $D(i,j) \in 2^A$, then the probability of $D \in D_S(U, A)$ is equal to

$$\frac{(2^{|U|} - 2)^{|A|}}{(2^{|A|})^{|U|\times|U|}}. \tag{4.13}$$

TABLE 6. Tabular representation of a soft set $S = (F, A)$.

U	e_1	e_2	e_3	e_4	e_5	e_6	e_7	σ_S
h_1	1	0	1	1	1	0	0	4
h_2	0	0	1	1	1	1	1	5
h_3	0	0	0	0	0	1	1	2
h_4	1	0	1	0	0	0	0	2
h_5	1	0	1	0	0	0	0	2
h_6	0	1	1	1	0	1	0	4

V. APPLICATIONS OF THE MATRIX OF DOMINANT SUPPORT PARAMETERS IN REPRESENTING THE SOFT DISCERNIBILITY MATRIX AND AN ALTERNATIVE ALGORITHM FOR COMPUTING THE ORDER REATIONS OF *S*

In this section we want to make a comparison between the matrix of dominant support parameters and the soft discernibility matrix in soft set theory given in [50]. We will show that the matrix of dominant support parameters can represent the soft discernibility matrix in a simple way and provide an alternative procedure for computing the order relation by choice values.

A. THEORIES FOR DECISION MAKING WITH THE MATRIX OF DOMINANT SUPPORT PARAMETERS OF SOFT SET S

Example 5.1: In order to make it clear, we will give our idea by a soft set example (*The soft set in Example 1 and shown in Table 1 of* [50]), where $U = \{h_1, h_2, \cdots, h_6\}$, $E = \{e_1, e_2, \cdots, e_7\}$, see TABLE 6. By [50], we have TABLE 7 and TABLE 8. By Definition 2.5 we have its matrix of dominant support parameters shown in Fig. 13.

TABLE 7, TABLE 8 and D_S give us a clear and intuitive comparision among the concepts discernibility matrix, soft discernibility matrix and the matrix of dominant support parameters. For information system, the discernibility matrix take 0 and 1 just as different symbols, which are used to distinguish the objects. The soft discernibility matrix took the form of discernibility matrix as foundation. But it pays attention to the order $1 > 0$ and represents these relation by addding well-defined superscripts. These superscripts in arbitrary entry of TABLE 8 can be divided into two parts. The matrix of dominant support parameters [41], [43] was inspired by the following idea: for each pair of objects u_i, u_j, we define $D_S(i,j)$ to be the set of parameters, for which u_i has a high value than u_j, i.e., $F(u_i, e) = 1$ and $F(u_j, e) = 0$.

It is easy to draw a close relation between the soft discernibility matrix and the matrix of dominant support parameters: the elements in $D_S(i,j) \cup D_S(j, i)$ is equal to the entry $D(i,j)$ $(i > j)$ of the soft discernibility matrix. In a word, the matrix of dominant support parameters divide $D(i,j)$ into two parts which correspond to the different superscripts. That is $e_s^t \in D(i,j)$ belongs to $D_S(i,j)$ if and only if $t = i$.

[50] gave a method for finding the order relation among objects by the Definition 2.3 with the soft discernibility

TABLE 7. The discernibility matrix for information system $S = (F, E)$ shown in TABLE 6.

C_i	C_1	C_2	C_3	C_4	C_5
C_1	$\emptyset$				
C_2	$\{e_1, e_6, e_7\}$	$\emptyset$			
C_3	$\{e_1, e_3, e_4, e_5, e_6, e_7\}$	$\{e_3, e_4, e_5\}$	$\emptyset$		
C_4	$\{e_4, e_5\}$	$\{e_1, e_4, e_5, e_6, e_7\}$	$\{e_1, e_3, e_6, e_7\}$	$\emptyset$	
C_5	$\{e_1, e_2, e_5, e_6\}$	$\{e_2, e_5, e_7\}$	$\{e_2, e_3, e_4, e_7\}$	$\{e_1, e_2, e_4, e_6\}$	$\emptyset$

TABLE 8. The soft discernibility matrix for information system $S = (F, E)$ shown in TABLE 6.

