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Information Allocation for Transport Switched Systems With ADT Method

XIAOLI JIANG¹, MINGYUE LIU¹, AND LINA LIU²

¹School of Mathematics and Physics, Bohai University, Jinzhou 121013, China

²School of Electronic and Information Engineering, Soochow University, Suzhou 215006, China

Corresponding authors: Xiaoli Jiang (jxls309@163.com) and Lina Liu (lln@suda.edu.cn)

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ABSTRACT In this work, an output feedback controller based on adaptive backstepping technique is considered for a class of transport information switched systems. A modified adaptive neural network control method is successfully built by combining the general approximation capability and dynamic surface control technique with the improved mean discrete time scheme. More specifically, a switched observer is constructed to reduce the conservatism caused by the use of a common observer, and by adopting the common coordinate transformation of all subsystems, it is proved that the overall closed-loop system is stable in the sense of semi-globally uniformly ultimately bounded in mean square, and steers the output to a small neighborhood of the origin. Finally, mass-spring-damping system with controller switching simulation studies are provided to demonstrate the validity of the proposed control format.

INDEX TERMS Neural networks structure, average dwell time, Internet of Things, transport information systems, mass-spring-damping.

I. INTRODUCTION

In recent years, latest trends in communication technologies have brought a rapid increase in the internet of things [1], [5], [11]. There has been considerable study interest in transport information systems from many researchers, because of many logistical systems such as networked systems [3], circuit and power systems [4] and robot systems [2] can be modeled as such systems. Switched transport systems constitute a special class of hybrid systems, which contain both continuous dynamics and discrete dynamics. The analysis and the synthesis of switched systems have drawn considerable attention in control and computer community in the past period of time [6]–[10]. The control theories of switched systems have been seen much progress of robotic, mechatronic and mechanical systems, gene regulatory networks, and switching power converters [12]–[14]. The widespread applications of switched systems are also motivated by the better performance via using a controller switching strategy. However, due to the inter action between continuous or discrete subsystems, switched dynamic may have a very complicated behavior [12], [15]–[17]. For example, switching between unstable subsystems can give rise to stability, while

switching between stable subsystems may lead to instability [12] and [15]. In all dynamic behaviors, there may be either good performance, bad performance or even unstable dynamics on the premise of ensuring the stability of the system, this work attempts to design an appropriate control strategy to make the system achieve better switching effect and optimized performance index. Hence, as for the switching systems, its motion pattern is richer and more complex than the previous single model, and it is also a challenging topic.

The controller design for stochastic switching systems with uncertain triangles has become a hot topic. Many relative results have been achieved by using the famous backstepping technique (see [18]–[20] and their references). The backward step design can effectively solve the stability and reliability of the tracking control of the lower triangular nonlinear systems [17], [21]–[32]. Dead zone and output constraints of the controller [31] were constructed for the uncertain stochastic system. An adaptive algorithm based on neural networks and tracking controller were established for the nonlinear problem of single input and single output. Recently, the stability analysis and global stability problems, have been extensively studied in [9], [33]–[35]. Stochastic switched systems can be effectively modeled as exchange systems in various fields [36], [37], etc. However, a switched system does

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not necessarily inherit the attributes of a single subsystem [38]–[40], for any switching signal, even if all subsystems have such properties, it is not necessarily establish the asymptotic stability of the switching system. For instance, several adaptive backstepping approaches have been presented for uncertain switched systems with lower-triangular form [41]–[44]. Among them, Tong et al [42], Long and Zhao [43], [44] presented adaptive neural output feedback controller. It should be noted that the aforementioned adaptive algorithms are only limited to the switched uncertain deterministic systems, they cannot be directly used in those switched ones with stochastic forms, this kind of problem has not been studied in depth. Therefore, how to design a stable scheme for switched stochastic systems is our main purpose.

In order to achieve the stability or certain performance, switching systems need to stay on each subsystem for a period of time before switching to another subsystem. These running times will form a switching sequence, which is the dwell time problem. For a class of continuous time switching systems with stable subsystems, Morse [45] first proposed the concept of dwell time. The minimum dwell time (MDT) was defined as the minimum of all dwell times. The average dwell time (ADT) was quantified when the average time interval was not less than τ_a in [46]. Zhai *et al.* [47] combined ADT technology with Lyapunov function and obtained the stability conditions of switching systems for unstable subsystems. The authors used ADT method to give switching rates for hybrid switching systems in [48]. Research has shown that the ADT scheme is an effective tool of designing stable switching signals [49]–[53]. All of the above work has obtained exponential stability, ADT reduces conservativeness and enhances flexibility of switching rules compared with dwell time (DT) and MDT. However, the above achievements were concentrated on linear systems, and most of the switching subsystems were fully stable. Less work has been done on lagging, time delay, including unstable subsystems, internal and external disturbances and pulses. To the best of our knowledge, no effort has been devoted to ADT control for switched systems in lower-triangular form with disturbances. Therefore, this work attempts to use ADT technology to deal with the switching problems of unstable subsystems, so as to obtain a control format with less conservativeness and more flexible switching strategy. In the process of controller design, a neural networks system is used to approximate unknown nonlinear function and a common coordinate transformation is chosen to avoid coordinate transformation of different subsystems. An output feedback scheme based on decentralized adaptive approximation is proposed. Compared with the existing design methods, the main advantages of this paper are as follows:

(i) By using ADT method and neural network tracking technique, sufficient conditions for adaptive output feedback scheme of switching system are given. A neural network output feedback controller based on adaptive backstepping is proposed for the lower triangular uncertain switched stochastic system.

(ii) In order to avoid the coordinate transformation of different subsystems, the common coordinate transformation of different subsystems is adopted in the backward operation of each step. By using the constructed switching observer, the unmeasurable state of the switching system is successfully estimated, which reduces the conservatism of the control design caused by the use of one general observer in each subsystem. The dynamic surface control (DSC) technique is successfully extended to switched stochastic systems by introducing a first-order filter in each step of the traditional backward extrapolation method.

(iii) The traditional ADT method in [41], [43] cannot be directly used to deal with the problem of adaptive neural network output-feedback controller design of switched uncertain stochastic nonlinear system since mathematical expectation, which is one of the important digital features of random variable, should be exerted in the proof process. To solve this challenging problem, we successfully expand the traditional ADT method in the deterministic case to the one in the stochastic case.

The rest of this work is composed as follows: We introduced some assumptions and preliminaries and described problem statements in section II, as well as output Feedback DSC Design and stability analysis were listed in this section III. Simulation results can be found in section IV, with concluding in section VI.

II. PROBLEM FORMULATION

We consider the switched systems by:

$$\begin{aligned} d\eta_i(t) &= (\eta_{i+1}(t) + f_{\sigma(t),i}(\bar{\eta}_i(t)) + g_{\sigma(t),i}(\eta(t)))dt \\ &\quad + h_{\sigma(t),i}(y(t))d\omega, \quad 1 \leq i \leq n \\ d\eta_n(t) &= (u_{\sigma(t)} + f_{\sigma(t),n}(\eta(t)) + g_{\sigma(t),n}(\eta(t)))dt \\ &\quad + h_{\sigma(t),n}(y(t))d\omega \\ y(t) &= \eta_1(t) \end{aligned} \quad (1)$$

where $\bar{\eta}_i(t) = [\eta_1(t), \eta_2(t), \dots, \eta_i(t)]^T \in R^i, i = 1, 2, \dots, n-1, \eta(t) = [\eta_1(t), \eta_2(t), \dots, \eta_n(t)]^T \in R^n$ denote state vectors of the system; $u_{\sigma(t)} \in R$ and $y \in R$ are the system input and output. Where $\sigma(t) : [0, +\infty) \rightarrow \mathbb{M} = 1, 2, \dots, N$ is the piecewise constant switching signal. For $k \in \mathbb{M}$, when $\sigma(t) = k$, we say that the k subsystem is jump and the other subsystems are not, $f_{k,i}(\cdot)$ are assumed to be unknown and locally Lipschitz nonlinear functions, and $g_{k,i}(\eta)$ ($i = 1, 2, \dots, n, k = 1, 2, \dots, m$) are external disturbances. For $\forall i = 1, 2, \dots, n, k = 1, 2, \dots, m$, assume that $h_{k,i}(y(t)) = y\varphi_{k,i}(y(t))$, and $\varphi_{k,i}(y(t))$ is a known smooth functions satisfying Local Lipschitz condition. ω stands for a standard Wiener process satisfying $E\{d\omega(t)\} = 0$.

Some definitions and lemmas needed in the research process will be described below.

Lemma 1 (Young's Inequality): For $\forall(x, y) \in R^2$ the following inequality holds

$$xy \leq \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\varepsilon^q}|y|^q$$

where $\varepsilon > 0, p > 1, q > 1$, and $(p-1)(q-1) = 1$.