C_i	C_1	C_2	C_3	C_4	C_5
C_1	$\emptyset$				
C_2	$\{e_1^1, e_6^2, e_7^2\}$	$\emptyset$			
C_3	$\{e_1^1, e_3^1, e_4^1, e_5^1, e_6^3, e_7^3\}$	$\{e_3^2, e_4^2, e_5^2\}$	$\emptyset$		
C_4	$\{e_4^1, e_5^1\}$	$\{e_1^4, e_4^2, e_5^2, e_6^2, e_7^2\}$	$\{e_1^4, e_3^4, e_6^3, e_7^3\}$	$\emptyset$	
C_5	$\{e_1^1, e_2^5, e_5^1, e_6^5\}$	$\{e_2^5, e_5^2, e_7^2\}$	$\{e_2^5, e_3^5, e_4^5, e_7^3\}$	$\{e_1^4, e_2^5, e_4^5, e_6^5\}$	$\emptyset$

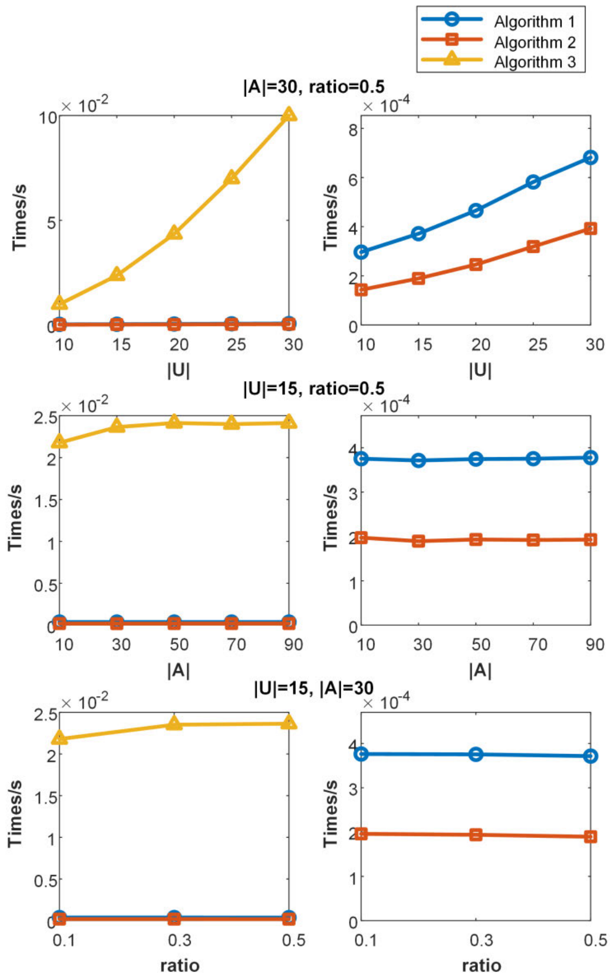

FIGURE 16. Comparative experimental results for Algorithm 1, Algorithm 2 and Algorithm 3.

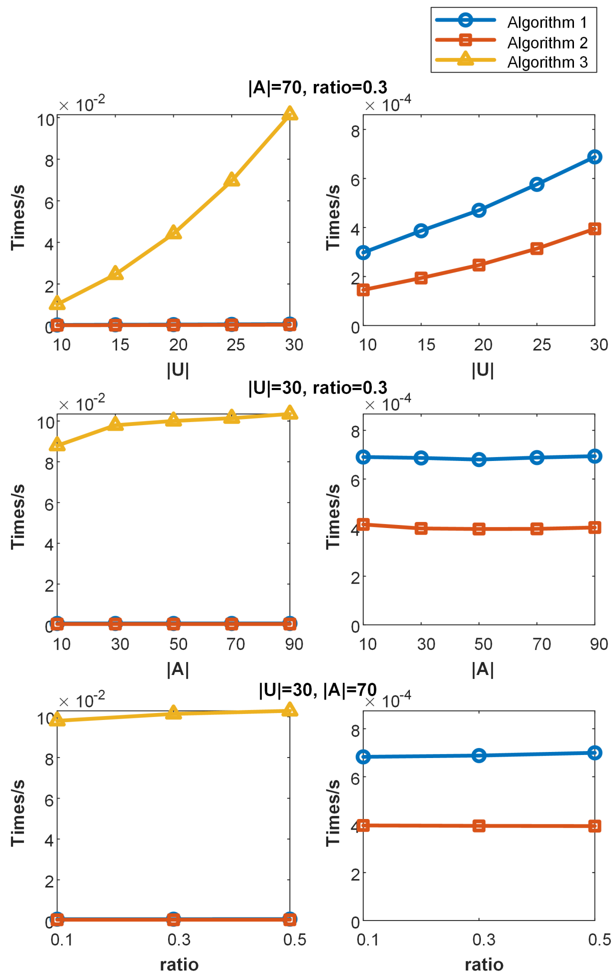

FIGURE 17. Comparative experimental results for Algorithm 1, Algorithm 2 and Algorithm 3.

matrix itself. It's an useful result. We list its algorithm as follows in TABLE 10, but we refer to [50] for more details.