Definition 1 [54]: For a switching signal $\sigma(t)$ and any $t > \tau > 0$, denote by $N_\sigma(\tau, t)$ the number of switchings of $\sigma(t)$ on the interval (τ, t) . We say that $\sigma(t)$ has an ADT τ_a if

$$N_\sigma(\tau, t) \leq N_0 + \frac{t - \tau}{\tau_a} \quad (2)$$

holds for $N_0 \geq 0$. $\tau_a > 0$, here N_0 is called the chatter bound.

Assumption 1: The external disturbances $g_{k,i}(\eta(t))$ are bounded and satisfy $g_{k,i}(\eta(t)) \leq \bar{g}_i$, where \bar{g}_i ($i = 1, 2, \dots, n, k = 1, 2, \dots, m$) are unknown constants.

A. STOCHASTIC STABILITY

Consider the stochastic system

$$d\eta(t) = f(\eta(t))dt + h(\eta(t))d\omega \quad (3)$$

where $\eta \in R^n$ is the system state, ω is r -dimensional standard Wiener process, and $f : R^n \rightarrow R^n$, $h : R^n \rightarrow R^n$ are locally Lipschitz functions and satisfy $f(0) = h(0) = 0$.

B. NNs APPROXIMATION

In this paper, approximation-based NNs will be used to approximate the unknown nonlinear function.

Lemma 2 [55]: For any continuous unknown smooth nonlinear function $f(x)$ over a compact set $\Omega \subset \mathfrak{N}^n$, there exists NNs $W^T S(x)$ such that for a desired level of accuracy ε

$$f(x) = W^T S(x) + \delta(x), \quad |\delta(x)| \leq \varepsilon \quad (4)$$

where $x \in \Omega \subset \mathfrak{N}^n$ is the input vector, $W = [\omega_1, \omega_2, \dots, \omega_l]^T$ is the weight vector, $l > 1$ is the number of the NNs nodes and $S(x) = [s_1(x), \dots, s_l(x)]^T$, with $s_i(x)$ being chosen as the commonly used Gaussian functions, which have the form

$$s_i(x) = \exp\left[-\frac{(x - \mu_i)^T(x - \mu_i)}{\varpi_i^2}\right], \quad i = 1, 2, \dots, l \quad (5)$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \dots, \mu_{in}]^T$ is the center of the receptive field and ϖ_i is the width of the Gaussian function.

III. OUTPUT-FEEDBACK ADAPTIVE CONTROLLER DESIGN

With the aid of radial basis function (RBF) NNs, we will obtain the main result of this paper, adaptive output-feedback neural tracking control is achievable by designing adaptive output-feedback controllers of subsystems and giving a class of switching signal with average dwell time, for the system (1).

A. OUTPUT FEEDBACK DSC DESIGN

Now, we will give the design procedure of a switched observer to estimate system state since state variables of the system (1) are not available. Design the switched observer as

$$\begin{cases} \dot{\hat{\eta}}_i(t) = \hat{\eta}(t)_{i+1} - l_{\sigma(t),i}\hat{\eta}_1(t), & i = 1, 2, \dots, n-1 \\ \dot{\hat{\eta}}_n(t) = u_{\sigma(t)} - l_{\sigma(t),n}\hat{\eta}_1(t) \end{cases} \quad (6)$$

where $\hat{\eta}_i(t)$ ($i = 1, 2, \dots, n$) are the estimates of $\eta_i(t)$ with $\tilde{\eta}_i(t) = \eta_i(t) - \hat{\eta}_i(t)$, for $\forall k \in \mathbb{M}$, u_k is the system input of (6),

the switching signal $\sigma(t)$ is the same as defined in (1), and $l_{k,i}$ $i = 1, 2, \dots, n, k = 1, 2, \dots, m$ are the design parameters such that the matrices

$$A_k = \begin{bmatrix} -l_{k,1} & & & \\ & \ddots & & \\ & & I_{n-1} & \\ -l_{k,n} & \cdots & & 0 \end{bmatrix}, \quad k \in M \quad (7)$$

Then combining (1) with (6), one gets

$$d\tilde{\eta}(t) = (A_{\sigma(t)}\tilde{\eta}(t) + F_{\sigma(t)} + G_{\sigma(t)})dt + H_{\sigma(t)}d\omega \quad (8)$$

where $F_k = [f_{k,1} + l_{k,1}y, \dots, f_{k,n} + l_{k,n}y]^T$, $G_k = [g_{k,1}, \dots, g_{k,n}]^T$ and $H_k = [h_{k,1}(y(t)), \dots, h_{k,n}(y(t))]^T = y[\varphi_{k,1}(y(t)), \dots, \varphi_{k,n}(y(t))]^T$. A_k is a strictly Hurwitz, which means that for any given positive definite symmetric matrices $Q_k > 0$, there exist some matrices $P_k > 0$ satisfying

$$A_k^T P_k + P_k A_k = -Q_k. \quad (9)$$

Therefore, the switched system can be rewritten as

$$\begin{aligned} d\tilde{\eta}(t) &= (A_{\sigma(t)}\tilde{\eta}(t) + F_{\sigma(t)} + G_{\sigma(t)})dt + H_{\sigma(t)}d\omega \\ dy(t) &= (\hat{\eta}_2(t) + \tilde{\eta}_2(t) + f_{\sigma(t),1}(\eta_1) + g_{k,1}(\eta(t)))dt \\ &\quad + h_{\sigma(t),1}(y(t))d\omega \\ d\hat{\eta}_i(t) &= (\hat{\eta}_{i+1}(t) - l_{\sigma(t),i}\hat{\eta}_1(t))dt, \quad i = 1, \dots, n-1 \\ d\hat{\eta}_n(t) &= (u_{\sigma(t)} - l_{\sigma(t),n}\hat{\eta}_1(t))dt \end{aligned} \quad (10)$$

where $y(t)$, $\eta_i(t)$, $l_{k,i}$, A_k , P_k , and Q_k are available for control design.

In order to avoid the problem of explosion of complexity. DSC approach is introduced in this part, based on the change of coordinates, as follows:

$$\begin{cases} e_1 = y(t) \\ e_i = \hat{\eta}_i(t) - \alpha_i, \quad i = 2, 3, \dots, n \end{cases} \quad (11)$$

α_i is the output variable of a designed first-order filter with the intermediate virtual control α_{i-1} and $u_{\sigma(t)} = \alpha_n$ is specified in the final step. In addition, $z_i = \alpha_i - \alpha_{i-1}$ is defined to denote the boundary layer error.

Similar to the traditional backstepping technique, the recursive design procedure contains n steps to steps $n-1$, the virtual controllers α_i of subsystems are constructed at Step n . Meanwhile, at each step, the RBF NNs $W_{ik}^T S_i(Z_i)$ is employed to approximate the unknown nonlinear function, unknown control signal $\hat{\alpha}_{ik}(Z_i)$. Before proceeding with the adaptive output-feedback neural tracking control, define first constants as

$$\theta_i = \max\{\|W_{ik}\|^2 : k \in \mathbb{M}\}, \quad i = 0, 1, 2, \dots, n \quad (12)$$

θ_i is an unknown constant because of $\|W_{ik}\|$ being unknown constants, $i = 0, 1, \dots, n, k \in \mathbb{M}$. Moreover, let $\hat{\theta}_i = \theta_i - \hat{\theta}_i$ ($i = 1, 2, \dots, n$), where $\hat{\theta}_i$ is the estimation of θ_i . In what follows, we will propose the backstepping-based DSC design procedure.

Step 1: For any $k \in \mathbb{M}$, it follows from (10) and (11) that

$$de_1 = (e_2 + \alpha_1 + z_2 + \tilde{\eta}_2(t) + f_{k,1} + g_{k,1})dt + h_{k,1}d\omega \quad (13)$$

Construct the stochastic Lyapunov function candidate as

$$V_{k,1} = \frac{1}{4}e_1^4 + \frac{\bar{b}_{k1}}{2r_1^2}\tilde{\theta}_1^2 \quad (14)$$

where $r > 0$ is a design parameter. According to (9) and (10), one has

$$\begin{aligned} \dot{V}_{k,1} \leq & e_1^3(e_2 + \alpha_1 + z_2 + \tilde{\eta}_2(t) + f_{k,1} + g_{k,1}) \\ & + \frac{3}{2}e_1^2 h_{k,1} h_{k,1}^T - \frac{\bar{b}_{k1}}{r}\tilde{\theta}_1 \dot{\hat{\theta}}_1. \end{aligned} \quad (15)$$

Afterwards, the NNs systems $W_{k,i}^T S(Z)$ is used to approximate the unknown nonlinear functions $f_{k,i} + l_{k,i}y(t)$ ($i = 1, \dots, n, k \in \mathbb{M}$) such that for $\forall \varepsilon_i > 0$

$$f_{k,i} + l_{k,i}y(t) = W_{k,i}^T S(Z) + \delta_{k,i}(Z), |\delta_{k,i}(Z)| \leq \varepsilon_i \quad (16)$$

where $Z = \eta(t)$ and $\delta_{k,i}(Z)$ are the approximation error. Further, according to the definition of F_k , we get

$$F_k(Z) = W_k^T S(Z) + \delta_k(Z), |\delta_k(Z)| \leq \varepsilon_0 \quad (17)$$

where $W_k = [W_k^T, \dots, W_{k,n}^T]$, $\delta_k(Z) = [\delta_{k,1}(Z), \dots, \delta_{k,n}(Z)]^T$, and $\varepsilon_0 > 0$ is a constant.