Now we try to propose another method for getting the order relation of objects with the matrix of dominant support parameters itself.

Definition 5.1: Suppose $S = (F, E)$ is a soft set over U, where $U = h_1, h_2, ..., h_m$. $C = \{C_i | i = 1, 2, \cdots, K, K <= m\}$ is the set of classifications according to [50], i.e., $\exists n, h_i, h_j \in C_n$ if and only if $\forall k = 1, 2, \cdots, |E|$,

TABLE 9. Algorithm 1 for decision making based on soft discernibility in [50].

Input: *A soft set* $S = (F, E)$ *over* U, *where* $U = h_1, h_2, \cdots, h_m$.
Output: *The order relation of all the objects.*
Step 1: *Compute the partition of U and the soft discernibility matrix* $D = D(C_i, C_j)_{i,j \leqslant m}$.
Step 2: *Denote* $D_1 = \{D(C_i, C_j) : d(C_i, C_j) = 2n, n \in N^+\}$ *and* $D_2 = \{D(C_i, C_j) : d(C_i, C_j) = 2n+1, n \in N^+\}$. *Select the items* D_1 *and* D_2 *from the soft discernibility matrix, respectively.*
Step 3: *For every element of* D_1, *we need to compare its* $|E^i|$ *with* $|E^j|$ *according to Property 7. If* $|E^i| = |E^j|$, *then the objects* $h_i \in C_i$ *and* $h_j \in C_j$ *are kept in the same* (**Note: here** $|E^i|$ **and** $|E^j|$ **are equal to** $D_S(i,j)$ **and** $D_S(j,i)$ **in our work, respectively**.) *decision class. Otherwise, there must exist an order relation between the objects* h_i *and* h_j, *that is, either* h_i *is superior to* h_j, *or the objects* h_j *is superior to* h_i.
Step 4: *Output the result of the step 3. If it is a global order relation for all of the objects in* U, *then the algorithm is end; otherwise, turn to step 5.*
Step 5: *Combine with the result of step 4, find the corresponding elements in* D_2 *to compare the order relation based on the Property 6. For the objects which lie in the same decision classes after the step 4, when compare them with the other objects, it only need to select one of them. And the selected objects must have the minimal cardinality in the set of soft discernibility parameters.*
Step 6: *Output the order relation among all the objects by combining the forth step with the fifth step.*

TABLE 10. Algorithm 2 for decision making algorithm by matrix of dominant support parameters.

Input: A soft set $S = (F, E)$ over U, where $U = h_1, h_2, ..., h_m$.
Output: Global order relation for all of the objects.
Step 1 Compute the partition of U and the matrix D_S;
Step 2 $\forall i = 1, 2, \cdots, m$, figure out $GET(i)$, $LOSE(i)$, $SCORE(i) = GET(i) - LOSE(i)$;
Step 3 Output global order relation for all of the objects in U by $\sigma(i) > \sigma(j)$ if and only if $SCORE(i) > SCORE(j)$.

$F(h_i, e_k) = F(h_j, e_k)$. $\forall i = 1, 2, \cdots, K$, define

$$GET(i) = \sum_{j=1}^{K} |D_S(i,j)|, \tag{5.1}$$

$$Lose(i) = \sum_{j=1}^{K} |D_S(j,i)|, \tag{5.2}$$

and

$$SCORE(i) = GET(i) - Lose(i). \tag{5.3}$$

Fig. 14 adds the GET and LOSE values to the D_S in the Fig. 13.

Theorem 5.1: Suppose $S = (F, E)$ is a soft set over U, where $U = h_1, h_2, \ldots, h_m$. $C = \{C_i | i = 1, 2, \cdots, K, K <= m\}$ is the set defined in Definition 5.1. Then we have

$$SCORE(C_i) - SCORE(C_j) = |C|(|D(i,j)| - |D(j,i)|), \tag{5.4}$$

where $|C|$ means the number of elements in C.