Therefore, according to Young's inequality and the definition of $\tilde{\eta}_2(t)$, we have (18)

$$\begin{aligned} e_1^3 e_2 &\leq \frac{3}{4}e_1^4 + \frac{1}{4}e_2^4 \\ e_1^3 z_2 &\leq \frac{3}{4}\varpi_{11}^{\frac{4}{3}}e_1^4 + \frac{1}{4\varpi_{11}^4}z_2^4 \\ e_1^3 \tilde{\eta}_2(t) &\leq \frac{3}{4}\varpi_{12}^{\frac{4}{3}}e_1^4 + \frac{1}{4\varpi_{12}^4}\|\tilde{\eta}_2(t)\|^4 \\ e_1^3 g_{k,1} &\leq \frac{3}{4}\varpi_{13}^{\frac{4}{3}}e_1^4 + \frac{1}{4\varpi_{13}^4}\bar{g}_1^4 \\ \frac{3}{2}e_1^2 h_{k,1} h_{k,1}^T &= \frac{3}{2}e_1^4 \varphi_{k,1} \varphi_{k,1}^T \leq \frac{3}{2}e_1^4 \varphi_1 \varphi_1^T \end{aligned} \quad (18)$$

where $\varpi_{1i} (i = 1, 2, 3)$ are positive design constants, φ_1 is a known smooth function satisfying $\varphi_1 \geq \varphi_{k,1} (k \in \mathbb{M})$.

By substituting (18) into (15), one has

$$\begin{aligned} \dot{V}_{k,1} \leq & e_1^3(\alpha_1 + f_{k,1} + \frac{3}{4}e_1 + \frac{3}{4}\varpi_{11}^{\frac{4}{3}}e_1 + \frac{3}{4}\varpi_{12}^{\frac{4}{3}}e_1 \\ & + \frac{3}{4}\varpi_{13}^{\frac{4}{3}}e_1 + \frac{3}{2}e_1 \varphi_{k,1} \varphi_{k,1}^T) + \frac{1}{4}e_2^4 + \frac{1}{4\varpi_{11}^4}z_2^4 \\ & + \frac{1}{4\varpi_{12}^4}\|\tilde{\eta}_2(t)\|^4 + \frac{1}{4\varpi_{13}^4}\bar{g}_1^4 - \frac{\bar{b}_{ik}}{r}\tilde{\theta}_1 \dot{\hat{\theta}}_1 \\ = & e_1^3(\alpha_1 + \bar{f}_{k,1}(Z_1)) + \frac{1}{4}e_2^4 + \frac{1}{4\varpi_{11}^4}z_2^4 \\ & + \frac{1}{4\varpi_{13}^4}\bar{g}_1^4 - \frac{\bar{b}_{ik}}{r}\tilde{\theta}_1 \dot{\hat{\theta}}_1 \end{aligned} \quad (19)$$

where $\bar{f}_{k,1}(Z_1) = f_{k,1} + (3/4)e_1 + (3/4)\varpi_{11}^{(4/3)}e_1 + (3/4)\varpi_{12}^{(4/3)}e_1 + (3/4)\varpi_{13}^{(4/3)}e_1 + (3/2)e_1 \varphi_1 \varphi_1^T$ with $\varpi_{1i} > 0$ ($i = 1, 2, 3$) being positive constants. Furthermore, by virtue

of Lemma 2, an NNs $W_{k,1}^T S_1(Z_1)$ can be employed to estimate $\bar{f}_{k,1}(Z_1)$, and then $\bar{f}_{k,1}(Z_1)$ can be rewritten as

$$\bar{f}_{k,1}(Z_1) = W_{k,1}^T S_1(Z_1) + \delta_{k,1}(Z_1), |\delta_{k,1}(Z_1)| \leq \varepsilon_1 \quad (20)$$

where $\delta_{k,1}(Z_1)$ denotes the approximation error, and $\varepsilon_1 > 0$ is a positive design parameter. Consequently, based on (12), we can get

$$\begin{aligned} e_1^3 \bar{f}_{k,1}(Z_1) &\leq e_1^3(W_{k,1}^T S_1(Z_1) + \delta_{k,1}(Z_1)) \\ &\leq \frac{1}{2a_1^2}\theta_1 e_1^6 S_1^T(Z_1)S_1(Z_1) + \frac{1}{2}a_1^2 \\ &\quad + \frac{3}{4}e_1^4 + \frac{1}{4}\varepsilon_1^4 \end{aligned} \quad (21)$$

where $a_1 > 0$ is a design parameter.

Therefore, we can obtain

$$\begin{aligned} \dot{V}_{k,1} \leq & e_1^3(\alpha_1 + \frac{3}{4}e_1 + \frac{1}{2a_1^2}\theta_1 e_1^6 S_1^T(Z_1)S_1(Z_1)) \\ & + \frac{1}{4\varpi_{11}^4}z_2^4 + \frac{1}{4\varpi_{13}^4}\bar{g}_1^4 - \frac{\bar{b}_{ik}}{r}\tilde{\theta}_1 \dot{\hat{\theta}}_1 + \frac{1}{4}e_2^4 \\ & + \frac{1}{4\varpi_{12}^4}\|\tilde{\eta}_2(t)\|^4 + \frac{1}{2}a_1^2 + \frac{1}{4}\varepsilon_1^4. \end{aligned} \quad (22)$$

Define the intermediate virtual control α_1 and the adaptation law $\dot{\hat{\theta}}_1$ as follows

$$\begin{aligned} \alpha_1 = & -\xi_1 e_1 - \frac{3}{4}e_1 - \frac{\hat{\theta}_1}{2a_1^2}e_1^3 S_1^T(Z_1)S_1(Z_1) \\ & - \frac{3(n-1)}{4}e_1(\varphi_1 \varphi_1^T)^2 \end{aligned} \quad (23)$$

$$\dot{\hat{\theta}}_1 = \frac{r}{2a_1^2}e_1^6 - \varrho_1 \hat{\theta}_1 \quad (24)$$

where ϱ_1 is a positive design parameter. Then, substituting (23) and (24) into (22) yields

$$\begin{aligned} \dot{V}_{k,1} \leq & \frac{1}{4}e_2^4 + \frac{1}{4\varpi_{11}^4}z_2^4 + \frac{1}{4\varpi_{13}^4}\bar{g}_1^4 + \frac{1}{2}a_1^2 \\ & + \frac{1}{4}e_1^4 + \frac{1}{4\varpi_{12}^4}\|\tilde{\eta}_2(t)\|^4 - \xi_1 e_1^4 \\ & - \frac{3(n-1)}{4}e_1^4(\varphi_1 \varphi_1^T)^2 + \frac{\bar{b}_{k1}\varrho_1}{r}\tilde{\theta}_1 \hat{\theta}_1. \end{aligned} \quad (25)$$

Step i ($2 \leq i \leq n-1$): In order to avoid repeatedly differentiating α_{i-1} , a new state variable α_i is introduced, and let α_{i-1} pass through a first-order filter with time constant κ_{i-1} to obtain α_i as

$$\kappa_{i-1}\dot{\alpha}_i + \alpha_i = \alpha_{i-1}, \alpha_i(0) = \alpha_{i-1}(0). \quad (26)$$

Then, we can get the dynamic equation of boundary layer error z_i as follows

$$dz_i = (-\frac{z_i}{\kappa_i} - \dot{\alpha}_{i-1})dt - \frac{\partial \alpha_{i-1}}{\partial y(t)}d\omega \quad (27)$$

with

$$\begin{aligned} \dot{\alpha}_{i-1} = & \frac{\partial \alpha_{i-1}}{\partial y(t)} (\hat{\eta}_2(t) + \tilde{\eta}_2(t) + f_{k,1} + g_{k,1}) \\ & + \sum_{j=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\eta}_j(t)} \dot{\hat{\eta}}_j(t) + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \\ & + \frac{1}{2} \frac{\partial^2 \alpha_{i-1}}{\partial y^2(t)} h_{k,1} h_{k,1}^T + \sum_{j=1}^{i-2} \frac{\partial \alpha_{i-1}}{\partial \alpha_{jf}} \dot{\alpha}_{jff}. \end{aligned} \quad (28)$$

Hence, from (10) and (11), we can obtain

$$de_i = (e_{i+1} + \alpha_i + z_{i+1} - l_{k,i} \hat{\eta}_1 + \frac{z_i}{\kappa_i}) dt. \quad (29)$$

Consider a stochastic Lyapunov function candidate in the form

$$V_{k,i} = V_{k,i-1} + \frac{1}{4} e_i^4 + \frac{1}{4} z_i^4 + \frac{\bar{b}_{k1}}{2r} \tilde{\theta}_i^2. \quad (30)$$

Then, we have

$$\begin{aligned} \dot{V}_{k,i} = & \dot{V}_{k,i-1} + e_i^3 (e_{i+1} + \alpha_i + z_{i+1} - l_{k,i} \hat{\eta}_1 + \frac{e_i}{\kappa_i}) \\ & + z_i^3 (-\frac{z_i}{\kappa_i} - \dot{\alpha}_{i-1}) - \frac{\bar{b}_{k1}}{r} \tilde{\theta}_i \dot{\hat{\theta}}_i \\ & + \frac{3}{2} z_i^2 (\frac{\partial \alpha_{i-1}}{\partial y(t)})^2 h_{k,1} h_{k,1}^T. \end{aligned} \quad (31)$$

Similar to (18), we can infer

$$\begin{aligned} e_i^3 e_{i+1} \leq & \frac{3}{4} e_i^4 + \frac{1}{4} e_{i+1}^4 \\ e_i^3 z_{i+1} \leq & \frac{3}{4} \varpi_{i1}^{\frac{4}{3}} z_i^4 + \frac{1}{4 \varpi_{i1}^4} z_{i+1}^4 \\ \frac{3}{2} z_i^2 (\frac{\partial \alpha_{i-1}}{\partial y(t)})^2 h_{k,1} h_{k,1}^T \leq & \frac{3}{4} (\frac{\partial \alpha_{i-1}}{\partial y(t)})^4 z_i^4 \\ & + \frac{3}{4} e_1^4 (\varphi_1 \varphi_1^T)^2. \end{aligned} \quad (32)$$

Furthermore, we obtain

$$\begin{aligned} \dot{V}_{k,i} \leq & e_i^3 (e_i + \alpha_i + \frac{3}{4} \varpi_{i1}^{\frac{4}{3}} e_i - l_{k,i} \hat{\eta}_1 + \frac{e_i}{\kappa_i}) \\ & + \sum_{j=2}^i z_j^3 (-\frac{z_j}{\kappa_j} - \dot{\alpha}_{j-1} + \frac{3}{4} (\frac{\partial \alpha_{j-1}}{\partial y(t)})^4 z_j) \\ & + \frac{1}{4} e_{i+1}^4 + \sum_{j=1}^i \frac{1}{4 \varpi_{j1}^4} z_{j+1}^4 + \frac{1}{4 \varpi_{12}^4} \|\tilde{\eta}_2(t)\|^4 \\ & - \sum_{j=1}^{i-1} x_j e_j^4 + \sum_{j=1}^{i-1} (\frac{1}{2} a_j^2 + \frac{1}{4} \varepsilon_j^4) \\ & + \sum_{j=1}^{i-1} \frac{\bar{b}_{k1} \varrho_j}{r} \tilde{\theta}_j \dot{\hat{\theta}}_j - \frac{\bar{b}_{k1}}{r} \tilde{\theta}_i \dot{\hat{\theta}}_i \\ & + \frac{1}{4 \varpi_{13}^4} \bar{g}_1^4 - \frac{3(n-i)}{4} e_1^4 (\varphi_1 \varphi_1^T)^2 \\ = & e_i^3 (\alpha_i + \bar{f}_{k,i}(Z_i)) + \frac{1}{4} e_{i+1}^4 + \sum_{j=1}^i \frac{1}{4 \varpi_{j1}^4} z_{j+1}^4 \end{aligned}$$

$$\begin{aligned} & + \sum_{j=2}^i z_j^3 (-\frac{z_j}{\kappa_j} - \dot{\alpha}_{j-1} + \frac{3}{4} (\frac{\partial \alpha_{j-1}}{\partial y(t)})^4 z_j) \\ & + \frac{1}{4 \varpi_{12}^4} \|\tilde{\eta}_2(t)\|^4 - \sum_{j=1}^{i-1} x_j e_j^4 + \sum_{j=1}^{i-1} \frac{\bar{b}_{k1} \varrho_j}{r} \tilde{\theta}_j \dot{\hat{\theta}}_j \\ & + \sum_{j=1}^{i-1} (\frac{1}{2} a_j^2 + \frac{1}{4} \varepsilon_j^4) - \frac{\bar{b}_{k1}}{r} \tilde{\theta}_i \dot{\hat{\theta}}_i \\ & + \frac{1}{4 \varpi_{13}^4} \bar{g}_1^4 - \frac{3(n-i)}{4} e_1^4 (\varphi_1 \varphi_1^T)^2 \end{aligned} \quad (33)$$

where $\bar{f}_{k,i}(Z_i) = e_i + \frac{3}{4} \varpi_{i1}^{\frac{4}{3}} e_i - l_{k,i} \hat{\eta}_1 + (\frac{z_i}{\kappa_i})$ with $\varpi_{i1} > 0$ being a positive constant.

What is more, by virtue of Lemma 3, we can employ an NNs $W_{k,i}^T S_i(Z_i)$ to estimate $\bar{f}_{k,i}(Z_i)$ can be rewritten as

$$\bar{f}_{k,i}(Z_i) = W_{k,i}^T S_i(Z_i) + \delta_{k,i}(Z_i), \quad |\delta_{k,i}(Z_i)| \leq \varepsilon_i \quad (34)$$

where $\delta_{k,i}(Z_i)$ denotes the approximation error and $\varepsilon_i > 0$ is a positive design parameter. Consequently, based on (12), we can get

$$\begin{aligned} e_i^3 \bar{f}_{k,i}(Z_i) \leq & e_i^3 (W_{k,i}^T S_i(Z_i) + \delta_{k,i}(Z_i)) \\ \leq & \frac{1}{2 a_i^2} \theta_2 e_i^6 S_i^T(Z_i) S_i(Z_i) + \frac{1}{2} a_i^2 \\ & + \frac{3}{4} e_i^4 + \frac{1}{4} \varepsilon_i^4 \end{aligned} \quad (35)$$

where $a_i > 0$ is a given design parameter. Now, choose the intermediate virtual control α_i and the adaptation law $\hat{\theta}_i$, respectively, as

$$\alpha_1 = -\xi_i e_i - \frac{3}{4} e_i - \frac{\hat{\theta}_i}{2 a_i^2} e_i^3 S_i^T(Z_i) S_i(Z_i) \quad (36)$$

$$\dot{\hat{\theta}}_i = \frac{r}{2 a_i^2} e_i^6 - \varrho_i \hat{\theta}_i \quad (37)$$

where ϱ_i is a positive design parameter.

$$\begin{aligned} \dot{V}_{k,i} \leq & \sum_{j=2}^i z_j^3 (-\frac{z_j}{\kappa_j} - \dot{\alpha}_{j-1} + \frac{3}{4} (\frac{\partial \alpha_{j-1}}{\partial y(t)})^4 z_j) \\ & + \frac{1}{4} e_{i+1}^4 + \sum_{j=1}^i \frac{1}{4 \varpi_{j1}^4} z_{j+1}^4 + \frac{1}{4 \varpi_{12}^4} \|\tilde{\eta}_2(t)\|^4 \\ & + \sum_{j=1}^i \xi_j e_j^4 + \sum_{j=1}^i (\frac{1}{2} a_j^2 + \frac{1}{4} \varepsilon_j^4) + \frac{1}{4 \varpi_{13}^4} \bar{\omega}_1^4 \\ & - \frac{3(n-i)}{4} e_1^4 (\varphi_1 \varphi_1^T)^2 + \sum_{j=1}^n \frac{\bar{b}_{ki} \varrho_j}{r} \tilde{\theta}_j \dot{\hat{\theta}}_j. \end{aligned} \quad (38)$$

Step n: In the final step, α_n is obtained by letting α_{n-1} pass through the following first-order filter with the design parameter $\kappa_n > 0$

$$\kappa_n \dot{\alpha}_n + \alpha_n = \alpha_{n-1}, \quad \alpha_n(0) = \alpha_{n-1}(0). \quad (39)$$