Proof: We can take a pair of objects for example. Assume $h_1 \in C_1$, and $h_2 \in C_2$ and it suffices to show that $SCORE(C_1) - SCORE(C_2) = |C|(|D(1,2)| - |D(2,1)|)$.

TABLE 11. Algorithm 3 for decision making algorithm with the first row and the first column in the matrix of dominant support parameters.

Input: A soft set $S = (F, E)$ over U, where $U = h_1, h_2, ..., h_m$.
Output: Global order relation for all of the objects.
Step 1 Compute the partition of U, the first row and the first column of the matrix D_S;
Step 2 $\forall i, j$, retrieve $D(i,j)$ and get the matrix D_S;
Step 3 $\forall i = 1, 2, \cdots, m$, compute $GET(i)$, $LOSE(i)$, $SCORE(i) = GET(i) - LOSE(i)$;
Step 4 Output global order relation for all of the objects in U by $\sigma(i) > \sigma(j)$ if and only if $SCORE(i) > SCORE(j)$.

$\forall e \in E$, we can get four situations as follows:

(1) When $F(h_1, e) = 1$, $F(h_2, e) = 1$, e contributes the same value to $GET(1)$ and $GET(2)$. And e contributes nothing to $LOSE(1)$ and $LOSE(2)$.

(2) When $F(h_1, e) = 0$, $F(h_2, e) = 0$, it can be derived that e contributes the same value to $LOSE(1)$ and $LOSE(2)$,and e contributes nothing to $GET(1)$ and $GET(2)$.

(3) When $F(h_1, e) = 1$, $F(h_2, e) = 0$, i.e, $e \in D(1,2)$, e contributes an advantage value $|C|$ for C_1 over C_2.

(4) When $F(h_1, e) = 0$, $F(h_2, e) = 1$, i.e, $e \in D(2,1)$, e contributes an advantage value $|C|$ for C_2 over C_1.

So we have $SCORE(C_1) - SCORE(C_2) = |C|(|D(1,2)| - |D(2,1)|)$.

It can be proved in the same way for an arbitrary pair of objects from different classes. The proof is over.

The matrix in expression (5.5) and the matrix in Fig. 15 give test examples for Theorem 5.1 with the soft set in TABLE 6.

$$\begin{pmatrix} |D(i,j)| - |D(j,i)| & C_1 & C_2 & C_3 & C_4 & C_5 \\ C_1 & 0 & -1 & 2 & 2 & 0 \\ C_2 & 1 & 0 & -3 & 3 & 1 \\ C_3 & -2 & 3 & 0 & 0 & -2 \\ C_4 & -2 & -3 & 0 & 0 & -2 \\ C_5 & 0 & -1 & 2 & 2 & 0 \end{pmatrix}. \tag{5.5}$$

Denote $\sigma(C_i) = \sigma(h)$, $h \in C_i$. According to [43], [50], it's easy to see that

$$\sigma(C_i) > \sigma(C_j) \text{ if and only if } |D(i,j)| > |D(j,i)|,$$
$$\sigma(C_i) = \sigma(C_j) \text{ if and only if } |D(i,j)| = |D(j,i)|.$$

So by Theorem 5.1 we have the following corollary.

Corollary 5.1: Suppose $S = (F, E)$ is a soft set over U, where $U = h_1, h_2, ..., h_m$. $C = \{C_i | i = 1, 2, \cdots, K, K <= m\}$ is the set defined in Definition 5.1. Denote $\sigma(C_i) = \sigma(h)$, $h \in C_i$. Then we have

$$\sigma(C_i) > \sigma(C_j) \Leftrightarrow SCORE(C_i) > SCORE(C_j),$$

and

$$\sigma(C_i) = \sigma(C_j) \Leftrightarrow SCORE(C_i) = SCORE(C_j)$$

TABLE 12. Comparative experimental results for time cost of Algorithm 1, Algorithm 2 and Algorithm 3(to be continued).