Then, we have

$$dz_n = \left(-\frac{z_n}{\kappa_n} - \dot{\alpha}_{n-1}\right)dt - \frac{\partial\alpha_{n-1}}{\partial y(t)}d\omega \quad (40)$$

where

$$\begin{aligned} \dot{\alpha}_{n-1} = & \frac{\partial\alpha_{n-1}}{\partial y(t)}(\hat{\eta}_2(t) + \tilde{\eta}_2(t) + f_{k,1} + g_{k,1}) \\ & + \sum_{j=2}^{n-1} \frac{\partial\alpha_{n-1}}{\partial \hat{\eta}_j(t)} \dot{\hat{\eta}}_j(t) + \sum_{j=1}^{n-1} \frac{\partial\alpha_{n-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \\ & + \frac{1}{2} \frac{\partial^2\alpha_{n-1}}{\partial y^2(t)} h_{k,1} h_{k,1}^T + \sum_{j=1}^{n-2} \frac{\partial\alpha_{n-1}}{\partial \alpha_{jf}} \dot{\alpha}_{jf}. \end{aligned} \quad (41)$$

According to (11), we have

$$de_n = (u_k - l_{k,n} \hat{\eta}_1(t) + \frac{z_n}{\kappa_n})dt. \quad (42)$$

Consider a stochastic Lyapunov function candidate in the form

$$V_{k,n} = V_{k,n-1} + \frac{1}{4}e_n^4 + \frac{1}{4}z_n^4 + \frac{\bar{b}_{k,1}}{2r}\tilde{\theta}_n^2. \quad (43)$$

Using the inductive argument above, we can construct an actual control an actual control input u_k for different subsystems and the adaptation law $\hat{\theta}_n$ as

$$u_k = \alpha_n = -x_n e_n - \frac{3}{4}e_n - \frac{\hat{\theta}_n}{2a_n^2} e_n^3 S_n^T(Z_n) S_n(Z_n) \quad (44)$$

$$\dot{\hat{\theta}}_n = \frac{r}{2a_n^2} e_n^6 - \varrho_n \hat{\theta}_n \quad (45)$$

where $\varrho_n > 0$ is a given design parameter.

Therefore, we can verify that

$$\begin{aligned} \dot{V}_{k,n} \leq & -\sum_{j=1}^n \xi_j e_j^4 + \sum_{j=1}^{n-1} \frac{1}{4\varpi_{j1}^4} e_{j+1}^4 + \sum_{j=2}^n z_j^3 \left(-\frac{z_j}{\kappa_j}\right. \\ & \left. - \dot{\alpha}_{j-1} + \frac{3}{4} \left(\frac{\partial\alpha_{j-1}}{\partial y(t)}\right)^4 z_j\right) + \sum_{j=1}^n \left(\frac{1}{2}a_j^2 + \frac{1}{4}\varepsilon_j^4\right) \\ & + \frac{1}{4\varpi_{13}^4} \bar{g}_1^4 + \sum_{j=1}^n \frac{\bar{b}_{k,1} \varrho_j}{r} \tilde{\theta}_j \hat{\theta}_j. \end{aligned} \quad (46)$$

Furthermore, let $-(z_j/\kappa_j) - \dot{\alpha}_{j-1} + 3/4(\partial\alpha_{j-1}/\partial y(t))^4 z_j = B_j(\cdot)$, and it can be deduced from [51] that $B_j(\cdot)$ is a continuous function satisfying $|B_j(\cdot)| \leq D_j$, D_j is a positive constant. This, we further get

$$\sum_{j=2}^n z_j^3 B_j \leq \frac{3}{4} \sum_{j=2}^n \zeta_j^{\frac{4}{3}} z_j^4 + \sum_{j=2}^n \frac{1}{4\zeta_j^4} D_j^4 \quad (47)$$

where $\zeta_j > 0$ is a constant. Moreover, appropriate parameter κ_j , ζ_j and $\varpi_{j-1,1}$ are chosen such that $-\left(\frac{1}{4\varpi_{j-1,1}^4} - \frac{3}{4}\zeta_j^{\frac{4}{3}}\right) \geq \zeta_j > 0 (j = 2, \dots, n)$.

B. STABILITY ANALYSIS

In this section, we provide the stability analysis of the corresponding closed-loop system. For the sake of simplicity, we first define

$$\begin{aligned} a_0 = & \min\{4x_j, \varrho_j, \frac{c_0}{r\lambda^2 \max(P_k)}, j = 1, 2, \dots, n, k \in \mathbb{M}\} \\ \mu = & \max\left\{\frac{\lambda \max(P_k)}{\lambda \min(P_l)}, k, l \in \mathbb{M}\right\}. \end{aligned} \quad (48)$$

Theorem 1: Under Assumption 1, suppose that for $1 \leq i \leq n$, $k \in \mathbb{M}$, all the unknown nonlinear function $f_{k,i}(\eta_i(t))$ can be approximated by NNs in the sense that the approximation errors ε_i is bounded. Consider the closed-loop system consisting of the switched system (1), the observer (6), the controller (44) with the intermediate virtual control (23) (36), and the adaptive laws (26), (39), and (47). For bounded initial conditions with and every switching signal $\sigma(t)$, all the signals in the closed-loop system are in probability SGUUB, and the output can be made arbitrarily small by choosing suitable design parameters.

Proof: The proof is divided into two parts. First of all in part (1), we will prove that the overall closed-loop system is semi-globally stable, and then the convergence of output will be verified in part (2).

(1): For stability analysis, we construct the following Lyapunov function for subsystems

$$V_k(X) = \frac{1}{4} \sum_{i=1}^n z_i^4 + \frac{1}{4} \sum_{i=2}^n e_i^4 + \frac{\bar{b}_{k,1}}{2r} \sum_{i=1}^n \tilde{\theta}_i^2, k \in \mathbb{M} \quad (49)$$

where $X = (z_1, \dots, z_n, e_2, \dots, e_n, \tilde{\theta}_1, \dots, \tilde{\theta}_n)^T$. It is obvious that we can find two function $\beta, \bar{\alpha} \in \kappa_\infty$, such that $\beta(\|X\|) \leq V_k(X) \leq \bar{\alpha}(\|X\|)$. Further, based on (49), we have $V_k(X(t)) \leq \mu V_l(X(t))$, $\forall k, l \in \mathbb{M}$.

For the terms $\frac{\bar{b}_{k,1} \varrho_j}{r} \tilde{\theta}_j \hat{\theta}_j$ ($j = 1, 2, \dots, n$) in (46), the following inequalities hold

$$\frac{\varrho_j}{r} \tilde{\theta}_j \hat{\theta}_j \leq -\frac{\varrho_j}{2r} \tilde{\theta}_j^2 + \frac{\varrho_j}{2r} \theta_j^2, \quad j = 1, 2, \dots, n. \quad (50)$$

Then, in view of (46) and (50), we have

$$\begin{aligned} \dot{V}_k \leq & -\sum_{j=1}^n \frac{\varrho_j}{2r} \tilde{\theta}_j^2 + \sum_{j=1}^n \left(\frac{1}{2}a_j^2 + \frac{1}{4}\varepsilon_j^4\right) \\ & + \frac{1}{4\varpi_{13}^4} \bar{g}_1^4 + \sum_{j=1}^n \frac{\varrho_j}{2r} \theta_j^2 + \sum_{j=2}^n \frac{1}{4\zeta_j^4} D_j^4 \\ \leq & -a_0 V_k + b_0 \end{aligned} \quad (51)$$

where $b_0 = \max_{k \in \Xi} \left\{ \sum_{j=1}^n \left(\frac{1}{2}a_j^2 + \frac{1}{4}\varepsilon_j^4\right) + \frac{1}{4\varpi_{13}^4} \bar{g}_1^4 + \sum_{j=1}^n \frac{\varrho_j}{2r} \theta_j^2 + \sum_{j=2}^n \frac{1}{4\zeta_j^4} D_j^4 \right\} > 0$

Further, it is easy to infer that $W(t) = e^{a_0 t} V_\sigma(t)(X(T))$ is piecewise differentiable along solutions of the system consists of (10), (26), (39), and (47). By (51), on each interval $[t_k, t_{k+1})$, one has

$$\dot{W}(t) = a_0 e^{a_0 t} V_\sigma(t)(X(t)) + e^{a_0 t} \dot{V}_\sigma(t)(X(t)) \leq b_0 e^{a_0 t}. \quad (52)$$

Taking $V_k(X(t)) \leq \mu V_l(X(t)) (\forall k, l \in L)$ into account, one has

$$\begin{aligned} W(t_{k+1}) &= e^{a_0 t_{k+1}} V_{\sigma(t_{k+1})}(X(t_{k+1})) \\ &\leq \mu e^{a_0 t_{k+1}} V_{\sigma_k}(X(t_{k+1})) \\ &= \mu W(t_k) \\ &\leq \mu [W(t_k) + \int_{t_k}^{t_{k+1}} b_0 e^{a_0 t} dt]. \end{aligned} \quad (53)$$

It follows immediately from integrating the inequality (53) from $k = 0$ to $k = N_{\sigma}(T, 0) - 1$ that

$$\begin{aligned} W(T^-) &\leq \mu^{N_{\sigma}(T,0)} [W(0) + \sum_{k=0}^{N_{\sigma}(T,0)-1} \mu^{-k} \int_{t_k}^{t_{k+1}} b_0 \\ &\cdot e^{a_0 t} dt + \mu^{-N_{\sigma}(T,0)} \int_{t_{N_{\sigma}(T,0)}}^T b_0 e^{a_0 t} dt]. \end{aligned} \quad (54)$$

For any $\delta \in (0, a_0 - (\ln \mu / \tau_a))$, because $\tau_a > (\ln \mu / a_0)$, one has $\tau_a > (\ln \mu / a_0 - \delta)$. By (2), it holds that

$$N_{\sigma(t)}(T, t) \leq N_0 + \frac{(a_0 - \delta)(T - t)}{\log \mu}, \quad \forall T \geq t \geq 0. \quad (55)$$

Notice $N_{\sigma}(T, 0) - k \leq 1 + N_{\sigma}(T, t_{k+1}), k = 0, 1, \dots, N_{\sigma}(T, 0)$, it means

$$\mu^{N_{\sigma}(T,0)-k} \leq \mu^{1+N_0} e^{(a_0-\delta)(T-t_{k+1})}. \quad (56)$$

In addition, since $0 < \delta < a_0$, we have

$$\int_{t_k}^{t_{k+1}} b_0 e^{a_0 t} dt \leq e^{(a_0-\delta)t_{k+1}} \int_{t_k}^{t_{k+1}} b_0 e^{\delta t} dt. \quad (57)$$

Then from (54) and (57), we can obtain

$$W(T^-) \leq \mu^{N_{\sigma}(T,0)} W(0) + \mu^{1+N_0} e^{(a_0-\delta)T} \int_0^T b_0 e^{\delta t} dt \quad (58)$$

which implies that

$$\begin{aligned} \alpha(\|X(T)\|) &\leq V_{\sigma(T^-)}(X(T^-)) \\ &\leq e^{N_0} e^{(\frac{\ln \mu}{\tau_a} - a_0)T} \bar{\alpha}(\|X(0)\|) \\ &\quad + \mu^{1+N_0} \frac{b_0}{\delta}. \end{aligned} \quad (59)$$

In view of (59) and $\delta > 0$, we can conclude that, if τ_a satisfies $\tau_a > \ln \mu / a_0$, then $\tilde{\eta}_j(t) \xi_j, \tilde{\theta}_j (j = 1, \dots, n)$, and $e_j (j = 2, \dots, n)$ are bounded by choosing bounded initial values. Because $\theta_j (j = 1, \dots, n)$ are constants, $\hat{\theta}_j (j = 1, \dots, n)$ are thus bounded. Furthermore, based on Assumption 1 and (11), $\hat{\eta}_j(t) (j = 1, \dots, n)$ are bounded. Moreover, according to the definition of $\tilde{\eta}_j(t) = \eta_j(t) - \hat{\eta}_j(t)$, we can infer that $\eta_j(t) (j = 1, \dots, n)$ are also bounded.

Thus, we conclude that all the signals of the corresponding closed-loop system are bounded under an arbitrary switching signal $\sigma(t)$ satisfying the ADT $\tau_a > (\ln \mu / a_0)$.

(2): On the other hand, for $\forall \zeta > 0$, we can obtain that the inequality $\mu^{1+N_0} (b_0 / \delta) \leq (1/4) \zeta^2$ holds by suitably choosing the matrices $Q_k, k \in L$, the design parameters ξ_j, ϱ_j ,

and r sufficiently large, and $a_j \varepsilon_j$ sufficiently small. Further, it follows from (59) that

$$\begin{aligned} \frac{1}{4} |e_1^4(T)| &\leq z^{N_0 \ln \mu} z^{(\frac{\ln \mu}{\tau_a} - a_0)T} \bar{\alpha}(\|X(0)\|) \\ &\quad + \mu^{1+N_0} \frac{b_0}{\delta} (1 - z^{-\delta T}), \quad \forall T > 0 \end{aligned} \quad (60)$$

which together with $\tau_a > (\ln \mu / a_0)$, indicates that

$$\lim_{t \rightarrow \infty} |e_1^4(t)| \leq 4 \mu^{1+N_0} \frac{b_0}{\delta} \leq \zeta^4. \quad (61)$$

The proof of Theorem 1 is completed here.

Remark 1: What is more, the designed controller u in (44) contains the neural networks basis functions, which will cause cumbersome calculations in realistic applications. Nevertheless, it should be also emphasized that $0 < S_i^T(Z_i) S_i(Z_i) \leq 1$ can be learned from the theories in [52].

From Remark 1, we realize that the basis functions in the controller (44) can be easily removed to further improve the results.

IV. SIMULATION EXAMPLE

In this section, two examples along with the respective numerical data and simulation results are presented to demonstrate the correctness and feasibility of the proposed control designs as well as to illustrate the system performances that can be achieved.

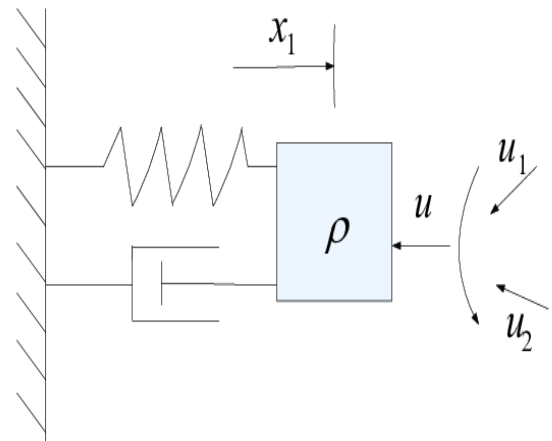


FIGURE 1. Mass-spring-damper system with controller.

Example 1: In order to prove the applicability and effectiveness of our proposed method, the following mass-spring-damping system with controller switching is considered (see Fig. 1):

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\frac{1}{\rho} f(x_1) - \frac{1}{\rho} g(x_2) + \frac{1}{\rho} u_k, \\ y &= x_1, \end{aligned} \quad (62)$$

where $f(x_1)$ and $g(x_2)$ are unknown smooth nonlinear functions with $f(0) = 0, g(0) = 0$, and ρ is an unknown positive constant. Further, it is supposed that $f(x_1)$ is stochastic

processes defined by $f(x_1) = \bar{f}(x_1) + h(x_1)\zeta(t)$, where $\bar{f}(x_1)$ are deterministic and $\zeta(t)$ is white noise degrading measurements. Then, we have the stochastic nonlinear systems below:

$$\begin{aligned} dx_1 &= x_2 dt, \\ dx_2 &= \left(-\frac{1}{\rho}\bar{f}(x_1) - \frac{1}{\rho}g(x_2) + \frac{1}{\rho}u_k\right)dt - \frac{1}{\rho}h(x_1)d\omega \\ y &= x_1 \end{aligned} \quad (63)$$

Moreover, suppose that we are only allowed to apply two prespecified candidate controllers $u_k = -\Delta f_k(x) + \rho v_k$, $k = 1, 2$ with $\Delta f_k(0) = 0$ to the system (63) and switch between them. Thus, we can obtain the following switched uncertain stochastic nonlinear system:

$$\begin{aligned} dx_1 &= x_2 dt, \\ dx_2 &= \left(v_2 - \frac{1}{\rho}(\bar{f}(x_1) + \Delta f_k(x)) - \frac{1}{\rho}g(x_2)\right)dt \\ &\quad - \frac{1}{\rho}h(x_1)d\omega, \\ y &= x_1. \end{aligned} \quad (64)$$

Next, the following switched observer is designed:

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 - l_{k,1}\hat{x}_1, \\ \dot{\hat{x}}_2 &= v_2 - l_{k,2}\hat{x}_1, \end{aligned} \quad (65)$$

where the design parameters are selected as $l_{1,1} = 2$, $l_{2,1} = 1$, $l_{1,2} = l_{2,2} = 4$. Apparently, the matrices A_1 and A_2 are Hurwitz, and we choose $Q_1 = 6I_2$, $Q_2 = 8I_2$. Then, we can find the positive-definite symmetric matrices

$$P_1 = \begin{bmatrix} 3 & -3 \\ -3 & 9 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 5 & -4 \\ -4 & 21 \end{bmatrix}$$

such that the equalities $A_k^T P_k + P_k A_k = -Q_k$ ($k \in \mathcal{I}$) are satisfied. It follows from (64) and (65) that:

$$\begin{aligned} de &= (A_k e + F_k)dt + H d\omega, \\ dy &= (\hat{x}_2 + e_2)dt, \\ d\hat{x}_2 &= (v_2 - l_{k,2}\hat{x}_1)dt, \end{aligned} \quad (66)$$

where $F_k = [l_{k,1}y, -\frac{1}{\rho}(\bar{f}(x_1) + \Delta f_k(x)) - \frac{1}{\rho}g(x_2) + l_{k,2}y]^T$, $H = -\frac{1}{\rho}h(x_1)$.