The number of objects	The number of parameters	The ratio of 1	average time cost for Algorithm 1 in [50] (10^{-2}s)	average time cost for Algorithm 2 (10^{-2}s)	average time cost for Algorithm 3 (10^{-2}s)
10	10	0.1	0.300	0.147	8.643
10	10	0.3	0.290	0.143	8.761
10	10	0.5	0.294	0.142	8.966
10	30	0.1	0.293	0.142	8.883
10	30	0.3	0.290	0.141	9.593
10	30	0.5	0.297	0.143	9.912
10	50	0.1	0.298	0.148	9.616
10	50	0.3	0.297	0.143	9.918
10	50	0.5	0.304	0.151	10.371
10	70	0.1	0.299	0.145	9.841
10	70	0.3	0.298	0.145	10.165
10	70	0.5	0.294	0.145	10.144
10	90	0.1	0.297	0.144	9.941
10	90	0.3	0.297	0.145	10.141
10	90	0.5	0.294	0.144	10.047
15	10	0.1	0.373	0.202	20.663
15	10	0.3	0.378	0.198	21.591
15	10	0.5	0.375	0.197	21.775
15	30	0.1	0.376	0.196	21.797
15	30	0.3	0.375	0.194	23.524
15	30	0.5	0.372	0.189	23.639
15	50	0.1	0.376	0.194	22.734
15	50	0.3	0.380	0.197	24.202
15	50	0.5	0.375	0.193	24.132
15	70	0.1	0.372	0.193	23.182
15	70	0.3	0.387	0.194	24.581
15	70	0.5	0.375	0.192	23.993
15	90	0.1	0.374	0.190	23.275
15	90	0.3	0.379	0.194	23.972
15	90	0.5	0.378	0.193	24.123
20	10	0.1	0.467	0.257	36.990
20	10	0.3	0.469	0.251	38.833
20	10	0.5	0.465	0.252	39.462
20	30	0.1	0.467	0.251	39.408
20	30	0.3	0.461	0.246	42.374
20	30	0.5	0.466	0.246	43.478
20	50	0.1	0.464	0.246	40.913
20	50	0.3	0.464	0.248	43.895
20	50	0.5	0.467	0.247	44.182

B. ALGORITHMS FOR DECISION MAKING WITH THE MATRIX OF DOMINANT SUPPORT PARAMETERS OF SOFT SET S

According to Theorem 5.1 and Corollary 5.1, we give **Algorithm 2** to compute the decision making of soft sets by its matrix of support parameters, see TABLE 10.

It is easy to show that $SCORE(1) = 9 - 6 = 3$, $SCORE(2) = 11 - 3 = 8, SCORE(3) = 5 - 12 = -7, SCORE(4) = 4 - 11 = -7, SCORE(5) = 9 - 6 = 3$. So we have $h_2 > h_1 = h_6 > h_3 = h_4 = h_5$. That's the same with the order given by choice values $\sigma(i)$.

TABLE 13. Comparative experimental results for Algorithm 1, Algorithm 2 and Algorithm 3(the continuation of TABLE 12).

The number of objects	The number of parameters	The ratio of 1	average time cost for Algorithm 1 in [50] (10^{-2}s)	average time cost for Algorithm 2 (10^{-2}s)	average time cost for Algorithm 3 (10^{-2}s)
20	70	0.1	0.466	0.245	42.621
20	70	0.3	0.470	0.247	44.166
20	70	0.5	0.466	0.246	44.171
20	90	0.1	0.464	0.244	42.944
20	90	0.3	0.469	0.247	44.303
20	90	0.5	0.472	0.247	44.396
25	10	0.1	0.586	0.346	60.667
25	10	0.3	0.584	0.334	62.690
25	10	0.5	0.587	0.327	63.873
25	30	0.1	0.573	0.322	61.674
25	30	0.3	0.577	0.316	68.540
25	30	0.5	0.582	0.319	69.901
25	50	0.1	0.581	0.326	66.867
25	50	0.3	0.580	0.317	69.332
25	50	0.5	0.575	0.313	69.438
25	70	0.1	0.570	0.315	67.155
25	70	0.3	0.576	0.314	69.583
25	70	0.5	0.571	0.311	69.411
25	90	0.1	0.575	0.317	68.621
25	90	0.3	0.574	0.312	69.681
25	90	0.5	0.574	0.310	69.588
30	10	0.1	0.688	0.431	84.077
30	10	0.3	0.690	0.413	87.921
30	10	0.5	0.690	0.410	89.917
30	30	0.1	0.690	0.410	90.099
30	30	0.3	0.687	0.397	97.922
30	30	0.5	0.682	0.392	100.043
30	50	0.1	0.676	0.398	94.217
30	50	0.3	0.680	0.394	100.046
30	50	0.5	0.671	0.387	99.629
30	70	0.1	0.683	0.397	98.028
30	70	0.3	0.688	0.395	101.396
30	70	0.5	0.700	0.394	102.898
30	90	0.1	0.691	0.397	100.695
30	90	0.3	0.694	0.400	103.441
30	90	0.5	0.692	0.391	103.016

Compared with the Algorithm in [50] (given in TABLE 9), **Algorithm 2** makes use of the matrix of dominant support parameters in a quantitative way. It is an alternative method for retrieving the order relations of S by D_S itself. It is simple and much more direct.

By the Filling Algorithm 4, once we have only the first row and the first column entries in matrix of dominant support parameters, we can firstly compute D_S itself, and then use Algorithm 2 to get the decision making order.

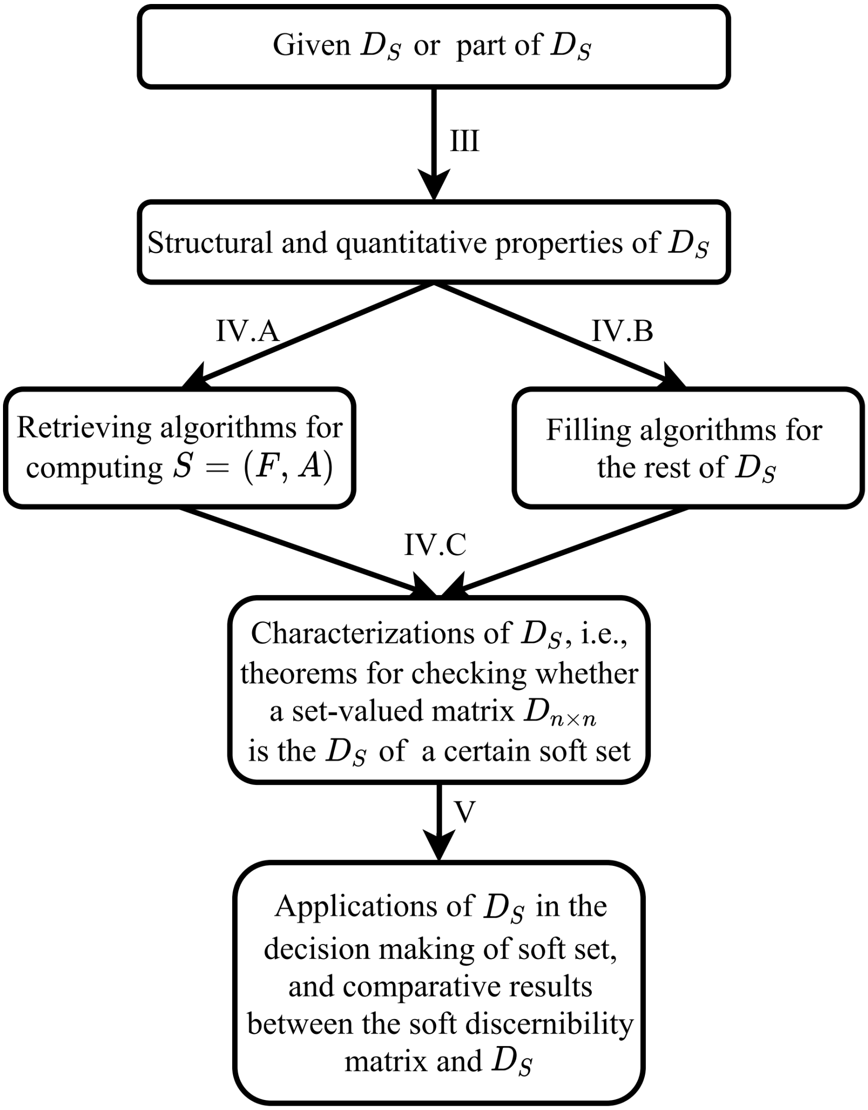

FIGURE 18. The contents that have been discussed in this article.

C. COMPARATIVE EXPERIMENTAL RESULTS FOR ALGORITHM 1, ALGORITHM 2 AND ALGORITHM 3

In this subsection we will show the results of comparative experiments among **Algorithm 1, Algorithm 2 and Algorithm 3** .

1) EQUIPMENT AND DATA GENERATION METHOD

(i) Our experiments are performed on PC with AMD Ryzen 5 3500U 2.10GHz CPU, 8G RAM and Win10 professional operating system.