From Theorem 1, the intermediate virtual controls, actual controller and adaptive laws are chosen as follows:

$$\begin{aligned} \alpha_1 &= -\xi_1 e_1 - \frac{3}{4}e_1 - \frac{\hat{\theta}_1}{2a_1^2}e_1^3 S_1^T(Z_1)S_1(Z_1) \\ &\quad - \frac{3}{4}e_1 \frac{1}{\rho^2}(\varphi_1 \varphi_1^T)^2, \\ v_2 &= \xi_2 e_2 - \frac{3}{4}e_2 - \frac{\hat{\theta}_2}{2a_2^2}e_2^3 S_2^T(Z_2)S_2(Z_2), \\ \dot{\hat{\theta}}_1 &= \frac{r}{2a_1^2}e_1^6 - \varrho_1 \hat{\theta}_1, \\ \dot{\hat{\theta}}_2 &= \frac{r}{2a_2^2}e_2^6 - \varrho_2 \hat{\theta}_2. \end{aligned} \quad (67)$$

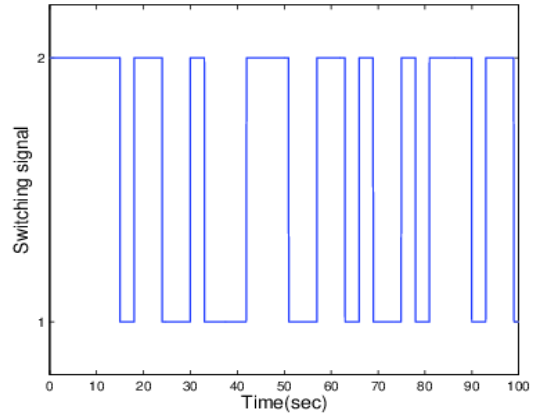


FIGURE 2. The time profile of the switching signal.

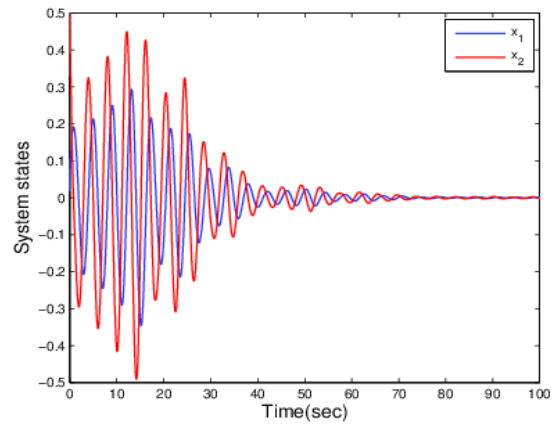


FIGURE 3. The time profile of x_1 and x_2 .

When $\rho = \frac{1}{4}$, $\bar{f} = 2x_1^2$, $\Delta f_1 = 2x_1^3 \sin(x_1 x_2)$, $\Delta f_2 = \frac{1}{2}x_2 \cos(x_1^2)$, $h = x_2^2 \cos(x_1^2)$, $g = \frac{1}{4}x_1 \sin(x_1)$, $\varphi_1 = \frac{1}{4} \sin(x_1)$, $\xi_1 = \xi_2 = 2$, $a_1 = a_2 = 1.6$, $\varrho_1 = \varrho_2 = 2$, $\varpi_{12} = \frac{\sqrt{2}}{2}$, and $\kappa_2 = 0.12$. The simulation is carried out under the initial conditions $[x_1(0), x_2(0), \hat{x}_1(0), \hat{x}_2(0), \hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0.02, 0.5, 0.1, 0.2, 0, 0]^T$, and a reasonable average dwell time τ_a showing in Fig. 2. Further, the simulation results are shown in Figs. 3-6.

Example 2: Consider the following stochastic nonlinear system:

$$\begin{aligned} dx_1 &= (x_2 + f_{k,1}(x_1) + g_{k,1}(x))dt + h_{k,1}(y(t))d\omega, \\ dx_2 &= (u_k + f_{k,2}(x) + g_{k,2}(x))dt + h_{k,2}(y(t))d\omega, \\ y(t) &= x_1, k = 1, 2, \end{aligned} \quad (68)$$

where

$$\begin{aligned} f_{1,1} &= \frac{4}{5}x_1, f_{2,1} = x_1, g_{1,1} = \frac{1}{10}x_1^3 \sin(x_1 x_2), \\ g_{2,1} &= 20 \sin(x_1 x_2), h_{1,1} = \frac{1}{20}x_1, h_{2,1} = \frac{1}{12}x_1, \\ f_{1,2} &= 4x_1^2, f_{2,2} = \frac{1}{2}x_1^2, g_{1,2} = 2 \sin(x_1 x_2), \varphi_1 = \frac{1}{12}, \\ g_{2,2} &= \frac{1}{100}x_1^3 \sin(x_2^2), h_{1,2} = \frac{1}{20}x_1, h_{2,2} = \frac{1}{12}x_1. \end{aligned}$$

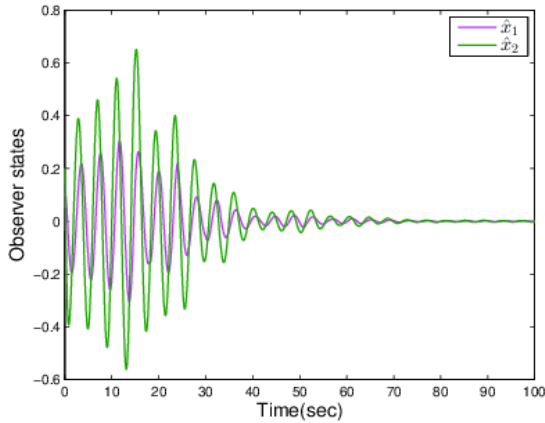


FIGURE 4. The time profile of \hat{x}_1 and \hat{x}_2 .

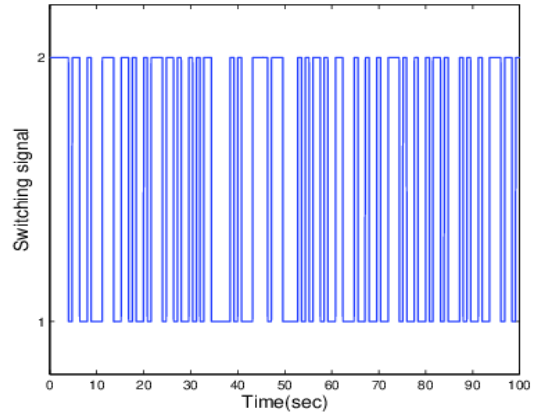


FIGURE 7. The time profile of the switching signal $\sigma(t)$.

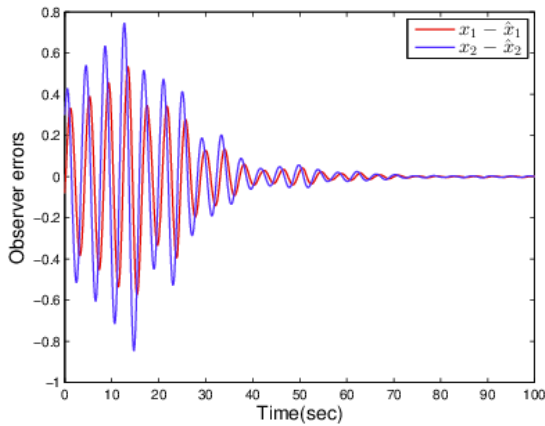


FIGURE 5. The time profile of $x_1 - \hat{x}_1$ and $x_2 - \hat{x}_2$.

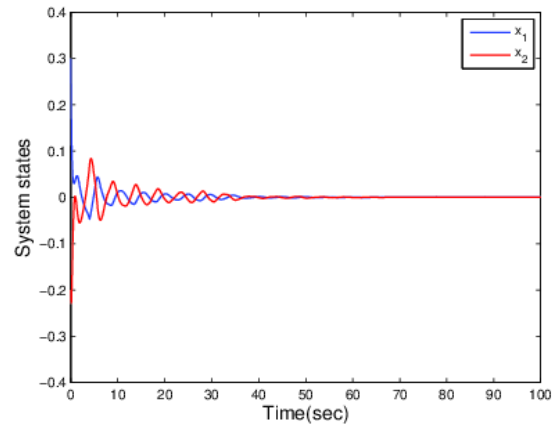


FIGURE 8. The time profile of x_1 and x_2 .

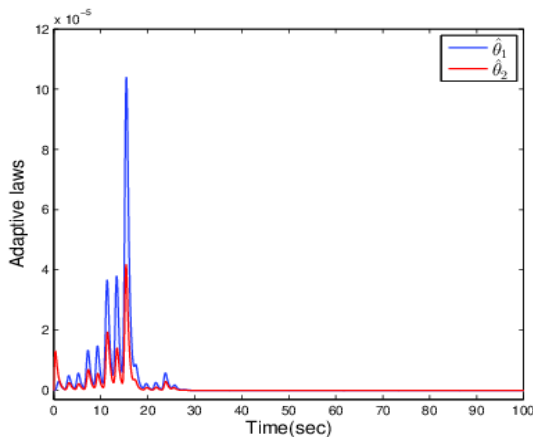


FIGURE 6. The time profile of $\hat{\theta}_1$ and $\hat{\theta}_2$.

For the system (68), the switched observer is given

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 - l_{k,1}\hat{x}_1, \\ \dot{\hat{x}}_2 &= u_k - l_{k,2}\hat{x}_1, \end{aligned} \quad (69)$$

and the design parameters are taken as $l_{1,1} = 2, l_{2,1} = 1, l_{1,2} = l_{2,2} = 4$, then the matrices A_1, A_2 are Hurwitz.

By choosing $Q_1 = I_2, Q_2 = 2I_2$, we can find the positive-definite symmetric matrices

$$P_1 = \begin{bmatrix} 250.2492 & -95.6755 \\ -95.6755 & 113.5835 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 366.7314 & -66.5925 \\ -66.5925 & 109.8747 \end{bmatrix}$$

such that the equalities $A_k^T P_k + P_k A_k = -Q_k (k \in \mathbb{M})$ are satisfied.

Based on Theorem 1, the intermediate virtual control α_1 , actual controller u and adaptive laws $\hat{\theta}_i (i = 1, 2)$ are defined in (23), (44) with $n = 2$, (26) and (47) with $n = 2$, respectively. The simulation is carried out when the initial conditions are chosen as $[x_1(0), x_2(0), \hat{x}_1(0), \hat{x}_2(0), \hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0.3, -0.2, 0.1, -0.4, 0, 0]^T$ and design parameters in adaptive fuzzy controller and in adaptive laws are chosen as $\xi_1 = \xi_2 = 2, a_1 = a_2 = 1.8, r = 1, \varrho_1 = \varrho_2 = 0.1$.

V. DISCUSSIONS

In the past few decades, the control of mechanical system in internet of things, chemical production, logistics and

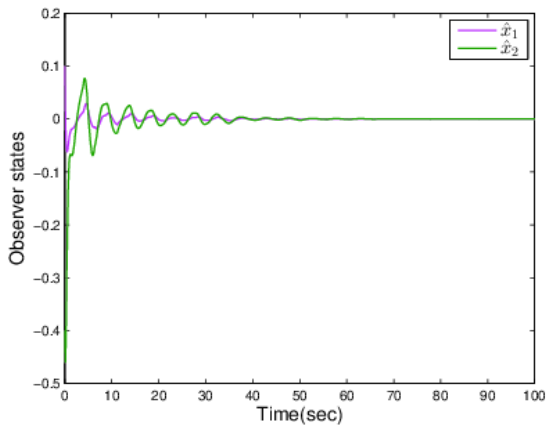


FIGURE 9. The time profile of \hat{x}_1 and \hat{x}_2 .

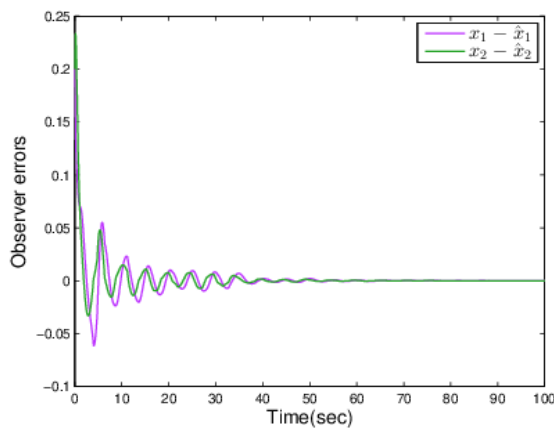


FIGURE 10. The time profile of $x_1 - \hat{x}_1$ and $x_2 - \hat{x}_2$.

transportation, automation control field has received great attention. One of the major control problems which has been well investigated in the internet of things is the velocity regulation of the motors in a mechanical vibration. Hence, we present a mass-spring-damper system with controller switching shown in Fig. 1. The obtained simulation results, which illustrated the achieved system performance of the mass-spring-damper system in the closed loop, are shown in Figs. 2-6. Finally, we can obtain the conclusion that all the charts clearly show a high quality, good control performance of the adaptive neural network control design, which makes the mechanical system better applied in the Internet of Things system. The simulation results are illustrated in Figure 7-11, respectively. Figure 7 demonstrates the time profile of the switching signal $\sigma(t)$. Figure 8 gives the time profile of x_1 and x_2 . Figure 9 shows the transmission signal time profile of \hat{x}_1 and \hat{x}_2 . The observer state is asymptotically stable with the change of transmission signal time. Figure 10 shows the transmission signal time profile of \hat{x}_1 and \hat{x}_2 . Figure 11 shows the transmission signal time profile of $\hat{\theta}_1$ and $\hat{\theta}_2$. From comparing the transmission signal time profile of Figure x_1 , \hat{x}_1 and the figure $x_1 - \hat{x}_1$, it can be seen

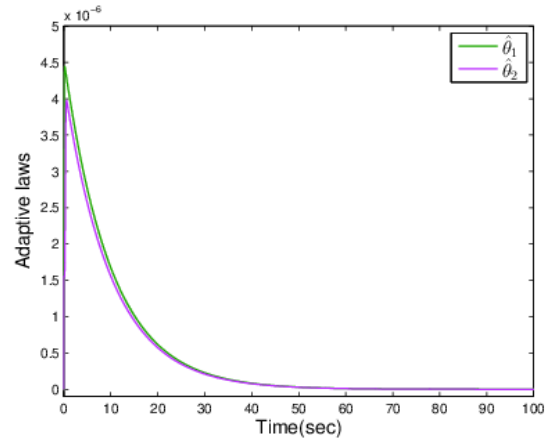


FIGURE 11. The time profile of $\hat{\theta}_1$ and $\hat{\theta}_2$.

that all transmission signals in the closed-loop structure are bounded and the states can track the given reference signals. We can conclude that the proposed transmission signal time control scheme is effective. Meanwhile, we demonstrated a remarkable closed-loop performance.

VI. CONCLUSION

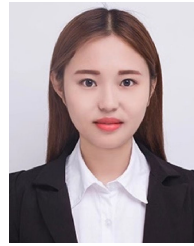
In this paper, we have presented a constructive method to handle the controller design problem by means of an adaptive NNs DSC design for a class of transport information switched nonlinear lower-triangular systems. In the design process, the Backstepping technique for nonlinear systems and DSC control are combined to give the design scheme of adaptive neural network control design. According to the ADT method to carry out the theoretical analysis, ensure that all the signals the resulting closed-loop system remain bounded in probability, and the output of the system converges to a small neighborhood of the origin with appropriate choice of design parameters. Then there are still some interesting control issues of switched systems remaining to be further considered. For examples, when the control direction is unknown, how to construct the controllers for switched systems? The question give us the inspirations and inspire us further to explore these problems in future research.

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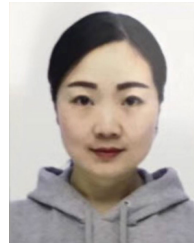
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MINGYUE LIU was born in Harbin, Heilongjiang, China, in 1996. She received the B.S. degree from the College of Information Engineering, Suihua University, in 2018. She is currently pursuing the master's degree in applied mathematics with the College of Mathematics and Physics, Bohai University, China, under the supervision Dr. X. Jiang. Her research interests include control decision and analysis of nonlinear systems.



XIAOLI JIANG was born in Suihua, Heilongjiang, China, in 1975. She received the M.S. degree from the College of Mathematics and Physics, Harbin Engineering University, in 2012, and the Ph.D. degree in system engineering from Harbin Engineering University in 2014. She is currently a Lecturer in Applied Mathematics with the College of Mathematics and Physics, Bohai University, China. Her research interests include system control decision and analysis, partial differential equations, nonlinear evolution equations, and dynamical systems.



LINA LIU was born in Mudanjiang, Heilongjiang, China, in 1985. She received the M.S. degree from the College of Mathematics and Physics, Harbin Engineering University, in 2011, and the Ph.D. degree in system engineering from Harbin Engineering University, in 2015. She is currently a Lecturer with the School of Electronic and information Engineering, Soochow University, China. Her research interests include artificial intelligence and data analyzing.

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