(ii) Our data are generated in the following way: Firstly, we use the rand function of MATLAB to generate a uniformly distributed matrix of random numbers in the [0,1] interval. Then the number less than or equal to N in the matrix is changed to 1 and the rest of the numbers are changed to 0. So we can get a matrix with a ratio of 1 to N.

2) MAIN CONTROLLING PARAMETERS IN OUR EXPERIMENTS

(i) The number of rows, i.e., the number of objects $|U|$.

(ii) The number of columns, i.e., the number of parameters $|A|$.

(iii) The ratio of value 1, i.e., the proportion of 1 values over $|U| \times |A|$

3) COMPARATIVE EXPERIMENTAL RESULTS FOR ALGORITHM 1, ALGORITHM 2 AND ALGORITHM 3

For each combination of parameters shown above, with respect to the same data set we run each algorithm for 100 times. The average time cost for Algorithm 1, Algorithm 2 and Algorithm 3 are listed in TABLE 12 and TABLE 13. Particularly we give two pictures FIGURE 16 and FIGURE 17. For FIGURE 16 we have the following explanations:

(i) Let $|A| = 30, |U| = 10, 15, \cdots, 30$. The ratio of 1 is equal to 0.5. The results of the time cost are shown at the top-level of FIGURE 16 (In FIGURE 16, the subfigure on the right side of each level is the partial display of the corresponding left one). With respect to $|U|$, the time cost decreases when $|U|$ increase, the time cost of Algorithm 3 is much longer when compared with Algorithm 1 and Algorithm 2. The time cost of Algorithm 1 is longer when compared with Algorithm 2.

(ii) Let $|U| = 15, |A| = 10, 20, \cdots, 90$. The ratio of 1 is equal to 0.5. The results of the time cost are shown at the middle-level of FIGURE 16. With respect to $|A|$, the time cost changes a little when $|A|$ increase. The time cost of Algorithm 1 is longer when compared with Algorithm 2.

(iii) Let $|U| = 15, |A| = 30$. The ratio of 1 is equal to 0.1, 0.3 and 0.5. The results of the time cost are shown at the bottom-level of FIGURE 16. With respect to the ratio of 1, the time cost changes a little when $|A|$ increase. The time cost of Algorithm 1 is longer when compared with Algorithm 2.

As to FIGURE 17, we have a similar result with FIGURE 16 shows.

VI. CONCLUSION AND FUTURE WORK

Fig. 18 gives the contents that have been discussed in this article.

A. THE MAIN RESULTS OF THIS PAPER

(1) The fundamental structural and quantitative properties are investigated, and these properties can help us in having a better understanding of the matrix of dominant support parameters.

(2) By using only part of the matrix of dominant support parameters, we can recompute the initial soft sets and fill the rest of the matrix itself. These algorithms are important from the aspect of knowledge representation and data mining.

(3) The proposed characterization theorems are helpful. With them we can judge which kind of set-valued matrices can be the matrices of dominant support parameters for certain soft sets or information systems.

B. LIMITATIONS OF OUR THEORY

(1) The matrix of dominant support parameters does not contain these parameters whose corresponding approximations are equal to $\varnothing$ or U. This problem should be solved. A potential way is to add the related parameters into the main diagonal of the matrix.

(2) The **Algorithm 1** computes D_1 and D_2. If $D_2 = \varnothing$, then it is implied than all the choice values of subjects are odd numbers or all the choice values of objects are even numbers. That is, if $\exists u_i, u_j, i \neq j, \sigma(u_i)$ is odd and $\sigma(u_j)$ is even, then $D_2 \neq \varnothing$. Our algorithms don't involve such information. So they need to be further improved.

(3) From the experimental results, we can we see that **Algorithm 3** costs much longer than **Algorithm 2** and **Algorithm 1** do. So we need to consider the following questions: (i) Do we have to retrieve all the entries of the matrix? (ii) Which ones should we retrieve and in which order?

C. FUTURE WORK

In the near future, we will make more research on the matrix of dominant support parameters. For example, as a future possible research direction we will upgrade the matrices of dominant support parameters in hypersoft sets [28] or soft sets combined with fuzzy set theory [16]–[25]. And we also will investigate the areas in which our theory and methods can be useful.

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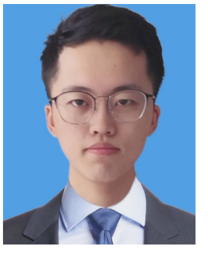

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